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ABSTRACT

In 1983, the Ohio State Board of Education responded to the public's expectations for schools by adopting minimum standards requiring competency-based education in mathematics on a 5-year cycle, of each school district's competency-based education program; and to publish an annual report on competency-based education in Ohio. This document is the State Board of Education's response to that mandate. The document is separated into seven sections. The introduction discusses the notions of the model curriculum, performance objectives, approaches to assessment, suggested intervention services, reporting requirements, on-site evaluations, and the required annual report. This is followed by a brief statement of philosophy. The third section presents curriculum strands from kindergarten to grade 12 in the following content areas: (1) patterns, relations, and functions; (2) problem solving strategies; (3) numbers and number relations; (4) geometry; (5) algebra; (6) measurement; (7) estimation and mental computation; and (8) data analysis and probability. The fourth section lists the subject objectives listed by strands in the previous section by grade level from kindergarten through grade 8, and by course for grades nine through twelve. The fifth section describes the performance objectives including categories, critical objectives, and performance objectives for the ninth grade proficiency test. The sixth section discusses suggested intervention services, and the last section discusses the development of standardized tests to measure compliance with the competencies. (MDH)

# MODEL COMPETENCY-BASED EDUCATION TASK FORCE

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# FOREWORD

With the adoption of the 1983 minimum standards, the State Board of Education responded to the public's increasing expectations for schools by requiring competency-based education in English composition, mathematics, and reading. As a result, the conditions necessary to promote a general education of high quality in all chartered schools have improved markedly. Such improvement is only possible when well-structured local and state leadership recognizes that the responsibility for providing direction to the system of education in the state must accommodate the flexibility necessary to establish educational programs that are responsive to local needs. Education in Ohio has benefited immeasurably as a result of this understanding.

At the same time, it is becoming increasingly important that the educational community be able to document, in language easily understood by the general public, the status of educational progress. In response to this need, the 118th General Assembly enacted Sections 3301.0715 and 3301.0716 of the Revised Code, which requires the board of education of each city, local, and exempted village school district to implement a competency-based education program for grades one through twelve of the district. The basic responsibilities for implementing the provisions of this competency-based education legislation are shared by the State Board of Education and the boards of education of city, local, and exempted village school districts. It is the responsibility of the State Board of Education to develop a model competency-based education program; to conduct on-site evaluations, on a five-year cycle, of each school district's competency-based education program; and to publish an annual report on competency-based education in Ohio to be distributed to school districts and the general public.

It is the responsibility of city, local, and exempted village school districts to implement competency-based education programs that include performance objectives for each grade level for composition, mathematics, and reading; instruction at each grade level designed to ensure that the specified performance objectives can be attained; provisions for periodic assessment of pupil performance to measure progress toward achieving the specified performance objectives; a program of intervention services for pupils who are failing to make satisfactory progress toward achieving the specified performance objectives; and written policies and procedures regarding the participation or exemption of handicapped pupils.

The major purpose of this legislation is to formalize and standardize rather than extend the competency-based education requirements specified in Minimum Standards for Elementary and Secondary Schools. This will be accomplished through an examination of each school district's competency-based education program for satisfactory comparison with the component elements included in

the model competency-based education programs adopted by the State Board of Education. The very important gains made by Ohio school districts through the implementation of competency-based education are in no way negated by this process. The quality of locally developed curriculum has never been better. These efforts are acknowledged and commended. We cannot, however, be satisfied with past and current successes. The need to design and implement curriculum that reflects important and dramatic changes in our society is clear and requires that we be responsive to the educational implications of those changes. These model programs have been designed to improve student achievement, to improve the quality of curriculum and instruction, and to strengthen school and community relationships through better communication. Appreciation is extended to those educators who contributed to the development of these competency-based education programs by sharing their time, expertise, and materials. Appreciation is also extended to staff members who worked untold hours to make these programs possible.

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# INTRODUCTION

In order to help school districts develop the elements of a competency-based education program and to enable the Ohio Department of Education to evaluate school district competency-based education programs, the State Board of Education has established a model competency-based education program. The model includes specification of all of the following for grades one through twelve:

1. A model curriculum for instruction in composition, mathematics, and reading for each grade level;

2. Recommended performance objectives in composition, mathematics, and reading for each grade level;

3. Recommended standardized tests suitable for measuring progress in meeting the recommended performance objectives in composition, mathematics, and reading for each grade level;

4. A recommended program of intervention services by grade level for pupils who do not make satisfactory progress toward achieving recommended performance objectives in composition, mathematics, and reading.

## MODEL CURRICULUM:

The model competency-based education program, including all prescribed elements, has been established for composition, mathematics, and reading by the State Board of Education subsequent to consultation with broadly representative advisory committees. The model curricula for composition, mathematics, and reading reflect generally accepted research bases, programmatic (K-12) scope, effective developmental processes, and relevant assessment practices. A major objective of competency-based education is to better guarantee correspondence among the written, implemented, and assessed curricula in Ohio schools. It cannot be assumed, however, that the translation of the written curriculum to the taught curriculum to the attained curriculum can be accomplished without a focused effort. That effort must begin with the development and implementation of curriculum and instruction based upon the most current research findings. The curriculum should be comprehensive in scope, and sequenced so as to provide developmentally appropriate instruction as necessary throughout the K-12 continuum. The model English Language Arts and Mathematics courses of study adopted by the State Board of Education provide Ohio school districts with such a focus.

## PERFORMANCE OBJECTIVES:

The pupil performance objectives for composition, mathematics, and reading have been generated from the model courses of study. These performance objectives represent the essential rather than the minimal knowledge and skills necessary to develop three critical dimensions of instructional development, i.e., literacy, communications/application, and career/specialization/vocation, that should guide the curricular/instructional program. Even the most clearly defined objectives, however, can provide only the structure necessary to achieve educational excellence. Instruction is the vital force in the process. The State Board of Education recognizes that instructional decision-making is best left in the hands of classroom teachers. Because competency-based education requires special attention, however, several examples of effective instructional activities for the achievement of selected pupil performance objectives have been suggested.

## ASSESSMENT:

In addition to instruction focused on student achievement of the specified performance objectives in composition, mathematics, and reading, competency-based education requires assessment of student progress. A clear distinction is made between districtwide tests required to be administered once in grades 1-4, once in grades 5-8, and once in grades 9-11; grade-level standardized testing; and ongoing assessment in the classroom. Assessment of student progress is a critically important component of competency-based education. Districtwide and grade-level accountability testing as well as ongoing classroom assessments are required. The use of assessment data for instruction, evaluation, intervention, guidance, and promotion must be specified by each school district in written guidelines.

Differences between accountability and instructional assessment are fundamental and necessary. Large-scale assessments, including districtwide tests for competency-based education, are best used to promote informed policy-making relevant to curricular programs. They are necessarily formal and objective, time- and cost-efficient, widely applicable, and centrally processed. The results must be in a form useful to policy makers. This may mean reducing complex processes to a single score. To ensure the credibility of accountability tests, both the incentives and the means to distort scores must be removed. This is best accomplished by limiting the use of the results of such efforts to monitoring the effectiveness of curricular/instructional programs. The achievement tests required of students in grades four, six, eight, and ten, as well as the ninth-grade proficiency test, may very well serve to meet both the districtwide grade-level testing requirements of competency-based education.

Informed decisions about individual students, including the need for intervention services, are best accomplished through assessment strategies

conducted at the classroom level. Current models of learning based on cognitive psychology contend that learners gain understanding when they construct their own knowledge and develop their own cognitive maps of the interconnections among concepts and facts. It is possible, therefore, to assess students' thinking processes in useful and undistorted ways. Teacher observations and other assessment activities implemented in the classroom may be less reliable (in a statistical sense) than standardized tests, but the accumulation of data gathered about individual students in the course of a school year has much more accuracy in terms of student learning. In short, it is essential that use is made of the wealth of data teachers themselves can provide about their students.

Teachers should not have to set aside good instruction to prepare students to take a test. Instead, good instruction itself should be the best preparation. The best assessments designed to support instruction may be characterized as informal, adapted to local context, locally scored, sensitive to short-term change in students' knowledge, and meaningful to students. Assessment tasks should be designed to closely resemble real learning tasks. Such assessments will incorporate tasks that have instructional value in themselves if students are provided with immediate, detailed, and complex feedback. Indicators of competence, such as those which follow, should be used as the bases for making decisions about individual student achievement of prescribed performance objectives.

Coherence of Knowledge. Assessment should tap the connectedness of concepts and the student's ability to access interrelated chunks of information. Student understanding should be demonstrably integrated and structured.

Principled Problem Solving. Assessment should focus upon the underlying principles and patterns needed to solve problems rather than the surface features of a task.

Knowledge Use. Complete understanding includes knowing the conditions that mediate the use of knowledge. Assessment should determine students' capacity to do so.

Automatized Skills. Assessment should determine the degree to which students integrate basic component skills into total performance.

Metacognitive or Self-Regulatory Skills. Assessment should determine whether students are able to monitor their own understanding, use strategies to make questions comprehensible, evaluate the relevance of accessible knowledge, and verify their own solutions.

## INTERVENTION SERVICES:

Recognizing that alternative or supplemental action designed to remediate, reinforce, or support student learning relative to the specified performance objectives will sometimes be necessary, suggested intervention services have been identified for mathematics at each grade level. The teacher must be able to identify the need for intervention, design the instructional form it will take, and implement the action. This requires a great deal of skill in classroom remediation, reinforcement, and enrichment techniques. Teachers must have the capacity to utilize content material for these activities, to instruct for specific skill/knowledge deficiencies, and to group students for special needs. The abilities to understand and use various diagnostic instruments, to analyze assessment data, and to teach prescriptively are critical elements of effective intervention.

## REPORTING REQUIREMENTS:

School districts are required annually by July 31 to collect, compile, and make available to the State Board of Education, upon request, separately for composition, mathematics, and reading, all of the following.

Copies of the written assessment instruments, by grade level, used during the preceding school year to determine student progress toward achieving the specified performance objectives, including information about the dates and methods of administration of the instruments and the methods of scoring or standards used for evaluating the results to determine whether or not students have made satisfactory progress toward achieving the objectives;

Data on the number and percentage of students, by grade level and by school building, who were shown by the assessment instruments not to have made satisfactory progress toward achieving the objectives during the preceding school year;

Information about the types, and a description of each type, of intervention services available to students who were shown by the assessment instruments not to have made satisfactory progress toward achieving the specified objectives; and data on the number of students who received each type of intervention service during the preceding school year, by grade level and by school building;

Estimates of the cost of providing intervention services to those students who were shown by the assessment instruments not to have made satisfactory progress toward achieving the objectives and who are not receiving intervention services, and the basis on which such costs were estimated;

Additionally, school districts must provide for making this information, excluding copies of assessment instruments, available for inspection by the public at the district board's offices, and for providing copies of the information to any person upon request and payment of a reasonable fee for the cost of reproducing the information.

**ON-SITE EVALUATION:**

At least once during every five-year period, the State Board of Education will conduct an on-site evaluation of schools to determine compliance with minimum standards for elementary and secondary schools. All elements of each district's competency-based education program will be reviewed as part of that evaluation process. An examination of each element of the district's competency-based education program for each grade level for comparison with the model curriculum, performance objectives, assessment instruments, and intervention programs for the corresponding grade level will be made. Based on that comparison, a rating of satisfactory or unsatisfactory will be assigned to each element of the competency-based education program for each grade level. An explanation of the reasons why each unsatisfactory rating was assigned will be provided.

**ANNUAL REPORT:**

The State Board of Education will publish an annual report of Ohio's competency-based education programs. The report will reflect any data received from school districts as well as the results from any on-site evaluations conducted during the preceding school year. Copies of the report will be sent to each district board of education, which will in turn make it available to the general public for examination at the district's offices. The district will make copies of the report available to any person upon request and payment of a reasonable fee for the cost of reproducing the report.

# PHILOSOPHY

The mathematics program should ensure that all students have an opportunity to become mathematically literate, are capable of extending their learning, have an equal opportunity to learn, and become informed citizens capable of understanding issues in a technological society. Knowledge of mathematics is an essential element in the development of the whole person.

Mathematics is more than a collection of concepts and skills to be memorized and mastered. Mathematics includes problem-solving, reasoning, and communicating, as well as valuing the breadth of its connections. Thus, an appropriate mathematics curriculum includes the investigation of the connections and interplay among various mathematical topics and their applications at every grade level.

All students can become mathematically powerful. They can learn to formulate and solve problems with a variety of strategies, to verify and interpret results, and to generalize solutions. Their understanding of mathematical concepts can enable them to identify and generate examples and nonexamples as well as recognize the various meanings and interpretations of concepts. They can learn to use models, diagrams, and symbols to represent concepts and to translate from one mode of representation to another. They can recognize when a mathematical procedure is appropriate and reliably and efficiently execute procedures, including appropriate methods of computation. They can verify the results of procedures as well as generate new procedures and extend or modify familiar ones.

All students should be provided access to the full range of mathematical topics. Knowledge of patterns, relations, and functions; of geometry and measurement; of probability and statistics; and of increasingly important topics in discrete mathematics are a necessary foundation for all students. Since students' interests, goals, and achievements change as they mature and advance through high school, the mathematics program should be designed to keep options open. While recognizing that individuals have different career objectives, and may well pursue careers as yet undefined, we further recognize that <u>all students</u> have the right to learn significant mathematics and to develop power over mathematical ideas.

The goals of the mathematics program are that all students:

- learn to value mathematics;
- become confident in their ability to do mathematics;
- become mathematical problem-solvers;
- learn to communicate mathematically; and
- learn to reason mathematically.

## CURRICULUM STRANDS

Mathematics programs must fit the needs of students who will live in a future requiring ability to solve problems and use technology. The intent for the model curriculum described here is to recognize this future in a practical, down-to-earth fashion. Such a curriculum provides a challenge to many schools and many teachers in that it requires change from current practice in both content and methods of instruction.

Mathematical experiences for students must recognize that current and future demands of industry and society go beyond shop-keeper arithmetic skills. Continued economic health and growth of business and industry depend on a work force that can use mathematics constructively and creatively in ordinary situations. In order to participate fully in society and to realize their potential, students must encounter a mathematics that challenges their abilities and capitalizes on their aspirations.

The heart of this document is a listing of content strands for key conceptual areas in mathematics. The strands are as follows:

Strand 1: *Patterns, Relations, and Functions*

Strand 2: *Problem Solving Strategies*

Strand 3: *Numbers and Number Relations*

Strand 4: *Geometry*

Strand 5: *Algebra*

Strand 6: *Measurement*

Strand 7: *Estimation and Mental Computation*

Strand 8: *Data Analysis and Probability*

The statements of content provide the conceptual underpinnings for mathematics. The content dimensions are described by grade level and correlate with the mandated proficiency tests and ability/achievement tests.

This document is designed to provide a base for curricular and instructional planning. Individuals—teachers, department chairs, curriculum supervisors, administrators, and school board members—responsible for the design of a graded course of study for mathematics will use it as a specification of what is needed for coverage of core mathematical ideas in the curriculum. Teachers implementing mathematics instruction in the classroom should examine this document to ascertain whether they are providing appropriate emphases and coverage for their students to perform successfully in state-mandated testing programs.

School board members and administrators must provide resources and equipment needed to support instruction and make the necessary policy decisions to maintain instruction for this core program. Commitment to an instructional program based on this model curriculum should guide the selection of texts, instructional materials, and equipment, and the planning of staff development activities.

The mathematics used in industry, business, and further schooling depends on technology. Equipment in the form of calculators and computers must be provided so that students can learn to use it correctly. The strands are organized with the assumption that calculators will be available for all levels of mathematics instruction. Calculators with graphing capabilities should be provided for all high school level instruction. Scientific calculators with an algebraic-operating system should be available to all middle or junior high school students. Elementary students need ready access to calculators at all times. Each classroom needs appropriate display equipment for computer-based instruction.

## Assumptions Guiding Content Selection and Instructional Processes

The design of the eight strands was guided by five assumptions that are necessary to successful implementation of the model curriculum. The assumptions reflect principles of learning, recent research about mathematics learning, and the best current thinking concerning instruction and content. Among the sources that served to guide the design of this model curriculum are the National Council of Teachers of Mathematics' *Standards for Curriculum and Evaluation of School Mathematics* (1989) and the National Research Council's *Everybody Counts* (1989). The goals inherent in these assumptions serve to specify the philosophical bases and dimensions of the strands. These principles should cut across all content and instruction.

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|---|---|
| **Assumption One:** | ***Thinking*** — Problem-solving and reasoning within a context of using mathematics should be the fundamental organizational theme in all strands. |
| **Assumption Two:** | ***Development*** — Curricular sequencing must reflect sound application of what is known about children's development. |
| **Assumption Three:** | ***Technology*** — Natural uses of technology in doing mathematics must be incorporated and learned by all. |
| **Assumption Four:** | ***Connections*** — Mathematical learning is more permanent and useable if the connections between ideas in different problem situations and across strands are stressed. |
| **Assumption Five:** | ***Communication*** — Learning mathematics is incomplete without significant attention to the development of communication skills. |

These assumptions provide a framework for the design of a graded course of study. The evidence is overwhelming that use of technology in the form of calculators and computers is important. Technology can produce more efficient learning; moreover, students must acquire a comfortable familiarity with technology that is used in the work place and in further study of mathematics, science, and business. Use of mathematics, both in and out of school, requires communication abilities, including reading, talking, and writing. Group activities and cooperative learning tasks that require communication of mathematical ideas significantly facilitate the acquisition of mathematical concepts. Problem solving and reasoning must stress the analysis of problem situations and the formulation of realistic moves toward a solution. Learning is more useful and easily remembered if new ideas are developed and motivated by the solution of a problem. Mathematical ventures result in learning only when activities reflect the maturity and past experiences of the student.

A graded course of study must emphasize these assumptions if a student's mathematical experience is to be more than merely adequate. A student must feel that mathematics provides a powerful but comfortable tool. Students must acquire independence and confidence in their abilities to make sense of mathematical situations and to use mathematics. Building this independence and confidence is among the most important outcomes of a mathematics program. However, it requires attention to what learners can do rather than to what they do not know.

Time is the most valuable commodity under the control of a teacher in planning for instruction. The assumptions and specifications of content in the eight strands recognize that control of time on task is crucial. For example, a problem-solving activity or a lesson that builds communication skills must have a mathematical purpose and be directed toward achieving content goals in a strand rather than serving a stand-alone objective. This is to say, a lesson may address one or more objectives from several strands. There has been no attempt made to avoid having an objective appear in more than one strand. The principles described in the assumptions must be manifested in efforts to achieve the particular goals of the eight individual core strands if the intent of the model curriculum is to be realized.

The dimensions of content and instruction contained in the strands and enunciated in the assumptions apply for all students in mathematics in Ohio schools. This document describes a core curriculum for grades K through 12. It is designed to further every student's opportunity to learn the mathematics contained in all strands. The core curriculum for a school must provide every student with the ideas reflected in the proficiency and competency-based testing programs. Ability or homogeneous grouping programs that deny selected students access to the ideas and skills found in one or more of the strands are educationally questionable. Most ability-grouping programs in the past have reduced the expectations of student performance for lower level tracks and have severely restricted the opportunity to learn for a large portion of students, particularly in middle school and beyond.

## Change and Your School's Program

Determining the mathematics curriculum is the responsibility of the local school system. Specification of a graded course of study in mathematics serves to provide the blueprint and conditions for selecting textbooks, acquiring equipment and materials to support instruction, designing staff development activities, designing testing programs, and judging the quality of the instructional program. In short, this is a document designed to help in the formulation and realization of a school mathematics program. It describes the type of comprehensive program needed to assure quality and excellence in mathematics for Ohio.

More changes will be needed in some schools than in others to satisfy the intent of the core recommendations of the strands . Where changes are needed, it is well to be systematic in assessing the capability for achieving change in the mathematics program. Some elements of the strands can be easily implemented, and some will require more incremental and systematically planned activities over a longer period of time in order to achieve implementation. It is important to focus on the readily accomplishable as the first priority and then shift to those elements requiring more extensive planning and development.

This Model Competency-Based Education Program is the standard against which the Ohio Department of Education will judge the content and quality of school districts' competency-based programs in mathematics. Some schools may aspire to a more comprehensive coverage of the content domains of mathematics than is represented in this program. The intent of the model is to provide a conceptually oriented program encompassing supportive use of technology to enhance the learning of mathematics that is both necessary and sufficient for the problem-solving demands of Ohio's citizens in industry and in other areas of endeavor.

The model mathematics curriculum is developed in eight strands that consist of objectives for all grade levels, from kindergarten to twelfth grade. The objectives are grouped by grade levels K-4, 5-8, and 9-12, and are related to the Curriculum and Assessment Standards developed by the National Council of Teachers of Mathematics. Comments that serve to clarify meanings are placed in the margin by some objectives.

The objectives have been correlated with the mandated ability/ achievement testing program. Test forms recommended for use at the fourth, sixth, and eighth grades in conjunction with the four most frequently used standardized achievement tests in Ohio were examined. Objectives which correspond to items on those tests are designated by an icon of children's blocks. Correlation has also been made with the outcomes assessed by the state-mandated ninth and twelfth grade proficiency tests. The icons are as follows:

 - achievement test

 - ninth grade proficiency test

 - twelfth grade proficiency test

It should be noted that at none of the grades (four, six, or eight) are all of the test objectives in this curriculum covered in time for fall testing. If a district desires to test only that which students have had the opportunity to learn, achievement/ability testing must necessarily be administered as late in the spring as possible.

## STRAND 1: PATTERNS, RELATIONS, AND FUNCTIONS

This strand enables students to see and explore regularities in the world about them by providing a set of experiences that are appropriate to the developmental stages of students. In the beginning, shapes and numbers, patterns and designs from different cultures, and changes in the rhythm and pitch of music are explored. The objectives are written in such a way that any kind of pattern observable in the environment is appropriate for investigation.

The study of patterns merges into the study of relationships. The focus becomes building mathematical models that will predict behaviors which have an exhibited pattern that students have recognized and described mathematically.

Finally, the distinction between relationships and functions is developed. Specific functions and their applications are explored with an emphasis on trigonometric functions in grade eleven.

There is a conscious attempt to weave patterns, relations, and functions throughout the entire curriculum. This theme "...begins in K-4, is extended and made more central in 5-8, and reaches maturity with a natural extension to symbolic representation and supporting concepts, such as domain and range, in grades 9-12."*

*Curriculum and Evaluation Standards for School Mathematics. Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 98.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES K—4

STANDARD 13:
PATTERNS AND RELATIONSHIPS

In grades K—4, the mathematics curriculum should include the study of patterns and relationships so that students can—

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 60.

Kindergarten

The student will be able to ...

1. represent a pattern using objects, drawings, rhythms, and body movements, and be able to describe a pattern verbally. — use bottle caps, foam packing, coins, clapping hands, etc.
2. replicate or copy a given pattern.
3. continue a pattern.
4. recognize a pattern in the environment.
5. sort objects according to an attribute such as size, color, shape, or weight. — use commercially prepared sets of objects plus objects collected by students.

Grade One

The student will be able to ...

1. represent, copy, and continue patterns. — include more complicated patterns than in kindergarten.
2. sort objects on multiple attributes. — use attribute blocks, etc.
3. identify and extend missing elements of repeating patterns and sequences of numbers.

Grade Two

The student will be able to ...

1. identify and extend patterns of objects and symbols.
2. recognize patterns in numbers and number combinations. — use addition charts through one hundred or monthly calendars . . .
3. use patterns to make generalizations and predictions by
   a) determining the rule and identifying missing numbers in a sequence of numbers; — emphasis on addition and subtraction
   b) determining the rule and identifying missing numbers in a table of number pairs; and

c) identifying missing elements in a pattern and justifying their inclusion.

