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ABSTRACT

This report describes the first year of an ongoing intervention study and the learning of a group of teachers who are working together to make changes in the ways they teach mathematics. The document presents a vision of mathematics education based on research in cognitive psychology which indicates that learners construct knowledge rather than being passive receivers of knowledge. When reflecting on their beginning efforts, the researchers found that several assumptions in their planned interventions were not borne out by the group's experience. The subject matter content (integers) posed major difficulties for several of the teachers. The teachers' interest in talking about their own practices strongly influenced the group's interactions. Collectively, the teachers and researchers created a learning community that was grounded in watching videotapes of mathematics teaching in a third grade classroom and discussing ideas about teaching and learning in ways that were different from their own experiences as teachers and students. Here, the researchers pose a set of conjectures that serve as a framework for the continuing collaboration of this group of educators' inquiry into non-traditional approaches in the teaching and learning of mathematics. (LL)

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National Center for Research on Teacher Learning

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[1]Formerly known as the National Center for Research on Teacher Education (1985-1990), the Center was renamed in 1991.

Abstract

In this report, the authors describe the first year of an ongoing intervention study and the learning of a group of teachers and researchers who are working together to make changes in the ways they teach mathematics. When reflecting on their beginning efforts, the researchers found that several assumptions in their planned interventions were not born out by the group's experience. The subject matter content (integers) they selected as an initial focus of study posed major difficulties for several of the teachers. The teachers' interest in talking about their own practices strongly influenced the group's interactions. Collectively, the teachers and researchers created a learning community that was grounded in watching videotapes of mathematics teaching in a third grade classroom and discussing ideas about teaching and learning in ways that were different from their own experiences as teachers and students. Here, the researchers pose a set of conjectures that serve as a framework for the continuing collaboration of this group of educators' inquiry into non-traditional approaches in the teaching and learning of mathematics.

LEARNING IN GOOD COMPANY: REPORT ON A PILOT STUDY

Helen Featherstone, Lauren Pfeiffer, and Stephen P. Smith

Over the past decade, various reports have identified serious deficiencies in mathematics education in the United States (MSEB, 1989: MSEB, 1990; McKnight, et. al., 1987; National Commission on Excellence in Education, 1983). The National Council of Teachers of Mathematics (NCTM) has responded to these calls for reform with the publication of the *Curriculum and Evaluation Standards* (1989) and the *Professional Standards for Teaching Mathematics* (1991). These documents present a vision of mathematics education based on research in cognitive psychology which indicates learners construct knowledge rather than being passive receivers of knowledge. The image that emerges from these documents is of classrooms where teachers are:

- Selecting mathematical tasks to engage students' interests and intellect;
- Providing opportunities to deepen their understanding of the mathematics being studied and its applications;
- Orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas;
- Using, and helping students use, technology and other tools to pursue mathematical investigations;
- Seeking, and helping students seek, connections to previous and developing knowledge;
- Guiding individual, small group, and whole-class work (NCTM, 1991; p. 1).

But changing what happens in classrooms is no easy matter. Even when reformers are able to agree about what sorts of alterations in teaching practices would fit with new understandings about the way children learn, they find it difficult to communicate their ideas to others. Most of today's adults—teachers, teacher candidates, parents, policy makers, and researchers—spent twelve years as students in conventional schools where teaching meant telling and learning meant memorizing facts and rules and practicing skills. This "apprenticeship of observation," as Dan Lortie (1975) calls it, forms the bedrock of their ideas about schooling. Teacher educators often manage to sprinkle some question marks and some new rhetoric around the old images, but when their students graduate and take jobs as regular teachers in schools whose norms resemble those of the schools in which they spent 12,000 hours as students, many slip back into familiar grooves.

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Stephen P. Smith, a doctoral candidate in teacher education at Michigan State University, is a research assistant with the National Center for Research on Teacher Learning and co-teaches mathematics in a first grade classroom. He is interested in children's mathematical ways of knowing and in the philosophy of mathematics. Additionally, his research includes studying the relationship between undergraduates' knowledge of subject matter and learning to teach.

It seems reasonable to hypothesize that vivid images of other sorts of teaching would help both prospective teachers and practicing teachers to imagine new approaches to teaching and to raise questions about these approaches. Many teacher educators are experimenting with narrative and cases in teacher education in the hope that stories and portraits of teachers coping with day-to-day dilemmas can evoke powerful thinking in aspiring teachers, but so far, the evidence is mixed (Shulman, 1992; Kleinfeld, 1992).

Deborah Ball and Magdalene Lampert both teach education courses at Michigan State University's (MSU) College of Education and mathematics in a public elementary school. Both have pioneered new approaches to mathematics teaching, centering their classes on a process of inquiry into meaning in a community of young "mathematicians." The emphasis is on discourse and making connections among important mathematical ideas, and students' central role in constructing argument (Lampert, 1985; Lampert, 1986; Ball, 1990a; Ball, 1990b). In the summer of 1989, they conceived a plan to use computer technology to provide prospective teachers with a host of flexible opportunities for exploring non-traditional ways of teaching mathematics. In part, Ball and Lampert were responding to practical pressures on their own elementary mathematics classes: colleagues in the College of Education liked to send their students to observe in Ball's and Lampert's classrooms because they found that such observations prompted valuable thought and discussion about the nature of mathematics and mathematical learning. The number of such requests sometimes swamped these third and fifth grade classrooms, and Ball and Lampert wondered whether there might be other ways to provide their own students and those of their colleagues with some of what they got from classroom visits.

In the summer of 1989, the National Science Foundation funded the Mathematics and Teaching Through Hypermedia (M.A.T.H.) project; permitting Ball, Lampert, and colleagues to document teaching and learning in their classrooms. Over the course of the next academic year they videotaped many mathematics lessons, as well as interviews with students, mathematicians, and mathematics educators. They saved and reproduced all student work, including homework, tests, and the journals in which students wrote during math class. Each day Ball and Lampert recorded their reflections on lessons in teaching journals and a team of graduate students kept fieldnotes on the mathematical and pedagogical issues raised in the lesson. During the following two years, Ball and Lampert worked with these graduate students to create videodiscs that would permit prospective teachers to explore some of these materials.

Because staff developers and teacher educators who wish to change the practices of experienced teachers face many of the same problems as do teacher educators working with prospective teachers, it soon occurred to Ball, Lampert, and others familiar with the M.A.T.H. materials that practicing teachers might also find them interesting and useful. Indeed, it seemed possible that these materials would offer teachers interested in learning more about alternatives to traditional mathematics teaching; unique opportunities to explore both teaching and learning in a non-traditional setting. While the ongoing necessity of managing multiple and (often conflicting) demands places real limits on, for example, teachers' exploration of their own students' thinking, the M.A.T.H. materials create opportunities for teachers to "stop the action" of a videotape and examine student notebooks, observers' fieldnotes, or the teachers' own reflections on the class. Potentially, the materials provide teachers both with a place to explore teaching and learning with some of the constraints of the classroom removed and an opportunity to construct an alternative "apprenticeship of observation" with a teacher who is exploring some non-standard approaches to math teaching that parallels those recommended by the NCTM *Standards*. Hoping to learn more about the ways in which explorations of these materials might facilitate the learning of experienced teachers, researchers at the Michigan State University College of Education proposed to study *Learning from Hypermedia* as part of the National Center for Research on Teacher Learning.

