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ABSTRACT

A pentagonal model, based on the star model of function understanding of C. Janvier (1987), is presented as a framework for the design and interpretation of research in the area of learning the concept of mathematical function. The five vertices of the pentagon correspond to five common representations of mathematical function: (1) graph; (2) table; (3) algebraic formula; (4) verbal description; and (5) situation. The 10 line segments forming the sides and diagonals of the pentagon represent translations between representations. As an example of the usefulness of the model as a framework for designing research and interpreting research data, a study of the understandings of 52 preservice elementary teachers concerning tasks involved in building and using linear mathematical models is presented. Objectives of the study were to determine if the concrete activity of data collection has a measurable effect on student performance on the model building and using tasks, and to test the usefulness of the model for framing research. Data collection activities appear to interfere with, rather than enhance, performance on these tasks. The model was found useful for framing and interpreting the research. It can be expected to be a useful aid in designing instruction. Five figures and two tables present study data. (SLD)

A Theoretical Framework for Research in Algebra: Modification of Janvier's "Star" Model of Function Understanding

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A pentagonal model, based on Janvier's (1987) "star" model of function understanding, is presented as a framework for design and interpretation of research in the area of learning the concept of mathematical function. The five vertices of the proposed pentagonal model correspond to five common external representations for mathematical function: graph, table, algebraic formula, verbal description, and situation. The ten line segments forming the sides and diagonals of the pentagon represent translations between representations. The twenty possible one-way translations are identified as source-to-target translations (Figure 1).

Insert Figure 1 about here

Mathematical function may be presented in either of two conceptualizations: (a) a mathematical perspective, based on the Dirichlet-Bourbaki set-theoretical definition (Vinner & Dreyfus, 1989), or (b) a scientific perspective (Sierpinska, 1992), where function is viewed as a relationship between variable magnitudes. Viewed within the pentagonal model, the mathematical perspective focuses on algebraic formulas, tables, and graphs, and on the corresponding translations among these three representations (Figure 2). The scientific perspective involves all five representations and situation-to-table, table-to-graph, graph-to-algebraic formula, and algebraic formula-to-verbal description translations (Figure 3).

Insert Figure 2 and Figure 3 about here

By comparing these two perspectives within the framework of the pentagonal model, problems associated with connecting representations and developing a comprehensive understanding of the function concept can be seen as inherent in the

Paper presented at the annual meeting of the American Educational Research Association Atlanta, GA, April 12, 1993.

ways mathematical function is taught. The model provides a means for (a) identifying gaps in curriculum design and instructional practices and (b) designing and interpreting research on learning and teaching the concept of mathematical function. As an example of the usefulness of the pentagonal model as a framework for designing research and interpreting research data, a study of preservice elementary teachers' understandings of tasks involving building, interpreting, and using linear mathematical models is presented. Further, it is proposed that the pentagonal model be used in three ways: (a) to re-analyze existing research on learning and teaching mathematical function, (b) to frame a unified plan of action for research directed at filling gaps in our understanding of how learners connect various representations to form stronger conceptions of mathematical function, and (c) to use the model as a vehicle for communication among educational researchers, curriculum specialists, and classroom teachers.

Research Example

Background for the Study

Elementary school science teachers are expected to teach children science process skills, including the processes associated with a scientific inquiry approach. Traditionally, science methods courses for preservice elementary teachers have stressed incorporating hands-on science experiences in elementary science instruction. However, preservice teachers are rarely asked to take the process beyond the data collection step. Therefore, elementary preservice programs have failed to adequately prepare teachers to instruct students in data analysis processes associated with a scientific inquiry approach.

The National Council of Teachers of Mathematics (1989) stresses the need for increased emphasis on functional relationships, data analysis, and problem solving in the elementary curriculum. Instructionally, these topics may be incorporated within a scientific inquiry approach to teaching science. There are four advantages to integrating science and mathematics instruction in this manner.

1. A major goal of school science, construction of verbal descriptions of relationships between real-world variables, may be facilitated through application of data analysis techniques involving several representations of mathematical function.

Paper presented at the annual meeting of the American Educational Research Association
Atlanta, GA, April 12, 1993.

2. A major goal of school mathematics, developing understanding of mathematical function and its representations and associated translation processes, may be facilitated by situating instruction within a scientific context.

3. The integration of science and mathematics via a scientific inquiry/data analysis approach may help students develop an understanding of the nature of science.

4. The integration of science and mathematics via a scientific inquiry/data analysis approach may help students appreciate the usefulness of mathematics in exploring our physical world.