Grade Three

The student will be able to ...

1. predict additional terms in a given pattern, describe how the pattern is created, and extend the pattern.
2. recognize multiplication patterns.
3. use patterns to make generalizations and predictions by
   a) determining the rule and identifying missing numbers in a sequence of numbers;
   b) determining the rule and identifying missing numbers in a table of number pairs; and
   c) identifying missing elements in a pattern and justifying their inclusion.
4. make a table of values to record the pairing of members of two sets, determine the relationship (rule) between each pair, and use the rule to generate additional pairs.

use objects as well as multiplication charts

include multiplication and division as well as addition and subtraction

This could be an opportunity for an informal use of variables to express the rule.

Grade Four

The student will be able to ...

1. explore and describe in words simple and complex patterns in nature, art, and poetry.
2. determine the rule and identify missing numbers in a sequence of numbers or a table of number pairs related by combinations of addition, subtraction, multiplication, and division.

A spreadsheet can be used by the teacher to develop these kinds of tables - students could also profit from learning to use a spreadsheet.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 5—8

STANDARD 8:
PATTERNS AND FUNCTIONS

In grades 5—8, the mathematics curriculum should include explorations of patterns and functions so that children can—

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships with tables, graphs, and rules;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- use patterns and functions to represent and solve problems.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 98.

## Grade Five

The student will be able to ...

1. investigate patterns that occur when changing numerators and denominators in equivalent fractions and describe these patterns verbally.
2. investigate the patterns of digits formed when fractions are changed to decimal form.
3. use patterns to explore the rules for divisibility.
4. investigate patterns formed by powers of ten using exponents and expanded form of numbers.
5. explore methods for finding the $n^{th}$ term of a simple sequence involving one operation and verbalize a procedure.
6. graph ordered pairs.
7. find missing terms of a sequence using powers.
8. explore and describe in words simple and complex patterns in music and science.

use a calculator to investigate patterns formed as a result of division of one-, two-, and three-digit numbers by 9 and 99 and formulate a rule to predict results

determine the relationship between position of digits and powers of ten

use a computer to demonstrate such as 1, 2, 4, 8, 16 . . .

## Grade Six

The student will be able to ...

1. build simple functions using concrete models and generate a corresponding rule.
2. explore the relation between doubling the side of a square and/or other regular figures and the corresponding increase in area.
3. explore mathematical expressions of relations observed in other curricular domains.
4. explore and describe in words simple and complex patterns in history and language arts.

e.g., adding blocks to a rectangular pattern, preserving shape

particularly those involving direct and inverse variation

## Grade Seven

The student will be able to ...

1. describe and represent relationships with tables, graphs, rules, and words.
2. describe, extend, analyze, and create a wide variety of patterns.
3. explore and symbolize direct and inverse variation.
4. extend the investigation of number patterns.
5. generate ordered pairs with and without a calculator to graph linear equations.
6. explore absolute value in the context of distance between points.
7. explore and describe in words simple and complex patterns in industrial technology and science.

use materials such as newspapers and current magazines

verbal - both written and oral, concrete, pictorial, graphs, and algebraic expressions

relate to distance, rate, time, and other physical phenomena

e.g. Pascal's triangle and the Fibonacci sequence

use a computer graphing program when available.

## Grade Eight

The student will be able to ...

1. use patterns and keys on the calculator to extend the concept of inverse operations.
2. use invented and conventional symbols to explain a function relation.
3. explore the right triangle relations sine, cosine, and tangent and their applications in measurement.
4. explore the effect of multiplying the dimensions of a simple shape or solid by a constant factor and relate to the change in area or volume.
5. explore and describe in words simple and complex patterns in the environment.

addition/subtraction, multiplication/division, powers/roots, etc.

finding inaccessible distances, etc.

e.g., make enlargements of pattern block shapes using those pieces where possible

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 9—12

STANDARD 6:
FUNCTIONS

In grades 9—12, the mathematics curriculum should include the continued study of functions so that all students can—

- model real-world phenomena with a variety of functions;
- represent and analyze relationships using tables, verbal rules, equations, and graphs;
- translate among tabular, symbolic, and graphical representations of functions;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions;

and so that, in addition, college-intending students can—

- understand operations on, and the general properties and behavior of, classes of functions.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 154.

Grade Nine

The student will be able to ...

1. model real-world phenomena with polynomial and exponential functions. — population growth, interest, depreciation, rate-time -distance, etc.
2. explore the relationship between zeros and intercepts of functions.
3. translate among tables, algebraic expressions, and graphs of functions.
4. use a graphing calculator or computer to generate the graph of a function.
5. explore the relation between a linear function and its inverse. — algebraic expression and graph

In addition, college-intending students will be able to ...

6. describe the general characteristics of polynomial functions and use them in problem-solving situations.

Grade Ten

The student will be able to ...

1. explore the conic sections and graph using a graphing calculator or computer. — locus definitions translated into algebraic expressions, physical models ...
2. apply trigonometric functions to problem situations involving triangles. — using right triangle relationships and Law of Sines and Law of Cosines.

In addition, the college-intending student will be able to ...

3. discover general relationships between the algebraic description of a conic, the kind of conic, and special properties of that conic. — applications such as solar ovens, satellite dishes, etc.

## Grade Eleven

The student will be able to ...

using a graphing calculator or computer software

1. explore periodic real-world phenomena using the sine and cosine functions.
2. analyze the effects of parameter changes on graphs.

e.g., $y = af(x)$, $y = f(x+h)-k$, etc.

3. use a graphing calculator or computer to graph functions.

In addition, the college-intending student will be able to ...

4. develop an understanding of rational and transcendental functions.

predict the general shape of graphs from their equations

5. understand the connection between trigonometric and circular functions.
6. use circular functions to model periodic real-world functions.

introduce the use of a computer algebra system

7. solve trigonometric equations and verify trigonometric identities graphically and analytically.
8. understand the connections between trigonometric functions and polar coordinates, exponential functions, logarithmic functions, complex numbers, and series.

## Grade Twelve

The student will be able to ...

1. model real-world phenomena with a variety of functions.

In addition, college-intending students will be able to ...

2. graph using polar coordinates .
3. explore graphs in three dimensions.

use a graphing calculator or computer

4. explore functions of several variables.

use a computer graphing package

5. explore recursive functions using spreadsheets and/or programming languages.

## STRAND 2: PROBLEM-SOLVING STRATEGIES

The purpose of mathematics is to solve problems, and this activity must permeate all the strands of mathematics. This strand provides an organized introduction of problem-solving techniques or strategies. It must rely on other strands for the content addressed by these strategies. Problem solving can be a powerful connecting link between strands.

Students should be required to explain and validate their process of solution. Among other skills, they should be able to describe false starts and their reasons for starting over. Ideas need to be effectively communicated by words (oral and written), diagrams, pictures, actions, mathematical symbols, or any other appropriate method. Evaluation of problem solving in the school environment should focus on the process as well as the solution.

Leaders in industry remind us that their problem situations many times can be solved only by teams of responsible employees rather than individuals. We must provide an opportunity for students to simulate this process by having them work together to determine solutions, and then contrasting and comparing solutions with those developed by other teams.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES K—4

STANDARD 1:
MATHEMATICS AS PROBLEM SOLVING

In grades K—4, the study of mathematics should emphasize problem solving so that students can —

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from everyday and mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem;
- acquire confidence in using mathematics meaningfully.

*Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 23.

Kindergarten

The student will be able to ...

1. use invented and conventional symbols to describe a problem situation.
2. look for a pattern of objects to predict the solution to a problem.
3. act out a problem situation.

Grade One

The student will be able to ...

1. select appropriate notation and methods for symbolizing the problem statement and the solution process. — concrete objects, pictures, mathematical symbols
2. look for a pattern (objects or pictures) to predict a solution to a problem.
3. role play to find the solution to a problem. — act out the situation and the solution
4. make tables to sort information.

Grade Two

The student will be able to ...

1. identify needed information to solve a problem.
2. select appropriate notation and methods for symbolizing the problem statement and the solution process. — concrete objects, pictures, arithmetic phrases, verbal explanation
3. look for patterns of numbers to predict a solution.
4. role play to find the solution to a problem.
5. make pictographs and bar graphs to sort information. — compare with data analysis strand
6. make repeated guesses to find a solution and check until a solution is found.
7. explain in words why a solution is correct.

## Grade Three

The student will be able to ...

graphs, concrete objects, pictures, mathematical symbols, written narrative, oral explanation

1. select appropriate notation and methods for symbolizing the problem statement and the solution process.
2. look for a pattern to predict a solution.
3. role play or use appropriate materials to find the solution to a problem.

many published activities have problems appropriately solved by this process

4. make and use a Venn diagram to collect and sort information.
5. extend the guess-and-check procedure by recording guesses and checks to help make better guesses until the solution is reached.
6. make a drawing of the information in the problem to clarify relationships.

comparison of different group efforts will usually provide this - the teacher may need to be directive here.

7. use more than one strategy to solve a given problem.
8. use a single strategy to solve different kinds of problems.
9. identify which questions could be answered given certain information.

this can do double duty in both mathematics and language arts

10. develop a convincing written argument for the correctness of a solution.

## Grade Four

The student will be able to ...

these ten activities should be a matter of routine by this grade level

1. use more than one strategy to solve a given problem.
2. use a single strategy to solve different kinds of problems.
3. make and use a table to record and sort information.

4. select appropriate notation and methods for symbolizing the problem statement and the solution process.
5. extend the application of previously learned strategies.
6. validate solution(s) to a problem.
7. generalize a problem-solving situation to other cases.

generalization is the foundation of mathematics

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 5—8

STANDARD 1:
MATHEMATICS AS PROBLEM SOLVING

In grades 5—8, the mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can—

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from situations within and outside mathematics;
- develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;
- verify and interpret results with respect to the original problem situation;
- generalize solutions and strategies to new problem situations;
- acquire confidence in using mathematics meaningfully.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 75.

Grade Five

The student will be able to ...

1. read a problem carefully and restate it without reference to the original problem.
2. read a problem carefully and identify subgoals that need to be attained in order to solve the problem.
3. expand the repertoire of appropriate notations and methods for symbolizing a problem statement and the solution process.
4. extend the application of previously learned strategies.
5. validate and generalize solutions to problems.

this develops skill in identifying essential factors of a problem

draw a picture, create an arithmetic phrase, use words

Grade Six

The student will be able to ...

1. extend the application of previously learned strategies.
2. expand the repertoire of appropriate notations and methods for symbolizing a problem statement and the solution process.
3. identify needed and given information in a problem situation as well as irrelevant information.
4. validate and generalize solutions.

explore use of algebraic symbols in addition to previous methods

Grade Seven

The student will be able to ...

1. use an open sentence (algebraic equation) to symbolize a problem situation and solve the equation to find a solution to the problem.
2. validate solutions to problems in a variety of ways.

as well as previously learned methods

an extremely useful technique particularly in the case of simpler numbers

a characteristic of good problem solvers - students can be helped to do this

3. rephrase a problem as a simpler problem to find a method of solution.
4. extend the application of previously learned strategies.
5. identify problems that are similar in structure.

Grade Eight

The student will be able to ...

a year for consolidating and strengthening students' problem solving techniques


1. select appropriate notation and methods for symbolizing the problem statement and the solution process.
2. extend the application of previously learned strategies to a wide variety of problems.
3. validate and generalize problem solutions.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 9—12

STANDARD 1:
MATHEMATICS AS PROBLEM SOLVING

In grades 9—12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can—

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 137.

Grades 9 - 12

The student will be able to solve problems using the following strategies:

this comprises an extended list of strategies developed in the K-8 years

they need to be subjected to the more advanced objectives in the 9-12 curriculum

1. Select appropriate notation
   - select appropriate methods for symbolizing the problem statement and the solution process.
2. Look for a pattern
   - recognize and extend patterns using a variety of materials.
   - recognize and extend patterns using counting and number sequences.
   - verbalize the procedure for finding the nth term of a simple sequence involving one or more operations.
3. Act it out
   - role play to find the solution to a problem.
   - use appropriate materials to act out the problem.
4. Make a graph
   - use a Venn diagram to collect and sort information.
   - make and read graphs of various kinds.
5. Guess and check
   - make a reasonable guess at a solution.
   - check the guess by applying it to the problem conditions.
   - record guesses and checks to make an adjustment.
   - adjust the guess and make another guess until the solution is reached.
6. Make a drawing or diagram
   - make a drawing of the information in the problem to clarify the relationships among the given information.

7. Make a model
   - represent a problem situation using concrete materials.
   - manipulate materials to arrive at a solution to the problem.
   - record the solution process in an appropriate form.
8. Construct a table
   - record data and computations in an organized manner.
   - use the table to identify missing data that are needed to solve the problem.
   - use the table to recognize and extend patterns to reach a solution to the problem.
9. Restate the problem
   - read the problem once in order to become familiar with the problem and what it involved.
   - reread the problem (more than once if needed) to focus on actions and related details.
   - state the problem in different words without reference to the original problem.
   - discuss the problem with classmates to determine a strategy or solution process.
10. Identify a subgoal
   - recognize a problem which will need more than one step in order to reach a solution.
   - identify the steps that need to be taken in the solution process.
   - follow those steps to reach a solution.
11. Identify given, needed, and extraneous information
   - focus on the question to determine what the problem is asking.
   - determine the information that is needed to solve the problem.

- determine what additional information is needed in order to solve the problem.
- ignore extraneous information.
- select other strategies that may be helpful in the solution process.

12. Account for all possibilities
    - use a table or pattern to help account for all possibilities.
    - eliminate possible solutions that do not fit the problem conditions.
13. Write an open sentence
    - use other strategies that will help lead to an open sentence
    - identify the unknown and use a variable to represent it.
    - identify the mathematical operations necessary to solve the problem.
    - represent the relationship between given and wanted information with an algebraic equation.
14. Solve a simpler or similar problem
    - start with simpler numbers than those given in the problem.
    - if necessary, continue with additional numbers leading to those given in the problem.
    - use the simpler numbers to find a pattern to solve the original problem.
    - simplify the problem situation and use it to provide insight into the more complex problem.
15. Work backwards
    - recognize that it is possible to begin with final conditions of an action to determine initial conditions.
    - use inverse operations to solve a problem.

## STRAND 3: NUMBERS AND NUMBER RELATIONS

This strand encompasses the whole of students' formal experience with number. It includes the development of number sense, of understanding of and skill at computation with numbers of all kinds, and explorations of certain aspects of number theory. This formal experience with numbers needs to be related to the informal, out-of-school number experiences of students.

This is an important part, but not the whole, of the mathematics curriculum. The objectives of this strand should be carefully correlated with, as well as used in, other strands such as problem-solving, estimation and mental computation, and measurement. These objectives are the workhorses of mathematics.

It is the purpose of this strand to help children develop number sense, an understanding of computation as well as skill, and a recognition of the pervasive influence of number in all of our lives. This cannot be achieved with an approach to teaching that is mainly drill and practice, nor can it be achieved if children cannot rapidly and accurately work with numbers. There must be a balance between skill and understanding and a freedom to use innovative methods.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES K—4

STANDARD 6:
NUMBER SENSE AND NUMERATION

In grades K—4, the mathematics curriculum should include whole number concepts and skills so that students can—

- construct number meanings through real-world experiences and the use of physical materials;
- understand our numeration system by relating counting, grouping, and place value concepts;
- develop number sense;
- interpret the multiple uses of numbers encountered in the real world.

STANDARD 7:
CONCEPTS OF WHOLE NUMBER OPERATIONS

In grades K—4, the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that students can—

- develop meaning for the operations by modeling and discussing a rich variety of problem situations;
- relate the mathematical language and symbolism of operations to problem situations and informal language;
- recognize that a wide variety of problem structures can be represented by a single operation;
- develop operation sense.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, pp. 38 and 41.

Kindergarten

The student will be able to ...

1. relate numbers to numerals.
2. separate (decompose), join (combine), and order sets of objects.
3. compare number of objects in two or more sets.
4. count
   a. by rote
   b. objects (1-1 correspondence)

dried lima beans painted on one side form the basis of an excellent Math Their Way activity

one more, one less, two more, and two less

Grade One

The student will be able to ...

1. continue all of the kindergarten objectives.
2. decompose, combine, order, and compare numbers.
3. estimate the number prior to counting and count objects
   a. using ten frames
   b. by twos, fives, and tens
   c. using ordinal numbers
4. skip count by twos, fives, and tens and count backwards.
5. develop the concept of addition and subtraction from situations in the environment, including joining, separating, and comparing sets of objects.
6. learn strategies for addition of whole numbers such as
   a. counting all
   b. counting on
   c. one more, one less
   d. two more, two less
   e. doubles
   f. doubles plus or minus one and doubles plus or minus two
   g. make ten
   h. using ten frames

place objects in the spaces of

first object, second object, etc.
beginning with a variety of numbers.

$9 + 3 = 9 + (1 + 2) = (9 + 1) + 2 = 12$

7. describe the operations of addition and subtraction in words.

pictures, arrows, etc. as well as arithmetic phrases

8. use invented symbols and conventional symbols to represent the operations of addition and subtraction.

introduce as a natural part of one's vocabulary

9. orally identify fractional parts (halves, thirds, fourths) of whole objects or sets of objects.

## Grade Two

The student will be able to ...

base ten blocks, bunches of craft sticks or soda straws, chains of paper clips, etc.

1. develop the concept of place value with concrete models of hundreds, tens, and ones.

pair items of a set, "odd balls" have one left over

2. develop the concept of odd and even using concrete materials.

rectangular strips are better than circular representations

3. use concrete models of fractions (halves, thirds, fourths, sixths) to investigate different physical representations for the same fractional parts of whole objects or sets of objects.

halves, thirds, fourths, sixths

4. illustrate fractional parts of whole objects or sets of objects.

divide the segments of the number line between whole numbers as in 3

5. place whole numbers and fractions on the number line.

6. develop the concept of multiplication and division by joining equivalent sets of objects or separating objects into equivalent sets.

7. learn strategies for the addition and subtraction of numbers such as

17 - 9 is the same as 17 - 10 + 1

also called equal addends method

a. compatible numbers
b. compensatory numbers
c. borrow and pay back (subtraction)
d. regrouping
e. using a calculator

8. develop skill in estimating sums and differences.

9. translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics.

10. find equivalent forms of numbers using hundreds, tens, and ones.

123 = 100 + 20 + 3 or
1 x 100 + 2 x 10 + 3

## Grade Three

The student will be able to ...


1. add and subtract numbers fluently using any strategy.
2. relate addition and subtraction statements to each other.

timed tests are inappropriate, but there are activities that encourage students to develop fluency

3. from situations created in the classroom
   a. develop models of multiplication and division (arrays)
   b. use invented and conventional symbols to represent multiplication and division
   c. describe multiplication and division in words

4. translate real-life situations involving multiplication and division into conventional mathematical symbols.

5. relate multiplication to skip counting.
6. multiply using the pocket multiplier.

a folded paper device that models the multiplication process

7. multiply and divide using a calculator.
8. recall multiplication and division facts through 12 x 12 using strategies such as:
   a. commutative property
   b. distributive property
   c. anchor facts
   d. squares

it is not important for students to know these terms in this grade

9. multiply using paper-and-pencil algorithms.

10. use conventional symbols to represent fractions.

11. order fractions on the basis of concrete materials.
12. develop the concept of tenths and hundredths using models.

the same models used for ones, tens, and hundreds, but with the hundreds model not identified as the unit

this gives a visual comparison of fractions and decimals

13. order whole numbers, fractions, and decimals (tenths and hundredths) on the number line.
14. translate freely between words and symbols in naming numbers.

compare to objective 2 in Grade Two

15. relate even numbers to division by two.
16. use the symbols $<$, $\leq$, $>$, $\geq$, and $=$ in describing order as well as the terms "at least" and "at most."

## Grade Four

The student will be able to ...

arrange a set of objects into rectangle

1. decompose numbers into factors using objects and translate using symbols.
2. identify prime and composite numbers.

also use diagrams

3. use physical models to represent fractions greater than one.
4. round fractions to zero, one-half, one, etc.
5. round numbers to the nearest ten and hundred and beyond.
6. order fractions using symbols.
7. find equivalent fractions with concrete materials and symbolically.
8. add and subtract fractions with concrete materials and symbolically.
9. develop concepts of tenths and hundredths using symbols.
10. add and subtract decimals.

two digit multipliers and one digit divisors

11. multiply and divide whole numbers fluently.
12. develop concepts of place value to include numbers through millions.
13. compare and order numbers with any number of digits.

families of facts

14. relate multiplication and division statements to each other.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 5—8

STANDARD 5:
NUMBER AND NUMBER RELATIONSHIPS

In grades 5—8, the mathematics curriculum should include the continuous development of number and number relationships so that students can—

- understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and scientific notation) in real-world and mathematical problem situations;
- develop number sense for whole numbers, fractions, decimals, integers, and rational numbers;
- understand and apply ratios, proportions, and percents in a wide variety of situations;
- investigate relationships among fractions, decimals, and percents;
- represent numerical relationships in one- and two-dimensional graphs.

STANDARD 6:
NUMBER SYSTEMS AND NUMBER THEORY

In grades 5—8, the mathematics curriculum should include the study of number systems and number theory so that students can—

- understand and appreciate the need for numbers beyond the whole numbers;
- develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers;
- extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers;
- understand how the basic arithmetic operations are related to one another;
- develop and apply number theory concepts (e.g., primes, factors, and multiples) in real-world and mathematical problem situations.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, pp. 87 and 91.

## Grade Five

The student will be able to ...

1. decompose numbers into factors, including prime factored form.
2. use the long division algorithm.
3. multiply and divide decimals.
4. find equivalent fractions.
5. add and subtract fractions.
6. order combinations of whole numbers, fractions, and decimals using the symbols $<$, $\leq$, $>$, $\geq$, and $=$ and by placing them on the number line.
7. explore order of operations relative to calculators with arithmetic and algebraic logic.
8. explore and use the idea of ratio relative to scaling.
9. explore the idea of squares and square root in the context of area of squares.
10. round, as appropriate to a problem situation, to the nearest thousand, hundred, ten, one, tenth, or hundredth.
11. explain in words the role of 0 and 1 as identity elements for addition and multiplication, respectively.
12. explain in words why order does not make a difference for addition and multiplication, but does for subtraction and division.

maximum of two-digit divisors relate to rules for fractions

An important skill, since both kinds of calculators are widely used.

relate to $a^2$ being read as a-squared

This implies rounding down or up.

## Grade Six

The student will be able to ...

1. compute with whole numbers, fractions, and decimals.
2. explore concepts of percent, ratio, and proportions in the context of real-world situations.
3. use proportions in a wide variety of applications.

Such as similarity of geometric figures and probability

4. investigate relations between ratios, proportions, and percents
5. round, as appropriate to a problem situation, to any digit.
6. change freely between fractions and decimals.
7. understand and describe in words the relations between addition, subtraction, multiplication, and division.
8. understand and describe in words how fractions and decimals expand the whole number system to the system of non-negative rational numbers.
9. be able to find a number between any two rational numbers. — for example, between $\frac{3}{5}$ and $\frac{4}{5}$
10. explore and explain when order does and does not make a difference for the four fundamental operations.
11. explore Roman numerals and contrast with the base ten number system. — Roman numerals are still frequently used to number pages.

Grade Seven

The student will be able to ...

