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ABSTRACT

A traditional view of mathematics learning suggests that basic fact knowledge provides a cornerstone for success in problem solving. But possibly, for some children, automated basic fact responses may be disadvantageous for their problem solving success. The purpose of this study was to investigate the relationship between basic fact knowledge and success in solving addition and subtraction word problems. The sample consisted of second graders (n=42) from an urban neighborhood school in a large metropolitan school district, 99% of whose students were from black American families. Results showed little or no correlation between students' basic fact and problem-solving performances. Analyses indicated two potentially interesting groups of students: (1) High Basic Fact/Low Problem Solving (n=6); and (2) Low Basic Fact/High Problem Solving (n=4). The results suggest that some beginning mathematics students are successful in solving various types of word problems without a strong fact background, and some beginners are unsuccessful in solving various types of word problems although they do have a strong basic fact background. This study supports a constructivist view of curriculum and suggests further investigation of the potential for damage to be done by curricula too focused on basic facts. Examples of word problem types and correlation and regression data are provided. (MKR)

Investigating the Role of Basic Facts in Early Mathematics Word Problem Solving

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A traditional view of mathematics learning suggests that basic fact knowledge provides a cornerstone for success in problem solving. As many adult users of mathematics examine their own problem solving processes, they may focus on the advantageous role that automated basic fact responses play for them in solving mathematical problems. But possibly, for some young learners of mathematics, this same automaticity may be disadvantageous for their problem solving success. The purpose of this paper is to investigate the relationship between young children's basic fact knowledge and their success in solving addition and subtraction word problems.

Background

The background literature for this topic comes from the combined research during the late 1970s and early 1980s by mathematics educators and cognitive psychologists who documented the informal mathematical concepts that young children develop before receiving formal instruction in school (Ginsburg, 1983). Specifically, in the area of addition and subtraction word problems, this literature indicated that most young children invent strategies for successfuly solving problems and that their invented strategies represent directly the structure, i.e., the action or setting, described in each problem (summary in Carpenter,

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1985). These invented structure-based strategies, that are both predictable and specific to addition and subtraction problem types, provide evidence that young children, before they have been to school, develop early powerful informal mathematical concepts.

However, additional literature from national mathematics assessments in the 1980s also documented a different picture: specifically, the contrast between young children's early insightful preinstructional structure-based strategies and those that older children exhibited after a few years of formal instruction. By the time children reach third grade, many appeared to use strategies that exhibited a search for and extraction of the basic fact within each word problem (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981). In essence, these older children, in their attempts to relate basic facts to the word problems, did not display the insights into problem structure that often are evident in children at younger ages.

The contrasting view of these two models of mathematics problem solvers, one who attends to and represents the structure of a word problem and the other who attempts to use a routinized response to solve a word problem, was captured by Briars and Larkin (1984) in their computer-generated program CHIPS. These authors designed CHIPS to solve the various word problem types by representing the problem structure. In their discussion, they suggested that CHIPS was successful because of its structure-based representational design whereas traditional school mathematics instructional was less successful because of its computational, or basic fact, focus.

The role played by routinized responses from whole number mathematics in the learning of decimal fractions by intermediate-aged students was investigated by Wearne and Hiebert (1988). Their results indicated that routinized responses from whole number skill practice may have been responsible for the older children's lack of reflection in developing concepts for decimal fractions. Specifically, they stated that, "Prior instruction that encouraged the routinization of syntactic rules seemed to interfere with, and prevented the adoption of, semantic analyses of the affected tasks" (p. 380).

The present study was designed to investigate the relationship between basic fact knowledge and word problem solving success in a sample of second grade students. The major focus of the study concerned whether the knowledge of basic facts facilitated or impeded students' success in solving addition and subtraction word problems. Of special interest were the performances of the following two groups of students: (a) High Basic Fact/Low Problem Solving--those who indicated strong knowledge of basic facts but poor success in solving word problems and, conversely, (b) Low Basic Fact/High Problem Solving--those who indicated weak knowledge of basic facts but good success in solving word problems.