Elementary teachers need to experience learning science and mathematics via a scientific inquiry/data analysis approach before they can reasonably be expected to teach elementary children using this approach. The treatment sessions utilized in this study were designed to engage preservice elementary teachers in activities involving building, interpreting, and using linear mathematical models based on sets of scientific data. Specifically, the study was designed to assess the effectiveness of the treatment sessions in helping preservice teachers connect the notion of describing relationships between two variables based on data collected in a physical science setting to what they already knew about linear mathematical functions in the form $y = m x + b$ from the study of algebra. This study was based on the Mathematical Association of America's (1991) recommendations for the mathematical preparation of teachers of mathematics, common standards: (a) connecting mathematical ideas, (b) building mathematical models, and (c) using technology.

Overview of the Study

The pentagonal model was used as a theoretical basis for framing an experimental study of the relationship between preservice elementary teachers' performance on tasks involving building, interpreting, and using linear mathematical models based on physical science data and whether or not the student participated in data collection tasks. Fifty-two elementary education majors enrolled at a small university in the southeastern region of the United States participated in this experiment by completing two, 2-hour workshops and a 50-minute, 36-item posttest. The 52 students were randomly assigned to one of two treatment groups. The 27 students in the "data collection" group were then randomly assigned to one of 13 experimental groups and the 25 students in the "no data collection" group were randomly assigned to

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one of 12 experimental groups. The students used TI-81 graphing calculators to analyze the relationships between four pairs of variables: (a) total mass of a liquid and its container (Y) versus the volume of liquid used (X), (b) total height from the table top to the water level in a beaker (Y) versus the volume of water in the beaker (X), (c) total mass of coins and the cup containing the coins (Y) versus the number of coins in the cup (X), and (d) the length of a spring (Y) versus the total mass of objects attached to the spring (X). Data analysis via TI-81 calculators included entering data from tables, constructing scatter plots, and determining the least squares linear regression model. For each mathematical model constructed, students identified the slope and y-intercept, including units of measure; constructed a contextual (situational) interpretation of the slope and y-intercept; and solved verbal problems using the model to predict outcomes.

Objectives

1. To determine if the concrete activity of data collection has a measurable effect on students' performance on tasks involving building, interpreting, and using linear mathematical models.
2. To test the usefulness of the pentagonal model as a model for framing research on designing instruction to increase connections students make among various aspects of the mathematical function concept.

Methodology

During two, 2-hour workshop sessions, students built, interpreted, and used linear mathematical models based on data sets from experiments that were carefully selected to involve (a) simple measurements, using common measuring instruments, that elementary school children could make, and (b) linear mathematical models, with little error variation, where both the slope and y-intercept have simple and clearly-recognizable physical interpretations. Time spent on data analysis, interpretation, and prediction activities was controlled to be the same for subjects in each treatment group. Since students were randomly assigned to the two treatment groups, the major difference in the two groups was that students in the "data collection" group collected data in the laboratory before analyzing the data, whereas the "no data collection" group worked on, presumably, non-interfering activities for a time period equivalent to the time students in the "data collection" group spent collecting data.

A 50-minute, 36-item posttest was administered to each student. Scores on the posttest were obtained by a blind, double grading procedure. Six students were selected to participate in the interview phase of this project. These interviews were conducted to provide additional insights into differences in group responses to treatment sessions and the posttest. Two-sample t-tests were conducted on the overall posttest score, each individual item score, and on 16 subscores of the posttest. Several repeated-measures MANOVAs were run on selected sets of posttest subscores.

<u>Results</u>

The "no data collection" group scored higher than the "data collection" group on the posttest, on all 16 subsets of the posttest considered in this analysis (Table 1), and on all 36 individual test items (Table 2). Inferentially, these differences in group means are significant, at an $\alpha = .05$ level, on the posttest, on 14 of the 16 subsets of the posttest considered in this analysis, and on 17 of the 36 individual posttest items. The mean posttest scores for "no data collection" group were significantly higher than the mean scores for the "data collection" group on each of the following:

1. the overall posttest ($p = .0023$);
2. items involving building, interpreting, and using mathematical models given data tables ($p = .0055$);
3. items involving building, interpreting, and using mathematical models given verbal descriptions ($p = .0140$);
4. items involving interpreting and using mathematical models given models as algebraic formulas ($p = .0033$);
5. items involving building mathematical models ($p = .0269$);
6. items involving interpreting mathematical models ($p = .0045$);
7. items involving using mathematical models ($p = .0031$);
8. items involving building, interpreting, and using mathematical models based on the same physical contexts utilized during treatment sessions ($p = .0015$); and

9. items involving building, interpreting, and using mathematical models based on physical contexts different from the contexts utilized during treatment sessions (p = .0091).

