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ABSTRACT

The scree test and its linear regression technique are reviewed, and results of its use in factor analysis and Delphi data sets are described. The scree test was originally a visual approach for making judgments about eigenvalues, which considered the relationships of the eigenvalues to one another as well as their actual values. The graph that is plotted resembles a mountainside where a base pile of rubble, or scree, is formed. The analysis determines which eigenvalues are salient (mountainside) and which are rubble (scree). A multiple linear regression (MLR) approach has been proposed that would include more data points than the usual scree test and could yield better results. The MLR test provides the same decision as does the visual scree test, but can be easily programed, using an approach in which the ordered eigen.alues are thought of as points in a scatterplot. Examples are presented of the use of the scree test and the MLR approach with Delphi technique data to help decide how many items or issues to retain in a Delphi study or survey. The MLR approach appears to be an effective analytical procedure for the scree test. It usually produces the same number or fewer factors than the visual scree test in factor analysis, but yields more items than the visual scree in factor analysis. Three tables and six figures illustrate the discussion. (Contains 15 references.) (SLD)

Using Linear Regression to Determine The Number of Factors to Retain in Factor Analysis and the Number of Issues to Retain in Delphi Studies and Other Surveys

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The scree test was introduced by R.B. Cattell (1966) as a method for separating trivial and non-trivial factors in factor analysis. Later, Zoski and Jurs (1990) explained how the scree test could be adapted to determine the number of concepts to retain in Delphi surveys. The renewed interest in applications of the scree test generated by the article led Zoski and Jurs (1993) to propose a multiple linear regression technique to replace the somewhat subjective procedures used in the scree test. The purpose of this paper is to review the scree test, its linear regression technique, and to describe the results of its use in factor analysis and Delphi data sets.

The Scree Test

The scree test was originally a visual approach for making judgment about eigenvlaues. The scree test considers the relationships of the eigenvalues to one another as well as their actual values. Eigenvalues are plotted in decreasing order on a graph where the eigenvalues are on the ordinate and the factors are on the abscissa. The resultant graph resembles a mountainside where the destructive growth of ice in repeated cycles of freezing and thawing of the surface water, known as riving, splits the rocks into smaller fragments. The continued ice wedging and splitting causes the rocks to break away and fall to the base where a pile of rubble, or scree, is formed. Therefore, the analyst must determine from the graph which of the eigenvalues are salient or mountainside, and which values are scree.

The scree test procedure requires drawing a straight line through the points associated with the smaller eigenvalues. The points near this line are judged trivial or scree, while the points above and to the left were judged to be non-trivial (Cattell 1978; Cattell & Vogelman 1977; Cattell & Jaspers 1967). Cattell and Vogelman (1977) and Cattell (1978) presented guidelines for this visual procedure.

These guidelines as summarized by Zoski and Jurs (1990) are:

1. Three sequential points form an undesirably low limit for drawing a scree.
2. The points on the part of the curve that one should consider scree should fit tightly.
3. The slope of the scree should not approach vertical. Instead, it should have an angle of 40° or less from the horizontal.

4. In the case of multiple screes falling below 40°, the first scree on the left is the arbitrator.
5. Generally, a sharp, albeit sometimes small, break in the vertical level exists between the last point of the curve and the left most point of the scree.

The Multiple Regression Approach

Gorsuch (1983) indicated that the scree test may not work well when there are multiple breaks in the eigenvalue curve, and that it might be difficult to justify one break over another. Gorsuch and Nelson (described by Gorsuch, 1983) developed an analytical method, having a rationale similar to that of the scree test, for determining the number of factors. The CNG scree test requires one to compare the slope of the first three roots with the slope of the next three roots. Then the slope of roots 2,3, and 4 is compared with the slope of the roots 5,6, and 7. This process continues so that all sets of three factors are compared. The number of factors is denoted where the difference between the slopes is greatest.

Zoski and Jurs (1993) suggested that using multiple linear regression (MLR) would include more data points than the CNG approach and could thus yield better results. The criterion for success would be through comparison with the visual determination.

