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ABSTRACT

A factor analysis study provided empirical evidence of the validity of the modified Mathematics Self-Efficacy Scale (MSES) and its three subscales: mathematics problems self-efficacy, mathematics tasks self-efficacy, and college courses self-efficacy. The MSES was administered to 522 undergraduate students from three state universities and internal consistencies were calculated for each of the three scales and the composite MSES using Cronbach's alpha. A principal component analysis led to the interpretation of a five-factor solution and a higher-order analysis provided evidence of a unitary structure to the MSES. Although the general structure of the MSES was validated, the study indicated specific constructs and inadequacies that need to be carefully analyzed by researchers using MSES. The college courses self-efficacy scale measured two separate constructs that have significantly different implications for differing substantive questions. The mathematics problems self-efficacy scale has limitations that need to be evaluated. Self-efficacy is a context-specific construct, and researchers who fail to evaluate the relationship between the performance task and the efficacy measure are likely to be frustrated by confounded relationships, ambiguous findings, and uninterpretable results. Contains 18 references. (Author/PDD)

The Mathematics Self-Efficacy Scale: A Validation Study

Thomas E. Langenfeld and Frank Pajares

University of Florida

Paper Presented to the annual conference of the American Education Research Association, April 1993, Atlanta, GA

RUNNING HEAD: Mathematics self-efficacy scale

Abstract

A factor analytic study provided empirical evidence of the validity of the modified Mathematics Self-efficacy Scale (MSES) and its three scales: mathematics problems self-efficacy, mathematics tasks self-efficacy, and college courses self-efficacy. A principal component analysis led to the interpretation of a five-factor solution and a higher-order analysis provided evidence of a unitary structure to the MSES. Although the general structure of the MSES was validated, the study indicated specific constructs and inadequacies that need to be carefully analyzed by researchers using the MSES. The college courses self-efficacy scale measured two separate constructs that have significantly different implications for differing substantive questions. The mathematics problems self-efficacy scale contained limitations that need to be evaluated prior to its use. In using the MSES, we caution researchers to carefully consider the nature of the performance task when considering an appropriate efficacy measure. Self-efficacy is a context-specific construct, and researchers who fail to carefully evaluate the relationship between the performance task and the efficacy measure are likely to be frustrated by confounded relationships, ambiguous findings, and uninterpretable results.

Social cognitive theory suggests that individuals' self-efficacy beliefs, "judgments of their capability to organize and execute courses of action required to attain designated types of performances" (Bandura, 1986, p. 391), are powerful predictors of behavior. Self-efficacy perceptions influence the choices people make, the effort they invest, and how long they persevere in the face of adversity (Bandura, 1977, 1982, 1986). The influence of self-efficacy beliefs has been investigated in areas as diverse as medicine, athletics, business, and psychology. In education, researchers generally have focused on the relationship between self-efficacy and academic performances or choices, and findings have had important implications for practitioners and school and career counselors (see Lent & Hackett, 1987).

Because of its special place in curricula and extensive prior research on variables such as gender differences and anxiety, mathematics has become a focus of study for self-efficacy researchers. Mathematics self-efficacy has been defined by Hackett and Betz (1989) as "a situational or problem-specific assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular [mathematical] task or problem" (p. 262). Researchers have investigated the relationship between math self-efficacy and a number of related variables, including career decisions, math anxiety, math self-concept, and mathematics performance or achievement (see Lent & Hackett, 1987; Pajares, 1993, for reviews).

Research has been hampered, however, by a lack of reliable and valid instruments with which to assess mathematics self-efficacy, and researchers typically have constructed their own instruments with varying degrees of success. Often, mathematics self-efficacy has been measured simply with one or two global and decontextualized items, something that Bandura (1986) warned is both inappropriate and of limited value. The Mathematics Self-Efficacy Scale (MSES) was created by Betz and Hackett (1982, 1983) to assess, with a greater degree of specificity, college students' confidence to solve mathematics problems, accomplish a series of mathematics-related tasks, and succeed in mathematics-related courses.

Since its creation, the MSES or one of its three scales have been used in a number of mathematics studies (e.g., Forbes, 1985; Hackett, 1985; Hackett & Betz, 1989; Lapan, Boggs, & Morrill, 1989; Lent, Lopez, & Bieschke, 1991; Randhawa, Beamer, & Lundberg, 1993; Pajares & Miller, 1993). Findings from these studies have offered valuable insights and have strengthened Bandura's (1986) arguments on the role of self-efficacy. For example, researchers have reported that mathematics self-efficacy is a stronger predictor of mathematics-related majors than are math anxiety or prior performance (Betz & Hackett, 1983) and that mathematics problem-solving self-efficacy is both a stronger predictor of related performance than either math anxiety, math self-concept, or prior mathematics experience and, as social cognitive theory predicts, mediates the effect of these variables on performance (Pajares & Miller, 1993). These findings, and their implications, must be viewed with caution, however, in light of the fact that neither the MSES nor any of its three scales has undergone validation analyses beyond the usual reporting of Cronbach's alpha.

