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ABSTRACT

This booklet explains the background, rationale, and issues addressed in the NCTM Curriculum and Evaluation Standards. A detailed vignette of a classroom lesson is presented as an example of what is envisioned for the future. A comparison of the differences between the reform urged by the Standards and the efforts of the "New Math" movement is described. Some issues discussed include: (1) the emphasis on problem solving and reasoning; (2) the importance of making connections between mathematics and other disciplines; (3) a continuing role for rote learning; (4) accepting calculators and computers in mathematics instruction; (5) revising assessment strategies; and (6) the role teachers need to play in mathematics education reform. (DE)

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The Metamorphosis of Mathematics Education

Each issue of FOCUS discusses a critical problem in education and the work Educational Testing Service and others are doing to cope with it. Most widely known for its standardized tests, ETS is also a nonprofit educational research organization.

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Writer and editor: Carol Gordon Carlson

Art Director: Paula Uribe
Designer: Robin H. Matlack
Illustrator: Geoffrey Moss
Production Coordinator: Fred Marx

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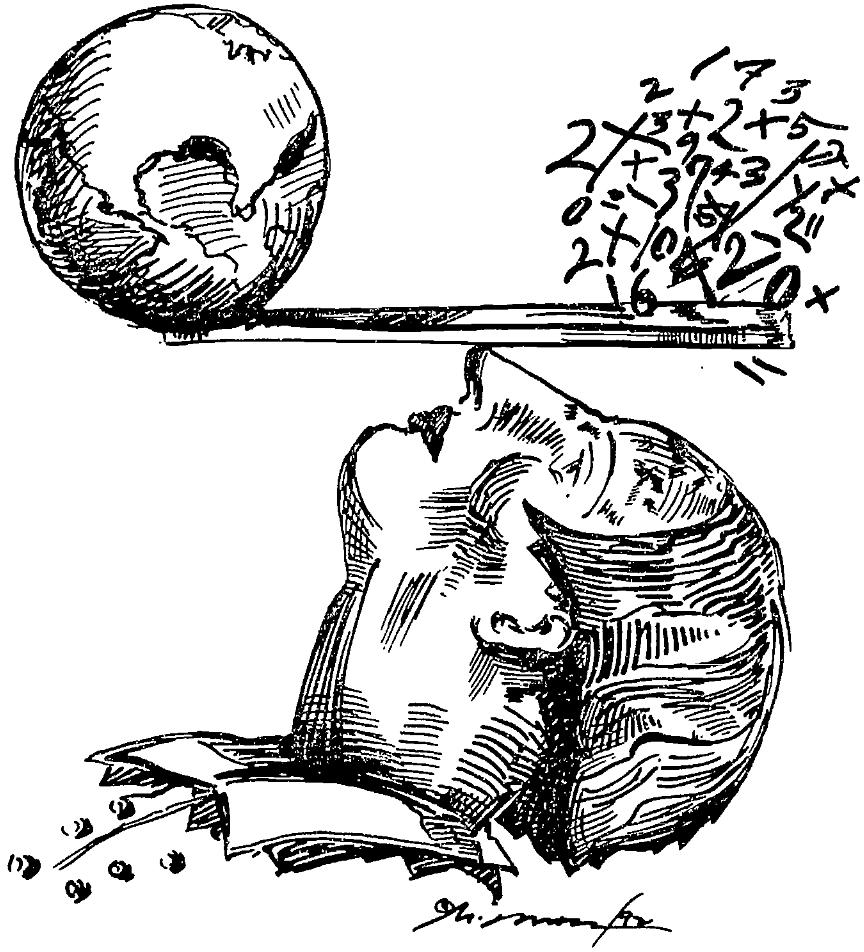
The Metamorphosis of Mathematics Education



A major international study of student performance, published last February, used the most advanced statistical theories and the latest assessment methods to prove once again what educators have known for decades — American students do poorly in mathematics.

The second International Assessment of Educational Progress (IAEP) reported that American 9-year-olds scored below students in 11 of 14 nations. American 13-year-olds in the study, who were ranked against students in 20 nations, outscored only those in Brazil, Jordan, Mozambique, and Portugal.

The hard-edged reality of these statistics is especially disheartening when contrasted with the idealistic goals of President Bush's America 2000 education plan, which calls for the United States to lead the world in mathematics by the next century.

Yet the assessment, conducted by Educational Testing Service, revealed no real surprises. It merely confirmed the results of numerous studies conducted during the past 40 years. These studies all show that the nation's math performance is deteriorating — a trend that may thwart America's efforts to achieve a competitive edge in the international marketplace.

One of the most recent reports calling for mathematics education reform is *Everybody Counts*, published in 1989 by the National Research Council. Due in part to the state of mathematics education in today's schools, the report says, "Three of every four Americans stop studying mathematics before completing career or job prerequisites. Most students leave school without sufficient preparation in mathematics to cope with either on-the-job demands for problem-solving or college expectations for mathematical literacy. Industry, universities, and the armed forces are thus burdened by extensive and costly demands for remedial education."

An earlier report was *The Underachieving Curriculum, Assessing School Mathematics from an International Perspective*, published in 1987. The report conta ıed the cheerless results of the Second International Mathematics Study and helped fuel the fire of concern over American mathematics achievement.

Among its observations:

• Declines in arithmetic achievement from 1964 levels raise "questions about the impact of the 'Back to Basics Movement' of the 1970s" and confirm the nation's "weak position."

• Twenty-five million children study mathematics in the nation's schools, but their time is "largely devoted to mastery of the computational skills which would have been needed by a shopkeeper in the year 1940 — skills needed by virtually no one today...."

Researchers have reported similar findings in *A Nation At Risk*, a landmark publication by the National Commission on Excellence in Education, published in 1983, and *An Agenda for Action*, which recommended a 10-year plan for revising school mathematics and was published by the National Council of Teachers of Mathematics (NCTM) in 1980.

Other significant reports were published in 1975, in 1963, and in 1959, just two years after Sputnik I rattled the American academic community.

But even Sputnik cannot be credited with initiating concerns about mathematics education. In the first half of the century, experts were already lamenting students' poor performance in mathematics.

The causes behind this continuing problem are numerous and varied, including:

• underprepared teachers and students
• outmoded textbooks and curricula
• uninspired teaching methods
• inadequate administrative support
• uninterested parents
• complex social problems

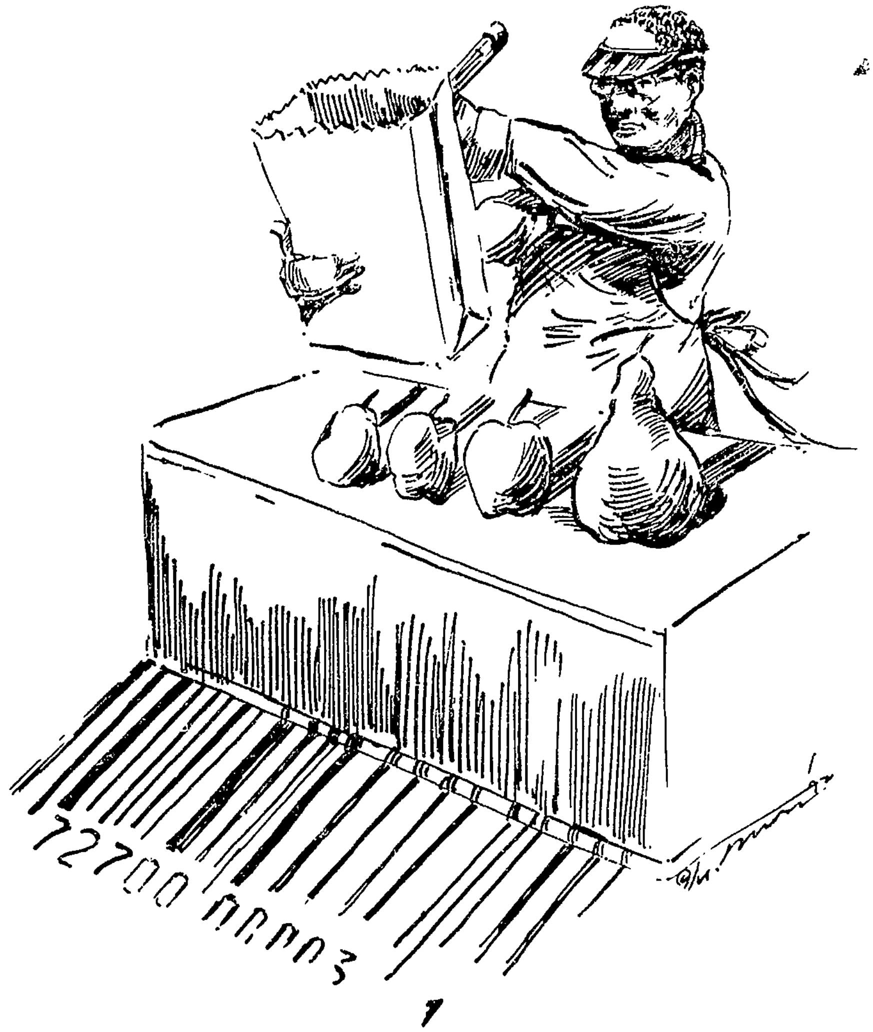
Often mixed into the distasteful brew are either bureaucratic inertia — the tendency for schools to reject anything that seems too different, faddish, untested, or risky — or overzealousness — a tendency to plunge headlong into education experiments without adequate planning or staff training.

