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ABSTRACT

Education researchers have long been concerned with finding the appropriate method for correlational analysis of hierarchical data. In recent years, the alternative of hierarchical linear modeling (HLM) has come into extensive use. HLM users typically center some or all student-level predictors either at the grand mean or at the school means. This procedure adds stability to the estimation process and leads to intercepts that are more readily interpretable. Centering also has the effect of changing the coefficients that are being estimated, and cannot be regarded as merely a technical device, when, in fact, it changes the research questions that are actually being asked. Some issues in centering are addressed theoretically and empirically. A combined equation for the two levels of modeling is presented, and some algebraic manipulation is used to show how each form of centering can be expected to modify the estimated coefficients. Data from the National Education Longitudinal Study are analyzed to investigate the effect of minority status on a mathematics achievement test score, comparing results based on centering with those based on raw data. Two tables illustrate the analyses. (SLD)

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SOME OBSERVATIONS ON THE EFFECT OF CENTERING ON THE RESULTS OBTAINED FROM HIERARCHICAL LINEAR MODELING

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SOME OBSERVATIONS ON THE EFFECT OF CENTERING ON THE RESULTS OBTAINED FROM HIERARCHICAL LINEAR MODELING

Education researchers have long been concerned with finding the appropriate method for correlational analysis of hierarchical data. With students nested within schools, for instance, it is not valid to enter both student-level and school-level predictors into a single, ordinary least squares (OLS) form of multiple regression. (One of the several problems with this traditional approach is that the number of degrees of freedom for the school-level coefficients is overstated, which increases the risk of Type I error when testing these coefficients.)

In recent years, the alternative of hierarchical linear modeling (HLM) has come into extensive use. This analytic procedure models the predictors at their correct levels, and can be thought of as a two-stage process in which:

1) First, student outcomes are modeled as a linear function of student predictors within each school;

2) Next, the coefficients from these school-level models are themselves modeled as a linear function of school-level predictors.

[REF TO RAUDENBUSH & BRYK]

In fact, however, the coefficients for the two levels of analysis are estimated simultaneously, using empirical Bayesian estimation.

HLM users typically center some or all student-level predictors, either at the grand mean or at the school means. This procedure is advised for two main reasons: It tends to add stability to the estimation process, and it leads to intercepts that are more readily interpretable.

Centering also has the effect of changing the coefficients that are being estimated. The changes can be large when certain conditions obtain, and can lead researchers to very different conclusions, relative to those that they would have reached had they not used centering. This is not to say that centering is invalid or that it should be avoided. The point to note is that this procedure cannot be thought of solely as a technical device; It represents, rather, a change in the research questions that are being asked. And, of course, when different questions are asked, different answers can be expected.

This paper addresses issues of centering, both theoretically and empirically. First, a combined equation for the two levels of modeling is presented, and some algebraic manipulation is used to show how each form of centering can be expected to modify the estimated coefficients. Next, data from the National Education Longitudinal Survey (NELS) are analyzed to investigate the effect of minority status on a mathematics achievement test score; the resulting based on centering are compared with those based on raw data.

In addition to the centering issues, the paper is also concerned with the differences that may be expected between HLM results and analogous OLS results. To pursue this question, the empirical work was conducted using OLS, as well as HLM.

A TWO-STAGE MODEL FOR MATHEMATICS ACHIEVEMENT

Consider the prediction of a mathematics achievement test score from minority status data. We are concerned here with individual test scores, but wish to take account of the fact that students are nested within schools, and that minority status can be measured at both the individual level and the school level.

The use of a hierarchical model is indicated. The model is expressed as follows:

Student level: $y_{ij} = \beta_{0i} + \beta_{1i} x_{ij} + e_{ij}$, where

y_{ij} = math score for student j in school i

x_{ij} = minority status for student j in school i

= 1 for minority, 0 for non-minority

School level:

$\beta_{0i} = \gamma_{00} + \gamma_{01} \bar{x}_i + e_i$

$\beta_{1i} = \gamma_{10} + \gamma_{11} \bar{x}_i + e'_i$, where

$\bar{x}_i$ = proportion minority students in school i

The equations from the two levels can be combined to yield:

$$y_{ij} = \gamma_{00} + \gamma_{10} x_{ij} + \gamma_{01} \bar{x}_{ij} + \gamma_{11} x_{ij} \bar{x}_i + e_{ij} + e_i + e'_i x_{ij}$$

We will focus our attention on the coefficients, and will not dwell on the error structure. It will suffice, for present purposes, to consider the equation for the expected value of the outcome:

$$(1) \quad \hat{y}_{ij} = \gamma_{00} + \gamma_{10} x_{ij} + \gamma_{01} \bar{x}_{ij} + \gamma_{11} x_{ij} \bar{x}_i$$

Note that formulating the problem as a two-stage model leads to the inclusion of an interaction term in Equation (1); γ_{11} can be thought of as the effect of a contextual variable (school-level proportion of minority students) on the expected difference between minority and non-minority students.

