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ABSTRACT

Recommending goals and standards to improve mathematics education at the two-year college and lower-division levels, this report focuses on core content themes for developmental mathematics, associate in applied science, and baccalaureate intending programs. Following introductory sections, chapter 2 describes the CORE, or common content themes for the entire mathematics curriculum, indicating that all students should have number sense, symbol sense, spatial or geometric sense, function sense, probability and statistical sense, and problem-solving sense. This section also establishes goals for introductory college mathematics courses, standards for student outcomes, standards for instructional strategies, and guidelines for faculty selection and program assessment. Chapter 3 discusses the goals of developmental mathematics programs, providing guidelines for instructional strategies and student placement, standards for student outcomes, and sample problems to demonstrate the objectives of the standards. Associate of Applied Science (AS) degree programs are the focus of chapter 4, describing CORE goals, instructional strategies, a model AS curriculum, and standards with sample problems, while chapter 5 provides curriculum standards and sample problems for associate programs that lead to a baccalaureate degree. The final chapter outlines procedures for developing these standards for two-year college mathematics and mechanisms to ensure implementation, including regional workshops highlighting examples of change. Includes 26 references. (ECC)

ED 362 248

STANDARDS FOR CURRICULUM

AND

PEDAGOGICAL REFORM

IN

TWO-YEAR COLLEGE AND LOWER DIVISION MATHEMATICS

Circulating Draft

October, 1993

The opinions expressed in this document do not necessarily reflect those of the AMATYC membership or either of the funding agencies.

Published by

American Mathematical Association of Two-Year Colleges

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JC 930 475

Curriculum and Pedagogical Reform in
Two-Year College and Lower Division Mathematics Project

Members of the Project Task Force

Task Force Co-Directors: Marilyn Mays and Karen Sharp
Facilitator: Dale Ewen
Editor: Don Cohen

Core
Co-Chairs: Nancy Angle* and Richelle Blair*
Members: Susan Forman, Judith Hector, William Thomas, Jr., Kathie Yoder

Developmental
Co-Chairs: Max Cisneros, Jr., and Betsy Darken
Members: Geoffrey Akst, Cheryl Cleaves, Edward Laughbaum*, William Thomas, Jr.
Advisor: Myrna Manly

AAS (Technical/Occupational)
Co-Chairs: Robert Kimball, Jr., and Susan Wood*
Members: Darrell Abney, Susan Forman, Margie Hobbs

Liberal Arts
Co-Chairs: Linda Boyd and Gregory Foley*
Members: Carol Edwards, Judith Hector, Dean Priest

Precalculus Group
Co-Chairs: James Leitzel* and Stephen Rodi
Members: David Dudley, Bobbie Righi, Sam White, Kathie Yoder

Consultants
Sol Garfunkel, Harvey Keynes, P. Uri Treisman

Note: *Members of the editorial panel.

(continued on the back inside cover)

IF YOU ARE ATTENDING THE NATIONAL CONFERENCES OF AMATYC, AMS/MAA, OR NCTM, PLEASE PLAN TO ATTEND THE HEARINGS FOR THIS DOCUMENT AND BRING THIS COPY OF THE DOCUMENT WITH YOU.

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READERS PLEASE NOTE

The readers should note that this circulating draft of *Standards for curriculum and Pedagogical Reform in Lower Division Mathematics* is clearly a document in transition. It contains several parenthetical comments and questions that need to be addressed by the writing teams before the finished document is produced.

AMATYC

American Mathematical Association of Two-Year Colleges

October, 1993

Letter to Reviewers:

This document is a work in progress. All readers are reviewers and are invited to contribute to the further development of what we hope will become a starchart to guide students, faculty and administrators into the twenty-first century in mathematics education.

The members of the Task Force which produced the document met for six days this summer to discuss major issues of content and format. Work since that time has taken place via mail, telephone, and e-mail. A non-circulating draft was sent in August to a few reviewers for comments. Task Force members and individuals associated with the project will be meeting with each other and interested mathematics educators throughout the fall and winter to discuss reviews and areas requiring additional work.

Your input to the discussion of the document is invited in several ways:

(1) Attend the related sessions at the national conferences of AMATYC, MAA, and NCTM. **Please bring this copy of the document with you to the meeting.** The sessions are for the purpose of answering questions and receiving reaction from reviewers. A schedule of these presentations is attached.

(2) Send your written comments and contributions as soon as possible but **by April 1, 1994,** at the latest to the Editor: Don Cohen, SUNY at Cobleskill, Cobleskill, NY 12043; Bitnet address: COHEN@SNYCOBVA

You are encouraged to submit the following in addition to reviews of the document:

(a) problems or examples (supply references if problems are not original with you)

(b) classroom methods or learning environments that have proven successful

(c) information about successful programs which incorporate the concepts of proposed standards

(d) ideas for the dissemination and implementation of the standards which are being developed

This information supplied by reviewers may be included in the standards document, in an appendix, or compiled into a companion document.

(3) Work in your AMATYC affiliate, MAA section, or NCTM or NADE regional meeting to collectively review the document, provide an organizational response, and suggest future actions.

This draft is being sent to the AMATYC membership, officers and chairs of selected committees of several professional organizations, and other friends of mathematics education. Please circulate it to your department, full-time and part-time faculty and teaching assistants, and encourage responses.

We gratefully acknowledge the efforts of all who have given their time and support and wish to thank the National Science and Exxon Education Foundations for funding.

We can no longer build bridges across gaps in mathematical education; we must go about transforming the learning process. That will happen only with the participation of the professoriate most directly involved. We look forward to hearing from you.

Sincerely,

Marilyn Mays

Marilyn Mays, Project Director

Scheduled conference presentations:

November 18, 1993, AMATYC National Conference in Boston
"Hearing on Reform on Curriculum and Pedagogy"
7:30 - 9:00 PM Independence Center in the Sheraton Boston Hotel and Towers

January 14, 1994, MAA National Conference in Cincinnati
"Hearing on the Project on Standards for Two-Year College and Lower Division Mathematics Below the Level of Calculus"
9:30 - 10:55 AM (Room to be announced)

April 15, 1994, NCTM National Conference in Indianapolis
"AMATYC Reports on Development of Standards for College Courses Below Calculus"
12:00 noon - 1:00 PM (Room to be announced)

PREFACE

Much has recently been written about curriculum and pedagogical reform in mathematics education. For example, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) presents revolutionary approaches to curriculum and pedagogy for the precollege level, *Moving Beyond Myths* (National Research Council, 1991) calls for dramatic changes that will "revitalize" undergraduate education and *Everybody Counts* (National Research Council, 1989) makes specific recommendations for transitions that schools must make in mathematics programs from kindergarten through graduate school. Furthermore, serious attention has been paid to calculus reform (see Crocker, 1990, for a description of calculus reform efforts). However, none of the major reform documents **specifically** addresses the curriculum and pedagogical reform that is needed in two-year college and lower-division mathematics programs below the level of calculus. The purpose of this document, *Standards for Curriculum and Pedagogical Reform for Two-Year Colleges and Lower Division Mathematics,* is to recommend goals and standards to improve mathematics education at this level.

While building upon the recommendations contained in the reform documents mentioned above, these standards are designed for students who may have some or all of the following characteristics:

1. often have inadequate mathematical preparation and require special instructional strategies,

2. are members of the diverse groups comprising the "new" student population (for example, minority students, older students, students who are parents, and students who work full-time) so no single instructional strategy will suffice,

3. are majoring in vocational/technology programs or in preprofessional programs where mathematical needs may differ from those needed by students majoring in mathematically related fields, and

4. have special learning needs.

The **CORE** chapter includes goals for introductory college mathematics, standards for instructional strategies, and standards for student outcomes. The goals and standards presented are themes that apply throughout the mathematics curriculum. The goals include increasing participation by all underrepresented groups, providing experiences that build student confidence as well as the ability to learn independently, and presenting mathematics as a developing human discipline. The standards include the use of appropriate technology, the use of interactive learning strategies, the active involvement of students in meaningful problem solving and the use of multiple approaches to problem solving. Recommended content themes include number sense, symbol sense, spatial sense, function sense, probability and statistics, and problem solving sense. Appropriate assessment strategies are also discussed.

The chapters that follow on developmental mathematics, Associate in Applied Science Degree (technical/vocation) programs and baccalaureate-intending programs extend the goals and standards presented in the **CORE** to each particular area.

It is of particular interest that while the standards included in this document are in the spirit of and reflect many of same beliefs as the NCTM *Standards* and other current reform documents, they are distinct and are focused on the needs of the particular group of students targeted. In particular,

- **Developmental Mathematics** strongly recommends that such programs simply not be repeats of courses that are offered at the high school level. Arithmetic, algebra, and geometry should be integrated into an in-depth applications driven curriculum. The goal of this curriculum is to prepare students for college-level introductory mathematics as efficiently as possible.

- **AAS Degree Programs** place strong emphasis on using real applications in the classroom. It is the responsibility of the instructors involved to customize material to meet the individual needs of students in their courses. For example, electronics majors can explore the voltage waves using oscilloscopes. Appropriate materials should be developed in collaboration with technology instructors or outside practitioners.

- **Baccalaureate-Intending Programs** provide curriculum standards for **all** students enrolled in programs that lead to bachelors' degrees. Heavy emphasis is placed on a common curriculum that includes functions, approximation and numerical estimation, modeling, logical reasoning, patterns, and probability and statistics. Students who are enrolled in major programs in such areas as the mathematical sciences, science, business and economics would then head toward the study of calculus, while students in such liberal arts disciplines as the humanities, elementary and middle grades education, nonscience secondary education and the social and behavior sciences would have the opportunity to study calculus but would probably head toward the concentrated study of statistics, decision making, and social choice.

In addition to the recommendations on instruction and content, chapters two through five include numerous examples that show how to implement the ideas that are presented. The examples are meant to be used and improved upon by our readers. The final chapter of the document explains how the task force plans on disseminating its recommendations to the mathematics community.

[THE COVER WILL BE COMPLETED LATER.]

The cover introduces the reader to a metaphor that guided the thoughts of the task force writing teams: a journey along a highway. Introductory college mathematics can be thought of as a journey for adult students. It is a journey along a highway that should provide access to higher education for all students.

The task force is indebted to each of the consultants who attended the Memphis meeting. Sol Garfunkel, Harvey Keynes, and Uri Treisman are "in-tune" with the needs of students who study introductory college mathematics. Their ideas truly helped with the preparation of this document.

It was the goal of the task force to present a document that would stimulate instructors to improve their courses. These standards and examples are not meant to be the final word. In fact, they are a starting point for your actions.

Don Cohen
Editor

HELP WANTED

The readers of this draft document are invited to contribute to the writing and design of the finished document in the following ways:

1. Send your suggestions on how the writing teams can improve the contents of this draft document to the editor by April 1, 1994. Examples that can be used to involve students in the type of mathematics education discussed in this document are especially welcome. Comments may be submitted on separate paper or as annotations on page copies.

2. The CPR Project Task Force has discussed the appropriateness of mentioning specific reform projects by name in the document. The Task Force feels that by using specific reform projects to exemplify the curricular and pedagogical standards contained in this document, we will be endorsing certain programs without having the appropriate research evidence to justify the endorsement. In addition, by mentioning some projects we might be slighting others.

   On the other hand, we feel that it is very appropriate to call our readers' attention to curricular and pedagogical innovations that appear "to work." In this regard, you are invited to send the editor very brief descriptions of innovative procedures that you use that exemplify the standards contained in this document. The writing teams may use your contributions within the exposition to offer ideas for the readers to test out for themselves. In addition, a list of your contributions will be included in an appendix. The contributors' names will be listed as contact persons so that interested readers can contact the contributors for additional information.

3. The preface of this draft document indicates that the cover design will be completed later. In this regard, you are invited to help us complete it. While the idea for the "journey along a highway" metaphor has been adopted, a specific graphic design or picture to illustrate the metaphor has yet to be developed. In addition to the cover design, art work is also needed for chapter introductions. If you are artistically talented and would like to contribute to the production of a final document that will "catch the eye" of its readers, please contact the editor to discuss your ideas.

Don Cohen, Editor
SUNY
Cobleskill, NY 12043
(518) 234-5116
E-Mail COHEN@SNYCOBVA

FOREWORD

Higher education is situated at the intersection of two major crossroads: an increasingly diverse and often academically underprepared group of students seeking entrance to postsecondary education, while the public and private sectors face growing needs for a workforce adequately prepared in the areas of mathematics, science, engineering, and technology. These disciplines, however, are experiencing decreases in the number of students entering mathematics as well as the number receiving degrees. According to *The Challenge of Numbers* (MSEB, 1990, p. 38) from 1966 to 1987, the number of entering college freshman expecting to major in mathematics decreased by approximately eighty percent. The number of bachelors' degrees also decreased sharply until 1981. There has been a gradual increase since that time, but the number of degrees awarded each year is still less than half of that in 1970. These challenges mandate consideration of revision and reform in what is being taught in the nations' institutions of higher learning and how it is being taught.

Increasing numbers of students come to postsecondary education unprepared for what is considered by many institutions as the gateway course--calculus. Many of those students are seeking careers in which there is no immediate need for calculus. All students, however, need mathematical preparation for their immediate goals which also open the gateway for achievement of long term goals that might be more mathematics-dependent.

The faculty who teach these students in two-year colleges and in four-year colleges and universities can play a major role in the reform effort taking place in undergraduate mathematics curriculum and pedagogy. Preparation of this document has been guided by the principle that faculty must help their students think critically, find motivation for the study of mathematics and an appreciation for its relevancy, and learn how to learn.

The purpose of *Standards for Curriculum and Pedagogy Reform in Two-Year College and Lower Division Mathematics* is to revitalize the content of the mathematics curriculum and to stimulate changes in instructional methods so that students will be engaged as active learners in worthwhile mathematical tasks. It represents a major effort of the American Mathematical Association of Two-Year Colleges (AMATYC), assisted by representatives of most of the national mathematics education organizations.

AMATYC's previous efforts to improve mathematics education at the two-year college level have taken the form of development of policy statements and guidelines. The most notable recent efforts have been *Guidelines for the Academic Preparation of Mathematics Faculty at Two-Year Colleges* (AMATYC, 1992) and *Guidelines for*

Mathematics Departments at Two-Year Colleges (AMATYC, 1993). AMATYC received impetus for additional efforts toward education reform when in August of 1991 the Mathematical Sciences Education Board (MSEB) convened a small planning group at the National Academy of Science to discuss two-year college mathematics education. Several members of the AMATYC leadership met with representatives from MSEB and others interested in two-year college mathematics education. This meeting and the subsequent efforts of the representatives of MSEB served to focus and launch the initiative.

The work of the planning committee led to the development of a multiple phase proposal designed to address curriculum and pedagogy reform initiatives at two-year colleges. The Exxon Education Foundation provided $50,000 to assist with the initiative; AMATYC committed substantial funding while seeking additional funds from other sources. Subsequently, the focus of the initiative was broadened to include all lower division mathematics education up to calculus, and the proposal was submitted to the National Science Foundation (NSF). Marilyn Mays, AMATYC President-elect, was designated Principal Investigator, as well as Karen Sharp, AMATYC President, and Dale Ewen, AMATYC Past President, as Co-Principal Investigators. A steering committee was formed with representation from the American Mathematical Society (AMS), Mathematical Association of America (MAA), MSEB, National Council of Teachers of Mathematics (NCTM), and the National Association for Developmental Education (NADE). This group met in February, 1993, in Washington, D.C., to plan the process for the development of a standards document. In March, 1993, the project received funding of approximately $80,000 from NSF.

The Steering Committee determined that the following principles should guide the deliberations of the Task Force:

- Fundamental changes are occurring in mathematics education as a result of the impact of the NCTM *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Professional Standards for Teaching Mathematics* (NCTM, 1991), the MSEB *Moving Beyond Myths* (National Research Council, 1991), calculus reform efforts, technological advances, and research into how students learn.

- Colleges and universities are preparing a diverse group of college students for further study and the world of work.

- While two-year college and lower division mathematics students are preparing for a multitude of future occupations, a common core of mathematical experiences, viewpoints, concepts, and skills needed by all students exists and should be identified.

- The manner in which students learn is inseparable from the content.

- Research regarding how students learn should be utilized fully in development of new pedagogical methods and in implementation of proven teaching techniques.

- The impact of technology both as a mode of instructional delivery and as a tool for learning requires a redefinition of the mathematics curriculum.

- Demands of the workplace require that all students become empowered citizens capable of critical thinking.

The Steering Committee further directed that the Task Force be divided into the following groups to address mathematics education from several perspectives:

Core	Common mathematical knowledge and experiences necessary for all students.
Developmental	Mathematical preparation specifically designed for students not prepared for college level courses.
AAS Degree (Technical/ Occupational)	Preparation for Associate in Applied Science degree students planning to continue in applied programs or enter the workforce upon attaining the two-year degree.
Liberal Arts Programs	Preparation for students seeking a degree in a nonscience major.
Precalculus	Preparation for students who will take calculus and seek a bachelor's degree.

The Curriculum and Pedagogical Reform in Two-Year College and Lower Division Mathematics Task Force was appointed by the National Steering Committee and assigned the responsibilities. The members of the Task Force and their respective assignments, along with the members of the Planning Group and Steering Committee, are listed on the insides of the front and back cover.

Several activities were completed in preparation for the meeting of the Task Force. Members of the Task Force reviewed the NCTM *Curriculum and Evaluation Standards for School Mathematics* and *Professional Standards for Teaching Mathematics*, the MSEB *Moving Beyond Myths*, and other references on mathematics reform provided by the Task Force Co-Directors. They wrote position statements detailing their vision of the curriculum, pedagogy, and assessment reforms needed. These were distributed to the Task Force prior to the meeting.

The Task Force met in June 1993 at State Technical Institute at Memphis to agree on a common vision and to draft this document, *Standards for Curriculum and Pedagogical Reform in Two-Year College and Lower Division Mathematics*. Those drafts were refined in the next few weeks by the participants. Editor Don Cohen pulled them together into a common format and produced a non-circulating draft which went back to the Task Force, to the National Steering Committee, and to a few reviewers and leaders of the mathematics education community. The document has been further refined. It is considered, however, by those of us who have been most involved in its development as an important first step. We encourage readers of this draft to take the next steps with us.

We are grateful to the National Science Foundation and the Exxon Education Foundation; to the members of the Planning Committee, the National Steering Committee, and the Task Force; and to the many friends of AMATYC for their support.

Karen Sharp
Project Co-Director
AMATYC President (1991-93)

Marilyn Mays
Project Director
AMATYC President (1993-95)

Dale Ewen
Project Co-Director
AMATYC President (1989-91)

CHAPTER 1 - Introduction

The Need for Change

Education, as well as life, is a journey. A century ago people prepared for their life's work by following in the footsteps of their parents, through formal or informal apprenticeship or education. The industrial revolution created new jobs and, consequently, precipitated the development of education for those jobs. Today's students must be educated for jobs that do not exist at this time. The world that today's students will live and work in is one that their parents may not even imagine. Education must be a preparation for the lifelong journey of learning.

As *A Challenge of Numbers* (MSEB, 1990) states, "More new jobs will require more postsecondary mathematics education. The rate of growth in mathematically based occupations is about twice that for all occupations" (p. 9). Yet, this publication tells us, "If current trends continue, these (socioeconomic and demographic) projections indicate reduced numbers of both college students and persons choosing mathematically based occupations" (p. 13). The first fact is dramatically illustrated by a conclusion of a publication that accompanied a teleconference hosted by Bill Moyers called "Investing in Our Youth". That publication states that, "In this complex and changing environment, the U.S. can no longer afford to waste the potential of even one [person]. In 1990 and beyond, three out of every four jobs will require education or technical training beyond high school. Projections for the year 2000 are that new jobs will require a work force whose median level of education is 13.5 years" (Palaich, Whitney, & Paolino, 1991, p. vii). Yet, at the same time, in 1990, the Children's Defense Fund (1990) indicated that of those who stay in school, half graduate (from high school) without reading, mathematics and science skills that would allow them to perform moderately complex tasks.

Everybody Counts (1989) asserts that more than ever before, Americans need to think for a living; more than ever before, they need to think mathematically. As the foundation of science and technology, mathematics provides the key to opportunity. This report also states that

> the mathematics curriculum is in need of extensive overhaul which must be implemented following national dialogue and a consensus on the changes to be implemented. Such changes should include a focus on the need for the learning of a significant common core of mathematics for all students, a student-centered learning environment rather than a teacher-centered lecture environment, a shift on the part of public attitudes from

indifference and hostility to recognition of the important role that mathematics plays in today's society, a shift from teaching routine skills to developing broad-based mathematical power by fostering the following skills in students:

- performing mental calculations and estimates with proficiency;
- solving problems requiring higher-order thinking skills;
- using the new technology proficiently;
- using tables, graphs, spreadsheets, and statistical techniques to organize, interpret, and present numerical information; and
- judging the validity of mathematical information presented by others. (pp. 81-83)

Furthermore, *Reshaping School Mathematics* (1990), includes the following recommendations:

- Mathematics education must focus on the development of mathematical power.
- Relevant applications should be an integral part of the curriculum.
- Each part of the curriculum should be justified on its own merits.
- Curricular choices should be consistent with contemporary standards for school mathematics.
- Mathematics instruction at all levels should foster active student involvement. (pp. 36-39)

Audience and Purpose of this Document

This document has been written for persons interested in the reform of introductory college mathematics education below the level of calculus. This statement, however, is subject to a variety of interpretations, depending upon the educational orientation of the reader. Both the phrase, "introductory college mathematics" and the students involved must be defined.

Many colleges, especially two-year colleges, serve diverse and unique student populations. While assisting students to obtain bachelors' degrees is part of the two-year college mission, the mission also includes offering two-year occupational degree programs for students whose imme 'ate goal is entering the workforce. In addition to serving recent high school graduates, many colleges serve significant numbers of returning adult students. These mature students usually commute and have a full- or part-time job, community involvements, and family responsibilities.

Two-year colleges and universities serve large numbers of students whose mathematics accomplishment is usually classified as precollege. Another large percentage of the student population need preparation for calculus. In all, seventy percent of course offerings in two-year college mathematics departments and approximately fifty percent of those at universities are below the level of calculus (Albers, D.J., Loftsgaarden, D.O., Rung, D.C., & Watkins, A.E., 1992). From these statistics it can easily be determined that this document is intended to address the needs of at least half of beginning college students.

This document attempts to identify the curriculum, pedagogy, and assessment strategies for students of introductory college mathematics below the level of calculus. The content of the current courses in which the students that comprise the targeted population are enrolled includes basic mathematics (arithmetic), algebra, geometry, and trigonometry.

Vision of Introductory Collegiate Mathematics

Reform of introductory collegiate mathematics education will be successful only if it is comprehensive and fully integrated throughout the targeted curriculum. Useful mathematical topics must be introduced in such a way that they build on each other logically. Technology must be used appropriately and to its fullest potential. Instructional delivery and assessment must be active, student focused, motivational experiences.

Content must be refined to be made more relevant to today's world and to reflect the needs of students and their future employers. The curriculum must stress the content the college student needs rather than attempting to repeat high school content in order to rectify any student deficiencies. Instead of attempting to teach numerous special cases that a student might need to know in the future, we must teach students how to prepare themselves for further mathematical study.