The MLR method usually provides the same decision as the visual scree test but can be easily be programmed. It uses an approach where the ordered eigenvalues are thought of as points in a scatterplot. One can then form two regression lines, one for the important factors and another for the scree or trivial factors. The decision about the number of factors to retain corresponds with the maximal differences between the two regression lines.

The process is as follows:

To use all of the eigenvalues, form these pairs of regression lines and compare them:

line 1 (points 1,2, and 3)	line 2 (points 4 through m)
line 3 (points 1,2,3, and 4)	line 4 (points 5 through m)
line 5 (points 1 through 5)	line 6 (points 6 through m)
•	•
•	•
•	•

line (m-2)(points 1,2,...,(m-3)) line (m-1) (points(m-2),(m-1) & m)

The slope of these regression lines will, of course, be negative and can be compared by the usual formulae (Howell, 1982, pp. 222, 239-240):

$$b = \frac{N\Sigma XY - \Sigma X \Sigma Y}{N\Sigma X^2 - \left(\Sigma X\right)^2}$$

$$t = \frac{b_1 - b_2}{s_{b_1 - b_2}}$$

with

$$s_{b_1 - b_2} = \sqrt{\frac{s^2_{Y \cdot X_1}}{s^2_{X_1}\left(N_1 - 1\right)} + \frac{s^2_{Y \cdot X_2}}{s^2_{X_2}\left(N_2 - 1\right)}}$$

and when homogeneity of error variances is assumed, one can pool:

$$s^2_{Y \cdot X} = \frac{\left(N_1 - 2\right)\left(s^2_{Y \cdot X_1}\right) + \left(N_2 - 2\right)\left(s^2_{Y \cdot X_2}\right)}{N_1 + N_2 - 4}$$

The salient factors are those with eigenvalues in the odd numbered line of the line pair where the t-test is maximized (highest value). The even numbered line of the pair denotes the scree line. Some analysts may choose not to include the last factor. Note that neither the CNG nor the multiple regression approach would be appropriate when there are only one or two factors.

Results With Factor Analysis Data

Two examples of the multiple regression approach were taken from Zoski and Jurs (1993). Both examples are originally from Tucker, Koopman & Linn (1969, p. 442). The eigenvalue plot for the first example is given in Figure 1, and the results from the multiple regression approach and the CNG approach are listed in Table 1. The data set was meant to have seven factors. The CNG approach yielded three factors and the multiple regression approach did yield the expected seven factors. Visual inspection of Figure 1 confirms that a seven factor solution is appropriate.

The data set for the second example (Tucker, Koopman & Linn, 1969, p. 442) was also intended to have seven factors, and a visual inspection of the scree plot in Figure 2 suggests that there are seven factors. The analysis presented in Table 2 indicates that the CNG approach yielded only three factors and the multiple regression approach yielded eight factors. This example shows that the multiple regression approach may not always agree with the results from a visual approach, but the technique seemed to work better than the CNG method for these data.