Methods

In this study, basic fact knowledge and word problem solution success were measured for 42 second-graders. The study used a post-hoc analysis of data that had been collected for an instructional treatment study (Bebout, 1992).

Sample

The sample consisted of 42 second-graders from two classrooms in an urban neighborhood school in a large metropolitan school district. Demographic features of the school indicated that it was representative of central city neighborhood schools nationwide: 99% of the students were from Black American families, with 76% of the students from families designated as low-income and with a school mobility rate of 33%. Similar to standardized measures in other urban schools, the school ranked in the 36th percentile on national standardized mathematics achievement tests (Cincinnati Public Schools, 1989).

Instruments of Measure

Three instruments of measure were used. These included timed number facts tests, group word problem tests, and individual problem solving interviews. The data generated by these three instruments provided both quantitative and qualitative measures for each child. Quantitative measures were based on correct number fact responses and on correct solutions to the word problems. Qualitative measures were based on the students' use of specific solution strategies during the interviews.

Number Facts Tests. The timed number facts tests consisted 2 five-minute tests of 49 addition and 49 subtraction number facts, respectively. A student's basic fact score was determined by the number of correct responses.

Group Word Problem Tests. The group word problem tests consisted of ten types of addition and subtraction word problems (examples in Table 1). For each problem, the students were asked to write a number sentence and to solve the problem. On this measure, a student's score was determined by the number of correct solutions on the seven more difficult problem types, specifically, the Change 3, 4, 5, 6, Combine 2, Equalize, and Compare problems.

(insert Table 1)

Individual Interviews. The individual interviews consisted of nine types of addition and subtraction word problems; for time purposes, the Combine 1 problem was not included. During the interview, the students were asked to model with blocks and to solve each problem. As in the Group Word Problem Test, a student's score was determined by the number of correct solutions on the seven more difficult word problem types.

Results

Correlation and regression analyses were used to quantify the relationships between the students' basic fact and problem solving performances. Correlation data are displayed in Tables 2a and 2b and regression data in Tables 3a and 3b and Figures 1a and 1b. Correlation analyses indicated a weak correlation of .32 ($p = .0360$) between each student's basic fact and interview problem solving performances and essentially no linear relationship, correlation of .14 ($p = .3608$), between the basic fact and group word problem performances. Regression analyses indicated two potentially interesting clusters of students who were either high basic fact/low problem solving or low basic fact/high problem solving.

(Insert Tables 2a, 2B, 3a, & 3b; Figures 1a & 1b)

High Basic Fact/Low Problem Solving

A group of six students indicated strong knowledge of basic facts (a score of 90 or more correct responses out of 98) but little success in solving the word problems (a score of 0 or 1 correct responses out of seven on either of the problem solving tests). The basic fact scores and problem solving performances are displayed in Table 4 for each student in this cluster. For the problem solving interviews, the number of appropriate strategies and correct solutions are shown; for the group word problem test, the number of correct number sentences and correct solutions are shown.

(insert Table 4)

During the problem solving interviews, three of these students used no appropriate strategies when trying to solve the seven word problem types. Their attempts included use of three inappropriate strategies predominatly: (a) extracting and adding the numbers stated in the problem; (b) repeating a number stated in the problem as the solution, or (c) making no attempt to solve the problem.

On the group word problem test, these six students wrote number sentences in which, for the most part, they had extracted and added the numbers stated in the problem. In a variation, they extracted and subtracted the problem numbers for the Change 4, Change 6, and Combine 2 problems; this strategy was successful for the Change 4 and Combine 2 problems but unsuccessful for the Change 6 problem.