Spicing the Mix

Many mathematics educators, however, are now counting on a dramatic new ingredient to spice up the mix. The new addition is a set of revised standards for elementary and secondary mathematics curricula and teaching, developed by the Commission on Standards for School Mathematics of the NCTM.

The new standards:

• are based on the most current research on educational and work force needs
• are realistic and applicable to students of all ages, nationwide
• were endorsed by 15 math associations, societies, conference boards, councils, institutes, etc. and supported by 25 professional organizations concerned with teaching, learning, and children.
• have the potential, if effectively employed, to level the playing field for minorities and women, who perform poorly in traditional mathematics coursework.



In 1986, the NCTM published *Curriculum and Evaluation Standards for School Mathematics*, which sets forth recommendations for revisions in what children should know and be able to do in mathematics from kindergarten through high school. The group has also developed innovative yet practical teaching methods and suggestions for teacher training, described in their publication, *Professional Standards for Teaching Mathematics*.

The introduction to *Curriculum and Evaluation Standards* states the rationale for the new standards:

"Schools, as now organized, are a product of the industrial age. In most democratic countries, common schools were created to provide most youth the training needed to become workers in fields, factories, and shops.... The educational system of the industrial age does not meet the economic needs of today. New social goals for education include (1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate. Implicit in these goals is a school system organized to serve as an important resource for all citizens throughout their lives.... As society changes, so must its schools."

The new goals for students in mathematics, as explained in the *Standards*, are: "(1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically."

The new standards also emphasize that in order to make mathematics education useful and accessible to more students, the central focus should be on problem solving. "Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned."

To do all of this, the standards call for classes that:

- are creative
- emphasize comprehension and problem solving, not just memorization
- train students to use calculators or computers effectively to enhance, not replace, knowledge of basic skills
- use manipulative materials to promote maximum comprehension

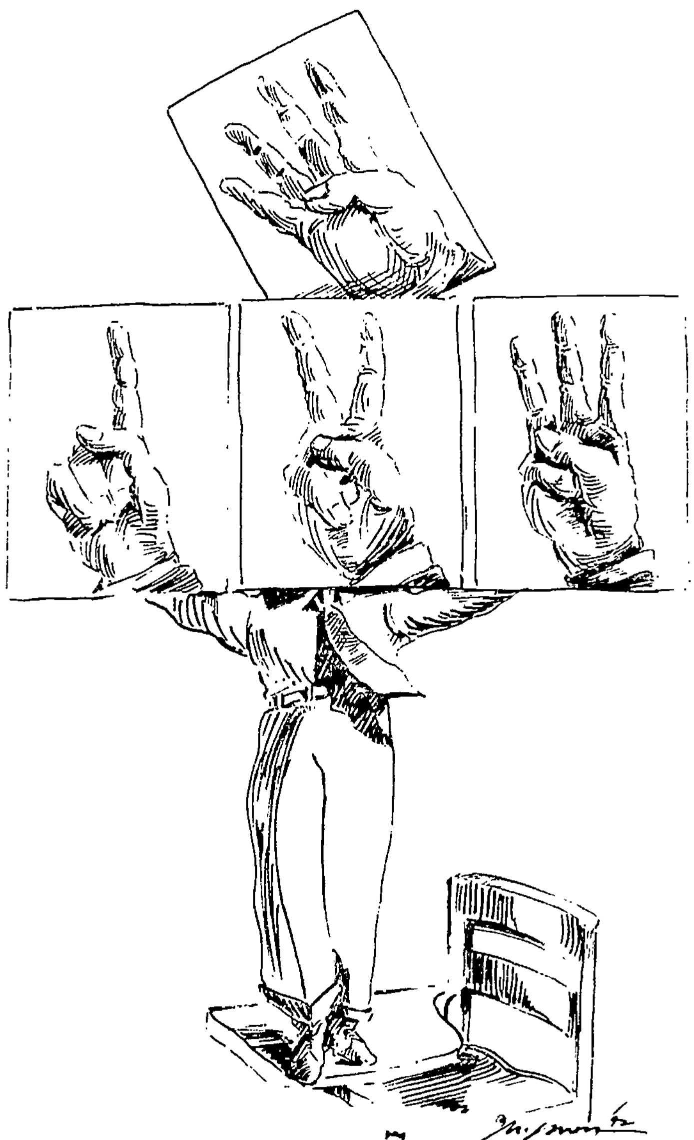
New Standards in Action

Already, many teachers have adopted these concepts. When they do, their classes often begin to look like the fifth grade advanced math class taught by George Tiefenthaler of the New Hope-Solebury Elementary School in Pennsylvania.

A morning spent in class with Mr. T., as his students call him, is like previewing the future — the class embodies the qualities that education leaders are seeking for all mathematics classes.

Mr. T. grabs the students' attention at the very start by standing on a chair at the front of the classroom, waving his basketball-player-sized hands like semaphores. This is one of Mr. T.'s games, and the children know that his hand positions signal that the students should add, subtract, multiply, or divide the number of fingers he is showing. With this deceptively simple strategy, Mr. T. transforms the drudgery of mental exercises into an appealing but challenging competition that requires concentration and a thorough knowledge of arithmetic skills.

After this warm-up review, Mr. T. gives the students a few problems



to work on their school-supplied calculators.

He then whisks the class into an exercise called "the problem of the day," instructing the students to work the problem alone first and then discuss it with classmates.

The problem states: "Your friend asks you for change for a dollar. You tell your friend that although you have $1.15 in change, you cannot make change for a dollar. Your friend says that you certainly can. Who is right? Explain."

Even before Mr. T. explains the ground rules, one bright student raises his hand and states, "But both people could be right."

"Right," says Mr. T., a smile crossing his face. The comment confirms that the children are learning the most important lesson of the year — there are many different ways to find an answer to a problem and, often, many different answers as well.

UP, DP, Do It!, DIMS

Before Mr. T. sets the class off in search of an answer, he reviews his recommended problem-solving steps. The words and symbols for the steps are written on paper-and-string mobiles, constructed by the students early in the year, now suspended from the ceiling and fluttering overhead.

The first step, symbolized by an arrow and the letters UP, is Understand the Problem. The second step is DP — Devise a Plan. The next step is represented on the mobiles by a Nike sneaker, symbolizing the Nike television commercials that feature the slogan, "Just do it!" For the students, this means to work the problem. Finally, the paper mobiles proclaim DIMS, which stands for, Does It Make Sense?

"Look at your answer and see if it makes sense," says Mr. T. "Is it possible for the boy to have change for a dollar?" "Yes," answers the class. He could have four quarters, one dime, and one nickel, or several other combinations of coins.

"Is it possible for him to *not* have change for a dollar?" "Yes." The class then comes up with several ways to have $1.15 in change and yet not have exact change for one dollar. The boy might, for instance, have nine dimes and one quarter, or three quarters and four dimes, or one half dollar, one quarter, and four dimes.