Insert Table 1 and Table 2 about here

Repeated measures MANOVAs, based on percentage scores, revealed only one group-by-subscore interaction. A Wilks' lambda value of .80 (F = 5.60; df = 1, 23; p = .0268) for the test of two-way interactions between (a) scores on tasks involving writing physical interpretations of slopes and (b) scores on tasks involving writing physical interpretation of y-intercepts indicated that there is a significant group-by-subscore interaction for these subscores. The experimental results are graphically summarized in Figure 4.

Insert Figure 4 about here

Research Conclusions

The results indicate that, within the limited time frame of the experimental treatment, data collection activities interfere with, rather than enhance, performance on tasks involving building, interpreting, and using linear mathematical models. The observed group differences may be the result of a combination of two factors: (a) conceptual versus procedural knowledge and (b) treatment time limitations. If the "data collection" group approached the modeling tasks conceptually and the "no data collection" group approached the tasks procedurally, then it is reasonable to expect that, due to treatment time limitations, the "no data collection" group might score higher on the posttest than the "data collection" group since it simply takes longer to develop an understanding of concepts than it does to develop skill in carrying out procedures. This proposed explanation is supported by comments made by students during individual interview sessions.

The pentagonal model was used to frame this research study. From a design perspective, the "data collection" group participated in tasks involving situation-to-table translations; the "no data collection" group did not participate in these tasks. Thus, the model provides a clear framework for considering the designed treatment difference (Figure 5). The pentagonal model was also used in interpreting the group mean results on various subsets of the posttest. Within the framework of the pentagonal model, each posttest task could be identified with one or more source-to-target translations. Comments made by students during individual interview sessions indicated that there might be a group difference in the translations used to complete a given task.

Discussion

The research project presented in this paper serves as an example of designing and interpreting research within the framework of the pentagonal model. The concept of mathematical function is a complex concept that may be viewed differently in (a) each of its representations and (b) each task involving translations between representations. Viewing representations, translations, and translation processes within the framework of the pentagonal model, provides a way to connect various aspects of the mathematical function concept. Making such connections seems crucial, not only for the student, but for teachers, curriculum specialists, and researchers. Making connections among so many, apparently diverse, ideas is not an easy task. Therefore, it seems imperative that instruction be specifically designed to increase the probability that students will develop multiple connections.

The pentagonal model is expected to prove a useful aid in designing instruction, first, by helping identify gaps in current instructional practices and, secondly, by providing a clear view of ways to link instructional units. A substantial body of research on learning mathematical function may be found within the research literature. The task of synthesizing the research results into a clear direction for curriculum reform is monumental without an appropriate framework for analyzing each piece as part of the whole. The pentagonal model may provide just that needed framework. Toward the goal of improving instruction in mathematical function for all students, it is proposed that (a) existing research on learning and teaching mathematical function be re-analyzed within the framework of the pentagonal model, (b) a unified plan of action for research be developed based on the model, and (c) the model be used as a vehicle for communication among educational researchers, curriculum specialists, and mathematics and science teachers.