In Spring 1991, we (Featherstone, Pfeiffer, and Smith) began to plan a pilot study which would involve inviting two teachers to explore the set of materials during the summer of 1991. Feeling that we could learn a considerable amount about the ways in which teachers might use and think

about the materials by carefully documenting the work of two teachers, we set about recruiting teachers. However, a variety of considerations led us to change our plans: the videodiscs that the M.A.T.H. team was creating would not be ready for some months; as we began to explore the M.A.T.H. materials, we realized that we needed to know more about the materials ourselves in order to help teachers to find productive paths through the vast databank that existed. We revised our plan, used the summer to explore one manageable part of the collection (documenting Ball's third grade classroom) and in the fall offered a group independent study, "Investigating Mathematics Teaching" (IMT), for teachers who wanted to explore the materials. By advertising this opportunity to students enrolled in the masters program in the College of Education, we hoped to attract a diverse group of teachers—including some who did not know Ball's reputation and perhaps some who were skeptical about (or ignorant of) the approaches to teaching math described in the NCTM *Standards*.

We learned from interviewing Ball that teachers who see videotapes of her math classes almost always ask how she establishes norms for productive classroom discourse about mathematics at the beginning of the school year. In consequence, we decided to focus the teachers' study on the first unit Ball taught in 1989-1990. As it happened, the first unit focused on integers—the third graders spent eight weeks exploring addition and subtraction of positive and negative numbers. Although we realized that few elementary teachers teach operations with negative numbers to eight year olds and that some might feel uncomfortable in this mathematical terrain, we reasoned that the teachers would grow comfortable with the mathematics as they watched Ball's students explore it. We also believed that most teachers would be curious enough about how Ball managed her early interactions with a group of children not familiar with this sort of teaching to rise above any transitory discomfort with the subject matter. We learned, as the quarter progressed, that we had greatly underestimated the strength of the aversion some teachers feel toward such abstract topics.

During the summer and fall, we studied the materials which document eight weeks of math instruction in Ball's classroom and created collections that would, we hoped, allow teachers to explore a variety of questions on their own. We housed these collections in four large cartons—which we called "toy boxes"—labelled *Curriculum*, *Representations*, *Student Thinking about Integers*, and *Teacher Exploration of Student Thinking*. Each box contained relevant videotaped segments, teacher and student journal entries, observer fieldnotes, and some articles. In September, as we put the finishing touches on these collections, we created and circulated a course description for the group independent study and sought help from colleagues in contacting teachers who might be interested in joining the group.

Putting the group together proved more difficult than we anticipated: The posters advertising the class generated only one inquiry; two teachers heard about the group through a summer course focussed on the NCTM *Standards*. During the last part of September we followed a host of leads. At almost the last minute we changed the day of meeting to accommodate several potential participants. By the first meeting we had recruited nine teachers who were interested enough to want to come to the first meeting. Only four of these teachers wanted to register for course credit towards a masters degree.

A Narrative History of the Pilot

We had put these collections together in the full expectation of exploring them with a group of teachers who knew little if anything about the NCTM *Standards*. In fact, however, eight of the nine teachers who came to the IMT group's first meeting were committed to teaching math in ways that were quite different from those they had experienced as students in schools. All eight of these teachers had at least heard of the NCTM *Standards*; three had studied them with some care.

During the first half of the first evening the teachers introduced themselves and wrote and talked about a math class they had recently taught and about the issues that were currently troubling them in their math teaching. Although

the teachers taught different grades (second to eighth) in a wide variety of schools—ranging from one in one of the state's most affluent suburbs to another in a poverty-stricken urban neighborhood—and mentioned different concerns, nods around the table suggested certain similarities among the group: several teachers remembered bad experiences with math as elementary and high school students and hoped to provide something better for the children in their charge; most were already trying to teach math in ways they had not experienced as students; most were finding this difficult and hoped to find some help in the group. A few were concerned by the practices of other teachers in their buildings, felt isolated, and hoped to persuade colleagues to consider other ways of teaching. All but one of the teachers had heard of Deborah Ball; several had met her or seen a videotape of one of her math lessons. The ninth teacher was a novice. She had heard neither of Ball nor of the NCTM *Standards*; she explained that she felt little enthusiasm either for math or for the idea of teaching it and hoped that the group might help her to address the deficiencies that she felt in this area.

During the first hour and a half of the meeting, the teachers raised a variety of questions and concerns: How can I develop a learning community in my classroom? How can I get students to really think, and not just give quick answers to questions? What role do I play as a teacher in a class where so much depends on student discourse? As each teacher described her (all the teachers were women) situation and her concerns, others around the table nodded; their concerns appeared to overlap considerably.[1]

During the second part of the three hour meeting, we introduced some of the M.A.T.H. materials: a videotape of a discussion that occurred in Ball's class on September 19, 1989, and the math journals of Ball's students for that day. After working alone and then as a group on the task that Ball had given her students that day ("Write number sentences equal to 10"), the teachers discussed what they would expect eight-year-olds to do with such an assignment and then examined the notebooks of Ball's students. The notebooks prompted considerable conversation about the patterns the teachers saw in individual notebooks and the evidence of collaboration among students.

Over the next forty minutes we watched the videotape, stopping the VCR occasionally to discuss what we were seeing. Fully twenty minutes of the third-grade discussion had focused on the question of whether 200-190=10, as one student asserted, or 190, as another argued. The class ended with this question unresolved; Ball facilitated the discussion, but remained neutral. The following excerpt from Smith's fieldnote gives a bit of the flavor of the discussion in the IMT group:

> There was a fair amount of quiet conversation while the [video]tape ran. No one seemed disturbed by others' talk. [The teachers] wondered who some of the kids were and if it was their notebook that they had. They also talked about certain of [Ball's] decisions and recognized some of the personalities as similar to their own students.
>
> After the tape there was general wondering about the kids: Were the [third graders who had not participated in the discussion] "into it"? Is it the curriculum that kept the kids so still for such a long time? What was the influence of the videotaping? Are the kids the same with [the teacher who teaches them language arts, social studies and science]? There also seemed to be consensus that it would be difficult to be as patient as [Ball] in certain instances and to stop at that point. There was a general desire to find out what happened next.

As the meeting drew to a close, Featherstone distributed copies of Ball's teaching journal for September 19th.

Two of the teachers decided not to return after the first meeting.[2] We began our second meeting with a discussion of the teachers' reactions to Ball's teaching journal. The journal carried us into questions about how much math a teacher needed to know to teach in the way that Ball was teaching, how teachers felt about leaving children hanging at the end of a discussion, and comparisons between Ball's teaching situation and their own. We then discussed where Ball, as the teacher of this class, might want to go next, given the multiple confusions of her students. Next, foreshadowing the content of the upcoming unit, we asked the teachers to talk about any memories they had of their own introduction to the idea that there were numbers below zero. Finally, we viewed and discussed another selection of videotape, this one selected from a class

Ball taught several days later. Ball was continuing to address the problem that had arisen on September 19; this time her students were working on the following problem:

> If Mrs. R. has 300 pencils and Miss L. has 190 pencils, how many more pencils does Mrs. R. have than Miss L.?

The structure of the IMT group's third meeting resembled that of its second: We continued to explore our ideas about and experiences with numbers less than zero—this time we worked both individually and collectively to design and analyze representations that we might use to teach third graders about negative and positive integers—and then watched and discussed another videotape. In this class, Ball introduced a representation she used for several weeks to introduce operations involving negative numbers. The representation consisted of a building with a ground floor, twelve "above ground" and twelve "below ground" floors and an elevator which traveled to all of the floors. As they watched Ball teach her students how to use number sentences to describe the movement of a little magnetic man up and down the building's elevator, the teachers noted that she was "much more directive" than she had been in previous lessons and discussed their observations.