Mr. T. caps off his class with one of the key components of the new mathematics standards — manipulatives. For the day's lesson, he passes out handfuls of black disks that look like flat, smooth checkers and creates groups of four to five students to work together to solve a problem. "No, you can't play poker with them," he says laughing, in answer to one student's comment.

He then hands out diagrams showing dots arranged in rows of varying lengths to form a diamond shape and instructs the students to arrange their disks in the pattern shown on the diagram. "The problem is," he says, "how many ways can you find to count them?" After the first few minutes he even gives the students the easiest method — count the dots one by one.

The disks are like a magnet for the children's hands. They cannot resist touching and moving them, arranging them into the pattern on the hand-out, then rearranging them again and again.

One boy calls out, "Why bother? There are only three ways we can do it." But Mr. T. merely says, "We'll see."

The noise level in the classroom rises, but Mr. T. does not call for quiet. The children are exploring, sharing, and comparing ideas, telling each other how to move the disks into new patterns.

After a while, Mr. T. asks the groups to share their discoveries with him, and he writes them all on the blackboard.

- First method: Change the arrangement from a diamond into a square. Then count how many rows have four disks (four of them) and how many rows have three disks (three), multiply out, and add the totals.
- Second method: Count the center row of the diamond (seven disks), and then count all the disks to the right of the center (nine). Because the pattern is symmetrical, there are as many disks to the left as to the right of the center line, so you can multiply nine by two and then add the seven of the center line.
- Third method: Group them in sets of five, and multiply by the number of groups (five times five).

The class comes up with several other methods as well, excited by the number of possibilities.

"What is the point? What is the teacher trying to tell us?" Mr. T. rhetorically asks the class. "There is more than one answer to the problem," he tells them. "Your homework assignment for tonight is to design a problem using 36 marbles, and to find as many ways as you can to count when you have 36."

Class is over for the day, and the children rocket off to the next subject.

New Insights into Learning

Mr. T.'s emphasis on solving problems through creative thinking distinguishes him from the more traditional mathematics teachers who emphasize rote memorization of arithmetic facts. While Mr. T. also requires his students to memorize basic facts, he adds additional activities that help them understand underlying concepts. The students, in other words, learn to reason mathematically — a skill they can use throughout their lives.

Mr. T.'s emphasis on reasoning and problem solving are essential elements of the new standards for mathematics.

Many elements of the new standards were developed as a result of new insights into learning itself, says Robert Davis, a noted mathematics educator and researcher at Rutgers University. In his 1984 book, *Learning Mathematics, The Cognitive Science Approach to Mathematics Education*, Davis writes that these insights were drawn "from Piaget's observations of children, from computer scientists' development of 'intelligent' computer programs, and from research into human information-processing capabilities."

"Twenty years ago, there was no extensive body of research that told you, for instance, how kids really think about fractions," confirms Richard Lesh, a research scientist at Educational Testing Service (ETS). In the absence of that research, "people with decent intuitions about such things tried to guess, but they missed the boat some of the time."

Since then, however, "people went out and watched kids solving real problems, watched them working with concrete materials," Lesh says. As a result, they found that some of the tenets of behavioral psychology, which were the foundation for much contemporary educational practice, were incorrect. Their research pointed them toward new understandings of human learning, now called cognitive psychology.

The difference, Lesh explains, is that "behavioral psychology was based on rules — everything got reduced to a rule of behavior, an action even, for the most part. Learning meant learning rules.

"Cognitive psychology is about the fact that people interpret the world using internal models, metaphors, and stories," Lesh says. People use these stories to help them understand and place in context their own experiences.

"Research in learning," states the report *Everybody Counts*, "shows that students actually construct their own understanding based on new experiences that enlarge the intellectual framework in which ideas can be created. Consequently, each individual's knowledge of mathematics is uniquely personal. Mathematics becomes useful to a student only when it has been developed through a personal intellectual engagement that creates new understanding. Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn."

"It's not that kids don't know anything and then we teach them something," Lesh explains. "They have a lot of ideas, and what we need to do if we want to teach them is to first make contact with the way they think, and then mold and shape those ways in directions we want to go."

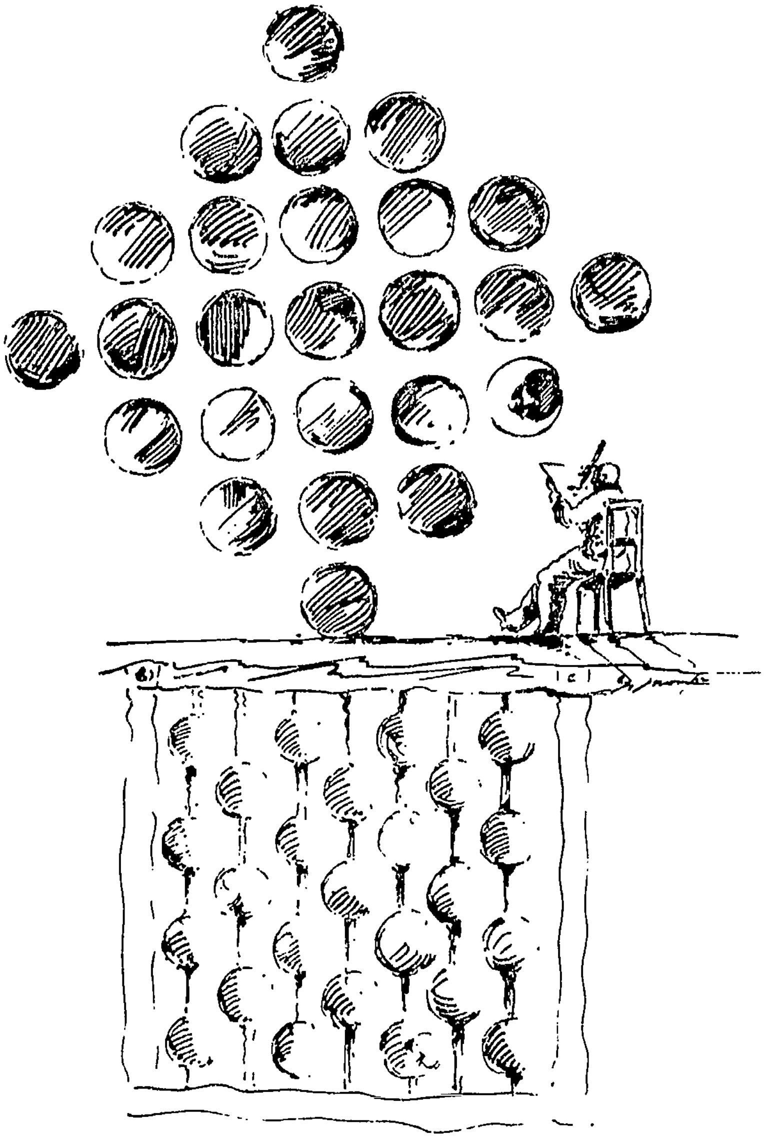
Making Connections

The recognition that mathematics is learned most effectively when it is placed in a personal context makes the new standards significantly distinct from current practice in mathematics education.

When students are able to link theory with reality, Lesh explains, they are better able to understand that mathematics is not just "a bunch of rules that people use for doing calculations with numbers. Mathematics is really about models for thinking about the world — models that deal with quantities, shapes, relationships, and other mathematical things."

At its best, mathematics education helps people understand these models so they can apply them in different situations.

In a course he conducts at Rutgers University, Lesh leads his students, mainly teachers, through a series of exercises to help them



better understand the foundations of mathematics.

"At the end of the course, the teachers end up saying something like, 'Math is really about models, and my job as a teacher is to help my kids construct those powerful models.' That's enormously different than, 'Math is a bunch of rules and I want my kids to do them flawlessly.' "

Davis agrees. During an interview at his office at Rutgers, he points out that mathematics "shouldn't be thought of as a few very specific things that you memorize and you do without understanding them — it's much more like an art or a craft."

As an example of the problem-solving nature of mathematics, Davis cites a teacher who developed an exercise that asked school children to gather several leaves of varying sizes and shapes. She would then ask the children to develop a method to compare the sizes of the leaves.

