DOCUMENT RESUME

ED 364 556 TM 020 703

AUTHOR Fleener, M. Jayne
TITLE Proportional Reasoning of Preservice Elementary Education Majors: An Epistemic Model of the Proportional Reasoning Construct.
PUB DATE Apr 93
NOTE 20p.; Paper presented at the Annual Meeting of the American Educational Research Association (Atlanta, GA, April 12-16, 1993).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *College Students; Difficulty Level; *Education Majors; Elementary Education; *Epistemology; Higher Education; Interviews; Knowledge Level; Learning; Models; Preservice Teacher Education; Problem Solving; Regression (Statistics); Scoring; Teaching Methods; Test Items; *Thinking Skills
IDENTIFIERS *Proportional Reasoning

ABSTRACT

Current research and learning theory suggest that a hierarchy of proportional reasoning exists that can be tested. Using G. Vergnaud's four complexity variables (structure, content, numerical characteristics, and presentation) and T. E. Kieren's model of rational number knowledge building, an epistemic model of proportional reasoning was constructed and tested with 1 male and 17 female preservice elementary education majors. A 12-item test of proportional reasoning was administered, followed by individual 30-minute interviews to probe general and specific reasoning and problem-solving approaches of students on a variety of proportional reasoning tasks. Data were collected in the fall of 1992. Item difficulty levels were calculated using a scoring rubric developed to take into account a variety of complexity and structural characteristics of the problems. Simple linear regression analysis revealed that the calculated scores did predict student performance with item difficulty accounting for approximately 30 percent of the variance of student performance on test items. Results support the model of a proportional reasoning construct consisting of levels of proportional reasoning and including contextual and problem features as relevant to success in performing proportional reasoning tasks. Understanding the levels of proportional reasoning may guide decisions concerning instructional sequencing and content. Seven figures are provided. (Contains 20 references.) (Author/SLD)

Proportional Reasoning of

Preservice Elementary Education Majors:

An Epistemic Model of the Proportional Reasoning Construct

M. Jayne Fleener
University of Oklahoma
College of Education
820 Van Vleet Oval
Norman, OK 73019
(405) 325-3604
FAX (405) 325-3242
DA5234@UOKMVSA.BACKBONE.UOKNOR.EDU

Running Head: Proportional Reasoning

Paper presented at the annual meeting of the American Educational Research Association (AERA), Atlanta, Georgia, April, 1993.

Abstract

Higher order reasoning and problem solving require the development of proportional reasoning. Current research and learning theory suggest an hierarchy of proportional reasoning that can be tested. Previous attempts to create an hierarchy of levels of proportional understanding have focused on a particular type of proportion task or a particular type of understanding.

Based upon Vergnaud's (1988) four complexity variables (structure, context, numerical characteristics, and presentation) and Kieren's (1988) model of rational number knowledge building, an epistemic model of the proportional reasoning construct was developed and tested on a group of eighteen preservice elementary education majors. A twelve item test of proportional reasoning was administered, followed by individual thirty minute interviews to probe general and specific reasoning and problem solving approaches of students on a variety of proportional reasoning tasks. Item difficulty levels were calculated using a scoring rubric developed to take into account a variety of complexity and structural characteristics of the problems. Simple linear regression analysis revealed the calculated scores did predict student performance ($p=.0014$) with item difficulty accounting for approximately 30% of the variance of student performance on test items. The results support the model of a proportional reasoning construct that consists of levels of proportional reasoning and includes contextual and problem features as relevant to success in performing proportional reasoning tasks.

Proportional knowledge building must be studied as a dynamic system of complexity relationships. Ultimately, understanding of the levels of proportional reasoning may guide decisions concerning instructional sequencing and content.

