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ABSTRACT

The analysis of covariance as a procedure for statistical correction of the effects for an extraneous variable, called a "covariate," is presented. An heuristic data set is used to make the discussion of the calculation of ANCOVA partitions easier to follow. A discussion of homogeneity of regression as an essential condition to be met when conducting ANCOVA is discussed. Data reliability and the interpretation of the residualized dependent variable as major issues when applying ANCOVA are also discussed. It is suggested that caution must be exerted when applying ANCOVA to statistically correct for differences due to a covariate. Five tables and two appendixes. (Contains 24 references.) (Author)

The Use of Analysis of Covariance: User Beware

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Abstract

The analysis of covariance as a procedure for statistical correction of the effects for an extraneous variable, called a covariate, is presented. An heuristic data set is used to make the discussion of the calculation of ANCOVA partitions easier to follow. A discussion of homogeneity of regression as an essential condition to be met when conducting ANCOVA is discussed. Data reliability and the interpretation of the residualized dependent variable as major issues when applying ANCOVA are also discussed. It is suggested that caution must be exerted when applying ANCOVA to statistically correct for differences due to a covariate.

The Use of Analysis of Covariance: User Beware

Analysis of variance methods and their analog (ANCOVA, MANOVA, MANCOVA) remain tools of choice for many researchers (Thompson, 1988). Willson (1980) examined the articles published in AERJ from 1969 to 1978 and found that ANOVA and ANCOVA were used in 41% of the articles. Elmore and Woehlke (1988) studied several volumes of educational journals published from 1978 to 1987 and found that ANOVA/ANCOVA were the most frequently used research methods. Though ANOVA methods are frequently used, ANCOVA is used much less often and has not been widely used in the published behavioral science research (Loftin & Madison, 1991; Thompson, 1992).

Analysis of covariance (ANCOVA) is used primarily as a procedure for statistical control of the effects of an extraneous variable, called a covariate, on the dependent measure (Hinkle, Weisman, & Jurs 1988; Keppel & Zedeck, 1989). Cohen (1968) states that a covariate is

> after all, nothing but an independent variable, which because of the logic dictated by the substantive issues of the research, assumes priority among the set of independent variables as a basis for accounting for Y variance. (p. 439)

ANCOVA integrates a regression analysis of the dependent variable with the covariate and an ANOVA on the adjusted (residual or error) scores on the dependent variable (Huitema, 1980; Loftin & Madison, 1991; Wildt & Ahtola, 1978). ANCOVA purports to control for the effects of the covariate by partitioning out the variance attributed to it (Hinkle, Wiersma, & Jurs, 1988). By statistically controlling for the variance attributed to the covariate, the error variance is hopefully reduced. In addition, the treatment effects can be clarified and the probability of obtaining statistically significant results will be increased (Loftin & Madison, 1991), if the assumptions required by ANCOVA are met.

However, this is of limited importance in and of itself. Thompson (1988) indicates that statistical significance "is not the end-all and be-all of research" (p. 100). Statistical significance is largely an artifact of sample size. Since the null hypothesis of no difference is almost always false, with a large enough sample the null hypothesis will always be rejected, indicating statistical significant results. In addition, statistical significance does not provide information about result importance or generalizability (Carver, 1978).

Figure 1 illustrates the partitioning of variance in ANCOVA. The area inside the circle represents the total variance of the dependent variable. The proportion of variance attributed to the treatment effects along with the variance attributed to the covariate are shown.

INSERT FIGURE 1 ABOUT HERE

Huitema (1980) indicates that ANCOVA has two advantages, when it is correctly applied. ANCOVA provides greater power against Type II error and reduces the bias caused by differences that exist among groups before experimental treatments are considered. However, due to the erroneous belief that ANCOVA will always provide "control" and "power", the method is used by researchers even in cases in which ANCOVA is not appropriate (Thompson, 1988).

Various conditions should be met in order to perform ANCOVA correctly (Elashoff, 1969; Bump, 1991). ANCOVA assumes a high correlation between the covariate and the dependent variable. If this condition is not met, the covariate will do little to reduce the error sum of squares, which is the primary objective of ANCOVA (Loftin & Madison, 1991).