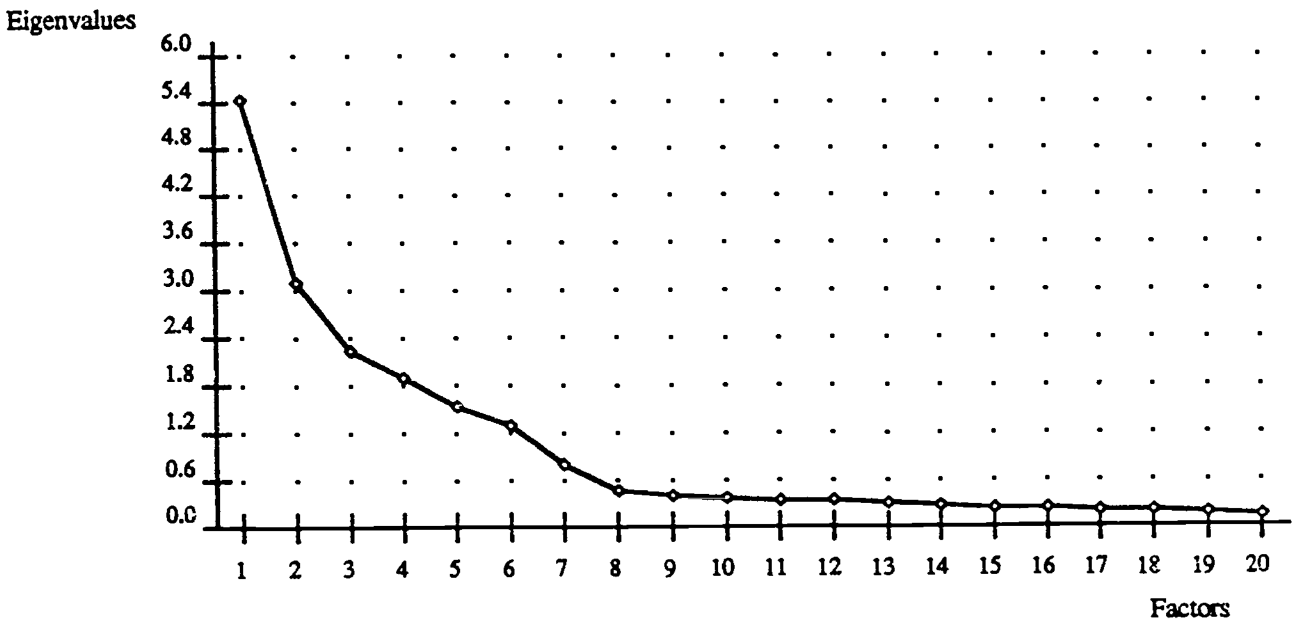

Figure 1 Scree plot from Tucker, Koopman and Linn (1969, p. 442, Middle 7)

Table 1

Comparison of Multiple Regression and CNG Approaches:

Example 1

# of factors	MR slope 1	slope 2	t	slope 1	CNG slope 2	difference
3	-1.595	-.084	6.346	-1.595	-.300	1.295*
4	-1.149	-.067	6.985	-.610	-.360	.250
5	-.904	-.051	7.327	-.360	-.415	.055
6	-.737	-.033	7.405	-.300	-.195	.105
7	-.651	-.023	7.665*	-.360	-.045	.315
8	-.590	-.022	7.563	-.415	-.030	.385
9	-.525	-.021	6.694	-.195	-.020	.175
10	-.465	-.021	5.507	-.045	-.025	.020
11	-.413	-.021	4.277	-.030	-.025	.005
12	-.367	-.020	3.176	-.020	-.025	.005
13	-.328	-.020	2.266	-.025	-.020	.005
14	-.295	-.019	1.555	-.025	-.020	.005
15	-.267	-.021	1.013	-.025	-.020	.005
16	-.243	-.020	.622	-.020	-.015	.005
17	-.222	-.025	.335	-.020	-.025	.005