THE EFFECT OF CENTERING THE INDIVIDUAL-LEVEL PREDICTOR

Equation (1) uses raw data, in the sense that the student-level predictor is coded in zero/one form. This is the traditional approach to estimating the coefficients by means of ordinary least squares (OLS) analysis.

In the hierarchical linear modeling (HLM) context, centering of student-level predictors is usually advised. Two centering transformations are in common use: centering at the grand mean, or at the group means (which in our case are school means). To gain some insight into the first of these options, let $\bar{\bar{x}}$ denote the grand mean of the minority status variable, i.e., the proportion of minority students across the total population of schools, and then rewrite Equation (1) as:

$$\begin{aligned}\hat{y}_{ij} &= \gamma_{00} + \gamma_{10}(x_{ij} - \bar{\bar{x}}) + \gamma_{10}\bar{\bar{x}} + \gamma_{01}\bar{x}_i \\ &\quad + \gamma_{11}(x_{ij} - \bar{\bar{x}})\bar{x}_i + \gamma_{11}\bar{\bar{x}}\bar{x}_i \\ &= (\gamma_{00} + \gamma_{10}\bar{\bar{x}}) + \gamma_{10}(x_{ij} - \bar{\bar{x}}) + (\gamma_{01} + \gamma_{11}\bar{\bar{x}})\bar{x}_i \\ &\quad + \gamma_{11}(x_{ij} - \bar{\bar{x}})\bar{x}_i \qquad (2)\end{aligned}$$

Thus, to the extent that the estimation process is unaffected by the centering transformation, we can expect grand-mean centering to:

1) Have no effect on the coefficients of either the student-level predictor or the interaction;

2) Change the intercept from γ_{00} to $\gamma_{00} + \gamma_{10}\bar{\bar{x}}$;

3) Change the coefficient of the school-level predictor from γ_{01} to $\gamma_{01} + \gamma_{11}\bar{\bar{x}}$.

The change to the school-level effect seems counterintuitive, since the centering operation affects only the student-level data, and is merely a linear transformation of those data. Still, it appears that grand-mean centering might make the difference between a statistically significant school effect and one that is not significant. (But note that this difference should occur only when a non-zero interaction effect is present.)

With school-mean centering, Equation (1) can be rewritten in the following way:

$$\hat{y}_{ij} = \gamma_{00} + \gamma_{10}(x_{ij} - \bar{x}_i) + \gamma_{10}\bar{x}_i + \gamma_{01}\bar{x}_i + \gamma_{11}(x_{ij} - \bar{x}_i)\bar{x}_i + \gamma_{11}(\bar{x}_i^2)$$

$$= \gamma_{00} + \gamma_{10}(x_{ij} - \bar{x}_i) + (\gamma_{10} + \gamma_{01})\bar{x}_i + \gamma_{11}(x_{ij} - \bar{x}_i)\bar{x}_i + \gamma_{11}\bar{x}_i^2 \quad (3)$$

It appears that, as with grand-mean centering, the coefficients for the student-level predictor and the interaction should not change, while the coeficient for the school effect should--in this case, from γ_{01} to $\gamma_{10}+\gamma_{01}$. The situation is further complicated, however, by a new term: the quadratic term in $\bar{x}_i$. Since this term is not part of the model, it can be expected to alter the coefficients in an unknown way.

The lack of equivalence between Equations (1) and (3) is not surprising, since group-mean centering, unlike grand-mean centering, is a more radical process than applying a linear transformation to existing predictors. The group-centered predictor is not a variant of the uncentered predictor; It is a new and different variable.

The analogues to Equations (2) and (3) can be developed for the case in which the school-level variable is substantively different from the student-level variable, rather than being an aggregation of the latter variable up to the school level. For grand-mean centering, the analogue to Equation (2) is essentially no different from that equation, suggesting that the effect of this type of centering can be predicted from the results of the uncentered model.

For group-mean centering, this is not the case. There is no useful way to group terms in the resulting equation, and the effect of centering is even less predictable than it was in the simpler situation.