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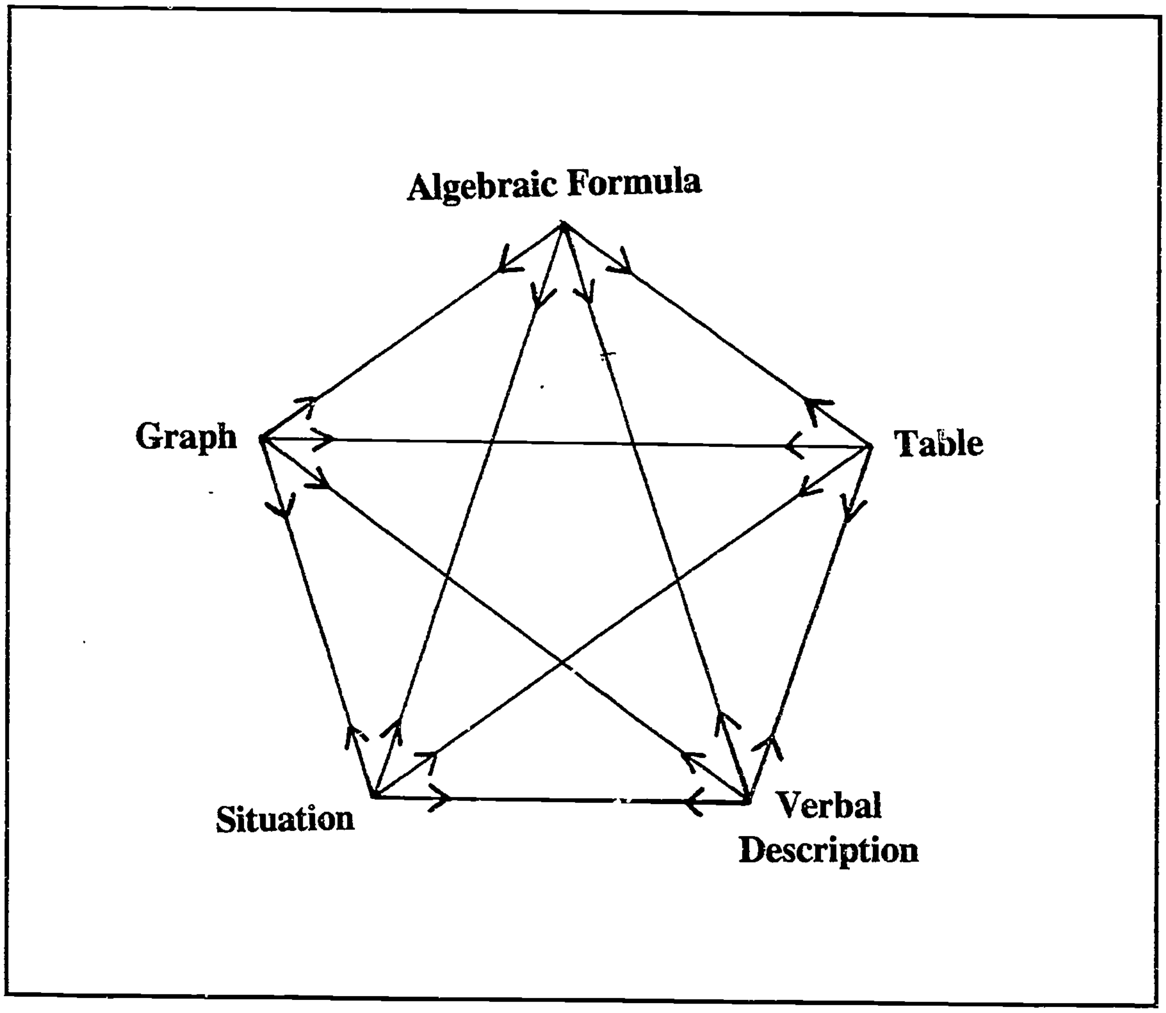

Figure 1. Pentagonal Model of Representations and Translations Between Representations for the Mathematical Function Concept

MATHEMATICAL PERSPECTIVE

A function is a correspondence between two non-empty sets A and B that assigns to each element of A one and only one element of B.

(Dirichlet-Bourbaki Concept of Function)