These experiences with mathematics should be designed to equip citizens to participate intelligently in civic affairs, help adults solve problems of everyday life, impart a major element of human culture, and prepare individuals for jobs, vocations, professions, and continued study. Introductory college mathematics should:

- introduce the student to the discipline of mathematics;
- emphasize the goals and techniques of the mathematics practitioner;
- deepen and broaden the student's mathematics experiences;
- engage the student in the discipline of mathematics; i.e., involve the student in exploring and in making and defending conjectures; and
- develop in students positive self-concepts with regard to mathematics ability.

Structure of the Document

The common mathematical experiences which all students should have, called the "CORE," is identified in Chapter 2. This is followed by a discussion of the particular appropriate mathematical experiences necessary for:

- Developmental Mathematics,
- Associate in Applied Science Degree Programs, and
- Baccalaureate-Intending Programs,

in Chapters 3 through 5, respectively. It should be noted that these classifications do not imply that there should be a strict separation of program areas. They denote the mathematics program in which students start but not necessarily where they finish. For example, many students enrolled in AAS degree programs at two-year colleges intend to transfer to four-year colleges to pursue bachelors' degrees.

Implementation of the standards is discussed in Chapter 6. This implementation will require a concerted effort by individual faculty, departments, colleges, and professional organizations. Local, state, and regional consortia will need to be formed in order to provide a framework for the development and dissemination of materials.

CHAPTER 2 - The CORE: Themes throughout the Mathematics Curriculum

Mathematics Education: A Life-Long Journey and the Need for Far-Reaching Reform

Mathematics is a vibrant, growing field of inquiry in which new knowledge is being developed daily by researchers throughout the world. Mathematics plays an ever larger role in all phases of society. It is being used in more ways by more people than ever before. For example, statistical methods are employed regularly by researchers testing hypotheses, by workers applying quality control in manufacturing, and by informed citizens who must evaluate information from the media in tabular, graphical, and report form in order to reach conclusions.

As the need to use mathematics becomes more pervasive, mathematics education is changing at all levels. This document acknowledges the important influence of visions for change already articulated in *Everybody Counts* (National Research Council, 1989) and National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). It will focus on introductory college mathematics and the call to revitalize undergraduate mathematics found in *Moving Beyond Myths* (National Research Council, 1991).

It is a basic premise of this document that educated adults must know and be able to do mathematics in order to be personally and professionally competent in this changing world. A much used metaphor compares mathematics education to a filter which year by yeaı eliminates students from the mathematics pipeline. As a result, some groups are underrepresented in fields requiring mathematical competence and some individuals fail to achieve quantitative literacy to become productive citizens. Postsecondary mathematics education must offer multiple entrance and exit points so that all students can be successful, including women and minority students who have been traditionally underrepresented in mathematics and science; students with special academic, economic, and physical needs; students moving directly from high school; and mature adults retraining for new careers or exploring lifelong learning opportunities.

This document proposes a new metaphor for mathematics education - that of a life-long journey involving the learning of mathematics at different life stages. On this journey, introductory college mathematics can be seen as one segment of a broad highway to success. Access to success for adults from many backgrounds requires multiple entrance ramps to the highway. Once on the highway, students should be able to change lanes easily. This document sets forth standards for introductory college mathematics programs that will make such a journey a reality.

Vision for Introductory Mathematics

Introductory college mathematics should develop students' mathematical powers beyond the level at which they enter, or which they attained in elementary and secondary education. Curriculum and pedagogical standards for two-year college and lower division mathematics must assure broadening of mathematical knowledge and development of mathematical depth and coherence. All courses in the college mathematics curriculum, regardless of level and sophistication, need to be based on a common set of mathematical themes, connecting threads, and shared methods of thinking and communicating. These common themes are called "The CORE." The CORE does not represent a rigid curriculum. Instead, it is the underlying philosophical set of assumptions and practices on which all college mathematics courses should be built.

Mathematics and its use should permeate all courses in the college curriculum. Every opportunity should be seized to reinforce the learning and utilization of mathematics across the curriculum. Institutions and individual faculty must take an active role in addressing the needs of diverse students, in providing a supportive environment for underrepresented groups, and in improving the curriculum and instructional strategies. Departments should make every effort to remove obstacles to success from the paths of all students. Postsecondary education should develop a wider range of connections among topics both within mathematics and between mathematics and other disciplines.

Adult students entering introductory college mathematics programs today bring a richness of diversity to our campuses that challenge educators to define clear goals and standards, develop innovative instructional strategies, and present mathematics in appropriate contexts. Introductory college mathematics should provide experiences for students that link previous mathematical experiences with mathematics necessary for success in a career, to be productive citizens, and to be equipped to pursue lifelong learning. The students served seek associate degrees and bachelors' degrees with majors in liberal arts, education, medicine, law, mathematics, the sciences, and engineering leading to a variety of careers. Today, and in the future, introductory college mathematics will be required to meet the needs of a diverse student population. Some young adults enter college after completing high school with a strong preparation in mathematics. Others decide on college with the minimal preparation specified by requirements for a diploma or after studying for a General Education Diploma (G.E.D.). Still other adults enter college after a number of years away from formal schooling. They have varying preparations in mathematics and varying levels of retention of earlier school experiences.

Recognizing this diversity in mathematical backgrounds and levels of competency, introductory college mathematics should **introduce** a diversity of students to **mathematics** as a **discipline**. It must:

- provide opportunities and experiences for all students to achieve the goals of the NCTM *Standards* (NCTM, 1989) and move beyond them.

- provide opportunities and experiences beyond high school at many levels for students to deepen their understanding of mathematics, their way of thinking about mathematics, and the techniques they use when doing mathematics.

At the conclusion of their lower division collegiate studies, all students should have developed certain general intellectual mathematical abilities as well as specific competencies and knowledge. Just as introductory college courses in psychology, chemistry, or history attempt to broaden a previous educational foundation, an introductory college mathematics program should invite students to consider the discipline of mathematics as a dynamic life-long journey. As presented in this chapter, the **CORE** outlines themes throughout the curriculum which provide the foundation on which success in advanced mathematics, general collegiate study, workforce training, and lifelong learning rests. This CORE of mathematical literacy is the basis for an informed and productive life.

Framework for Mathematics Standards

The CORE for mathematics in the two-year college and lower division outlined in this document is consistent with frameworks presented in earlier mathematics reform initiatives. In order to achieve the national goal to be first in the world in mathematics by the year 2000, new standards for collegiate mathematics should embrace the goals of the national mathematics community.

For example, *Reshaping School Mathematics* (MSEB, 1990) presented a practical philosophy for an improved mathematics curriculum at the K-12 level by outlining the following goals

- A Practical Goal: To help individuals solve problems of everyday life.

- A Civic Goal: To enable citizens to participate intelligently in civic affairs.

- A Professional Goal: To prepare students for jobs, vocations, or professions.

- A Cultural Goal: To impart a major element of human culture.

The Mathematical Sciences Education Board in *Moving Beyond Myths* (National Research Council, 1991) addressed undergraduate mathematics education reform with these goals:

- Effective undergraduate mathematics instruction for all students.
- Full utilization of the mathematical potential of women, minorities, and the disabled.
- Active engagement of college and university mathematicians with school mathematics, especially in the preparation of teachers.

In developing and defining standards for introductory college mathematics, the five general goals for all students in the NCTM *Curriculum and Evaluation Standards* (NCTM, 1989) are to be extended and developed more deeply. Those general goals for all students are

- learning to value mathematics,
- becoming confident in one's own ability,
- becoming a mathematical problem solver,
- learning to communicate mathematically, and
- learning to reason mathematically.

These sets of goals listed above, while not exhaustive, form the framework within which the standards for mathematics in the two-year college and lower division are designed. Accomplishing these goals will require the cooperative efforts of institutions, the entire college faculty, professional societies, the private sector, and legislators. The time for a new curriculum and for new teaching methods is now.

Curriculum standards for mathematics in the two-year college and lower division for the CORE, and for developmental, technical/vocational, and baccalaureate-intending programs are outlined in this document. In this chapter, standards for the CORE are presented, along with appropriate instructional strategies, student outcomes, and suggested content. This foundation for continued learning will enable **all** students to widen their views of the nature and value of mathematics and to become more productive citizens.

Goals for Introductory College Mathematics

Goals for all introductory college mathematics courses extending beyond the goals of elementary and secondary education are:

Goal 1: **Introductory college mathematics will increase participation (in mathematics and in careers using mathematics) for all students -- including women, minorities, and students with learning difficulties, differing learning styles, and language and socialization difficulties who have been traditionally underrepresented.**

Efforts must be made to attract and retain persons in mathematics who have not traditionally sought careers in mathematics, science, engineering, and related areas. By the year 2000, the U.S. economy is expected to create more than 21 million new jobs, most of which will require postsecondary education and the use of mathematics (MSEB, 1990). Jobs requiring mathematical skills, such as data analysis, problem solving, pattern recognition, statistics, and probability, are growing at nearly double the rate of overall employment. In the past, white males usually filled such jobs. By the year 2000, over 40% of new workforce entrants will be minorities and immigrants (National Center on Education and the Economy, 1990). It is the challenge and the opportunity of the mathematics community to empower this new workforce to succeed in the changing work environment.

Goal 2: **Introductory college mathematics will provide rich, deep experiences that encourage independent, nontrivial exploration in mathematics, build tenacity, and reinforce confidence in the ability to use mathematics appropriately and effectively.**

"To function in today's society, mathematical literacy ...is as essential as verbal literacy" (MSEB, 1989, p. 7). One needs to understand basic mathematical ideas to read the daily newspaper and cope with our technological society. This mathematical literacy, or numeracy, must go beyond the ability to do arithmetic. A productive citizen needs to be able to read and interpret graphs, tables, and mathematical information useful for making decisions. "Mathematical literacy is essential as a foundation for democracy in a technological age" (MSEB, 1989, p. 8). In addition, students must be convinced that they can succeed at mathematics, and that they can, on their own and with the cooperation of their colleagues, make mathematical choices in the workforce.

Goal 3: Introductory college mathematics will present mathematics as a devel oping human discipline and demonstrate its connections to other disciplines.

Mathematics can no longer be viewed as a discrete discipline unconnected to other disciplines or careers. The integration of science, engineering, business, and mathematics should provide career opportunities and also develop the curiosity to explore other areas and enrich individual lives. Mathematics is now viewed as the science of patterns, rather than a set of procedures (MSEB, 1991). Applications of mathematics are found in the social, biological, and behavioral sciences, as well as in finance, political analysis, marketing, art, music and other related areas.

Goal 4: Introductory college mathematics will illustrate the power of mathematical thinking as a foundation for independent, lifelong learning.

Mathematics is all around us. Every individual uses mathematics in some way, consciously or unconsciously. Introductory college mathematics should help students to view mathematics as a common everyday activity used throughout life, not as a difficult, complicated process whose skills are possessed by a select few or used in select instances. In order to become productive and dynamic citizens in a complex technological society, students must develop an appreciation for the learning and power of mathematics and develop dynamic problem-solving and decision-making skills. This will allow them to participate fully in our democratic society where all individuals are valued decision makers (National Science Board Commission on Precollege Education in Mathematics, Science, and Technology).

Standards for Content Themes

College graduates, whatever their major, are likely to make multiple career changes during their lives. They need to experience the breadth of mathematics while exploring selected applications in depth. These mathematical problems should be chosen from contexts appropriate to adult lives. Standards to guide the selection of specific content experiences should focus upon the following:

Standard 1: Students will learn important mathematics knowledge through mathematical modeling applied to real world situations.

Standard 2: **Students will engage in problem solving both in the context of applied situations and in extending knowledge of mathematical theory.**

Standard 3: **Students will extend logical reasoning skills in activities which ask them to make and test conjectures, formulate counterexamples, follow logical arguments, judge the validity of arguments, and construct valid arguments.**

Content Themes

Mathematics instruction should provide for the achievement of the CORE goals and standards. Mathematical power must be developed through the in-depth study of specific mathematical content with decreased attention to rote manipulation. This power provides adult students with both the important factual information needed for more advanced study in mathematics and other disciplines, as well as the necessary problem solving strategies that adults need to function as productive workers and citizens. Citizens need to possess a certain level of "mathematical intuition" - a mathematical way of "seeing" the world around them - for their own satisfaction and for success in today's society. Mathematical intuition is developed through an understanding of mathematical content themes that permeate all programs and courses and are presented through appropriate instructional strategies.

College graduates, whatever their major, are likely to have multiple career challenges. They need to experience the breadth of mathematics while exploring selected applications in depth. These applications should be from contexts appropriate to adult lives. Students also need to be helped to generalize this mathematical knowledge to related contexts. Teaching mathematics with an emphasis on context places less emphasis on the learning of unrelated, but important procedures and facts to be used later in courses. Likewise, practice of skills for which little or no purpose has been established must be de-emphasized.

The specific content in each theme will vary greatly depending on which program the student enrolls: developmental, vocational-technical, or baccalaureate-intending. The suggested topics within each theme outlined below are by no means an exhaustive list. Whatever the actual content or level of a specific course, the themes below should permeate the introductory college mathematics curriculum and provide experiences that increase or develop the following abilities in students:

Theme 1: Number sense

Students should be able to perform operations, reason and draw conclusions from numerical information. Experiences in the representation of arithmetic operations, development of operation sense, and the ability to draw inferences should be made available in every course (MSEB, 1990). Number sense at the college level should go beyond basic operations to include the ability to estimate reliably, to judge the reasonableness of numerical results, to understand orders of magnitude, and to think proportionally. Suggested course topics include pattern recognition, data representation and interpretation, estimation, proportionality, and comparison.

Theme 2: Symbol Sense

All students at the college level should be able to move beyond concrete numerical operations to the use of abstract concepts and the use of symbols to solve problems. Students should be able to represent mathematical situations symbolically and utilize the appropriate methods or technology to form conjectures about the problem. Suggested course topics include measurement, derivation of formulas, translation of a real-life problem into a mathematical sentence, and the solution of equations by graphical and algebraic methods.

Theme 3: Spatial or Geometric Sense

Students should possess the powers of mathematical visualization to solve problems and interpret graphs. Students should develop a spatial sense including the ability to draw one, two, and three dimensional objects and extend the concept to n dimensions. Students should be able to visualize, compare, rotate, and revolve objects under consideration. Suggested course topics include comparison of geometric objects in real-life, graphing, prediction from graphs, measurement, and vectors.

Theme 4: Function Sense

At the college level, students should be able to move beyond a basic familiarity of functions to the ability to generalize about families of functions and their behaviors. Students should be able to create mathematical models to solve real-life problems and to predict future behavior. Students should be able to interpret functional relationships between two or more variables, connect such relationships when presented in data sets, and to transform functional information from one representation to another. Suggested course topics include generalization about families of functions, use of functions to solve real-life problems, and the behaviors of functions.

Theme 5: Probability and Statistical Sense

Students should be able to organize, represent, and summarize data, to draw conclusions or make predictions from data, and to assess the relative chances of certain events happening. Suggested course topics include data interpretation, measures of uncertainty, and probability and sampling distributions.

Theme 6: Problem Solving Sense

Students should be able to use a wide variety of problem solving strategies, intellectual risk-taking rather than simple procedural approaches, persistence, and the ability to recognize inappropriate assumptions. Suggested course topics include numerical problem solving, pattern recognition, tables, charts, case studies, graphical solution, mathematical modeling, and discrete mathematics.

In general, the specific content themes above should concentrate on increasing the mathematical power of students, focus attention on problem solving of real-life problems, decrease the value of routine procedures and manual skills, increase the use of technology, utilize alternative forms of assessment and techniques for communicating mathematics, and actively involve the student in all activities.

Reform in mathematics education requires reform in curriculum and in the topics or themes presented to students. In order to accomplish the above goals and help students achieve the above standards, the following changes are recommended in mathematics curricula:

Topics to Receive Increased Attention:	Topics to Receive Decreased Attention:
ARITHMETIC	
Estimation, pattern recognition, opportunities for reasoning, drawing inferences, open-ended problems, using oral and written communication to describe solutions.	Memorization, manipulative skills and routine operations in whole numbers, fractions, and decimals.

Topics to Receive Increased Attention:	Topics to Receive Decreased Attention:
ALGEBRA	
Understanding the concept of an equivalence relation and the meaning of a variable, problem solving using a variety of methods, connections between mathematics, science and other disciplines, use of technology.	Memorization and manipulative skills such as factoring, simplification of radical expressions, routine calculations of the quadratic formula. Coin, work, or meaningless word problems.
GEOMETRY	
Integration across topics. Graphical insight into the behavior of functions. Visual representation of statistical data, visual representation of area, probability trees, visual presentation of financial time lines, linear programming, graph theory, and logic circuits.	Analytic geometry as a separate course. Emphasis solely on formal proofs as a means of establishing geometric relationships.
FUNCTIONS	
Integration across topics in a course and among courses. Functions that are constructed as models of real-world problems representative of situations encountered by adults. Connections among a problem situation, its model as a function in symbolic form, and the graph of that function. Function equations expressed in standardized form as checks on the reasonableness of graphs produced by graphing utilities. Connecting observations of functional behavior such as where a function increases, decreases, achieves a maximum, or changes concavity to the situation modeled by the function.	Algebraic procedures with radical expressions, factoring, rational expressions, logarithms, and exponents. Separate and unconnected units on linear, quadratic, polynomial, radical, exponential, and logarithmic functions. Pencil and paper evaluation of functions.

Topics to Receive Increased Attention:	Topics to Receive Decreased Attention:
PROBABILITY	
Modeling problems with experimental and theoretical probability including estimations using simulations. Random variable and the probability distributions including binomial, uniform, normal, and chi-square. Statistical process control and its basis in the normal distribution.	Emphasis on selection of a correct formula and its use before underlying intuitions are developed from problem situations.
STATISTICS	
Charts, tables, and graphs that summarize data from real-world situations as a source of inferences. Curve fitting to predict from data including transformation of data when needed. Measures of central tendency, variability, and correlation and the effects of data transformations on these measures. Carrying out of experimental design and sampling to study problems. Testing hypotheses using appropriate statistics procedure. Use of statistical software.	"Cookbook" approaches to applying statistical computations and tests which fail to focus on the logic behind the processes.
DISCRETE AND FINITE MATHEMATICS	
Mathematics of finance including real problems on how to finance a college education, a new car, a home, and retirement. Matrices and their applications to models involving systems of equations and linear programming. Graph theory and algorithms as a means of solving problems.	"Cookbook" approaches to financial formulas, linear programming, etc.

However, as emphasized in Project Kaleidoscope's (1991) plan for strengthening undergraduate science and mathematics education *What Works: Building Natural Science Communities*, content themes should not be couched within courses that merely repeat the content and approach used in corresponding high school courses. While there may be nothing wrong with the content and approaches used for high school students, "First year students should instead plunge directly into a supportive community of learners where each student is expected to learn and is openly respected for what he or she already knows...By moving ahead to new ideas, with support to fill in missing pieces as needed, each student will grow in expertise and confidence" (p. 58).

Standards for Student Outcomes

Success in the technological, information based society of the 21st century will depend on possessing mathematical competencies that were not needed in the mechanical society of the early 1900s (MSEB, 1991). "More and more jobs - especially those involving the use of computers - require the capacity to employ sophisticated quantitative skills" (*National Research Council*, 1991, p. 11). Expectations of mathematics instructors and students must include the development of problem solving skills, mathematical intuition, and the mathematical power necessary to achieve career goals. Introductory college mathematics experiences and activities must provide opportunities for students to achieve the following standards:

Standard 1: Students will possess the mathematical tools to recognize, analyze and solve problems using numerical, graphical, and symbolic approaches to create mathematical models of real-life situations.

For success in our technological society, students must go beyond the mastery of basic operations to a real understanding of how to use mathematics, the meaning of the answers and how to interpret the answers (MSEB, 1989).

Standard 2: Students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw logical conclusions from general principles.

Students must learn that success in mathematics depends more on sustained effort than on innate ability (National Research Council, 1991). Opportunities must be offered to foster student's success in mathematics and develop confidence and independent thinking.

Standard 3: Students will be able to choose the appropriate tools and technology to solve mathematical problems and judge the reasonableness of the results.

The impact and power of calculators and computers can not be understated. "Those who use mathematics in the workplace - accountants, engineers, scientists - rarely use paper-and-pencil procedures anymore...Electronic spreadsheets, numerical analysis packages, symbolic computer systems, and sophisticated computer graphics have become the power tools of mathematics in industry" (MSEB, 1989, p. 1). It is imperative that students develop an ability to use technology appropriately and interpret the results it presents.

Standard 4: Students will be able to work effectively in groups and communicate about mathematics both orally and in writing.

The development of mathematical literacy is achieved with an understanding of signs, symbols, and vocabulary of mathematics. This can be best accomplished when students have an opportunity to read, write, and discuss mathematical problems and concepts (NCTM, 1989). As students learn to speak and write about mathematics, they develop mathematical power and become effective employees in the workplace.

Standards for Instructional Strategies

Instructional strategies used in introductory college mathematics courses have a dramatic impact on the success with which adult students develop a depth of understanding in mathematics. Many researchers in mathematics education agree that special attention needs to be given to student-constructed knowledge, the knowledge that students bring with them to class (MSEB, 1990). Collegiate mathematics faculty must become more aware of the research on the teaching and learning of mathematics and adopt a constructivist approach (see Crocker, 1991) toward mathematics education. While constructivist theories may be interpreted differently by different educators, constructivism is based on the premise that knowledge is something that students must construct for themselves. It cannot be "given" to them. In addition, faculty must invent and utilize instructional strategies for adult students that extend those outlined in the NCTM *Professional Standards for Teaching Mathematics* (NCTM, 1989). Standards for instructional strategies to be used in all introductory college mathematics courses are:

Standard 1: The mathematics instructor will use appropriate technology, naturally and routinely, in the teaching of mathematics.

Technology is changing the way mathematicians do mathematics around the world. Mathematics faculty must adapt to this new reality and adopt new strategies to guide students to necessary competencies and mathematical power to be competitive in graduate school science and mathematics and in the workforce. Almost every research study on the use of calculators or computers in the classroom, reported that "the performance of groups using calculators equaled or exceeded that of control groups denied calculator use" (MSEB, 1990, p. 22). Computers, hand-held calculators, educational television, computer-based telecommunications, video discs, and other technological tools and related software should be common place in collegiate classrooms (*Educating Americans for the 21st Century*, 1990; *A Call for Change*, MAA, 1991; *UME Trends*, May, 1993; and Foley, 1990).

Standard 2: The mathematics instructor will foster interactive learning through writing, reading, speaking, and collaborative activities. Learning activities should include projects and apprenticeship situations that encourage independent thinking and require sustained effort and time.

Communication skills necessary in our technological, information-based society require the ability to read, write, and communicate about mathematics and in mathematical terms. Mathematics instructors must adopt instructional strategies that develop communication skills in mathematics for all students, within a context of real-life application problems. Strategies such as cooperative learning experiences (see Crocker, 1992); oral and written reports presented individually or in groups; maintenance of journals; open-ended projects; business apprenticeships; and alternative assessment strategies such as essay questions and portfolios, need to be employed in college classrooms (*A Call for Change*, MAA, 1991; NCTM, 1989).

Standard 3: The mathematics instructor will actively involve students in meaningful mathematics problems which build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics.

Mathematics can no longer be presented as isolated rules and procedures. Students and employers are demanding that problems and projects be relevant to the individual's experiences or anticipated career. Students must have the opportunity to observe the power of scientific and mathematical investigation and see first hand the application to their lives. The responsibility for making mathematics relevant and meaningful is the collective responsibility of faculty, administrators, and producers of instructional materials.