ANCOVA also requires that the covariate must be unaffected by the independent variable(s) and thus, explains different portions of the total variance of the dependent variable. Also in ANCOVA, the residualized dependent variable is assumed to be normally distributed for each level of the independent variable, and the variances of the residualized dependent variable for each level of the dependent variable are assumed to be equal. Next ANCOVA assumes a linear relationship between the covariate and the dependent variable. A linear relationship implies that a change in magnitude on the covariate is presumed to cause a proportional change on the dependent variable at each level of the covariate. Finally, ANCOVA requires that the regression slopes between the covariate and the dependent variable must be parallel for each independent variable group. This is known as homogeneity of regression.

The purpose of the present paper is to explain the computational processes of ANCOVA partitions. A discussion of homogeneity of regression as an essential condition to be met when conducting ANCOVA is presented. Data reliability and the interpretation of the residualized dependent variable as major issues when applying ANCOVA are also discussed.

Computing the adjusted sum of squares for ANCOVA

Like ANOVA, ANCOVA is often used to test whether group means differ. However, in the ANCOVA case, the means have been adjusted for differences between the groups on the covariate(s) (Huck, Cormier, & Bounds, 1974).

As previously discussed, ANCOVA combines regression analysis and analysis of variance (ANOVA) when adjusting sum of squares for the variance attributed to the covariate(s) (Wildt & Ahtola, 1978). Table 1 presents a general summary table for ANCOVA.

INSERT TABLE 1 ABOUT HERE

The calculation of ANCOVA sum of squares involves several steps (Hinkle, Wiersma, & Jurs, 1988). Table 2 presents an heuristic data set used to make the calculation of ANCOVA partitions easy to follow.

INSERT TABLE 2 ABOUT HERE

The adjusted total sum of squares ($SOS_{T'}$) is defined as the total sum of squares after removing the variance attributed to the covariate(s). Thus,

$$SOS_{T'} = SOS_T(1 - r^2_T)$$

where SOS_T is the total sum of squares from the ANOVA on the dependent variable and r_T is the correlation between all scores on the dependent variable and the covariate. These calculations are presented in Table 3.

INSERT TABLE 3 ABOUT HERE

The SAS commands to perform ANOVA/ANCOVA are presented in Appendix A. Table 4 presents an ANOVA summary table for the data set, as a basis for comparison with the results for the same data once a covariance correction is involved.

INSERT TABLE 4 ABOUT HERE

The adjusted within group sum of squares is defined as,

$$SOS_{W'} = SOS_W(1 - r^2_W)$$

where SOS_W is the within group sum of squares from the ANOVA on the dependent variable and r_W is the pooled correlation coefficient between the scores on the dependent variable(s) and the covariates(s).

The calculation of the adjusted between group sum of squares involves several somewhat complicated steps. For the purpose of this paper, the $SOS_{B'}$ will be calculated as,

$$SOS_B' = SOS_{T'} - SOS_{W'}$$

where $SOS_{T'}$ is the adjusted total sum of squares and $SOS_{W'}$ is the adjusted within sum of squares.

The sum of squares for the covariate is computed by subtracting the adjusted sum of squares total (SOS_T') from the total sum of squares (SOS_T),

$$SOScov = SOS_T - SOS_T'$$

Table 5 presents a summary table for the ANCOVA partitions for the data set.

INSERT TABLE 5 ABOUT HERE

Computing the ANCOVA test statistic and testing significance

The adjusted sum of squares partitions can be used to test the statistical significance of difference in group means after adjustment for the covariate(s) is made (Hinkle, Wiersma, & Jurs, 1988). The test statistic for ANCOVA (F) is the ratio of the adjusted between group mean square (MS_B') and the adjusted within group mean square (MS_W'),

$$F = MS_B'/MS_W'$$

To test significance, the obtained value of F (i.e., "observed F", "F calculated", or "F ratio") is compared to the critical value of F, obtained form a table of critical values found in most statistics books. The critical value of F is obtained using the adjusted between group sum of squares (SOS_B') degrees of freedom for the numerator and the adjusted within group sum of squares (SOS_W') degrees of freedom for the denominator at an alpha level predetermined by the researcher.