1. represent percent by proportions and algebraic equations and solve for missing terms. — This use of variables makes applications of percent easier.
2. solve problems and make applications involving percent.
3. solve and use proportions. — Simple and compound interest, percent increase and decrease, radioactive decay, etc.
4. develop the concept of integers using concrete models, including number lines, and in the context of real-world situations.
5. compare, order, and determine the equivalence of whole numbers, fractions, decimals, percents, and integers.
6. expand understanding of place value to include bases other than ten. — particularly bases used in computer science
7. find square root using a calculator.

a good lab activity

8. explore the concept of pi by comparing the measure of the diameter and circumference of circles.
9. explore interpretations of addition and multiplication that are different for whole numbers and fractions.
10. develop and apply theories about primes, factors, and multiples in real-world and mathematical problem situations.

Students need to recognize when a calculator shifts into scientific notation.

11. explore powers and scientific notation as alternate ways of writing numbers and in the context of calculators.

## Grade Eight

The student will be able to ...

fraction, decimal, percent, exponential, prime factored form, and scientific notation

1. understand, represent, and use numbers written in a variety of equivalent forms in real-world and mathematical problem-solving situations.

Colored tiles or chips where one color represents positive numbers and another color represents negative numbers make useful models.

2. develop an understanding of operations with integers using the number line and other models of integers.
3. represent numerical relationships in one- and two-dimensional graphs.

ideas of closure and identity elements are important

4. understand the real number system and describe it in words.

use diagonals of rectangles

5. construct segments to represent irrational numbers such as the square roots of 2, 3, 5, etc.
6. locate certain irrational numbers on the number line and find an irrational number between any two given numbers.
7. understand and describe in words how the negative numbers expand the non-negative rationals to the rational number system.

8. give a meaningful explanation for the impossibility of division by zero.
9. explore interesting topics such as abundant, deficient, or perfect numbers; triangular and square numbers; cubes; palindromes; factorials; and Fibonacci numbers.
10. solve problems and make applications involving percentages.

An important opportunity for all students.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 9—12

STANDARD 4:
MATHEMATICAL STRUCTURE

In grades 9—12, the mathematical curriculum should include the study of mathematical structure so that all students can—

- compare and contrast the real number system and its various sub-systems with regard to their structural characteristics;
- understand the logic of algebraic procedures;
- appreciate that seemingly different mathematical systems may be essentially the same;

and so that, in addition, college-intending students can—

- develop the complex number system and demonstrate facility with its operations;
- prove elementary theorems within various mathematical structures, such as groups and fields;
- develop an understanding of the nature and purpose of axiomatic systems.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 184.

Grade 9-12

The student will be able to ...

1. compare, order, and determine equivalence of real numbers.
2. estimate answers, compute, and solve problems involving real numbers.

and, in addition, college-intending students will ...

3. compare and contrast the real number system, the rational number system, and the whole number system.
4. extend their understanding to the complex number system and develop facility with its operations.

# STRAND 4: GEOMETRY

Geometry is an important component of the mathematics curriculum at every level. It helps in exploring, describing, and understanding the world in which we live. This strand includes objectives that require students to investigate, experiment, and explore with physical materials, drawings, and computers. Students also communicate the results of their investigations with verbal descriptions or drawings. The K—8 part of the strand is designed to help children develop spatial sense and develop facility with the language of geometry before the more formal study of geometry takes place at the secondary level. Something more than a superficial study of geometric shapes and terminology is essential from the earliest school experience.

The strand begins with recognition of whole shapes in the classroom and in the environment and proceeds through analysis of the properties of shapes and their transformations to the expression of geometric properties algebraically. While informal proof or validation is used in the early grades, more formal proof is delayed until the high school grades.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES K—4

STANDARD 9:
GEOMETRY AND SPATIAL SENSE

In grades K—4, the mathematics curriculum should include two- and three-dimensional geometry so that students can—

- describe, model, draw, and classify shapes;
- investigate and predict the results of combining, subdividing, and changing shapes;
- develop spatial sense;
- relate geometric ideas to number and measurement ideas;
- recognize and appreciate geometry in their world.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 48.

## Kindergarten

The student will be able to ...

1. identify common shapes in the environment. — such as triangles, rectangles, circles
2. manipulate, color, fold, and create simple geometric shapes.
3. use a computer program, such as Logo (appropriately modified), to explore paths and construct simple geometric shapes.

4. identify common solids in the environment. — such as spheres, cylinders, cubes, etc.

## Grade One

The student will be able to ...

1. continue all of the kindergarten objectives with more detail, greater depth, and with different materials.
2. combine shapes to form a given shape. — e.g., cover shapes with tangram pieces or pattern blocks

3. identify two-dimensional shapes on three-dimensional objects. — rectangles as the faces of a retangular solid, etc.
4. compare two-dimensional shapes describing similarities and differences using appropriate standard and non-standard language.
5. explore situations by manipulating shapes, measuring, and counting.
6. use mathematically correct names of pattern block pieces. — hexagon, trapazoid, triangle, rhombus, square

## Grade Two

The student will be able to ...

1. investigate congruence and symmetry. — use models, drawings, geoboards, computer graphics, and miras

2. investigate perimeter using concrete models.

3. explore paths, simple closed curves, and the ideas of interior and exterior.
4. explore tangram and pattern block puzzles.

compare a cube, rectangle, solid, cylinder, and sphere

5. compare three-dimensional objects describing similarities and differences using appropriate standard and non-standard language.

Grade Three

The student will be able to meet any previous objective, and in addition ...

1. explore properties of geometric figures and relationships by measuring, coloring, folding, cutting, making models, and using tiles and geoboards.
2. investigate angles using models, paper folding, drawings, and computer graphics. 
3. build a solid to match a given solid using cubes.
4. describe a three-dimensional object from different perspectives. 
5. investigate area by covering regions with standard and non-standard units. 

face, vertex, etc.

6. use mathematically correct names for common geometric figures. 

e.g., the number of squares on a checkerboard

7. identify and count common overlapping figures in the environment. 

Grade Four

The student will be able to ...

1. investigate reflection, rotations, and translations of geometric figures using concrete objects.
2. identify parallel lines, perpendicular lines, and right angles in geometric figures and the environment. 

3. build models that illustrate intersecting lines, parallel lines, perpendicular lines, and right angles.

also skew lines

4. determine properties of two-dimensional figures and compare shapes according to their characterizing properties.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 5—8

STANDARD 12:
GEOMETRY

In grades 5—8, the mathematics curriculum should include the study of the geometry of one, two, and three dimensions in a variety of situations so that students can—

- identify, describe, compare, and classify geometric figures;
- visualize and represent geometric figures with special attention to developing spatial sense;
- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- understand and apply geometric properties and relationships;
- develop an appreciation of geometry as a means of describing the physical world.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 112.

Grade Five

The student will be able to ...


1. compare and contrast angles in relation to right angles. — know acute and obtuse
2. construct circles with a given center and/or a given radius.

3. encounter and use appropriate vocabulary relative to circles. — arcs, chords, sectors, radius, diameter, $\pi$, etc.

4. build models of previously encountered shapes and figures and describe the process in words. — two and three dimensions

5. explore concepts of similarity by enlarging shapes with pattern blocks, geoboards, and computer graphics.
6. explore patterns that result from combinations of reflections, rotations, and translations of geometric figures.

Grade Six

The student will be able to ...


1. measure angles in geometric figures and explore relationships between angle measure and other characteristics of the figures. — supplementary, complementary, vertical, etc.
2. estimate the measure of angles and draw angles that approximate given measures.
3. identify and distinguish among similar, congruent, and symmetric figures.

4. visualize and show the results of a rotation, translation, reflection, or stretching . — sketch, use graph paper, paper folding, computer graphics. . .
5. build models of three-dimensional figures such as pyramids, cones, prisms with polygonal bases, and investigate the properties associated with those figures.

6. explore properties that can be used to characterize or contrast different classes of figures.

a square is a rectangle, is a rhombus, is a parallelogram, is a quadrilateral...

7. recognize, classify, and use characteristics of lines and simple two-dimensional figures.

## Grade Seven

The student will be able to ...

inclusion is an important relationship

1. explore and verbalize relationships between different kinds of figures.

2. explore and describe procedures for changing one figure or shape to another .
3. develop minimum sets of properties that describe a geometric figure.
4. develop definitions of common geometric figures.
5. build the model of a figure given top, side, and front views.

congruent triangles, sum of angles in triangle, etc.

6. validate fundamental geometric theorems using manipulative materials and informal arguments.

7. visualize and describe the results of folding geometric figures.

8. use separation of rectangles as an area model for the distributive property.

## Grade Eight

The student will be able to ...

relate to ratio and proportion

1. calculate missing measurements of similar figures.

2. investigate the relationships between angles formed when parallel lines are cut by a transversal, using diagrams and computer graphics.
3. sketch three-dimensional figures from different perspectives.

4. graph similar figures, reflections, translations, and rotations on a co-ordinate plane.
5. explore linear relationships graphically using graph paper and computer or calculator graphics.
6. extend experiences validating fundamental geometric theorems.

use the geometric supposers


7. find perimeters (circumferences) and areas of polygons (circles)
8. explore uses of the Pythagorean theorem.


9. find surface areas and volumes of rectangular solids.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 9—12

### STANDARD 7
### GEOMETRY FROM A SYNTHETIC PERSPECTIVE

In grades 9—12, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can —

- interpret and draw three-dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships;
- deduce properties of, and relationships between, figures from given assumptions;

and so that, in addition, college-intending students can—

- develop an understanding of an axiomatic system through investigating and comparing various geometries.

### STANDARD 8
### GEOMETRY FROM AN ALGEBRAIC PERSPECTIVE

In grades 9—12, the mathematics curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that all students can—

- translate between synthetic and coordinate representations;
- deduce properties of figures using transformations and using coordinates;
- identify congruent and similar figures using transformations;
- analyze properties of Euclidean transformations and relate translations to vectors;

and so that, in addition, college-intending students can—

- deduce properties of figures using vectors;
- apply transformations, coordinates, and vectors in problem solving.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, pp. 157 and 161.

Grade Nine

The student will be able to ...

1. create and interpret drawings of three-dimensional objects.
2. represent problem situations with geometric models and apply properties of figures.
3. describe and apply the properties of similar and congruent figures.
4. apply the Pythagorean theorem.
5. demonstrate an understanding of angles and parallel and perpendicular lines.

Grade Ten

The student will be able to ...

1. explore inductive and deductive reasoning through applications to various subject areas.
2. translate between synthetic and coordinate representations.
3. identify congruent and similar figures using transformations with computer programs.
4. deduce properties of figures using transformations and using coordinates.
5. use deductive reasoning.
6. explore compass and straight edge constructions in the context of geometric theorems.

and in addition, college-intending students will ...

7. demonstrate an understanding of and ability to use proof.
8. use a variety of techniques of proof, such as synthetic, transformational, and coordinate proofs.

including understanding terms such as undefined term, postulate, theorem, definition, hypothesis, conclusion, and necessary and sufficient conditions.

9. use a variety of proof formats, including the T- (or two-column) proof and the paragraph proof.
10. explore different strategies of proof.
11. investigate different proofs of theorems.

use axioms for a finite geometry

12. develop an understanding of an axiomatic system.

Grade Eleven

The student will be able to ...

1. apply transformations and coordinates in problem solving.
2. represent problem situations with geometric models and apply properties of figures.

and, in addition, college-intending students will ...

3. deduce properties of figures using vectors.
4. analyze properties of Euclidian transformations and relate translations to vectors.
5. apply vectors in problem solving.
6. further develop understanding of axiomatic systems by investigating and comparing various geometries.

## STRAND 5: ALGEBRA

All students need algebra. Algebra is critical to success in school mathematics for two primary reasons:

1. Algebraic thinking serves to consolidate arithmetic skills and understandings.
2. Algebraic thinking equips students with the capability to succeed in mathematics beyond elementary school.

Algebraic thinking is a component of most mathematics used beyond secondary school.

Success in algebra is too important to be left until middle or secondary school experiences. Patterns of thinking must be introduced in intuitively familiar numerical settings before children encounter their first formal course in algebra. Students should build basic algebraic ideas and skills by making generalizations about numerical problem situations.

This strand provides a developmentally sensible approach that features use of variables in describing dependency relationships, highlights attention to graphs and their interpretations, and promotes key ideas from a base of problem-solving experiences. A focus on moving among different representations of algebraic ideas establishes a base for featuring communication and connections in the classroom as well as providing for a sounder approach to learning.

Students are expected to learn how to use the technology of modern industry and of learning institutions beyond the secondary level. Calculators with algebraic operating systems should be introduced early. It should be remembered that the curriculum works better if students have access to graphing calculators and computers for secondary school mathematics. The use of technology provides for more efficient and better learning of algebraic concepts in addition to building necessary skills and understandings for life beyond school.

Kindergarten

The student will be able to ...

| | |
|---|---|
| 1. use and understand the language of logic in talking about problem situations and solutions to problems. | if, then, since, because, etc. |

Grade One

The student will be able to ...

| | |
|---|---|
| 1. use and understand the language of logic in describing problem situations and solutions to problems. | if, then, since, because,not, all, some... |
| 2. use invented symbols and letters to describe the joining, separating, and comparing of bunches of objects. | where a + b = 5 3 + c = d given a find b, given c find d, or find related values for a,b, c,d |
| 3. model a problem situation using a number phrase or sentence. | |

Grade Two

The student will be able to ...

| | |
|---|---|
| 1. explain in words thinking strategies for making computations. | addition and subtraction |
| 2. understand the use of letters in statements such as a-b = 6 and find a or b when one is given. | discuss constraints on values for a and b. |
| 3. model a problem situation using numbers and/or letters. | |

Grade Three

The student will be able to ...

| | |
|---|---|
| 1. explain in words thinking strategies for making computations. | multiplication and division as well as addition and subtraction |
| 2. explore calculator keys other than ⊟ that perform an operation. | e.g., enter 2 + 3 and determine which keys will produce 5. |
| 3. understand the use of letters in statements such as ab=12 or 3c=d and find a when b is given, etc. | discuss constraints on values for a,b,c,d |

## Grade Four

The student will be able to ...

if, then, since, because, not, all, some,...

1. use and understand the language of logic in describing the inverse relationships between addition and subtraction, multiplication and division.

variable used as a number that is not known yet.

2. use variables in mathematical expressions to represent problem situations.

statement: 3 + 4 x 5
key press : 3 [+] 4 [X] 5 [=]
display: 3 3 4 7 5 35

3. symbolize a keying sequence on a calculator with arithmetic logic to execute the computation of an arithmetic phrase and predict the display as each key is pressed

variable used to hold the place for a number that may be changed.

4. explore using variables to generalize arithmetic statements

discuss constraints on a,b,c,d for each statement.

5. explore tables that describe arithmetic relationships.
6. understand the use of letters in statements such as a/b = 4 or c/2 = d and find a when b is given, etc.

*from the NCTM Standardss:*

## CURRICULUM STANDARDS FOR GRADES 5—8

STANDARD 9:
ALGEBRA

In grades 5—8, the mathematics curriculum should include exploration of algebraic concepts and processes so that students can—

- understand the concepts of variable, expression, and equation;
- represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships;
- develop confidence in solving linear equations using concrete, informal, and formal methods;
- investigate inequalities and nonlinear equations informally;
- apply algebraic methods to solve a variety of real-world and mathematical problems.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 102.

## Grade Five

The student will be able to ...

phrase: 3 + 4 x 5
keys: 3 [+] 4 [x] 5 [=]
display: 3 3 4 4 5 23

1. symbolize a keying sequence on a calculator with algebraic logic to execute the computation of an arithmetic phrase and predict the display as each key is pressed.

an introduction to order of operations

2. explain in words differences between calculators with arithmetic logic and algebraic logic.
3. use variables to describe arithmetic processes.
4. interpret tables that describe problem situations.

## Grade Six

The student will be able to ...

a more formal use of the distributive property

1. use the distributive property in arithmetic computations.
2. construct tables to describe a problem situation.
3. use a variable to describe a generalization from a problem situation.
4. symbolize, using variables, the relations between addition, subtraction, multiplication, and division.
5. explore the use of parentheses on a calculator to change results of a computation.
6. solve linear equations using concrete representations.

## Grade Seven

The student will be able to ...

1. use parentheses accurately to group numbers for applying operations.
2. apply formulas to problem situations.

3. describe problem situations involving ratios, proportions, and percents with algebraic expressions.
4. solve linear equations with one variable by working backward (relate to inverse operations).
5. evaluate algebraic expressions (simple substitutions). — include powers, parentheses, quotients, etc.
6. interpret graphs of problem situations describing linear, quadratic, and exponential relationships.
7. construct graphs describing problem situations and assign and label scales to axes of graphs appropriately.
8. relate ratio and proportion concepts to variation situations, direct and inverse.

Grade Eight

The student will be able to ...

1. use and relate tables, graphs, and equations to solve problem situations involving exponential growth and decay, simple interest, and compound interest. — use a spreadsheet to model these situations, or create tables using a calculator
2. explain in words the meaning of the expression "solution of an equation." — distinguish between solution as a process and solution as the number that makes an equation true
3. investigate solutions to pairs of simultaneous equations.
4. solve linear inequalities in one variable. — including integers in problem statement/solution
5. interpret problem situations described by linear inequalities in words and graphically.
6. find the distance between two points in the coordinate plane. — relate to Pythagorean theorem
7. explore and interpret the concepts of slope and intercept as characteristics of linear functions. — relate to pitch of roof, grade of highway, etc.
8. describe and solve variation situations, direct and inverse, algebraically and graphically.

another aspect of the distributive property

computer, calculator, paper and pencil, or mental arithmetic/ estimation techniques.

9. factor mathematical expressions involving a common factor.
10. select appropriate tools and/or techniques for computation..
11. solve simple number sentences and use formulas.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 9—12

STANDARD 5:
ALGEBRA

In grades 9—12, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can—

- represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
- use tables and graphs as tools to interpret expressions, equations, and inequalities;
- operate on expressions and matrices, and solve equations and inequalities;
- appreciate the power of mathematical abstraction and symbolism;

and so that, in addition, college-intending students can—

- use matrices to solve linear systems;
- demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 150.

## Grade Nine

The student will be able to ...

1. describe problem situations by using and relating numerical, symbolic, and graphical representations.
2. use the language and notation of functions in symbolic and graphing settings.

relate to factors of an expression when possible

3. recognize, relate, and use the equivalent ideas of zeros of a function, roots of an equation, and solution of an equation in terms of graphical and symbolic representations.
4. describe and use the logic of equivalence in working with equations, inequalities, and functions.

including systems of equations, the intermediate value property, finding zeros of functions. . .

5. develop graphical techniques of solution for problem situations involving functions.

polynomial, exponential, rational, ...

6. explore and describe characterizing features of functions.
7. make arguments and proofs in algebraic settings.

use in mental computations
49 x 51 = (50-1) (50+1)
= 2500-1
= 2499

8. factor the difference of two squares.
9. determine slope, midpoint, and distance.

and, in addition, college-intending students will ...

discuss restrictions on domain and range.

10. explore and combine rational functions.

such as grouping, difference of two squares, trinomials, and sums and differences of cubes.

11. explore factoring techniques.
12. solve quadratic equations by factoring and formula.
13. set up and solve linear equations.
14. solve systems of linear equations with two variables.

Grade Ten

The student will be able to ...

1. describe geometric situations and phenomena using variables, equations, and functions.
2. describe measures of central tendency, mean, median, mode, and variance, algebraically and graphically.
3. represent inequalities on the number line and in the coordinate plane.

use a computer statistical package, a spreadsheet, or a calculator with statistical functions.

and, in addition, college-intending students will ...

4. use coordinate arguments in making geometric proofs.
5. symbolize transformations of figures and graphs.
6. explore the geometric basis for the functions of trigonometry.
7. graph linear functions.

Grade Eleven

The student will be able to ...

1. develop and use vectors to represent direction and magnitude including operations.
2. use polar and parametric equations to describe, graph, and solve problem situations.
3. represent sequences and series as functions both algebraically and graphically.
4. explore recursive functions and procedures using spread sheets and other computer utilities and use notations appropriate to these problem situations.
5. describe and solve algebraic situations with matrices.

use computer packages

compare Strand 1, Grade 11, Objective 5

use a computer program or a calculator that manipulates matrices

graphical representatives

6. describe and use the inverse relationship between functions including exponential and logarithmic.
7. analyze and describe the errors and sources of error that can be made when using computers and calculators to solve problems.

this means a student must be knowledgeable about all of these

8. decide when a problem situation is best solved using a computer, calculator, paper and pencil, or mental arithmetic/estimation techniques.

and, in addition, college-intending students will ...

9. explore relationships between complex numbers and vectors.
10. make arguments concerning limits, convergence and divergence in contexts involving sequences, series, and other types of functions.
11. represent transformations in the plane with matrices.
12. contrast and compare the algebras of the rational, real, and complex numbers with characteristics of a matrix algebra system.

Taylor's services, for example

13. construct polynomial approximations of a function over specified intervals of convergence.
14. examine complex numbers as zeros of functions.
15. translate verbal statements into symbolic language.
16. simplify algebraic expressions.
17. use the laws and exponents (including scientific notation).

Grade Twelve

The college-intending student will be able to ...

1. expand and extend the idea of vectors and linear algebra to higher dimensional situations.

2. use the idea of independent basis elements for a vector space and associated fundamental concepts of finite dimensional linear algebra. relate to matrices
3. develop and communicate arguments about limit situations.
4. use matrices to describe and apply transformations.
5. develop and use polar and parametric equations to represent problem situations.
6. explore proofs by mathematical induction.



75

# STRAND 6: MEASUREMENT

In establishing a firm foundation in the basic underlying concepts and skills of measurement, students need to understand the attribute to be measured as well as what it means to measure. It is recommended that students experience a variety of activities that focus on comparing objects directly, comparing them with various units, and counting the units. It is crucial that they understand the process of measuring as well as gain an awareness of the sizes of units. Decisions can be made about the sizes of objects by looking at, feeling, or comparing objects directly. These experiences provide opportunities to build appropriate vocabulary associated with measurement in a natural way rather than in a contrived, artificial manner. Special attention should be given to measurement of real objects. Looking at pictures in textbooks cannot substitute for measuring real objects. Children learn to measure by measuring. Activities should also include everyday situations which require only an estimate. For example, they need to recognize that the driving time from Dayton to Columbus is about an hour or that the cost of a movie is between four and five dollars.

Since students encounter measurement ideas both in and out of school, well-chosen measurement activities should provide for a dynamic interaction between students and their environment. Ideally, the study of measurement should show the usefulness and practical applications of mathematics. In addition, the students' need to communicate about various measurement activities should highlight the importance of standard units and common measurement systems. The measurement concepts that were introduced and taught in grades K—4 should be reinforced and extended in grades 5—8. Any mathematics curriculum should focus on teaching for understanding and not overemphasize the rote, mechanical memorization of a select number of meaningless formulas. According to Brownell's meaning theory, students must understand what they are learning if learning is to be permanent. One way students develop an understanding of measurement is to use measuring tools and to measure real-life objects. This theory supports students' working with many manipulative materials as they perform these measurement activities.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES K—4

STANDARD 10:
MEASUREMENT

In grades K—4, the mathematics curriculum should include measurement so that students can—

- understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;
- develop the process of measuring and concepts related to units of measurement;
- make and use estimates of measurement;
- make and use measurements in problem and everyday situations.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 51.

Kindergarten

The student will be able to ...

| | |
|---|---|
| 1 compare and order objects of different lengths, weights, and capacities. | Use pencils, hands, heights of children, etc. A crude balance can be made from a coat hanger. |
| 2. model the results of joining or separating lengths, weights, and capacities. | Fill different size cans with blocks, water, beans, etc. |
| 3. order events based on time. | |
| 4. count pennies and nickels to 10 cents. | use real coins, not pictures |

Grade One:

The student will be able to ...