Standard 4: The mathematics instructor will model the use of multiple approaches - numerical, graphical, symbolic, and verbal - to solve a variety of problems.

Mathematical power includes the ability to solve many types of problems. Solutions to complex technological problems require a variety of techniques and the ability to work through open-ended problem situations (Pollak, 1987). Collegiate mathematics instructors must provide rich opportunities for students to explore complex problems and guide them to solutions through multiple approaches.

The Faculty

The instructional process must be designed and implemented by knowledgeable, caring, and effective faculty. The AMATYC "Guidelines for the Academic Preparation of Mathematics Faculty at Two-Year Colleges" details the characteristics of effective faculty (p. 3). According to this document, effective teachers are reflective, creative, and resourceful. They use a variety of instructional methods and respond to the needs of the particular class and students they are teaching. They model behaviors they wish their students to exhibit.

While it is not the purpose of this document to offer a complete set of guidelines for the operation of mathematics departments, it should be noted that the CPR Project Task Force fully supports the AMATYC's *Guidelines for Mathematics Departments at Two-Year Colleges* (AMATYC, 1993) and the Mathematical Association of America's *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences* (MAA, 1993). In particular,

- Faculty members should be aware of advances being made in both mathematics content and educational methods. Those who teach at two-year colleges should have a minimum of a master's degree that includes at least 18 hours of graduate work in mathematics.

- Of considerable concern is the growing dependence of mathematics departments on part-time or adjunct faculty and teaching assistants. When graduate students are used to teach mathematics courses they should be closely supervised by regular faculty members. Furthermore, while adjunct faculty can bring special expertise to the classroom, excessive use of adjuncts can contribute to the overloading of full-time faculty with regular department duties. The minimum qualifications for adjuncts should be the same as for full-time faculty, and they should be closely supervised.

- Classes must be held in a suitable classroom environment. Classes must be of small size (a maximum of 30) to enhance the opportunity for the use of interactive learning strategies. Classrooms must be equipped so that computer instructional material and calculator outputs can be displayed. Computer laboratories should be available for student use.

- Adequate support services outside of class must be made available to students. Support services should include: faculty who are available in their offices on a regular basis to help students, learning centers that offer professional and peer tutoring, and technology specialists who can help students in computer laboratories.

- Professional development opportunities focusing on mathematical content and pedagogical strategies appropriate for adult students should be available to all full- and part-time faculty.

Assessment

As curriculum standards, instructional strategies, and student outcomes change, effective standards for testing, assessment, and accountability must follow. A new national understanding of assessment standards will be necessary built upon the fundamental principles presented in *Measuring Up* (MSEB, 1993):

- Tests should measure what's worth learning, not just what's easy to measure.

- Progress depends on constant correction based on feedback from assessment.

- Schools (and colleges) are accountable, both to taxpayers and to students.

The higher education mathematics community must ensure that assessment reflects innovative instructional strategies, the unique characteristics of adult students, and students' varied learning styles.

Goals for mathematics assessment outlined in *For Good Measure* (MSEB, 1992) are appropriate for the adult student, as well as those in grades K-12, and should guide testing and assessment:

- Assessments will be aligned with the mathematical knowledge, skills, and processes that the nation needs all of its students to know and be able to do.

- Assessment practices will promote the development of mathematical power for all students.

- A variety of effective assessment methods will be used to evaluate outcomes of mathematics education.

- Adequate accountability systems will be used to assess mathematics.

- Guidelines will be developed for judging the quality of all forms of mathematics assessments.

- Mathematics teachers and school administrators will be proficient in using a wide variety of assessment methods for improving the learning and teaching of mathematics.

- The public will become better informed about assessments and assessment practices.

A new vision of what mathematics assessment should be and its appropriate role in mathematics education reform needs to be formulated based upon diverse student abilities, and the needs of students, faculty, and institutions. New assessment frameworks need to be developed to assess the new mathematics that is to be taught. Ways to use the assessment information to support change in mathematics teaching and learning need to be designed and implemented. Assessment must be equally valid and appropriate for all groups. Mathematics assessment needs to take a new form as we move into the 21st century.

CHAPTER 3 - Developmental Mathematics

Introduction

A critical part of the revitalization of introductory college mathematics is the inclusion of a dramatically reformed developmental mathematics curriculum. Developmental mathematics is the portion of a college's mathematics curriculum designed for students not yet ready for college-level mathematics, traditionally defined as college algebra and beyond. The developmental curriculum must develop mathematical intuition along with a relevant base of knowledge, challenge our students even as it builds their confidence, and provide experiences which bridge the gap between classroom learning and real-world applications. People should emerge from this curriculum with the ability and confidence to use mathematics effectively in their multiple roles as students, workers, citizens and consumers. The trademark of the new curriculum must be lively classrooms with students enthusiastically engaged in activities designed to enhance their understanding of mathematics and their ability to solve problems mathematically.

This new vision for developmental mathematics is based on a reasonable assumption that there will be a continuing need for developmental mathematics programs, which currently exist at most postsecondary institutions. It is likely that higher education will continue to serve older students with faded mathematical backgrounds, younger students with inadequate secondary school preparation, and students with learning disabilities and other special circumstances. In addition, the *gradual* implementation of the NCTM *Standards* (NCTM, 1989) in the K-12 mathematics curriculum will add to the diversity of students' backgrounds, especially with regard to their experiences with technology. The central role of developmental mathematics is to expand the educational opportunities and broaden the career options of these students.

It is important to reevaluate the relationship between secondary mathematics and developmental mathematics. The developmental curriculum is strongly related to the K-12 curriculum, and in fact the NCTM *Standards* have had a major impact on our vision of a new developmental mathematics curriculum. However, at the same time it is important to recognize that developmental programs must not simply replicate the high school experience. There is a subtle but critical difference between building a curriculum around students' needs and building it around their deficiencies. The greater time constraints, the more focused career interests and broader experiences of adult learners, as well as the goals and expectations of postsecondary institutions, necessitate the creation of standards specifically designed for college students. The evaluation of developmental mathematics programs and the assessment of students should be based on these standards, not those of the secondary curriculum.

Goals

While the CORE goals described earlier apply to developmental mathematics as well as other programs, the following goals highlight the special needs of developmental students.

Goal 1: **The developmental mathematics curriculum will emphasize the development of mathematical understandings and relationships.**

The essence of mathematics at any educational level, including the developmental, is not rote memorization of "basic skills" but rather a way of thinking which involves logical reasoning, analytical problem solving, and conceptual understanding. Learning mathematics should be a sense-making experience which develops students' abilities to conduct independent explorations.

Goal 2: **The developmental mathematics curriculum will develop students' confidence in their ability to use mathematics appropriately and efficiently so they will become effective and independent users of mathematics.**

Such confidence is especially critical for many developmental students whose fear of mathematics is so strong that it stifles their ability to use mathematics. Both self confidence and independence must be fostered through positive experiences which seriously engage students in mathematical activities. Developmental programs should be structured so that even students who do not plan to proceed to college level courses will profit from their experiences.

Goal 3: **The developmental mathematics curriculum will reflect the impact of technology on the curriculum.**

Technological advances are not only impacting how we teach but also what we teach in mathematics. Developmental students must be prepared for different mathematics and mathematics-related curricula and a world transformed by technology, rather than being taught outmoded skills which no longer have relevance.

Goal 4: Developmental mathematics programs will increase participation of students in mathematics and mathematics-related disciplines, especially women, minorities and other underrepresented groups.

This is central to the purpose of developmental mathematics. Because of factors such as the increasing trend of adults returning to school, the time element involved in reforming elementary and secondary mathematics education, the continuing needs of students in special circumstances, and the inevitability of systemic and individual failures, a large and highly diverse set of students will continue to need developmental mathematics. Current trends indicate that this will include a disproportionately high number of women and minorities. Thus, developmental programs play a key role in providing equity and access.

Goal 5: Developmental mathematics programs will provide multiple entry and exit points appropriate for the student populations and programmatic needs of each institution.

The diverse mathematical requirements of various degree programs, along with dissimilar backgrounds and divergent interests of college students, create a strong argument for a well-designed placement system and a flexible developmental mathematics curriculum. First, placement procedures should include review of high school records, results of placement tests and/or college entrance examinations, and an opportunity for students to discuss or appeal their placement with mathematics faculty. Second, curricular requirements should delineate between general expectations for all students and needs for specific disciplines and majors. For example, many traditional intermediate algebra courses have been designed to prepare students for calculus. Other courses may be more suitable for students who do not need calculus.

Instructional Strategies

While the level of mathematical sophistication differs from developmental mathematics to introductory college mathematics, the instructional standards given in the CORE still apply.

- Technology should be used naturally and routinely.

- Students will learn interactively through writing, reading and collaborative activities. These activities should include projects that encourage independent thinking and require sustained effort and time.

- Students will work on meaningful mathematics problems and real-world applications.

- Students will use multiple approaches to solve problems.

It is especially important to note that the use of technology is just as important in developmental mathematics as it is in college level mathematics. The developmental curriculum should place particular emphasis on how and when to use technology in balance with other tools such as mental work, paper and pencil, and manipulatives. Developmental mathematics students must be prepared for the radically revised college mathematics curriculum, and they should not be handicapped by having learned outmoded skills in passive classes.

The importance of meaningful applications must also be stressed. While skill development is an important part of the developmental curriculum, the latter must also emphasize the usefulness of mathematics in other disciplines and the workplace, and its effectiveness as a tool in daily life and for responsible citizenship. In addition, the appropriate use of technology, mental work and other tools for solving problems is best illustrated in the context of meaningful applications. Often in developmental mathematics, applications have been curtailed either because they are perceived to be too time consuming or too difficult. This trend must be reversed.

Finally, while certain adjustments may be appropriate, the developmental curriculum must also incorporate the use of writing exercises and other creative instructional activities to promote critical thinking.

Instructional strategies must be implemented by caring faculty who want to teach at the developmental level. Teaching at this level requires special qualities from instructors who

- are knowledgeable mathematicians who have demonstrated understanding of the special instructional needs of developmental students,

- are available for extra assistance, and

- are ready and willing to play a special mentoring role in addition to teaching.

Faculty teaching developmental mathematics must provide careful academic advice, be flexible about ways in which students can meet course requirements, and simultaneously provide support and demand commitment from their students. In addition, developmental instructors need the support of a learning center as a source of tutoring and all other forms of student assistance, especially for times when the instructor is not available. Learning centers should also provide a setting for students to work in groups.

Placement

The placement of students into developmental courses is of special concern. Students must understand that being placed in developmental courses is not a punishment nor is it done without serious thought about their overall mathematics education. Rather, taking developmental mathematics courses must be perceived as an opportunity to ensure success in college level courses. Mandatory placement procedures should include a review of students' high school records and college entrance examinations, up-to-date mathematics placement tests, and opportunities for students to meet with mathematics faculty to discuss their placement. Final decisions on the proper placement of mathematics students must be the responsibility of the mathematics department.

Changes in Emphasis in the Developmental Curriculum

The underlying theme of these developmental standards is to prepare our students to become effective users of mathematics. Developmental mathematics programs which are committed to this vision are expected to have the curricular changes described in the following table.

INCREASED ATTENTION	DECREASED ATTENTION
functional sense	
data sense	
spatial sense	
probability sense	
active involvement of students experiencing mathematics	passive listening
appropriate use of technology throughout the curriculum	pencil and paper drill
problem solving and multi-step problems	one-step single answer problems
mathematical reasoning	memorization of facts and algorithmic procedures
conceptual understanding	skill building
real-world applications	contrived exercises
an integrated curriculum	isolated topic approach
topics developed in context	development of topics out of context
use of technology to aid in conceptual development	long hand simplification of polynomial, rational, exponential, and radical expressions; factoring; and solving equations and inequalities
students making choices on the method used to solve a problem	students being given "a" method for solving a problem
number sense, mental arithmetic and estimation	arithmetic drill exercises
diverse and frequent assessment, in-class as well as out of class	tests and a double-weighted final exam as the sole assessment
variety of teaching strategies	lecturing

The Organization of the Developmental Curriculum

It is not the intent of this document to offer specific course syllabi. The latter should be developed locally using the standards recommended herein. However, it is important to note that the traditional sequence of developmental courses is not the only option. In particular, a curriculum using an integrated approach may have major advantages. While mathematics includes a certain amount of hierarchial structure, it is a flexible and interconnected discipline. For instance, students do not need to be thoroughly competent in arithmetic to do algebra, nor in algebra to study the behavior of functions, nor in function theory to use technology. In addition, many problems can be approached more effectively by using a combination of numeric, symbolic, and geometric approaches. It may be especially advantageous to use an integrated approach in developmental mathematics, not only because it is an effective way to revisit familiar material, but also because it lays a foundation for using mathematics effectively.

Furthermore, it is important for developmental students to progress to college-level mathematics as soon as possible. This does not mean that students should be rushed through a superficial program. In fact, some students may have to study developmental mathematics for an extended period of time. However, an in-depth integrated approach may be more efficient than the traditional approach because it mitigates the need for repeated review.

Standards for Student Outcomes

Standard 1: Students will develop a sense of numeracy.

Numeracy, knowing and using numbers, is at the heart of developmental mathematics, yet remains an elusive concept to define and characterize. It is the lack of numeracy that gives rise to many of the cases of math anxiety in adults; they don't feel comfortable with numbers. To develop numeracy, a program must address the issue of building confidence so that students can be in control of the numbers in their lives.

With today's technological advances, developing students' skills with computational exercises should no longer be the major emphasis in developmental mathematics classrooms. Today's student must be in command of a variety of computational techniques: mental (including estimation), paper and pencil, and calculator. To be in command, the student must develop the insight to be able to choose which method is appropriate to the situation and the skills to be confident in using each method. Embedded in each method is the need for knowing basic number facts. The ability to recall basic facts immediately is the basis upon which mental arithmetic rests, algorithmic procedures depend and estimation techniques are built. While total mastery of the facts is desired, this is no longer a prerequisite to the study of other mathematical topics. In fact, study of other topics will provide opportunities to reinforce the learning of basic facts as well as to increase the students' belief that mastery of these facts is important.

Another facet of numeracy is the ability to connect the abstractions both of number and of the operations performed with them to real-wold phenomena. For adult learners with varied life experiences, the applications can serve as motivators for certain topics and also provide the means by which concepts are made clear.

Becoming numerate is only part of the larger study of mathematics, but it can be used as an introduction to some of the "big ideas" of the discipline. The developmental mathematics program should strive to connect the rules of arithmetic to the properties of real numbers and algebraic expressions.

Additionally, by encouraging students to recognize patterns in even the simplest number relations, a developmental program should provide the student with a basis upon which to build mathematical thinking skills.

The developmental mathematics curriculum should include experiences with real numbers, their properties and operations, so that students will be able to accomplish the following objectives.

Objective 1. Students will be able to use mental arithmetic, paper and pencil algorithms, and calculator as computation tools in solving mathematical problems.

The following investigation shows the value of an estimate.

Problem

The diameter of a bicycle tire is 2.25 feet. How far will the bicycle travel when the wheel goes around one time? Five times? Compare the results when solving this problem in three ways:

a. use 3 as an estimate for π and solve mentally;

b. use 3.14 as an estimate for π and solve with paper and pencil;

c. use the π key on the calculator and solve with technology.

Are any of the answers exact? Describe a situation when each result would be called for, including the exact answer.

Objective 2. Students will be able to develop both number sense and operation sense to build confidence needed to use multiple representations of numbers and equivalent forms of a problem, and to estimate the result of a computation.

Confidence with numbers is indicated when students substitute fractional, decimal and percent equivalents readily, recognize how the inverse operations undo each other, and describe reasonable results.

Problem

A Typical Student Monthly Budget

Housing	Food	Utilities	Tuition/Books	Entertainment	Clothes/Misc.
$240	$160	$80	$320	$40	$120

a. What is the total monthly budget?

b. What fractional part of the total budget is spent on housing? Express this also in percent form.

c. Repeat question "b" using the remaining categories: food, utilities, etc.

d. To check the accuracy of your computations, find the sum of all the fractions found in questions "b" and "c". Repeat this for the percents. What should the sum be? If there are differences, explain them.

e. Construct a bar graph to illustrate the budget information. Make it so that the maximum height of any bar is 10 cm high. Determine the length of each bar. What should the sum of the lengths of the bars be?

f. Construct a circle graph to illustrate the budget information. Determine the size of the central angle for each wedge using the fractions found in questions "b" and "c". Repeat this process using the percents you found earlier. Which method do you prefer?

g. Compare this sample budget with your own personal budget. For which categories (housing, food, etc.) do you spend a greater portion of your total?

Objective 3. Students will be able to extract the mathematical content and relationships (especially proportional relationships) present in real world everyday problems, especially problems from their prospective career fields.

Problem

Decide if a proportional relationship is present in the following problems.

a. In one small office within a corporation, 6 out of 9 management positions are held by women. If that rate holds throughout the corporation, how many of the 180 management positions are held by women?

b. If one inch is equivalent to 2.54 centimeters, 7.25 centimeters are equivalent to how many inches?

c. The length of a side of a square is 3 m; it's area is $9m^2$. What is the length of a side of a square whose area is $36m^2$?

Objective 4. Students will be able to construct a coherent understanding of the concepts which underlie the algorithms of arithmetic.

While traditional computation algorithms provide an efficient and universally applicable set of procedures to compute answers, they are often not user-friendly and have become a source of anxiety for the learner who has had trouble with mathematics. Alternate ways to compute provide the adult learner with a "new" method and afford the opportunity to connect an arithmetic procedure to the properties of real numbers.

Problem

The following example, using the principle of compensation for subtracting mixed numbers, can be connected to the properties of additive inverse and additive identity. To find the answer to the problem,

$$3\frac{1}{7}-1\frac{5}{7},$$

traditional algorithms require regrouping, or "borrowing". Using compensation, the problem is made easier by writing an equivalent one,

$$3\frac{3}{7}-2.$$

This is accomplished by first determining that adding 2/7 to 1 5/7 makes it a whole number, which is easier to subtract. The same quantity is then added to the 3 1/7 so that the equivalence is maintained. In effect, the same number is added and subtracted, leaving the result unchanged.

Problems

Compare exercise A to exercise B and find the pattern:

GROUP A	GROUP B
$3\frac{1}{3}-1\frac{2}{3}$	$3\frac{2}{3}-2$
$5\frac{2}{5}-3\frac{3}{5}$	$5\frac{4}{5}-4$
$6.2-4.8$	$6.4-5$

Questions:

a. Which exercise in each row is easier for you to work? Why?

b. For each pair of exercises, what was added to the second number of the A exercise so that it equalled the second number in the B exercise?

c. Notice that the same number that was added to the second number (see above) was also added to the first number. Discuss why this technique gives different problems which have the same answer. Try to convince your fellow group members that it will work for all such problems. (No more borrowing!)

Standard 2: Students will be able to use the concept of function as a central theme throughout the study of developmental mathematics.

The functional approach to mathematics should be introduced at the arithmetic level, and enhancement should continue throughout the entire developmental mathematics curriculum. While analysis of functions is the mainstay of mathematics in the first two years of college, the developmental curriculum provides the perfect setting for introducing the function concept as well as low level analysis of a variety of elementary functions. The world abounds with simple functional relationships; they should be used to help students understand the ideas of rate of change, increasing or decreasing, maximum or minimum, positive or negative, zeros, and domain and range. Graphical analysis of functions gives new understanding to numerical analysis and makes connections between the different representations of functional relationships.

With a solid foundation in the study of functional behaviors, the transition can be made to solving equations. Equations are solved by finding the zeros of the related function. Solving inequalities implies finding where the related function is positive or negative. Factoring of polynomials can be taught using the zeros of the related polynomial function. Checking answers to symbol manipulation exercises can be accomplished by comparing numeric or graphic representations of the exercise and the answer. Symbol manipulation can be minimized because numerical or graphical methods of solving problems often require very little manipulation.

Creating a mathematical model of real-world data can be accomplished by comparing the behavior of the data with the behavior of known functions. Geometric transformations or relationships between function parameters and behaviors, as well as educated guessing and technology to improve the guesses, can be used by developmental students to create relatively accurate models. Interesting discussions center around the validity of such student-generated models.

The developmental mathematics curriculum should include investigations, explorations, and projects that will enhance the objects that follow.

Objective 1. Students will be able to compute numerical values for a variety of functions.

The computation of numerical function values begins the learning spiral for understanding the concept of a function. Examples should be drawn from various types of functions including linear, quadratic, absolute value, square root, exponential and rational. Such exercises have the additional advantage of improving computational skills in a meaningful context.

Problem

A server at the gourmet restaurant Slow Eddies earns $80 per week in salary and averages $7.56 in tips per table ($t$) served. His total wages can be found from the expression $7.56t + 80$. Calculate the servers wages when he has served the following number of tables.

t	7	8	9	10	11	12	13	14
7.56t+80								

Similar calculations should follow from functions like: quadratic, absolute value, square root, exponential, and rational. At the arithmetic level, tables should be developed by calculating the expression for each given value. At the algebra level, the table making feature of the calculator should be used.

Objective 2. Students should be able to plot and interpret graphs in the coordinate plane.

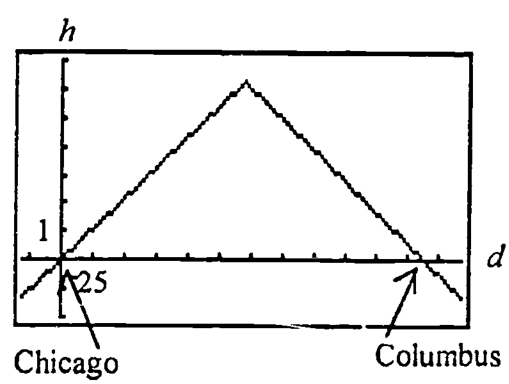
Graphical representations of functions provide powerful insight into analyzing mathematical relationships. The actual construction of graphs is a relatively simple exercise for students who have analyzed functions from a numerical perspective. While the plotting of points that make up the graph of a function can be done on traditional graph paper, plotting points electronically on the graphing calculator is good motivation and maintains a similar learning experience. Simply moving the cursor around the coordinate plane quickly shows the student when the coordinates of a point are positive, negative, or zero. If sufficient time is used to analyze functions from a numerical perspective, students make the transition to plotting the pairs of numbers on the coordinate plane with little difficulty.

Interpreting the graph of a function is a more complicated task, but its usefulness justifies introducing this topic at the developmental level. Approaching the topic from a contextual setting is critical.

Problem

Consider the following example of a mathematical model of the flight plan for an airplane flying from Chicago to Columbus. Because it is a short trip and the airline management requires an efficient (fuel saving) flight plan, the flight path of the plane is an inverted V. With a ground distance of 290 miles and a maximum height of 33,000 feet (6.25 miles) in the middle of the trip, the model is

$$h = -\frac{6.25}{145}|d - 145| + 6.25$$



and the graphical representation is shown above. Providing students with this absolute value function out of context seems to maximize the difficulties in graphical representation minimizes the difficulties in answering the following questions:

a. Is the airplane rising (height increasing) as it leaves Chicago, or is it rising as it arrives in Columbus? Note the importance of reading the graphical representation from left to right. Without the contextual setting, students have a very difficult time telling when a function is increasing or decreasing because they don't read from left to right.

b. When is the height of the airplane zero? What are the zeros of the function? It is not always obvious to students that when a function is zero, the graphical representation is on the x-axis (the d-axis).

c. If the TRACE key is used, what is the interpretation of the TRACE numbers on the screen relative to the flight plan?

d. When is the airplane at a maximum height? What is the maximum height? The answers to these questions become obvious when asked in the context of real-world situations.

e. When is the airplane above ground? (When is the function positive?) Should the model show the plane below ground? Why? What are the function values when the model shows the plane under ground?