There is no formula for figuring out such a problem, Davis explains. Students must devise their own models and methods to find an answer. The goal of the new standards for mathematics is to train students for creative problem solving of this sort, so they can do more than just calculate number problems on a ditto sheet.

The need for reality-based mathematics is not a new concept, Davis says. "It's been developing for a long time."

"The fact that people have been learning these abstract procedures in a meaningless way," Davis says, "is not new knowledge — we've known it for ages. And the fact that you ought to deal with math in a meaningful context is truly not a new idea. It certainly goes as far back as John Dewey, and I don't think he created it, either."

The system of using rote memorization developed in the days when children generally were brought up to follow their parent's profession, Davis theorizes. "In those days, doctors were mostly the sons of doctors and farmers were typically the sons of farmers. If you grew up on a farm, you knew what farmers do, and things didn't really change that fast for a long time."

A farmer's son knew that to put up a fence around a field, he would have to know a formula to determine the perimeter, Davis explains. He would use another formula to determine the area of the field, so he could estimate how much seed he needed. The numbers themselves stood for concepts familiar to students from their daily lives, making the ideas both easier to understand and to learn.

"Kids in school nowadays," Davis says, "do not know the context, and they probably are not going to follow a parent's occupation, or they find out the occupation is different by the time they get into it. They can't count on the context being there."

Yet long after the context was removed, the education system continued using the old methods because they had worked so well in years past. "But without the context, the numbers by themselves are meaningless. You don't understand what you are actually doing," Davis says.

The context argument is purely conjecture, Davis emphasizes, but it makes sense in light of recent research. Davis cites the example of a researcher who would give people problems in a class situation and later present a similar problem in a situation such as grocery shopping or adjusting recipes. The researcher found that even people who could not work the problems in a classroom could often figure them out when they were presented in a real-life context.

"There's no doubt, I think, that these contexts are very powerful," Davis emphasizes.

"If you start looking at what you do with math, you start seeing that you typically use it in a context. People don't come up to you in the ordinary world, and say, 'What's this divided by that,'" Davis says. "They will ask a question such as, 'We've got this many kids going on a school trip — how many school buses do we need?'

"So really, what we're arguing for is that math should be in context, that it should involve manipulable materials, that people should learn to think through problems that no one showed them how to do. Cer-

tainly no one showed me anything about refinancing a mortgage," Davis says.

Playing Tug-of-War with Rote

Despite the collaborative process that the NCTM used in developing its new standards for mathematics education, some educators still object to its goals and methods.

This dispute is part of what Davis, in his book, *Learning Mathematics*, calls a continuing tug-of-war in mathematics education "between a 'drill-and-practice' and 'back-to-basics' orientation that focuses primarily on memorizing mathematics as meaningless rote algorithms, versus an approach based on 'understanding' and 'making creative use' of mathematics."

Although current research supports the creative approach to education, in practice the tug-of-war is still being won by the traditional model despite the fact that "it is quite clear that present pract[illegible] at an ineffective extreme," states the report *Everybody Counts*.

Advocates of the new standards, of course, believe that the balance of power in the tug-of-war will gradually change as both schools and colleges shift away from "classrooms of passive students who are expected to sit and absorb rules which appear as arbitrary dicta from on high...."

In their place, they hope to see classes that emphasize "broad-based mathematical power," says *Everybody Counts*.

"Mathematical power requires that students be able to discern relations, reason logically, and use a broad spectrum of mathematical methods to solve a wide variety of non-routine problems. The repertoire of skills which now undergird mathematical power includes not only some traditional paper-and-pencil skills, but also many broader and more powerful capabilities."

Some educators are still pulling the tug-of-war rope for rote memorization, but in many cases, their objections arise from a failure to fully understand the standards.

One retired math teacher described his views in a letter to the editor of the *NJEA (New Jersey Education Association) Review* last fall (November 1991).

"You have to crawl/walk before you can run" the letter states. "If formulas aren't memorized, there will be no basis for the mathematical reasoning. If there is no mechanistic answer finding, there will be no conjecturing, inventing, and problem solving. If you don't know a body of so-called isolated concepts and procedures, there won't be any connecting mathematics and its applications. Judicious use of old-fashioned rote memory and drill are as necessary today as they were in generations past."

The standards, however, recognize the need for "a certain amount of basic memorization," explains John Dossey, professor of mathematics at Illinois State University and former president of the NCTM.

"We expect kids to know basic facts and operations as much today as we have at any time in our history. What we're saying is that what children ***do*** learn, they need to connect with the real world, too."

In general, objections to the new standards are unusual, Dossey says. "You'll find a few people, but I think there is general consensus because of the way the standards were developed. There just is no really identified large group of resistance out there.

"Personally, although I was very involved in developing the standards, there are places I can point to and say, 'I don't agree with that,' but, overall, I wouldn't have the slightest hesitation for my child to complete a program using the new standards."

Different from "New Math"

For all of their potential, today's new mathematics reforms are still often misunderstood by the public and by some educators who may be less involved in such professional organizations as the NCTM. In part, perhaps, this is because they remember another "new math," associated with ambitious mathematics education reform programs of the late 1950s through the 1970s.

There are, however, a "whale of a lot of differences," as Dossey says,

between today's mathematics reforms and the so-called "new math."

"The 'new math' probably tried to throw out too much and to start from scratch, including throwing out the established ways of dealing with basic operations," explains Nancy Cole, vice president of Educational Testing Service and a member of the Mathematical Sciences Education Board. "That's how it got the name 'new math'"

In contrast, the current reform movement builds on existing approaches and knowledge and expands out from them.

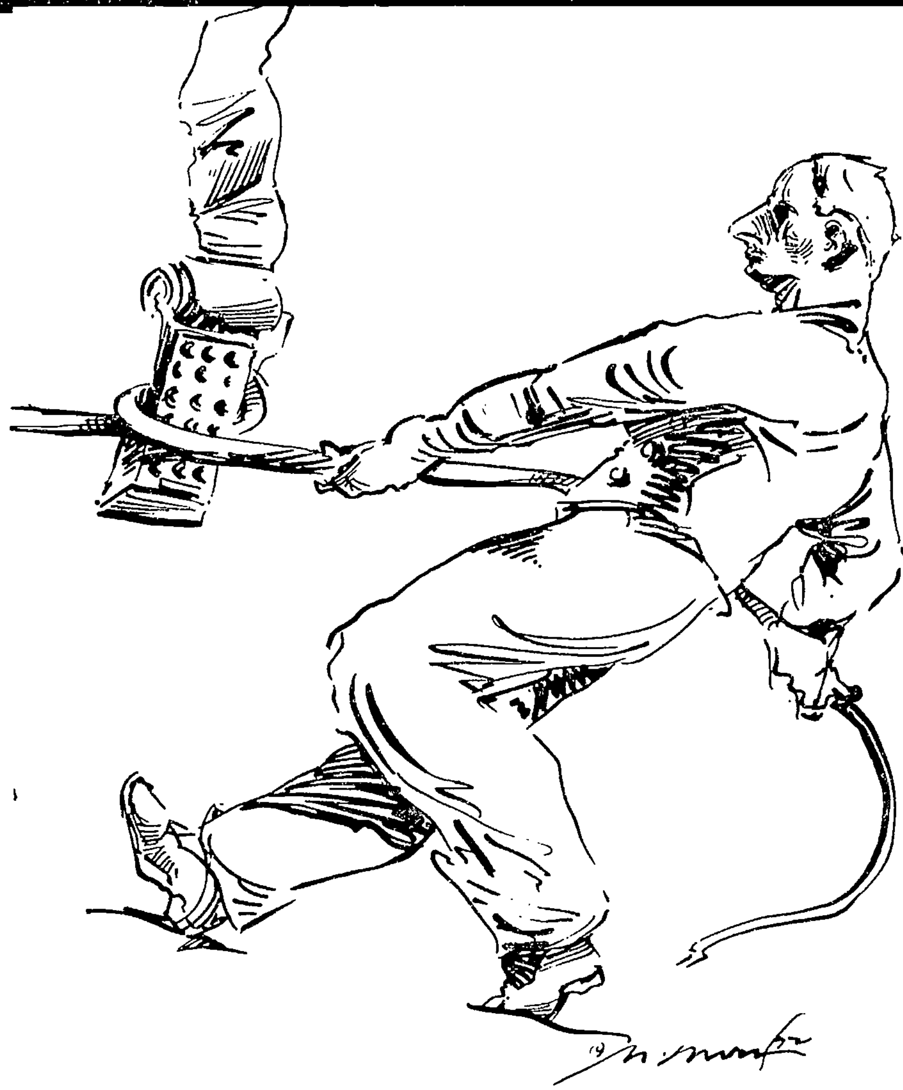
Today's mathematics also takes into account the fact that "children come to school with a certain amount of mathematical background, and you're better off trying to build their mathematical understanding on that background, as opposed to just throwing rules and procedures at them, out of context," says Beverly Whittington, an ETS senior examiner in the College Board program.