Background

Understanding of rational number concepts and facility with proportional reasoning and computations are essential for continued mathematical development and success in the study of higher mathematics and science (Lawson & Bealer, 1984; Wollman & Lawson, 1978; Heller, Ahlgren & Post, 1989; Lesh, R., Post, T. & Behr, M., 1988). Proportional reasoning is a pivotal scheme in the acquisition of formal reasoning ability (Inhelder and Piaget, 1958), a significant predictor of science achievement (Mitchell and Lawson, 1988), and may be dependent on the internalization of the linguistic elements of argumentation (Lawson, Lawson, and Lawson, 1984) and therefore may be related to the ability to communicate mathematically (NCTM, 1989). Upper elementary and secondary students are not, however, proficient with nor accustomed to the reasoning required to develop proportional thinking (Lindquist, 1989; NCTM, 1989).

More needs to be known about proportional reasoning structures so effective teaching/learning environments can be provided. Although research evidence supports the idea that proportional reasoning is not the same across tasks and situational characteristics (see for example, Heller, et al., 1989), much of the current proportional reasoning literature attempts to isolate a variety of complexity variables associated with proportion problem features rather than considering proportional reasoning to be an hierarchically complex construct.

This paper presents the theoretical underpinnings for an epistemic model of the general proportional reasoning construct. Based upon review of the research literature on proportional reasoning, construct theory, and Piagetian genetic epistemology, an hierarchy of levels of proportional reasoning within a general proportional construct is proposed. Context variables such as familiarity with the subjects/objects in the problem and structural features of proportion tasks such as length, consumption and intensivity used in the equivalence relation are included in developing the hypothesized hierarchy. In addition to context variables and structural features, presentation type and numerical characteristics will be factored in to the predictive model. The model was tested with 18 preservice elementary education majors. A comparison of actual verses predicted performance on 12 proportional reasoning tasks will lend support to an hierarchical view of a general proportional reasoning construct.

Proportionality: Construct or Skill?

In school mathematics, the foundations for developing rational number concepts and proportional reasoning traditionally are laid during the intermediate grades, namely in grades 5-8. Opportunities to reason and problem solve using proportional and probabilistic reasoning are encouraged and of central focus starting in the middle grades (NCTM, 1989). Proportion problems involve an equivalence relation between two ratios. Frequently, however, the development of proportional reasoning strategies is subverted as algorithmic solution strategies are taught; namely the cross-multiply-and-divide algorithm (Lesh, Post, Behr, 1988). Textbooks group (ratio) proportion problems by structural type and treat the progression of proportional problem solving proficiency as concept acquisition rather than general reasoning attainment. Most instruction, therefore, treats proportional problem solving as a skill to be mastered rather than a general reasoning construct.

The model purported in this paper assumes proportional reasoning is a general reasoning construct and not a skill to be mastered. Kieren's (1988, p. 165) model of rational number knowledge building offers insight into how general reasoning constructs can be construed and used for knowledge construction. Foundational to any mathematical knowledge, according to Kieren, is a "constructive mechanism", similar to Vergnaud's idea of a conceptual field (1988), and Piaget's general reasoning constructs (Piaget, 1969). The cognitive structure is the lens through which reality is viewed by the student. Students can learn to perform proportional problem solving acts, but in the absence of an underlying cognitive structure, future success and understanding are limited.

Proportional Reasoning Construct

Research findings suggest proportional reasoning may evolve gradually (Lesh, Post, and Behr, 1987; Karplus et al., 1983). The model of proportional reasoning tested in this study utilizes Vergnaud's four complexity variables: structure, context, numerical characteristics, and presentation (Vergnaud, 1988), and Kieren's model of rational number knowledge building (Kieren, 1988). The ability to perform a particular proportional reasoning task is dependent upon structural and developmental characteristics of the proportional reasoning construct as well as situational features of the problem itself. Context and problem features, however, are aspects of the mathematical knowledge building framework specifically related to the proportional reasoning construct. (See Figure 1).

Insert Figure 1 here

The proportional reasoning knowledge component, according to this model, has six levels (see Figure 1) with inverse/compensatory proportional reasoning at the highest level and number magnitude at the lowest level. Each of these levels and sample problems are discussed below. (See Figure 2 for sample problems.)