When the observed value of F exceeds the critical value of F, the null hypothesis of no difference among the adjusted means can be rejected, implying that the result is statistically significant. If the observed value of F is less that the critical value of F, then the null hypothesis is not rejected.

Obtaining statistically significant results is not indicative of results importance, replicability, or generalizability (Carver, 1978). It only indicates that a decision to reject or not reject the null hypothesis has been taken (Haase & Thompson, 1992; Thompson, 1993). The only conclusion that can be drawn at this point is that at least one pair or combination of adjusted means differs. To determine which pair or combination of pairs differ, a post hoc analysis must be conducted (Hinkle, Wiersma, & Jurs, 1988). However, Thompson (1988) discusses the limitations of post hoc comparisons and the advantages of a priori or planned comparisons, and can also be used in ANCOVA. A priori or planned comparisons provide more power against Type II error and force the researcher to be more thoughtful in conducting research.

Keppel and Zedeck (1989) discuss a multiple regression approach to ANCOVA involving a hierarchical strategy "in which the covariate is the first variable (or set of variables) to be entered into the analysis and the vectors representing the independent variable are entered next" (p. 457). For a thorough discussion of this approach see Keppel and Zedeck (1989).

<u>The homogeneity of regression assumption</u>

As previously discussed, various conditions should be met to perform ANCOVA correctly. ANCOVA requires that the slope of the regression line is the same for all treatment groups. This is what is so called <u>homogeneity of regression assumption</u> (Wildt & Ahtola, 1978). Loftin and Madison (1991) argue that "this is exactly where most applications of ANCOVA fail, since researchers quite often have truly non equivalent <u>K</u> groups for which the regression slopes indeed are different".

What ANCOVA does, according to Thompson (1992), is to create a single pooled regression equation ignoring group assignment, to calculate the adjustment in the dependent variable using the covariate(s). This pooled equation is created by assuming that the equations are the same across groups and that an "average" equation can be used for all subjects ignoring group membership (Bump, 1992). Then an ANOVA, not ignoring groups, of the deviation of the residualized scores from the regression line is performed (Cliff, 1987).

The homogeneity of regression is legitimate if and only if the regression equation of the groups have parallel slopes (Huck, Cormier, & Bounds, 1974; Thompson, 1992). This assumption requires that the "b" weights applied to the covariate(s) be reasonably equal across each groups. That is, any adjustment in the covariate(s) will result in the same proportionate adjustment in the dependent variable for each K level of the independent variable.

Two other conditions ought to be met in order to perform ANCOVA correctly. One deals with the reliability of the covariate(s). As Thompson (1992) points out

> researchers often incorrectly presume that the characteristic of reliability inures to tests, when in fact reliability is a characteristic of a given set of

> data collected using a given time from a given set of subjects using a given protocol (p.xii).

Due to this erroneous belief, many researchers do not check and do not report the reliability of their data. Loftin and Madison (1991) argue that the covariate(s) used must be especially reliable, "or one will end up potentially adjusting sampling error with measurement error, and creating a mess" (p. 145).

The other condition involves the interpretation of the residualized dependent variable. As discussed, ANCOVA is used to correct for the effects of a covariate(s) on the dependent variable (Huck, Cormier, & Bounds, 1974). Then the residual is analyzed, thus partitioning the effects of the treatment. However, as Thompson (1992) states, the use of statistical correction may be dangerous especially when using multiple covariates. It may result in the analysis of a dependent variable that no longer makes sense.

<u>Summary</u>

The analysis of covariance is used to statistically correct for the effect of an extraneous variable. The purpose is to adjust for initial group differences before the treatment is applied. However, several conditions should be met when applying ANCOVA. Due to the erroneous belief that ANCOVA will always provide "control" and "power", the method is applied even when is not appropriate. Caution should be exerted when applying ANCOVA as a method for statistical correction, especially when using multiple covariates; the researcher may end with a dependent variable that does not make any sense. The reliability of the data and the interpretation of the residual are issues of concern when applying statistical correction methods.

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Table 1.