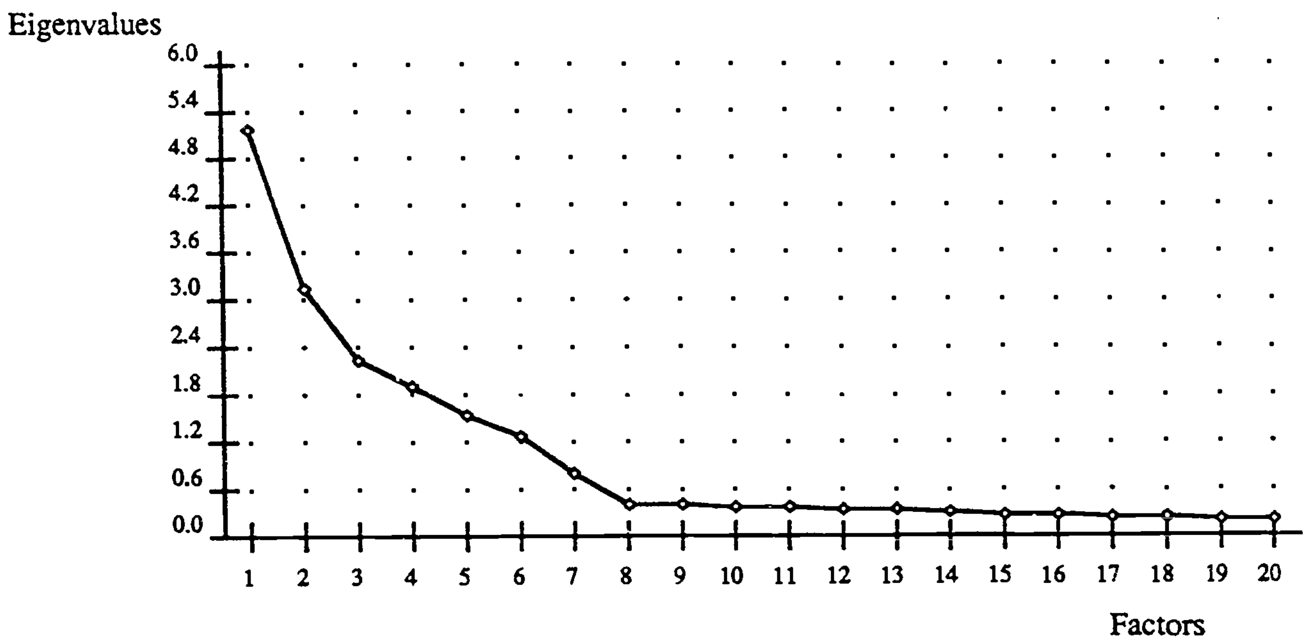

Figure 2 Scree Plot from Tucker, Koopman

& Linn (1969, p. 442 Formal 7)

Table 2

Comparison of Multiple Regression and CNG Approaches:

Example 2

# of factors	MR slope 1	slope 2	t	slope 1	CNG slope 2	difference
3	-1.475	-.081	5.855	-1.475	-.315	1.160*
4	-1.071	-.063	6.818	-.610	-.365	.245
5	-.850	-.047	7.522	-.345	-.440	.095
6	-.702	-.029	7.944	-.315	-.210	.105
7	-.625	-.018	8.369	-.365	-.015	.350
8	-.574	-.018	8.443*	-.440	-.010	.430
9	-.513	-.018	7.401	-.210	-.010	.200
10	-.455	-.195	5.974	-.015	-.025	.010
11	-.403	-.019	4.554	-.010	-.025	.015
12	-.358	-.018	3.341	-.010	-.020	.010
13	-.320	-.178	2.365	-.025	-.015	.010
14	-.287	-.177	1.611	-.025	-.020	.005
15	-.260	-.018	1.047	-.020	-.020	.000
16	-.235	-.015	.647	-.015	-.020	.005

Additional Factor Analysis Data Sets

Because the preliminary results suggested that the linear regression approach had promise, the approach was applied to twenty-five data sets from the literature. Table 3 contains a summary of the findings.

Table 3
Results Across Data Sets

	Number of Factors		
Data Source	Visual Scree	Linear Regression	CNG
Cliff (1970)			
US 600	4	6	5
CS 100	4	5	5
CS 600	4	5	5
CD 600	3	3	3
Linn (1968)			
Formal R-utilities	4	3	3
Formal C-I	4	3	3
Formal R-SMC	4	3	3
ACB - utilities	3	3	3
ACB - C-I	3	3	3
ACB R-smc	4	3	3
Thurstone - r- utilities	3	3	3
Thurstone - CI	3	3	3
Thurstone - R - SMC	3	3	3
Harman - R - utilities	4	3	3
Harman - CI	3	3	3
Harman - R - SMC	4	3	3
Tucker, Koopman & Linn (1969)			
Formal 442	7	8	3
Middle	7	7	3
Simulation	7	4	3
Formal 443	7	5	3
Middle	6	5	3
Simulation	6	4	3
Formal 444	7	3	3
Middle	4	3	3
Simulation	4	3	3

It is clear from the results in Table 3 that there are many instances where the visual approach, the multiple regression approach, and the CNG method produce the same number of retained factors. However, there are also some instances where the approaches yield different results and these analyses imply that the linear regression approach produces results that more closely approximate those produced by a visual scree test.