Insert Figure 2 here

Magnitude proportional reasoning is also known as relative magnitude number sense, rational number meaning and relations (NCTM, 1989), or linear coordinate subconstruct of fractions (Behr et al., 1983). The presence of magnitude proportional reasoning is often determined by one's ability to place rational numbers on a number line, ordering rational numbers, or performing the basic operations on rational numbers. As part of the rational number construct, this component of the proportional reasoning construct serves as a link with prior mathematical knowledge.

Discrete quantity proportional reasoning is the next level of proportional thought. Just as early number concepts build on discrete quantity measures including counting, so the proportional reasoning construct builds upon discrete quantity proportions.

Continuous quantity proportional reasoning builds upon the length measurement construct of continuous quantity measures and comes after discrete quantity proportional understanding. Consumption proportional reasoning requires intensive quantity relationships (Swartz, 1988) where at least one element of the ratio relationship is a discrete quantity measure, while the other is a non-length continuous quantity measure. Because conservation of length occurs early in the stages of concrete operational thought, continuous quantity proportional reasoning, including ratio measures involving lengths, occurs before consumption proportional reasoning.

Ratio measures occur in the next highest level of proportional reasoning and involve intensive quantity relationships where each element of the ratio is a continuous quantity measure. Speed, velocity, and density are all examples of intensive quantities which would comprise a ratio proportion problem.

Compensatory or inverse proportional relationships occur at the highest level of proportional reasoning. These structures are sometimes called multiplicative and the balance scale is the prototype of problem types requiring compensatory proportional reasoning. The reasoning required for compensatory proportional reasoning is particularly difficult due to the inverse relationship among the variables.

The proportional reasoning construct occurs within a mathematical knowledge-building framework (see Figure 1) and cannot be separated from that framework. In order to test general levels of proportional reasoning, context and problem feature variables must be considered in addition to proportional reasoning requirements. A general scoring rubric was developed that would take into account structural, contextual, numerical, and presentation characteristics of problem situations and is discussed below.

Procedures

Subjects for this study were enrolled in an intermediate/middle school mathematics methods course for elementary education majors the final semester before student teaching. Eighteen students participated in this study with 17 female participants and one male participant. Data for this study were collected in the fall of 1992.

Participants took the researcher designed Qualitative and Quantitative Test of Proportional Reasoning (QQTPR), the Test of Logical Thinking (TOLT) (Tobin & Capie, 1981), and participated in individual student interviews. The TOLT was used to determine developmental levels of students. Individual student scores of proportional reasoning according to the TOLT were not used in analysis of the model since aggregate tendencies, rather than individual differences, were desired for determining the fit of the proposed model. Test scores on the TOLT ranged from 0 to 10 indicating a wide range of concrete to formal operational thought, which is necessary for assessing the epistemic proportional reasoning construct (Niaz, 1991).

Individual interviews lasted approximately thirty minutes and began by having students rank test items on the QQTPR according to perceived difficulty level. Students took the QQTPR approximately three to four weeks before the individual interviews took place and did not have their original written performance before them as they did their rankings. The general problem features, including presentation and numerical characteristics, context variables, and structural attributes, were discussed as students were asked to explain their choice of problem rankings from easiest to most difficult. Students were then asked to solve or discuss solution strategies for some of the problems. Different problems were worked by various participants according to their individual rankings and prior performance on the written QQTPR. The researcher-interviewer pre-selected particular problems for individuals to work based on ambiguities or solution strategies used on their written work as well as in response to their rankings.

The QQTPR included qualitative and quantitative questions of six types of proportion problems. Four types of ratio proportions (length, quantity/mixture, speed, and consumption/production), relative rational number magnitudes, and product proportion (compensatory/balance) problems were included in the test for a total of six structurally different types of proportion questions. (See Heller et. al., 1989 for definitions of ratio proportions and Harel, et. al., 1992 for a discussion of ratio verses product proportional reasoning tasks.) (See Figure 3 for a content analysis of the QQTPR.)

Questions on the QQTPR were scored according to the four complexity variables: structure, context, numerical characteristics, and presentation mode. (See Figure 4 for scoring rubric and sample items scored.) Composite item scores ranged from 4 to 15.