The topic of functional behaviors requires a thorough investigation in the context of a physical situation and usually requires more than one example.

Objective 3. Students will be able to create and recognize a variety of function patterns.

Pattern recognition for functional behavior may best be taught using student collected data such as the monthly kilowatt hours of electricity used by their college. These data can be analyzed numerically for periodicity, maximum and minimum values, increasing or decreasing, domain and range, and average rate of change from one month to another. Once introduced in this context, other typically studied functions can be analyzed using the numeric representation.

The first step in being able to develop a mathematical model for real-world data is pattern recognition. Pattern recognition is accomplished by introducing students to the numerical and graphical representations of a few elementary functions. Simple numerical and graphical analysis of functions quickly leads to model building.

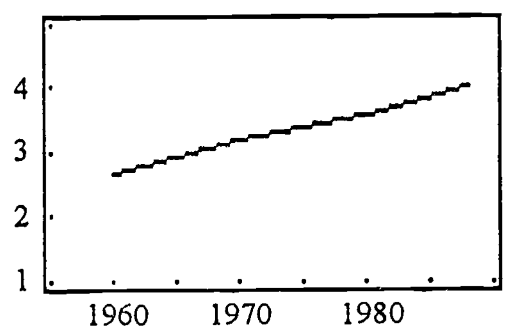
Problem

Consider the following example of daily garbage generation per person in the United States.

Year (t)	1960	1970	1980	1988
Garbage (g) in pound	2.7	3.2	3.6	4.0

Source: U.S. Environmental Protection Agency, 1990.

This particular data gives rise to a variety of possible models and provides interesting classroom discussion of the proper choice, why one choice might be better than another, and what model should be used if, for example, recycling becomes mandatory. Is the per person per day garbage generation pattern increasing in a linear fashion? If it doesn't follow a linear pattern, what functional pattern models the data? Of course a visualization is an immense help and can be developed using electronic graph paper on the graphic calculator. This approach is especially useful because students can compare the graph of the actual data with the graph of their proposed mathematical model and make adjustments in the parameters as needed. Students learn that educated guessing is a practical and sometimes simple problem-solving strategy. Other students learn that blind guessing is a very time consuming problem-solving strategy and rarely leads to an acceptable solution.



Objective 4. Students will be able to use multiple representations of functions to solve problems.

The time for exclusive use of one-answer and one-method problems is over. The tools are available for using multiple methods and they can be successfully used by developmental students. It is entirely appropriate for the educator to demonstrate the solution to a problem using three different representations of the function required to solve the problem. Students must learn which method is best for them to use when solving a variety of problems. They must be given the choice of methods when they attempt to solve problems.

Problem

Consider the problem of determining the speed of a car as it travels between two broad white stripes painted on the freeway at a distance of 1/4 mile between stripes. The mathematical model needed to find the speed in miles per hour is the function $v = 900/t$, where t is measured in seconds.

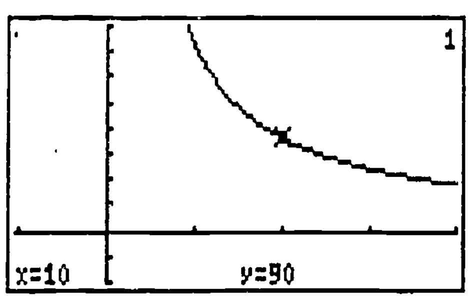
a. If the highway patrol officer clocks the time at 15 seconds, how can she find the speed of the car? (The analytical method may be called for.)

b. If there are cars crossing the marked path at a high frequency, how can she quickly determine their speed? (The numeric representation of the model might be better.)

t	6	7	8	9	10	11	12	13	14	15	16
v	150	129	113	100	90	82	75	69	64	60	56

(t in seconds, v in mph)

If the officer creates the table above, she has the infomation she needs to make rapid decisions. She can see immediately that if the time is 13 seconds or larger, she shouldn't bother sending a cruiser, if the speed limit is 65 mph. Finally, the best approach may be to use the graphical representation of the mathematical model on an integer window. The TRACE key will immediately give the speed of a car with a known time. Like the numeric representation, she has much more information available to use in decision making. Unlike the static numeric representation, the TRACE key will pan both left and right, and it provides an interesting piece of evidence in presenting a speeding case before a judge.



Objective 5. Students will be able to analyze functions and their behaviors.

The absolute value function provides for very interesting behavior and models a variety of relationships. Since it is not analyzed to any great extent at other mathematical levels and the graphing calculator provides easy access to an investigation of its behaviors, the absolute value function is a natural choice for study at the developmental level. Examples taken from the context of student familiarity will develop a deeper understanding of the mathematical behaviors than will an example where no physical meaning is attached to the variable or function.

[PROBLEM IS NEEDED HERE]

Objective 6. Students will be able to make connections between the parameters in a function and the related behavior of the function.

Functions in standard form lend themselves to being easily analyzed because students can make connections between the behaviors exhibited in the numeric or graphic representation and the parameters of the symbolic representation. Confirmation of many estimated behaviors determined numerically or graphically is made by the symbolic representation. For example, after determining numerically and/or graphically that the functions $y = -2|x-3|+5$, $y = -2|x-3|+1$, and $y = -2|x-3|-6$ have maximum values of 5, 1, and -6 respectfully, students are convinced that $y = -2|x-3|+19$ has a maximum value of 19 without ever verifying numerically or graphically.

Analyzing the connections between parameters and behaviors starts the learning spiral for further topics such as geometric transformation and model building. While transformations are an efficient method for building a mathematical model of real-world data, students can be introduced to using behaviors as a tool for model building at a much earlier time in their developmental education.

Problem

If the flight plan model from the problem for objective two were given as,

x	20	50	120	135	140	145	150	165	190	250	290
y	.862	2.16	5.17	5.82	6.03	6.25	6.03	5.39	4.31	1.72	0

students see that the maximum height of the airplane according to the chart is 6.25 miles and when they graph the data, they see that the shape is that of the absolute value function $y=d|x-e|+f$, and that f is 6.25. Since e is the horizontal coordinate of the maximum value, they also know that e is 145. The value of d is a little more complicated because the concept of rate of change is more complicated; however, a quick calculation of $\Delta y/\Delta x$ gives the rate of ascent of the plane. Finally, when students know how the parameters control the behavior, they know that the value of d should be negative.

Objective 7. Students will be able to solve equations and inequalities using the zeros of the related function.

Technology has not only minimized the need for students to learn complicated symbol manipulation, it has also allowed for a greatly reduced emphasis on solving equations and inequalities by analytical methods. At the same time, technology encourages the conceptual development of solutions to equations and inequalities by relating the solution to the zeros of the associated function.

Using the function approach to teaching mathematics requires a totally different perspective be used when solving equations and inequalities. In the absence of knowledge of function behaviors, solving equations and inequalities means using the proper algorithm to find a value for the variable that makes the statement true. Solving equations after an analysis of function behavior means finding the zeros of the related function. Solving inequalities means finding where a function is positive or negative. Two new methods for solving equations and inequalities are immediately available, the numeric and graphical methods.

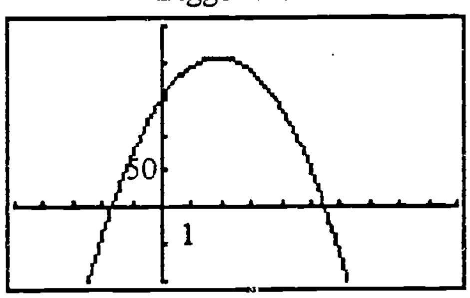
Developmental students now have options that have not been accessible before. When asked to solve an equation or inequality, they can make the choice to solve by the numerical, graphical, or analytical method. This may be especially beneficial to developmental students because they often think of mathematics as a set of algorithms that they have to memorize and follow in order to solve equations and inequalities. They seem to be overwhelmed by the number and length of such algorithms. At last, when choices are available they find they are in control of how to solve a problem. If they cannot remember the specific analytic algorithm, they may choose between the numeric or graphic methods. The numeric and graphic methods remain constant for all of the equations and inequalities they will ever solve. Gone is the need to solve page after page of equations - just so the student can "practice" the associated analytic algorithm. Now, once the graphic and numeric algorithms are mastered, and behaviors are remembered, they can always be used to solve any equation or inequality (assuming a solution exists). Before an equation is solved, the visualization of the function related to the equation tells the student how many possible real solutions there are, and which ones make sense in the problem being solved.

Problem

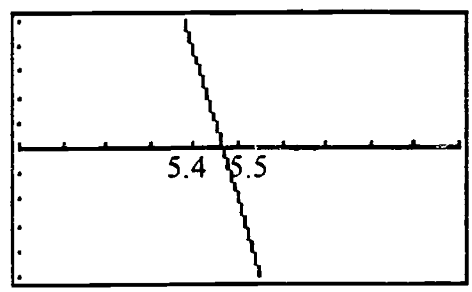
A ball thrown upward from a height of 150 feet at an initial velocity of 60 feet per second will hit the ground t seconds after it is thrown. When will it hit the ground if the equation that models this time is $-16t^2 + 60t + 150 = 0$?

Sample Solution - Graphical Method:

Bigger Picture



ZOOM-IN



The ZOOM-IN graph shows a useful solution of 5.45 with an error of less than 0.1.

Sample Solution - Numerical Method:

Bigger Picture

t	0	1	2	3	4	5	6	7
height	150	194	206	186	134	50	-66	-214

The time is between 5 and 6. Make a new table from 5 to 6 using a Δx of 0.1.

Zoom-In

t	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7
height	50	39.8	29.4	18.6	7.4	-4	-15.8	-27.8

The ZOOM-IN table shows a time of of 5.45 with an error of less than 0.1.

Objective 8. Students will be able to use geometric transformations to manipulate the graphical representation of a function so that students can model real-world relationships with functions.

In addition to developing mathematical models by adjusting the parameters in the symbolic representation of the model to match the known behaviors of the given data, students should be able to apply geometric transformations of a reflection about the x-axis, a stretch or shrink, and a vertical or horizontal translation to a parent function in order to find the symbolic representation of a model. This method reinforces the parameter method and bridges the gap to further study of transformations in higher level courses. Doing the parameter method first helps students to understand the concept of transformations.

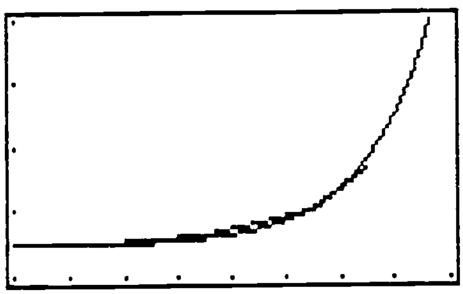
Problem

Develop a mathematical model for the following data.

t (years)	1725	1775	1825	1875	1925	1975
C (Carbon Dioxide)	275	278	283	293	300	335

Carbon Dioxide is in parts per million. Source: World Resources Institute

How is the level of CO_2 is increasing? The graphical representation will not only give a clue about how it is increasing, but also the general "shape" of the data. From this information a general behavior model can be selected such as $C = 1.017^t$. This can be transformed by a horizontal translation of 1725 and a vertical translation of 275 to get the model $C = 1.017^{t-1725} + 275$. The graphs of data and model are below.



Using the graphing calculator to graph the data on top of the proposed model of the data, students can make adjustments in the transformations and then re-graph to compare the new model to the original data. This process is continued until the student is satisfied with the model. Once the model has been developed, questions can be asked that require the use of the model, challenge the validity of the model, or query the behavior of the model. The questions include

a. Describe a situation under which your model of the carbon dioxide level would not apply. What other limitations are there to your model?

b. Using your mathematical model, how much carbon dioxide will be in the atmosphere in 1995? In 2050?

c. Does your model have a maximum? If yes, when and what?

d. What is the average rate of change from 1725 to 1825? From 1925 to 1975?

e. Can you make the domain of your model [1725, 2050]?

Standard 3. Students will be able to translate problem situations into their symbolic representations.

Algebra continues to serve as both the primary language for communicating mathematical ideas and one of five areas of the developmental mathematics curriculum. Nevertheless, mathematics education related to algebra is changing under the pressure of recent technological advances -- graphing utilities (graphing calculators and function plotting software) and computer algebra systems.

Because of the speed and power of graphing utilities, their introduction into the developmental classroom results in increased emphasis being placed on graphing as opposed to analytic techniques of solving a wide range of problems. On the more philosophical level, technology fosters a broadening of the definition of what constitutes the answer to an algebra problem. The answer need no longer be in closed form as long as it can be computed to any desired degree of accuracy.

Algebra systems, now widely available on computers and beginning to be introduced on handheld calculators, reduce the need for an algorithm-based algebra curriculum. With their introduction into the classroom, complex examples are done by machine, while simpler expressions and equations are still handled by traditional pencil-and-paper algorithms. To check answers computed on machine, students will need to master skills in mental algebra analogous to the estimation skills which have rapidly gained importance in calculator-based arithmetic (e.g., $(5x^3 - 2x^2 + 7x + 5)(2x^4 - 7x - 3)$ has as its leading and constant terms $10x^7$ and -15, respectively).

During this transitional period, many students will be entering college who will need to be taught mathematics using these new tools. Developmental mathematics programs will have to assume a large share of this responsibility.

Using a graphing utility or a computer algebra system permits more study of challenging and motivating real-world examples than does the traditional pencil-and-paper approach. It also erodes the significance of the traditional hierarchy of equations with the concomitant development of techniques to solve each.

The fundamental algebraic concepts that should be included in the developmental program follow.

Objective 1. The developmental mathematics curriculum should provide students with the opportunity to learn fundamental algebraic concepts and properties, stressing a functional and multi-representational approach to these topics.

Problems

1. Solve the equation $|x-5| = |3x + 5|$ a) numerically, b) graphically and c) algebraically

2. Explain what is meant by an extraneous root. Give an example of an extraneous root.

3. By studying the following table of values, find the relationship between x and y.

x	1	2	3	4
y	6	11	16	21

4. Consider the functions 2^x and x^2,

   a. Explain the difference in meaning between them.

   b. Graph these functions. Describe these graphs in words.

   c. Solve the equation $2^x = x^2$.

Objective 2. The development mathematics curriculum should provide students with the opportunity to learn the algorithms for manipulation of algebraic expressions, but reduce the level of difficulty and the time spent teaching these skills as compared with the traditional classroom.

Problems

1. Explain the difference between the following two properties and give examples of each: $a^x \bullet b^x = (ab)^x$ and $(a^x)^y = a^{xy}$

2. Simplify: a) $4^{-0.5}$ b) $(2x + 3y)^2$

3. Mentally compute (1,001) (999).

Objective 3. The developmental mathematics curriculum should provide students with the opportunity to learn a range of algorithms -- paper-and-pencil, mental, and machine-based -- to solve equations, systems of equations, and inequalities.

Problems

1. Consider a quadratic equation of the form $ax^2 + bx + c = 0$. How many different real roots can an equation of this form have? Give an example of each possibility and graph your example. In about 50 words, discuss the general situation.

2. Consider an equation of the form $y = ax + b$. What does the graph of this function look like? Give an example of each of the following functions, show its graph, and in about 50 words, discuss the general situation.

a	b	example
0		
large positive		
small positive		
large negative		
small negative		
	0	
	large positive	
	small positive	
	large negative	
	small negative	

Objective 4. The developmental mathematics curriculum should provide students with the opportunity to learn methods of applying algebra to solve real-world problems.

Problems

1. Suppose we know that the isotope carbon-14 has a half-life of about 5600 years and a living body radiates approximately 918 rays per gram from carbon-14 per hour. If a fossil gives off about 7 rays per gram from carbon-14 each hour, find the age of the fossil.

2. The table below (adapted from *Collaborative Explorations for Algebra*, Pepe, Ray and Langkamp, Seattle Central Community College, 1993) shows some winning distances for the Olympic long jump from 1948 to 1988.

Year	Men	Women
1948	25.7	18.7
1956	25.7	20.8
1964	26.5	22.2
1972	27.5	22.3
1980	28.3	23.2
1988	28.6	24.3

a. Plot the points, estimate the coordinates of the point where the two lines intersect and explain the significance of this point.

b. Find the equations for the lines of best fit for each data set using your graphing calculator.

c. Calculate the point of intersection of these lines algebraically. How close is it to your graphical estimate.

d. If the women's event had been held at the first recorded Olympics in 776 BC, what does your model predict for the women's long jump distance?

Standard 4: Student will develop a spatial and measurement sense.

Geometry, the study of shapes and their properties, has wide applicability in human experience. Geometrical problems arise in recreational activities, as well as in such practical areas as construction, business trades, the sciences, and the arts. Geometry provides powerful visual models of numerical and algebraic concepts. The study of geometric ideas and relationships can also develop students' ability to reason analytically and draw logical conclusions. For all of these reasons geometry must be included in the developmental mathematics curriculum. Likewise the curriculum must include the study of measurement, which is inextricably linked to geometry.

Several difficulties must be addressed regarding the inclusion of geometry in the developmental curriculum. First, geometry typically has not been included in this curriculum, and if it has been studied at all, it has usually been limited to a formula-driven approach to perimeter, area and triangle properties. Second, it is sometimes assumed that geometry at the developmental level should emulate traditional Euclidean geometry class, with emphasis on axioms, hypotheses and proofs. As indicated by the curricular guidelines listed below, neither of these approaches are models for the reformed curriculum. Instead the curriculum is intended to develop a geometrical intuition which can be used as a tool in understanding practical situations as well as mathematical relationships. To achieve this goal, geometry must be taught in *context*, using real-world applications and making explicit connections with other mathematical topics.

The study of geometry and measurement may be particularly effective if these topics are interwoven with other subjects in the developmental curriculum. They provide a particularly useful vehicle for reinforcing numerical concepts, practicing estimation and becoming facile with basic number facts. Geometry and algebra are also interconnected: geometry provides intuitively useful visualizations of various algebraic topics, while algebra summarizes many geometrical properties. The study of geometry must certainly incorporate the use of technology. It provides an optimal setting for developing students' understanding of significant digits and other issues of accuracy. Integrating the study of geometry and measurement with other subjects may also have the advantage of efficiency, alleviating problems connected with expanding the scope of the developmental curriculum.

The developmental mathematics curriculum will include the study of geometry and measurement so that students can meet the objectives outlined below.

Objective 1. Students will be able to use both the U.S. Customary System and the International (Metric) System of measurements in problems and everyday situations.

Problems

a. Determine the average number of gallons of oil a service station may expect to use per month for automobile oil changes, if the average car requires five quarts of oil and approximately 15 cars are serviced per day.

b. A runner runs a 5 km race in 21.3 minutes. What is her rate in m/s?

Objective 2. Students will be able to apply principles of accuracy and precision.

Problem

Calculate the volume of a soft drink can. Discuss the assumptions you made and the accuracy of your answer. Compare your answer with that on a soft drink can.

Objective 3. Students will be able to use basic properties of angles and triangles.

Problem

A 30 ft pole is to be braced with a wire.

a. What practical information do you think you will need to determine how much wire is needed?

b. Making reasonable guesses about these parameters, estimate how much wire may be needed.

Objective 4. Students will be able to apply concepts of perimeter, area, volume and weight for basic shapes.

Problems

a. How many square feet of carpeting are needed for an L-shaped classroom? Students can take appropriate measurements, reach decisions about how to deal with doorways and so on, and obtain information from stores about cost of carpeting.

b. Using scissors, demonstrate how the formula for the area of a parallelogram is based on the area of a related rectangle.

c. Construct a track with 15 m diameter semicircular ends so that the track is 200 m long.

d. Given a can of tennis balls (three to the can), which is larger, the height of the can or the circumference?

Objective 5. Students will be able to make connections between algebra and geometry.

Problems

a. Explore the effect on an area when a fixed perimeter is altered.

b. Develop a table of information and write a symbolic expression for area as a function of the fixed perimeter and a variable width. (*Mathematics Teacher*, March 1993).

c. Wheat is stored in cone-shaped piles. If we know the value of a bushel of wheat, how can we estimate the value of the wheat in a certain pile? Note that in this problem, the radius and height of the cone must be found indirectly from the measurable circumference and slant height. (*Mathematics Teacher*, March 1993.)

d. Illustrate the product $(x + y)^2$ using rectangles.

Standard 5. Students will be able to analyze data and use probability in order to make inferences about real-world situations.

It is important that developmental mathematics students be prepared for their multiple roles as students, workers, citizens, and consumers by having the ability to analyze data and make informed decisions. The real-life experiences of the developmental mathematics students can provide the basis for meaningful investigations and easy access to technology and will facilitate data manipulations and storage.

The concepts of central tendency and variation should be introduced into the curriculum within the context of these real-life investigations. Algorithms should receive attention with focus on how the algorithm reinforces the understanding of the concept.

Today's business world has become highly dependent on data analysis in the decision-making process. In addition, the mathematics used to analyze problems of uncertainty is receiving increased attention in the work place. Developmental mathematics students should be introduced to problems on uncertainty through experimentation.

The developmental mathematics curriculum should include experiences with data analysis and probability in real-world situations so that students will be able to attain the following objectives.

Objective 1. Students will be able to collect, process, and display data so that usable information is represented.

Data are often summarized with charts, tables, or graphical representations such as circle, bar, and line graphs. Students should be actively involved in gathering data, organizing the data through frequency distributions, histograms, and frequency polygons. The data to be gathered, organized, and analyzed should be interesting and relevant to adult students.

Sampling procedures should be important considerations in the data-gathering process. Students should understand the concept of random sampling and the consequences of bias in sampling procedures, and limited samples.

Problem

Form student groups of three or four and ask each group to design a statistical project in which data (at lease 50 scores) are gathered using appropriate sampling procedures, organized with frequency distributions, histograms, and frequency polygons. Circle and line graphs may also be appropriate ways to represent the data. Examples of data which may be gathered include: credit hours enrolled per student and time of day, movies viewed by students within last month, ways students in various majors spend out-of-class time. Students may also be able to work with the institution's research officer to conduct studies (relating to students) that are relevant to the college operation.

Objective 2. Students will be able to make inferences based on data analysis, applying measures of central tendency, variability, and correlation.

Many students have a basic understanding of an arithmetic average of numerical data, but this concept should be broadened to include other statistical averages of data. The mean, median, and mode should be examined as ways of identifying the characteristics of a set of data. Measures of central tendency should be combined with measures of variation or dispersion for a more complete analysis of data. It is important that students understand the need to look at the various statistics of a set of data before making inferences. One statistic in isolation can give misleading information.

Problem

Students should find the measures of central tendency for the data gathered in the previous problem. They should describe the mean, median, and mode for their data set and explain the implications of these three measures for the particular set of data. Further, students should find the standard deviation and variation and explain the implications of these two measures.

Objective 3. The student will be able to solve real-world problems by designing and conducting a statistical experiment, and by interpreting and communicating the outcome.

Problem

Student working groups can be formed to set up realistic statistical experiments, gather the data, analyze the data, and prepare a written and verbal report. Examples of problems that may be studied are effectiveness of advertising media by companies, the relationship between various locations of the same food chain and the most frequently ordered menu items, the relationship of various study strategies to course outcome (grade). Students should be encouraged to interview "live" businesses to determine research needs of the business or to "sell" a particular research design that would provide valuable information to the business.