"That doesn't mean the children are left on their own to understand math," Dossey explains. "The teacher and the curriculum provide them with experiences that help them shape their understanding in a progressive fashion."

The curriculum, he says, helps them "work to create their own understanding. They learn not just because the teacher tells them it's so, but because they see it and wrestle with it on their own."

The goals of today's math make it perfectly suited to this type of experiential teaching, agrees Whittington. "The old 'new math' was more theoretical. The idea then was to give people an idea of the structure of mathematics — it was more pure mathematics.

"Today, people are talking about putting things in a real-life context and being able to relate mathematics to other fields. I think that's to help the children transfer their mathematics concepts more easily.



so they can see the worth of mathematics in their lives."

Another feature of "new math," Dossey points out, is that it aspired to "create mental masters overnight." Because the reforms were spurred by an urgent need to catch up with the Soviet Union's scientific advances, they tried to make "overnight" changes, he says, "like taking a template and placing it down on the grades and tracing around it." The system basically told the students, "Here's the structure you need, now learn it" with the goal of "pumping the stuff in quick."

Today's mathematics reforms, in contrast, are being introduced into curricula slowly. "If it happens too quickly then there's a problem," Dossey says.

"Here, we're talking about a staged change, over a decade, perhaps, so that teachers are comfortable with it. The children are not expected to change overnight, but to change as they move through the system."

Perhaps the most basic difference between the two movements, Davis points out, is that while today's reforms were developed with a unified structure, the "new math" was not. He contends, in fact, that while mathematical theory did, indeed, support many of the mathematics reforms from the '50s through the '70s, the term "new math" is erroneous — promulgated not by educators, but by the media.

The single common element among these reforms, Davis says, agreeing with Cole, was that "every one of them wanted to replace the existing math book with something else." Otherwise, he says, the reform efforts were quite diverse.

"If one were to believe popular accounts, one might conclude that something known as 'the new mathematics' was created in the late 1950s and early 1960s, that it was tried in U.S. schools, and that it failed disastrously," Davis writes in *Learning Mathematics*.

"This is all quite wrong.

"In the 1950s and 1960s, *many* different school curricula were created.... These programs were not all similar — some were mainly abstract, some used mainly concrete learning experiences; some stayed close to traditional content, some explored very different content; some dealt only with mathematics, some combined mathematics and science, some approached mathematics mainly *through* science; some assumed considerable rote learning, others assumed very little; some emphasized 'discovery learning,' others used it little or not at all; some undertook extensive teacher education programs, some took an opposite route and sought to prepare so-called 'teacher-proof' material."

Clearly, this was no unified set of standards that could be easily categorized as "new math."

In fact, Davis writes, "Most schools experienced little or no change."

Not only was there no single "new math," there also was no complete and disastrous failure, Davis writes. "In those few schools where they were extensively and carefully implemented, the best of the new curricula produced very pronounced *gains* in student performance. The fact is that they did *not* fail; on the contrary, they were markedly successful.... "

With the combination of some significant successes and some major problems, the overall results of the curricular reforms characterized as "new math" were "quite uneven," concludes the report *Everybody Counts*.

What Made it Happen?

"A major difference between the new math and the reforms being suggested now is that this time around, educators worked carefully on building a broad consensus," explains ETS's Cole.

"I think there must have been a critical mass of really concerned, respected, and wise mathematics educators. It wasn't a single person, but it was a strong cluster of them that saw the possibilities for changing mathematics education and changing the political processes necessary to have an impact.

"One of their early steps was to bring many professional groups together to discuss curriculum, which gave them a lot more power. Then they went to the National Academy of Sciences and in 1985, got the academy to start the Mathematical Sciences Education Board (MSEB) to be a coordinating board for all the professional groups."

The National Council of Teachers of Mathematics, which took the lead in creating the standards, had strong teacher and college-level mathematics educator leadership, Cole says. At the same time, the MSEB worked with the groups to have the standards endorsed.

"It was just a lot of really strongly committed people willing to work very hard that made it happen," Cole says.

Adding impetus to the movement for change, Dossey says, were two reports on the nation's failing mathematics achievement levels:

- *A Nation At Risk*, published in 1983 by the National Commission on Excellence in Education, "really shook up the education community," Dossey says.
- *The Underachieving Curriculum, Assessing School Mathematics from an International Perspective*, published in 1987, showed that neither the "new math" reforms nor the "back-to-basics" movement that followed had improved American mathematics achievement levels.

Spreading the Message

Despite its drawbacks, the "new math" taught educators some valuable lessons. Among the most important, states *Everybody Counts*, is that "...any successful effort to improve mathematics curricula and instruction in the schools will require an extensive public information campaign that reaches all the varied constitutencies of mathematics education."

The lack of communication was a major roadblock for the "new math" reforms. As Davis explains, "there was never any intelligent discussion of it." Very little was written about "new math" when it was first introduced, so few educators clearly understood the rationale behind the programs and few participated in discussions with their peers or with the public about how best to accomplish their goals.

Today's reforms have an advantage in this area because many professional organizations were involved in their development. The

"Unless texts... change dramatically to reflect the standards' philosophical underpinnings... there will be little, if any, change in the way that math is taught...."

NCTM mailed an executive summary of its *Standards* reports to every principal and school board president in the nation, Dossey says.

But even with national support, Cole points out, dissemination problems remain. "Not every teacher belongs to NCTM, or is active, so you've got just a huge gap between these national groups who are doing very constructive things and very busy teachers in classrooms who may not even know that some of this is going on."

To reach a wider audience, Cole is a member of a state coalition examining various education activities for teachers and parents, ranging from public TV programs to pamphlets for teachers about using the standards in the classroom. Called the New Jersey State Mathematics Coalition, the group will collect and disseminate information about successful programs in use nationwide. The committee is also opening contact with school administrators "to get a feel for how we can help them and how they can help us."

"Administrator groups are concerned about reform, just like teacher groups are at this stage, so the climate's right for mutual help," Cole says. "Up to this point, we've done very little with principals, superintendents, and administrators, but we're beginning to realize that they're very important groups to reach."

Committees similar to the New Jersey Coalition are being formed in all 50 states at the initiative of the MSEB.

The advocates of change will still be fighting an uphill battle, however, until the textbooks are in agreement, Dossey says. "There are already some textbooks that are moving in that direction, but textbooks will only change when the market demands."

To discuss the need for revised texts, the NCTM met with the School Division of the Association of American Publishers last March. During the meeting, NCTM members told the textbook publishers that, "Unless texts and other materials change dramatically to reflect the standards' philosophical underpinnings...there will be little, if any, change in the way that math is taught," said an article in *Education Week*.

The article quotes Glenda Lappan, a mathematics professor at Michigan State University, as telling other university researchers, "If we don't put our focus on writing better materials for teachers, we will have missed the boat. We will not serve the needs of teachers until we write better materials from which teachers can learn."

Accepting Calculators and Computers

One element of the new mathematics reforms that has received considerable attention in both scholarly and popular literature is the recommendation that new technologies be included in mathematics curricula.

Because these technologies are now integral to mathematics, the NCTM charged the committee developing the new standards to, "Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields."