Insert Figures 3 and 4 about here

Simple linear regression analyzed best fit between the model's predicted difficulty level and student composite scores on individual items. Student responses to each of the 29 individual test items (12 questions and their subparts) were scored as either right (1) or wrong (0) and average scores for each item were calculated and used to generate the regression formula (see Figure 5). A scatter plot was then used to identify test items on which student performance either fell way below or way above the predicted level.

Results

Regression analysis indicates the predicted item difficulty level matched the actual student performance on the QQTPR ($p=.0014$) (see Figure 7 for summary table), with item difficulty accounting for about 30% of the variance of student performance on items. Negative beta weights support the contention that as problem difficulty level increases according to the four complexity variables, success levels will go down.

An analysis of selected QQTPR items will offer (a) possible explanations for why student performance fell below or above predicted student performance on certain items and (b) suggestions for necessary modifications to the scoring rubric used to predict difficulty level of test questions. Items 6a, 7a, 2b, 3a, 12a, and 9b, will be examined since the response patterns to these questions fell the furthest from the regression curve. Figure

5 shows the scatter plot of item responses related to the regression curve. Items discussed are represented on the plot as closed circles. The regression equation calculated to compare group correct response rates with predicted rates was:

$$y = -.035x + 1.045, \text{R-squared: } .321$$

(See Figure 6 for the predicted and actual scores analyzed.)

Student performance on item 9b fell below the predicted level of performance with a correct response rate of only 4 out of 18 or 22%. The question asked was:

9b Ben runs an 8 minute mile. Beth runs a 6 minute mile. Suppose Beth and Ben are running on a 1 mile track. If they both start at the same time, when will Beth lap Ben? Explain.

Complexity analysis of this item gave a difficulty score of 13 (structure=5, presentation=2, context=1, and numerical complexity=5) which would suggest a 59% correct response rate (y=-.035(13) + 1.045). Written student performance revealed that only four students were able to do this problem with nine students not even attempting an answer. This problem had the highest non-response rate of any of the other items on the QQTPR. One of the students who got the correct answer to this problem used a build-up strategy where completed laps were charted. It is debatable whether this constitutes a proportional approach to this problem (Lesh, Post, & Behr, 1988). Another student drew two circles and apparently kept track of laps mentally as minutes were enumerated. The other two who got the right answer to this question showed no work to indicate strategy used.

Of the students who attempted an answer to this problem but came up with an incorrect solution, all took numbers found in the problem and performed an arbitrary calculation. For example, one student divided 6 by 8 to get 75%, then answered 3/4 of a mile. Another used an additive approach by suggesting that since there was a two minute difference in running rate, Beth would lap Ben in the 3rd lap.

Analysis of individual item rankings by participants indicated that students felt this question was the second hardest problem on the test, just behind a chemical mixture problem. (In actuality, students performed well on the chemical mixture problem with a 2/3 correct response rate.) During the interviews, many of the subjects indicated that they had no idea how to approach the Beth and Ben race problem.

"I'm not even sure how to get started on that one!"

Other students indicated their difficulty came with trying to represent the problem.

"I just remember I kept on erasing it. I kept on drawing the diagram over and over, trying to sketch it out where they were and how fast they were moving."

"This is the part where I was totally lost. This is the one I kept on trying to draw something."

Finally, some students expressed confusion over the underlying mathematics involved.

> "I don't know how to set it up. I mean ...I understand it, I just don't know how to do the right thing to figure it out."

Although the model predicted this would be one of the harder problems, the lack of general format or familiar solution strategy was underestimated. The context complexity was ranked at one but it seems questionable that running a race around a track was familiar to many of these students.

Problem 12a was another question that student performance was well below the predicted level with only 8 out of 18 students responding correctly. Problem 12a was:

> 12a Ralph and Mary were making lemonade for the science club picnic. The recipe called for 4 cups of sugar for 6 quarts of lemonade. If they need to make 16 quarts of lemonade, how many cups of sugar will they need? Show your work.