Student experiences should be rich in activities that solve real-world problems using statistics. These activities should include the designing and conducting of a statistical experiment that relates to a specific problem. After the data are gathered, organized, and analyzed, the results should be interpreted and the outcomes communicated with a written or oral report, including visual representations.

Objective 4. The student will be able to represent and solve problems of uncertainty by using experimental and theoretical probability.

Problems of uncertainty should be investigated through experimentation. Experimental results and theoretical probabilities should be compared and analyzed.

Problem

Since many locations throughout the nation now permit gaming activities, an excellent topic for students is to experiment with die tossing, card dealing, and other probabilities related to the gaming industry. These experiments can then be compared with the theoretical probabilities. Experimentation with coin tosses can be used to illustrate binomial distributions. Newspapers may also be used to find data that can be used to compute probabilities. Data may also be obtained from various government sources such as the Census Bureau.

Objective 5. The student will be able to use the normal curve and its properties to make conjectures about sets of data that are normally distributed.

By comparing the characteristics of data sets that are normally distributed with those that are not normally distributed, students can enhance their understanding of the meaning of "normal distribution" and "normal curve." Actual test grades for a class may be listed as one set of data. This set, if not normally distributed, can be compared with a set of similar data that are normally distributed. Students should be able to construct a set of data that are normally distributed and show that the set is normally distributed through computational techniques. Students should be encouraged to study data that come from their major field of study.

Connections with many areas of mathematics as well as other disciplines can be made through probability. Students should understand the basis for making predictions and the underlying assumptions associated with predictions. An understanding of normal distributions, the normal curve, and its properties can illustrate the practice of making predictions based on specific criteria and assumptions.

Standards for Instructional Strategies

The standards for instructional strategies for developmental mathematics include those given in the chapter on the CORE; namely

- technology will be used naturally and routinely;
- interactive learning will be fostered, and projects that encourage independent thinking and require sustained effort will be assigned;
- examples and assignments will include meaningful problems;
- multiple approaches to problem solving - numerical, graphical, symbolic, and verbal - will be used to solve problems.

In addition to these CORE standards, developmental educators must use strategies that provide for the special needs of students enrolled in developmental courses. These special needs include provision for overcoming learning disabilities and math anxiety. For example, faculty may provide opportunities for students to take tests individually rather than in a class setting and may allow students additional needed time to complete a test.

[SPECIFIC EXAMPLES OF INSTRUCTIONAL STRATEGIES PARTICULARLY APPROPRIATE FOR DEVELOPMENTAL MATHEMATICS ARE NEEDED HERE]

Assessment

While standards for assessment are not being developed in this document, assessment must be viewed as an integral part of instruction. Mathematics educators must use assessment strategies that support learning. Students in developmental mathematics should be expected to write essays, do boardwork, do group projects, and make oral presentations, in addition to being evaluated by the more traditional quizzes, homework assignments, and tests. Good assessment practices should be indistinguishable from good instructional practices. The assessment instrument or method is not the end product of learning, but rather part of the learning process.

[A FEW EXAMPLES OF NONTRADITIONAL ASSESSMENT TECHNIQUES PARTICULARLY APPROPRIATE FOR STUDENTS IN DEVELOPMENTAL MATHEMATICS PROGRAMS ARE NEEDED HERE]

Summary

If the current reform of introductory college mathematics is to succeed, more than just the curriculum must be changed. Reform efforts must also address the issues of teaching and the assessment of learning. The standards produced must therefore, not only address what mathematics is taught, but how it is taught, and how the results are measured.

The developmental mathematics classroom may be the perfect environment to begin the implementation of these three components of the reform effort. Developmental mathematics must not be a mere "rehash" of the traditional K-12 experience, but rather a fresh approach to the study of mathematics at the collegiate level. Students enrolling in college should expect that their new collegiate experience in mathematics will acknowledge their experience as mature adults and that the instructor will draw upon it as part of the mathematics program.

In the introduction to the developmental mathematics curriculum, the vision of students emerging from the developmental mathematics classroom with the ability and confidence to use mathematics effectively as "students, workers, citizens and consumers" is promoted. In order to accomplish this, students must come to expect "lively classrooms where they will be enthusiastically engaged in activities designed to enhance their understanding of mathematics... and their ability to solve problems mathematically."

CHAPTER 4 - Associate in Applied Science Degree Programs

Students in Associate of Applied Science (AAS) degree programs enroll in varied vocational or technical curricula designed to prepare them for the workplace. With a few exceptions, AAS curricula cluster into four categories: health-related, business, engineering and science, and service-related. AAS students demonstrate the typical diversity found in two-year college students including an increased age at matriculation, a focus on short-term goals, and the combining of academics with work and/or family responsibilities.

Although AAS students may not plan initially to earn bachelors' degrees, opportunities for transfer are facilitated by the enhanced current or proposed mathematics curriculum found in AAS programs. Students enrolled in certificate programs or in associate degree programs without a general education requirement are not specifically referred to herein. However, the essence of the vision, curricular model, and instructional practices suggested in this document should be adapted to mathematics courses for these students whenever possible.

Vision

Students in AAS programs are preparing to become members of an ever-changing workplace and to function effectively in diverse work environments. More than any other group of collegiate mathematics students, their experiences should enable them to become flexible workers who can adapt to the needs of their jobs. They need to develop strategies that enable them to learn different or additional mathematics independently in order to change from one job to another. While an AAS student's primary mathematical emphasis should be its application to his or her chosen curriculum, such students should also learn to appreciate mathematics. Development of the ability to reason and analyze situations from mathematics and other disciplines will assist AAS students in becoming better-informed citizens and consumers.

In AAS programs, mathematics is presented as a necessary underpinning which supports problem-solving activities. The study of the proof and structure of mathematics, while still included, is de-emphasized. Connections are made within areas of mathematics and between mathematics and applications in other fields. Various mathematical threads, for example applied geometry, are woven throughout an integrated curriculum which extends beyond secondary school or foundational mathematics experiences and enriches students' understanding of the role of mathematics in vocational and technical curricula.

The amount of mathematics required by various AAS programs varies greatly. Decisions concerning how much mathematics is required for a particular program of study should be made by each college after review of accrediting agency guidelines and programs at other colleges and consultation with the mathematics faculty and experts from the field. All students, however, should cover the CORE content. While some programs require students to take one "liberal arts" type course, other programs may require mathematics through calculus. Course proliferation due to excessive custimization should be minimized wherever possible.

For those programs which require calculus, the courses leading up to calculus should be at a level of rigor and content that would adequately prepare students for the calculus in the Associate in Science (AS) program. These courses should be designed to follow a logical progression of study from one topic to another without outdated classifications such as algebra, trigonometry, or geometry.

Content should be emphasized, or de-emphasized, with technology in mind. Technology must be used naturally in the curriculum, particularly in problem solving, data analysis, statistical investigations, modeling, and graphical analysis.

Mathematics content in programs which do not require calculus should be taught at a level of rigor commensurate with collegiate mathematics. Courses which are designed to provide the student with a survey of mathematics should include content based on the standards in Chapter 5 of this document. For example, courses in mathematics for AAS business students should include applications using fractions and decimals, although the teaching of operations with fractions and decimals should take place at the developmental level. Courses which are designed to provide the student with a survey of mathematics should include meaningful content, such as discussed in Chapter Five (Baccalaureate-Intending Programs) of this document.

As the changes in mathematics instruction presented herein come about, it may appear that there is little difference between precalculus mathematics for AS programs and precalculus mathematics for AAS programs. However, the emphasis on preparation for the workplace for AAS students will influence both content and delivery of their mathematics. Also, the applications used to set the context for the learning of mathematics are more frequent and of a technical nature.

Goals

While the CORE goals for introductory college mathematics apply to all mathematics curricula, the following list of curriculum goals points out the special needs of students enrolled in AAS degree programs.

The mathematics curriculum for students in AAS programs will

- increase students' awareness of the relevance of mathematics in the workplace and the world;
- apply mathematics to a variety of fields leading to preparation for the workplace;
- provide regular opportunities to explore mathematics using a variety of manipulatives, measuring devices, models, calculators, and computers; and
- enable students to become better and more organized researchers -- able to find information, select resources, identify and interview experts.

Instructional Strategies

It is expected that the curriculum for students enrolled in AAS programs will be presented using the standards for instructional strategies given in the CORE.

Namely,

1. Instructors will use appropriate technology, naturally and routinely in teaching mathematics.

2. Instructors will foster interactive learning through writing, reading, and collaborative activities. Learning activities should include projects and apprenticeship situations that encourage independent thinking and require sustained effort and time.

3. Instructors will involve students in meaningful mathematics problems.

4. Instructors will model the use of multiple approaches (numerical, graphical, symbolic and verbal) to solve problems.

Instructors have the responsibility for adapting the various strategies to meet the needs of students enrolled in particular AAS programs. For example, the instructor teaching mathematics to electronics majors can design a laboratory experience which explores sine waves of voltage using an oscilloscope. Furthermore, classroom experiences with equipment specific to a technology area may be team taught with a technology instructor or prepared in consultation with a practitioner.

Qualified mathematics educators employed in industry who wish to continue their associations with students while working as adjunct instructors can be a valuable source of expertise in real industrial applications. While excessive dependence on adjunct instructors and teaching assistants can have detrimental effects, as was noted in the CORE portion of the document, individuals with work experience as well as teaching experience can enhance mathematics instruction for AAS students.

Furthermore, it is particularly important for technology students to learn to work in groups. Through frequent group activities, effective teamwork skills can be developed to prepare students for the teamwork necessary in the workplace. Finally, group projects should include the reading of technical materials and the writing of technical reports.

For any instructional strategy to be effective, the instruction and the classroom environment must be characterized by

- caring instructors who acknowledge and value the rich and varied experiences of their students;
- instructors who have high expectations of students regardless of race, gender, socioeconomic status, or disability;
- instructors who are mathematicians and who can develop sound instructional strategies and content to meet the special needs of AAS students;
- instructors who are available to students outside of the classroom to offer help; and
- a climate that is nonthreatening and encourages students to ask questions and to take risks.

Curriculum Model

Similarly, the content themes included in the CORE must be included within the mathematics curriculum presented to AAS students. The content reflects and builds upon the recommendations included in the NCTM *Standards* (NCTM, 1989).

Specifically,

- Number sense
- Symbol sense
- Spatial sense
- Function sense
- Probability and Statistics sense
- Problem solving sense

The idea of "sense" here is of particular importance. The programs of AAS students may require different emphasis in those students' study of mathematics. Mathematical intuition developed through an understanding of the content and how it may be applied to solving problems will serve the needs of these students and provide a basis for future study. In addition, AAS students should have special emphasis placed on

- reading technical charts and graphs;
- reading and learning from other technical materials;
- working with formulas, computationally and algebraically;
- problem solving with real applications; and
- regular use of appropriate and field-specific technology.

Specific curricula must be tailored to the various programs within health-related, business, engineering and science-related, and service-related areas. Students will begin to study the mathematics required for their AAS programs at a level determined by their institutions. The amount of mathematics studied (beyond the CORE content) is dependent upon the program of study, the student population, advisory group recommendations, and accrediting agency guidelines. It is not the intent that each curriculum should have its own mathematics course. Rather, programs with similar mathematical needs may enroll students in a common mathematics course in which applications and student projects may be program-specific. At many institutions, successful Tech Prep programs exist which assist institutions in establishing guidelines for quality mathematics for various curricula. As shown in Figure 1, completion of the AAS program creates a gateway into the workplace and informed citizenry and consumership.

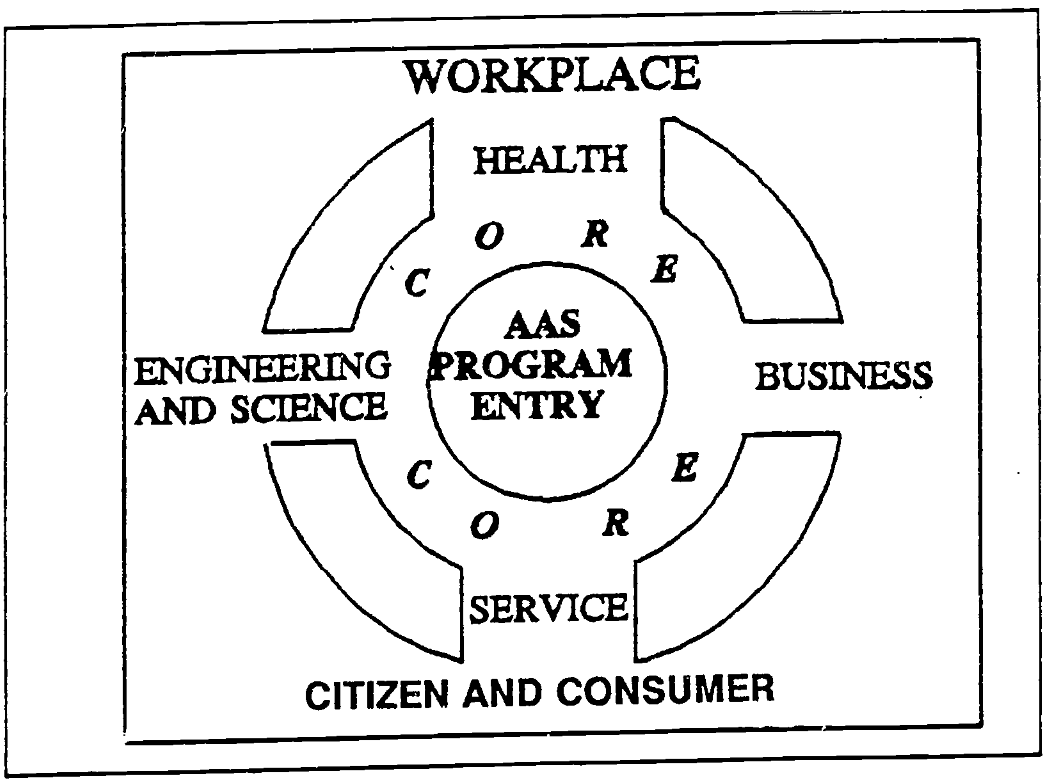


Figure 1: Routes to success through AAS programs.

For students not meeting the mathematics entry level in the AAS programs, each institution should design a developmental course or sequence of courses designed to help AAS and other students to

- begin their study of mathematics at a point where success is probable; and
- exit the preparatory course(s) prepared to enroll and complete successfully the mathematics required in their fields of study.

The curriculum model for the mathematics education of AAS students is applications-based. As indicated in the SCANS Report for America 2000, Learning a Living: A Blueprint for High Performance (SCANS, 1992), "Teaching should be offered 'in context,' that is, students should learn content while solving realistic problems" (p. xvi). Mathematical topics are motivated and enhanced by both short classroom activities as well as extended projects. Short classroom activities (examples follow) may be used in small group settings, demonstrations, presentations, or discovery lessons. Extended projects (examples follow) are activities which require the students (in groups or individually) to select and use appropriate mathematics to solve nonroutine problems. Projects may involve the use of technology and require the student to gather data and other information, organize and analyze the data, and interpret the results. Projects are generally assigned to be completed over an extended period of time and involve work both inside and outside the classroom.

The applications which motivate the mathematics in AAS programs derive from the particular field of study, from the students' needs, interests, and experiences, and from other real world examples. Applications should

- be developed in consultation with practitioners in the field,
- be revised and updated regularly, and
- incorporate emerging technologies.

Completion of the curriculum should enable students to learn additional mathematical or technical material at some future time.

Standards for Student Outcomes

Standard 1: **Students will have the mathematical tools to analyze and solve problems, using numerical, graphical, and symbolic approaches, and modeling.**

The focus of the educational experience for AAS students is problem solving. Mathematics that reflects this emphasis will be more meaningful to these students. Problem solving strategies include posing questions, organizing information, drawing diagrams, analyzing situations through using trial and error, graphing, and modeling, and drawing conclusions by translating, illustrating, and verifying results. The problems that are used for educational purposes should be meaningful and of interest to the students involved.

In order to produce good problem solvers, the mathematics courses in AAS programs should equip students with necessary mathematical tools. The amount of mathematics required in AAS programs varies greatly from institution to institution and even within the institution from program to program. Each institution should determine the amount and nature of mathematics required in each curriculum, in collaboration with various outside groups such as accrediting agencies, advisory groups, and state governing boards.

An important accomplishment of the successful mathematics student is to be able to transfer mathematical skills to specific technical applications. By engaging in activities that promote problem solving and critical thinking, the student should become more capable of making connections to specific technical applications. All students should understand the idea of a mathematical model, should be able to find one or more models to fit data, and should be able to evaluate models for appropriateness. Students should be comfortable and confident using numerical, graphical and symbolic approaches to solving problems.

The examples that follow each standard serve to illustrate the ideas expressed in that standard.

Example 1: Interpreting Data from a Graph

Objective: Students will be able to read graphs and extract meaningful information from graphs.

The curve in Figure 2 indicates that the open-loop gain of an operational amplifier is 100,000. This means that an input-signal voltage would be amplified 100,000 times at the output if there were no feedback resistor.

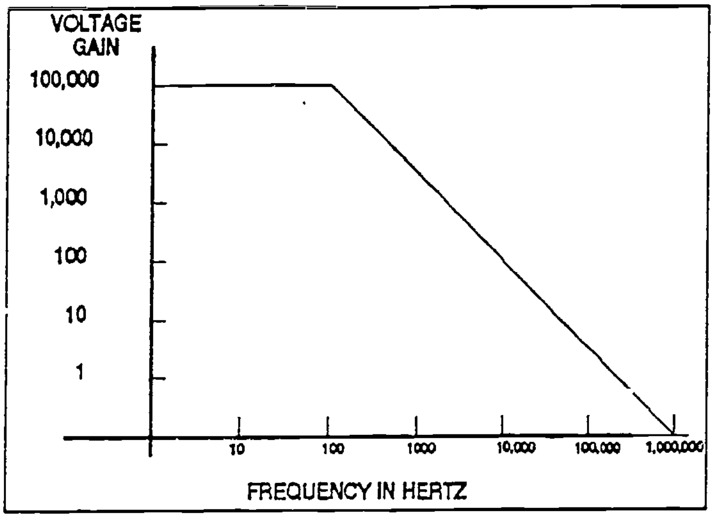


Figure 2. An open loop response curve of an operational amplifier.

a. Find a piece-wise function to model this curve.

b. Describe the domain.

Extension

c. The bandwidth of an amplifier is the range of frequencies between the points where the gain drops to 0.707 times the maximum gain. Find the bandwidth for this example.

Example 2: Modeling Numerical Data

Objective: Students will be able to make a table of values given a model in the form of an equation and construct a mathematical model given numeric data.

The following formula models the apparent temperature T (wind chill) for various wind velocities, *v*, in miles per hour at 32 degrees Fahrenheit.

$$T = 91.4 - \frac{297\,(10.45 + 6.68\sqrt{v} - 0.447\,v)}{110}$$

a. Make a table of values for this function using wind velocities of 5, 10, 15, ..., 40.

b. Use the table of values in part a. to determine the change in apparent temperature as the wind increases from 10 mph to 15 mph? From 25 mph to 30 mph?

c. Using the calculations in the table above for 25 mph and 30 mph only, estimate the apparent temperature for 27.5 mph. Do you think this is an exact answer? Why or why not?

d. Do you think T is a linear function of v? Why or why not?

Wind Velocity (mph)	Temperatures (°F)									
	35	30	25	20	15	10	5	0	-5	-10
5	33	27	21	16	12	7	0	-5	-10	-15
10	22	16	10	3	-3	-9	-15	-22	-27	-34
15	16	9	2	-5	-11	-18	-25	-31	-38	-45
20	12	4	-3	-10	-17	-24	-31	-39	-46	-53
25	8	1	-7	-15	-22	-29	-36	-44	-51	-59
30	6	-2	-10	-18	-25	-33	-41	-49	-56	-64
35	4	-4	-12	-20	-27	-35	-43	-52	-58	-67
40	3	-5	-13	-21	-29	-37	-45	-53	-60	-69

Table 1. Apparent Temperature (Wind Chill) As a Function of Wind Velocity and Temperature

<u>Extension</u>

e. While the formula for *T* in terms of *v* only finds the apparent temperature for 32 degrees, Table 1 above shows the wind chill for various wind velocities at different temperatures. Find a model for any one row and any one column of the table.

Example 3: Various Methods of Solving Problems

Objective: Students will be able to solve problems graphically, analytically, and using technology.

Two forces are acting on an object. One force is 450 Newtons at an angle of 70 degrees. A second force is 100 Newtons at an angle of 330 degrees. Find the magnitude and direction of the resultant force.

a. Solve the problem graphically using both the triangular and parallelogram methods for adding vectors.

b. Solve the problem analytically using both the component method and the law of sines.

c. Solve the problem with a calculator or computer using parametric equations. Graph these equations. Trace to find the end point of the resultant vector.

```
Xt1 = 450 cos(70)
Yt1 = 450 sin(70)

Xt2 = 100 cos(-30)
Yt2 = 100 sin(-30)

Xt3 = Xt1 + Xt2
Yt3 = Yt1 + Yt2
```

Parametric Equations Used to Solve for the Resultant

In comparing answers obtained through the methods listed above, instructors can guide students through a discussion of the relative importance of differences in answers and which method is most appropriate to use.

Standard 2: Students will have the confidence to access and use needed mathematics and other technical information independently, form conjectures from an array of specific examples, and draw conclusions from general principles.

Today's graduates will most likely change jobs many times and careers several times before leaving the work force. Since requirements of the workplace are changing dramatically, the mathematics for AAS programs must provide students with a sufficient background in mathematical skills, confidence, and motivation to be independent learners for future needs.

For example, technicians must be able to locate and retrieve information from technical manuals, graphs or reports. In addition, they must be able to identify and extend patterns and use experiences and observations to make conjectures.

Example 4: Access and Use Technical Information

Objective: Students will be able to use mathematics together with technical data to solve a problem.

The object pictured in Figure 3 is made of silver and has a circular rod running completely through it. The brass rod is 22.5 inches long with a diameter of 3.5 inches.

a. Find the density of each material in lb/ft^3. There are several sources from which to find this information. A physics book should have a list of materials with their densities.

b. Find the volume of the compound object.

c. Find the weight of the compound object.

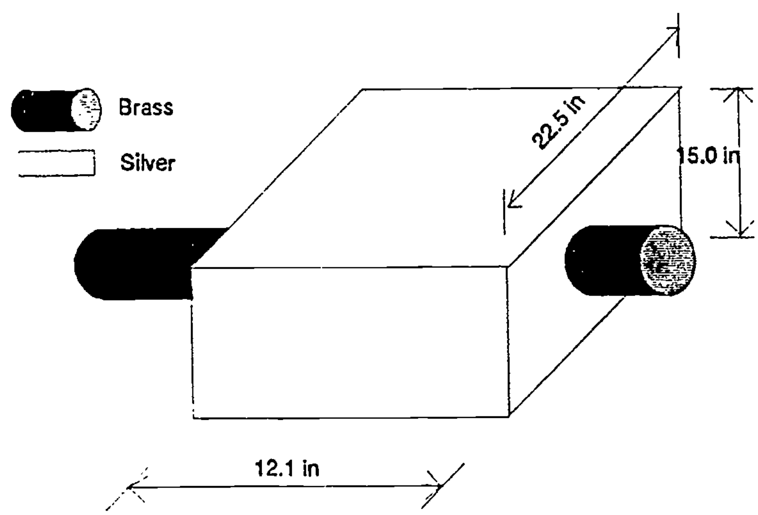


Figure 3.