The empirical work to be presented here is based on models that incorporate both types of school-level predictor: The aggregated version of the student-level measure of student minority status, and a school-level measure of faculty minority status. Note that the equations developed above are based on one predictor at each level, and cannot be expected to hold for more complicated models. They may, however, provide useful approximations to the results that are obtained empirically.

METHODOLOGY

The issues discussed above were investigated empirically, using data from the National Education Longitudinal Survey (NELS), a national longitudinal survey conducted by NCES. The survey is based on a two-stage design. For the base year, schools were sampled at the first stage, and then 8th grade students were sampled within schools. For the analyses to be presented here, the sample was restricted to ....

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All analyses included the following variables:

Outcome:	A (continuous) score on a mathematics achievement test
Student-Level Predictors:	Dummy variable for Black/non-Black Dummy variable for Hispanic/non-Hispanic Continuous measure of socioeconomic status Continuous measure of absenteeism/tardiness
School-Level Predictors:	Proportion of minority students Proportion of minority faculty Dummy variable for urban location Dummy variable for rural location Proportion of low-SES students

Note that, with this model, student-level minority status is treated as two variables: a dummy variable for Black, and a second dummy variable for Hispanic.

These variables were incorporated into HLM analyses in the following way:

1) The intercepts from the student-level analyses were modeled as functions of all school-level variables;

2) The slopes for the Black and Hispanic variables were modeled as functions of the two minority-proportion school-level variables;

3) The SES and absent/tardy variables were treated as fixed effects.

Thus, in addition to the main effects at the two levels of analysis, four interactions are included in the model: each of the student-level minority variables crossed with each of the school-level minority-proportion variables.

Three HLM runs will be presented: one with the two student-level minority variables uncentered, one with these variables grand-mean-centered, and one in which they are group-mean centered. (The other two student-level variables were group-mean centered for all runs.)

These three setups were also investigated using OLS.

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RESULTS: STUDENT MINORITY STATUS PREDICTORS

The coefficients and associated p-values are shown in Table 1, for all terms involving minority status and for all six of the models that are under consideration.

The results of the HLM analyses and the OLS analyses are essentially the same, and we will discuss these results in terms of the numerical values from the HLM analyses. (Note, though, that the similarity of the two methods, with regard to the p-values as well as the coefficients, is itself of some interest.)

Moving from no centering to grand-mean centering has virtually no effect on the coefficients. This is consonant with Equation (2), because the interactions in the uncentered model are close to zero and are not significant. However, the small change observed in the school-level coefficient (from 0.034 to 0.050) is positive, and this is consistent with the fact that the interaction coefficients (and the grand-mean proportion of minority students) are positive.

Going from no centering to group-mean centering increases the school-level effect from a non-significant value (gamma = 0.034, p = 0.783) to a significantly negative value (gamma = -0.187, p = 0.040). This change follows from Equation (3), since the (highly significant) student-level effect is incorporated into the school-level effect when group-mean centering is used.

RESULTS: FACULTY MINORITY STATUS PREDICTOR

The models that have been tested all contain a main effect for the school-level variable measuring the proportion of minority faculty, and for the interactions involving this variable and the two student-level minority status variables. The results of the analyses, for these three variables, are shown in Table 2.

The effect of group-mean centering on the coefficient of the school-level variable is again apparent, although this effect is much more pronounced for the HLM analysis than for the OLS analysis. It is important to note that Equation (3) holds only for the situation in which the school-level variable is the aggregated version of the student-level variable. When the two variables are substantively different, as is the case here, the effect of centering cannot be predicted in any simple way.

With the student-level minority status variables uncenterd, the HLM model shows a negative relationship between proportion of minority teachers and mathematics score (gamma = -0.176, p = 0.024). With group centering, the relationship is attenuated and is no longer significant (gamma = -0.053, p = 0.222). The directionality of the change seems surprising. The student-level effect, rather than augmenting the school-level effect, tends to cancel it.