Using the Scree Test in Delphi Studies and Surveys

Delphi studies are efforts to reach consensus by an iterative process of issue identification and prioritization. The objective of the Delphi method was "to obtain the most reliable consensus of opinion of a group of experts" (Dalkey & Helmer, 1963 p. 458).

The first round of a Delphi process consists of having participants identify the key issues in some area of concern, usually via a mail survey, for example, each person is asked to provide five issues or items on the survey topic. Upon the receipt of the responses, the similar issues are grouped together. The compiled lists are sent back to the respondents so they may reconsider their suggestions in light of the suggestions of other participants. The reconsidering is round two, where respondents vote for their preferences from the complied list. In the third round, and in any subsequent rounds, the respondents receive the collated responses including the total number of votes received for each item. The respondents reconsider their choices in light of the group results. Usually, confidentiality is maintained and only the pooled results are available to the participants thereby avoiding confrontation between experts. After three or more rounds, a consensus will usually emerge about the key issues.

Deciding how many issues to retain as salient in a Delphi study clearly parallels the problem of deciding how many factors to retain in a factor analysis. Zoski and Jurs (1990) suggested that the scree test could be used to help with this decision. The only necessary alteration in the scree technique is that the graph would have the percentage of respondents who endorsed an issue on the ordinate, and the issues in descending order of endorsement on the abscissa. This technique has been very successfully used in several studies (Zoski, 1990; Kosarchyn, 1990; Condray, 1993).

Results With Delphi Data

The first example of the MLR technique with Delphi data is taken from Zoski (1989). Members of a professional organization of Educational Technologists were surveyed to find the most important research needs for the 1990's. A large number of research needs was identified and the scree test was used to identify those with the highest priority. Figure 3 contains the data on which the scree test was based. Forty-seven of the 150 research needs were retained using this method. Further grouping became apparent and twelve items were determined priorities. The regression approach yielded 62 important factors out of 156 items but the CNG indicated only 5.

The second example (Condray, 1993) was an attempt to establish a research agenda for a campus ministry. The Delphi technique was used to limit the number of endorsed agenda items to a manageable number of high priority items. Approximately 130 items were produced by the respondents, Mainline and Evangelical Protestants. The data in Figures 4 and 5 show the results for two groups of participants. In each of these Figures, only 65 points were plotted as the rest of the points were linear; however all points were included in the calculations. For the Mainline Protestants the CNG approach gave three, whereas the regression gave 29 items and the visual approach indicated 23 salient items. For the Evangelicals, the CNG test had less than three and the regression approach yielded 32 items. The visual scree produced 18 items.

Finally, we offer an example of an application of the scree test to typical survey data, not from a Delphi study. Kosarchyn (1990) asked school nurses across the nation to identify the health needs of Hispanic elementary school children. A long list of health needs were generated but the scree test (Figure 6) allowed Kosarchyn to focus upon the most important health needs. The visual approach produced 9 important items out of 50, the regression approach yielded 11 and the CNG approach retained three items.

Although the application of the scree test to Delphi and survey data produced some discrepancies between the visual and the linear regression results, the regression techniques did yield a manageable number of items through a non-arbitrary technique. The scree test application is not limited to eigenvalues, it works well with survey data.

Discussion

The scree test is a very useful data reduction technique. It helps the user decide how many factors to retain in a factor analysis and it can be used to help decide how many items or issues to retain in a Delphi study or survey. The multiple linear regression approach appears to be an effective analytical procedure for the scree test. Judging from the results of the present analyses, the MLR approach usually produced the same number or fewer factors than did the visual scree test in factor analysis. However, the MLR approach yielded more items than the visual scree did when applied to survey data. The characteristics, strengths, and weaknesses of the MLR approach will be revealed only after the technique is used with a wide variety of data sets.