1. recognize and count a collection of coins including pennies, nickels, dimes, and a quarter and determine its value.

2. measure lengths using nonstandard units, centimeters, and inches. — Frequent units are paper clips, hands, blocks, etc.
3. indicate the approximate size of the units, inch and centimeter, without using a ruler. — The units for base ten blocks are cubic centimeters and some blocks are inch cubes.

4. tell time to the hour and the half-hour.

5. explore capacity using cups, pints, quarts, and liters and recognize these measuring units. — Liter (or two liter) soda bottles are helpful here in comparing.
6. explore weight using pounds and kilograms and recognize these measuring units. — A familiar weight should be a pound of butter or margarine. Verify the saying, "A pint is a pound."

Grade Two

The student will be able to ...

1. explore length, capacity, and weight by selecting and using appropriate metric and conventional units such as centimeter, inch, liter, cup, pint, quart, kilogram, and pound. — Exploration should include comparisons between and within systems

2. count collections of coins including pennies, nickels, dimes, quarters, and half dollars and compare values.

Relate five minute intervals to skip counting by five.

3. tell time to the nearest five-minute interval on digital and dial timepieces.
4. relate time on dial and digital timepieces.

## Grade Three

The student will be able to ...

Notice that smaller units are introduced. Relationships need to be established.

1. continue explorations of length, capacity, and weight, and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce, and fractional parts of each.
2. count collections of coins and bills which include one, five, and ten dollar bills and compare values.

Maintain a daily log of temperatures and determine the differences from day to day.

3. explore common temperatures using both Fahrenheit and Celsius scales.
4. read time on digital and dial timepieces and determine amount of time elapsed.

Meter sticks and yard sticks should be readily available.

5. illustrate the approximate size of units (inch, centimeter, meter, and yard).

Using different tools for these measurements will help the children remember the difference between perimeter and area.

6. use string, tiles, and blocks to explore perimeter, area, and volume .
7. make change using coins.

## Grade Four

The student will be able to ...

1. choose an appropriate unit and measure lengths, capacities, and weights in U.S. standard and metric units.

2. measure and determine perimeters and areas of simple straight line figures and regions without using formulas.

Review the use of string to measure. Children may discover that using a ruler is more accurate.

3. make reasonable estimates of lengths under 10 meters.
4. read temperature measurements in °C and °F.

Children should be able to determine the scale on the thermometer.

5. relate the number of units that measure an object to the size of the unit as well as to the size of the object.

Explore length, capacity, and weight of an object with different size units.

6. make change using coins and bills.
7. use mental, paper-and-pencil, and physical strategies to determine time elapsed.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 5—8

STANDARD 13:
MEASUREMENT

In grades 5—8, the mathematics curriculum should include extensive concrete experiences using measurement so that students can—

- extend their understanding of the process of measurement;
- estimate, make, and use measurements to describe and compare phenomena;
- select appropriate units and tools to measure to the degree of accuracy required in a particular situation;
- understand the structure and use of systems of measurement;
- extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass;
- develop the concepts of rates and other derived and indirect measurements;
- develop formulas and procedures for determining measures to solve problems.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 116.

## Grade Five

The student will be able to meet any previous objective, and, in addition ...

1. choose an appropriate unit and measure lengths, widths, or capacities to a specified degree of precision in U.S.standard or metric measurement.
2. convert, compare, and compute with common units of measure within the same measurement system.
3. determine what to measure and measure in order to determine perimeters, areas, and volumes of simple shapes and solids.
4. make reasonable estimates of lengths, weights, and capacities.

Projects that involve measuring large regions such as playgrounds as well as small objects such as paper clips help make mathematics meaningful.

Notice that this is a more formal approach to these measurements, but does not presume use of formulas.

## Grade Six

The student will be able to meet any previous objective and, in addition ...

1. select and use appropriate units and devices to measure length, area, volume, and weight.
2. explore and use formulas to compute areas and perimeters (circumferences) of common polygons (polygonal regions) and circles (circular regions).
3. convert, compare, and compute with common units of measure within the same measurement system.
4. measure angles using a protractor.

A wide variety of projects of use around the school or home can bring meaning to these concepts.

Computation with denominate numbers is an important skill.

## Grade Seven

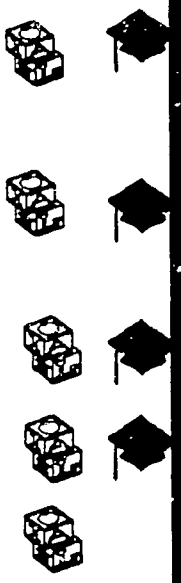
The student will be able to meet any previous objective and, in addition ...

All of these objectives can best be realized in the framework of interdisciplinary activities.

include angle measurement

Use in an application situation such as planning the amount of carpet to buy.

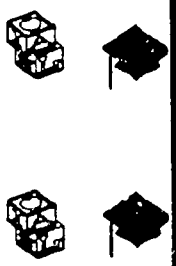
1. select and compute with appropriate standard or metric units to measure length, area, volume, weight, capacity, time, money, and temperature
2. make appropriate judgments regarding accuracy and precision.
3. make reasonable estimates of measurements.
4. state and apply area formulas for the following regions: circular, rectangular, parallelogram, trapezoidal, and triangular.
5. apply volume formulas for the following: prisms, cylinders, and spheres.
6. determine formulas for surface area.
7. explore measurement of time relative to time zones.
8. determine what to measure and measure to calculate perimeters, areas, and volumes.



Grade Eight

The student will be able to meet any previous objective and, in addition ...

1. measure and compute perimeter for irregular polygonal figures.
2. compute area for regular polygonal regions, other composite figures, and lattice (geoboard) figures.
3. make appropriate measurements and compute volume of solids such as prisms, cylinders, pyramids, and cones.
4. recognize and use the concepts of significant digits and accuracy in measurements.
5. recognize and use the concepts of precision and relative error in measurements.



6. successfully utilize a ruler and protractor for specific measurement tasks.
7. read a scale on a measurement device to the nearest mark and make interpolations where appropriate.
8. make change using the process of addition.

The protractor should be used in relation to an understanding of acute and obtuse angles.

The thrust of the measurement strand at the 9—12 level is to apply the competencies in the context of other disciplines. Students, at these levels, will be pursuing studies in the traditional academic programs and courses (e.g., mathematics, physics, physical science, chemistry, biology, geology, earth science, and others) as well as in vocational education trades (e.g., carpentry, automotive service, horticulture, agriculture, electronics) and related business education programs. It is critical to integrate measurement competencies within courses of study in the sciences (social, biological, and physical-environmental) and vocational education, including the world of work. Consequently, there are specific competencies for these areas of concentration.

Grades 9—12:

The students will be able to ...

| | |
|---|---|
| 1. estimate and use measurements | capacity, length, area including surface area of solids, and volume. |
| 2. understand the need for measurement and the probability that any measurement is accurate to some designated specification. | |
| 3. understand and apply measurements related to power and work | e.g., watt, joule, kilowatt, horsepower. |
| 4. understand and apply measurement concepts of distance-rate-time problems and acceleration problems only with real-world experiments. | |
| 5. use real experiments to understand and solve reaction-time situations. | |
| 6. use real experiments to investigate areas of elasticity, heat, sound, electricity, magnetism, light, acceleration (e.g., falling bodies, force), velocity, energy , and gravity. | charge, electric field, rotation |
| 7. use real-world problem situations involving mass and weight. | |
| 8. use real-world problem situations involving simple harmonic motion. | e.g., velocity, acceleration, force, period |
| 9. establish ratios with and without common units. | |
| 10. construct and interpret maps, tables, charts, and graphs as they relate to real-world mathematics. | |
| 11. understand and solve rate-change problems. | e.g., mass over specified intervals of time, height changes over a given interval of time; kilometers per hour and miles per hour) |
| 12. understand and solve right triangle relationships as they relate to measurement—specifically those that deal with the Pythagorean Theorem. | |
| 13. graph and interpret ordered pairs. | e.g., longitude, latitude, changes in temperature, application of Pythagorean distance formula, "taxicab distances." |

e.g., computing the total cost of X items at $Y and T items at $S

e.g., simple interest, compound interest, inflation, decay of elements, amount of light or sound through filters of different thicknesses, population growth

e.g., home mortgages, car payments—why do auto sales persons emphasize monthly payments rather than the total cost, loan payments, life insurance—term vs. whole life.

growth of forests

e.g., nominal, ordinal, ratio, and interval

14. compute total sales from a variety of items.
15. comprehend and compute rates of growth or decay.
16. comprehend, compute, and interpret real problems involving annuities.
17. develop an ability to identify real problems and provide possible solutions to these problems.
18. express and apply different types of measurement scales.
19. determine area and volume.

## STRAND 7: ESTIMATION AND MENTAL COMPUTATION

Estimation is a skill useful in the daily life of children as well as adults. Their study of mathematics should include specific instruction in ways to estimate, when it is appropriate to estimate, and how close an estimate is reasonable. Estimates are important when determining whether the result of a computation or the answer to a problem is reasonable. Frequently, an estimate is more appropriate than an exact answer.

.Estimates are important both in computation and in measurement. There is some overlap in the objectives included in this strand and in the measurement strand. A single lesson might include objectives from each of these strands.

Practice in mental computation provides children an opportunity to explore different combinations of numbers and alternative strategies for working with numbers. It is an important incentive for internalizing work with manipulatives.

A combination of estimation and mental computation helps children become more flexible in applying computation skills and better able to select a method of computation most appropriate for a given situation.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR K—4

STANDARD 5:
ESTIMATION

In grades K—4, the curriculum should include estimation so students can—

- explore estimation strategies;
- recognize when an estimate is appropriate;
- determine the reasonableness of results;
- apply estimation in working with quantities, measurement, computation, and problem solving.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 36.

Kindergarten

The student will be able to ...

1. estimate the number of items in a set by comparing to a set where the number of items is known.
2. tell whether a collection of objects is less than, the same as, or more than a given number of objects, both when the objects are arranged in a pattern and when they are randomly placed.
3. estimate the measure (length, capacity, weight) of an object given a non-standard unit of reference.
4. recognize whether a given estimate is reasonable or not.
5. recognize situations for which an estimate is sufficient.

given a stack of ten unifix cubes, estimate the number of cubes in a different stack

about how long is the table in "Robert hands"?

Grade One

The student will be able to ...

1. compare two sets of objects to determine whether there are about the same number of items in each set when the objects are arranged randomly or in a familiar pattern.
2. make a reasonable estimate for addition or subtraction statements.
3. estimate the value of a given collection of coins.
4. explore situations for which an estimate is appropriate.
5. mentally count on to determine a sum.
6. mentally count back to determine differences.

these activities help internalize work with concrete objects

## Grade Two

The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. recognize when the number of items in one set falls between the number of items in two other sets.
3. recognize when the number of items of one set is closer to a second set than a third set.

this makes the transition from concrete objects to symbolization with numbers

4. extend the estimation skills in objectives 2 and 3 to numbers.

46 + 23 is about 4 tens and 2 tens or 6 tens or 60

5. make estimates in addition and subtraction using front-end digits.
6. determine whether a calculated solution is reasonable by using estimation.

centimeter, inch, liter, cup, pint, quart, kilogram, and pound

7. estimate length, capacity, and mass.

think 46 + 23 = 40 + 20 + 6 + 3 = 69

8. use left-to-right addition to refine estimates obtained by using front-end digits.

strategies are listed in the number and number relations strand

9. use the strategies for addition and subtraction mentally, without reference to actual objects.

## Grade Three

The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. use front-end digits to estimate addition with several addends.
3. round factors and use multiples of ten to estimate products.
4. explore the use of estimation in problem solving and know when an estimate is appropriate.
5. add strings of numbers mentally by finding groups of tens.

2 + 17 + 18 + 23 + 5 =
(2 + 18) + (17 + 23) + 5 =
20 + 40 + 5 =
65

6. subtract mentally using multiples of ten.

| | |
|---|---|
| 7. explore multiplication using the strategies without reference to actual objects. | e.g., 56 - 32 = 56 - 30 - 2 = 26 - 2 = 24 and 47 - 29 = 47 - 30 + 1 = 17 + 1 = 18 |

Grade Four

The student will be able to ...

| | |
|---|---|
| 1. perform and extend the objectives listed in previous grades. | |
| 2. round to find the approximate sum or difference of numbers and compare the result to that obtained by using front-end digits. | |
| 3. explore estimates of sums and differences and determine whether they are greater than or less than the exact sum or difference. | |
| 4. explore estimates of products and quotients and determine whether they are greater or less than products or quotients found by a calculator. | |
| 5. use compatible numbers to estimate in division. | |
| 6. use estimates to determine the reasonableness of results in problem solving. | change 340/6 to 360/6, since 6 is a factor of 36 |
| 7. use compensatory numbers in adding. | |
| 8. look for combinations that make ten or one hundred. | 62 + 28 = 60 + 30 = 90 |
| 9. explore combinations of quarters and relate to sums and differences with 25. | 125 + 45 + 75 = 125 + 75 + 45 = 245 |
| 10. use left-to-right multiplication and the distributive property to find an exact answer without paper and pencil. | 5 x 38 = 5 x 30 + 5 x 8 = 150 + 40 = 190 |

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR 5—8

STANDARD 7:
COMPUTATION AND ESTIMATION

In grades 5—8, the mathematics curriculum should develop the concepts underlying computation and estimation in various contexts so that students can—

- compute with whole numbers, fractions, decimals, integers, and rational numbers;
- develop, analyze, and explain procedures for computation and techniques for estimation;
- develop, analyze, and explain methods for solving proportions;
- select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods;
- use computation, estimation, and proportions to solve problems;
- use estimation to check the reasonableness of results.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 94.

Grade Five

The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. use compatible numbers to estimate in addition.
3. round fractions to 0, 1/2, and 1 and use these values to estimate sums and differences of fractions. — 7/8 + 11/12 is about 2
4. round mixed numbers to the nearest whole number to estimate sums and differences of mixed numbers.
5. use front-end digit and rounding strategies for estimating sums and differences of decimals.
6. use compatible numbers to add or subtract mentally.
7. halve and double factors to find the product. — 25 x 236 = 50 x 118 = 100 x 59 = 5900
8. use place value and trailing zeroes to mentally divide when the known numbers are multiples of powers of 10.

Grade Six

The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. estimate the sum of several close addends by estimating an average and multiplying the average by the number of values. — 3 + 5 + 7 + 9 + 10 average is about 7, five numbers 7 x 5 is 35
3. estimate the sum or difference of mixed numbers by adding or subtracting the whole numbers and then adding or subtracting the fractions using their closest value, 0, 1/2, or 1.
4. estimate the product or quotient of mixed numbers by rounding them to whole numbers. — 1-7/8 times 2-1/2 is about 4

5. estimate the product or quotient of decimal numbers by rounding them to a single decimal place and then performing the operation.
6. look for compatibles in multiplication and division to help perform these operations mentally.
7. use estimation to eliminate choices in multiple-choice tests.

an excellent opportunity to review a variety of objectives from other strands

Grade Seven

The student will be able to ...

1. perform, refine, and extend the objectives listed in previous grades.
2. adjust fractional number and decimal estimates in all operations.
3. estimate with percents, using 1%, 10%, and 50%, and multiples of these numbers.
4. use fractions, decimals, and percent equivalents interchangeably in making estimates.
5. estimate the square root of a given number to the nearest whole number or range of whole numbers.
6. use estimation to eliminate choices in multiple-choice tests.

a 15% tip on a bill of $11.83 would be 10% plus half of 10% or about $1.20 plus .60 or $1.80.

the square root of 50 is between the square root of 49 and 64 or between 7 and 8 and is closer to 7

Grade Eight

The student will be able to ...

1. perform, refine, and extend the objectives listed in previous grades.
2. use fractions, decimals, and percent equivalents interchangeably in making estimates.
3. estimate the square root of a given number to the nearest whole number or range of whole numbers.
4. use estimation to determine the reasonableness of results in all problem solving.

33% of 360 is about 1/3 of 360 or 120

5. extend mental computation to the solution of simple equations.
6. use estimation to eliminate choices in multiple-choice tests.
7. use estimation to determine reasonableness of problem solutions.

to solve $2a + 7 = 49$, think 2a must be 42 so a is 21.

The main purpose of the estimation and mental computation strand at the 9-12 level is to continue the habitual use of these skills and to apply them to the more advanced topics in other strands. Estimation aimed at assessing the reasonableness of results is of continuing importance, as mathematics is used in other disciplines.

Grades 9-12

The student will be able to ...

1. use estimation to eliminate choices in multiple-choice tests.
2. use estimation to determine reasonableness of problem situations in a wide variety of applications.
3. estimate the shape of graphs of various functions and algebraic expressions.
4. use mental computation when computer and calculator are inappropriate.

## STRAND 8: DATA ANALYSIS AND PROBABILITY

Understanding of the processes of data collection, representation, and interpretation is crucial in today's world. It is essential that students develop the ability to summarize and analyze data, to draw inferences, and to make accurate predictions and judgments based on that analysis.

Students should be given ample opportunities to examine real-world situations. This strand provides an excellent opportunity for students to become active participants in mathematics learning. The vast variety of potential topics for investigation provides the opportunity for considering a variety of interests and abilities, and thus makes the classroom a motivating and exciting place to be.

The study of data analysis should start in the early grades with the comparison, classification, and organization of objects, then proceed to the collection, organization, and representation of more abstract data through the use of tables, charts, graphs, and maps. Finally, students should learn to analyze the data, note trends, draw conclusions, and make predictions.

It is essential that students understand how statistics can be manipulated, and recognize the potential uses and misuses of statistical information.

The availability of computing technology presents today's students with the ability to perform statistical computations and comparisons which would have proved too unwieldy just a few short years ago.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES K—4

STANDARD 11:
STATISTICS AND PROBABILITY

In grades K—4, the mathematics curriculum should include experiences with data analysis and probability so that students can—

- collect, organize, and describe data;
- construct, read, and interpret displays of data;
- formulate and solve problems that involve collecting and analyzing data;
- explore concepts of chance.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, p. 54.

Kindergarten

The student will be able to ...

1. describe objects or pictures according to distinct attributes.
2. classify objects or pictures and use given categories or attributes in sorting.
3. create a floor/table graph by arranging actual objects in appropriate categories.

the development of verbal skills is important

use attribute blocks or any common objects

Grade One

The student will be able to ...

1. use given categories in sorting information.
2. create a picture graph (each picture representing a single unit) by drawing a picture of a floor/table graph, as mentioned in the kindergarten objectives.
3. explore meanings of a picture graph by making identifications, comparisons, and predictions.
4. collect data and record by tallying.
5. identify events that are sure to happen, events that are sure not to happen, and those we cannot be sure about.

children may suggest categories, but some will need help

Grade Two

The student will be able to ...

1. collect and organize data and represent with a picture or bar graph.
2. develop a variety of categories for sorting information.
3. explore picture and bar graphs (scaled by one) by making identifications, comparisons, and predictions.
4. identify information on a labeled picture map.

## Grade Three

The student will be able to ...

1. read and interpret pictographs in which pictures represent more than a single unit.
2. create, read, and interpret tables and charts.
3. explore bar graphs (scaled by one) by making identifications, comparisons, and predictions.

students need to develop independence in interpreting symbols from contextual clues

4. identify information on a labeled picture map using a picture-symbol key.
5. collect and record data on the frequency of events.
6. investigate, display, and record all possible arrangements of a given set of objects.

abstract data from all of them

7. translate freely among pictographs, tables, charts, and bar graphs.

## Grade Four

The student will be able to ...

1. collect data and create a picture or bar graph representing the data.
2. make predictions and modify them as additional data are collected.
3. read and interpret diagrams and time lines.

scaling of graphs becomes an important focus

4. explore picture and bar graphs by making identifications, comparisons, and predictions, and use them to solve application problems.
5. investigate, display, and record all possible arrangements of a given set of events.

use spinners, coin tossing, random selection, etc.

6. find simple experimental probabilities.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 5—8

STANDARD 10:
STATISTICS

In grades 5—8, the mathematics curriculum should include exploration of statistics in real-world situations so that students can—

- systematically collect, organize, and describe data;
- construct, read, and interpret tables, charts, and graphs;
- make inferences and convincing arguments that are based on data analysis;
- evaluate arguments that are based on data analysis;
- develop an appreciation for statistical methods as powerful means for decision making.

STANDARD 11:
PROBABILITY

In grades 5—8, the mathematics curriculum should include explorations of probability in real-world situations so that students can—

- model situations by devising and carrying out experiments or simulations to determine probabilities;
- model situations by constructing a sample space to determine probabilities;
- appreciate the power of using a probability model by comparing experimental results with mathematical expectations;
- make predictions that are based on experimental or theoretical probabilities;
- develop an appreciation for the pervasive use of probability in the real world.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, pp. 105 and 109.

## Grade Five

The student will be able to ...

1. explore the effect of changing scales on bar graphs.
2. select a scale and create a line graph.
3. identify the ordered pair for a point on a labeled grid.

geographical maps

4. identify a direction, distance, and/or location using a political map containing a key, a scale, and a compass.
5. explore the concept of average and calculate the arithmetic mean of a given set of numbers.

Use fractions, ratios, and percents to describe the probability of a given event.

6. determine experimental and theoretical probabilities.
7. make predictions based on experimental or theoretical probabilities.

## Grade Six

The student will be able to ...

1. collect data and create a circle graph.
2. explore circle graphs and use them to solve application problems.

make identifications, comparisons, predictions

3. read, interpret, and use tables, charts, maps, and graphs to identify patterns, note trends, and draw conclusions.
4. explore the concept of average and calculate the arithmetic mean and the mode of a given set of numbers.
5. explore changes in the mean and the mode when some data are changed.
6. construct a tree diagram to list alternatives and procedures.
7. read and construct scale drawings.
8. investigate probabilities for the possible outcomes of a simple experiment.

9. make predictions of outcomes of experiments based on theoretical probabilities and explain actual outcomes.

Grade Seven

The student will be able to ...

1 collect data and create the appropriate type of graph and use the appropriate scale.
2. create, read, and interpret tables, charts, diagrams, and maps.
3. identify the ordered pair for a point on a labeled coordinate plane.
4. calculate and explore relationships between the mean, median, mode, and range of a given set of numbers.
5. explore permutations and combinations and the relationships between them.
6. make logical inferences from statistical data.
7. detect misuses of statistical or numerical information.
8. develop and interpret frequency tables.
9. compute averages.

Grade Eight

The student will be able to ...

1. collect data and create appropriate graphs to illustrate.
2. make identifications, comparisons, and predictions, and solve application problems using picture, bar, circle, and line graphs.
3. find the mean, mode, median, and range of a set of data and use them in application problems.
4. detect misuses of statistical or numerical information.

A spreadsheet can be used to create simulations to broaden students' experiences.

5. use elementary notions of probability.
6. explore the role of sampling and collecting data in making a statistical argument.

*from the NCTM Standards:*

## CURRICULUM STANDARDS FOR GRADES 9—12

### STANDARD 10:
### STATISTICS

In grades 9—12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can—

- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
- use curve fitting to predict from data;
- understand and apply measures of central tendency, variability, and correlation;
- understand sampling and recognize its role in statistical claims;
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes;
- analyze the effects of data transformations on measures of central tendency and variability;

and so that, in addition, college-intending students can —

- transform data to aid in data interpretation and prediction;
- test hypotheses using appropriate statistics.

### STANDARD 11:
### PROBABILITY

In grades 9—12, the mathematics curriculum should include the continued study of probability so that all students can—

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- use simulations to estimate probabilities;
- understand the concept of a random variable;
- create and interpret discrete probability distributions;
- describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed;

and so that, in addition, college-intending students can —

- apply the concept of a random variable to generate and interpret probability distributions including binomial, uniform, normal, and chi square.

*Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1989, pp. 167 and 171.

Grade Nine

The student will be able to ...

data from real-world situations

1. organize data into tables, charts, and graphs.
2. understand and apply measures of central tendency, variability, and correlation.

and, in addition, college-intending students will be able to ...

3. transform data to aid in data interpretation and prediction.

Grade Ten

The student will be able to ...

1. use curve fitting to predict from data.
2. use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty.
3. use computer simulations and random number generators to estimate probabilities.

and, in addition, college-intending students will be able to ...

4. test hypotheses using appropriate statistics.
5. read, interpret, and use tables, charts, and graphs to identify patterns, note trends, draw conclusions, and make predictions.
6. determine probabilities of events involving unbiased objects.

Grade Eleven

The student will be able to ...

1. use sampling and recognize its role in statistical claims.
2. design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.

and, in addition, the college-intending student will be able to ...

3. describe, in general terms, the normal curve and use its properties.

Grade Twelve

The student will be able to ...

1. create and interpret discrete probability distributions.
2. understand the concept of random variable.

and, in addition, the college-intending student will be able to ...

3. apply the concept of a random variable to generate and interpret probability distributions, including binomial, uniform, normal, and chi square.

# CURRICULUM BY GRADE LEVEL AND COURSE

The subject objectives listed by strands in the previous section are listed here by grade level for grades kindergarten through eight and by course for grades nine through twelve.

For high school level courses, a district may choose to continue with an integrated approach to mathematics for both college-intending and non-college-intending students. A course for college-intending students will include all objectives for the indicated grade level, but a course for non-college-intending students will include only the core objectives for that grade level. Such a course will be less theoretical, but will address a wider variety of applications than one for college-intending students. A high school course which consists primarily of K-8 objectives is inappropriate.

An example of selected objectives for some of the more usual college preparatory courses is provided for districts that choose to continue a non-integrated mathematics program. A district may wish to combine objectives in some way or provide courses that go beyond the objectives listed here.



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# KINDERGARTEN

**Strand One**
The student will be able to ...

1. represent a pattern using objects, drawings, rhythms, and body movements, and be able to describe a pattern verbally.
2. replicate or copy a given pattern.
3. continue a pattern.
4. recognize a pattern in the environment.
5. sort objects according to an attribute such as size, color, shape, or weight.

**Strand Two**
The student will be able to ...

1. use invented and conventional symbols to describe a problem situation.
2. look for a pattern of objects to predict the solution to a problem.
3. act out a problem situation.

**Strand Three**
The student will be able to ...

1. relate numbers to numerals.
2. separate (decompose), join (combine), and order sets of objects.
3. compare number of objects in two or more sets.
4. count
   a. by rote
   b. objects (1-1 correspondence)

**Strand Four**
The student will be able to ...

1. identify common shapes in the environment.
2. manipulate, color, fold, and create simple geometric shapes.
3. use a computer program, such as Logo (appropriately modified), to explore paths and construct simple geometric shapes.
4. identify common solids in the environment.

**Strand Five**
The student will be able to ...

1. use and understand the language of logic in talking about problem situations and solutions to problems.

**Strand Six**
The student will be able to ...

1. compare and order objects of different lengths, weights, and capacities.
2. model the results of joining or separating lengths, weights, and capacities.
3. order events based on time.
4. count pennies and nickels to 10 cents.

**Strand Seven**
The student will be able to ...

1. estimate the number of items in a set by comparing to a set where the number of items is known.
2. tell whether a collection of objects is less than, the same as, or more than a given number of objects, both when the objects are arranged in a pattern and when they are randomly placed.
3. estimate the measure (length, capacity, weight) of an object given a non-standard unit of reference.
4. recognize whether a given estimate is reasonable or not.
5. recognize situations for which an estimate is sufficient.

**Strand Eight**
The student will be able to ...

1. describe objects or pictures according to distinct attributes.
2. classify objects or pictures and use given categories or attributes in sorting.
3. create a floor/table graph by arranging actual objects in appropriate categories.

# GRADE ONE

**Strand One**
The student will be able to ...

1. represent, copy, and continue patterns.
2. sort objects on multiple attributes.
3. identify and extend missing elements of repeating patterns and sequences of numbers.

**Strand Two**
The student will be able to ...

1. select appropriate notation and methods for symbolizing the problem statement and the solution process.
2. look for a pattern (objects or pictures) to predict a solution to a problem.
3. role play to find the solution to a problem.
4. make tables to sort information.

**Strand Three**
The student will be able to ...

1. continue all of the kindergarten objectives.
2. decompose, combine, order, and compare numbers.
3. estimate the number prior to counting and count objects
   a. using ten frames
   b. by twos, fives, and tens
   c. using ordinal numbers
4. skip count by twos, fives, and tens and count backwards.
5. develop the concept of addition and subtraction from situations in the environment, including joining, separating, and comparing sets of objects.
6. learn strategies for addition of whole numbers such as
   a. counting all
   b. counting on
   c. one more, one less
   d. two more, two less
   e. doubles
   f. doubles plus or minus one and doubles plus or minus two
   g. make ten
   h. using ten frames

7. describe the operations of addition and subtraction in words.
8. use invented symbols and conventional symbols to represent the operations of addition and subtraction.
9. orally identify fractional parts (halves, thirds, fourths) of whole objects or sets of objects.

**Strand Four**
The student will be able to ...

1. continue all of the kindergarten objectives with more detail, greater depth, and with different materials.
2. combine shapes to form a given shape.
3. identify two-dimensional shapes on three-dimensional objects.
4. compare two-dimensional shapes describing similarities and differences using appropriate standard and non-standard language.
5. explore situations by manipulating shapes, measuring, and counting.
6. use mathematically correct names of pattern block pieces.

**Strand Five**
The student will be able to ...

1. use and understand the language of logic in describing problem situations and solutions to problems.
2. use invented symbols and letters to describe the joining, separating, and comparing of bunches of objects.
3. model a problem situation using a number phrase or sentence.

**Strand Six**
The student will be able to ...

1. recognize and count a collection of coins including pennies, nickels, dimes, and a quarter and determine its value.
2. measure lengths using nonstandard units, centimeters, and inches.
3. indicate the approximate size of the units, inch and centimeter, without using a ruler.
4. tell time to the hour and the half-hour.
5. explore capacity using cups, pints, quarts, and liters and recognize these measuring units.
6. explore weight using pounds and kilograms and recognize these measuring units.

**Strand Seven**
The student will be able to ...

1. compare two sets of objects to determine whether there are about the same number of items in each set when the objects are arranged randomly or in a familiar pattern.
2. make a reasonable estimate for addition or subtraction statements.
3. estimate the value of a given collection of coins.
4. explore situations for which an estimate is appropriate.
5. mentally count on to determine a sum.
6. mentally count back to determine differences.

**Strand Eight**
The student will be able to ...

1. use given categories in sorting information.
2. create a picture graph (each picture representing a single unit) by drawing a picture of a floor/table graph, as mentioned in the kindergarten objectives.
3. explore meanings of a picture graph by making identifications, comparisons, and predictions.
4. collect data and record by tallying.
5. identify events that are sure to happen, events that are sure not to happen, and those we cannot be sure about.

# GRADE TWO

**Strand One**
The student will be able to ...

1. identify and extend patterns of objects and symbols.
2. recognize patterns in numbers and number combinations.
3. use patterns to make generalizations and predictions by
   a) determining the rule and identifying missing numbers in a sequence of numbers;
   b) determining the rule and identifying missing numbers in a table of number pairs; and
   c) identifying missing elements in a pattern and justifying their inclusion.

**Strand Two**
The student will be able to ...

1. identify needed information to solve a problem.
2. select appropriate notation and methods for symbolizing the problem statement and the solution process.
3. look for patterns of numbers to predict a solution.
4. role play to find the solution to a problem.
5. make pictographs and bar graphs to sort information.
6. make repeated guesses to find a solution and check until a solution is found.
7. explain in words why a solution is correct.

**Strand Three**
The student will be able to ...

1. develop the concept of place value with concrete models of hundreds, tens, and ones.
2. develop the concept of odd and even using concrete materials.
3. use concrete models of fractions (halves, thirds, fourths, sixths) to investigate different physical representations for the same fractional parts of whole objects or sets of objects.
4. illustrate fractional parts of whole objects or sets of objects.
5. place whole numbers and fractions on the number line.
6. develop the concept of multiplication and division by joining equivalent sets of objects or separating objects into equivalent sets.

7. learn strategies for the addition and subtraction of numbers such as
   a. compatible numbers
   b. compensatory numbers
   c. borrow and pay back (subtraction)
   d. regrouping
   e. using a calculator
8. develop skill in estimating sums and differences.
9. translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics.
10. find equivalent forms of numbers using hundreds, tens, and ones.

**Strand Four**
The student will be able to ...

1. investigate congruence and symmetry.
2. investigate perimeter using concrete models.
3. explore paths, simple closed curves, and the ideas of interior and exterior.
4. explore tangram and pattern block puzzles.
5. compare three-dimensional objects describing similarities and differences using appropriate standard and non-standard language.

**Strand Five**
The student will be able to ...

1. explain in words thinking strategies for making computations.
2. understand the use of letters in statements such as a-b = 6 and find a or b when one is given.
3. model a problem situation using numbers and/or letters.

**Strand Six**
The student will be able to ...

1. explore length, capacity, and weight by selecting and using appropriate metric and conventional units such as centimeter, inch, liter, cup, pint, quart, kilogram, and pound.
2. count collections of coins including pennies, nickels, dimes, quarters, and half dollars and compare values.
3. tell time to the nearest five-minute interval on digital and dial timepieces.
4. relate time on dial and digital timepieces.

**Strand Seven**
The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. recognize when the number of items in one set falls between the number of items in two other sets.
3. recognize when the number of items of one set is closer to a second set than a third set.
4. extend the estimation skills in objectives 2 and 3 to numbers.
5. make estimates in addition and subtraction using front-end digits.
6. determine whether a calculated solution is reasonable by using estimation.
7. estimate length, capacity, and mass.
8. use left-to-right addition to refine estimates obtained by using front-end digits.
9. use the strategies for addition and subtraction mentally, without reference to actual objects.

**Strand Eight**
The student will be able to ...

1. collect and organize data and represent with a picture or bar graph.
2. develop a variety of categories for sorting information.
3. explore picture and bar graphs (scaled by one) by making identifications, comparisons, and predictions.
4. identify information on a labeled picture map.

# GRADE THREE

**Strand One**
The student will be able to ...

1. predict additional terms in a given pattern, describe how the pattern is created, and extend the pattern.
2. recognize multiplication patterns.
3. use patterns to make generalizations and predictions by
   a) determining the rule and identifying missing numbers in a sequence of numbers;
   b) determining the rule and identifying missing numbers in a table of number pairs; and
   c) by identifying missing elements in a pattern and justifying their inclusion.
4. make a table of values to record the pairing of members of two sets, determine the relationship (rule) between each pair, and use the rule to generate additional pairs.

**Strand Two**
The student will be able to ...

1. select appropriate notation and methods for symbolizing the problem statement and the solution process.
2. look for a pattern to predict a solution.
3. role play or use appropriate materials to find the solution to a problem.
4. make and use a Venn diagram to collect and sort information.
5. extend the guess-and-check procedure by recording guesses and checks to help make better guesses until the solution is reached.
6. make a drawing of the information in the problem to clarify relationships.
7. use more than one strategy to solve a given problem.
8. use a single strategy to solve different kinds of problems.
9. identify which questions could be answered given certain information.
10. develop a convincing written argument for the correctness of a solution.

**Strand Three**
The student will be able to ...

1. add and subtract numbers fluently using any strategy.
2. relate addition and subtraction statements to each other.

3. from situations created in the classroom
   a. develop models of multiplication and division (arrays).
   b. use invented and conventional symbols to represent multiplication and division.
   c. describe multiplication and division in words.
4. translate real-life situations involving multiplication and division into conventional mathematical symbols.
5. relate multiplication to skip counting.
6. multiply using the pocket multiplier.
7. multiply and divide using a calculator.
8. recall multiplication and division facts through 12 x 12, using strategies such as
   a. commutative property
   b. distributive property
   c. anchor facts
   d. squares
9. multiply using paper-and-pencil algorithms.
10. use conventional symbols to represent fractions.
11. order fractions on the basis of concrete materials.
12. develop the concept of tenths and hundredths using models.
13. order whole numbers, fractions, and decimals (tenths and hundredths) on the number line.
14. translate freely between words and symbols in naming numbers.
15. relate even numbers to division by two.
16. use the symbols $<$, $\leq$, $>$, $\geq$, and $=$ in describing order as well as the terms "at least" and "at most."

**Strand Four**

The student will be able to meet any previous objective, and in addition ...

1. explore properties of geometric figures and relationships by measuring, coloring, folding, cutting, making models, and using tiles and geoboards.
2. investigate angles using models, paper folding, drawings, and computer graphics.
3. build a solid to match a given solid using cubes.
4. describe a three-dimensional object from different perspectives.
5. investigate area by covering regions with standard and non-standard units.
6. use mathematically correct names for common geometric figures.
7. identify and count common overlapping figures in the environment.

**Strand Five**
The student will be able to ...

1. explain in words thinking strategies for making computations.
2. explore calculator keys other than = that perform an operation.
3. understand the use of letters in statements such as ab=12 or 3c=d and find a when b is given, etc.

**Strand Six**
The student will be able to ...

1. continue explorations of length, capacity, and weight, and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce, and fractional parts of each.
2. count collections of coins and bills which include one, five, and ten dollar bills and compare values.
3. explore common temperatures using both Fahrenheit and Celsius scales.
4. read time on digital and dial timepieces and determine amount of time elapsed.
5. illustrate the approximate size of units (inch, centimeter, meter, and yard).
6. use string, tiles, and blocks to explore perimeter, area, and volume .
7. make change using coins.

**Strand Seven**
The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. use front-end digits to estimate addition with several addends.
3. round factors and use multiples of ten to estimate products.
4. explore the use of estimation in problem solving and know when an estimate is appropriate.
5. add strings of numbers mentally by finding groups of tens.
6. subtract mentally using multiples of ten.
7. explore multiplication using the strategies without reference to actual objects.

**Strand Eight**
The student will be able to ...

1. read and interpret pictographs in which pictures represent more than a single unit.
2. create, read, and interpret tables and charts.
3. explore bar graphs (scaled by one) by making identifications, comparisons, and predictions.
4. identify information on a labeled picture map using a picture-symbol key.
5. collect and record data on the frequency of events.
6. investigate, display, and record all possible arrangements of a given set of objects.
7. translate freely among pictographs, tables, charts, and bar graphs.

# GRADE FOUR

**Strand One**
The student will be able to ...

1. explore and describe in words simple and complex patterns in nature, art, and poetry.
2. determine the rule and identify missing numbers in a sequence of numbers or a table of number pairs related by combinations of addition, subtraction, multiplication, and division.

**Strand Two**
The student will be able to ...

1. use more than one strategy to solve a given problem.
2. use a single strategy to solve different kinds of problems.
3. make and use a table to record and sort information.
4. select appropriate notation and methods for symbolizing the problem statement and the solution process.
5. extend the application of previously learned strategies.
6. validate solution(s) to a problem.
7. generalize a problem-solving situation to other cases.

**Strand Three**
The student will be able to ...

1. decompose numbers into factors, using objects and translate using symbols.
2. identify prime and composite numbers.
3. use physical models to represent fractions greater than one.
4. round fractions to zero, one-half, one, etc.
5. round numbers to the nearest ten and hundred and beyond.
6. order fractions using symbols.
7. find equivalent fractions with concrete materials and symbolically.
8. add and subtract fractions with concrete materials and symbolically.
9. develop concepts of tenths and hundredths using symbols.
10. add and subtract decimals.
11. multiply and divide whole numbers fluently.
12. develop concepts of place value to include numbers through millions.
13. compare and order numbers with any number of digits.
14. relate multiplication and division statements to each other.

**Strand Four**
The student will be able to ...

1. investigate reflection, rotations, and translations of geometric figures using concrete objects.
2. identify parallel lines, perpendicular lines, and right angles in geometric figures and the environment.
3. build models that illustrate intersecting lines, parallel lines, perpendicular lines, and right angles.
4. determine properties of two-dimensional figures and compare shapes according to their characterizing properties.

**Strand Five**
The student will be able to ...

1. use and understand the language of logic in describing the inverse relationships between addition and subtraction, multiplication and division.
2. use variables in mathematical expressions to represent problem situations.
3. symbolize a keying sequence on a calculator with arithmetic logic to execute the computation of an arithmetic phrase and predict the display as each key is pressed
4. explore using variables to generalize arithmetic statements
5. explore tables that describe arithmetic relationships.
6. understand the use of letters in statements such as a/b = 4 or c/2 = d and find a when b is given, etc.

**Strand Six**
The student will be able to ...

1. choose an appropriate unit and measure lengths, capacities, and weights in U.S. standard and metric units.
2. measure and determine perimeters and areas of simple straight line figures and regions without using formulas.
3. make reasonable estimates of lengths under 10 meters.
4. read temperature measurements in ·C and ·F.
5. relate the number of units that measure an object to the size of the unit as well as to the size of the object.
6. make change using coins and bills.
7. use mental, paper-and-pencil, and physical strategies to determine time elapsed.

**Strand Seven**
The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. round to find the approximate sum or difference of numbers and compare the result to that obtained by using front-end digits.
3. explore estimates of sums and differences and determine whether they are greater than or less than the exact sum or difference.
4. explore estimates of products and quotients and determine whether they are greater or less than products or quotients found by a calculator.
5. use compatible numbers to estimate in division.
6. use estimates to determine the reasonableness of results in problem solving.
7. use compensatory numbers in adding.
8. look for combinations that make ten or one hundred.
9. explore combinations of quarters and relate to sums and differences with 25.
10. use left-to-right multiplication and the distributive property to find an exact answer without paper and pencil.

**Strand Eight**
The student will be able to ...

1. collect data and create a picture or bar graph representing the data.
2. make predictions and modify them as additional data are collected.
3. read and interpret diagrams and time lines.
4. explore picture and bar graphs by making identifications, comparisons, and predictions, and use them to solve application problems.
5. investigate, display, and record all possible arrangements of a given set of events.
6. find simple experimental probabilities.

# GRADE FIVE

**Strand One**
The student will be able to ...

1. investigate patterns that occur when changing numerators and denominators in equivalent fractions and describe these patterns verbally.
2. investigate the patterns of digits formed when fractions are changed to decimal form.
3. use patterns to explore the rules for divisibility.
4. investigate patterns formed by powers of ten using exponents and expanded form of numbers.
5. explore methods for finding the n-term of a simple sequence involving one operation and verbalize a procedure.
6. graph ordered pairs.
7. find missing terms of a sequence using powers.
8. explore and describe in words simple and complex patterns in music and science.

**Strand Two**
The student will be able to ...

1. read a problem carefully and restate it without reference to the original problem.
2. read a problem carefully and identify subgoals that need to be attained in order to solve the problem.
3. expand the repertoire of appropriate notations and methods for symbolizing a problem statement and the solution process.
4. extend the application of previously learned strategies.
5. validate and generalize solutions to problems.

**Strand Three**
The student will be able to ...

1. decompose numbers into factors, including prime factored form.
2. use the long division algorithm.
3. multiply and divide decimals.
4. find equivalent fractions.
5. add and subtract fractions.
6. order combinations of whole numbers, fractions, and decimals using the symbols $<$, $\leq$, $>$, $\geq$, and $=$ and by placing them on the number line.
7. explore order of operations relative to calculators with arithmetic and algebraic logic.

8. explore and use the idea of ratio relative to scaling.
9. explore the idea of square and square root in the context of area of squares.
10. round, as appropriate to a problem situation, to the nearest thousand, hundred, ten, one, tenth, or hundredth.
11. explain in words the role of 0 and 1 as identity elements for addition and multiplication, respectively.
12. explain in words why order does not make a difference for addition and multiplication, but does for subtraction and division.

**Strand Four**
The student will be able to ...

1. compare and contrast angles in relation to right angles.
2. construct circles with a given center and/or a given radius.
3. encounter and use appropriate vocabulary relative to circles.
4. build models of previously encountered shapes and figures and describe the process in words.
5. explore concepts of similarity by enlarging shapes with pattern blocks, geoboards, and computer graphics.
6. explore patterns that result from combinations of reflections, rotations, and translations of geometric figures.

**Strand Five**
The student will be able to ...

1. symbolize a keying sequence on a calculator with algebraic logic to execute the computation of an arithmetic phrase and predict the display as each key is pressed.
2. explain in words differences between calculators with arithmetic logic and algebraic logic.
3. use variables to describe arithmetic processes.
4. interpret tables that describe problem situations.

**Strand Six**
The student will be able to meet any previous objective, and, in addition ...

1. choose an appropriate unit and measure lengths, widths, or capacities to a specified degree of precision in U.S.standard or metric measurement.
2. convert, compare, and compute with common units of measure within the same measurement system.

3. determine what to measure and measure in order to determine perimeters, areas, and volumes of simple shapes and solids.
4. make reasonable estimates of lengths, weights, and capacities.

**Strand Seven**
The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. use compatible numbers to estimate in addition.
3. round fractions to 0, 1/2, and 1 and use these values to estimate sums and differences of fractions.
4. round mixed numbers to the nearest whole number to estimate sums and differences of mixed numbers.
5. use front-end digit and rounding strategies for estimating sums and differences of decimals.
6. use compatible numbers to add or subtract mentally.
7. halve and double factors to find the product.
8. use place value and trailing zeroes to mentally divide when the known numbers are multiples of powers of 10.

**Strand Eight**
The student will be able to ...

1. explore the effect of changing scales on bar graphs.
2. select a scale and create a line graph.
3. identify the ordered pair for a point on a labeled grid.
4. identify a direction, distance, and/or location using a political map containing a key, a scale, and a compass.
5. explore the concept of average and calculate the arithmetic mean of a given set of numbers.
6. determine experimental and theoretical probabilities.
7. make predictions based on experimental or theoretical probabilities.

# GRADE SIX

**Strand One**
The student will be able to ...

1. build simple functions using concrete models and generate a corresponding rule.
2. explore the relation between doubling the side of a square and/or other regular figures and the corresponding increase in area.
3. explore mathematical expressions of relations observed in other curricular domains.
4. explore and describe in words simple and complex patterns in history and language arts.

**Strand Two**
The student will be able to ...

1. extend the application of previously learned strategies.
2. expand the repertoire of appropriate notations and methods for symbolizing a problem statement and the solution process.
3. identify needed and given information in a problem situation as well as irrelevant information.
4. validate and generalize solutions.

**Strand Three**
The student will be able to ...

1. compute with whole numbers, fractions, and decimals.
2. explore concepts of percent, ratio, and proportions in the context of real-world situations.
3. use proportions in a wide variety of applications.
4. investigate relations between ratios, proportions, and percents.
5. round, as appropriate to a problem situation, to any digit.
6. change freely between fractions and decimals
7. understand and describe in words the relations between addition, subtraction, multiplication, and division.
8. understand and describe in words how fractions and decimals expand the whole number system to the system of non-negative rational numbers.
9. be able to find a number between any two rational numbers.

10. explore and explain when order does and does not make a difference for the four fundamental operations.
11. explore Roman numerals and contrast with the base ten number system.