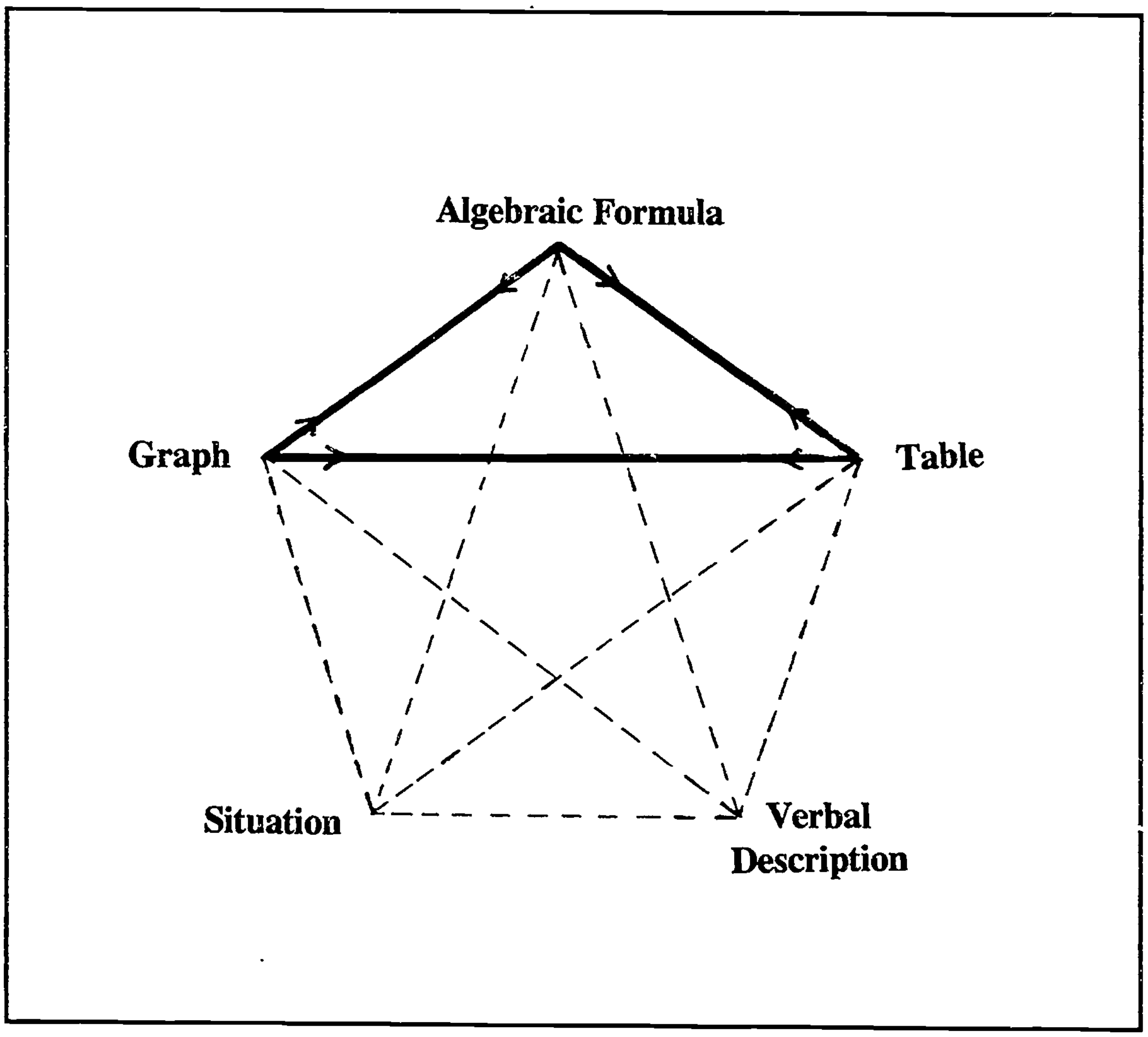

Figure 2. The Function Concept from a Mathematical Perspective

SCIENTIFIC PERSPECTIVE

> Maybe, in teaching, functions should first appear as models of relationships. This is how they came into being in history. They were tools for description and prediction. If we assume that the meaning of a concept lies in the problems and questions that gave birth to it, and we wish that our students grasp the meaning of the notion of function, then this seems to be a quite reasonable claim to make.
>
> — Sierpinska, 1992