There are probably other ways to use the scree test. Altschuld and Thomas (1991) listed the criteria for its application to various kinds of data. Zoski and Jurs (1991) indicated the potential adaptability of the scree test for unidirectional linear variables on the internal or ratio scale. Creative data analysts will surely identify new ways to use this old technique.

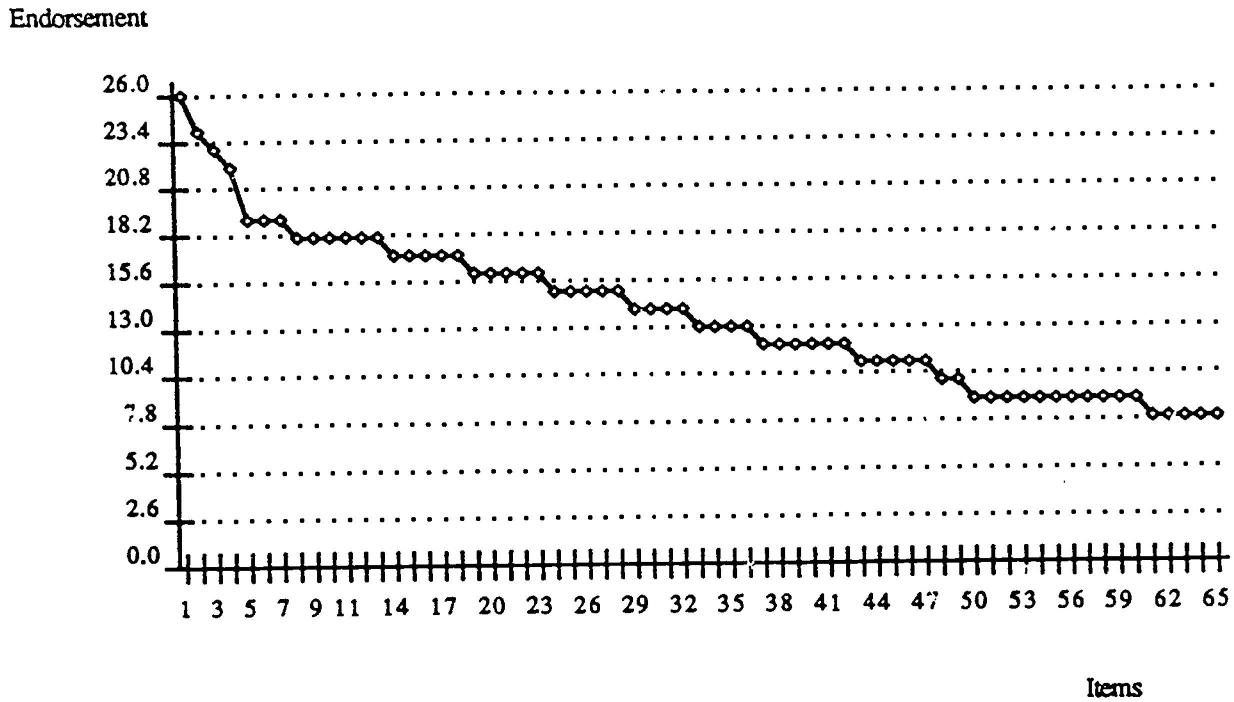

Figure 3 Scree plot from Zoski (1989)