**Strand Four**
The student will be able to ...

1. measure angles in geometric figures and explore relationships between angle measure and other characteristics of the figures.
2. estimate the measure of angles and draw angles that approximate given measures.
3. identify and distinguish among similar, congruent, and symmetric figures.
4. visualize and show the results of a rotation, translation, reflection, or stretching.
5. build models of three-dimensional figures, such as pyramids, cones, and prisms, with polygonal bases and investigate the properties associated with those figures.
6. explore properties that can be used to characterize or contrast different classes of figures.
7. recognize, classify, and use characteristics of lines and simple two-dimensional figures.

**Strand Five**
The student will be able to ...

1. use the distributive property in arithmetic computations.
2. construct tables to describe a problem situation.
3. use a variable to describe a generalization from a problem situation.
4. symbolize, using variables, the relations between addition, subtraction, multiplication, and division.
5. explore the use of parentheses on a calculator to change results of a computation.
6. solve linear equations using concrete representations.

**Strand Six**
The student will be able to meet any previous objective and, in addition ...

1. select and use appropriate units and devices to measure length, area, volume, and weight.
2. explore and use formulas to compute areas and perimeters (circumferences) of common polygons (polygonal regions) and circles (circular regions).

3. convert, compare, and compute with common units of measure within the same measurement system.
4. measure angles using a protractor.

**Strand Seven**
The student will be able to ...

1. perform and extend the objectives listed in previous grades.
2. estimate the sum of several close addends by estimating an average and multiplying the average by the number of values.
3. estimate the sum or difference of mixed numbers by adding or subtracting the whole numbers and then adding or subtracting the fractions using their closest value, 1, 1/2, or 1.
4. estimate the product or quotient of mixed numbers by rounding them to whole numbers.
5. estimate the product or quotient of decimal numbers by rounding them to a single decimal place and then performing the operation.
6. look for compatibles in multiplication and division to help perform these operations mentally.
7. use estimation to eliminate choices in multiple-choice tests.

**Strand Eight**
The student will be able to ...

1. collect data and create a circle graph.
2. explore circle graphs and use them to solve application problems.
3. read, interpret, and use tables, charts, maps, and graphs to identify patterns, note trends, and draw conclusions.
4. explore the concept of average and calculate the arithmetic mean and the mode of a given set of numbers.
5. explore changes in the mean and the mode when some data are changed.
6. construct a tree diagram to list alternatives and procedures.
7. read and construct scale drawings.
8. investigate probabilities for the possible outcomes of a simple experiment.
9. make predictions of outcomes of experiments based on theoretical probabilities and explain actual outcomes.

# GRADE SEVEN

**Strand One**
The student will be able to ...

1. describe and represent relationships with tables, graphs, rules, and words.
2. describe, extend, analyze, and create a wide variety of patterns.
3. explore and symbolize direct and inverse variation.
4. extend the investigation of number patterns.
5. generate ordered pairs with and without a calculator to graph linear equations.
6. explore absolute value in the context of distance between points.
7. explore and describe in words simple and complex patterns in industrial technology and science.

**Strand Two**
The student will be able to ...

1. use an open sentence (algebraic equation) to symbolize a problem situation and solve the equation to find a solution to the problem.
2. validate solutions to problems in a variety of ways.
3. rephrase a problem as a simpler problem to find a method of solution.
4. extend the application of previously learned strategies.
5. identify problems that are similar in structure.

**Strand Three**
The student will be able to ...

1. represent percent by proportions and algebraic equations and solve for missing terms.
2. solve problems and make applications involving percent.
3. solve and use proportions.
4. develop the concept of integers using concrete models, including number lines, and in the context of real-world situations.
5. compare, order, and determine the equivalence of whole numbers, fractions, decimals, percents, and integers.
6. expand understanding of place value to include bases other than ten.
7. find square root using a calculator.
8. explore the concept of pi by comparing the measure of the diameter and circumference of circles.
9. explore interpretations of addition and multiplication that are different for whole numbers and fractions.

10. develop and apply theories about primes, factors, and multiples in real-world and mathematical problem situations.
11. explore powers and scientific notation as alternate ways of writing numbers and in the context of calculators.

**Strand Four**
The student will be able to ...

1. explore and verbalize relationships between different kinds of figures.
2. explore and describe procedures for changing one figure or shape to another .
3. develop minimum sets of properties that describe a geometric figure.
4. develop definitions of common geometric figures.
5. build the model of a figure given top, side, and front views.
6. validate fundamental geometric theorems using manipulative materials and informal arguments.
7. visualize and describe the results of folding geometric figures.
8. use separation of rectangles as an area model for the distributive property.

**Strand Five**
The student will be able to ...

1. use parentheses accurately to group numbers for applying operations.
2. apply formulas to problem situations.
3. describe problem situations involving ratios, proportions, and percents with algebraic expressions.
4. solve linear equations with one variable by working backward (relate to inverse operations).
5. evaluate algebraic expressions (simple substitutions).
6. interpret graphs of problem situations describing linear, quadratic, and exponential relationships.
7. construct graphs describing problem situations and assign and label scales to axes of graphs appropriately.
8. relate ratio and proportion concepts to variation situations, direct and inverse.

**Strand Six**
The student will be able to meet any previous objective and, in addition ...

1. select and compute with appropriate standard or metric units to measure length, area, volume, weight, capacity, time, money, and temperature.
2. make appropriate judgments regarding accuracy and precision.

3. make reasonable estimates of measurements.
4. state and apply area formulas for the following regions: circular, rectangular, parallelogram, trapezoidal, and triangular.
5. apply volume formulas for the following: prisms, cylinders, and spheres.
6. determine formulas for surface area.
7. explore measurement of time relative to time zones.
8. determine what to measure and measure to calculate perimeters, areas, and volumes.

**Strand Seven**
The student will be able to ...

1. perform, refine, and extend the objectives listed in previous grades.
2. adjust fractional number and decimal estimates in all operations.
3. estimate with percents, using 1%, 10%, and 50%, and multiples of these numbers.
4. use fractions, decimals, and percent equivalents interchangeably in making estimates.
5. estimate the square root of a given number to the nearest whole number or range of whole numbers.
6. use estimation to eliminate choices in multiple-choice tests.

**Strand Eight**
The student will be able to ...

1. collect data and create the appropriate type of graph and use the appropriate scale.
2. create, read, and interpret tables, charts, diagrams, and maps.
3. identify the ordered pair for a point on a labeled coordinate plane.
4. calculate and explore relationships between the mean, median, mode, and range of a given set of numbers.
5. explore permutations and combinations and the relationships between them.
6. make logical inferences from statistical data.
7. detect misuses of statistical or numerical information.
8. develop and interpret frequency tables.
9. compute averages.

# GRADE EIGHT

**Strand One**
The student will be able to ...

1. use patterns and keys on the calculator to extend the concept of inverse operations.
2. use invented and conventional symbols to explain a function relation.
3. explore the right triangle relations sine, cosine, and tangent and their applications in measurement.
4. explore the effect of multiplying the dimensions of a simple shape or solid by a constant factor and relate to the change in area or volume.
5. explore and describe in words simple and complex patterns in the environment.

**Strand Two**
The student will be able to ...

1. select appropriate notation and methods for symbolizing the problem statement and the solution process.
2. extend the application of previously learned strategies to a wide variety of problems.
3. validate and generalize problem solutions.

**Strand Three**
The student will be able to ...

1. understand, represent, and use numbers written in a variety of equivalent forms in real-world and mathematical problem-solving situations.
2. develop an understanding of operations with integers, using the number line and other models of integers.
3. represent numerical relationships in one- and two-dimensional graphs.
4. understand the real number system and describe it in words.
5. construct segments to represent irrational numbers such as the square roots of 2, 3, 5, etc.
6. locate certain irrational numbers on the number line and find an irrational number between any two given numbers.
7. understand and describe in words how the negative numbers expand the non-negative rationals to the rational number system.
8. give a meaningful explanation for the impossibility of division by zero.

9. explore interesting topics such as abundant, deficient, or perfect numbers; triangular and square numbers; cubes; palindromes; factorials; and Fibonacci numbers.
10. solve problems and make applications involving percentages.

**Strand Four**
The student will be able to ...

1. calculate missing measurements of similar figures.
2. investigate the relationships between angles formed when parallel lines are cut by a transversal, using diagrams and computer graphics.
3. sketch three-dimensional figures from different perspectives.
4. graph similar figures, reflections, translations, and rotations on a coordinate plane.
5. explore linear relationships graphically using graph paper and computer or calculator graphics.
6. extend experiences validating fundamental geometric theorems.
7. find perimeters (circumferences) and areas of polygons (circles)
8. explore uses of the Pythagorean theorem.
9. find surface areas and volumes of rectangular solids.

**Strand Five**
The student will be able to ...

1. use and relate tables, graphs, and equations to solve problem situations involving exponential growth and decay, simple interest, and compound interest.
2. explain in words the meaning of the expression "solution of an equation."
3. investigate solutions to pairs of simultaneous equations.
4. solve linear inequalities in one variable.
5. interpret problem situations described by linear inequalities in words and graphically.
6. find the distance between two points in the coordinate plane.
7. explore and interpret the concepts of slope and intercept as characteristics of linear functions.
8. describe and solve variation situations, direct and inverse, algebraically and graphically.
9. factor mathematical expressions involving a common factor.
10. select appropriate tools and/or techniques for computation.
11. solve simple number entences and use formulas.

**Strand Six**
The student will be able to meet any previous objective and, in addition ...

1. measure and compute perimeter for irregular polygonal figures.
2. compute area for regular polygonal regions, other composite figures, and lattice (geoboard) figures.
3. make appropriate measurements and compute volume of solids such as prisms, cylinders, pyramids, and cones.
4. recognize and use the concepts of significant digits and accuracy in measurements.
5. recognize and use the concepts of precision and relative error in measurements.
6. successfully utilize a ruler and protractor for specific measurement tasks.
7. read a scale on a measurement device to the nearest mark and make interpolations where appropriate.
8. make change using the process of addition.

**Strand Seven**
The student will be able to ...

1. perform, refine, and extend the objectives listed in previous grades.
2. use fractions, decimals, and percent equivalents interchangeably in making estimates.
3. estimate the square root of a given number to the nearest whole number or range of whole numbers.
4. use estimation to determine the reasonableness of results in all problem solving.
5. extend mental computation to the solution of simple equations.
6. use estimation to eliminate choices in multiple-choice tests.
7. use estimation to determine reasonableness of problem solutions.

**Strand Eight**
The student will be able to ...

1. collect data and create appropriate graphs to illustrate.
2. make identifications, comparisons, and predictions, and solve application problems, using picture, bar, circle, and line graphs.
3. find the mean, mode, median, and range of a set of data and use them in application problems.
4. detect misuses of statistical or numerical information.
5. use elementary notions of probability.
6. explore the role of sampling and collecting data in making a statistical argument.

# GRADE NINE - INTEGRATED

**Strand One**
The student will be able to ...

1. model real-world phenomena with polynomial and exponential functions.
2. explore the relationship between zeros and intercepts of function.
3. translate among tables, algebraic expressions, and graphs of functions.
4. use a graphing calculator or computer to generate the graph of a function.
5. explore the relation between a linear function and its inverse.

In addition, college-intending students will be able to ...

6. describe the general characteristics of polynomial functions and use them in problem-solving situations.

**Strand Two**

Selected objectives as appropriate.

**Strand Three**

Selected objectives as appropriate.

**Strand Four**
The student will be able to ...

1. create and interpret drawings of three-dimensional objects.
2. represent problem situations with geometric models and apply properties of figures.
3. describe and apply the properties of similar and congruent figures.
4. apply the Pythagorean theorem.
5. demonstrate an understanding of angles and parallel and perpendicular lines.

**Strand Five**
The student will be able to ...

1. describe problem situations by using and relating numerical, symbolic, and graphical representations.

2. use the language and notation of functions in symbolic and graphing settings.
3. recognize, relate, and use the equivalent ideas of zeros of a function, roots of an equation, and solution of an equation in terms of graphical and symbolic representations.
4. describe and use the logic of equivalence in working with equations, inequalities, and functions.
5. develop graphical techniques of solution for problem situations involving functions.
6. explore and describe characterizing features of functions.
7. make arguments and proofs in algebraic settings.
8. factor the difference of two squares.
9. determine slope, midpoint, and distance.

and, in addition, college-intending students will ...

10. explore and combine rational functions.
11. explore factoring techniques
12. solve quadratic equations by factoring and formula.
13. set up and solve linear equations.
14. solve systems of linear equations with two variables.

**Strand Six**

Selected objectives as appropriate.

**Strand Seven**

Selected objectives as appropriate.

**Strand Eight**

The student will be able to ...

1. organize data into tables, charts, and graphs.
2. understand and apply measures of central tendency, variability, and correlation.

and, in addition, college-intending students will be able to ...

3. transform data to aid in data interpretation and prediction.

# GRADE TEN - INTEGRATED

**Strand One**
The student will be able to ...

1. explore the conic sections and graph using a graphing calculator or computer.
2. apply trigonometric functions to problem situations involving triangles.

In addition, the college-intending student will be able to ...

3. discover general relationships between the algebraic description of a conic, the kind of conic, and special properties of that conic.

**Strand Two**

Selected objectives as appropriate.

**Strand Three**

Selected objectives as appropriate.

**Strand Four**
The student will be able to ...

1. explore inductive and deductive reasoning through applications to various subject areas.
2. translate between synthetic and coordinate representations.
3. identify congruent and similar figures using transformations with computer programs.
4. deduce properties of figures using transformations and using coordinates.
5. use deductive reasoning.
6. explore compass and straight edge constructions in the context of geometric theorems.

and in addition, college-intending students will ...

7. demonstrate an understanding of and ability to use proof.
8. use a variety of techniques of proof, such as synthetic, transformational, and coordinate proofs.
9. use a variety of proof formats, including the T- (or two-column) proof and the paragraph proof.

10. explore different strategies of proof.
11. investigate different proofs of theorems.
12. develop an understanding of an axiomatic system.

**Strand Five**
The student will be able to ...

1. describe geometric situations and phenomena using variables, equations and functions.
2. describe measures of central tendency, mean, median, mode, and variance, algebraically and graphically.
3. represent inequalities on the number line and in the coordinate plane.

and, in addition, college-intending students will ...

4. use coordinate arguments in making geometric proofs.
5. symbolize transformations of figures and graphs.
6. explore the geometric basis for the functions of trigonometry.
7. graph linear functions.

**Strand Six**

Selected objectives as appropriate.

**Strand Seven**

Selected objectives as appropriate.

**Strand Eight**
The student will be able to ...

1. use curve fitting to predict from data.
2. use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty.
3. use computer simulations and random number generators to estimate probabilities.

and, in addition, college-intending students will be able to ...

4. test hypotheses using appropriate statistics.
5. read, interpret, and use tables, charts, and graphs to identify patterns, note trends, draw conclusions, and make predictions.
6. determine probabilities of events involving unbiased objects.

# GRADE ELEVEN - INTEGRATED

**Strand One**
The student will be able to ...

1. explore periodic real-world phenomena using the sine and cosine functions.
2. analyze the effects of parameter changes on graphs.
3. use a graphing calculator or computer to graph functions.

In addition, the college-intending student will be able to ...

4. develop an understanding of rational and transcendental functions.
5. understand the connection between trigonometric and circular functions.
6. use circular functions to model periodic real-world functions.
7. solve trigonometric equations and verify trigonometric identities graphically and analytically.
8. understand the connections between trigonometric functions and polar coordinates, exponential functions, logarithmic functions, complex numbers, and series.

**Strand Two**

Selected objectives as appropriate.

**Strand Three**

Selected objectives as appropriate.

**Strand Four**
The student will be able to ...

1. apply transformations and coordinates in problem solving.
2. represent problem situations with geometric models and apply properties of figures.

and, in addition, college-intending students will ...

3. deduce properties of figures using vectors.
4. analyze properties of Euclidian transformations and relate translations to vectors.

5. apply vectors in problem solving.
6. further develop understanding of axiomatic systems by investigating and comparing various geometries.

**STRAND FIVE**

The student will be able to ...

1. develop and use vectors to represent direction and magnitude including operations.
2. use polar and parametric equations to describe, graph, and solve problem situations.
3. represent sequences and series as functions both algebraically and graphically.
4. explore recursive functions and procedures using spread sheets and other computer utilities and use notations appropriate to these problem situations.
5. describe and solve algebraic situations with matrices.
6. describe and use the inverse relationship between functions, including exponential and logarithmic.
7. analyze and describe the errors and sources of errors that can be made when using computers and calculators to solve problems.
8. decide when a problem situation is best solved using a computer, calculator, paper and pencil, or mental arithmetic/estimation techniques.

and, in addition, college-intending students will ...

9. explore relationships between complex numbers and vectors.
10. make arguments concerning limits, convergence and divergence in contexts involving sequences, series, and other types of functions.
11. represent transformations in the plane with matrices.
12. contrast and compare the algebras of the rational, real, and complex numbers with characteristics of a matrix algebra system.
13. construct polynomial approximations of a function over specified intervals of convergence.
14. examine complex numbers as zeros of functions.
15. translate verbal statements into symbolic language.
16. simplify algebraic expressions.
17. use the laws and exponents (including scientific notation).

**Strand Six**

Selected objectives as appropriate.

**Strand Seven**

Selected objectives as appropriate.

**Strand Eight**
The student will be able to ...

1. use sampling and recognize its role in statistical claims.
2. design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.

and, in addition, the college-intending student will be able to ...

3. describe, in general terms, the normal curve and use its properties.

# GRADE TWELVE - INTEGRATED

**Strand One**
The student will be able to ...

1. model real-world phenomena with a variety of functions.

In addition, college-intending students will be able to ...

2. graph using polar coordinates .
3. explore graphs in three dimensions.
4. explore functions of several variables.
5. explore recursive functions using spreadsheets and/or programming languages.

**Strand Two**

Selected objectives as appropriate.

**Strand Three**

Selected objectives as appropriate.

**Strand Four**

Selected review objectives from grades 9-11.

**Strand Five**
The college-intending student will be able to ...

1. expand and extend the idea of vectors and linear algebra to higher dimensional situations.
2. use the idea of independent basis elements for a vector space and associated fundamental concepts of finite dimensional linear algebra.
3. develop and communicate arguments about limit situations.
4. use matrices to describe and apply transformations.
5. develop and use polar and parametric equations to represent problem situations.
6. explore proofs by mathematical induction.

**Strand Six**

Selected objectives as appropriate.

**Strand Seven**

Selected objectives as appropriate.

**Strand Eight**
The student will be able to ...

1. create and interpret discrete probability distributions.
2. understand the concept of random variable.

and, in addition, the college-intending student will be able to ...

3. apply the concept of a random variable to generate and interpret probability distributions, including binomial, uniform, normal, and chi square.

# ALGEBRA I

| Strand | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Grade 9 | Alg I | Selected Items | Selected Items | | Alg I | Selected Items | Selected Items | |
| Grade 10 | Alg I | | | | Alg I | | | |
| Grade 11 | | | | | | | | |
| Grade 12 | | | | | | | | |

Include all objectives from Strand One, grades nine and ten, as well as all objectives from Strand Five, grades nine and ten. In addition, include selected items from other strands, particularly two, three, six, and seven.

# GEOMETRY

| Strand | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Grade 9 | | Selected Items | Selected Items | GEO | | Selected Items | Selected Items | |
| Grade 10 | | | | GEO | | | | |
| Grade 11 | | | | GEO | | | | |
| Grade 12 | | | | | | | | |

Include all objectives from Strand Four plus selected objectives from other strands.

# ALGEBRA II

| Strand | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Grade 9 | | Selected Items | Selected Items | | | Selected Items | Selected Items | Alg II |
| Grade 10 | | | | | | | | Alg II |
| Grade 11 | Alg II | | | | Alg II | | | |
| Grade 12 | | | | | | | | |

Include objectives from Strand One, grade eleven, Strand Five, grade eleven, and Strand Eight, grades nine and ten, plus selected items from other strands.

# ADVANCED MATHEMATICS

| Strand | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Grade 9 | | Selected Items | Selected Items | | | Selected Items | Selected Items | |
| Grade 10 | | | | | | | | |
| Grade 11 | | | | | | | | Adv Math |
| Grade 12 | Adv Math | | | | Adv Math | | | Adv Math |

Include objectives from Strand One, grade 12, Strand Five, grade 12, and Strand Eight, grades 11 and 12.

# CALCULUS

Many students will be ready for the study of calculus during the twelfth grade. Schools implementing a calculus course should participate in the Advanced Placement Calculus program and require that students take the Advanced Placement (AP) examination. The AP program provides carefully and thoroughly detailed syllabi for two different calculus courses--the AB course and the BC course. The syllabi are sufficient for purposes of planning.

# PERFORMANCE OBJECTIVES

## INTRODUCTION

The goal of a competency-based education program is to ensure that every student is achieving at grade level and that each student is challenged to the limit of his/her abilities. Some words of warning about the relation of competency to reaching criteria on pupil performance objectives. All of us forget. Students forget. What does this mean for a teacher of mathematics operating within the context of competency-based education and proficiency examinations? It must be remembered that a student who has exhibited a competency while a unit is being taught may not remember the concept or skill when tested months later on a critical competency or proficiency test. The teacher who has behaved responsibly is not at fault in such a situation.

The model curriculum has been designed, in part, to account for how memory works. We think in terms of the three Rs of memory — Recognition, Recall, and Reconstruction. Recognition is simply looking at something and realizing that it is familiar, something seen before. Recall is the automatic response from a stimulus that causes one to do something without having to think. Each of these two types of memory require a relatively low level of mental functioning. Reconstruction is different in that it means that one may retrieve an idea by building it up from other ideas available in memory. For example, if asked how many windows are in a house or apartment, most people reconstruct the number of windows by imagining being present in each room and combining the number of windows to obtain the total. In mathematics, if one is asked to do something not remembered, the procedure can be reconstructed provided one can remember a related idea and build on it. Research evidence indicates that students who have had instruction that stressed connections between and among ideas and focused on problem solving are much more capable of reconstructing ideas. Reconstruction memory is the most powerful and important kind of memory.

Additionally, each strand is designed to provide opportunity for teachers to provide experiences that maintain and extend objectives established in earlier grades. If students keep ideas current through use, the ideas will remain available in memory. Using and building on ideas is more effective than activities specifically designed for purposes of review.

The timing of assessment of the pupil performance objectives is critically important. Assessment of student understanding and mastery of concepts prior to extensive practice in the use of the concept is essential. Assessment in all categories should be timed so that long-term rather than short-term retention is judged.

## CATEGORIES OF PERFORMANCE OBJECTIVES

The critical objectives selected to become pupil performance objectives fall into four different categories, each with a different type of assessment. The categories are ...

### A. Knowledge and Skill

The student should be able to recall certain basic mathematical facts and perform certain procedures rapidly and accurately. Objectives in this category are most appropriately assessed by a paper-and-pencil test with a criterion that demands a high percentage of accuracy. Such a criterion is possible when students are given repeated opportunities to reach the required degree of accuracy. It is important to note that emphasis on rapid and accurate performance of procedures is inappropriate if correct underlying mathematical concepts have not yet been formed. When a student is having difficulty implementing procedures, it may be due to incomplete or incorrect understanding of concepts. Check to see if this is the case.

The basic construct of a knowledge and skill performance objective is:

Given a skill/knowledge task, the student will complete the task with an accuracy of _____%.

For example,

Given a set of fractions, the student will add, subtract, multiply, and divide them as directed with an accuracy of 90%.