Complexity analysis of this item gave a difficulty score of 8 (structure=2, presentation=2, context=1, and numerical complexity=3) which would have a predicted success rate of 77% ($y=-.035(8)+1.045$). Scored written student responses on this item may have underestimated general proportional reasoning ability. Of the 10 students whose responses were not considered correct, 6 had solutions between 10 and 11 cups. Some of the students made rounding errors, and a few made computational errors with fractions. One had the problem set up as a ratio then answered "About 11" which for practical purposes is sufficiently close.

During interviews, students did not find this problem problematic and ranked it as the fifth easiest problem. Transcripts confirmed the role of experience in solving this problem as students indicated familiarity with this sort of problem solving.

> "I think ...I just made up my own little steps to it. Step-by-step, I used my own formulas. I have always done things different than anyone else. I will take the long way.
>
> I've done this sort of thing ...I mean, I make lemonade all the time.

This problem may have allowed for more varied solution attempts as students had more confidence and familiarity with the contextual parameters of this problem. Likewise, many individuals who use similar reasoning for performing cooking tasks may not find the need for arriving at the exact solution.

If the six students whose answers were close to the correct answer (between 10 and 11 cups) had their responses counted correct, the model's calculated value (.77) would have exactly matched the performance level of the students. So, although this problem appeared to fall outside of the acceptable error range, the error occurred due to the item scoring rubric and not the complexity schema for predicting problem difficulty.

Problems 6a and 7a fell in exactly the same place above the prediction line indicating performance on these items was better than expected. Student performance on these items was better than on any other item with 17 out of 18 students answering these questions correctly. These items were:

6a Suppose in a large 16 ounce bag of M&M's there are the following number of each color of M&M's: 12 red, 24 light brown, 24 dark brown, 16 yellow, and 18 green. If you purchased an 8 ounce bag, how many M&M's would you expect to have?

7a If you double a 2X3 rectangle along each dimension, what would the dimensions be of the new rectangle?

The complexity ratings for each of these problems was 9; (2,2,1,4) and (3,2,2,2) respectively, for a predictability correct response ratio of 73% (y=-.035(9)+1.045). Closer analysis of student responses to question 6a makes it unclear whether solution strategies involved proportional problem solving strategies. Several of the students whose answers were scored correctly simply halved each color and added the numbers together. It has been argued that this approach to the problem would not indicate proportional thought (Lesh, Post & Behr, 1988). Because these items can be solved without proportional reasoning, little can be determined about a general proportional reasoning construct. These items neither detract from nor lend support to the hypothesized hierarchy and complexity rating for mathematical knowledge building.

Student performance on items 2b and 3a likewise fell above predicted levels. These items both referred to compensatory relationships:

2b A teeter totter is 10 feet long with the fulcrum exactly in the middle of the teeter totter. If George weighs twice as much as his brother Gerald, how could the boys sit to make the teeter totter balance? (Include a diagram and explain your answer.)

3a A meter stick is balanced at its natural balance point on a fulcrum.
100 gram weight is placed 20 cm to the left of the fulcrum. Where would a 200 gram weight be placed if the stick is to be balanced again? Explain your reasoning.

These items received complexity ratings of 11 (6,2,1,2) and 12 (6,2,2,2) respectively which would have predicted correct response rates of 66% (y=-.035(11)+1.045) and 63% (y=-.035(12)+1.045). Response analysis of item 3a revealed 5 of the 17 subjects who responded correctly supplied a correct diagram but could not describe their thinking. Probing during interviews revealed the same difficulty. Although they were scored as correct, their responses indicate that this type of proportional reasoning is still evolving for them and suggest that the complexity rating for this item may be correct.

Responses to item 2b indicate the complexity rating scoring rubric may have underestimated the role experience plays in developing compensatory proportional understanding. As several of the students revealed during the interview:

On the teeter totter, I can imagine, well, you know, I could picture that!

Although the compensatory relationship has been shown to be difficult for students, the physical familiarity of students with balancing on teeter totters may suggest the need for levels or dimensions of familiarity or experience to be factored in to the complexity equation.