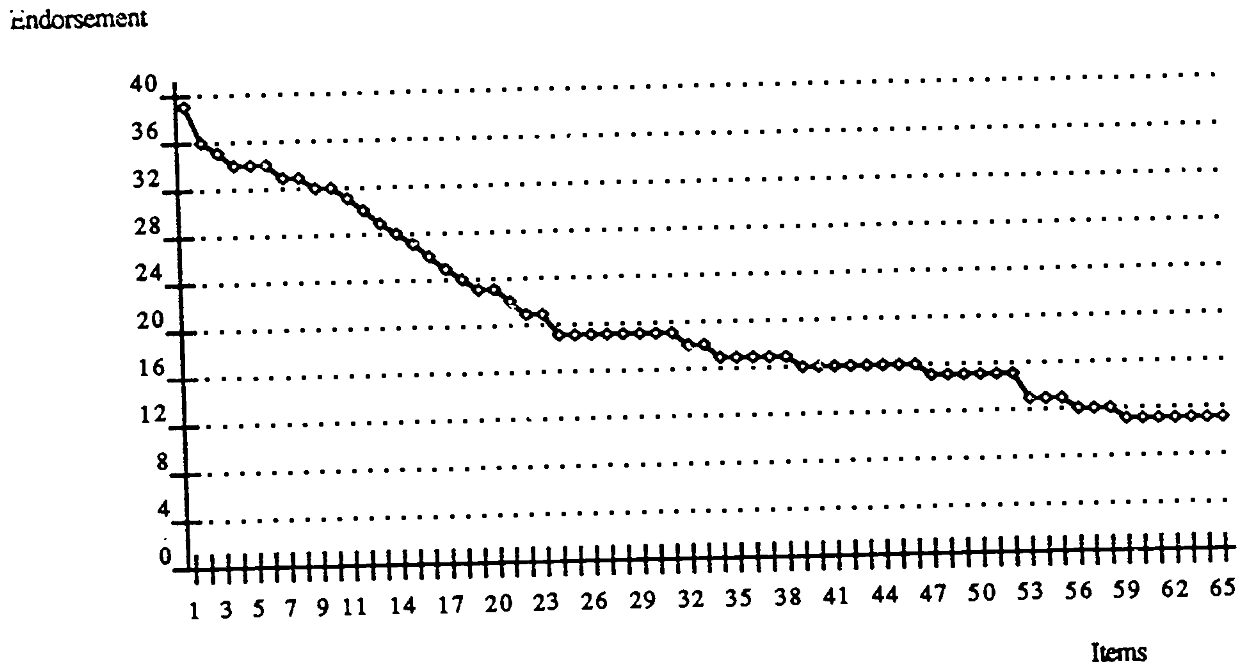

Figure 4 Scree plot from Condray (1993, Mainline Protestant)