Yet the use of calculators has long been debated. Advocates say that calculator use will pare the time spent on routine computations and help students concentrate on mathematical concepts and applications. Critics contend that calculators will allow students to avoid learning the basics of mathematics they will need beyond their school years.

"The mathematics professional community now is quite well unified that calculators and computers will, in the future, be like paper and pencil," says Cole. "We need to teach kids when to use them, when not to use them, and how to use them effectively. It is essential that those kinds of tools be part of mathematics education." The public often misunderstands how educators plan to incorporate calculators in the classroom, Cole says

As the report *Everybody Counts* states, "Calculators and computers for users of mathematics, like word processors for writers, are tools that simplify, but do not accomplish, the work at hand. Thus, our vision of school mathematics is based on the fundamental mathematics students will need, not just on the technological training that will facilitate the use of that mathematics."

Contrary to the fears of many, the report says, there is "no evidence" that the availability of calculators and computers "makes students dependent on them for simple calculations."

"As long as the primary focus is their use as a tool, they should pose no threat to mathematics education," says Chancey Jones, principal measurement specialist at ETS.

In fact, he notes, today's calculators have already progressed far beyond simple arithmetic functions. "They can be a teaching instrument because the students can see on the screen what it means to approach a limit or draw the curve of an equation. It's right in front of them; they can interact with the material. For all practical purposes, a lot of the graphing calculators today are really minicomputers."

The public's fears are based on a general misunderstanding of mathematics itself, says Cole. "They think of mathematics as learning to add, but that's not mathematics — that kind of operation is a very small and insignificant piece of mathematics. One of the big problems we fight is that people have this notion that mathematics is mainly those operations."

Calculators are not just for working computations in class; they affect society at large, points out Lesh. "The calculator and computer relieve you of doing the tedious computation, but those tools also introduce whole new things that you need to understand, such as graphics-based communication."

Because graphics are now so easy to produce on calculators and computers, their use has increased enormously in fields such as business. Now routine features in popular newspapers and magazines, graphics are used to illustrate a variety of issues such as trends or cost-effectiveness. Graphics have vastly improved the way people can communicate mathematical ideas, Lesh says, and they therefore need to be included in mathematics classrooms.

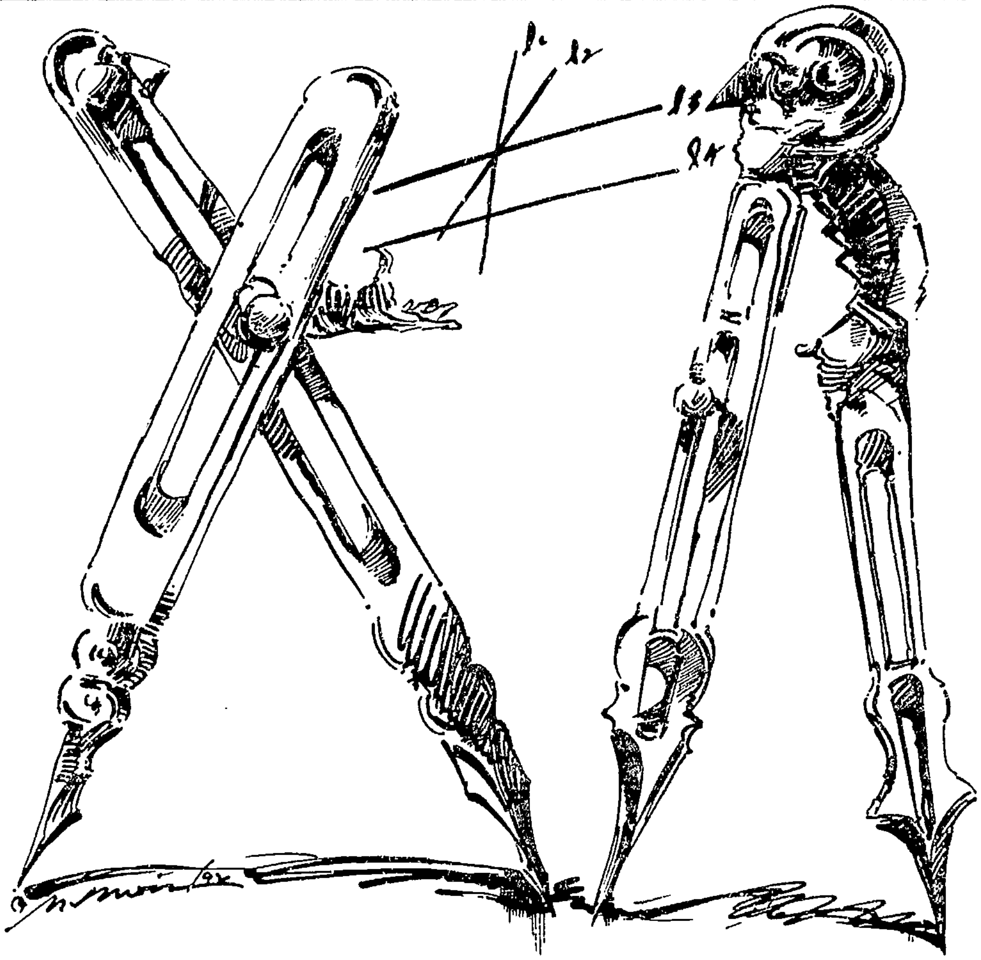
Revised Assessments

The new standards for mathematics call for changes in the way mathematics is taught as well as the content that is emphasized. Although a significant amount of traditional material remains in the recommended curriculum, the standards require new assessment strategies, points out James Braswell, principal measurement specialist at ETS. Braswell heads the mathematics areas of the Preliminary Scholastic Aptitude Test (PSAT) and the Scholastic Aptitude Test (SAT).

"The standards have deemphasized some of the more mechanical sorts of things that students have spent a lot of time on in the past," he says. "They emphasize other things — for example, working with data, making graphical representations of data, and making connections between different disciplines, such as geometry and algebra."

All this means major, if gradually introduced, revisions in standardized assessments, Braswell says. "We have made it a point to get all of our test developers familiar with the standards so they know what is being emphasized and what is being deemphasized, but change will have to be gradual, giv our operating parameters."

Available testing time, validity, and reliability are among the parameters that must be considered. "With the SAT, we have to come up with a reliable measure of what students can do in an hour. There are two possible approaches: you can pose three or four fairly



complex problems, so you have all of your eggs in just a few baskets, or you can come up with a lot of little problems and hope that based on the responses, you can make some inference about the larger picture. That's the way we're doing it now, and I think that's probably the way it will be for a while, but advances in computer-delivered testing and expert systems hold promise for the future."

The large standardized tests are also obliged to reflect current teaching practices and curricula, points out ETS's Jones. "The standards are being widely distributed, but it's going to be a long time until we know whether or not they have a real impact in the classroom."

A more complex problem confronting test developers is validity, Braswell says. "One of the objectives of the standards is providing students with the opportunity to solve more realistic problems, problems that go beyond a quick answer, and problems that people can work on in groups. The standards point out that if you go out in business and industry, you find people working in teams and collaborating.

"But that's hard to do on a test," Braswell points out. "When someone is taking the SAT, we frown on a collaborative effort taking place — how would you know who did the work? Yet the new standards emphasize how people learn from a collaborative process, not necessarily who gets credit for what level of involvement. In a classroom, if two people work together, even if one person does most of the work, the other person has learned something as a result of that collaborative effort."

So far, Braswell says, very little new material has been developed that meets all the objectives of the standards in terms of tasks the students have to do, texts that include and emphasize these tasks, or model test questions that reflect the standards.

In an effort to accommodate the new standards, ETS is already redesigning its largest testing program, the SAT. The new tests will be introduced in the spring of 1994.

Among the changes:

• Some new questions will require students to produce their own responses, instead of choosing the standard multiple-choice answers.

• Calculators will be permitted (although not required).
• The assessment will place increased emphasis on interpretation of data and applied mathematics.

"What we ought to strive for in the testing programs is a real balance of both well-structured open-ended questions and multiple-choice questions that allow students to demonstrate their abilities and capabilities," says Jones. "I don't want people to think that there's no value at all to multiple-choice questions. They have served us well. And there are complications of validity and reliability when you use only open-ended questions."