Low Basic Fact/High Problem Solving

Four students in particular indicated a weak knowledge of basic facts (a score of 60 or less correct responses out of 98) but good success in solving the word problems (a score of 5 or more correct responses out of seven problems on either of the problem solving tests). The basic fact scores and problem solving performances are displayed in Table 4 for each student in this cluster. In the same manner as the previous cluster, for the problem solving interviews, the number of appropriate strategies and correct solutions are shown; for the group word problem test, the number of correct number sentences and correct solutions are shown.

One of these four students had a strong performance in the problem solving interview by using appropriate strategies on all but one of the problems. The three other students had their strong performances on the group word problem tests; one student in particular excelled in writing complete number sentences with correct solutions, while the remaining two students did very well in solving the problems.

Discussion

The weak relationships demonstrated in this study between second-graders' knowledge of basic facts and their successes in solving addition and subtraction word problems support the arguments that (a) some beginning mathematics students are successful in solving various types of word problems without a strong basic fact backgound and (b) some beginners are unsuccessful in solving various types of word problems although they do have a strong basic fact backgound. Although the results of the study do not determine whether the knowledge of basic facts facilitates or impedes the success of all young students in solving addition and subtraction word problems, the results do implicate two important clusters of students: those who succeed in problem solving without using basic fact responses and those who may be impeded in problem solving because of their routinized rote fact responses. In this study, these two clusters of outliers comprised nearly 25% of the students in this study, beginning mathematics students in two typical urban classrooms.

In addition to its significance for individual students and their present/future success in mathematics problem solving, the results of this study have potential significance for early mathematics curriculum issues. This study supports a constructivist view of curriculum as classroom instruction that enables young mathematics learners to construct/invent their insightful and potentially powerful mathematics problem solving strategies. The study also suggests substantial reasons for concern about the potential damage that basic fact curricula may inflict. The damage may be inflicted not only on the small cluster of students identified in this study as High Basic Fact/Low Problem Solving folks but also on additional young students; these additional populations include those students who, because of their low scores on basic fact tests, may be labeled in the minds of their teachers, publicly "charted" to their peers, and ultimately perceived by themselves to be low achievers in mathematics.

The children who participated in this study, young urban Black American students, are a crucial population to reach with mathematics for their future educational, financial, political, and social power. At the current time, most of the students in this sample, like their sisters and brothers in similar school settings, are receiving traditional Chapter 1 instructional materials that focus on basic skill acquisition rather than on solving mathematics problems. The conventional wisdom perspective, that knowledge of basic facts is the cornerstone for successful problem solving, is reflected not only in classroom settings in the forms of curriculum materials, assessment instruments, and activities, but also beyond the classroom level in the forms of policies from administrators, directives from parent groups, musings of higher mathematicians, and positions of certain schools of psychology. Results of this study suggest strongly that the role of basic facts be further investigated as a potentially hazardous cornerstone for young children's mathematical problem solving.

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Table 1
Examples of Addition & Subtraction Word Problem Types

Type	Text
Change 3	Jane had 8 puzzles She got more puzzles for her birthday Then she had 13 puzzles How many did she get for her birthday?
Change 4	Ruth had 15 pennies She gave some of them to her friend Then she had 7 pennies How many did she give to her friend?
Change 5	Jackie had a box of crayons She put 9 more crayons in the box Then she had 14 crayons How many crayons were in the box at the start?
Change 6	Some kids were in the swimming pool 5 of them had to go home Then there were only 7 kids in the pool How many kids were in the pool at the start?
Combine 2	There were 15 balloons 9 balloons were green and the rest were blue How many balloons were blue?
Equalize	Sue found 6 peanuts Her brother found 13 peanuts How many more peanuts does Sue have to find to catch up with her brother?
Compare	Mary had 8 cupcakes Louis had 14 cupcakes How many more does Louis have than Mary?