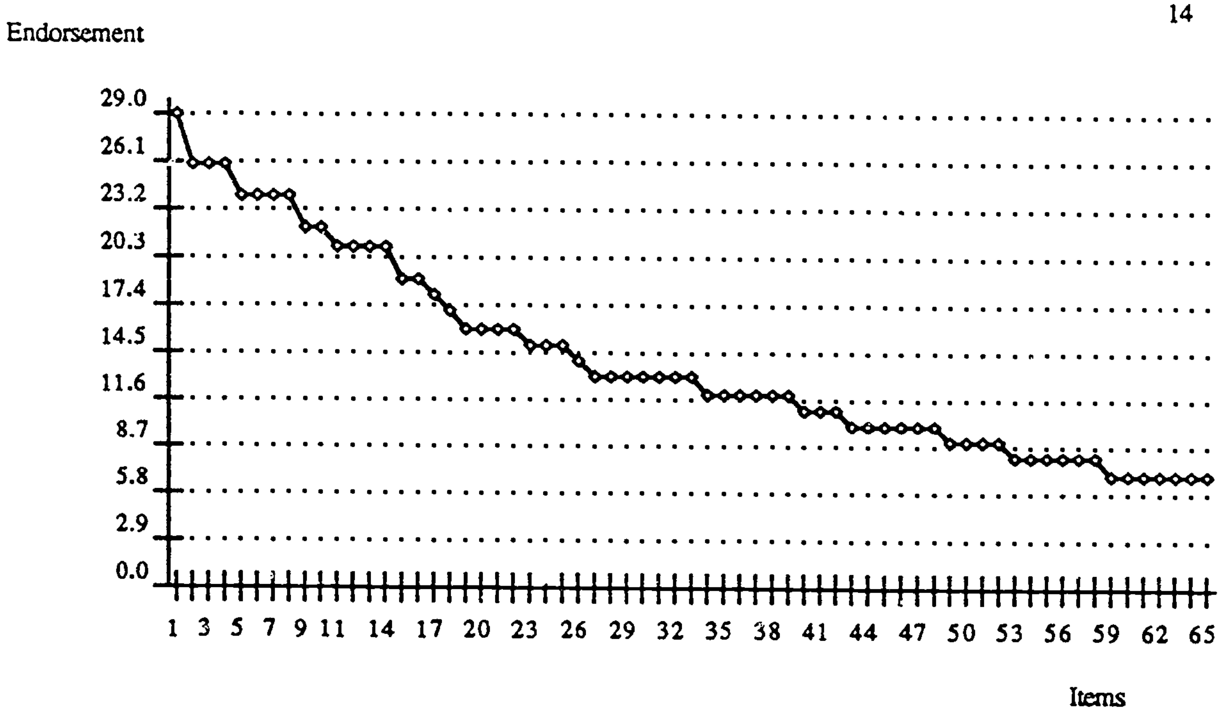

Figure 5 Scree plot from Condray (1993, Evangelical Protestant)

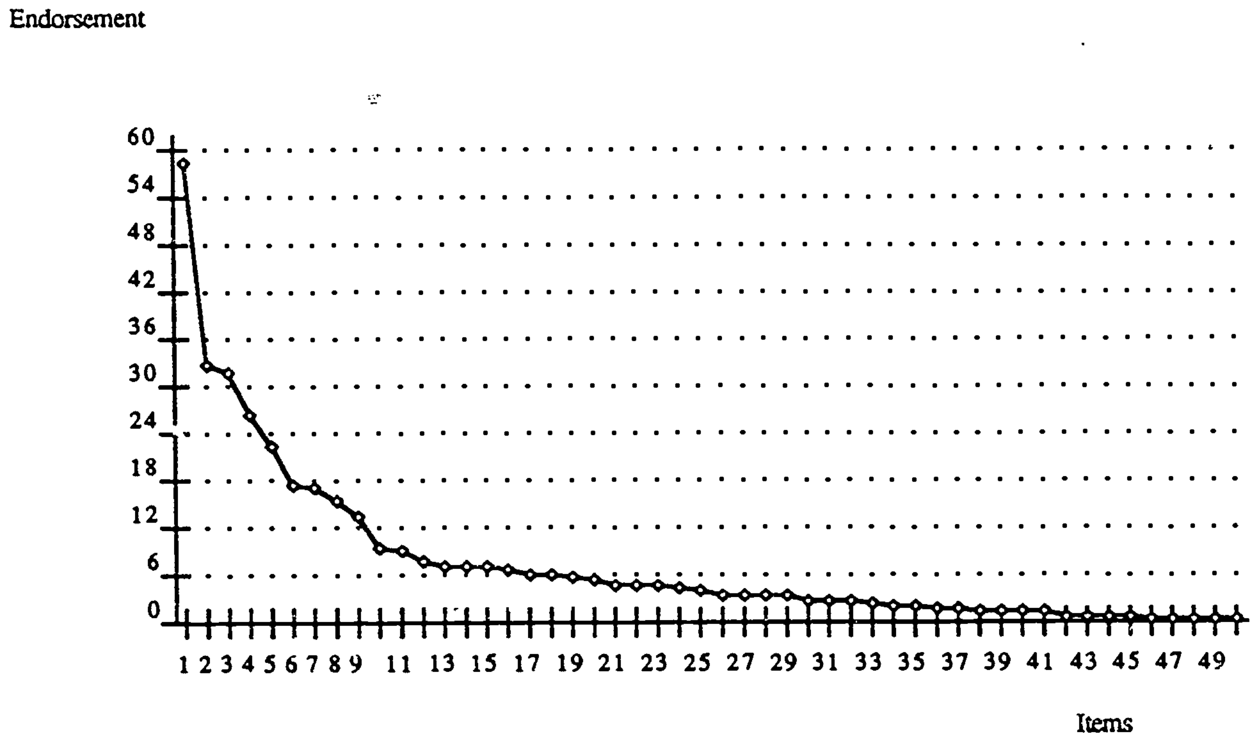

Figure 6 Scree plot from Kosarchyn (1990)

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