A good example of an assessment that is balanced in terms of a variety of item formats and is responsive to the changing directions in mathematics education, Jones says, is the 1992 National Assessment of Educational Progress (NAEP) Mathematics Assessment. Conducted by ETS, the assessment was administered to students in grades 4, 8, and 12.

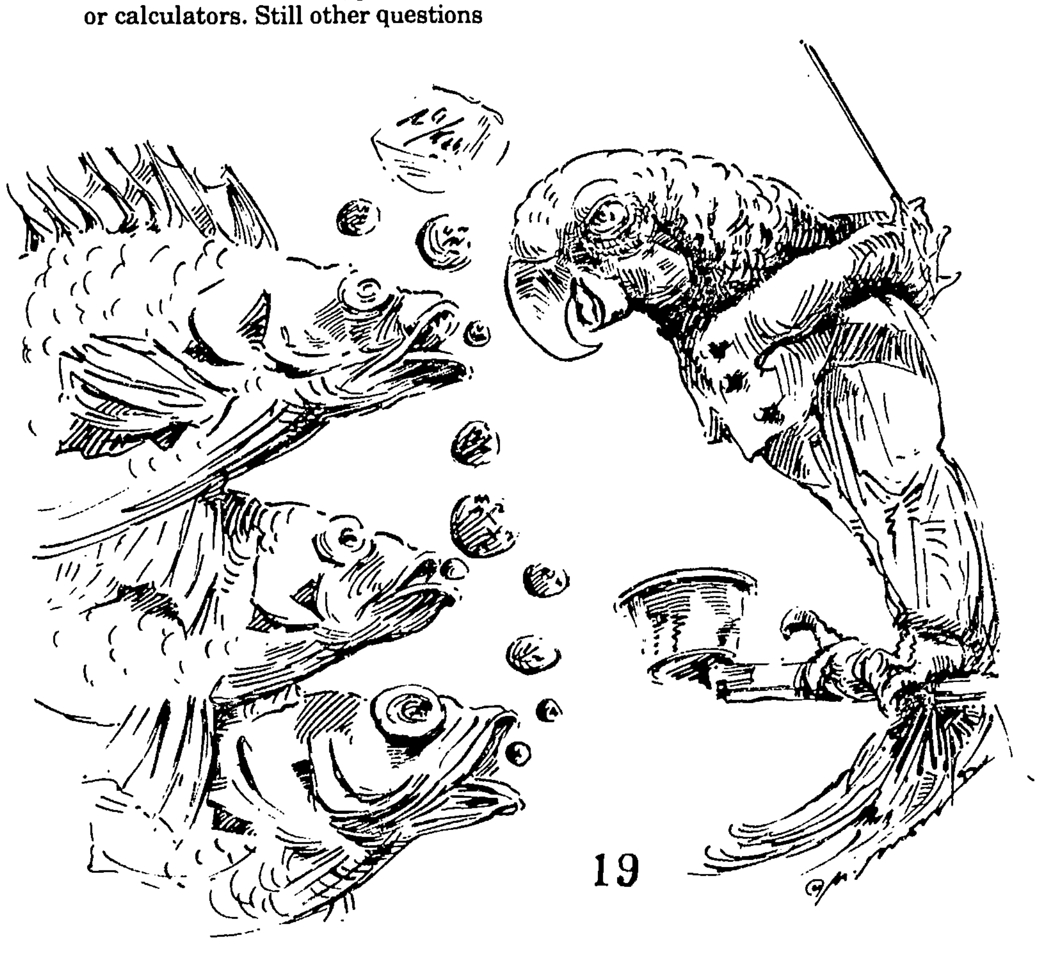
More than one-third of the questions on the assessment required the students to construct their own reponses, which took about 40 percent of their total time. Some of these questions required the students to work with manipulatives or to use rulers, protractors, or calculators. Still other questions asked students to provide reasons to justify their solutions.

The assessment was unique because it included, for the first time, several extended constructed-response tasks at each grade level and allowed students to demonstrate their level of mathematics understanding within a specific context, Jones says.

This item type enhanced the assessment by providing more detailed information about the students' approaches to problem solving, the breadth and depth of their mathematical understanding, and their ability to solve nonroutine problems that required higher-order thinking skills.

Classroom practice, of course, is still the key to effecting the changes required by the standards. "The standards put a fair amount of the burden on the schools themselves to deal with this issue, because they realize that it's up to the schools to have their own internal, comprehensive assessment plan," says Braswell.

"In a classroom situation, students can have whatever time they need. They don't have to go to a test center under standardized conditions. Students can work cooperatively on projects that take several days to complete. So it is likely



that schools will assume the lion's share of the burden for the larger assessment projects.

"We have to make sure that as a testing organization, our evaluation is consistent with their efforts. We don't want to suggest that schools should be responsible for these comprehensive, insightful, creative projects while testing organizations emphasize topics like factoring — that would be totally out of alignment. But there are interesting and challenging problems that we can pose, too, that would be in line with the standards," Braswell says.

"What we hope for is that if the schools do their job, and if we do our job, then test scores will go up and everyone — the public, employers, colleges — will say, 'The students know a lot more today, they don't just reach in and pull out these little canned algorithms that they've learned. They know how to think, and to solve problems, and they can operate in a late 1900s environment.'"

Coordinated Programs

The dilemma of assessments also goes beyond determining who is the best person in the talent pool, Lesh says. "We need to develop the talented people that we find, but we also need to find more of these talented people."

Working for the past two years with a grant from the National Science Foundation, Lesh has developed a new mathematics assessment system that is the prototype for a new ETS product called Packets and is designed to mesh with the new standards for mathematics education.

The Packets system is a series of "math-rich newspapers," Lesh explains. The four-page tabloid-sized newspapers include reprinted articles from real newspapers that discuss math-related topics, such as grocery prices or business trends.

Teachers can use the newspapers as classwork, as homework, or as assessments. They meet the new standards in that:

- They are based in real-life situations.
- They provide opportunities for the teachers to explore students' problem-solving skills.
- They are interesting, which is especially helpful for students who do not perceive themselves as math-oriented.

"We'd like the teachers to take this material and use it to help identify kids with a different sort of mathematics talent," Lesh says. He explains that pilot tests have shown that this nontraditional program often helps teachers identify talented students who do not show up in traditional assessments, particularly girls and minority students.

Algebridge is another innovative assessment program that is especially beneficial for these students. Developed by ETS and the College Board, Algebridge was designed to help students master the change of thinking required in algebra — a subject that is often a stumbling block for college-bound students.

The Algebridge program materials "actually promote learning rather than just measuring the end results of the instructional process," says an article in *ETS Developments*.

"Algebra is a critical gateway through which high school students must pass if they are to pursue college-preparatory course work," says Paul Ramsey, Algebridge project director. By easing the transition from the concrete concepts of arithmetic to the abstract thinking of algebra, Algebridge makes the path to college smoother, particularly for the average or at-risk student.

A new program called Pacesetter has also been initiated by the College Board and ETS to improve instruction by coordinating curriculum and assessment. The first Pacesetter offering will be in mathematics and is scheduled for pilot testing in 1993-94. It will be followed by offerings in English, science, world history, and Spanish.

The Pacesetter course represents the fourth year of a high school mathematics curriculum, as outlined in the NCTM's *Standards*. Algebridge, in contrast, focuses on the earliest aspects of high school mathematics, explains Ernie Kimmel, executive director of Academic Services in the College Board Division at ETS.

The NCTM standards, Kimmel points out, provide general suggestions for teaching methods and course content, but do not provide teachers with specific recommendations for curriculum or assessments. Teachers traditionally base their coursework on the textbook they use and therefore "frequently choose to use the very superficial tests provided with the textbooks," Kimmel says.

The Pacesetter program will provide a better alternative, he says, because it was developed by a task force of school and college mathematics teachers chosen in consultation with the NCTM and the Mathematical Association of America.

Pacesetter mathematics includes an outline of course con nt, "meaty" coordinated assessments that require students to apply their knowledge, and coordinated teacher development opportunities.