In this example, students are challenged to become independent learners by locating and using a proper source for necessary data. Students must understand the shape of the object from its drawing, determine appropriate formulas, and understand relationships among linear dimensions, volume, density, and weight.

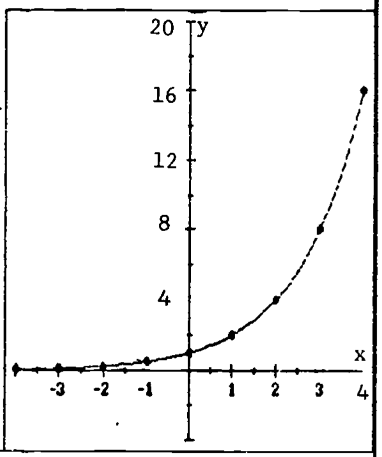
Example 5: Drawing Conclusions

Objective: Students will be able to use mathematics to gather data, make and test conjectures, and draw conclusions.

Consider the following pairs of graphs in Figures 4 and 5.

$y=2^x$

X	Y
-4	.8625
-3	.125
-2	.25
-1	.5
0	1
1	2
2	4
3	8
4	16



$y=LOG(2^x)$

X	Y
-4	-1.20412
-3	-.90309
-2	-.60206
-1	-.30103
0	0
1	.30103
2	.60206
3	.90309
4	1.20412

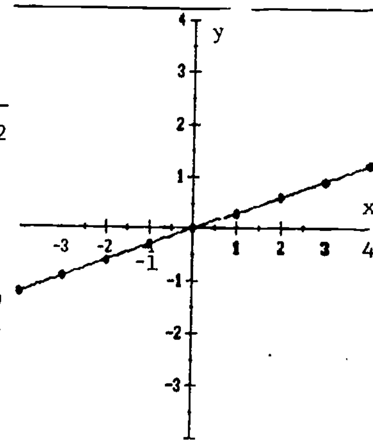
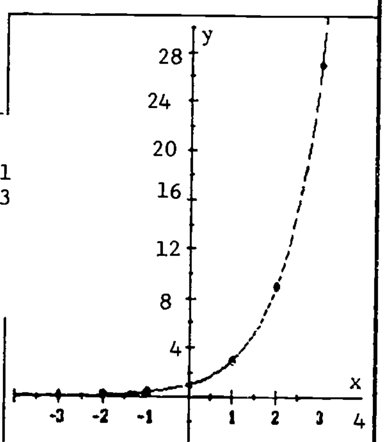


Figure 4.

$y=3^x$

X	Y
-3	.837037
-2	.1111111
-1	.3333333
0	1
1	3
2	9
3	27



$Y=LOG(3^x)$

x	y
-3	-1.431364
-2	-.9542425
-1	-4.771213
0	0
1	.4771213
2	.9542425
3	1.431364

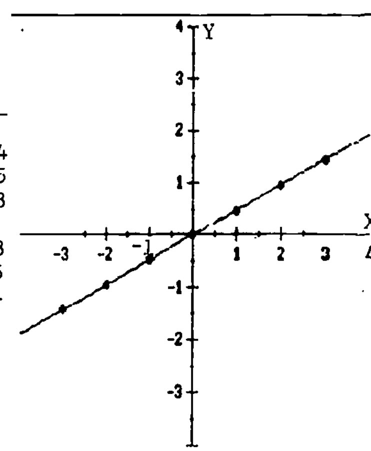


Figure 5.

In each pair of figures, the graph on the left is the graph of an exponential function and the graph on the right is the graph of the logarithm of the function. A table of values for each graph is also included.

a. Find the slope of the line in each of the right-hand graphs.

b. Compare the slope of the line in each right-hand graph to the base of the corresponding exponential function sketched in each left-hand graph. What conjecture can you make from your observations?

Extension

c. Consider the pairs of functions $y = x^2$ and $\log y = \log(x^2)$, and $y = x^3$ and $\log y = \log(x^3)$.

Graph each pair of functions. What conjecture can you make about the relationship of the graph of a power function and the graph of the logarithm of that power function?

Example 6: Forming Conjectures

Objective: Students will observe differences and similarities in various graphs and communicate their observations in writing.

According to the laws of physics, the height of a projectile thrown directly upward with velocity v_0 and initial height h_0 can be modeled by the equation

$$s(t) = -16t^2 + v_0t + h_0$$

Observe the differences in the graphs of this function where $h_0 = 5$ and v_0 assumes the values 15, 25, 35. Describe the differences and the similarities in the three graphs in a paragraph using complete sentences.

Standard 3: Students will choose appropriate tools and technology to solve mathematical problems and judge the reasonableness of the results.

Graduates of AAS programs will need to use mathematics with facility in their jobs and integrate their mathematics knowledge with the requirements of their employers and particular technology areas. Students will need to be able to solve problems effectively and efficiently. They will need to choose appropriate mathematical methods and technology for solving each problem that they encounter in the classroom as well as on the job. The question of whether to use mental estimates, paper and pencil algorithms, or calculator or computer technology must be answered effectively by AAS students. Calculator technology may include nongraphing or graphing calculators, while computer technology may include the use of existing software such as a spreadsheet, database, graphing utility, computer algebra system, or computer aided design or even a self-written program. Furthermore, effective instruction in mathematics courses, both in AAS programs and in the core curriculum, should nurture an intuitive sense of the reasonableness of results. Students should develop the competence to check both their work and that of others for reasonableness so that they may work interdependently when sharing information is necessary. In cases where there are several correct answers which meet the requirements of the problem, the student, working with others, must learn how to make decisions based on sound mathematics as well as economics, politics, and even personal values.

Example 7: Using Available Tools

Objective: Students will be able to display and explain graphically and numerically why an approximation is good for x-values close to zero.

The value of $\cos x$ can be approximated by the series of terms

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$$

Show that as the number of terms used in the expression increases, the value of the expression approximates $\cos x$ for x-values close to zero. Let:

$$y_1 = \cos x$$

$$y_2 = 1-\frac{x^2}{2}$$

$$y_3 = 1-\frac{x^2}{2}+\frac{x^4}{24}$$

$$y_4 = 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}$$

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a. Method #1: Graphically

Graph each equation shown above on the same graphing calculator or computer screen. Notice that the graphs converge near $x = 0$. Thus, the value of the expression $\cos x$ is close to the value on each curve that approximates $\cos x$.

b. Method #2: Numerically

Construct a spreadsheet that illustrates numerically the values of each expression that approximates the value of
$\cos x$.

The spreadsheet will show that as the number of terms used in the approximation increases, the value of the approximation gets closer to the value of $\cos x$ for x-values close to zero. Use values of x between -1.0 and 1.0 in increments of 0.1.

In solving this problem, students must understand the graphical significance of the various approximating curves as well as the trends in the numeric data in the spreadsheet.

Example 8: Find an Appropriate Solution

Objective: Students will understand a real-life problem situation and make and test conjectures about its solution. With the support afforded by technology, students experiment to determine the best solution to a problem.

Chlorine is used to control micro-organisms in the water of a pool. Too much chlorine produces burning eyes; too little, and the slime moves in. Here are some facts about pool care:

- Chlorine dissipates in reaction to bacteria and to the sun at a rate of about 15% of the amount present per day.
- The optimal concentration of chlorine in a pool is from 1 to 2 parts per million (ppm), although it is safe to swim when the concentration is as high as 3 ppm.
- It is normal practice to add small amounts of chlorine every day to maintain a concentration within the 1 to 2 ppm ideal.

a. Use a calculator to find the amount of chlorine (in ppm) remaining each day for 10 days, if the level at time zero is 3 ppm and no more is added.

b. Graph the concentration of chlorine (in ppm), *c*, as a function of time, *t*, for the data determined in part a. Find the interval of time over which the chlorine level is optimal for humans.

c. If chlorine is added every day, another model is necessary. Use a spreadsheet to model this system for a 21 day period when the concentration is 3 ppm at time zero.

   i. Try adding 1 ppm each day. Clearly that is too much, but does the pool water turn to chlorine? What is the largest amount of chlorine attainable?

   ii. Try adding 0.1 ppm everyday. Does this process yield ideal conditions in the long run?

   iii. Find a daily dosage that stabilizes the concentration of chlorine at 1.5 ppm.

In solving this problem, students must understand thoroughly the parameters of the situation--the facts about pool care. Through experimentation afforded by technology, students can design and determine an effective solution to the chlorine problem and judge whether their solution is reasonable.

Standard 4: Students will be able to work effectively in groups and communicate about mathematics both orally and in writing.

Employees need to have the ability to communicate orally and in writing, and to work effectively in groups. In technology areas supported by mathematics, students must work to develop these important lifelong skills in the mathematics classroom. Frequent instructional activities which include writing about mathematical concepts using appropriate terminology (and using appropriate word processing technology) will provide students with the needed practice. Oral communication can be enhanced through in-class reports to the whole class. Effective group work will include discussions where a plan for carrying out a group assignment is discussed and developed, responsibilities are shared, and individual efforts are balanced and evaluated against the goals of the group as a whole. Written and oral group project reports should be an integral part of the process.

Example 9: Group Activity Modeling

Objective: Students will be able to make conjectures and test them in a group setting. Based on data, they will make decisions on how to derive a model and what factors should be included in the model. In a group, sufficient data will be gathered to describe and test the model.

Students are divided into groups and given the following equipment: a weight set, string, and a stopwatch. Each group is asked to construct a pendulum that will have a certain number of cycles per every 10 seconds. The group records the changes made in each pendulum and the effect each change had on the number of cycles per second. Once this task is completed the group is asked to develop enough data to determine the function which models the frequency of the pendulum. The group will make an oral report to the class on their findings.

Students are given little direction. Group-decision making, observation, and experimentation will lead eventually to rejection of all but the correct solution. The group designs the conduct of the experiment, records observations, makes and tests conjectures, and formulates an oral presentation on the findings.

Example 10: Medication Model

Below is the numeric representation of the amount of the prescription drug Digoxin in a typical patient's blood stream over a 31-week time period. The symbolic representation of the mathematical model you are to develop should reflect the amount of Digoxin in the blood stream at any time during the 31-week period when the patient is on the medication. The Digoxin is measured in nanograms per milliliter of blood. You may use any arithmetic combination of functions to develop your symbolic representation of the function. There are many different correct responses to many of the questions since there are many different acceptable mathematical models.

Table 2. Blood Levels of Digoxin by Weeks

t	0	.5	1.	1.5	2	8	25	31	31.5	32	32.5
D	0	1	2	2.5	2.5	2.5	2.5	2.5	2	1	0

t = time in weeks
D = amount of Digoxin in nanograms per ml

a. What type of function can be used as a general behavior model? That is, as you study the data, what does it most behave like?

b. What function can you use to restrict the domain to real numbers larger than or equal to 0 units and less than or equal to 32.5 units?

c. What is the symbolic representation of your first attempt at a model?

d. What adjustments to your first model can you make to develop a better model?

e. What is the symbolic representation of your second attempt at a model?

f. What adjustments to your second model can you make to develop a better model?

g. What is your final mathematical model for the Digoxin level? As you continued to improve your model, how many times did you make adjustments to get your final model?

h. Describe a situation under which your model of the Digoxin would not apply. List any other limitations on your model.

i. Using **your** model, what amount of Digoxin is present at week 20? At week 31.25? How long has the patient been on Digoxin if the blood level is 1.5 nanograms per ml?

This problem has no one correct answer. Students are challenged to derive and support an appropriate model. There is no obvious answer, therefore students are required to formulate and substantiate their choice of solution. Extensions to this problem include a discussion of how good is a good model and further questioning of the student to extract greater understanding of the solution.

Example 11: Automobile Purchase vs. Lease Decision

Develop a three-year automobile lease or purchase plan for your employer and submit a written report containing your recommendations and justification.

Scenario: Students work in small groups. They will formulate an action plan, establish criteria for decision-making, and gather data and other necessary information.

Learning activities: Students must do research and make decisions based on the employer's needs. This will involve exploring purchase and lease plans with various options and locating, collecting, organizing, analyzing, and evaluating data. Using teamwork, each group arrives at a conclusion and determines key points to include in the justification.

Procedure: In class, groups present their plans, interim reports, and final reports. Outside class, students work together to research the problem and prepare the reports.

Student Outcomes: Upon successful completion of this project, students experience a sense of accomplishment, increased knowledge of their community, and awareness of the value of time management and teamwork. They learn about leadership and delegating tasks within their groups. The mathematics they use will include percentages, comparisons, estimation, statistics, and probability. Tools to support the exploration necessary for this project may include a spreadsheet and a calculator. [*Consortium*, , COMAP]

Example 12: Data Analysis

Any model found using data analysis should be tested to see if it is a good model by looking at both the goodness of fit and at the residuals. Mathematical models are seldom exact, and the imperfections are of great concern. The amount by which a model misses the correct prediction is known as a residual. A residual is the actual value minus the predicted value. A very good mathematical model does not miss the data points used to generate it by a significant amount. However, it is also true that a good model will not create a pattern in its residuals. That is, the residuals should be randomly distributed. An appropriate method for checking the patterns in residuals is to plot them sequentially, either above or below a horizontal axis, according to whether they are positive or negative.

Consider each of the following data sets, its linear model, and the plot of its residuals. What conclusions can you draw from the residual plot?

Figure 6 shows the actual data points. Linear regression, (or a median-median line) is used to find a line of best fit. Even though there was a correlation of .97, this line may not be the correct model.

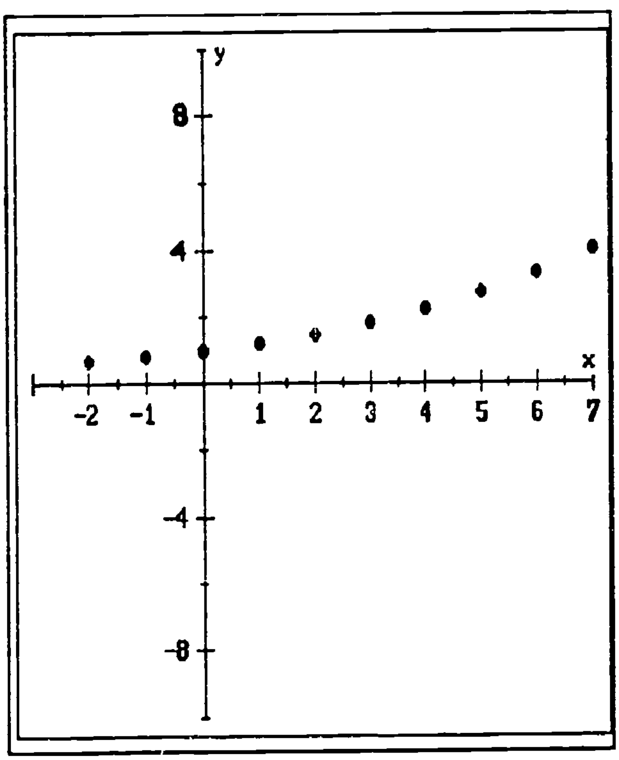


Figure 6

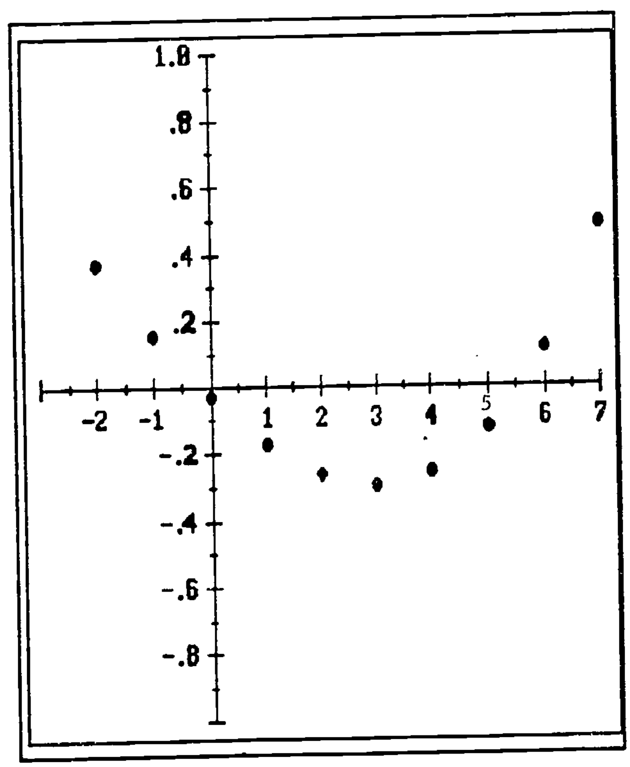


Figure 7

The plot of the residuals, Figure 7, shows a pattern. The student must see a pattern and re-express the data using some other model. The function in this example is $y = e^{x/5}$.

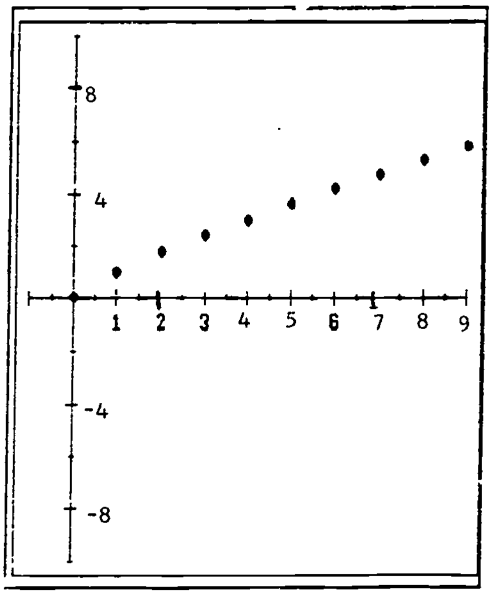


Figure 8

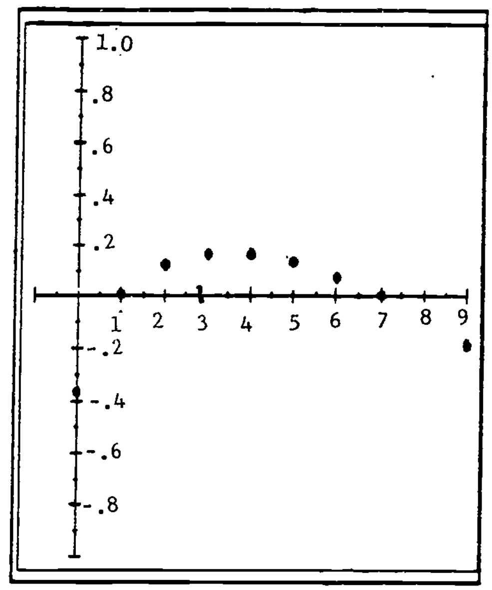


Figure 9

Figure 8 is the plot of the actual data points. Linear regression was used to find the line of best fit. The correlation was .99. But, a plot of the residuals (Figure 9) shows a pattern and indicates the need for further analysis. The function used to find the data points was $y = x^{0.8}$.

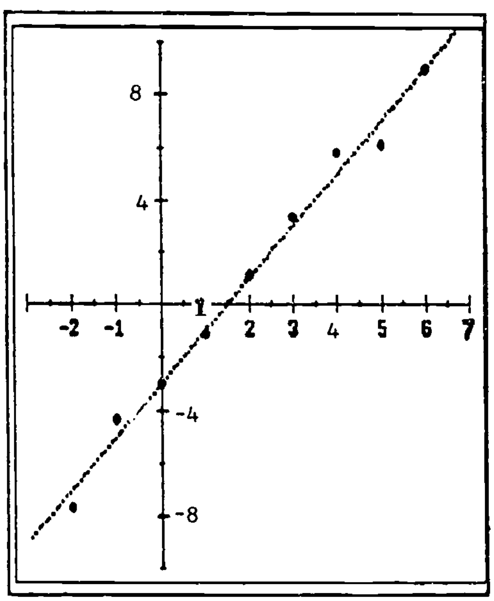


Figure 10

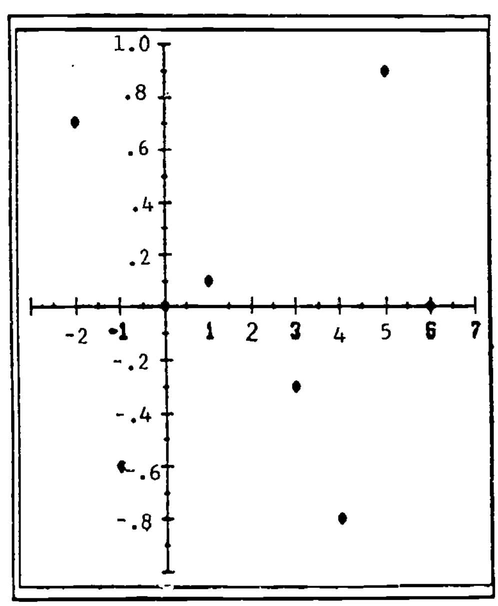


Figure 11

The data points in Figure 10 above were generated randomly. The line of best fit is also shown and it appears that the line is a fairly accurate model. A plot of the residuals in Figure 11, reveals a random displacement with no pattern. We will accept this linear model.

Students should be able to test the appropriateness of a model by examining the residuals. A program to plot the residuals on a TI-81 is shown in the appendix. [WILL APPEAR IN FINAL DOCUMENT] [*Consortium*, Spring 1992, COMAP]

Assessment

The level of expectation required of AAS graduates must be high in order for them to attain the skills and knowledge needed to be productive workers and to prepare for lifelong learning. Helping students meet this level of expectation will require new classroom strategies, changed standards of performance for faculty and students, and new assessment methods. Assessment routinely should build excellence into the educational process by providing regular feedback to the student and the instructor about learning and instruction.

Assessment tools will need to reflect different instructional strategies. A variety of methods will have to be developed to assess student progress, instruction and the curriculum. Assessment will be in the areas of mathematics content and problem-solving as well as in students' ability to communicate, work in groups, and read technical materials. The effectiveness of the curriculum depends on its ongoing revision and revitalization based on regular evaluations by mathematicians, mathematics educators, and practitioners in relevant fields. Periodic discussions should be held with graduates of AAS programs and their employers to determine the effectiveness of these programs in meeting business and industry needs of the community.

Implementation

Successful implementation of the curriculum and instruction standards contained herein requires institutions to provide release time and other resources to support the professional development of faculty. As stated in Moving Beyond Myths (National Research Council, 1991), faculty need to think as deeply about how to teach as about what to teach. Resources must be provided so that faculty can attend meetings, seminars, workshops, and short courses. It is critical that institutions acknowledge the importance of ongoing faculty development and reward faculty for their active involvement in curriculum and pedagogy reform.

The content organization of the mathematics curriculum for AAS students envisioned in this document presents a radical departure from most current practices. Faculty who teach mathematic courses in AAS programs need to interact with experts in the various fields to develop challenging, relevant applications for classroom use.

Establishing business/educational alliances will insure the currency and appropriateness of the curriculum. This will be further reinforced by open communication with accrediting agencies (see Figure 12).