By the fourth meeting of the IMT group we felt that the teachers had had sufficient introduction to the materials to do some explorations on their own. Wishing both to learn more about the range of ways in which they might use the materials if they were not harnessed to the group and to give them time to pursue individual agendas, we invited the teachers to work either alone or in small groups during the first hour and a half of each of the next four meetings. During the final hour and a half of these four meetings we discussed various issues together; during three of the four, we also watched some videotape.

The last three meetings of the fall quarter had a somewhat different structure: All were focused in some way on individuals. During the first, we met with Deborah Ball to discuss both questions that teachers wanted to ask her and the questions that she wanted to raise with them about the teaching of integers. During the ninth and tenth meetings we heard reports from individuals who had undertaken projects and discussed issues that arose from their reports. In each of these meetings we also discussed, in different ways, our own work as a group. At the end of the eighth meeting, after Ball's departure, teachers talked about aspects of the work we were doing together and what they were finding helpful or confusing. Toward the end of the ninth and tenth meetings we discussed our future as a group. We asked: Which teachers wanted to continue to meet? What might we do if we did continue to meet? Which materials might we explore? How often ought we meet?

In January, we reassembled, having agreed that we would get together for two and a half hours every other Thursday—a total of six times—during the ten-week winter quarter. Four of the teachers had declared that they definitely wanted to continue; the other three had expressed uncertainty, mentioning a variety of personal and professional concerns. We were surprised, therefore, when all seven teachers appeared at the first January meeting, looking excited and refreshed by their two week holiday break. Based on what the teachers had told us in November and December about what was useful to them, we introduced a new structure: we would begin each meeting with "current events"—a chance for teachers to raise issues that had come up for them over the previous fortnight in their schools or classrooms; we would move away from integers and begin to explore the unit on fractions that Ball had taught at the end of the 1989-1990 school year; we would look at the videotapes in this unit through lenses derived from the NCTM *Professional Standards for Teaching Mathematics*. In fact, this structure guided our meetings for the remainder of the school year.

We launched the first of the two January meetings with some old business. In laying the ground rules for group membership in early October, Featherstone had said that everyone in the group should plan to do some sort of "project;" partly in order to reduce the "distance" between teachers and researchers, she had added that she would herself do a project and that she would report on her progress in December, at the same time others were doing so. However, because the teachers' presentations had filled the agendas of the two December meetings, she postponed her presentation until after the holiday. Since none of the

teachers arrived at the first January meeting with a story or a problem, Featherstone presented her project. This presentation focused on close analysis of the thinking of two students in Ball's class who revised their thinking during one mathematics class and was built around half an hour of videotape. The subsequent discussion of the tape drew on what Featherstone was able to point out about the interactions that seemed to play into the revisions the two children made in their thinking, on two pages from the NCTM *Standards*, on Ball's teaching journal for the day, and on observations that two teachers made about differences between Ball's teaching and their own.

Several teachers arrived at the second January meeting with stories and questions for the group. One reported on a lesson she had taught in her own class as a result of the discussion that had taken place at the group's last meeting (See *The Attack of the Killer Elevens*, p. 10.). Another spoke about adopting a new questioning strategy as a result of the same discussion. We watched a segment of videotape in which Ball interacted quite differently, and at considerable length, with an individual student and discussed different responses to this interaction. In her journal, Pfeiffer noted that conversation in the IMT group seemed to have changed, and that the tapes seemed to be serving the group in a new way:

> I made a note during Karen's[3] story and the discussion it prompted that this "feels" great—the teachers are talking about their practice. It took us a long time to get here, but the quality of the talk is different now. One difference I can point to is that they are making more and greater detailed connections to their own students and their own practice.

Because the teachers responded in quite different ways to what they could see of Ball's work with this student, the videotape prompted discussions of the multiple ways in which students react to prolonged one-on-one conversation with their teacher, about the notion of there being "more than one way to teach," about the struggle to balance listening and questioning. Half an hour after the official end of the group meeting, half the teachers were still in the room talking either to one another or to one of the researchers.

We have defined the end of January as the end of the pilot phase of our work because it marked the end of our second round of interviews with the teachers. In many ways, however, it felt like a beginning. The IMT group had a new sort of life. It was no longer a course that the teachers attended for graduate credit—it had never been a course for several of them—although one of the teachers did complete a separate independent study project for graduate credit. It was now a voluntary association of teachers and researchers—the former committed to teaching "against the grain" (Cochran-Smith, 1991) and the latter committed to learning more about possible paths into this teaching—who came together to learn. Over the course of the next few months the "feel" of the group continued to change, as did the nature of the conversation, the use of the M. A. T. H. materials, and the practice of many of the teachers. Analysis of the later history of the group will have to wait, however, for later reports. In what follows we will consider first some of what did and did not happen during this pilot phase of the study, and then a few of the answers teachers gave when we asked them to talk about any changes they had made in their teaching of mathematics.

Reflecting on the Pilot

As we revisit the data generated between October 1 and February 1, several points stand out: The work was difficult in unexpected ways, perhaps because the assumptions that we made about what we were studying and what would happen were not always born out by experience; the group itself is a major resource for the teachers; the M.A.T.H. materials raised many questions for discussion.

Difficulties and Surprises

The work we did during the project's pilot phase, though serving as a critical foundation, was more difficult than it has been in later phases of the project. Our preliminary analysis of the data that we gathered between October and January leads us to several observations about the sources of these difficulties. First, the subject matter of the unit we had chosen to focus on posed major difficulties for several of the teachers. We had not anticipated the extent of these difficulties and we failed to respond in ways that were helpful at the time. Second, we

made a number of assumptions about the ways in which the teachers would use the materials we placed at their disposal that were not born out by experience.

Struggles with negative numbers. The decision to focus on a unit involving operations with positive and negative integers plunged the group into perilous mathematical terrain. Few elementary school math textbooks do more than touch on the existence of numbers below zero. Neither state nor local curriculum guidelines pressed the teachers in the IMT group to get into this territory in the elementary school. Not even the mathematically ambitious *Curriculum and Evaluation Standards* of the National Council of Teachers of Mathematics recommends teachers explore this territory in elementary grades. None of the teachers in the IMT group had taught about operations with negative integers. Close examination of this unit did not offer them concrete help in thinking about specific mathematical ideas that they needed to teach and that they wanted help in thinking about.

The teachers, therefore, had no immediate practical reason to focus on the substance of the mathematics that Ball was teaching. In addition, at least three of the teachers felt uncomfortable about their own mastery of this subject matter (see Featherstone, Smith, et. al.). Some felt that their own school experiences with this curricular territory represented all that they hoped to avoid in their own teaching: They had learned the rules for solving textbook examples involving below-zero numbers without ever understanding the reasons for those rules.

We assumed that the chance to design and discuss the merits and limitations of a variety of representations of negative numbers, together with the chance to explore and discuss the uses Ball and her third graders made of two of these representations, would help the teachers to move about comfortably in this mathematical terrain. We were wrong. Even in December one teacher wrote in her journal:

> I desperately wanted to "understand" their [negative numbers] purpose and not just a process by which to use them. I still haven't discovered that. . . . But I have been struck by the struggle I was going through and the sense of frustration I was experiencing. I didn't want to go to class. I didn't want to do the project. I really wanted to stop coming. I began to tune out.

Assumptions about the ways in which the teachers would want to use the materials. We made at least three false assumptions about the ways in which the teachers would want to use time and materials. First, we assumed that the teachers would want to use the materials to explore on their own questions about teaching and learning of mathematics that were important to them. Second, we assumed that they would want access to the materials outside of the group in order to continue these explorations. Third, we assumed that they would find the collections we had put together during the summer useful for these explorations.