The purpose of our study was to provide an analysis of the factoral composition and psychometric properties of the MSES. To this end, we administered the MSES and a mathematics performance instrument to a sample of college students in an attempt to: (a) provide normative data and internal consistency estimates for the total scale and each individual scale, (b) find factor analytic support for the MSES, (c) compare MSES scores to a measure of mathematics performance, and (d) make recommendations regarding possible revision of the MSES. Clearly, if the MSES will continue to be used in studies of mathematics self-efficacy, it is important that its psychometric properties be more clearly understood and subjected to careful analysis.

Method

Subjects

Participants in the study consisted of 522 undergraduate students from three state universities in Florida, Texas, and North Carolina. Graduate students administered the instrument to students primarily within College of Education classes. Over 30 separate majors were represented. The sample included 349 women and 173 men.

Instrument

The MSES contains 52 items within three scales: solution of mathematics problems, completion of mathematics tasks used in everyday life, and performance in college courses requiring knowledge and mastery of mathematics. Each of the 52 MSES items is rated on a 10-point Likert scale, with a score of 9 indicating complete confidence and 0 indicating no confidence.

We made three modifications to the MSES for this study. The 10-point Likert rating scale was reduced to a 5-point rating scale to improve the ease of administration and scoring. Comparable reliability figures using the 5-point rating scale were found in a pilot study (Langenfeld & Pajares, 1993). From the pilot study, we also concluded that a math tasks self-efficacy item asking subjects to rate their confidence to use a slide rule was outdated and inappropriate. Consequently, we deleted the item, and we substituted onw requesting subjects to rate their confidence to use a scientific calculator.

The math problems scale of the MSES was adapted from the Mathematics Confidence Scale (MCS) created by Dowling (1978), who utilized preliminary forms in constructing her final two forms of mathematics self-efficacy and performance. Betz and Hackett (1983) selected one of the preliminary forms rather than Dowling's final instrument. Because the final version possessed stronger psychometric qualities, we replaced Betz and Hackett's scale with Dowling's problems scale. This alteration enhances both the validity of the MSES and the integrity of Dowling's original intention. Following the completion of the MSES, participants were asked to solve the 18 items of the Mathematics Problems Performance Scale (MPPS). Dowling (1978) created these items in multiple choice format to correspond to the 18 items of the math problems self-efficacy scale. That is, they were the identical items contained in the efficacy instrument.

Data Analysis

Internal consistency estimates were calculated for each of the three scales and for the composite MSES using Cronbach's alpha. The internal consistency of the 18 mathematics problems was estimated through KR 20. The 52-item MSES was factor analyzed using a principal component common factor solution. Communalities were estimated through an iterative procedure using a variable's largest correlation without regard to sign with any other variable in the matrix and iterating twice (Gorsuch, 1985). The number of factors was determined using Cattell's scree test (1966) and the requirements of simple structure (Gorsuch, 1985). Due to the anticipated intercorrelation of factors, a promax rotation was utilized. Factor scales were established based upon factor patterns assigning an item to a factor scale if its loading on the factor was greater than or equal to .35. The extracted factors were then analyzed and interpreted, in part, comparing the relationship of each of the factors to students' performance on the 18 mathematics problems. The intercorrelation matrix of the extracted factors was analyzed to determine possible higher-order factors using a principal component model. In higher-order factor analysis, communality estimates affect the solution to a greater extent than in first-order factor analysis. Consequently, a more conservative procedure was used to estimate the communalities. The squared multiple correlations were used for the initial estimation with two iterations. A factor loading greater than or equal to .35 was considered sufficient to assign a first-order factor to a higher-order factor.

Results

The internal consistency of the three scales was estimated to be .90 for mathematics problems self-efficacy, .91 for mathematics tasks self-efficacy, and .91 for mathematics courses self-efficacy. Cronbach's alpha for the total MSES was .95. The internal consistency of the 18 mathematics problems was estimated to be .74. These reliability estimates are consistent with previous estimates found for the MSES (Betz & Hackett, 1983; Langenfeld & Pajares, 1993).

Analysis of the eigenvalues, the scree plot, and the principles of simple structure indicated a five factor solution. The factors were rotated and their intercorrelations were calculated. The factor intercorrelations are reported in Table 1. The five factor promax solution accounts for 85% of the cumulative variance.

Insert Table 1 here

The first factor, containing 17 of the 18 mathematics tasks self-efficacy items, accounted for 56% of the total variance. Composition of the subscale based on Factor I is reported in Table 2. Items associated with this factor deal with specific mathematics-related tasks found in everyday life and often require examinees to evaluate their competence against a stated standard (e.g., balance your checkbook without a mistake). The one item that failed to load on Factor I was the newly created item asking subjects to rate their ability to use a scientific calculator.

Insert Table 2 here

The second factor, containing 15 of the 18 mathematics problems self-efficacy items, accounted for 11% of the total variance. Composition of the subscale based on Factor II is reported in Table 3. Items associated with Factor II reflect three components of mathematics (arithmetic, geometry, and algebra) and in varying contexts. Item 13, an algebraic story problem, double loaded on Factor I and Factor II.