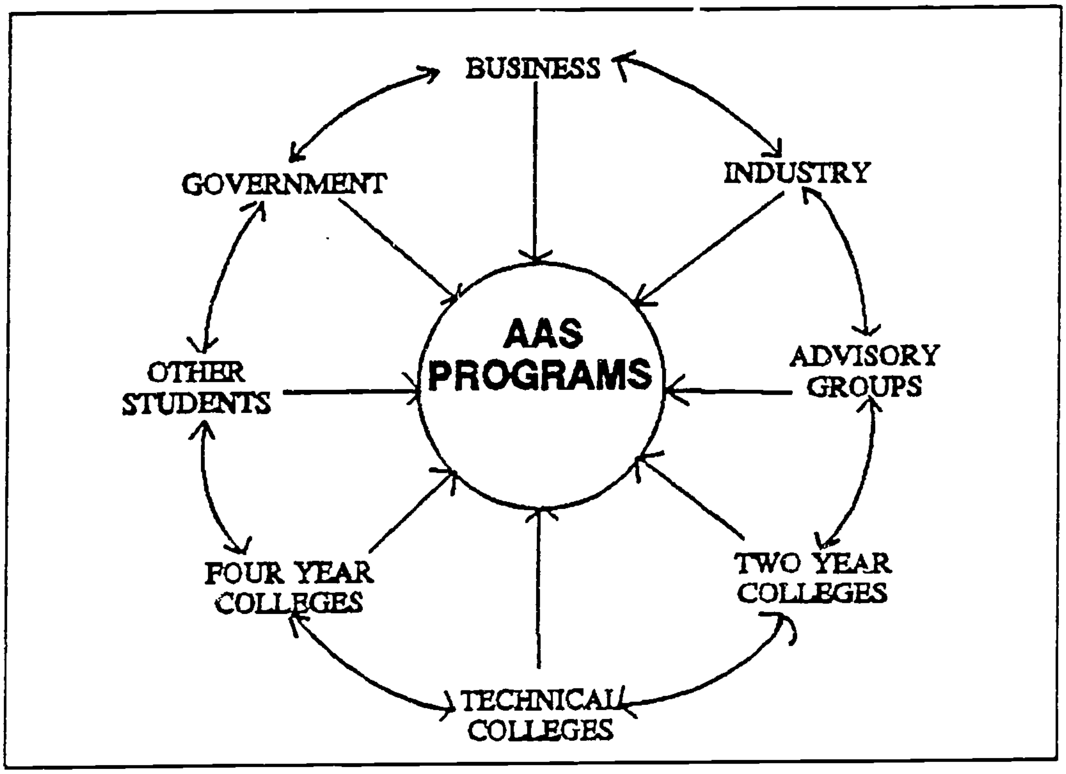


Figure 12
Information Flow for an Effective Learning Environment

The success of the applications-based curriculum model will (in large part) depend on the availability of quality instructional materials. There must be a coordinated national effort to identify sources of appropriate materials, such as COMAP. Existing materials must be evaluated to determine their applicability to mathematics education in AAS programs and new materials developed as needed. Information about high quality applications-based curriculum materials and how they can be obtained must be widely disseminated. These materials will be complemented by others developed locally.

The implementation components described above are not without cost. It is imperative that academic institutions, business and industry, and government agencies "invest in change" (SCANS, p. 44) in order to implement these standards for collegiate mathematics and make them a reality.

CHAPTER 5 - Baccalaureate-Intending Programs

[THIS CHAPTER ADDRESSES CURRICULUM STANDARDS FOR BOTH MATHEMATICS RELATED MAJOR PROGRAMS AND LIBERAL ARTS DISCIPLINES. ARE THESE AUDIENCES DISTINCT ENOUGH TO WARRANT DISCUSSION IN SEPARATE SECTIONS?]

This chapter addresses the mathematical needs of college students who are enrolled in programs that lead to bachelors' degrees. At two-year colleges such students would be in either Associate in Arts or Associate in Science programs. The programs are described here as "baccalaureate-intending" to denote the fact that they lead to bachelors' degrees, although the students in the programs may have no intentions of pursuing such degrees after an initial period of two-year college or lower division enrollment.

Vision

Mathematics at the undergraduate level is changing in colleges and universities across the country. In response to the many sets of recommendations about the revitalization of undergraduate mathematics and guidelines describing the nature of undergraduate programs and departments in the mathematical sciences, faculty at two-year, four-year, comprehensive, and research oriented colleges and universities are carefully considering steps toward change. For example, with major support from a National Science Foundation grant, the University of Illinois hosted a conference for 75 leading mathematics educators from all levels of mathematic education on *Preparing for the New Calculus* in April 1993. The conference was organized under the assumption that calculus reform is now a reality and high schools and colleges must prepare students in new ways to meet the needs of the new calculus. Reports on this conference will be forthcoming from the MAA. Furthermore, the National Council of Teachers of Mathematics has produced two documents, *Curriculum and Evaluation Standards for School Mathematics* (NCTM,1989) and *Professional Standards for Teaching Mathematics* (NCTM,1991) that, over the next 10 years, will dramatically affect the background preparation of students enrolling in institutions of higher education. At the same time, there is wide-spread activity aimed at changing ways of teaching calculus and other mathematics courses at the undergraduate level. For example, technology has profoundly affected the way professionals do mathematics and, therefore, must be integrated into the teaching and learning of mathematics. Mathematics is a vibrant and growing field. Both the content and applications of mathematics are expanding to more fields, in the scientific as well as in liberal arts disciplines. Students, as they enter their chosen careers or begin to function as informed citizens, will discover an increasing need for understanding mathematics. Within this changing climate, colleges and universities are enrolling students who have completed more traditional programs at the secondary level than those recommended in the NCTM documents mentioned above, students who have been away from formal school experiences for a period of years, and students who, despite dedicated and committed teachers, have not yet mastered basic mathematical competencies.

The following set of standards is developed for two groups of students. The first includes students who are enrolled in programs in the mathematical sciences, science, engineering, computer science, business, and economics. The second includes students who are enrolled in programs in the liberal arts disciplines. Such disciplines include the humanities, elementary and middle grades education, nonscience secondary education, and the social and behavioral sciences. The standards provide for a mathematic component that is diverse and meaningful in order to prepare adults for work, as well as for life-long learning that meets the cultural and intellectual challenges of the 21st century. Courses designed to meet the experiences described in the Curriculum Standards should reflect the background, talents, and intended career choices of the audience.

These standards are addressed to institutions and their faculties as guidelines for developing introductory college mathematics programs that will enable students to complete major programs in their chosen fields successfully. The experiences described in the standards that follow are based on the assumption that all students have the ability to

- reason proportionally;
- perform basic algebraic manipulations;
- solve equations and inequalities symbolically and obtain approximate solutions graphically;
- solve systems of linear equations symbolically and obtain approximate solutions graphically;
- understand and apply basic properties of plane geometric figures (triangles, circles, quadrilaterals), including such concepts as area, perimeter, volume, similarity (including scaling), and congruence;
- visualize, describe, and roughly sketch figures in three-dimensions; and
- understand, apply, and interpret measures of central tendency.

Individual institutions (or coalitions of institutions) have the responsibility to formulate and describe particular courses, course sequences, or other instructional strategies that enable students to develop the background preparation described above.

Instructional Strategies

College students bring to their study of mathematics diverse backgrounds of previous experiences and levels of understanding of concepts. Mathematics is internalized differently by these students and they construct new meanings in individual ways. They progress in their learning by exploring problem situations that are based on interesting real world content that emphasize interrelations among topics and the connectedness of different fields within mathematics.

In order to help students move toward developing solid mathematical understanding, learning experiences should be provided so that students will

- carry out projects, individually or in groups, in real world context;
- draw conclusions from appropriately designed sets of examples and then develop additional examples to refine their conclusions;
- construct physical models;
- participate in collaborative classwork to explore and develop concepts;
- communicate orally and in writing of in cooperative groups, in whole-class discussion and in formal presentations; and
- develop the ability to persevere in working on problems involving new ideas or novel contexts.

In particular, experiences in mathematics at the lower division level must include many and varied opportunities for students to participate in collaborative exploration, oral and written discussion, and the use of technology in the investigation of concepts and in solving real problems. These experiences must be provided in an environment of mutual respect and cooperation among students and faculty. It is critical that a sense of community be established among these emerging scholars. Many students at this level have fragile confidence in their own ability and are tentative about entering disciplines based on mathematical understanding. They will need support and careful advising from caring and concerned faculty.

A "revised" undergraduate mathematics curriculum is still under development, but certain aspects of change are already clearly indicated. Hence, we have made some assumptions about the mathematical content and context that baccalaureate-intending students might encounter in later experiences during their major programs. These assumptions include that, in their later experiences, students will have

- regular access to appropriate technology;
- applications which include probabilistic and statistical thinking and the analysis of distribution functions;
- linear algebra as a requirement in various major programs in the sciences;

- experiences in probability, statistics, linear programming, matrices, counting, and combinatorial reasoning as requirements for those in business/economics areas.

Curriculum Standards

As noted in the CORE, the goals for introductory college mathematics are to

- provide rich, deep experiences that encourage independent, nontrivial exploration in mathematics, build tenacity, and reinforce confidence in the ability to use mathematics appropriately and effectively;
- present mathematics as a developing human discipline and demonstrate its connections to other disciplines;
- illustrate the power of mathematical thinking as a foundation for independent lifelong learning and problem analysis; and
- increase participation in mathematics and in careers using mathematics, particularly by members of underrepresented groups -- women, minorities, and students with learning difficulties, differing learning styles, and language and socialization difficulties.

In keeping with these goals, all experiences designed to meet the standards in this section should be presented to students in ways that model the instructional strategies identified in the CORE standards of Chapter 2 and make use of available technology to enhance students' learning and doing of mathematics.

The standards presented in this chapter reflect key concepts, understandings, and abilities students must develop in their introductory experiences. Especially for those pursuing majors in the liberal arts disciplines, opportunities to enlarge their perspectives beyond the view that mathematics is purely a tool to be used in problem solving will need to be included. Students should view mathematics as a human endeavor, rich in historical and cultural contributions. Thus, it is our expectation that all courses designed to meet these standards will contain components that lead students to

- recognize the aesthetic components of mathematics;
- value mathematics as an ongoing human endeavor and appreciate the contributions it makes to social and economic development;
- develop a historical perspective of mathematics and the contributions made to the discipline by all groups in our society.

[COMMENTS ARE NEEDED ON THE ROLES OF MATHEMATICAL RIGOR AND INTUITION IN THE PRESENTATION OF COURSE CONTENT]

STANDARD 1: Build Knowledge of Functions

Functions are an important foundation for almost all aspects of further mathematical study and related sciences and applications. Students will encounter the use of functions in all aspects of their continued study. The introductory college mathematics program should place emphasis on building students' capacity to deal with functional concepts.

Students will have broad experience with the concept of a function so that they will be able to

- categorize and organize functions into families and explore their properties;
- analyze functions graphically and numerically in each of these categories;
- understand and use the algebra of functions; and
- model functions by constructing charts, tables, and graphs that summarize data from real world situations and make inferences based on the results.

Categories of functions should include linear, power, trigonometric (periodic), exponential, logarithmic, polynomial, and rational functions. There should be less emphasis placed on polynomials of higher degree and rational functions at this level of experience. Trigonometric and periodic functions should receive less attention for students intending major fields of study in business or economics. However, faculty should note that in business examples periodic models may occur when dealing with data of a seasonal nature.

The analysis of functions should involve verbal descriptions, tables of data, and analytic approaches. Students should build understanding of the connections among a problem situation, its model as a function in symbolic form, and the resulting graph. This study should include investigation of the properties of circular functions and local and global behavior in the graphs of functions. Attention to finding zeros of the function, finding intervals where the function is increasing or decreasing, descriptions of concavity, and where functions achieve a maximum or minimum should be part of this experience.

Working with the algebra of functions, students should gain experience developing sums, products, and composition of relevant functions as they relate to general graphical approaches. Attention should also be paid to discussion of fundamental properties of inverse functions.

Example 1. Building Ideas of the Circular Functions

Among the most important skills required in reform calculus courses is the ability to model real world problems in a mathematical setting. Frequently, this involves sketching a picture which turns out to be the graph of a function. Students need a considerable amount of practice with this process long before they get to the detailed study of particular functions like the sine and cosine. The skillful teacher will provide those experiences early in a course. Here is such an example.

Assumption: Students know what the Cartesian coordinate system is and have had practice in plotting points. Students are beginning to study the idea of function and have had some experience modeling simpler physical situations. Students need not have studied trigonometry. No reference needs to be made to trigonometry at this stage.

Collaborative learning: The complexity of this problem, with the many possible places where a student could "miss the idea," makes it an excellent candidate for group work in a classroom by small teams of students (3 or 4 to a team) or as a group project students might work on over a two-class time interval. Since it is not likely that teams of students will arrive at the same graphical model, after a 15 to 30 minute time period the instructor might have pairs of teams compare their models or have individual teams present and justify their model to the class.

Physical Modeling: The instructor might have a physical model of the problem available in the classroom for students to look at and use. Or, each learning group might have one or more members of its team construct such a model.

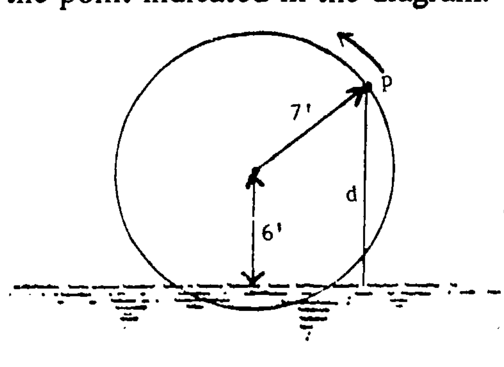
Writing: This activity provides opportunity for students to analyze their own thinking in writing and to present written reports with appropriate justification.

Materials: Students should be provided with prelined rectangular graph paper and a worksheet containing suggestive questions like the ones outlined below. The worksheet also provides a framework for students to record conjectures and answers.

The Problem

[Adapted from an example in Trigonometry: Functions and Applications (2cd ed.) by P. A. Forester, Addison-Wesley, , p. 86.

A grist mill has a waterwheel like the one in the figure below. It turns at six revolutions per minute. This means that, after one minute, point P will again be resting on top of itself but will have moved around the wheel six times. It takes P two seconds to move from its starting position indicated in the figure to the top of the wheel where it has its greatest height above the water. You want to draw a graph of the function which describes (models) *P*'s distance (d) above the water at each moment of time t. Assume P starts at time $t = 0$ at the point indicated in the diagram.



Questions to help you think about the problem and its model.

a. Labeling axes: Input/output or domain/range of model function.

i. How should you label the horizontal axis of the graph?

ii. How should you label the vertical axis of the graph?

b. Getting the first point of the model.

i. Can you estimate P's height above the water at time $t = 0$ from the picture? Give two numbers between which the height must certainly lie at the start of the rotation? Which of the two values is closer to P at the starting $t = 0$?

ii. Plot a point on your graph which approximately represents the pair (time, distance) information at $t = 0$.

iii. As P begins to rotate counterclockwise, will its height above the water increase or decrease? Therefore, will your graph (from the first point you plotted) go right and up, right and down, left and up, left and down, right and stay level, or left and stay level?

c. Getting a second point for the model.

i. What is P's location on the wheel when it reaches its greatest height above water?

ii. At what time after the start of the measurements will P reach its greatest height above the water for the first time?

iii. Plot a point on your graph which approximately represents the paired (time, distance) information when P reaches its greatest height above the water for the first time.

d. Connecting the first and second point in the model.

i. How should you connect the first two points on your graph? Will this connection be a straight line? Why or why not? If it is not straight, what is a reasonable form for the piece of the graph connecting these two points?

e. Getting a third point on the model.

i. At what time will P reach its greatest height above the water for the **second** time?

ii. Plot a point on your graph which approximately represents the paired (time, distance) information when P reaches its greatest height above the water for the second time.

f. Connecting the second and third point.

i. Now consider how to fill in your graphical model between your second and third points. In figuring this out, you might first ask yourself what happens to P's distance above the water after P passes through the point at the top of the circle for the first time?

ii. As P rotates past its high point for the first time, will it reach a position where its distance above the water is zero? That is, will the graph cross the horizontal axis?

iii. Will P reach a position where its distance above the water is negative? That is, will the graph dip below the horizontal axis?

iv. As P rotates through the water the first time, does its distance above the water become negative and then become positive again? That is, will your graphical model reach a low point, turn back up, and recross the horizontal axis?

v. Use the information you have gained to fill in your graph between the second and third points. Will the connection be a straight line? Why or why not? If it is not a straight line, will the connection be cupped up or cupped down?

g. Sketching the entire model.

i. Assuming the wheel keeps spinning indefinitely, how long does it take to make each revolution? What happens to P after it reaches its greatest distance above the water for the second time?

ii. In light of the physical behavior of the wheel, what is the form of your graph as it extends to the right of the vertical axis, beyond the piece that you have already graphed?

iii. What does your graph look like to the left of the vertical axis? Why?

h. Draw some conclusions from the model.

i. Look at the second and third points again (i.e., the points plotted in parts b and c). Can you use the times at which these high points were reached to determine the time when P is at its low point for the first time?

ii. From this information or other information in the problem, can you tell how far above or below the water P is when it reaches its low point for the first time?

iii. How many seconds after the start will P reach its low point for the **second** time?

iv. How many seconds after start will P come back to its original position for the first time?

v. Will P's distance from the water ever be greater than 13?

vi. If you think of your model as the graph of a function, are the outputs (range values) of the function ever negative? If so, give a physical interpretation of what this means with respect to P and the wheel.

vii. Draw the horizontal line $d = 6$ on the same axes as your graph. Assuming the wheel turns exactly 6 times, how many times will your graphical model cross the line? That is, in exactly six rotations of the wheel, how many times will P be exactly 6 feet above the water?

i. Follow-up essay.

Compare your model to the correct model provided by your instructor. If you correctly identified the model, write a short essay (two pages maximum) explaining any ideas you had (either from the questions provided or from your own analysis) that helped you see the correct model. If your thinking went astray, explain why in the essay. In doing so, you might want to imagine you are writing a short set of directions to students who will be doing this same problem next term, forewarning them where they could go wrong.

STANDARD 2. Build Knowledge of Approximation and Numerical Estimation

Not all problems have exact answers but many problems can be approached through approximation techniques. Students need to develop early acquaintance with these techniques and procedures that allow them to gain power in solving problems.

Similarly, estimation is a qualitative approach to problem solving. The study of complex phenomena is often more clear by applying approximations and estimations in restricted, simpler cases. This is a basic strategy in honing problem solving skills. Students must have many and varied experiences in which they

- select and use computational techniques -- mental computation, paper-and-pencil algorithms, calculator algorithms, computer methods -- appropriate to obtaining and verifying numerical results for specific problems; and
- develop and use estimation, approximation, and numerical techniques as a fundamental tool of problem solving.

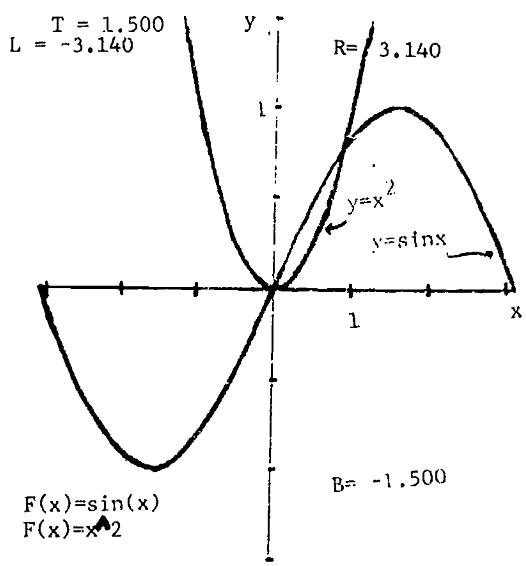
Such experiences will include exploring techniques for approximating zeros and limits of functions, for approximating asymptotic behavior and end behavior of functions and for finding areas and volumes. Through their investigations, students will develop better understanding of how basic arithmetic operations are interrelated, use in practical ways the order relations in the number systems, and represent numbers in various equivalent forms (integer, fraction, decimal, percent, scientific notation).

Example 2. Building Ideas of Trigonometric Equations

The Problem

a. Solve the equation $\sin x = x^2$.

Use a graphing calculator or a computer to graph $y = \sin x$ and $y = x^2$ in the viewing rectangle [-3.14, 3.14] by [-1.5, 1.5]. Notice that the curves intersect at two points. One of these is the origin. Use your zoom and trace features to approximate the x value at the other point.



b. Approximate the x-intercepts for the function

$$f(x) = \cos 3x + \cos x.$$

Again use a graphing utility to sketch the function in the interval $[0,2\pi]$. Use your zoom and trace features to approximate the x-intercepts. Check these approximations by solving the $\cos 3x + \cos x = 0$ algebraically.

STANDARD 3. Build Knowledge of Modeling Processes

To build a more diverse set of strategies for approaching problems, students need to develop a wide range of mathematical tools. Real world applications and their mathematical models provide the motivation for learning specific mathematical techniques. It is through suitably developed models that students see the ability of mathematics to predict the occurrence of events and to make informed decisions.

In developing problem solving approaches, students should

- apply elementary curve-fitting techniques to model and solve problems;
- use polynomial, exponential, and logarithmic functions and matrix approaches to model problem contexts; and
- develop various geometrical approaches to modeling problems.

Students need a variety of experiences to meet the intent of this standard. These should include the following specific geometrical tools:

- coordinate approaches: rectangular and polar, including use of parametric equations;
- transformational approaches in graphs: reflection, translations, stretching, and symmetry properties;
- vector approaches to graphs and figures;
- trigonometric approaches: right triangle trigonometry, use of law of sines and cosines, techniques for solving trigonometric equations; and
- three dimensional graphing techniques; concrete and informal experiences with solids formed by revolving shapes about a fixed axis.

Example 3. Building Ideas of Matrix Operations

Technology: It is assumed that students will use a calculator or computer software that can perform matrix calculations in this example.

Mathematics in Context: The purpose of the next problem is to introduce students to matrices and matrix operations. Notice that the new ideas are introduced within the context of an application. Furthermore, rather than having the instructor **tell** the students how to perform matrix multiplication and matrix addition, the problem is designed to allow students to either figure out an appropriate algorithm for themselves, or to ask the instructor for guidance as the need arises. In either case, the student is an active participant in the learning process.

The Problem

The owner of the New York Yankees baseball team, has developed a uniform incentive clause for hitters on his team. He has decided to pay them $100 for each time at bat, $400 for each run scored, $300 for each hit and $500 for each run batted in. Find the sports section in the newspaper and calculate the value of the incentive clause for the first six hitters in the Yankees' lineup. If it is not baseball season, go to the library and find an old one. If all else fails, here is a sample box score you can use:

	ab	r	h	bi
James cf	3	1	0	1
Boggs 3b	6	2	3	2
O'Neill lf	5	1	2	0
Tartabull rf	5	2	2	4
Nokes dh	4	1	1	2
Maas 1b	2	0	1	0

These are from a game played on Wednesday, May 19, 1993, that the Yankees won 11-6. When first typing these statistics, the headings "ab r h bi" were not included. Why wasn't that necessary? Experience reading lots of box scores shows that the information is always in the same order. That is, the position in the table of numbers conveys the kind of information. Hence, it is known that the third number in the fifth row is the number of hits that Matt Nokes got in the game without referring to the heading.

Nokes receives 4*$100 + 1*$400 + 1*$300 + 2*$500 = $2100.

a. Find the value of the other players' incentive clauses.