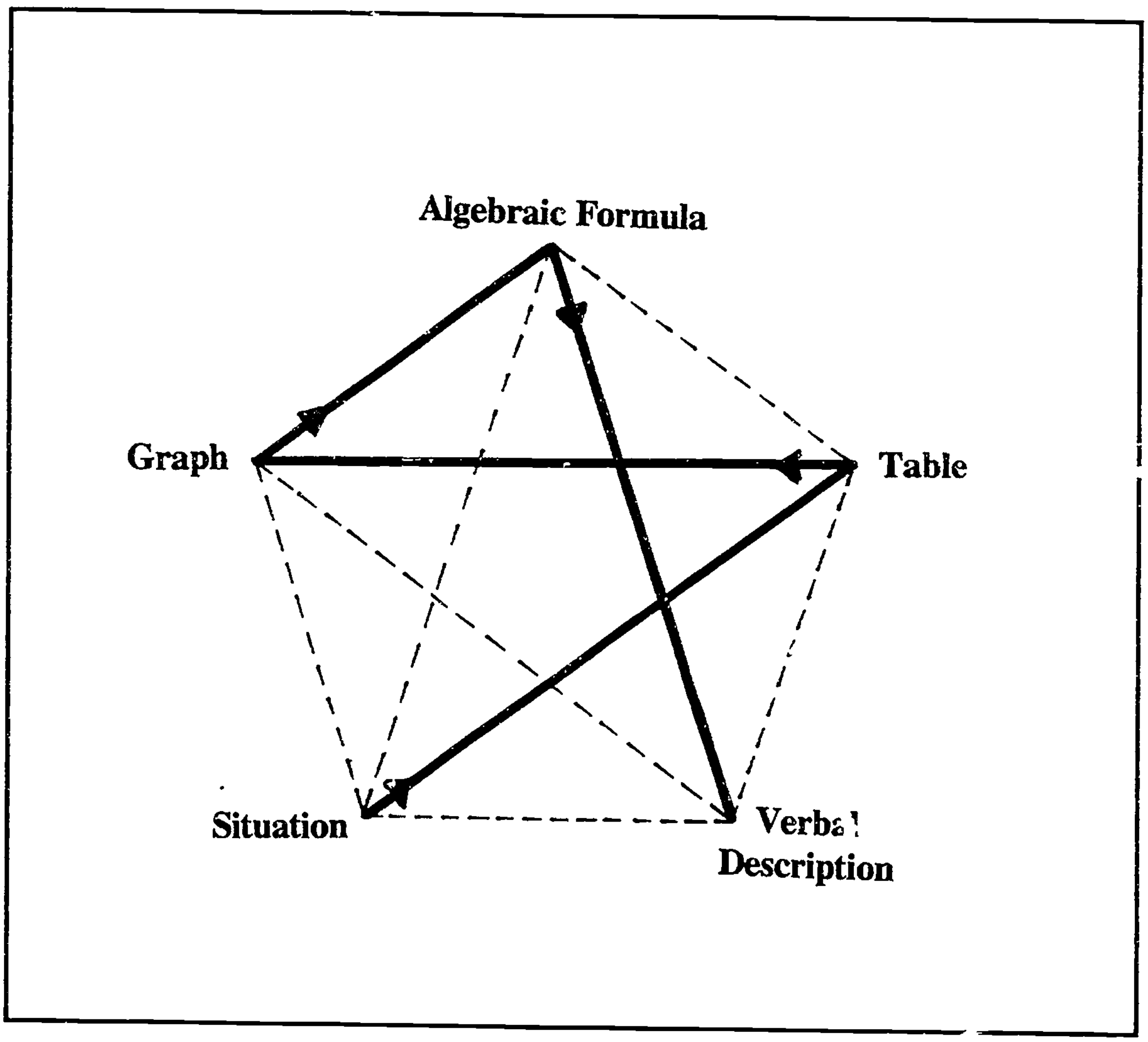

Figure 3. The Function Concept from a Scientific Perspective

Table 1

Posttest Results by Selected Subtests: Group Means, t-Statistics, and p-Values (df = 23)

Subtest (# Items)	Group A Mean (% Correct)	SD_A	Group B Mean (% Correct)	SD_B	t-value	p-value
POST (36)	13.18 (36.61)	5.65	20.28 (57.33)	4.63	-3.42	.0023
DATASETS (12)	5.36 (44.66)	2.05	7.65 (63.77)	1.65	-3.06	.0055
VERBAL (12)	4.51 (37.55)	2.39	6.95 (57.93)	2.18	-2.66	.0140
ALGEBRAIC (12)	3.31 (27.62)	1.98	5.67 (47.28)	1.57	-3.28	.0033
BUILD1 (2)	1.60 (80.13)	.42	1.88 (93.75)	.31	-1.82	.0813
BUILD2 (2)	.65 (32.69)	.51	1.07 (53.47)	.44	-2.16	.0417
BUILD (4)	2.26 (56.41)	.80	2.94 (73.61)	.63	-2.36	.0269
SLOPE1 (6)	2.41 (40.17)	1.00	3.75 (62.50)	.87	-3.56	.0017
INTERCEPT1 (6)	2.93 (48.82)	1.24	3.90 (64.93)	1.06	-2.09	.0482
SLOPE2 (6)	1.45 (24.15)	.92	2.85 (47.57)	1.27	-3.19	.0041
INTERCEPT2 (6)	1.63 (27.24)	.87	2.23 (37.15)	.74	-1.83	.0801
INTERPRET (24)	8.42 (35.10)	3.33	12.73 (53.04)	3.51	-3.14	.0045
USEY (6)	2.06 (34.40)	1.54	3.62 (60.30)	1.02	-2.94	.0073
USEX (2)	.44 (21.79)	.44	.99 (49.31)	.26	-3.75	.0010
USE (8)	2.50 (31.25)	1.91	4.60 (57.55)	1.30	-3.31	.0031
FAMILIAR (18)	7.51 (41.74)	3.19	11.60 (64.47)	2.38	-3.61	.0015
UNFAMILIAR (18)	5.67 (31.48)	2.59	8.67 (48.19)	2.68	-2.85	.0091