Table 2a.
Correlation for Basic Fact Scores x Interviews

Corr. Coeff. X_1: BASIC FACTS Y_1: SOL.INT

Count:	Covariance:	Correlation.	R-squared:
42	15.91	.32	.11

Table 2b.
Correlation for Basic Fact Scores x Group Problem Solving Tests

Corr. Coeff. X_1: BASIC FACTS Y_2: SOL.PRE

Count:	Covariance:	Correlation.	R-squared:
42	7 56	14	02

Figure 1a.
Regression Display for Basic Fact Scores x Interviews

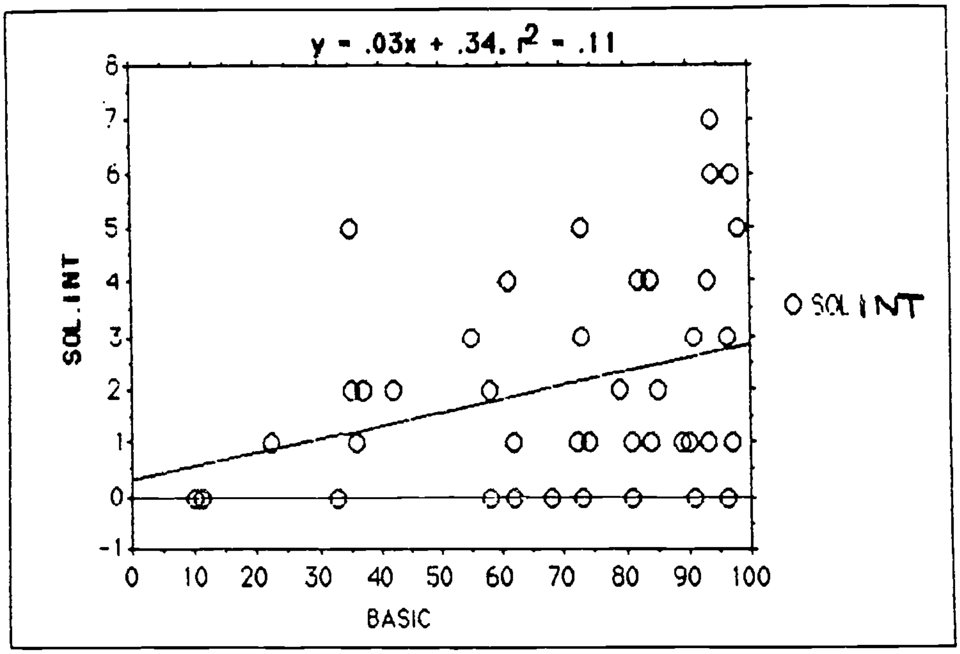

Table 3a.
Regression Data for Basic Fact Scores x Interviews

Simple Regression X_1: BASIC FACTS Y_1: SOL.INT

Count:	R:	R-squared:	Adj. R-squared:	RMS Residual.
42	.32	.11	.06	1.87

Analysis of Variance Table

Source	DF:	Sum Squares:	Mean Square:	F-test:
REGRESSION	1	16.38	16.38	4.71
RESIDUAL	40	139.24	3.48	p = .036
TOTAL	41	155.62		

Beta Coefficient Table

Variable:	Coefficient:	Std. Err.:	Std. Coeff.:	t-Value:	Probability:
INTERCEPT	.34				
SLOPE	.03	.01	.32	2.17	.036

Confidence Intervals Table

Variable:	95% Lower:	95% Upper:	90% Lower:	90% Upper:
MEAN (X,Y)	1.51	2.68	1.61	2.58
SLOPE	1.72E-3	05	.01	.04

Figure 1b.
Regression Display for Basic Fact Scores x Group Problem Solving Tests