In fact, although the teachers complied politely with our suggestions that they break into groups to look at tapes or explore the contents of the boxes at the beginning of the fourth, fifth, and sixth meetings, their comments at the last December meeting do not suggest that they found these opportunities for individual exploration particularly valuable. Nor did we hear much animated conversation during this part of the fall meetings. Nor did the teachers vie eagerly for opportunities to report on what they had seen or the questions these explorations had raised for them. Audiotapes of these sessions picked up comparatively little conversation; many of the conversations that did occur were initiated by the researchers. No teacher ever spontaneously initiated exploration of any of the boxes of materials; rarely did any teacher ask to borrow any of the materials in these boxes. The materials the teachers did ask to borrow were copies of Ball's teaching journal for other parts of the year, and videotapes of particular lessons—usually tapes that we had viewed at some session that they had had to miss or tapes relating to content that they wished to teach themselves. Only one teacher used the materials to explore a particular question.

Across the year, the teachers did, however, use the materials in a variety of other ways:

- As a source of mathematics problems,
- To plan a unit,
- To deepen their own thinking about the complexity of subject matter while teaching a unit,
- As a source of representations,
- As a source of ideas and models for questioning students' thinking
- As a source of questions about their own teaching

The Group as a Key Resource

As we dig more deeply into our data, we are struck by the ways in which these three assumptions connect to what we now see as a meta-assumption about our entire enterprise: Our data suggests that as we put together our collection of materials over the summer and planned for the pilot study in the early fall, we saw ourselves planning to study an unknown number of *individual* teachers as they explored the M.A.T.H. materials. In fact, however, our data suggests that the *group* became increasingly important to participants, and that participants found resources that they needed in the *collective* conversation. We have only just begun to analyze the changes in the functioning of the group over time; we will comment only briefly here on what we have been able to see about the way the group functioned as a resource, and the way we, as the group's leaders, accommodated to our changing perception of the role of group conversation in our enterprise.

An excerpt from Pfeiffer's fieldnote on the IMT group's second meeting suggests both that participants were beginning to see the group as a resource and that we were somewhat surprised either by this development or by the form it took. Pfeiffer's comment followed a description of a conversation about one teacher's perception that neither her principal nor her colleagues shared her ideas about mathematics teaching (see *"I Can't Believe She Spent Three Days on One Problem,"* p. 9):

> I'm getting the sense that the teachers have a strong need for the opportunity to talk to other teachers ("pioneers") who are trying to change mathematics teaching, at least as much as they have a need to watch tape and talk about pedagogy. At this point the conversation felt more like a support group.

Four weeks later, one of the teachers provided a startling image of these Thursday evening meetings:

> [Laura] commented last night that what she gets from watching [Ball] is lots of questions that she asks herself. She also said that coming to the group was, for her, like going to church and being renewed. She always comes away with affirmation and confidence to continue pressing ahead in her efforts to teach differently. These comments seem related to her struggle in feeling isolated in her [district].

Two weeks after that, when we explicitly asked teachers to talk about what was useful to them and what was not, most focused on their particular interests—how Ball asked questions, how she made sense of student thinking, for example—rather than on particular group activities, but two explicitly identified the group conversation as being helpful. Both commented that they were not sure *why* it was helpful; one added, "It helps to know that you aren't the only one who is thinking [a particular way about something]." In the interviews we completed in January, several others pointed to aspects of the group conversation as a key facilitator of their growth (see *"It's a Huge World of 10 Out There!"* p. 14).

Raising Questions

Reviewing our data for the four months of the pilot, we are overwhelmed by the number, range, and urgency of the questions these materials raised for teachers. It would be overwhelming to try to list *all* the questions raised by the materials and discussed by the teachers either in group meetings or in their journals. Instead we have picked three issues that three different teachers raised during our second meeting, as representative of questions connected directly to the materials they examined.

"I'm feeling less urgency." At the conclusion of our first meeting we had distributed photocopies of the journal entry that Ball made after the discussion of 200-190. We began the second meeting with an open-ended invitation for reactions to the journal. Jennifer Stark, a novice teacher in the group, commented that after reading the journal entry she was "feeling less urgency" about resolving the issue of 200-190 than she had at the conclusion of our previous session. She explained that she had felt worried and upset by the idea of Ball's students leaving the classroom either believing that 200-190=190 or uncertain about what it equalled. Stark's comment suggests the wealth of questions that the videotape could raise for teachers who are new to this kind of teaching. For example: Will students learn anything from grappling with a math problem that they cannot solve? Will they learn incorrect approaches from watching another student do a math problem in a non-standard way if the teacher does not pronounce this approach incorrect? Will confusion harm them?

One of us asked Stark *why* she was feeling more comfortable about leaving things "hanging;" Stark said that she wasn't sure. When another teacher, Laura Morrow, suggested that perhaps it was because she was coming to value the process above the product, Stark looked uncertain.

Stark also reported that she had given her own students—like Ball, she taught third grade—Ball's open-ended problem: "Write number sentences that equal 10." She reported with excitement that her students came up with many of the same equations and patterns as Ball's students. We wondered what role, if any, her interactions with her own students around this open-ended assignment played in making her more comfortable with Ball's students' confusions.

"I felt jealous of how she could put labels on kids' thinking." After several other teachers had talked about the thoughts and feelings that they had as they read Ball's journal entry for September 19, Karen Miller commented, "I felt jealous of how she could put labels on kids' thinking." In her journal Ball had written:

> [T]his problem is interesting, in part, because their ability to reason mathematically about it depends on their understanding of the compare meaning of subtraction. [Betsy] was reasoning based on take-away model.

Ball's labeling displays an understanding of several mathematical meanings of subtraction and indicates that she believes that the differences between the compare meaning and the take away meaning have pedagogical implications. Miller's comment suggests the way in which reading Ball's journal opened up questions about the role of different kinds of knowledge in doing the sort of teaching Ball was doing. When Featherstone responded to Miller's comment by asking whether it was important for teachers to be able to label things, Morrow responded that she found that labels helped her to identify patterns and look at them over time.

"I Can't Believe She Spent Three Days on One Problem." Somewhat later the same evening, Smith described the way in which Ball had followed up on the September 19 discussion by giving the students several related problems. He explained that the tape we were about to watch showed part of the second day of discussion of one of these two problems. Laura Morrow interrupted his introduction. Pfeiffer's fieldnote describes the interaction this way:

> [Smith] began to introduce the segment of tape we were about to view. [Morrow] interrupted him to try to express how she was feeling in the moment. She was thinking about [Smith's] comments on the different days and different problems the class worked (200-190 one day; 100-90 another day; 300-290 another day) and she was overwhelmed to think that her principal would not understand or support spending three days on three problems. This led to a discussion by the whole group about [Morrow's] teaching circumstances and the lack of support she has in her school and district for change in the way mathematics is taught. It was dramatic that [Morrow] needed to vent her feelings about this and the other teachers were interested in how she was dealing with it. [A second teacher] described how the teachers in her school blame the new state mandated achievement test because it is designed for the high level students and it is trying to trick the students. [A third teacher] raised the question, What do we do first to open others up to think differently about this?

Interestingly, Morrow brought this conversation up spontaneously seven months later, in the context of another group discussion. At that time she commented that in October she had been shocked that Ball would spend three days on one problem, and that now it was shocking for her to imagine that she had felt this way only seven months before. This later comment suggests that she herself was as troubled by the idea of spending nearly a week on one problem as she imagined her principal would be. The juxtaposition of her later comment with our October fieldnote suggests (as do some of our other data) that the questions and comments teachers make about Ball's teaching represent real concerns. However, sometimes these concerns are framed as *other* people's issues.