An item that would assess this objective might be:

Add, subtract, multiply, or divide the following pairs of fractions as indicated -

1. $\frac{7}{9} + \frac{5}{9}$
2. $4\frac{4}{5} - 2\frac{3}{5}$
3. $\frac{37}{40} \times \frac{8}{12}$
4. $2\frac{2}{3} + 3\frac{1}{2}$
5. $\frac{4}{5} - \frac{1}{2}$
6. $\frac{7}{25} + \frac{27}{100}$
7. $\frac{3}{5} + \frac{13}{60}$
8. $2\frac{4}{9} + 4$
9. $4\frac{1}{8} \times 5\frac{1}{3}$
10. $\frac{13}{8} - \frac{4}{8}$

## B. Concept

Of critical importance in the educational process is the determination of whether students are developing correct concepts. Assessment of student concepts is difficult using paper-and-pencil testing. The teacher, as a knowledgeable professional, is best prepared to determine which students have sufficient understanding of a mathematical idea or procedure. This may be most easily accomplished through observation, questioning, and/or interview. Concept assessment may be formal or informal. It may occur as a planned activity, in the context of knowledge and skill assessment, or within a classroom teaching situation. The principal use of concept assessment is to assist the teacher in planning appropriate instruction.

The basic construct of a concept performance objective is as follows:

The student will demonstrate an understanding of _____ by (showing, modeling, explaining in words, ...).

For example,

The student will demonstrate an understanding of place value:

a. by modeling specified numbers with base ten blocks,
b. by identifying the numbers which immediately precede and follow a specified number, and
c. by indicating which digit of a given number has the greatest value.

Appropriate questions to ask could be

- Why did you ...?
- How do you know ...?
- Are you sure ...?
- Could you explain this to ...?
- What else ...?

## C. Problem solving/application

While correct answers in problem-solving or application situations are always important, for instructional purposes the processes used by the learner are of greater importance. A method of assessment that gives an indication of the student's understanding of concepts and his/her ability to make connections in mathematics, solve problems, and verify results as well as perform correct computations is a holistic scoring of the student's problem-solving process. The basis of holistic scoring is a description of specific expectations of responses at several levels of proficiency (rubric) and examples of student responses that typify each level (sometimes referred to as anchor papers).

An example of a rubric is

**0 Points**
- Nothing is done, except to recopy the problem
- Only an incorrect answer is given

**1 Point**
- Some interpretation of the given data is made
- Clearly the student is on the wrong track
- There is no evidence that a second strategy was tried
- No subgoal was reached

**2 Points**
- Evidence of some understanding of the problem
- A strategy that would work was started
- A subgoal is reached
- A correct answer is given, but it is impossible to tell how it was reached

**3 Points**
- Appropriate solution strategy but the answer is wrong
- Correct answer but the solution process is not clear

**4 Points**
- Correct strategy
- Copying error caused incorrect answer
- Correct answer but no attempt to justify

**5 Points**
- Correct strategy
- Correct answer that has been justified
- Considerable insight shown in approach to problem

The basic construct of a problem-solving/application performance objective is as follows:

Given a mathematical or application situation involving _____, the student will write a solution and receive at least a score of ___ as determined by the district holistic scale.

or

Given a mathematical project involving _____, the student will complete the project, describe the procedure either orally or in written form, and display the results. A score of ___, as determined by an appropriate rubric, is required.

For example,

> Given an application involving percentages, the student will write an explanation of the process used to find the required solution and receive a score of ___, as determined by the district holistic scale.

An item for this objective might be:

> Sue and four of her friends plan to order school jackets from Schoolhouse Catalogue Company. The cost of each jacket is $60.00, but if 4 or more are purchased, a saving of 1/3 the cost of each jacket is realized. If the sales tax is 5.5% and shipping is 8%, how much would each jacket cost?

### D. Mathematical Disposition

Confidence, willingness to undertake and persevere in mathematical tasks, and valuing mathematics for its influence in their lives are important characteristics of successful students of mathematics. The habit of monitoring and reflecting on their own performance, making sense of their mathematics, and being flexible, insightful, and inventive in doing mathematics are also marks of competent students. To assess these mathematical dispositions, a combination of student self-report questions and a teacher observation checklist should be compared. Teachers should observe students on an ongoing basis and record significant events, either good or bad. Checklists should be updated on a regular basis, at least at the end of each school year. Students that exhibit a lack of any of these characteristics should be given additional support, encouragement, and attention.

Sample questions for these kinds of performance objectives are as follows:

Do I (or does the student, if the teacher is observing)

like to do mathematics?
stick with problems?
check solutions to see if they are reasonable?
think that mathematics is important for me?
work cooperatively with others?
think up different ways to do problems?

## CRITICAL OBJECTIVES

The following have been selected from the objectives in the model curriculum. While all objectives are important and must be taught and assessed, critical objectives are sufficiently important to require a special kind of assessment and intervention in a competency-based program.

These objectives do not relate to minimum competency, but are identified as those objectives critical to the mathematical development of students. As such, a performance objective which assesses whether students have a correct concept, or can use the concept with knowledge and skill, or in a problem solving or application situation should be developed for each critical objective. In the case of critical objectives which are also outcomes assessed by the Ninth Grade Proficiency Test, performance objectives are provided at all three levels (see p. 179).

**Kindergarten**

The student will ...

1-3. continue a pattern.
2-3. act out a problem situation.
3-1. relate numbers to numerals.
3-3. compare number of objects in two or more sets.
3-4a. count objects.
4-1. identify common shapes in the environment.
4-4. identify common solids in the environment.
6-1. compare and order objects of different lengths, weights, and capacities.
8-3. create a floor/table graph by arranging actual objects in appropriate categories.

**Grade One**

The student will ...

1-2. sort objects on multiple attributes.
2-1. select appropriate notation and methods for symbolizing the problem statement and the solution process.
3-2. decompose, combine, order, and compare numbers.
3-5. develop the concept of addition and subtraction from situations in the environment, including joining, separating, and comparing sets of objects.
3-6. learn strategies for addition of whole numbers such as
   a. counting all
   b. counting on
   c. one more, one less
   d. two more, two less
   e. doubles
   f. doubles plus or minus one and doubles plus or minus two
   g. make ten
   h. using ten frames
3-7. describe the operation of addition and subtraction in words.
4-3. identify two-dimensional shapes on three-dimensional objects.
5-3. model a problem situation using a number phrase or sentence.

6-1. recognize and count a collection of coins including pennies, nickels, dimes, and a quarter and determine its value.
6-2. measure lengths, using non-standard units, centimeters, and inches.
7-3. estimate the value of a given collection of coins.
8-5. identify events that are sure to happen, events that are sure not to happen, and those we cannot be sure about.

**Grade Two**

The student will be able to ...

1-2. recognize patterns in numbers and number combinations.
2-1. identify needed information to solve a problem.
3-1. develop the concept of place value with concrete models of hundreds, tens, and ones.
3-4. illustrate fractional parts of whole objects or sets of objects.
3-7. learn strategies for the addition and subtraction of numbers such as
   a. compatible numbers
   b. compensatory numbers
   c. borrow and pay back (subtraction)
   d. regrouping
   e. using a calculator
3-9. translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics.
4-5. compare three-dimensional objects describing similarities and differences using appropriate standard and non-standard language.
5-3. model a problem situation using numbers and/or letters.
6-1. explore length, capacity, and weight by selecting and using appropriate metric and conventional units such as centimeter, inch, liter, cup, pint, quart, kilogram, and pound.
6-3. tell time to the nearest five-minute interval on digital and dial timepieces.
7-5. make estimates in addition and subtraction, using front-end digits.
8-3. explore picture and bar graphs (scales by one) by making identifications. comparisons, and predictions.

**Grade Three**

The student will be able to ...

1-3. use patterns to make generalizations and predictions by
   a) determining the rule and identifying missing numbers in a sequence of numbers;
   b) determining the rule and identifying missing numbers in a table of number pairs; and
   c) identifying missing elements in a pattern and justifying their inclusion.

2-5. extend the guess-and-check procedure by recording guesses and checks to help make better guesses until the solution is reached.
3-1. add and subtract numbers fluently using any strategy.
3-3. from situations created in the classroom
   a. develop models of multiplication and division (arrays)
   b. use invented and conventional symbols to represent multiplication and division
   c. describe multiplication and division in words.
3-8. recall multiplication and division facts through 12 x 12 using strategies such as
   a. commutative property
   b. distributive property
   c. anchor facts
   d. squares.
3-16. use the symbols $<$, $\leq$, $>$, $\geq$, and $=$ in describing order as well as the terms "at least" and "at most."
4-4. describe a three-dimensional object from different perspectives.
5-1. explain in words thinking strategies for making computations.
6-2. count collections of coins and bills which include one, five, and ten dollar bills and compare values.
6-5. illustrate the approximate size of units (inch, centimeter, meter, and yard).
7-5. add strings of numbers mentally by finding groups of tens.
8-7. translate freely among pictographs, tables, charts, and bar graphs.

**Grade Four**

The student will be able to ...

1-1. explore and describe in words simple and complex patterns in nature, art, and poetry.
2-3. make and use a table to record and sort information.
3-6. order fractions using symbols.
3-8. add and subtract with concrete materials and symbolically.
3-10. add and subtract decimals.
3-11. multiply and divide whole numbers fluently.
3-13. compare and order numbers with any number of digits.
4-2. identify parallel lines, perpendicular lines, and right angles in geometric figures and the environment.
4-4. determine properties of two-dimensional figures and compare shapes according to their characterizing properties.
5-3. symbolize a keying sequence on a calculator with arithmetic logic to execute the computation of an arithmetic phrase and predict the display as each key is pressed.
6-2. measure and determine perimeters and areas of simple straight line figures and regions without using formulas.
6-6. make change using coins and bills.

6-7. use mental, paper-and-pencil, and physical strategies to determine time elapsed.
7-3. explore estimates of sums and differences and determine whether they are greater than or less than the exact sum or difference.
8-6. find simple experimental probabilities.

**Grade Five**

The student will be able to ...

1-2. investigate the patterns of digits formed when fractions are changed to decimal form.
2-2. read a problem carefully and identify subgoals that need to be attained in order to solve the problem.
3-3. multiply and divide decimals.
3-4. find equivalent fractions.
3-6. order combinations of whole numbers, fractions, and decimals using the symbols $<$, $\leq$, $>$, $\geq$, and $=$ and by placing them on the number line.
3-7. explore order of operations relative to calculators with arithmetic and algebraic logic.
3-10. round, as appropriate to a problem situation, to the nearest thousand, hundred, ten, one, tenth, or hundredth.
4-1. compare and contrast angles in relation to right angles.
4-4. build models of previously encountered shapes and figures and describe the process in words.
5-4. interpret tables that describe problem situations.
6-3. determine what to measure and measure in order to determine perimeters, areas, and volumes of simple shapes and solids.
7-3. round fractions to 0, 1/2, and 1 and use these values to estimate sums and differences of fractions.
8-5. explore the concept of average and calculate the arithmetic mean of a given set of numbers.

**Grade Six**

The student will be able to ...

1-2. explore the relation between doubling the side of a square and/or other regular figures and the corresponding increase in area.
2-3. identify needed and given information in a problem situation as well as irrelevant information.
3-1. compute with whole numbers, fractions, and decimals.
3-6. change freely between fractions and decimals.
3-7. understand and describe in words the relations between addition, subtraction, multiplication, and division.
4-2. estimate the measure of angles and draw angles that approximate given measures.

4-4. visualize and show the results of a rotation, translation, reflection, or stretching.
4-7. recognize, classify, and use characteristics of lines and simple two-dimensional figures.
5-6. solve linear equations, using concrete representations.
6-3. convert, compare, and compute with common units of measure within the same measurement system.
7-5. estimate the product or quotient of decimal numbers by rounding them to a single decimal place and then performing the operation.
8-3. read, interpret, and use tables, charts, maps, and graphs to identify patterns, note trends, and draw conclusions.
8-9. make predictions of outcomes of experiments based on theoretical probabilities and explain actual outcomes.

**Grade Seven**

The student will be able to ...

1-1. describe and represent relationships with tables, graphs, rules, and words.
2-1. use an open sentence (algebraic equation) to symbolize a problem situation and solve the equation to find a solution to the problem.
3-3. solve and use proportions.
3-4. develop the concept of integers using concrete models, including number lines, and in the context of real-world situations.
3-5. compare, order, and determine the equivalence of whole numbers, fractions, decimals, percents, and integers.
4-3. develop minimum sets of properties that describe a geometric figure.
4-6. validate fundamental geometric theorems using manipulative materials and informal arguments.
5-3. describe problem situations involving ratios, proportions, and percents with algebraic expressions.
5-5. evaluate algebraic expressions (simple substitutions).
6-1. select and compute with appropriate standard or metric units to measure length, area, volume, weight, capacity, time, money, and temperature.
6-4. state and apply area formulas for the following regions: circular, rectangular, parallelogram, trapezoidal, and triangular.
6-6. determine formulas for surface area.
7-3. estimate with percents, using 1%, 10%, and 50%, and multiples of these numbers.
8-2. create, read, and interpret tables, charts, diagrams, and maps.
8-9. compute averages.

**Grade Eight**

The student will be able to ...

1-3. explore the right triangle relations sine, cosine, and tangent and their applications in measurement.
2-3. validate and generalize problem solutions.
3-1. understand, represent, and use numbers written in a variety of equivalent forms in real-world and mathematical problem-solving situations.
3-2 develop an understanding of operations with integers using the number line and other models of integers.
3-10. solve problems and make applications involving percentages.
4-1. calculate missing measurements of similar figures.
4-7. find perimeters (circumferences) and areas of polygons (circles).
4-9. find surface areas and volumes of rectangular solids.
5-11. solve simple number sentences and use formulas.
6-2. compute area for regular polygonal regions, other composite figures, and lattice (geoboard) figures.
6-3 make appropriate measurements and compute volumes of solids such as prisms, cylinders, pyramids, and cones.
6-7. read a scale on a measurement device to the nearest mark and make interpolations where appropriate.
7-2. use fractions, decimals, and percent equivalents interchangeably in making estimates.
8-3. find the mean, mode, median, and range of a set of data and use them in application problems.
8-5. use elementary notions of probability.

**Grade Nine**

The student will be able to ...

1-3. translate among tables, algebraic expressions, and graphs of functions.
1-4. use a graphing calculator or computer to generate the graph of a function.
3-1. compare, order, and determine equivalence of real numbers.
3-2. estimate answers, compute, and solve problems involving real numbers.
4-3. describe and apply the properties of similar and congruent figures.
4-4. apply the Pythagorean theorem.
4-5. demonstrate an understanding of angles and parallel and perpendicular lines.
5-3. recognize, relate, and use the equivalent ideas of zeros of a function, roots of an equation, and solution of an equation in terms of graphical and symbolic representations.
5-5. develop graphical techniques of solution for problem situations involving functions.
5-9. determine slope, midpoint, and distance.
8-1. organize data into tables, charts, and graphs.

8-2. understand and apply measures of central tendency, variability, and correlations.

In addition, the college-intending student will be able to ...

1-6. describe the general characteristics of polynomial functions and use them in problem-solving situations.
5-11. explore factoring techniques.
5-12. solve quadratic equations by factoring and formula.
5-13. set up and solve linear equations.
5-14. solve systems of linear equations with two variables.

**Grade Ten**

The student will be able to ...

1-2. apply trigonometric functions to problem situations involving triangles.
4-5. use deductive reasoning.
4-6. explore compass and straight edge constructions in the context of geometric theorems.
5-3. represent inequalities on the number line and in the coordinate plane.
8-1. use curve fitting to predict from data.
8-3. use computer simulations and random number generators to estimate probabilities.

In addition, the college intending student will be able to ...

4-8. use a variety of techniques of proof, such as synthetic, transformational, and coordinate proofs.
4-12. develop an understanding of an axiomatic system.
5-4. use coordinate arguments in making geometric proofs.
5-7. graph linear functions.
8-5. read, interpret, and use tables, charts, and graphs to identify patterns, note trends, draw conclusions, and make predictions.
8-6. determine probabilities of events involving unbiased objects.

**Grade Eleven**

The student will be able to ...

1-2. analyze the effects of parameter changes on graphs.
5-1. develop and use vectors to represent direction and magnitude including operations.
5-6. describe and use the inverse relationship between functions including exponential and logarithmic.
8-2. design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes.

In addition, the college-intending student will be able to ...

1-5. understand the connection between trigonometric and circular functions.
1-7. solve trigonometric equations and verify trigonometric identities graphically and analytically.
4-4. analyze properties of Euclidian transformations and relate translations to vectors.
4-6. further develop understanding of axiomatic systems by investigating and comparing various geometries.
5-10. make arguments concerning limits, convergence and divergence in contexts involving sequences, series, and other types of functions.
5-14. examine complex numbers as zeros of functions.
5-15. translate verbal statements into symbolic language.
5-16. simplify algebraic expressions.
5-17. use the laws of exponents (including scientific notation).
8-3. describe, in general terms, the normal curve and use its properties.

**Grade Twelve**

The student will be able to ...

1-1. model real-world phenomena with a variety of functions.
8-2. understand the concept of random variable.

In addition, the college-intending student will be able to ...

1-5. explore recursive functions using spreadsheets and/or programming languages.
5-3. develop and communicate arguments about limit situations.
5-4. use matrices to describe and apply transformations.
5-5. develop and use polar and parametric equations to represent problem situations.
5-6. explore proofs by mathematical induction.
8-3. apply the concept of a random variable to generate and interpret probability distributions, including binomial, uniform, normal, and chi square.

# PERFORMANCE OBJECTIVES
# (NINTH GRADE PROFICIENCY TEST OUTCOMES)

Since passing the Ninth Grade Proficiency Test will determine whether or not a student will graduate from high school, the outcomes assessed on that test have been included verbatim at grade levels five through eight in the model curriculum. It is important that students not only have the opportunity to learn the mathematics described in the proficiency test outcomes, but that their skill and understanding be assessed in the classroom, and appropriate intervention provided. Accordingly, the outcomes are also included (along with other objectives) in the list of performance objectives. Since outcomes are assessed on the proficiency test at three different levels of use, they should be assessed in the classroom at the same three levels. The proficiency test uses a multiple-choice format and necessarily sacrifices depth of assessment for ease and economy of scoring. Such a compromise is neither necessary nor appropriate for a competency program.

Performance objectives based on the proficiency outcomes and sample items to assess them at each level are ...

**Grade Five**

3-10. round, as appropriate to a problem situation, to the nearest thousand, hundred, ten, one, tenth, or hundredth.

A. Given a set of numbers, the student will round them to the specified place value with 100% accuracy.

Round each number to the nearest thousand, hundred, and ten.

| | | | |
|---|---|---|---|
| 1045.356 | _____ | _____ | _____ |
| 43,679.03 | _____ | _____ | _____ |
| 23,457,223 | _____ | _____ | _____ |

Round each number to the nearest one, tenth, and hundredth.

| | | | |
|---|---|---|---|
| 2345.667 | _____ | _____ | _____ |
| 347 | _____ | _____ | _____ |
| 63,421.045 | _____ | _____ | _____ |

B. The student will demonstrate understanding of the concept of rounding by

a. selecting a number and describing the process for determining when to change a digit to the next larger and when to leave it unchanged,
b. explaining how to determine which digits are replaced by zeros, and
c. giving an example of a situation when it is appropriate to round up to a whole number, even though the decimal part of the number is less than one-half.

C. Given a problem/application situation that involves rounding, the student will solve the problem and achieve a score of ___ according to the district holistic scale.

A group of soldiers need to be transported by truck to a distant camp. Trucks can carry 36 soldiers at a time. How many trucks are needed if they can make two trips each and there are 300 soldiers to be transported? Explain your solution process and justify your result.

**Grade Six**

3-1. compute with whole numbers, fractions, and decimals.

A. Given a set of whole numbers, fractions, and decimals, the student will add, subtract, multiply, and divide them with 90% accuracy.

$4376\frac{1}{3}+296=$ $\quad$ $3.651-2.7=$

$425\div 15=$ $\quad$ $71\div 0.6=$

$2\frac{2}{3}+7\frac{1}{4}=$ $\quad$ $23.7+1.42=$

$4.35\times 0.16=$ $\quad$ $2\frac{1}{3}\times\frac{1}{2}=$

$5\times 21\frac{1}{4}=$ $\quad$ $5\frac{2}{5}-2\frac{1}{4}=$

B. The student will demonstrate understanding of computation with whole numbers, fractions, and decimals by

a. giving examples of situations where computation with each kind of number would be required,
b. explaining the relation between adding fractions and adding decimals,
c. explaining a procedure for multiplying mixed numbers, and
d. relating multiplication of fractions to a rectangular area model.

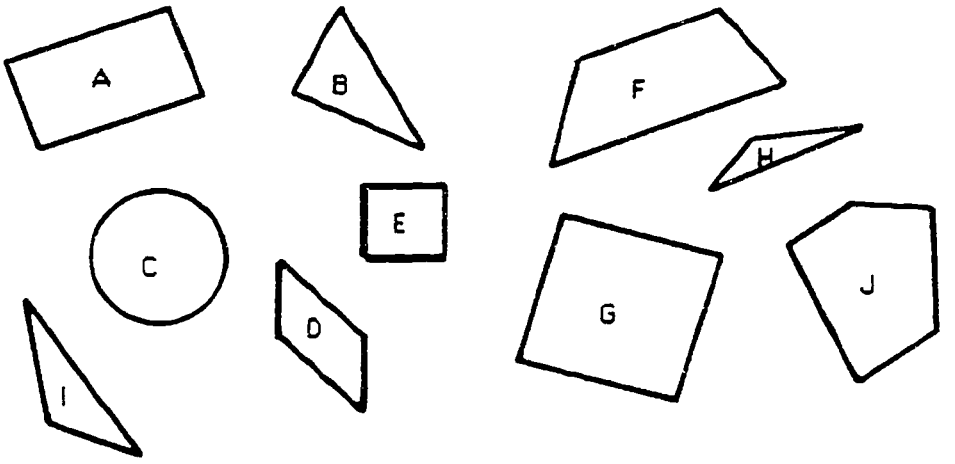
C. Given a problem/application situation involving computation with whole numbers, fractions, and/or decimals, the student will solve the problem and achieve a score of ___ on the district holistic scale.

Marveena bought an assortment of cheeses at the market. She bought 2.3 pounds of cheddar, 1.7 pounds of swiss, and 3/4 of a pound of feta cheese. The plastic bag she carried was guaranteed to hold four pounds. Should she use that bag to carry the cheese? Defend your answer.

4-7. recognize, classify, and use characteristics of lines and simple two-dimensional figures.

A. Given a set of labeled figures consisting of triangles, quadrilaterals, rectangles, squares, parallelograms, and trapezoids, the student will identify them by a correct mathematical name and indicate lines that are perpendicular and lines that are parallel with 100% accuracy.

Identify each figure and list figures that have parallel lines and those that have perpendicular lines.

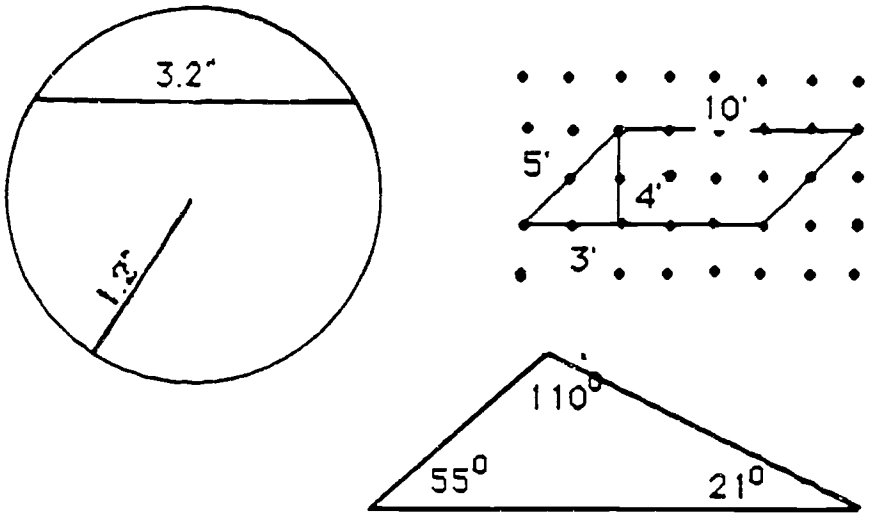



B. The student will demonstrate understanding of simple two-dimensional figures by

a. describing the relationship between squares and rectangles, rectangles and parallelograms, and parallelograms and trapezoids, and
b. give examples of such figures in the environment.