General summary table for ANCOVA

Source	*SOS*	*df*	*MS*	*F*	*Fcv*
Covariate		1			
Between		k-1			
Within		n-k-1			
Total		n-1			

$SOS_{COVARIATE}$ = SOS_{TOTAL} times r2 between the covariate and the dependent variable

k= number of groups

n = sample size

$MS_{cov} = SOS_{cov}/df_{cov}$

$MS_{B'} = SOS_{B'}/df_{B'}$

$MS_{B'} = SOS_{W'}/df_{W'}$

$F = MS_{B'}/MS_{W'}$

Table 2.

Data set and analysis for ANCOVA example

Group	Acievement Score (Y)	IQ Covariate
1	60	100
1	63	102
1	66	104
1	69	103
2	62	104
2	63	109
2	67	104
2	71	117
3	65	102
3	68	117
3	72	108
3	76	105

Group	n	Mean (Y)	Std Dev (Y)	Mean (Cov)	Std Dev (Cov)
1	4	64.50	3.87	102.25	1.71
2	4	65.75	4.11	108.50	6.14
3	4	70.25	4.79	108.00	6.48

Table 3.

<u>Calculation of ANCOVA partitions</u>

$$
\begin{aligned}
SOS_T' &= SOST\ (1 - r^2_T) \\
&= 237.67\ (1-.41(.41)) \\
&= 237.67\ (.8341) \\
&= 198.24
\end{aligned}
$$

$$
\begin{aligned}
SOS_W' &= SOS_W\ (1 - r^2_W\ {}^*) \\
&= 164.50\ (1 -.32(.32)) \\
&= 164.50\ (1- .1037) \\
&= 164.50\ (.8963) \\
&= 147.45
\end{aligned}
$$

*see Appendix B for r_W calculation

$$
\begin{aligned}
SOS_B' &= SOS_T' - SOS_W' \\
&= 198.24 - 147.5 \\
&= 50.79
\end{aligned}
$$

$$
\begin{aligned}
SOScov &= SOST - SOS_T' \\
&= 237.67 - 198.24 \\
&= 39.42
\end{aligned}
$$

Table 4.

ANOVA summary for data set

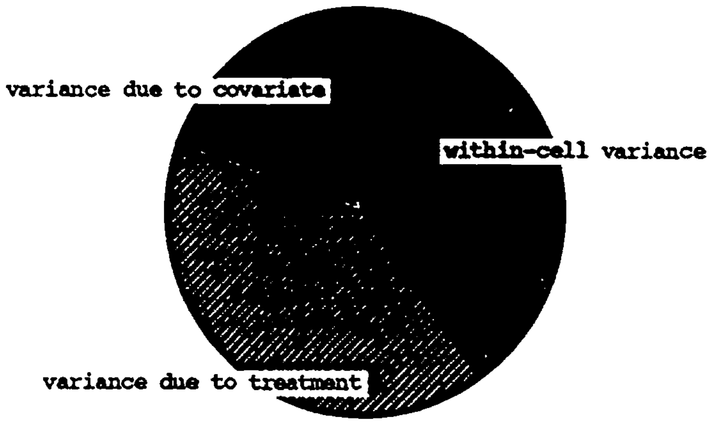
Source	*SOS*	*df*	*MS*	*F*	*Fcv*
Between	73.17	2	36.58	2.00	4.26
Within	164.50	9	18.27		
Total	237.67	11			

Table 5.

ANCOVA summary table for data set

Source	*SOS*	*df*	*MS*	*F*	*Fcv*
Covariate	39.42	1	39.42		
Between	*50.79*	*2*	*25.40*	*1.38*	*4.46*
Within	*147.45*	*8*	*18.47*		
Total	237.67	11			

Figure 1
Partitioning of variance in ANCOVA



Note: This diagram presumes that the covariate does not overlap at all the variance due to treatment