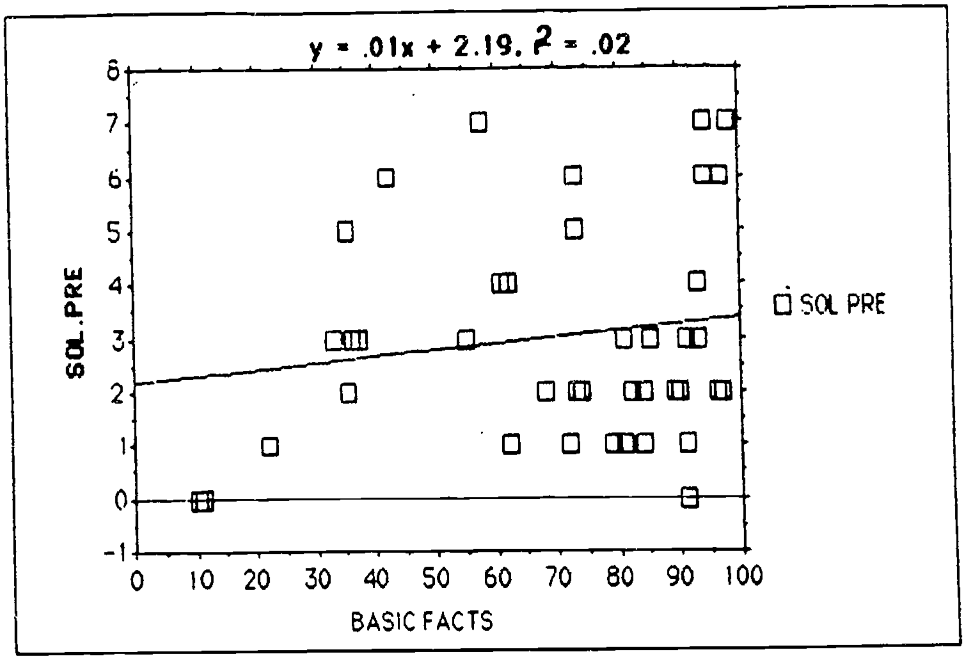

Table 3b.
Regression Data for Basic Fact Scores x Group Problem Solving Tests

Simple Regression X1: BASIC FACTS Y2: SOL.PRE

Count:	R:	R-squared:	Adj. R-squared:	RMS Residual:
42	.14	.02	-3.56E-3	2.08

Analysis of Variance Table

Source	DF:	Sum Squares:	Mean Square:	F-test:
REGRESSION	1	3.7	3.7	.85
RESIDUAL	40	173.27	4.33	p = .3608
TOTAL	41	176.98		

Beta Coefficient Table

Variable:	Coefficient:	Std. Err.:	Std. Coeff.:	t-Value:	Probability:
INTERCEPT	2.19				
SLOPE	.01	.01	.14	.92	.3608

Confidence Intervals Table

Variable:	95% Lower:	95% Upper:	90% Lower:	90% Upper:
MEAN (X,Y)	2.37	3.67	2.48	3.56
SLOPE	-.01	.04	-.01	.03

Table 4.
Selected Student Scores on Basic Facts and Problem Solving.

Student ID	Basic Fact Score	Problem Solving			
		Interview		Group Test	
		strategy	solution	sentence	solution
High Basic Facts/Low Problem Solving					
BF 1	97	0	1	2	2
BF 2	96	0	0	2	2
BF 3	93	3	1	1	3
BF 4	91	6	3	1	1
BF 5	91	0	0	0	0
Low Basic Facts/High Problem Solving					
PS 1	35	6	5	1	2
PS 2	35	3	2	4	5
PS 3	42	2	2	2	6
PS 4	58	0	0	7	7

Table 4.
Selected Student Scores on Basic Facts and Problem Solving.

Student ID	Basic Fact Score	Problem Solving			
		Interview		Group Test	
		strategy	solution	sentence	solution
High Basic Facts/Low Problem Solving					
BF 1	97	0	1	2	2
BF 2	96	0	0	2	2
BF 3	93	3	1	1	3
BF 4	91	6	3	1	1
BF 5	91	0	0	0	0
Low Basic Facts/High Problem Solving					
PS 1	35	6	5	1	2
PS 2	35	3	2	4	5
PS 3	42	2	2	2	6
PS 4	58	0	0	7	7