Conclusions

Higher order reasoning and problem solving require the development of proportional reasoning. Seldom are proportion tasks understood beyond the instrumental application of the "cross-multiply and divide" algorithm. Past attempts to "teach" proportional reasoning have included grouping (ratio) problem types and teaching rules and methods for problem solutions rather than focusing on developing proportional reasoning abilities. Before a general approach to developing proportional reasoning is possible, however, the relationships among the levels of proportional reasoning and the other complexity variables (context, presentation, and numerical) must be understood. Current research and learning theory suggest an hierarchy of proportional reasoning that can be tested. Previous attempts to create an hierarchy of levels of proportional understanding have focused on a particular type of proportion task or a particular type of understanding. This proposed hierarchy considers qualitative as well as quantitative proportional understanding. Proportional knowledge building must be studied as a dynamic system with all four structures considered in combination, including intensivity and compensatory relationships. Ultimately, understanding of the levels of proportional reasoning may guide decisions concerning instructional sequencing and content.

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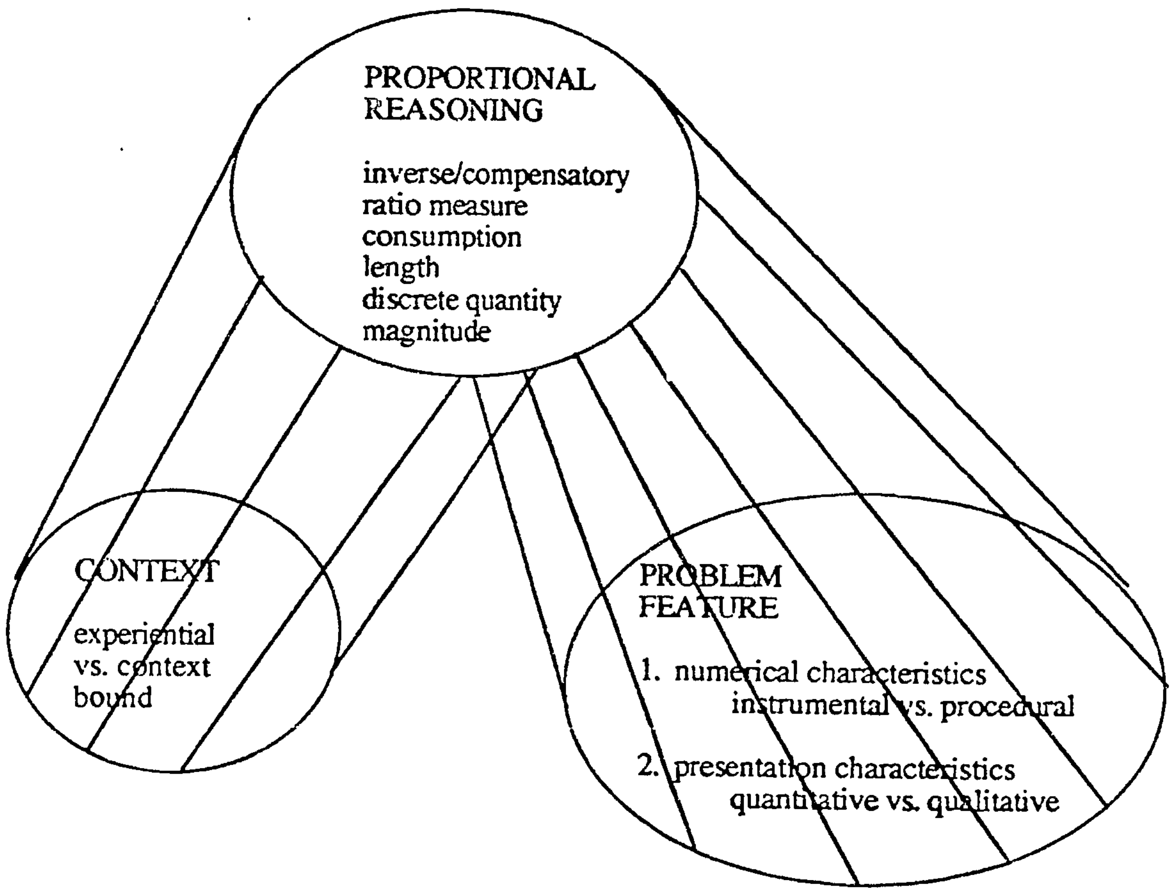

Figure 1: Proportional reasoning construct within a mathematical knowledge building framework

Sample Magnitude Problem:

Place each number below on the number line provided. If a number cannot be placed on the number line, circle it and explain why it cannot be put on the number line.