Although the three comments and questions described in this section only begin to suggest ways in which the M. A. T. H. materials raised questions and focused discussion, they do give some sense of the ways in which discussions moved back and forth between Ball's practice and that of the teachers, and of the range of questions the videotapes and teaching journal raise for a group of practicing teachers. They suggest how centrally the teachers' daily work in schools conditioned the ways in which they reacted to the materials and framed questions for discussion.

"Has participating in the group influenced your practice?" In January we interviewed all group members a second time, in order to learn as much as possible about the teachers' experiences with the group and to assess what impact, if any, participation had had on them. These interviews were a turning point, in some sense, for the researchers, because all of the teachers who were teaching math[4] indicated that their teaching of math had been influenced by the IMT group in ways that they considered significant. We had not expected this: when we planned the group independent study we thought that exploring the M.A.T.H. materials might influence teachers' thinking, but we did not expect it to change their practice. We might have expected other teachers under other circumstances to tell us that they had changed their practice because they might think that this was what we wanted to hear. However, our experiences with the IMT teachers had not led us to expect that they would tell us what they thought we wanted to hear just to be polite. Indeed, we had been quite shaken by journal letters like these earlier in the term:

From one teacher, in mid-October:

> I don't like the way this class is going. I am unclear as to the purpose of the class. What is going on. I don't like the idea of being studied. Are "they" studying us? For what? What do they want to know? Helen asked about use of time? What was she saying? She didn't like the way we used the time tonight? No one said anything. . . . I didn't really like the conversation I had with Steve either.

From another teacher, a week later:

> I'm very frustrated with the math class. I guess I don't see what the purpose is. These discussions don't seem interesting to me . . . they seem to drag out and go no place. . . . I also am having trouble with the negative numbers. To me they seem like non-numbers and how do we teach them if they don't exist.

In what follows we want to describe and reflect on what two teachers said about changes in their own teaching. Their comments suggest the complexity of the task that confronts teachers who are trying to make the sorts of changes in their practice that the NCTM recommends.

The Attack of the Killer Elevens

Karen Miller had transformed her literacy instruction five years before she joined the IMT group, moving from a program built around basal readers and skills instruction to one based on the principles of "whole language." However, she had continued to teach *math* fairly traditionally ("although," she notes, "[a colleague with whom she planned these changes] and I kept saying to each other, there must be some way to extend these ideas into our math teaching") until the summer of 1989 when she saw in a workshop a videotape of Deborah Ball teaching math in new and unfamiliar ways. She was attracted to what she saw. That fall, she began to make some changes in the way she taught math; by the following year she had

stopped using a textbook altogether. In Marilyn Burns' books on mathematics teaching (e.g., Burns, 1992) she found useful problems and help planning units.

In the spring of 1991 Miller joined a math study group of teachers and student teachers run by an MSU graduate student. Here she found encouragement and concrete help for rethinking her math teaching. Near the end of the school year she heard about the IMT project through an MSU faculty member with whom she was working on another project and expressed an interest in participating. The idea of watching video tapes of Ball's teaching "really attracted" her. However, she had never felt confident about her knowledge of mathematics and, when she and Featherstone discussed the group on the phone in September, she noted that she was greatly intimidated by the idea that the IMT group would be exploring a unit that focused on operations involving negative numbers. Nor did exposure to the material allay her distress: throughout the fall quarter she expressed concerns about this subject matter.

When Featherstone interviewed Miller in her classroom in January, she was surprised to hear her claim that participation in the group "is having a lot of effect on my teaching that I hadn't expected. Watching Debbie Ball teach and reading her journals," she added, "you can't help but raise questions for yourself." The group has provided a "constant influx of ideas." When Featherstone asked her to say more about the changes in her practice, she offered to tell a story about two recent math lessons.

A week earlier, in our first meeting of the winter term, the IMT group had viewed and discussed a section of videotape in which Ball and her students discuss possible approaches to the following problem: "If Mrs R. has 300 stickers and Miss L. has 190, how many more stickers does Mrs R. have than Miss L. ?" In the 25-minute long class discussion, Ball's third graders struggle to decide among four different answers that students in the class have gotten to the problem: 290, 210, 110, and 10. At the beginning of this conversation most of the third graders agree with the reasoning of a classmate who writes the problem on the board

$$\begin{array}{r} 300 \\ -190 \\ \hline 290 \end{array}$$

and explains, "Zero take away zero is zero. Nine take away zero, you can't do it so you write down the 9. Three take away one is two."

Returning to questions the teachers had raised in October when they viewed a videotape featuring a similar problem (see above), the IMT group had focused on two children who changed their minds during the discussion. Once again we tried to figure out why Betsy's written presentation carried more weight than the other approaches that class members took to the problem. In the course of our discussion one teacher wondered aloud what might have happened if Ball had asked the students to compare 30 and 19 rather than 300 and 190. Would the written representation of the problem still hold sway, or would students' intuitive sense of number take over when the numbers were smaller and more familiar?

Intrigued by the question, Miller decided to conduct an investigation in her own class. She posed a question similar to Ball's, substituting 30 for 300 and 19 for 190.

> But I didn't stop there. I said "use your mini-computer[5] to figure this out, and then explain how you figured it in your journal. And then, if you finish, here are some other problems." I mean, I knew I had to have something else for them to do while some of the other kids finished the problem. So I gave them a whole series of problems. And as I got to creating them I thought, "oh, I'll do a pattern and they'll all come out the same and I'll see what they do with it."

After the students had worked on these problems independently for a while they reconvened as a group and looked at the first problem together. They agreed without much difficulty that the answer was 11. So, "we finished with it, and everyone was feeling pretty good about

it, except that one of my students, Lisa, tried to talk about how 30 take away 19 and 50 take away 39, which were the only two problems she had done, were the same." In trying to articulate this Lisa came up to the board and she wrote

$$\begin{array}{rr} 50 & 39 \\ -30 & -19 \\ \hline 20 & 20 \end{array}$$

"And she's seen a pattern! . . . Which I thought was interesting. So, I decided to pursue that with the kids." The next day, Miller gave them a series of problems like this:

$$\begin{array}{rrrrr} 30 & 60 & 90 & 40 & 80 \\ -19 & -49 & -79 & -29 & -69 \end{array}$$

asking the third graders to work individually on the problems and to look for patterns. Several students responded immediately that they knew that all the answers were 11. Miller said that this was fine; they could just write down 11 and then start looking for patterns.

When her class reconvened, students talked excitedly about the patterns they saw: they noticed that they were adding first 10, then 20, etc. to both the top and the bottom number, that all the top numbers ended in zero and the bottom numbers ended in 9, etc. Then they got interested in what would happen if they added some number that was not a multiple of 10 to the top and the bottom numbers. They tried 7 and were surprised to see that the difference was still 11. They became much engaged with trying to find something that they could add that would break the strangle hold of 11—"the attack of the killer 11," they called it. They tried lots of numbers with, of course, no luck. Then Nathaniel called out excitedly, "Eleven! Let's try 11." There was a murmur of excited approval. As a group, the third graders appeared to be convinced that if they added 11 to both the 19 and the 30, the difference could not continue to be 11. They were again astonished by the results of their arithmetic:

$$\begin{array}{rrr} 19 & 30 & 41 \\ +11 & +11 & -30 \\ \hline 30 & 41 & 11 \end{array}$$

Finally Cindy got Miller's attention: "Mrs. Miller, I've had my hand up for an *hour* and you *never* call on me." After making her way to the front of the room, she turned triumphantly toward her classmates: "You're all wrong. 60-49 isn't 11: It's 29!! See [writing it on the board]: 0 take away 9, you can't do it so you write 9. 6 take away 4 is 2. 29! So, if you add 30, you get 29 not 11!"