Insert Table 3 here

The third factor, containing 7 of the 16 mathematics courses self-efficacy items and the task-related item dealing with use of a scientific calculator, accounted for 9% of the variance. Composition of the subscale based on Factor III is reported in Table 4. Items associated with

Factor III, other than the calculator item, dealt with examinees' confidence to perform at a "B" or better level in college mathematics courses. Item 15, confidence in advanced calculus, double loaded on Factor III and Factor V.

Insert Table 4 here

The fourth factor, containing 7 of the 16 mathematics courses self-efficacy items, accounted for 5% of the total variance. Composition of the subscale based on Factor IV is reported in Table 5. All of these items concerned examinees' confidence to perform satisfactorily in nonmathematics courses that required applications of basic math skills (e.g., business administration and zoology).

Insert Table 5 here

The fifth factor, containing two of the mathematics problems self-efficacy items and one courses self-efficacy item, accounted for 4% of the total variance. Composition of the subscale based on Factor V is reported in Table 6. The two problems self-efficacy items tested geometry, although the two problems were cognitively different. Item 16 measured students' confidence to solve a geometric story problem.

Item 18 required students to recall a specific geometric term and therefore measures memory skills more than mathematical ability. Both items require specific information and are dependent on an individual's recent study of geometry. The mathematics courses self-efficacy item, advanced calculus, double loaded with Factor III.

Insert Table 6 here

Three of the 52 items failed to load on any of the five factors. These were item 9 of the problems self-efficacy scale, and items 3 and 11 of the courses self-efficacy scale. Item 9 of the problems scale was an algebraic problem with two unknown variables, making it conceptually more difficult than the other problems on the scale. Items 3 and 11 of the courses scale, confidence to perform satisfactorily in statistics and accounting, were marginally related to both Factor III and Factor IV. The loadings for statistics on Factor III and Factor IV were .32 and .22, respectively, and the loadings for accounting on Factor III and Factor IV were .30 and .29, respectively.

Reliability estimates for each of the five factors reflected the reliabilities of the three scales for factors I, II, and III; however, they were considerably lower for factors IV and V. Cronbach's alpha for each of these were as follows: Factor I - .91, Factor II - .90, Factor III - .91, Factor IV - .82, and Factor V - .56.

The higher-order factor analysis suggested a single underlying factor represented in the MSES. All of the factors substantially loaded on the single factor as follows: Factor I - .71, Factor II - .63, Factor III - .57, Factor IV - .60, and Factor V - .61. These findings confirm the presence of an underlying construct measured by the full 52-item scale.

The correlations of the five factors and examinees' score on the 18-item MPPS indicated a relationship between mathematics self-efficacy and performance, but more significantly, they corroborated Bandura's belief in the need for self-efficacy instruments to be directly related to the criterial task. Students demonstrated a stronger relationship between their responses that loaded on Factor II, math problems self-efficacy, and the MPPS. This illustrated the contextual nature of self-efficacy and the need for instrument specificity. The correlations between the five factors and the MPPS were as follows: Factor I - .38, Factor II - .62, Factor III - .52, Factor IV - .33, and Factor V - .41.

Discussion

The results of our study support the general proposal that the MSES is a multidimensional measure of mathematics self-efficacy with reliable and valid scales. The results also suggest direction for revision to strengthen each of the three scales. Although one higher-order factor was identified, for self-efficacy research purposes, the constructs represented by the first-order factors provide valid measures consistent with social cognitive theory.

The math tasks self-efficacy scale emerged as Factor I with 17 of 18 items clearly loading on this factor. The failure of item 5 to load with the other items in the scale may be attributed to its lack of specificity. Careful revision of the item asking students to rate their confidence to use a scientific calculator to perform a specific mathematical task (e.g., calculate the cosine of 50) would make the item more consistent with the other items in the scale.

Although 15 of 18 items loaded on the second factor representing math problems self-efficacy, we do not endorse simply dropping the three problematic items. The reduction of items in the problems self-efficacy scale and, subsequently, the MPPS could create difficulties for researchers interested in analyzing relationships in this domain. Because Dowling (1978) developed two alternate forms of the MCS, additional items related to the same mathematical areas could be identified and substituted. The items in Table 7 compare favorably with items 9, 16, 18 of the math problems self-efficacy scale.

Insert Table 7 here

Our analysis demonstrates that the courses self-efficacy scale consists of two factors, one that contains mathematics courses and another that contains nonmathematics courses requiring knowledge and mastery of mathematics. To understand this phenomenon, it is instructive to recall that Betz and Hackett's (1983) initial investigations of the role of self-efficacy involved the influence of mathematics self-efficacy expectations on the selection of science-based college majors. That focus was later broadened to include mathematics-related majors. For this reason, the inclusion of science and mathematics-related courses was deemed particularly appropriate. In doing so, however, it seems clear that two types of courses were included in the scale, and this is made evident by the factor analysis. In addition, the courses self-efficacy scale asks students to provide a judgment of their confidence to earn a B or better in a series of college courses. They are then presented with course titles such as Algebra I, Algebra II, and Geometry on the one hand and Biochemistry, Philosophy, and Physiology on the other. It is easy to see how a student's mind might very well harken images of high school coursework when confronted with the courses on Factor III and of college courses when looking at those from Factor IV.