.025, 1.67, -1.5, 7/8, 1.25, 13/7, .3, 5/4

Sample Discrete Quantity Problem:

Suppose in a large 16 ounce bag of M&M's there are the following number of each color of M&M:

12 red, 24 light brown, 24 dark brown, 16 yellow, 18 green

If you purchased an 8 ounce bag of M&M's, how many M&M's would you expect to have?

Sample Length (continuous quantity) Problem:

You have decided to construct a playhouse for your children that is an exact replica of your own house. Suppose the front windows of your house are 72 inches (height) by 24 inched (width). What would be the width of the playhouse window if the height of the playhouse window was 12 inches?

Sample Consumption Problem:

As a smart shopper, you have gathered the following information about name-brand products and store-brand equivalents. That information has been written in the table below:

Product	Name-Brand Weight	Price	Store-Brand Weight	Price
Wheat Crackers	9 oz.	$1.36	12 oz.	$1.79
Peanut Butter	16 oz.	$1.66	12 oz.	$1.15

Which is the better buy in wheat crackers?

Sample Ratio Measure Problem:

Density is defined as the ratio of mass to volume (D=MV). The density of substance B is twice that of substance A. If 100 ml of substance A has a mass of 1000g, what mass would 100 ml of substance B have? Why?

Sample Compensatory Problem:

A meter stick is balanced at its natural balance point on a fulcrum. A 100 gram weight is placed 20 cm to the left of the fulcrum. Where would a 200 gram weight be placed if the stick is to be balanced again?

Figure 2: Sample proportion tasks by increasing level of difficulty

	PRESENTATION TYPE		
	QUANTITATIVE		QUALITATIVE
PROBLEM STRUCTURE	INSTRUMENTAL*	PROCEDURAL*	
Number Relations and Magnitudes	1c-a 1c-b 1c-c		1a 1b 1d
Discrete Quantity/ Mixture		6a(f)+ 6b(f)+ 8a(c)+ 8c(c)+ 12a(f)+ 12b(f)+	6c(f)+ 8b(c)+
Ratio/Length		4b(f)+ 5b(c)+ 7a(c)+ 7b(c)+	4a(f)+ 5a(c)+
Consumption/ Production		11b(f)+	11a(f)+
Speed/Density Ratio		9a(f)+ 9b(f)+ 10a(c)+ 10b(c)+	
Compensatory/ Inverse Proportion		2b(f)+ 2c(f)+ 3a(c)+ 3b(c)+	2a(f)+

* NUMERICAL CHARACTERISTICS (Instrumental vs. procedural)

+ CONTEXT (f=familiar, c=context bound)

FIGURE 3 Item analysis of QQTPR by four complexity variables: Structure, Context, Numerical Characteristics, and Presentation Mode

STRUCTURE VARIABLES
1 Relations/Magnitude
2 Discrete Quantity
3 Length
4 Consumption
5 Speed/Density Ratio
6 Compensatory

CONTEXT VARIABLES
1 Familiar
2 Context Bound

PRESENTATION MODE
1 Qualitative
2 Quantitative

NUMERICAL CHARACTERISTICS
0 No computation
1 Instrumental
2 Procedural - simple proportion (1:1, 1:2, 1:3)
3 Procedural - whole number proportion (e.g. 2:3)
4 Procedural - rational number proportion or extraneous information
5 Procedural - requires abstract symbolization

+1 to any problem that requires multi-step solution strategies or for which no ready solution strategy is available (possibility of total +2)

SAMPLE ITEM SCORING

QUESTION	STRUCTURE	PRESENTATION MODE	CONTEXT	NUMERICAL CHARACTERISTICS	TOTAL
1a	1	1	2	1	5
1b	1	1	2	1	5
1c	1	2	2	1	6
1d	1	1	2	0	4
2a	6	1	1	0	8
2b	6	2	1	2	11
2c	6	2	1	2	11
5a	3	1	2	0	6
5b	3	2	2	3	10
8a	2	2	2	4	10
8b	2	1	2	0	5
8c	2	2	2	5	11