James =	__________	Tartabull =	__________
Boggs =	__________	Nokes =	__________
O'Neill =	__________	Maas =	__________

Matrices

A baseball box score is an example of a matrix. A matrix is a rectangular array (arrangement) of numbers. If you enter the box score as a 6 x 4 matrix **A** in your calculator or computer and similarly enter the incentive clause as a 4 x 1 matrix **B**, then

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 0 & 1 \\ 6 & 2 & 3 & 2 \\ 5 & 1 & 2 & 0 \\ 5 & 2 & 2 & 4 \\ 4 & 1 & 1 & 2 \\ 2 & 0 & 1 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 \\ 4 \\ 3 \\ 5 \end{pmatrix}$$

Matrix **A** is called a 6 x 4 matrix because it has 6 rows and 4 columns. The "6" and "4" give the dimensions of the matrix.

b. Use your calculator or computer to find the product [A][B]. How does the 6 x 1 product matrix compare with the calculations made in part a?

c. Based on your observations here, explain how matrix multiplication works? Try your ideas out by multiplying the following on your calculator and comparing it to your hand or mental calculations. Make a statement about the relative dimensions of the factor matrices and the size of the resulting product.

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 4 & 1 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 4 & 1 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 5 & 6 \\ 2 & 3 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 5 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 5 & 6 \\ 2 & 3 \end{pmatrix}$$

Explain any "strangeness" you observe here.

Generalizations and Extensions

e. In basketball box scores, some negative statistics are included such as turnovers and personal fouls, along with positive ones like points scored and assists. How would you include a distinctive subclause to the incentive clause that would punish someone with too many turnovers or fouls? Find a basketball box score and set up the two matrices, the performance matrix and the incentive clause matrix, so that your calculator can do the multiplication for you.

f. In a series of games, you can calculate the value of the incentive clause for each game and then add them together. As an alternative, explain how the calculator could be used to add the performance statistics matrices before the multiplication for the incentive clause. In other words, explain how matrix addition should work.

g. Compare and contrast matrix addition with real number addition. Also, compare and contrast matrix multiplication with real number multiplication. Include a discussion of the various properties such as associativity, commutativity, existence of identities and inverses, and the distributive property.

[A PROBLEM INVOLVING THE FITTING OF A PROPOSED FUNCTION (MODEL) TO EXPERIMENTALLY GATHER DATA USING GRAPHING METHODS MAY BE APPROPRIATE HERE.]

STANDARD 4. Build Knowledge of Logical Reasoning

Logic and well developed arguments are essential ingredients of mathematical discourse. Students need to have these basic understandings as they draw conclusions and explore the validity of conjectures. The introductory college mathematics program provides a rich opportunity for students to begin developing this fundamental mathematical capacity.

The collegiate experience must include experiences in which students

- develop and regularly apply inductive and deductive reasoning with use of logical quantifiers, frame examples and counterexamples, and explore the meaning and roles of mathematical identities; and
- build appreciation of the power of mathematical abstraction and symbolism.

As part of developing these abilities, students should be able to recognize and use basic forms of argument, differing approaches to proof (direct and indirect methods), the powerful role of examples and counterexamples in the analysis of conjectures and be able to verify (either algebraically or graphically) various mathematical identities. In these investigations, opportunity should be available for students to develop further their use of algebraic symbolism as an efficient and powerful way to describe relationships.

Example 4. Making and Testing Conjectures about Graphs

The following problem offers students the opportunity to make and test conjectures about the changes that will be produced in the graph of a standard function by incorporating the absolute value operation in the pattern of the function. Assume that the students have previously studied translations, reflections, and expansions of standard graphs.

The Problem

Part I: Applying the absolute value operation before the characteristic operation.

Directions: Sketch a graph for each function. On the same axes dash in the standard graph produced by o. itting the absolute value operation.

1. $f(x) = |x|$
2. $g(x) = |x|^2$
3. $h(x) = |x|^3$
4. $k(x) = \frac{1}{|x|}$
5. $t(x) = \sqrt{|x|}$
6. $m(x) = e^{|x|}$

Conclusions: Describe how applying the absolute value operation after the characteristic operation affects the standard graph.

Part II: Applying the absolute value operation after the characteristic operation.

Directions: Sketch a graph for each function. On the same axes dash in the standard graph produced by omitting the absolute value operation.

1. $G(x) = |x^2|$
2. $H(x) = |x^3|$
3. $T(x) = |\sqrt{x}|$
4. $M(x) = |e^x|$
5. $K(x) = \left|\frac{1}{x}\right|$

Conclusions: Describe how applying the absolute value operation after the characteristic operation affects the standard graph.

Part III: Combining the absolute value operation with functions whose graphs have been shifted.

Directions: Sketch a graph for each function. On the same axes dash in the graph produced by omitting the absolute value operation.

1. $f_1(x) = |x^2 - 4|$
2. $f_2(x) = |x^2 - 5x + 6|$
3. $g_1(x) = 2^{|x|} + 3$
4. $g_2(x) = 2^{|x|} - 4$

Conclusions: Do your descriptions from Part I and Part II explain **how to** produce the graphs of f_1 and g_1 from their standard graphs? Do your descriptions give a way to produce the graphs of f_2 and g_2 from their standard graphs? Give steps to produce the graph.

Part IV: Generalizing your conclusions.

Directions: Use the steps you listed in Part III to sketch the graph of each function. Don't use any other method. Now by checking points or using your graphing calculator determine if the graph is correct.

1. $h_1(x) = \frac{1}{|x-3|}$
2. $h_2(x) = \frac{1}{|x-3|}$
3. $k_1(x) = \ln|x+1|$
4. $k_2(x) = \ln|x-3|$

Conclusions: If your steps did not cover these cases, explain why. Re-do your descriptions to cover these more general cases.

STANDARD 5. Building Knowledge of Patterns and Number Sense

Recognition and use of patterns, including those dealing with aspects of the very large and the very small, permeate many aspects of mathematics. Numerical techniques that have always played a role in estimation and measurement take on new significance with the increasing use of technology in the solving of problems. Patterns of growth and rates of change are described using functions based on number relationships.

Throughout their introductory college mathematics program, students must

- recognize the analysis of patterns as an ongoing theme in mathematics and be able to develop basic properties of number sequences in the context of searching for patterns; and
- investigate limiting processes by examining asymptotic behavior of functions, infinite sequences and series, and the area under curves.

The process of looking for patterns and making, explaining, and verifying the properties of sequences arising from such patterns provide students with opportunities to enhance their use of mathematical language and symbolism and further develops their reasoning and communication skills. Patterns explored should include examples that give rise to recursive functions and procedures that require iteration. Various concepts from elementary number theory (e.g., primes, factors, division algorithm) provide background skills and contribute interesting areas for additional exploration.

Example 5. Machine Scheduling

A common problem that arises in an area of applied mathematics called "operations research" is that of machine scheduling.

THE PROBLEM

[This example is taken from *Applied Management Science* by R. Hesse, G. Woolsey, SRA, 1980, pp. 15-16.]

For our purposes, let's consider jobs that must be processed by two machines - machine A and machine B - and the jobs must first be processed by machine A. The jobs vary in the amount of time required on each machine, as shown below (time is in hours).

Job	Machine A	Machine B
1	3	6
2	7	2
3	4	7
4	5	3
5	7	4

In what order should the jobs be scheduled so that all five jobs finish in the least amount of time? If the jobs are processed in the order received as indicated above, it would take 30 hours to finish the five jobs.

Job	Machine A		Machine B		
	In	Out	In	Out	
1	0	3	3	9	
2	3	10	10	12	must wait until the 10th hour
3	10	14	14	21	
4	14	19	21	24	must wait until the 21st hour
5	19	26	26	30	

If the jobs were processed in the order 1-3-4-2-5, it would still take 30 hours to finish. Is a better order possible?

One can investigate with various patterns to see how the total processing time is affected. One quickly discovers that if Machine A processes short jobs earlier, the overall processing time is usually, but not always, decreased. Also, one finds that if Machine B processes its short jobs later, overall processing time is usually, but not always, decreased.

An hour table combining both of these methods is required. The following algorithm was developed by C.M. Johnson (see Optimal Two and Three Stage Production Schedules with Set-Up Times Included, *Naval Research Logistics Quarterly,* Vol. 1, 1954, pp. 61-68).

Johnson Algorithm

1. Find the smallest overall time in the chart.

2. If it appears in column A, schedule the job as SOON as possible for machine A.

3. If it appears in column B, schedule the job as LATE as possible for machine B.

4. Cross off that job and go back to step one until all jobs are scheduled.

Example 6. Building Ideas of Patterns and Number Sense

If I = the amount of interest earned
P = the principal or present value
r = the annual interest rate in decimal form
t = the length of the investment period in years,

the simple interest formula is: $I = Prt$.

THE PROBLEMS

a. If you start with $2,000 and deposit it into an account paying at the rate of 6% per year, how much interest is earned after one-quarter of a year? If the interest is automatically deposited back into the account, what is the amount or balance in the account after one-quarter of a year?

b. In an account in which interest is compounded quarterly, the interest earned every quarter is deposited back into the account and the interest earned for the following quarter is based on this new amount. Complete the table below to show the balance of an account where the starting amount is $2,000 and the interest is 6% compounded quarterly:

Number of Corresponding Periods	Balance
0 (beginning)	2,000.00
1 quarter	2,030.00
2 quarters	2,060.45
3 quarters	
4 quarters (1 year)	
5 quarters	
6 quarters	
7 quarters	
8 quarters (2 years)	

Did you remember that you are dealing with money and that more than two decimal places is not meaningful?

An Extension of the Problem: Abstraction

c. Let P represent the principal (the starting balance). Then the balance in the account after one period is

$$P_1 = P_0 + I_0, \text{ where } I_0 = P_0rt$$

since

$$P_1 = P_0 + P_0rt, \text{ or } P_1 = P_0(1 + rt)$$

the balance after two compounding periods is

$$P_2 = P_1 + I_1 = P_1 + P_1rt = P_1(1 + rt), \text{ or}$$
$$P_2 = P_0(1 + rt)(1 + rt) = P_0(1 + rt)^2$$

i. What is the value of P_3 in terms of P_0, r, and t? _________

ii. What is the value of P_8 in terms of P_0, r, and t? _________

iii. After n compounding periods the value of the investment is P_n. What is the value of P_n in terms of P_0, r, and t? _________

iv. Use mathematical induction to prove that after n compounding period $P_n = P_0(1 + rt)^n$.

v. Use the compound interest formula determined above to check the balance after two years (8 quarters) in part b.

vi. Suppose that First National Bank offers an interest rate at 5% per year compounded quarterly, while Federal Savings offers a rate of 4% per year compounded daily. Which do you think will result in a higher balance if $2,000 is invested for 20 years? Explain. Actually find the balance for such an investment in each bank. Use $t = 1/365$ and $n = (20)(365)$ for daily compounding. Neglect leap years?

A Further Extension of the Problem: Annuities

d. Many people save by making regular investments at fixed time intervals. Such plans are called "annuities." Suppose that you deposit $2,000 every year into an account. Furthermore, suppose that the account compounds annually instead of quarterly. That is, the value of t in the compound interest $Pn = P_0(1 + rt)^n$ formula is 1 and the value of the exponent in the formula for an investment of n years is simply n.

i. Let's deposit $2,000 today into an account that pays 5% compounded annually. How much will be in that account when you are ready to retire at age 65? ________ Let's deposit the same amount in the same kind of account next year. How much will be in that account when you reach age 65? ________ Continue to follow this strategy until you reach age 65. How would you calculate the amount in the combined accounts? ________ Can you find a short-cut? Look for a pattern.

ii. Let's set up a table (assume you are now 20 years old):

Deposit made at age ...	Value of account at age 65
20	$\$2,000(1 + .05)^{45}$
21	$2,000(1 + .05)^{44}$
22	$2,000(1 + .05)^{43}$
.	.
.	.
.	.
n	$2,000(1 + .05)^{(65-n)}$
.	.
.	.
.	.
63	2,205
64	2,100
65	2,000

The straight forward method for determining the value of the annuity at age 65 is to add up the amounts in the second column.

How would you write a program for your calculator that finds the accumulated value of these accounts?

iii. Let's develop an annuity formula to save time.
Let R_o be the amount you deposit each year and r be the annual interest rate. Let's assume that you make one last deposit when you reach 65 and let's re-examine the table above.

Deposit made at age ...	Value of account at age 65
65	R_o
64	$R_o(1 + r)$
63	$R_o(1 + r)^2$
.	.
.	.
.	.
n	$R_o(1 + r)^{(65-n)}$
.	.
.	.
.	.
20	$R_o(1 + r)^{45}$

Letting F_n be the future value of the annuity account after n year,

$$F_n = R_o + R_o(1 + r) + R_o(1 + r)^2 + ... + R_o(1 + r)^k + ... + R_o(1 + r)^n$$

where n is the number of years that we pay into the annuity.

Multiplying both sides by $(1 + r)$ yields... you tell me. "Subtracting" the first equation from the second and solving for F_n, the future value, we get a nice formula for F_n in terms of R_o, r, and n.

To "officially" use the formula you have just obtained, you must also verify it using mathematical induction. Do so.

Using $F_n = R_o[(1 + r)^{n+1}-1] /r$, with R_o = \$2,000, $r = 0.05$, and $n = 48$, find the future of the annuity. If all the deposits were made into the same account, how would the calculations be affected?

Generalizations

e. The preceding is an example of the use of geometric sequences.

i. What exactly is a geometric sequence? How do you find the sum of the first several terms of such a sequence? Find some other examples of geometric sequences. Does this have anything to do with arithmetic sequences? What is an arithmetic sequence? What is the sum of the first several terms of an arithmetic sequence?

ii. How would you have to change the formula in the annuity if you deposited $500 each quarter and the interest was compounded quarterly? How much does this deposit scheme change the value of the account at age 65?

iii. The inverse problem to the annuity is called "amortization." It is related to the future value of a lump sum deposit along with the stream of payments as in an annuity. Find out about amortization and make a report to the group about your discoveries. How does this relate to buying a car?

STANDARD 6. Building Knowledge of Probability and Statistics

Topics in chance and risk confront today's citizens in many ways - from newspaper articles to decisions in their daily lives. Therefore, students will need a strong understanding of the elements of probability and statistics.

Students should have experiences in their introductory college mathematics programs in which they have opportunities to

- plan and conduct sampling experiments to develop an appreciation for randomness;
- compute empirical and theoretical probabilities, and
- understand and use normal and binomial distributions for modelling data.

In building these experiences, students should be expected to systematically collect, organize, an describe data presented in different ways. They should establish empirical probability based on data they have collected and relate it to the theoretical probability based on the nature of the underlying sample space. In the context of searching for solutions to presented problems, they should make predictions based on their experimental work. Through these means, students can develop an appreciation for probabilistic and statistical methods as powerful means for decision making.

Example 7. Probability, Data Sets, and Statistics

People earn a living working as statisticians. Such work involves developing research question, designing research projects, gathering data from samples, tabulating and analyzing the data, making probabilistic inferences about populations, and writing research reports. Students who study statistics, even at the introductory level, should become involved in working as statisticians in addition to working a traditional textbook problems.

THE PROBLEM

The class is given the following scenario:

Assume that the class is asked for help by the college newspaper at No Ka Oi Community College. The newspaper plans on running a series of articles describing the "typical" student attending the college. The class must decide on

a. the type of information that needs to be gathered,
b. the design of the survey instrument so that the information that is gathered will lend itself to a statistical analysis,
c. how the sample is to be selected so that it will be representative of the entire population, and
d. tabulation and analysis procedures.

Procedures:

The instructor can solicit suggestions for the type of information that needs to be gathered from the entire class. The class can than be divided into working groups of three or four students each. Each group can design and implement a research procedure for studying one type of information describing a typical student. Upon completion of the statistical analysis each group will prepare a research report.

Points of emphasis during the class discussion should include

a. problems associated with privacy of information (if the college has a human subject research committee, special permission may be required),
b. sampling procedures so that the data is not biased.

[MONTE CARLO METHODS USE PROBABILITY TO APPROXIMATE SOLUTIONS TO MATHEMATICAL PROBLEMS. IT MAY BE APPROPRIATE TO INCLUDE A PROBLEM WHERE THE AREA UNDER A CURVE IS FOUND USING MONTE CARLO METHODS HERE.]

Program Structure

Institutions and their faculties need to design carefully a framework for providing students with the curricular experiences described in the standards. This framework could be a course or sequence of courses that will meet the needs of each of the two groups of students considered in this chapter. Certainly, all students must be competent in basic algebra, geometry, and trigonometry. After that, those students in strongly mathematics related areas will prepare for the study of calculus and other courses beyond the introductory level. Liberal Arts students may study courses that emphasize statistics, decision making, and social choice. Topics such as difference equations, recursive functions, and chaos that reflect contemporary developments in mathematics should be included in course development.

Departments are encouraged to form regional consortia to consider innovative and creative ways for implementing the curriculum standards.

[TO THE READERS: IT APPEARS THAT COURSES IN WHAT IS **TRADITIONALLY** KNOWN AS "LIBERAL ARTS MATHEMATICS" (SET THEORY, NUMBER THEORY, LOGIC, ...) ARE BECOMING LESS POPULAR WITH OUR STUDENTS. WHAT ARE YOUR THOUGHTS ON THE APPROPRIATE PROGRAM STRUCTURE FOR THE IMPLEMENTATION OF THE CURRICULUM STANDARDS? EXAMPLES OF APPROPRIATE STRUCTURES WOULD BE USEFUL HERE.]

CHAPTER 6 - Next Phases

Development of Faculty and Materials

This standards document is intended to provide a framework for reform. Adoption and implementation of the reforms outlined require a systemic effort. They must be understood and supported at every level of higher education. The review process for the document was designed to increase awareness of the need for standards as well as to refine the standards. Dissemination and consensus building will be critical ongoing processes.

Formal Announcement of the Document

AMATYC, with assistance from other participating organizations, plans to announce the publication of this document just prior to the AMATYC Conference in Tulsa in November 1994. This announcement will attract national attention to the issue of implementing reform in two-year college and lower division mathematics and thereby provide support and impetus to the mathematics faculty concerned. Further benefits from this announcement are to

- focus national attention on the importance of two-year colleges in achieving reform in mathematics;
- clarify the mission of two-year college and lower division mathematics with regard to mathematics education reform; and
- set forth workable national and local strategies for implementation of reform.

Networks of Institutions

All AMATYC affiliates have been asked to consider both participation in the development of the standards document and implementation strategies and to develop a response or outline of activities. Organization of local consortia of two-year colleges and universities will be encouraged through state and regional mathematics groups. These consortia would provide the framework for development and dissemination of materials, professional development, institutional research on instructional effectiveness, and development of grant proposals for local, state, or national assistance for mathematics education.

Proposed Regional Workshops

Maximum potential for change would result from the dissemination of the standards through a series of regional workshops attended by teams of faculty and administrators from institutions in the region. Funding will be sought for the workshops, each of which will involve approximately 200 participants attending in teams, and each state represented will have a sufficient number of attendees to ensure that a working group will continue the efforts beyond the workshop itself. Each professional development workshop will last approximately three days.

The purpose of these workshops will be to inform a wider audience of the proposed pattern for change, to provide examples of quality change already begun, to develop processes of assessment appropriate to the standards, to create teams or consortia of two-year schools and four-year schools to jointly work together for reform, and to enhance the ability of two-year college mathematics faculty to obtain funding for reform projects. The workshops will lay the groundwork for systemic change at two-year colleges and in lower division mathematics. At each workshop, participant teams will form consortia, set goals, and plan for the continuing work.

The format and agenda of the regional workshops ultimately will be defined by regional planning groups, but will satisfy the following guidelines:

- promote alliances for carrying out the work of reform and of curriculum content and pedagogical approaches in the "real world in real time";

- focus on systemic change as well as provide real help to individual teachers in their classrooms and administrators in their institutions;

- encourage change in attitudes as well as practices;

- model what we want to see happening in two-year college and lower division classrooms by engaging participants actively both in mathematics and in content and pedagogy;

- encourage cooperation with other educational and societal institutions;

- address effective and creative uses of technology; and

- initiate self-sustaining follow-up activities.

Participants will be selected as teams rather than as individuals. Teams will make a commitment to report periodically on their progress. In addition, teams must commit to forming consortia with other teams to work on state-wide or regional projects. In this way, the persons attending the regional workshops become a curriculum and pedagogy reform action force for change.

The entire sequence of events will enhance the stature of two-year colleges in the reform agenda and advance the undergraduate mathematics reform effort by

- stimulating faculty to make changes in their classrooms;
- enabling teams of educators to initiate changes at their colleges;
- developing partnerships to assist in the implementation of systemic curricular and pedagogical reform; and
- providing opportunities for funding agencies to become aware of the reform efforts and their value.

Development of Materials

Development of materials that will provide careful guidance to mathematics faculty in their efforts toward educational reform is a necessary step if a program or project is to have long-term influence. Many faculty will not have the benefit of professional development opportunities or a support structure at their institutions. They need instructional materials that are functional without extensive previous knowledge of new pedagogy. Learning should be exciting and challenging for both the student and the faculty member.

Development of materials may occur through one of the structures mentioned in this section. It may also occur through the traditional means of publishing companies and manufacturers of software. Organizations should attempt to educate these commercial entities about mathematics reform below the level of calculus. They, in turn, can educate faculty about selling the idea of mathematics reform through instructional materials.

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Members of the Planning Group
Convened by the Mathematical Sciences Education Board

Ray Shiflett
MSEB Executive Director

Michael Clapp
MSEB Associate Executive Director

Julian Weisglass
MSEB Consultant

Calvin Moore
Member MSEB Executive Board

David Pierce
President, American Association of Community Colleges

Lynn Steen
Member MSEB Executive Board

John Romo
Vice President for Academic Affairs
Santa Barbara Community College

Max Cisneros, Jr.
Albuquerque Technical & Vocational Institute

Cheryl Cleaves
State Technical Institute at Memphis

Ron David
DeKalb College

Carol Edwards
St. Louis Community College at Florissant Valley

Dale Ewen
Parkland College

Susan Forman
MSEB, Director of College and University Programs

Wanda Garner
Cabrillo College

Marilyn Mays
North Lake College

Karen Sharp
Mott Community College

Members of the National Steering Committee

John Bradley
American Mathematical Society

Linda Boyd
DeKalb College

Cheryl Cleaves
State Technical Institute at Memphis

Max Cisneros
Albuquerque Technical & Vocational Institute

Don Cohen
SUNY at Cobleskill

Dale Ewen
Parkland College

Susan Forman
MSEB, Director of College and University Programs

Wanda Garner
Cabrillo College

Margie Hobbs
State Technical Institute at Memphis

Julie Keener
Central Oregon Community College

Genevieve Knight
Coppin State College

James Leitzel
University of Nebraska

Robert Malena
Community College of Allegheny County-South

Marilyn Mays
North Lake College

Dean Priest
Harding University

Janet Ray
Seattle Central Community College

Stephen Rodi
Austin Community College

Karen Sharp
Mott Community College

Christine Thomas
Benjamin Bannekar H.S.

P. Uri Treisman
University of Texas at Austin

Julian Weissglass
University of California
Santa Barbara

Assisting with the Steering Committee Meeting were:

Richelle Blair
Ohio Board of Regents

Marilyn Hala
National Council of Teachers of Mathematics

Robert Kimball
Wake Technical Community College

Ray Schiflett
Former Director of MSEB