The clear implication of these results is that researchers are well advised to consider the nature of the performance task when considering an appropriate efficacy measure. Bandura (1986) warned that judgments of self-efficacy are task-specific and that different ways of assessing confidence will differently correspond to the assessed performance. For this reason, measures of self-efficacy should be tailored to the criterial task being assessed and the domain of functioning being analyzed. This particular warning has often gone unheeded in educational research, however, where assessments of self-efficacy have often borne little resemblance to the criterial task with which they have been compared. For example, it is not especially useful to compare self-efficacy judgments regarding accomplishing mathematics-related tasks or successfully completing mathematics-related courses with a performance measure that requires the solving of mathematics problems. The mismeasurement of self-efficacy is a recurring theme in educational research, often producing poorly defined constructs, confounded relationships, ambiguous findings, and uninterpretable results.

It is with this warning in mind that we must consider what value any decontextualized instrument offers researchers who are interested in investigating the role of such a context-specific construct as self-efficacy. The problems self-efficacy scale of the MSES provides a measure of students' judgments of their ability to solve those particular 18 problems. Its value is therefore limited to its ability to predict performance of the problems themselves. A more generalized measure of mathematics problems self-efficacy can be obtained only by assessing students' judgments of their capability to solve problems included in a broader, more reliable and valid performance measure -- for example, a norm-referenced measure with strong psychometric properties, high external validity, and numerous alternate forms.

Along these same lines, students' judgments of their capability to succeed in college mathematics courses should prove more predictive of their choice of mathematics majors than should judgments of their capability to succeed in courses such as biochemistry. This latter judgment, in turn, is more instructive if we wish to assess the role of self-efficacy on the choice of science-related courses. Neither judgment may prove particularly useful in understanding a myriad of mathematics-related tasks. Surely there are more than a few biochemists and mathematicians who are profoundly baffled by the intricacies of the forms provided by the Internal Revenue Service.

It is not altogether easy, then, to see what value the full-scale score provided by the MSES may have. Such a score decontextualizes self-efficacy and transforms it into a personality trait -- precisely what Bandura (1986) warned must not be done (pp. 5-12). For this reason, findings such as those reported by Randhawa, Beamer, and Lundberg (1993) must be interpreted with caution and skepticism. If, as Bandura argued, self-efficacy assessment must conform to the criterial task, what outcome measure can be compared to a global score comprised of confidence to succeed in mathematics courses, complete math-related tasks, and solve specific mathematics problems?

Understanding its components, however, is both useful and informative. Its four factors suggest that different judgments are being tapped, and each may be used depending on the criterial task. Bandura wrote (1977) that, because the definition of self-efficacy is quite straightforward, efficacy questionnaires in academic areas are adequate operational measures of the "conviction that one ca[illegible] successfully execute the behavior required to produce the outcomes" (p.93). If the MSES is to continue to be used as such a measure, social cognitive theory and mathematics researchers are best served by understanding not only the different components of the instrument but the circumstances under which the use of each of them is most appropriate and useful.

Table 1

Intercorrelations of Factors

Factor	I	II	III	IV	V
I	1.00				
II	.521	1.00			
III	.330	.435	1.00		
IV	.485	.285	.370	1.00	
V	.452	.370	.365	.423	1.00

Table 2

Factor I: Mathematics Tasks Self-Efficacy Loadings

Item	Abbreviated Statement	Loading
TSE 1	Add two ... large numbers ... in your head.	.42
TSE 2	Determine ... sales tax ...	.70
TSE 3	Figure out how much material ...	.44
TSE 4	Determin how much interest ...	.63
TSE 6	Compute ... car's gas mileage.	.46
TSE 7	Calculate recipe quantities ...	.48
TSE 8	Balance your checkbook ...	.49
TSE 9	Understand how much interest ...	.64
TSE 10	Figure out how long it will take to travel ...	.50
TSE 11	Set up a monthly budget	.59
TSE 12	Compute your income taxes ...	.53
TSE 13	Understand a graph ...	.48
TSE 14	Figure out how much you would save ...	.68
TSE 15	Estimate your grocery bill ...	.62
TSE 16	Figure out which of two ... jobs is better ...	.70
TSE 17	Figure out the tip ...	.71
TSE 18	Figure out how much lumber ...	.43

Table 3

Factor II: Mathematics Problems Self-Efficacy Loadings

Item	Abbreviated Statement	Loading
PSE 1	In a certain triangle .... What is the sum ..?	.56
PSE 2	About how many times larger ..?	.61
PSE 3	.... Find the largest number.	.41
PSE 4	.... Determine the position of the points...	.58
PSE 5	... find x when y = 10.	.64
PSE 6	.... Which decimal would most closely..?	.65
PSE 7	If P = M + N, then ..?	.59
PSE 8	The hands of a clock ...	.43
PSE 10	How far apart are two towns ..?	.50
PSE 11	.... how much change should he receive?	.41
PSE 12	.... According to this formula, how much ..?	.38
PSE 13	Mary's average .... What is the highest possible..?	.35
PSE 14	3 4/5 - 1/2 =	.64
PSE 15	In an auditorium, ... Multiply (x + 1) (y + 1).	.48
PSE 17	Set up the problem ...	.52