FIGURE 4 Scoring Rubric for QQTPR Items using Complexity Variables

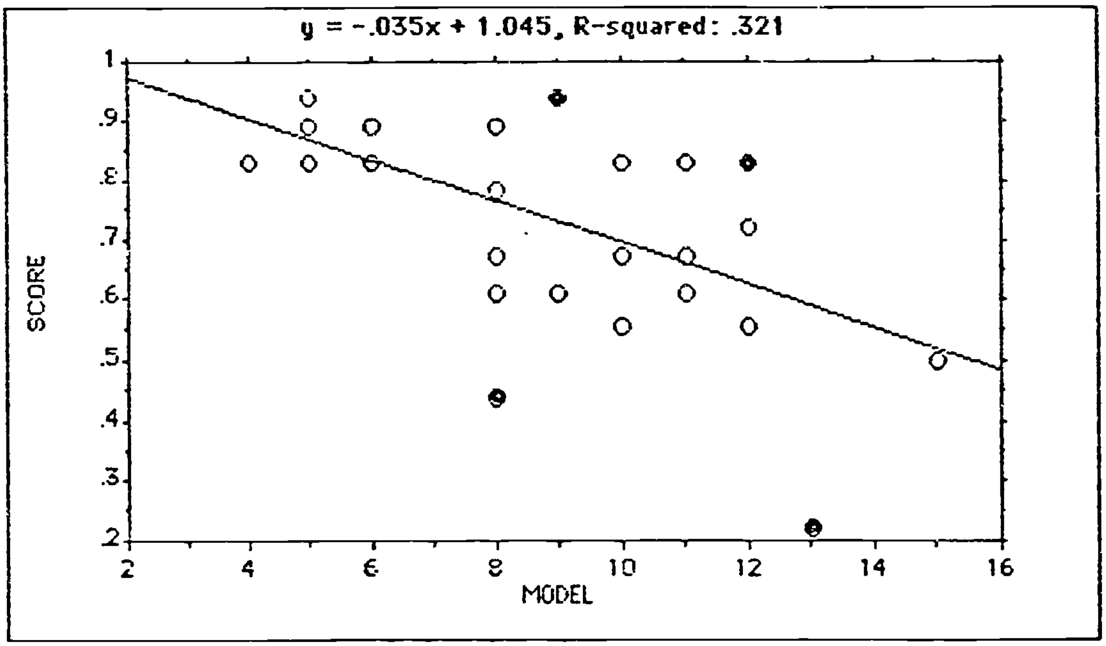

FIGURE 5 Linear Regression Line of Expected vs. Actual Performance

	MODEL	SCORE
	X_i	Y_i
1	5	.89
2	5	.89
3	6	.83
4	4	.83
5	8	.89
6	11	.83
7	11	.61
8	12	.83
9	15	.50
10	5	.94
11	8	.78
12	6	.89
13	10	.56
14	9	.94
15	9	.61
16	8	.61
17	9	.94
18	10	.83
19	10	.67
20	5	.83
21	11	.61
22	12	.56
23	13	.22
24	12	.72
25	11	.67
26	10	.83
27	12	.72
28	8	.44
29	8	.67

FIGURE 6 Data for Regression Analysis

Simple Regression X_1: MODEL Y_1: SCORE

DF:	R:	R-sqrd:	Adj. R-sqrd:	Std Err:
28	.567	.321	.296	.145

Summary Table

Source:	DF:	SS:	MS:	F-test:
Regression	1	.267	.267	12.77
Residual	27	.564	.021	p=.0014
Total	28	.831		

Beta Coefficient Table

Parameter:	Value:	Std. Err.:	Std. Val.:	t-Val:	Prob.:
Intercept	1.045				
Slope	-.035	.01	-.567	3.574	.0014

FIGURE 7: Summary Chart of Regression Analysis