The third graders stared at the numbers on the board, and then many agreed! Some didn't. All this despite the fact that Miller had worked extensively with regrouping only a few months earlier and, with most of the students (she had taught second grade the previous year), the year before as well. Gregory disagreed. He said that you *could* take 9 from 0, and that, when you did this, you got -9.

At the end of this story, Miller told Featherstone that despite her deep uncertainties about her understanding of negative numbers, she now felt she "would love to teach" this topic. But she did not feel she could do it without help.

In a later conversation, reflecting on the story and the way it represented for her the sorts of changes the IMT group was prompting her to make, Miller explained that the biggest change she thought that she had made during the fall quarter was "listening to the kids."

> Going into "the killer elevens" so deeply, I was doing something different. [If I had given this problem a few months before] I would have stopped earlier. When they said it was 11, I would have just assumed that they knew the answers. I would have thought, "Why go on?" I wouldn't have believed that something else was going to come out of it. But by this time I had a different kind of trust in them as learners.
>
> Actually, I was floored when they knew that the answer was 11. But then I thought, "That's okay, it isn't just right answers we are looking for. It's their thinking."
>
> And that's really critical—the idea that I did stick with it.

Miller went on to explain that her lack of knowledge of mathematics had been a central issue from the first. She knew that Deborah Ball knew much more math than she did. She wanted

to teach in the way she saw Ball teaching, in the ways the NCTM *Standards* advocate, but much of what she heard about this kind of teaching seemed to assume a strong knowledge of subject matter. During the fall, as she watched tapes of Ball and her students engaging with mathematics and discussed what she saw with other teachers, she began to see another road into this material; she began to listen more carefully to her students, to feel that if she created the space, the norms, and the time, the children would, as a community, extend the problems she offered them in interesting and educationally productive ways. They would lead the way into important mathematical questions. In November, December, and January she was beginning to play with this hypothesis. By May she was fairly certain that she was finding interesting routes into the mathematics by following the children:

> "Do you have to know the mathematics in order to teach this way?" was always the question. Now I don't think the answer is "yes" or "no."

The changes in her teaching that Miller identifies here—her deepening interest in what children had to say about mathematical ideas and problems, her willingness (indeed, eagerness) to stick with a problem over several days—seems to connect directly to the fall term activities of the IMT group: the watching and discussion of videotapes of Deborah Ball's classroom. For as the teachers and researchers watched these tapes together they had the leisure, as outsiders, to stop the action and comment on children's thinking and confusions. During the day, in their own classrooms, the teachers rarely had this luxury; they had to act. In the evening, watching the tapes, we asked each other questions. When we could not decide what a child had said, or what he had meant, we rewound the tape and watched the interchange a second time. Over and over again, like Ball herself, the group noticed that third graders thought in unexpected ways about mathematics, that they generated surprising insights on some subjects, and displayed surprising confusions on others.

Miller's story shows a teacher listening with new ears, and great excitement, to her student's ideas about mathematics and learning new things about what they do and do not know. It shows her doing a kind of research in her classroom—research on the thinking of her eight-year-old students, and research, by inference, on Ball's students. For the questions Miller raises with her students grow immediately out of questions raised in the IMT group both about the thinking of Ball's students, and about pedagogy: How could a teacher move third graders beyond "300-190=290?" How could she challenge them to challenge the authority of their (incorrect) algorithm? And this research brings Miller interesting information both about her own students and about Ball's. It also feeds back into the conversations in the group, for the following week Miller brought the charts her students made and described the conversation she had with her students in detail, noting that the children who agreed with Cindy's written formulation—30-19=29—were among her strongest math students. Like most good research, Miller's has raised new uncertainties while shedding some light on the original question.

Miller's story also gives us a glimpse of a teacher taking several different kinds of steps to deal with limited knowledge of mathematics. To begin with Miller chooses an interesting question, offers it to her students, listens carefully to their ideas, and then, the next day, extends the problem in order to expose more of their thinking. Even though she remains unsure that *she* knew how to extend their thinking, she is becoming increasingly confident that, given time, encouragement, and a reasonably interesting problem, the children will push themselves and one another into mathematically interesting ground.

But the conversations about "the killer elevens" are only the beginning. Miller fastens onto Gregory's observation ("You *can* subtract nine from zero. You get negative nine.") as an opportunity to confront her own feelings of inadequacy about subject matter head on. She asks for help in teaching the content which plagued her during the fall quarter. She is convinced, she tells Featherstone, that she can come to understand this chunk of mathematics if she gets some help in teaching it. She is constructing a way to confront the problem of subject matter knowledge that fits with what she has believed about herself as a math teacher for a long time. "I always believed," she tells Featherstone, "that I was a good math teacher, because math wasn't easy for me, so I understood the children's

difficulties and confusions." Having already constructed an image of teacher-as-fellow-learner for herself, she may find this approach to learning math particularly congenial.

A postscript to this story: With Featherstone's help, Miller taught a short unit on negative integers. This was followed by a unit on fractions in which Miller thoughtfully probed her own understandings of a subject she had always seen as straightforward, as well as those of her students. At the end of the year she saw major changes in her own understanding of several areas of mathematics—changes that grew out of, and along with, her teaching.

"It's a Huge World of 10 Out There!"

When we interviewed Connie Marsh a few weeks later, we asked her the same question we had asked Karen Miller: Could she see changes in her math teaching when she compared what she was doing now to what she was doing at the beginning of the school year? She nodded emphatically:

> Yes, I can. I think I'm more aware of the discourse that goes on in my room, and the things I'm asking. Much more aware. And I think I'm a lot closer to what I see Debbie [Ball] doing. [laughs.] Not very close, but closer.
>
> I'm more aware of the questions I pose through the curriculum, but also the questions I pose to them individually.
>
> But also, I'm not jumping in as soon. I'm still jumping in [more laughter]. But I'm holding back more and letting them *struggle* with some of the things. That's something I've gotten much more comfortable with. Much more comfortable. I always said, "Now you write down 23." Now I'm saying, "Wait a minute. That's taking it all away from them." Now I'm letting them do some of that. That's okay. And it's okay if somebody gets confused and doesn't know where to begin. That gives us a place to start from.

After describing a bit about what is involved in helping a child to figure out how and why he is confused, and in trying to make his journey out of confusion an educational one, she began to talk about another important change in her relationship to the work she is doing with her students in math:

> What has been a real awakening for me, I think, as much as anything, is the relationships in number. I really never saw much relationship before. I mean, addition's addition and carrying was related to addition and borrowing was related to subtraction. But now the world of number is really exciting for me. When I see the combinations of numbers that [the students] get with the mini-computer or the combinations that they got with the problem that Debbie [Ball] gave, "What's 10?" and some of the things they are coming up with. And I always thought 10 was 6+4 and that there were certain facts. . . . But it's a huge world of 10 out there, it's a whole world of all different numbers and I always looked at it as a very narrow thing.
>
> And that is really growing for me this year. It's exciting. It really is. . . . Part of this has come from this class.
>
> And now I'm thinking about, How does that fit in here? How do I let my kids experience some of that? And how do I bring them along? . . . Sometimes it's overwhelming, when I really stop to look at it and think "Where am I going?"

Like Miller, Marsh sees herself listening to her students in a different way. And like Miller, she finds that listening to her students carries her more deeply into the subject matter. She describes her journey in powerful language: she is "awakening;" she now sees a "huge world of number," a world that she previously found narrow, constricted, and rigid in a way that seemed to exclude much of what her students had to say about it. And as she herself awakens to the pleasures of contemplating and exploring that world, she sees the pleasures that these explorations offer her students: She restrains herself from *telling*—"Now you write down 23"—because she feels that this kind of guidance denies them a pleasure she now values. "Wait a minute," she tells herself, "That's taking it all away from them."