Table 1. Results of HLM and OLS analysis of the effect of minority status on mathematics achievement score

Model		HLM		OLS	
		Coeff	Prob	Coeff	Prob
NO CENTERING	Stdnt-lev (B)	-3.612	(.000)	-3.706	(.000)
	Stdnt-lev (H)	-1.418	(.037)	-1.543	(.016)
	Sch-lev	0.034	(.783)	-0.021	(.842)
	Interact (B)	0.026	(.913)	0.133	(.526)
	Interact (H)	0.027	(.899)	0.129	(.468)
GRAND-MEAN CENTERING	Stdnt-lev (B)	-3.611	(.000)	-3.706	(.000)
	Stdnt-lev (H)	-1.418	(.037)	-1.543	(.016)
	Sch-lev	0.050	(.639)	0.058	(.492)
	Interact (B)	0.025	(.915)	0.133	(.526)
	Interact (H)	0.027	(.890)	0.129	(.468)
GROUP-MEAN CENTERING	Stdnt-lev (B)	-3.198	(.000)	-3.097	(.000)
	Stdnt-lev (H)	-2.199	(.009)	-2.129	(.016)
	Sch-lev	-0.187	(.040)	-0.163	(.013)
	Interact (B)	-0.209	(.488)	-0.221	(.481)
	Interact (H)	0.101	(.677)	0.112	(.661)

Stdnt-lev (B) = 1 for Black non-Hispanic, = 0 otherwise

Stdnt-lev (H) = 1 for Hispanic, = 0 otherwise

Sch-lev = proportion minority students in school

Interact (B) = interaction between Stdnt-lev (B) and Sch-lev

Interact (H) = interaction between Stdnt-lev (H) and Sch-lev

Table 2. Results of HLM and OLS analysis of the effect of the proportion of minority faculty on mathematics achievement score

Model		HLM		OLS	
		Coeff	Prob	Coeff	Prob
NO CENTERING	Sch-lev	-0.176	(.024)	-0.141	(.049)
	Interact (B)	0.227	(.017)	0.171	(.051)
	Interact (H)	0.159	(.078)	0.121	(.150)
GRAND-MEAN CENTERING	Sch-lev	-0.063	(.179)	-0.053	(.132)
	Interact (B)	0.227	(.017)	0.171	(.051)
	Interact (H)	0.159	(.077)	0.121	(.150)
GROUP-MEAN CENTERING	Sch-lev	-0.053	(.222)	-0.052	(.073)
	Interact (B)	0.316	(.007)	0.321	(.008)
	Interact (H)	0.171	(.106)	0.156	(.159)

Sch-lev = proportion minority faculty in school

Stdnt-lev (B) = 1 for Black non-Hispanic, = 0 otherwise

Stdnt-lev (H) = 1 for Hispanic, = 0 otherwise

Interact (B) = interaction between Stdnt-lev (B) and Sch-lev

Interact (H) = interaction between Stdnt-lev (H) and Sch-lev

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Grand-mean centering also affects the coefficient of the school-level measure, reducing it to -0.063, which is not significantly different from zero ($p = 0.179$). This change is predictable, since Equation (2) is valid, regardless of whether or not the school-level variable is an aggregated version of the student-level variable. The change is attributable to the significantly positive interaction effects, which partially cancel the original negative effect of the school-level variable.

CONCLUSIONS

As suggested by Equations (2) and (3), centering can alter the conclusions that the researcher draws from the data analysis. For models in which the same variable--student minority status, in our case--is being considered at both levels of analysis, the school--level component can assume signifilcance or lose significance as a result of centering.

For the particular dataset and models considered here, a researcher using uncentered or grand-mean centered student minority status would conclude that it is only the individual student's minority status that affects that student's mathematics achievement. But a second researcher, choosing to group-mean center the same variable, would conclude that the average minority status of the student's school also has a significant effect. Neither conclusion is incorrect. Each one, however, is tied to the centering upon which it is based.

Using Equation (2), the researchers can, to some extent at least, predict each other's results. But this is seldom done. The normal procedure is to select one or another centering option for reasons that are either technical or, if substantive, only defensible in vague terms, and then to interpret the results of the analysis without regard to the centering.

With regard to the significance of average faculty minority status, all researchers would agree that this variable interacts with the individual student's minority status to influence mathematics achievement. A researcher using uncentered data would also conclude that that the faculty variable itself affects achievement, while researchers using either of the centering options would not observe such an effect. Again, differences in centering lead to different findings; in this case, to findings that are more difficult to reconcile on an algebraic basis.

The above remarks are based on HLM analysis, but apply to OLS analysis as well; The effects of centering are not limited to HLM, but apply to more traditional forms of regression as well. Finally, it is interesting to note that no strong or consistent difference was found between HLM and OLS with regard to significance level of school predictors. It was expected, since OLS overstates the degrees of freedom for these predictors, that this procedure would be too lenient in finding significant relationships. However, among the six school-level tests shown in Tables 1 and 2, it is only the group-centered model shown in Table 2 for which this phenomenon is clearly present.

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