All the material is based on what professional educators in the subject area feel that students should know and be able to do by the end of their high school years. Some students who take accelerated mathematics courses in seventh or eighth grade may take the Pacesetter course even before their final year of high school.

Modeled after the College Board's successful Advanced Placement (AP) program, Pacesetter Mathematics will be appropriate for students preparing to go on to calculus in AP courses or in college; for college-bound students interested in quantitative fields such as statistics, accounting, or science; or for students planning to enter the work force.

"The challenge, of course, is that Pacesetter seeks to raise the academic achievement levels of the broadest possible range of students," Kimmel says. "This course represents the task force's best conception of a fourth-year course that fills that need, while at the same time cross-referencing and making specific the relevant portions of the NCTM framework."

Taking a cue from the strategies used to develop the new standards for mathematics, the College Board is collaborating with a number of professional subject-matter organizations to develop the Pacesetter courses and assessment materials, which will include both classroom assessments during the course to aid instruction and end-of-course assessments on concepts and skills.

Teachers: The Key to Success

Clearly, the success of today's mathematics reforms rests with teachers. "The best assessment or curriculum materials in the world won't do any good unless we do something about teacher education," Lesh emphasizes.

The sad fact, however, according to the NCTM, is that teachers are not yet receiving the support and training they need to initiate changes in the classroom.

Yet the need for such training is acute. According to *Everybody Counts*, of the nation's 200,000 secondary school mathematics teachers, "over half do not meet current professional standards for teaching mathematics. Probably no more than 10 percent of the nation's elementary school teachers meet contemporary standards for their mathematics teaching responsibilities."

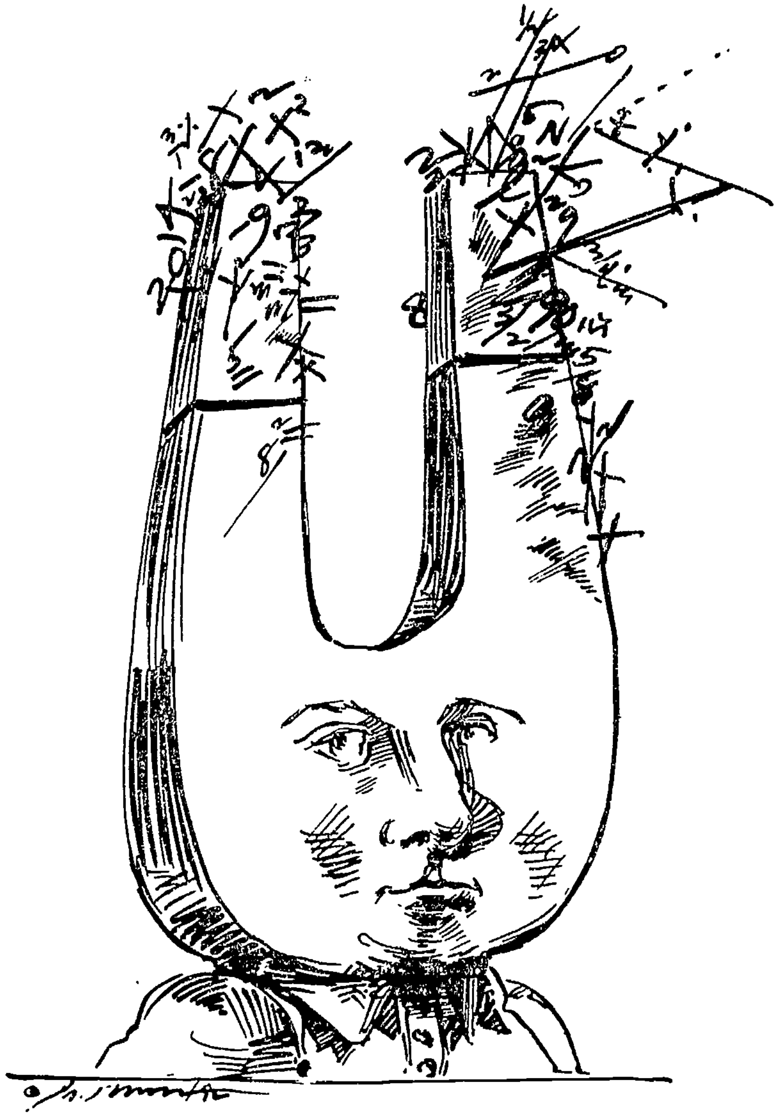
One reason for the dilemma is the inadequate supply of mathematics teachers, says *Everybody Counts*. "For nearly two decades, the number of students receiving degrees in mathematics has been declining, falling roughly 50 percent from its peak in the early 1970s."

Another reason, the report says, is "deficit financing of intellectual capital. When demand for mathematics in universities increased sharply during the last decade, most institutions responded either by increasing class size or by hiring underqualified temporary teachers."

The result has been that high schools "have been filling classroom positions with teachers whose qualifications are substandard." Elementary school teachers are also inadequately prepared, says the report. "...Typically, they take only one of the four courses in mathematics recommended as appropriate preparation for teaching elementary school mathematics."

As the "new math" experiment proved, short-term methods will not solve so deep a problem. Real change requires long-term, long-range, planned, and consistent training for teachers.

"The success of the standards depends on two things," says Jones. "How well they will be carried off by the classroom teacher, and how well the classroom teacher will be supported to be able to do it. That means formal training. And it



means monetary support for that training."

Davis and his staff at Rutgers University now run one of the few teacher training programs in the country that attempts to provide the sort of in-depth, on-the-job training that seems to be needed, but he finds that his program is frequently misunderstood.

"Our business here is what's called teacher development, and it's becoming clear to me that people don't realize what this is like. People will ask for a three-day workshop for teachers to train them in the new material, but that's not even in the right ball park. Our people have worked with some teachers for seven years now," Davis says.

"I tell people that this is a lot more like psychoanalysis than it is like telling somebody a new recipe. I can tell you a new recipe fast, but teachers have to reflect on what they do, and whether there's some other way to do it. What often happens now is that people concentrate on the more trivial aspects of it. I think that people have been underestimating the magnitude of the task."

"School districts aren't usually able to provide the kind of in-service support teachers need," Jones notes, "even if their leadership may embrace the concept. And if they don't come up with the program to support teachers and bring them along, then it's not as likely to be as successful.

"The standards are not a cure-all in and of themselves — it takes a lot of patience and a lot of time, and a firm and long commitment to be able to realize all of this."

Teachers should recognize, however, that they do not need to be experts in the new standards in order to begin using them, Cole points out. "The teachers will be learning, too, but in doing so, they can show their students that learning is not just a child's activity, it's every person's activity.

"I think if students realized the teachers are learning, just as they are trying to learn, it would set a different notion of what teaching and learning is," she says.

When children see that the teachers are experimenting with and learning new information, Cole says, "It would change the notion of teachers telling students what is right and what is wrong. After a while, you begin to realize that right-wrong is not totally the question.

"Looking at different aspects of a complex problem is one of the things the more complex performance exercises are able to explore. Instead of asking,'Is it right?' we need to ask instead, 'Is this a good enough solution, or would there have been a better way to do this?'"

"My view is that elementary teachers make a big, big difference," says Jones. "I think just by their attitude about mathematics, they can set a tone very early that is hard to reverse. My hat's off to them — they have to teach everything. But it would be nice if they could give math a fair shake.

"There are workshops and programs available for elementary teachers, but a large percentage of elementary school teachers can't take advantage of them. Their local school boards don't send them to those meetings. There are good materials and lots of resources out there, but many classroom teachers simply don't have access to them."

The Mathematical Sciences Education Board addressed this dilemma in its 1991 publication *Counting On You*, which was directed to school boards, administrators, and legislators.

"The key is *teacher professionalism*," the report states, "the assumption by teachers of more complete responsibility for the quality of education and the simultaneous provision by society of the respect and support teachers require to get the job done....

"To ensure that mathematics education in our schools is of the highest caliber, we must have well-prepared teachers who have the ability and authority to change within reasonable bounds the nature of their own roles and the nature of their classroom environments.... as schools evolve from a model with teachers as hired hands to one in which teachers function as professional educators, they should welcome the challenge to implement national standards for mathematics education."

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