Marsh's answers to our question carried us into the *aesthetics* of the changes she was making and experiencing. She delighted in the mathematical vistas that were opening before her. She rejoiced, as well, at what she was glimpsing of her student's knowledge and their thinking:

> I'm amazed at how much knowledge kids really have. I'm always amazed. I don't think a class goes by that I'm not amazed. They know *a lot*. There are a lot of them who know a lot about a lot of things. And you don't discover that unless you let them share and let them talk.
>
> It goes back to being open to change. I keep thinking, I'm struggling so hard right now, but I'm not *preparing* for something: this is life right now. This is part of it. If I can't get what I want out of it right now, I've got to make some changes. Because I'm not preparing for anything: I've got to appreciate what this is. For some reason that really hits for me. Because I hear so many people say, "I'm preparing for retirement," "I'm preparing for this." But you aren't going to do it then *either*. THIS IS LIFE. You aren't preparing to graduate, you aren't preparing for anything. This is *it*. What can you get out of *this* right now, and what can we do with *this*?

Marsh is describing here a complex spiraling interaction between her own developing skills as a listener, her growing commitment to listening, her mounting excitement both about mathematics and about her students' ideas, and her new-found belief that life, both for her and for her students, needs to be lived and celebrated in the present. She sees new practices fostering new discoveries and strengthening new beliefs. When she invites her students to explore number combinations on the mini-computer, their explorations open up new vistas on the world of number for her. Her changing vision of mathematics as an exciting and approachable landscape suggests the potential benefits of new explorations. Like David Hawkins (1974), she describes a triangular relationship between herself as a teacher, her students, and the subject of mathematics, a relationship in which she celebrates her students' discoveries not only because she cares about them and their learning, but because their investigations bring her new insights about mathematics, and new pleasures from the subject.

Marsh's final comments remind us of the role of pleasure, delight, and excitement in teacher change. Much research documents the difficulty teachers experience in changing their teaching. Many researchers have analyzed the critical role of collaboration and support during the process of change (e.g., Little, 1982). Even more explicitly than Miller's story, Marsh's comments identify the fundamental necessity for intrinsic satisfactions. For Miller makes it plain that if she did not enjoy what she was doing, if she was not pleased and excited by the changes in her classroom, by the new frontiers she was glimpsing, she would not be doing what she is doing.

The juxtaposition of her comments suggests that she is speaking about her students' learning as well as her own. As Miller's comments make clear, presiding over an exciting mathematics discussion can make teachers feel as though they are throwing an intellectual party instead of holding students' noses to an academic grindstone. Both Miller and Marsh appear to be saying here that one reason that teachers will pursue the difficult business of change ("I'm struggling so hard right now") is that it makes classroom life more fun for everyone.

Marsh's insistence on the importance of intrinsic satisfactions seems peculiarly relevant to the reform of mathematics teaching, for here, perhaps even more than in other areas of the elementary curriculum, teachers, principals, parents and curriculum committees justify today's drudgery by pointing toward tomorrow's presumed necessities. Third grade teachers start math period with timed tests of multiplication facts so their students will be prepared to do three digit multiplication in fourth grade, compute with decimals in fifth grade, and factor quadratic equations in ninth grade. More than in language arts, art, or social studies, students are assumed to need the skill taught in one grade in order to solve the problems they will encounter in the next grade. Even the National Council of Teachers of Mathematics justifies their ringing call for reform in elementary school teaching by urging the importance of preparing children for future careers in science and technology (NCTM, 1989).

After Marsh had described some of the changes that she had seen over the preceding four months both in her practice and in her thinking, we asked her to think about the role the IMT group had played in helping her to make these changes.

> It's okay to grapple, as much as anything. It's validating this feeling of "how to really go about this?" And it's okay: It's okay not to have the right question; It's okay to say "Good for you!" and then to say, "Oops! Did I just say 'Good for you!'?" And *they* [the other teachers in the group] validate that. That was really reassuring.

At first glimpse Marsh's comment suggests that the impact of the group has been less on her practice than on her experience of that practice: She *feels* better about the problems she encounters with her teaching. In fact, however, her comment points to far broader issues, issues central to teacher learning.

Teaching is, among other things, a theatrical performance; teachers, like actors, have to command the attention of a large crowd of other people. And, even though they are always outnumbered, they also have to persuade these other people to do their bidding. During their thirteen years as students in elementary and secondary schools, prospective teachers learn that the consent of the governed is not automatic: students obey some teachers and take others with more than a grain of salt. More often than not, the teachers who command the attention and obedience of their students look considerably more assured than those who do not. For many aspiring teachers, all the evidence points to one conclusion: Teaching comes easily and naturally for competent teachers; only the incompetents have to struggle.

Induction into the profession does surprisingly little to dispel this notion. Unlike actors, who see other actors struggle to master a role during rehearsals, teachers mostly see other teachers performing their public role or relaxing. Many struggle to master routines and manage behavior during their first year or two on the job, but they do this mostly privately. Without vivid images of other *competent* teachers struggling to meet the multiple demands of their jobs, they are likely to conclude that these struggles ought to be transitory, and that if they have "the right stuff" they will soon manage the demands of children and classrooms fairly easily.

The prophesy that teaching will not involve struggle for more than a year or two is often self-fulfilling: If we believe that good teachers do not have to struggle, we feel incompetent—and deeply threatened—every time we try something new and difficult. These feelings of incompetence make the struggles that come inevitably with any serious effort to change teaching even more frightening and unpleasant. They reduce the likelihood that a teacher will continue to try to pull off difficult teaching. Since the sorts of things the teachers in the IMT group were trying to do are very difficult, requiring constant juggling of priorities and wrestling with intransigent dilemmas, support for the notion that good teaching involves struggle is a key issue for any group of teachers who are trying to teach in ways more like those urged in the NCTM *Standards*.

This is a key point in understanding what teachers need in order to pull off the kinds of teaching that reformers are urging: Most people recognize that teachers need support of many kinds in order to cope with the inevitable challenges and difficulties; what is less obvious is that they need support in embracing the idea that struggle is okay, that it does not call their competence into question.

It may not be an accident that, as the group validates *for Marsh* the value of "grappling" with teaching problems, she begins to see value *for her students* in struggling with difficult mathematics. Indeed, the connections she appears to be drawing between her discoveries about her students' knowledge, the necessity of being open to change, and the necessity of inherent satisfactions suggests that she enjoys her struggles and she believes her students enjoy theirs.

Conjectures

It is customary for researchers to close a report on a research project with a "conclusion" in which they explain what they think they have learned. Many of the lessons we are now

drawing from this study are so tentative that we feel more comfortable with a term we have borrowed from the classrooms of our teacher colleagues in the IMT group. When children in Karen Miller's or Laura Morrow's classroom observe a pattern in the results they have been getting as they worked alone or as a group on a problem, they often raise their hands to say, "I want to propose a conjecture." They then describe the pattern they see in the examples they have worked on and suggest how they think it may extend to other situations. Teacher and children then work together to frame the conjecture in words that everyone in the class understands. This done, children may spend time evaluating the conjecture, either by looking for examples and/or counter examples, or by trying to figure out why it might or might not be true. If the conjecture withstands this sort of examination—if children find many examples supporting the generalization and no exceptions, for instance, or if some find arguments for its validity that make sense to their classmates—the conjecture becomes, over time, an accepted tenet of this community of "mathematicians."