Appendix A

SAS program listings for ANCOVA job

```
TITLE 'ANCOVA';
DATA D1; INFILE ABC;
INPUT GROUP 1 DEPVAR 3-4 COV 6-8;
PROC PRINT;
     TITLE 'DATA PRINT OUT';
PROC CORR;
PROC SORT;BY GROUP;
PROC MEANS;BY GROUP;
PROC GLM;
CLASSES GROUP;
     MODEL DEPVAR COV = GROUP;
MEANS GROUP/TUKEY;
PROC GLM;
CLASSES GROUP;
     MODEL DEPVAR = COV GROUP GROUP*COV;
PROC GLM;
CLASSES GROUP;
     MODEL DEPVAR = COV GROUP;
LSMEANS GROUP/PDIFF;
MEANS GROUP;
```

Source: Hinkle, Wiersma, & Jurs (1988)

APPENDIX B
Calculation of r_W

Group	X	Y	XY	X^2	Y^2
1					
	100	60	6000	10000	3600
	102	63	6426	10404	3969
	104	66	6864	10816	4356
	103	69	7107	10609	4761
Sum	409	258	26397	41829	16686
2					
	104	62	6448	10816	3844
	109	63	6867	11881	3969
	104	67	6968	10816	4489
	117	71	8307	13689	5041
Sum	434	263	28590	47202	17343
3					
	102	65	6630	10404	4225
	117	68	7956	13689	4624
	108	72	7776	11664	5184
	105	76	7980	11025	5776
Sum	432	281	30342	46782	19809

$$r_W = \frac{\Sigma\ (\ n_K\ \Sigma XY_K - \Sigma X_K\ \Sigma Y_K\ )}{\sqrt{\{\Sigma\ [\ n_K\ \Sigma X_K{}^2 - (\Sigma X_K)^2\ ]\}\ \{\Sigma\ [\ n_K\ \Sigma Y_K{}^2 - (\Sigma Y_K)^2\ ]\}}}$$

$\Sigma\ (\ n_K\ \Sigma XY_K - \Sigma X_K\ \Sigma Y_K\ )$

n_1	ΣXY_1	-	ΣX_1	ΣY_1
4	26397		409	258
4	26397			105522
	105588			105522
				66

n_2	ΣXY_2	-	ΣX_2	ΣY_2
4	28590		434	263
4	28590			114142
	114360			114142
				218

n_3	ΣXY_3	-	ΣX_3	ΣY_3
4	30342		432	281
4	30342			121392
	121368			121392
				-24

$\Sigma = 66 + 218 + -24 = \mathbf{260}$

$$\sqrt{\{\Sigma\ [\ n_K\ \Sigma X_K^2 - (\Sigma X_K)^2\ ]\}\ \{\Sigma\ [\ n_K\ \Sigma Y_K^2 - (\Sigma Y_K)^2\ ]\}}$$

n_1	ΣX_1^2	$-$ $(\Sigma X_1)^2$	n_1	ΣY_1^2 $-$	$(\Sigma Y_1)^2$
4	41829	409	4	16686	258
4	41829	167281	4	16686	66564
	167316	167281		66744	66564
		35			180

n_2	ΣX_2^2	$-$ $(\Sigma X_2)^2$	n_2	ΣY_2^2 $-$	$(\Sigma Y_2)^2$
4	47202	434	4	17343	263
4	47202	188356	4	17343	69169
	188808	188356		69372	69169
		452			203

n_3	ΣX_3^2	$-$ $(\Sigma X_3)^2$	n_3	ΣY_3^2 $-$	$(\Sigma Y_3)^2$
4	46782	432	4	19809	281
4	46782	186624	4	19809	78961
	187128	186624		79236	78961
		504			275

$\Sigma = 35 + 452 + 504 = 991$ $\quad$ $\Sigma = 180 + 203 + 275 = 658$

$991 \times 658 = 652078$

$652078^{.5} =$ **807.5134**

$$r_W = \frac{\Sigma\ (\ n_K\ \Sigma XY_K - \Sigma X_K\ \Sigma Y_K\ )}{\sqrt{\{\Sigma\ [\ n_K\ \Sigma X_K^2 - (\Sigma X_K)^2\ ]\}\ \{\Sigma\ [\ n_K\ \Sigma Y_K^2 - (\Sigma Y_K)^2\ ]\}}}$$

$= 260 \ / \ 807.5134 = 0.321976$