Table 4

Factor III: Mathematics Courses Self-Efficacy Loadings

Item	Abbreviated Statement	Loading
CSE 1	Basic college math	.42
CSE 5	Calculus	.61
CSE 7	Algebra II	.85
CSE 9	Geometry	.46
CSE 13	Algebra I	.74
CSE 14	Trigonometry	.59
CSE 15	Advanced Calculus	.52
TSE 5	Use of a ... calculator	.39

Table 5

Factor IV: Nonmathematics Courses Requiring Math-Related Skills Self-Efficacy

Item	Statement	Loading
CSE 2	Economics	.49
CSE 4	Physiology	.56
CSE 6	Business administration	.45
CSE 8	Philosophy	.73
CSE 10	Computer science	.37
CSE 12	Zoology	.70
CSE 16	Biochemistry	.49

Table 6

Factor V: Course-Specific Mathematical Skills Loadings

Item	Abbreviated Statement	Loading
PSE 16	A ferris wheel .... Find the measure in degrees ...	.55
PSE 18	Two circles in the same plane ... are called ____	.45
CSE 15	Advanced Calculus	.54

Table 7

Recommended Items for the Mathematics Problems Self-Efficacy Scale

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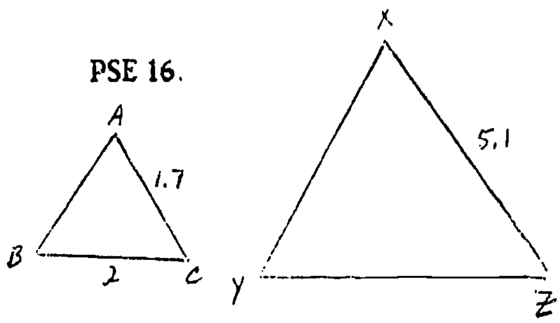
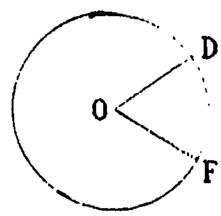
PSE 5. Multiply (1 + x) (1 + y).

PSE 16.



The two triangles shown at the left are similar. Thus, the corresponding sides are proportional, and AC/BC = XZ/YZ. If AC = 1.7, BC = 2, and XZ = 5.1, find YZ.

PSE 18. If the circumference is 24, and the length of arc DF is 4, what is the measure in degrees of central angle DOF?



(Dowling, 1978, p. 246).

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Thomas E. Langenfeld and Frank Pajares

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Participants in the study consisted of 522 undergraduate students from three state universities in Florida, Texas, and North Carolina. Graduate students administered the instrument to students primarily within College of Education classes. Over 30 separate majors were represented. The sample included 349 women and 173 men.

Instrument

The MSES contains 52 items within three scales: solution of mathematics problems, completion of mathematics tasks used in everyday life, and performance in college courses requiring knowledge and mastery of mathematics. Each of the 52 MSES items is rated on a 10-point Likert scale, with a score of 9 indicating complete confidence and 0 indicating no confidence.

We made three modifications to the MSES for this study. The 10-point Likert rating scale was reduced to a 5-point rating scal. to improve the ease of administration and scoring. Comparable reliability figures using the 5-point rating scale were found in a pilot study (Langenfeld & Pajares, 1993). From the pilot study, we also concluded that a math tasks self-efficacy item asking subjects to rate their confidence to use a slide rule was outdated and inappropriate. Consequently, we deleted the item, and we substituted onw requesting subjects to rate their confidence to use a scientific calculator.

The math problems scale of the MSES was adapted from the Mathematics Confidence Scale (MCS) created by Dowling (1978), who utilized preliminary forms in constructing her final two forms of mathematics self-efficacy and performance. Betz and Hackett (1983) selected one of the preliminary forms rather than Dowling's final instrument. Because the final version possessed stronger psychometric qualities, we replaced Betz and Hackett's scale with Dowling's problems scale. This alteration enhances both the validity of the MSES and the integrity of Dowling's original intention. Following the completion of the MSES, participants were asked to solve the 18 items of the Mathematics Problems Performance Scale (MPPS). Dowling (1978) created these items in multiple choice format to correspond to the 18 items of the math problems self-efficacy scale. That is, they were the identical items contained in the efficacy instrument.