It is in the spirit of these learning communities that we offer some of our current thoughts. Our conversations and observations suggest some hypotheses, some ideas we intend to continue to explore. We hope that other teachers, researchers, teacher educators, and staff developers will join us in these deliberations.

Our conjectures are of three sorts: Conjectures about working teachers' uses of the M.A.T.H. materials; conjectures about the development of a group of experienced teachers who are supporting each other in the study of their mathematics teaching; conjectures about teacher learning.

Since six of the seven teachers joined the group because of their interest in learning more about approaches to math teaching that were different from those they had experienced as students, we feel unqualified to speculate about how teachers who are rigidly committed to traditional teaching might use the M.A.T.H. materials. We focus our conjectures on teachers who are already disposed to open the Pandora's box of questions about teaching and learning in mathematics.

Paralleling mathematics, where theorems are conjectures whose proof often requires the proof of a series of sub-conjectures (lemmas) first, we follow several of our main conjectures (■) with a set of supporting conjectures (•).

Conjectures concerning working teachers' uses of the M.A.T.H. materials:

■ Teachers will use the M.A.T.H. materials to start and sustain an inquiry into their own practice.

The data from this pilot study indicate that, over and over again, explorations of the M.A.T.H. materials prompted the teachers in the IMT group to raise questions about their schools and classrooms. Our observations suggest that the teachers were less likely to "stop the clock" in order to look at a student's notebook (though one teacher did use the materials to make a sustained inquiry into the thinking and learning of one student), than to connect what they were seeing in Ball's classroom to their own teaching, or to their own students' learning and behavior. The supporting conjectures below emerge from our observations of this group and our interviews with participants.

- Teachers who are actively trying to change their math teaching, to bring it into closer alignment with the NCTM *Standards* welcome the opportunity to watch videotapes of another teacher with similar goals and to discuss them with others who are engaged in a parallel struggle.

- Teachers who are trying to teach "against the grain" (Cochran-Smith, 1991) need and value conversation about this kind of teaching generally, and about specific issues related to curriculum and pedagogy. The M.A.T.H. materials are useful partly because of the ways in which they feed the conversation.

- These teachers welcome the opportunity to read another teacher's teaching journal after viewing the videotape of a lesson. Reading the journal raises new questions about teaching and learning and broadens the scope of their thinking as they analyze the lesson.

- If explicitly and routinely offered the opportunity to look at students' written work, teachers will examine, make thoughtful observations, and raise interesting questions about this work. They may be less likely, however, to seek out the children's written work than either videotapes of classes or documentation of the teacher's thinking.

- Teachers may use the M.A.T.H. materials in a wide variety of ways, including: direct appropriation of problems, direct appropriation of strategies for managing class discussions and teaching dilemmas, to plan a unit.

Conjectures about the development of a group of experienced teachers who are supporting each other in making changes in the way they think about the teaching and learning of mathematics.

- Teaching in non-standard, non-traditional ways—especially when this means changing the way one teaches—alters teachers' personal and professional relationships.

In order to understand how and why teachers change their practice, we need to think about the psychic rewards and the psychic costs of teaching. In a review of research on the cultures of teaching, Feiman-Nemser and Floden (1986) report that teachers put relationships with colleagues near the top of the list of personal professional satisfactions—higher, on average, than they put relationships with students. Our data suggest that when a teacher seeks to make dramatic changes in the way she teaches, she may endanger relationships with colleagues and increase the satisfaction she gets from her students. Many first person accounts of unconventional teaching lead to a similar hypothesis (e.g., Kohl, 1967; Herndon, 1965).

- It may threaten relationships with colleagues in their building, placing the teacher at risk of real isolation.
- It may increase the satisfactions a teacher gets from her work with students.
- Satisfactions offered by other professional relationships connected to the changes (for example, the IMT group) are related directly to the teachers' effort to question traditional math teaching.
- Traditionally, teachers have very little access to the idea that "good teaching" involves struggle. Participation in a group that legitimates struggle provides support for engaging in difficult and professionally risky practices.

Conjectures concerning teacher learning:

- The dichotomy between a "support group" and a "study group" is a false one.

The audiotapes of the IMT group provide considerable evidence that the teachers derived support from the group and from their interactions with the researchers. Our reading of the group's history indicates to us that this sense of support, this belief that they were part of a professional community that cared about their struggles and shared goals, emboldened teachers both to ask questions and to take risks in their classrooms. Because of its endemic uncertainties, all teaching poses questions. But acknowledging and embracing these questions takes courage; emotional support can, therefore, promote both intellectual inquiry and critical changes in teaching practice.

- For some teachers who are committed to making changes in their mathematics teaching of the sort recommended by the NCTM *Standards*, knowledge of mathematics is a significant issue and one that is not easily resolved.

As we have said, at least two of the teachers in the IMT group continued to feel uncomfortable with negative numbers through December. During the pilot, we saw no evidence that the activities we planned significantly improved this situation. We believe that we underestimated the strength of their aversion to this mathematical terrain. We believe it would be easy for others to make the same mistake. Our classroom observations and our conversations with the teachers since December however, have lead us to believe that the teachers themselves have found in their classrooms a more congenial path into the subject matter.

- Teaching mathematics in ways that involve students in discourse about significant mathematical issues, while both difficult and occasionally frightening for teachers who feel they know little mathematics, can help such teachers to deepen their understanding of the subject matter (see Featherstone, Smith, et.al.).

Many elementary school teachers know frighteningly little mathematics. The NCTM *Standards* argue that in order to facilitate changes in the teaching of school mathematics, we need to equip prospective teachers with a deeper understanding of the subject matter. But it isn't clear how to affect this change. To begin with, many

practicing elementary school teachers have neither the impulse nor the opportunity to take more college courses in mathematics. In addition, recent research (Ball, 1988; McDiarmid and Ball, 1988; Ball and McDiarmid, 1989) indicates that simply taking more mathematics courses won't necessarily give prospective teachers the *kinds* of knowledge and dispositions they need for teaching. There is not evidence that more of the same will make the difference. If such teachers wish to improve their mathematics teaching, what can they do? This was a burning question for three of the seven teachers in the IMT group, all of whom were attracted to the type of teaching they saw described in the NCTM *Standards* but felt poorly prepared in mathematics. Karen Miller's case suggests that some teachers may be able to learn some mathematics in the company of their students and that the impulse to serve these students better may provide them the motivation to do so. Two other teachers in the IMT group tell stories of deepening their understanding of an area of mathematics as they taught it. These three teachers travelled somewhat different roads into understanding, but their experiences (which we are now investigating further) suggest the importance of exploring this conjecture carefully.

- Watching videotape and discussing it with others can be a powerful learning experience for teachers.

- Conversations in which teachers speculate together about student thinking may influence teachers to teach more adventurously and to create space in their classrooms for long exploratory conversations about mathematics.

These conjectures and lemmas serve as a framework for our ongoing collaboration with the IMT group. They stand as an interim report of what we think we are learning and help us in setting the direction of our next steps. We hope they will also feed the ongoing conversation on teacher learning.

Notes

[1]A note on this apparent overlap and on the different perspectives of participants and researchers: although the researchers saw great overlap, one of the participants was considerably intimidated by the introductions and concluded that the other teachers were "way ahead of me." So intimidated and out of place did she feel, that she returned only with great reluctance.

[2]These teachers lived more than fifty miles from the university and were heavily involved in other professional development activities.

[3]All teacher and student names given are pseudonyms.

[4]One participant, who considers herself a potential mathematics teacher, taught social studies and language arts in a departmental middle school.

[5]A mini-computer is a teacher-made instructional tool created to assist learners in doing and understanding arithmetic operations with integers.

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