Table 2

Posttest Results by Item: Group Means, t-Statistics, and p-Values

Item	Group A Mean	SD_A	Group B Mean	SD_B	t-value	p-value
P1	.820	.240	.958	.144	-1.72	.0989
P2	.506	.265	.736	.181	-2.51	.0194
P3	.269	.260	.451	.356	-1.47	.1550
P4	.532	.282	.771	.225	-2.33	.0290
P5	.455	.346	.590	.212	-1.17	.2560
P6	.340	.265	.556	.228	-2.17	.0402
P7	.782	.249	.917	.195	-1.50	.1479
P8	.391	.191	.646	.129	-3.88	.0008
P9	.186	.181	.479	.310	-2.92	.0077
P10	.558	.291	.729	.198	-1.71	.1017
P11	.077	.188	.125	.226	-.58	.5674
P12	.442	.423	.694	.407	-1.52	.1431
P13	.436	.351	.653	.261	-1.74	.0947
P14	.391	.260	.625	.272	-2.20	.0381
P15	.410	.237	.694	.274	-2.78	.0106
P16	.532	.242	.764	.200	-2.60	.0161
P17	.494	.222	.611	.239	-1.28	.2150
P18	.455	.315	.764	.273	-2.61	.0156
P19	.218	.249	.417	.289	-1.85	.0776
P20	.321	.240	.535	.267	-2.11	.0458
P21	.353	.330	.563	.264	-1.75	.0940
P22	.365	.300	.479	.291	-.96	.3462
P23	.295	.304	.375	.225	-.74	.4649
P24	.237	.347	.472	.316	-1.77	.0908
P25	.429	.183	.625	.199	-2.56	.0176
P26	.154	.217	.438	.304	-2.70	.0128
P27	.500	.306	.688	.241	-1.69	.0144
P28	.237	.240	.444	.237	-2.17	.0405
P29	.333	.373	.660	.220	-2.64	.0148
P30	.218	.249	.576	.265	-3.49	.0020
P31	.372	.217	.583	.163	-2.74	.0117
P32	.077	.188	.229	.249	-1.73	.0962
P33	.442	.208	.465	.202	-.28	.7824
P34	.077	.188	.083	.195	-.08	.9339
P35	.256	.251	.472	285	-2.01	.0560
P36	.218	.227	.410	.260	-1.97	.0611