Data Analysis

Internal consistency estimates were calculated for each of the three scales and for the composite MSES using Cronbach's alpha. The internal consistency of the 18 mathematics problems was estimated through KR 20. The 52-item MSES was factor analyzed using a principal component common factor solution. Communalities were estimated through an iterative procedure using a variable's largest correlation without regard to sign with any other variable in the matrix and iterating twice (Gorsuch, 1983). The number of factors was determined using Cattell's scree test (1966) and the requirements of simple structure (Gorsuch, 1983). Due to the anticipated intercorrelation of factors, a promax rotation was utilized. Factor scales were established based upon factor patterns assigning an item to a factor scale if its loading on the factor was greater than or equal to .35. The extracted factors were then analyzed and interpreted, in part, comparing the relationship of each of the factors to students' performance on the 18 mathematics problems. The intercorrelation matrix of the extracted factors was analyzed to determine possible higher-order factors using a principal component model. In higher-order factor analysis, communality estimates affect the solution to a greater extent than in first-order factor analysis. Consequently, a more conservative procedure was used to estimate the communalities. The squared multiple correlations were used for the initial estimation with two iterations. A factor loading greater than or equal to .35 was considered sufficient to assign a first-order factor to a higher-order factor.

Results

The internal consistency of the three scales was estimated to be .90 for mathematics problems self-efficacy, .91 for mathematics tasks self-efficacy, and .91 for mathematics courses self-efficacy. Cronbach's alpha for the total MSES was .95. The internal consistency of the 18 mathematics problems was estimated to be .74. These reliability estimates are consistent with previous estimates found for the MSES (Betz & Hackett, 1983; Langenfeld & Pajares, 1993).

Analysis of the eigenvalues, the scree plot, and the principles of simple structure indicated a five factor solution. The factors were rotated and their intercorrelations were calculated. The factor intercorrelations are reported in Table 1. The five factor promax solution accounts for 83% of the cumulative variance.

Insert Table 1 here

The first factor, containing 17 of the 18 mathematics tasks self-efficacy items, accounted for 56% of the total variance. Composition of the subscale based on Factor I is reported in Table 2. Items associated with this factor deal with specific mathematics-related tasks found in everyday life and often require examinees to evaluate their competence against a stated standard (e.g., balance your checkbook without a mistake). The one item that failed to load on Factor I was the newly created item asking subjects to rate their ability to use a scientific calculator.

Insert Table 2 here

The second factor, containing 15 of the 18 mathematics problems self-efficacy items, accounted for 11% of the total variance. Composition of the subscale based on Factor II is reported in Table 3. Items associated with Factor II reflect three components of mathematics (arithmetic, geometry, and algebra) and in varying contexts. Item 13, an algebraic story problem, double loaded on Factor I and Factor II.

Insert Table 3 here

The third factor, containing 7 of the 16 mathematics courses self-efficacy items and the task-related item dealing with use of a scientific calculator, accounted for 9% of the variance. Composition of the subscale based on Factor III is reported in Table 4. Items associated with

Factor III, other than the calculator item, dealt with examinees' confidence to perform at a "B" or better level in college mathematics courses. Item 15, confidence in advanced calculus, double loaded on Factor III and Factor V.

Insert Table 4 here

The fourth factor, containing 7 of the 16 mathematics courses self-efficacy items, accounted for 5% of the total variance. Composition of the subscale based on Factor IV is reported in Table 5. All of these items concerned examinees' confidence to perform satisfactorily in nonmathematics courses that required applications of basic math skills (e.g., business administration and zoology).

Insert Table 5 here

The fifth factor, containing two of the mathematics problems self-efficacy items and one courses self-efficacy item, accounted for 4% of the total variance. Composition of the subscale based on Factor V is reported in Table 6. The two problems self-efficacy items tested geometry, although the two problems were cognitively different. Item 16 measured students' confidence to solve a geometric story problem.

Item 18 required students to recall a specific geometric term and therefore measures memory skills more than mathematical ability. Both items require specific information and are dependent on an individual's recent study of geometry. The mathematics courses self-efficacy item, advanced calculus, double loaded with Factor III.

Insert Table 6 here

Three of the 52 items failed to load on any of the five factors. These were item 9 of the problems self-efficacy scale, and items 3 and 11 of the courses self-efficacy scale. Item 9 of the problems scale was an algebraic problem with two unknown variables, making it conceptually more difficult than the other problems on the scale. Items 3 and 11 of the courses scale, confidence to perform satisfactorily in statistics and accounting, were marginally related to both Factor III and Factor IV. The loadings for statistics on Factor III and Factor IV were .32 and .22, respectively, and the loadings for accounting on Factor III and Factor IV were .30 and .29, respectively.

Reliability estimates for each of the five factors reflected the reliabilities of the three scales for factors I, II, and III; however, they were considerably lower for factors IV and V. Cronbach's alpha for each of these were as follows: Factor I - .91, Factor II - .90, Factor III - .91, Factor IV - .82, and Factor V - .56.

The higher-order factor analysis suggested a single underlying factor represented in the MSES. All of the factors substantially loaded on the single factor as follows: Factor I - .71, Factor II - .63, Factor III - .57, Factor IV - .60, and Factor V - .61. These findings confirm the presence of an underlying construct measured by the full 52-item scale.