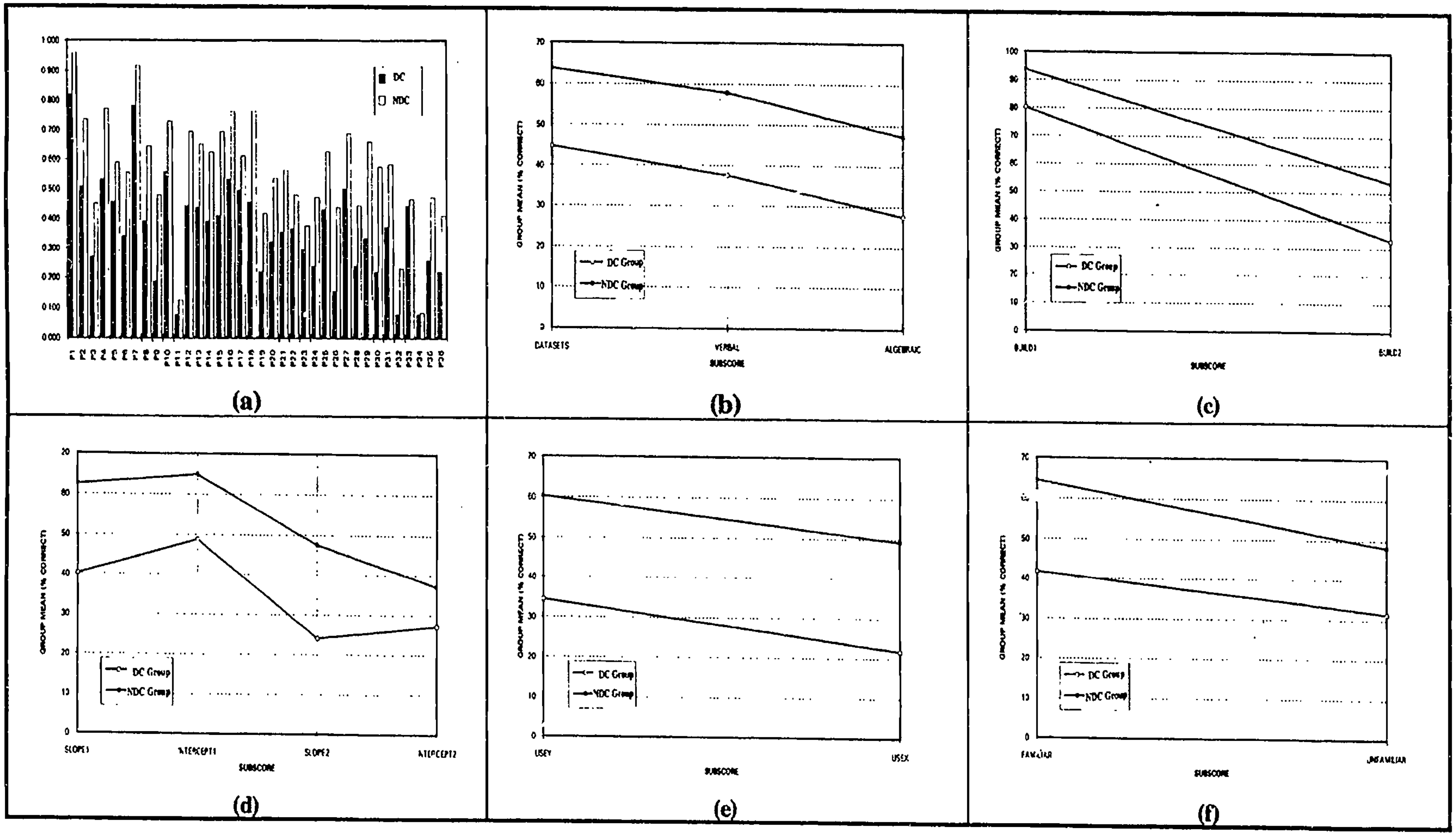

Figure 4. Graphical Representation of Posttest Results. (a) Item-by-Item Analysis; (b) Subscores Based on Starting Point of the Problem; (c) Building Mathematical Models Subscores; (d) Identifying and Interpreting Slope and y-Intercept Subscores; (e) Using Mathematical Models Subscores and (f) Context Familiarity Subscores.

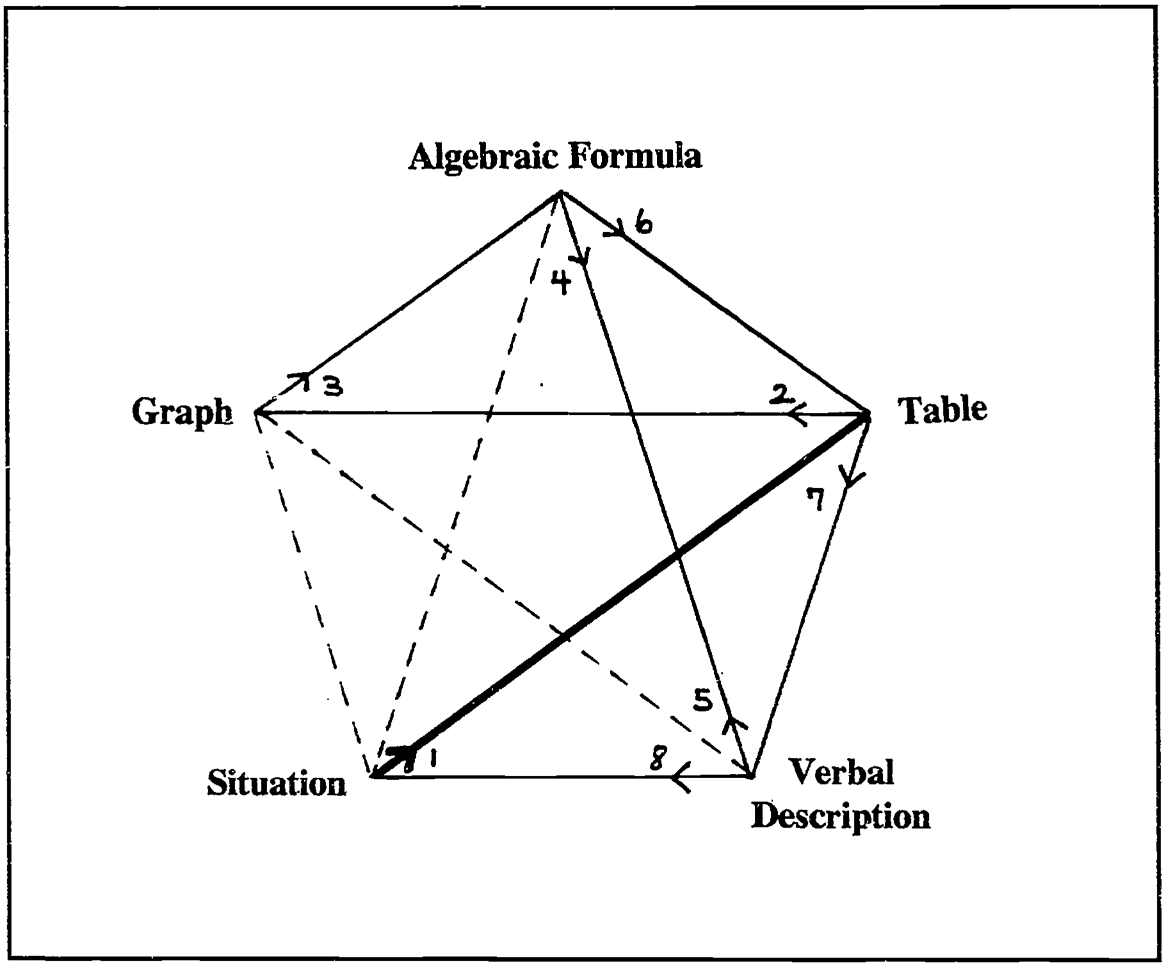

Figure 5. Translations Involved in Treatment Tasks. The Situation-to-Table Translation was Incorporated in Tasks for the "Data Collection" Group but not in Tasks for the "No Data Collection" Group