The correlations of the five factors and examinees' score on the 18-item MPPS indicated a relationship between mathematics self-efficacy and performance, but more significantly, they corroborated Bandura's belief in the need for self-efficacy instruments to be directly related to the criterial task. Students demonstrated a stronger relationship between their responses that loaded on Factor II, math problems self-efficacy, and the MPPS. This illustrated the contextual nature of self-efficacy and the need for instrument specificity. The correlations between the five factors and the MPPS were as follows: Factor I - .38, Factor II - .62, Factor III - .52, Factor IV - .33, and Factor V - .41.

Discussion

The results of our study support the general proposal that the MSES is a multidimensional measure of mathematics self-efficacy with reliable and valid scales. The results also suggest direction for revision to strengthen each of the three scales. Although one higher-order factor was identified, for self-efficacy research purposes, the constructs represented by the first-order factors provide valid measures consistent with social cognitive theory.

The math tasks self-efficacy scale emerged as Factor I with 17 of 18 items clearly loading on this factor. The failure of item 5 to load with the other items in the scale may be attributed to its lack of specificity. Careful revision of the item asking students to rate their confidence to use a scientific calculator to perform a specific mathematical task (e.g., calculate the cosine of 50) would make the item more consistent with the other items in the scale.

Although 15 of 18 items loaded on the second factor representing math problems self-efficacy, we do not endorse simply dropping the three problematic items. The reduction of items in the problems self-efficacy scale and, subsequently, the MPPS could create difficulties for researchers interested in analyzing relationships in this domain. Because Dowling (1978) developed two alternate forms of the MCS, additional items related to the same mathematical areas could be identified and substituted. The items in Table 7 compare favorably with items 9, 16, 18 of the math problems self-efficacy scale.

Insert Table 7 here

Our analysis demonstrates that the courses self-efficacy scale consists of two factors, one that contains mathematics courses and another that contains nonmathematics courses requiring knowledge and mastery of mathematics. To understand this phenomenon, it is instructive to recall that Betz and Hackett's (1983) initial investigations of the role of self-efficacy involved the influence of mathematics self-efficacy expectations on the selection of science-based college majors. That focus was later broadened to include mathematics-related majors. For this reason, the inclusion of science and mathematics-related courses was deemed particularly appropriate. In doing so, however, it seems clear that two types of courses were included in the scale, and this is made evident by the factor analysis. In addition, the courses self-efficacy scale asks students to provide a judgment of their confidence to earn a B or better in a series of college courses. They are then presented with course titles such as Algebra I, Algebra II, and Geometry on the one hand and Biochemistry, Philosophy, and Physiology on the other. It is easy to see how a student's mind might very well harken images of high school coursework when confronted with the courses on Factor III and of college courses when looking at those from Factor IV.

The clear implication of these results is that researchers are well advised to consider the nature of the performance task when considering an appropriate efficacy measure. Bandura (1986) warned that judgments of self-efficacy are task-specific and that different ways of assessing confidence will differently correspond to the assessed performance. For this reason, measures of self-efficacy should be tailored to the criterial task being assessed and the domain of functioning being analyzed. This particular warning has often gone unheeded in educational research, however, where assessments of self-efficacy have often borne little resemblance to the criterial task with which they have been compared. For example, it is not especially useful to compare self-efficacy judgments regarding accomplishing mathematics-related tasks or successfully completing mathematics-related courses with a performance measure that requires the solving of mathematics problems. The mismeasurement of self-efficacy is a recurring theme in educational research, often producing poorly defined constructs, confounded relationships, ambiguous findings, and uninterpretable results.

It is with this warning in mind that we must consider what value any decontextualized instrument offers researchers who are interested in investigating the role of such a context-specific construct as self-efficacy. The problems self-efficacy scale of the MSES provides a measure of students' judgments of their ability to solve those particular 18 problems. Its value is therefore limited to its ability to predict performance of the problems themselves. A more generalized measure of mathematics problems self-efficacy can be obtained only by assessing students' judgments of their capability to solve problems included in a broader, more reliable and valid performance measure -- for example, a norm-referenced measure with strong psychometric properties, high external validity, and numerous alternate forms.

Along these same lines, students' judgments of their capability to succeed in college mathematics courses should prove more predictive of their choice of mathematics majors than should judgments of their capability to succeed in courses such as biochemistry. This latter judgment, in turn, is more instructive if we wish to assess the role of self-efficacy on the choice of science-related courses. Neither judgment may prove particularly useful in understanding a myriad of mathematics-related tasks. Surely there are more than a few biochemists and mathematicians who are profoundly baffled by the intricacies of the forms provided by the Internal Revenue Service.

It is not altogether easy, then, to see what value the full-scale score provided by the MSES may have. Such a score decontextualizes self-efficacy and transforms it into a personality trait -- precisely what Bandura (1986) warned must not be done (pp. 5-12). For this reason, findings such as those reported by Randhawa, Beamer, and Lundberg (1993) must be interpreted with caution and skepticism. If, as Bandura argued, self-efficacy assessment must conform to the criterial task, what outcome measure can be compared to a global score comprised of confidence to succeed in mathematics courses, complete math-related tasks, and solve specific mathematics problems?

Understanding its components, however, is both useful and informative. Its four factors suggest that different judgments are being tapped, and each may be used depending on the criterial task. Bandura wrote (1977) that, because the definition of self-efficacy is quite straightforward, efficacy questionnaires in academic areas are adequate operational measures of the "conviction that one can successfully execute the behavior required to produce the outcomes" (p.93). If the MSES is to continue to be used as such a measure, social cognitive theory and mathematics researchers are best served by understanding not only the different components of the instrument but the circumstances under which the use of each of them is most appropriate and useful.

Table 1

Intercorrelations of Factors

Factor	I	II	III	IV	V
I	1.00				
II	.521	1.00			
III	.330	.435	1.00		
IV	.485	.285	.370	1.00	
V	.452	.370	.365	.423	1.00

Table 2

Factor I: MathematicsTasks Self-Efficacy Loadings

Item	Abbreviated Statement	Loading
TSE 1	Add two ... large numbers ... in your head.	.42
TSE 2	Determine ... sales tax ...	.70
TSE 3	Figure out how much material ...	.44
TSE 4	Determin how much interest ...	.63
TSE 6	Compute ... car's gas mileage.	.46
TSE 7	Calculate recipe quantities ...	.48
TSE 8	Balance your checkbook ...	.49
TSE 9	Understand how much interest ...	.64
TSE 10	Figure out how long it will take to travel ...	.50
TSE 11	Set up a monthly budget ...	.59
TSE 12	Compute your income taxes ...	.53
TSE 13	Understand a graph ...	.48
TSE 14	Figure out how much you would save ...	.68
TSE 15	Estimate your grocery bill ...	.62
TSE 16	Figure out which of two ... jobs is better ...	.70
TSE 17	Figure out the tip ..	.71
TSE 18	Figure out how much lumber ...	43

Table 3

Factor II: Mathematics Problems Self-Efficacy Loadings

Item	Abbreviated Statement	Loading
PSE 1	In a certain triangle .... What is the sum ..?	.56
PSE 2	About how many times larger ..?	.61
PSE 3	.... Find the largest number.	.41
PSE 4	.... Determine the position of the points...	.58
PSE 5	... find x when y = 10.	.64
PSE 6	.... Which decimal would most closely..?	.65
PSE 7	If P = M + N, then ..?	.59
PSE 8	The hands of a clock ...	.43
PSE 10	How far apart are two towns ..?	.50
PSE 11	.... how much change should he receive?	.41
PSE 12	.... According to this formula, how much ..?	.38
PSE 13	Mary's average .... What is the highest possible..?	.35
PSE 14	3 4/5 - 1/2 =	.64
PSE 15	In an auditorium, ... Multiply (x + 1) (y + 1).	.48
PSE 17	Set up the problem ...	.52

Table 4

Factor III: Mathematics Courses Self-Efficacy Loadings

Item	Abbreviated Statement	Loading
CSE 1	Basic college math	.42
CSE 5	Calculus	.61
CSE 7	Algebra II	.85
CSE 9	Geometry	.46
CSE 13	Algebra I	.74
CSE 14	Trigonometry	.59
CSE 15	Advanced Calculus	.52
TSE 5	Use of a ... calculator	.39

Table 5

Factor IV: Nonmathematics Courses Requiring Math-Related Skills Self-Efficacy

Item	Statement	Loading
CSE 2	Economics	.49
CSE 4	Physiology	.56
CSE 6	Business administration	.45
CSE 8	Philosophy	.73
CSE 10	Computer science	.37
CSE 12	Zoology	.70
CSE 16	Biochemistry	.49

Table 6

Factor V: Course-Specific Mathematical Skills Loadings

Item	Abbreviated Statement	Loading
PSE 16	A ferris wheel .... Find the measure in degrees ...	.55
PSE 18	Two circles in the same plane ... are called ____.	.45
CSE 15	Advanced Calculus	.54

Table 7

Recommended Items for the Mathematics Problems Self-Efficacy Scale

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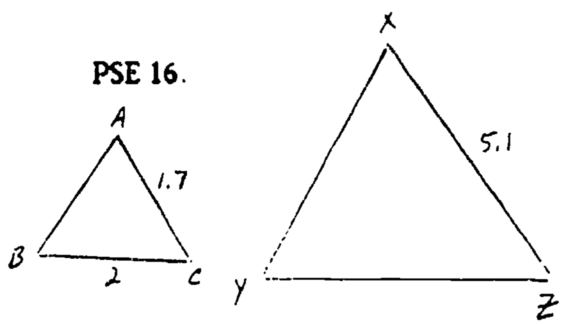
PSE 5. Multiply (1 + x) (1 + y).

PSE 16.

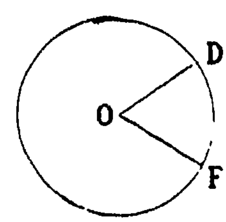


The two triangles shown at the left are similar. Thus, the corresponding sides are proportional, and

AC/BC = XZ/YZ.

If AC = 1.7, BC = 2, and XZ = 5.1, find YZ.

PSE 18. If the circumference is 24, and the length of arc DF is 4, what is the measure in degrees of central angle DOF?



(Dowling, 1978, p. 246).

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