C. Given a problem/application situation involving lines and/or simple two-dimensional figures, the student will solve the problem and achieve a score of ____ as measured on the district holistic scale.

The following figures are not drawn to scale. Discuss the indicated measurements.




6-3. convert, compare, and compute with common units of measure within the same measurement system.

A. Given a set of computation items involving units of measure, the student will perform the computation with 90% accuracy.

Find the following:

1. 2 ft 11 in
   + 7 ft 10 in

2. 3 yd 2 ft
   + 8 yd 1 ft

3. 1 h 25 min
   + 3 h 19 min

4. 3 qt 1 pt
   + 2 qt 1 pt

5. 4 m 13 cm
   + 3 m 46 cm

6. 14 ft 3 in
   - 8 ft 8 in

7. 5 yd 8 in
   x 2

8. 25 h 4 min 50 s
   - 10 h 55 min 5 s

9. 4 qt
   - 1 qt 1 pt

10. 6 m 83 cm
    - 2 m 94 cm

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B. The student will demonstrate understanding of units of measure by

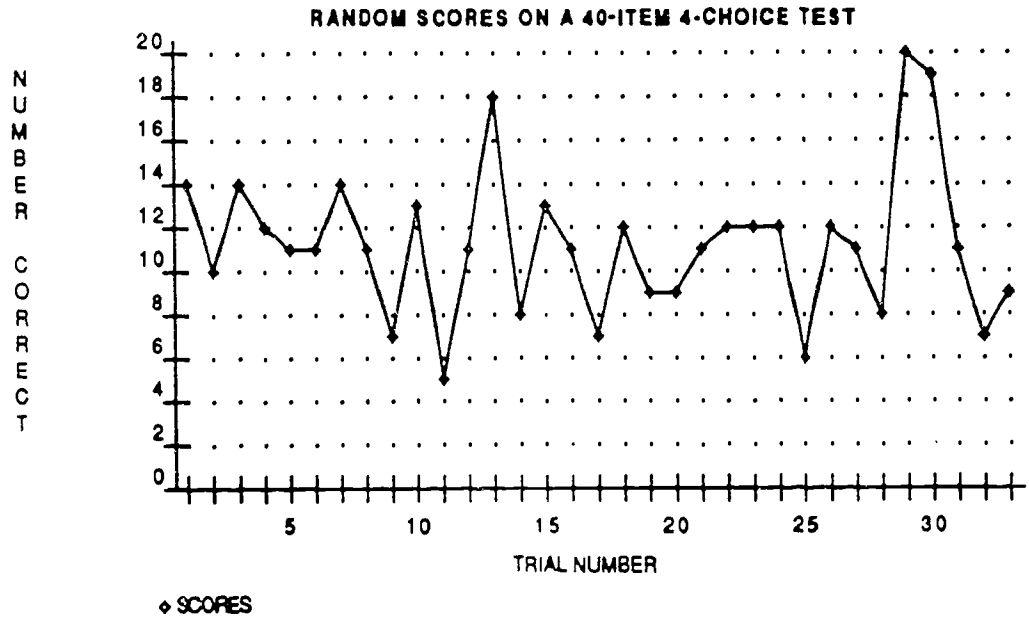
a. describing the process of computing with denominate numbers and
b. comparing to place value and work with fractions.

C. Given a mathematical or application problem involving comparing, converting, and/or computing with units of measure, the student will solve the problem and achieve a score of ___ on the district holistic scale.

A class at Union School is given old books for recycling. The students are told that they have to bundle the covers separately. A stack of 40 books is one meter high. If covers are 0.25 cm thick and each cover is removed, find the height of the stack of pages of the books in centimeters. Validate your answer.

8-3. read, interpret, and use tables, charts, maps, and graphs to identify patterns, note trends, and draw conclusions.

A. Given a table, chart, map, or graph, the student will be able to read information from it with 100% accuracy.




The expected chance score on this test is 10. Of the first 10 trials, how many scores were exactly 10? How many were below 10? How many trials are indicated on the graph? How many of these are exactly 10? How many are above 10? If there were 10,000 trials, about what would the average score be?

B. The student will demonstrate understanding of tables, charts, maps, and graphs by

a. comparing the advantages and disadvantages of graphs and tables,
b. giving an example of a misleading graph, and
c. explaining how to convert from one form to another.

C. Given a mathematical or application problem situation involving the use of tables, maps, charts, or graphs, the student will solve the problem and achieve a score of ___ on the district holistic scale.

The table gives information about the length, width, and perimeter of some rectangles that have a length three units greater than the width. What is the length of a rectangle that has a perimeter of 23 units? Justify your answer.

| width | length | perimeter |
|---|---|---|
| 3 | 6 | 18 |
| 4 | 7 | 22 |
| 5 | 8 | 26 |
| 6 | 9 | 30 |

**Grade Seven**

3-3. solve and use proportions.

A. Given three terms of a proportion, the student will find the fourth with an accuracy of 100%.

Find the missing term in the following proportions

$\frac{a}{3} = \frac{4}{12}$ $\qquad$ $\frac{4}{5} = \frac{c}{7}$

$\frac{2}{b} = \frac{4}{14}$ $\qquad$ $\frac{6}{7} = \frac{8}{d}$

B. The student will demonstrate understanding of proportion by selecting a correct proportion to use in an application situation and explaining why it is correct.

If a 5" x 7" picture is enlarged so that the longer side is 21", what is the proportion used to find the shorter side? Can you write this proportion with the ratio 5/7 or 7/5?

C. Given an application situation involving proportions, the student will write a solution and receive a score of ___ as measured by the district holistic scale.

Twenty tagged fish are released into a pond. Some time later a fisherman catches seven fish. Two of these are tagged. About how many fish are now in the pond? Write out your solution completely.

3-6. compare, order, and determine equivalence of fractions, decimals, percents, whole numbers, and integers.

A. Given a set of fractions, decimals, percents, whole numbers, and integers, the student will order them as requested with 100% accuracy.

Place the following numbers in order from least to greatest

.205, 2/3, 45%, .37, 3/5, 25.5%

B. The student will demonstrate an understanding of relative size of fractions, decimals, percents, whole numbers, and integers by placing them on a number line and giving an explanation upon request.

C. Given an application or mathematical problem situation involving comparison of numbers in a variety of forms, the student will write a solution and achieve a score of ___ on the district holistic scale.

At the store, a 16-pound container is filled in the following manner — 25% raisins, 2 pounds of pecans, one-half pound of dates, and 3.5 pounds of walnuts. The remainder consists of peanuts. Arrange the items in order from least to greatest weight. Defend your answer.

5-5. evaluate algebraic expressions (simple substitutions).

A. Given algebraic expressions and values for the variable(s), the student will evaluate the expressions with 90% accuracy.

Evaluate the following if a = 4, b = 3 and c = 2

| | | | |
|---|---|---|---|
| 1. | $a(b+3c^2)$ | 6. | $(a+2b)^2$ |
| 2. | $\frac{a+b}{c+4}$ | 7. | $2ab^2$ |
| 3. | $2ab+3c$ | 8. | $(a+b)(a-b)$ |
| 4. | $a^2b-c^2$ | 9. | $a-\frac{b}{2c}$ |
| 5. | $a^2+3a+4$ | 10. | $1.3a+1.5c$ |

B. The student will demonstrate understanding of algebraic syntax by describing a procedure to evaluate an expression and answering questions as required.

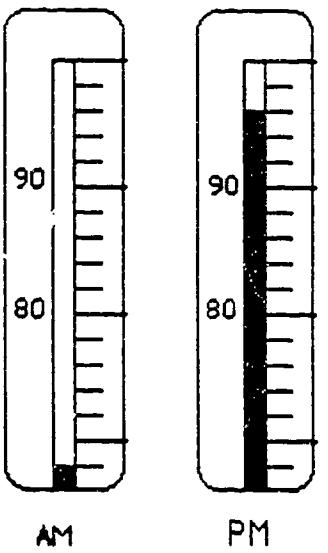
C. Given a problem-solving/application situation involving the evaluation of an algebraic expression, the student will find the solution and achieve a score of ___ on the district holistic scale.

List a sequence of calculator key presses to evaluate the expression $a + 3b$ (when $a = 4$, $b = 5$) for a calculator with algebraic logic and for one with arithmetic logic. Compare the results and justify your answer.

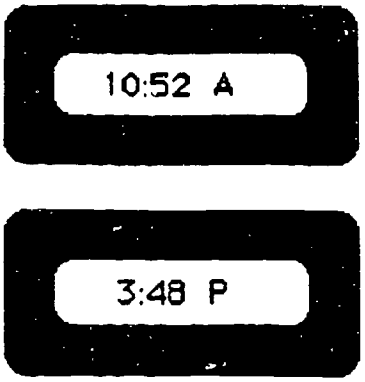
6-1. select and compute with appropriate standard or metric units to measure length, area, volume, weight, capacity, time, money, and temperature.

A. Given items that include various measurements, the student will select the appropriate measures and determine length, area, volume, etc., as indicated, 90% of the time.

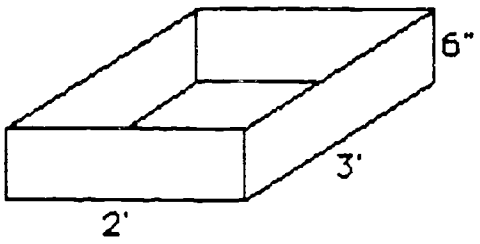
The thermometers represent temperatures on a day in August.



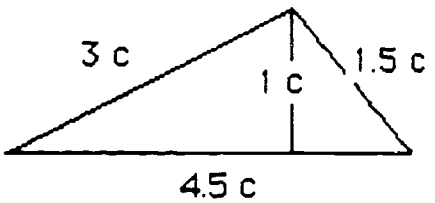

What is the difference between the high and low temperature?



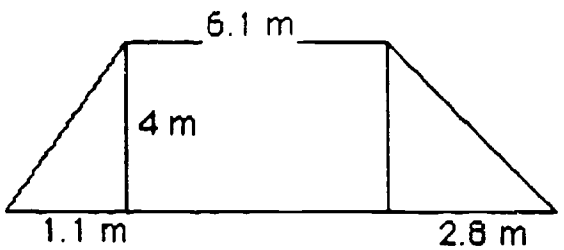

The timepieces above indicate when the radio was turned on and off. How long was it on?




How many cubic feet will the box above contain? (Look at the units carefully)




What is the area and the perimeter of the figure above?




What is the perimeter of the figure above?

B. The student will demonstrate understanding of selection and computation with standard and metric units by

a. explaining the selection of units in several examples, and
b. explaining how to correct incorrect examples of computation.

C. Given a mathematical or application problem situation involving selection and computation with standard or metric units to determine length, area, and/or volume, the student will solve the problem and achieve a score of ___ on the district holistic scale.

One wall of a building is 80 feet long and 16 feet high. A one-gallon can of paint covers up to 450 square feet. How many cans of paint are needed to paint the wall? If each can of paint costs $16.50, find the total cost of paint for the wall. Verify your answer.

8-9. compute averages.

A. Given several sets of numbers, the student will find the average of each set 100% of the time.

Find the mean and median of each set of numbers:

1. 30, 40, 23, 17, 10
2. 0.4, 5.1, 3.5, 6
3. 16, 17, 18, 19, 20
4. 21, 21, 19, 19, 20
5. 152, 316, 232
6. 35500, 36500

B. The student will demonstrate understanding of average by

a. estimating an average of a set of numbers and giving reasons for that choice, and
b. explaining whether the mean, mode, or median is the most representative average for a given situation.

C. Given a problem/application situation involving averages, the student will solve the problem and achieve a score of ___ on the district holistic scale.

Pedro has a 75% average on four tests. He needs an 80% average to earn a B-. Can he raise his average to 80% by taking one more test? Justify your answer.

**Grade Eight**

3-10. solve problems and make applications involving percentages.

A. Given a variety of exercises involving rate, base, and percentage where one is missing, the student will find the missing number with 90% accuracy.

Do the following:

1. What is 31% of 90?
2. 20 is what % of 40?
3. 12 is 8% of what?
4. What is 10% of 25?
5. 8.5% of $40,000 = ____
6. 24 is what percent of 30?
7. Find 30% of 90.
8. 12 is 20% of what?
9. 82% of ? = 205
10. What percent is 64 of 48?

B. The student will demonstrate understanding of percent by

a. illustrating various percents on a 10 x 10 grid,
b. comparing percents to fractions, and
c. using a model to find percent ( e.g., 75% of 48 marbles)

C. Given a problem-solving/application situation involving percent, the student will solve the problem and achieve a score of ___ on the district holistic scale.

Congratulations! You have just won this week's lottery. Your winnings total $1,000,000. You place your winnings in a bank Certificate of Deposit for 18 months, and the Certificate has an annual rate of 8% compounded monthly. How much money will you have at the end of the eighteen months? You may use a calculator, but you must show the keying sequence. Validate your answer.

4-7. find perimeters (circumferences) and areas of polygons (circles).

A. given the dimensions of several polygons and circles, the student will find the perimeters (circumferences) and areas in 90% of the cases.

Find the following:

1. What is the area of a circle that has a diameter of 4 centimeters?
2. Find the perimeter of a 4" x 8" rectangle.
3. Find the area of a 2.5' x 3.6' rectangle.
4. What is the area of a circle that has a radius of 3.1 inches?
5. Find the circumference of a circle with a diameter of 4.3 meters.
6. What is the area of a rectangle that has a length of 4.1 m and a width of 6.2 m?
7. Find the perimeter of a rectangle that has a length of 5.5' and a width that is half as great as the length.

8. Find the circumference of a circle with a radius of 4.3".
9. A rectangle has a length that is three more than its width. If the width is 4', what is the area? The perimeter?

B. the student will demonstrate understanding of perimeter and area by

a. showing how to use a grid to find the area of a figure,
b. showing how to use a hand trundle wheel to find perimeter,
c. explaining the difference between linear units and square units, and
d. show how to find area by dividing a figure into parts.

C. Given a problem-solving/application situation involving perimeter and area, the student will find a solution and achieve a score of ___ on the district holistic scale.

Pat has three pieces of 8.5" x 11" colored construction paper. She needs to make six rectangles that are 4" x 9". Does she have enough paper? Verify your answer.

4-9. find surface areas and volumes of rectangular solids.

A. Given the dimensions of several rectangular solids, the student will find the surface areas and volumes in 90% of the cases.

Find the indicated areas and volumes:

1. A rectangular box is 2' x 3' x 3'. How many square feet of cardboard will be needed to make the box, including a top? How many cubic feet will it hold?

2. How many cubic centimeters can be placed in a box that is 3 centimeters on each edge? How much material will be needed to make the box if a top is not needed?

B. The student will demonstrate understanding of surface area and volume of rectangular solids by

a. modeling volume, using cubes, and
b. showing which faces have the same area.

C. Given a problem-solving/application situation involving surface area and/or volume of a rectangular solid, the student will solve the problem and achieve a score of ___ on the district holistic scale.

A 5" x 7" x 11" container is half full. How many more one-inch cubes can it hold? Verify your answer.

5-11. solve simple number sentences and use formulas.

A. Given linear equations and/or formulas, the student will solve the equations and use the formulas correctly in 90% of the cases.

Find the following:

1. If $2a - 3 = 15$, find a
2. If $3b + 4 = b + 18$, find b
3. If $P = 2(b + h)$, $P = 24$, $h = 5$, find b.
4. If $.5c = 10$, find c
5. When $a = 6$ and $b = 7$, and $c = 2a + 3b$, what does c equal?
6. Find a if 75% of a is 150.
7. If $25b = 13b + 24$, find b
8. When $a = 0.6$ and $b = 0.13$, and $c = a - b$, what does c equal?
9. The sum of a and b is 67. The value of b is 2. What is a?
10. If $c - 5 = 13$, find c.

B. The student will demonstrate understanding of equations by

a. using a model (a balance or colored chips) to demonstrate the solution of an equation.
b. explaining the relation between the model and a symbolic solution.

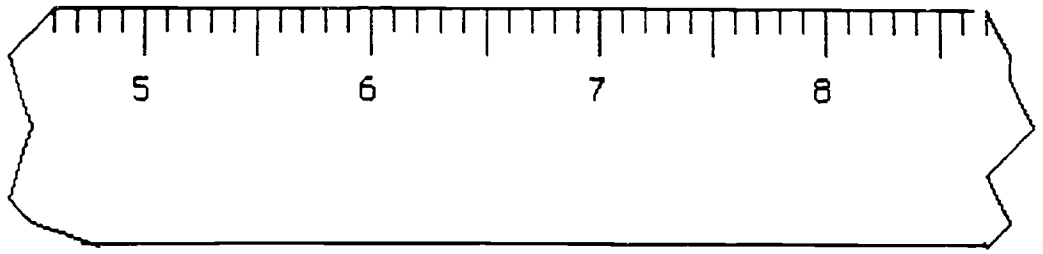
C. Given a mathematical or application problem involving equations or formulas, the student will solve the problem and achieve a score of _____ on the district holistic scale.

A class in Paris was communicating with a class in Cleveland, using a new telecommunications system. They shared the information that the temperature in Paris was 20° and the temperature in Cleveland was 68°. What was the difference in temperatures? (Recall that Paris uses the Celsius scale and Cleveland uses Fahrenheit. The conversion formula is $C = 5(F-32)/9$ ). Explain the process you used to determine your answer and justify your result.

6-7. read the scale on a measurement device to the nearest mark and make interpolations where appropriate.

A. Given a scale with divisions (not including zero), the student will determine the value of a division with 100% accuracy.

What is the value of each division on this inch ruler?

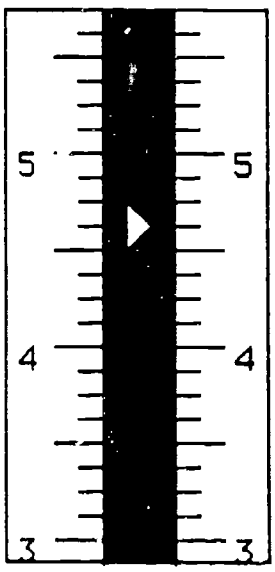



B. The student will demonstrate understanding of scales on measurement devices by

   a. explaining how to determine the value of subdivisions on the scale, and
   b. demonstrating the process for different scales.

C. Given a mathematical or application problem that requires reading a measurement scale, the student will solve the problem and achieve a score of ___ on the district holistic scale.

Kurt was weighing nails at the hardware store. He placed ten large nails in the pan of the scale (which measures in ounces) and noted the position of the marker. Approximately how many nails of this size would be in a pound package? Justify your answer.

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8-5. use elementary notions of probability.

A. Given an event (rolling of a die, etc.), the student will list all possible outcomes and the probability of a particular outcome.

List all possible telephone area codes (three-digit numbers) where only the digits 1, 2, and 3 are used. What is the probability that a random selection will produce the area code 123?

B. The student will demonstrate understanding of simple probability by:

a. explaining an organized process for determining possible outcomes, and
b. explaining how to use the possible outcomes to determine the probability of a given outcome.

C. Given a mathematical or application problem situation involving simple probability, the student will solve the problem and achieve a score of ____ on the district holistic scale.

Edwina tossed a coin five times, and each time it landed heads. What is the probability that it will land heads the next time? Explain your answer.

# INTERVENTION SERVICES

Recognizing that alternative or supplemental action designed to remediate, reinforce, or support student learning relative to the specified performance objectives will sometimes be necessary, suggested intervention services have been identified for mathematics at each grade level. Two very important assumptions need to be remembered:

- intervention must always be tied to assessment, and
- intervention is a shared responsibility.

These assumptions undergird any successful intervention program. The tie between intervention and assessment seems obvious. To plan and implement intervention procedures or strategies without first assessing student performance would be illogical. The relationship between these two important concepts, however, is neither direct nor simple. A competency-based program developed by individuals who understand human learning, curriculum development, and mathematics will include intervention and assessment components which are interrelated, which build upon each other, and which are not necessarily linear. It is important to remember that assessment may be formal or informal, but it should always indicate to the teacher whether intervention is necessary. This does not mean that a single assessment will necessarily indicate the need for and/or kind of intervention that should be provided. Intervention programs need to be based on the full-range of assessments that are included in a district's competency-based education program.

It is widely understood and accepted that "intervention is a shared responsibility." How this responsibility is shared, and with whom, however, is not as generally understood. In the broadest sense, intervention is the responsibility of all individuals who care about student achievement. Minimally, intervention should be structured through three successive levels: the classroom, the building, and the district. Ideally, these structures would involve students, teachers, parents, and building and district administrators. When a student's need for intervention cannot be satisfactorily addressed by the regular classroom teacher, it will be necessary to have building and district options available. Building-level options might include interclass grouping, intervention assistance teams, tutorial programs, and resource/intervention rooms and teachers. District-level options might include summer school programs, in-term extra hours programs (e.g., Saturday School), and required remedial academic courses. Provisions for intervention services, including adequate resources and appropriate staff development, should be made at all three levels.

## Classroom-Level Intervention

The primary responsibility for providing intervention, nevertheless, rests with the classroom teacher. The teacher must be able to identify the need for intervention, design the instructional form it will take, and implement the action. This requires a great deal of skill in classroom remediation, reinforcement, and enrichment techniques. The teacher must have the capacity to utilize content material, instruct for specific skills/knowledge deficiencies, and group students for special needs. The ability to understand and use various diagnostic instruments, analyze assessment data, and teach prescriptively are critical elements of effective intervention. A teacher who is astute, creative, and knowledgeable in the areas of mathematics and pedagogy is the key to an intervention program which meets the needs of students. While courses of study and lesson plans focus upon group outcomes, intervention must focus upon the individual student. Intervention in the classroom can take place during a lesson, after a lesson, at the end of a unit, after an assessment, or at the end (or beginning) of a grade level. Any or all of these models accommodate one-to-one teacher-student interaction, as well as various tutoring models. The most effective intervention point is during the initial lesson.

There are many paradigms for instruction/intervention at the classroom level. Three of the most commonly observed patterns (though not necessarily the most educationally sound) are characterized by (1) whole-group instruction followed by remediation of students who "don't get it"; (2) whole-group instruction followed by collaborative groups; and (3) group problem-solving strategies. These three patterns represent some of the most typical classroom teaching models. One-to-one teacher-student interaction, tutoring situations other than those involving classroom teachers, and situations where students discover and explore individually on computers represent just a few of the many other instructional models that are not as readily observable.

An exemplary "whole-group lesson" is designed to cause students to think about the ideas that were presented, stimulate internalization of those concepts, and elicit feedback as to how well the new concepts or techniques are being understood. Good whole-group instruction models many of the components of effective communication. It is important that instruction be viewed as interactive among teachers and students. Students must not view themselves as receptacles to be filled by the teacher, and teachers must not see themselves as founts of knowledge whose only responsibility is to spew forth that knowledge in clear and motivating lessons. Understanding comes through dialogue. Students have at least half the responsibility for learning in any instructional setting, including large-group lectures. Even so, learners respond at various rates and with varying levels of understanding. Individual student responses will provide teachers with opportunities to extend, amplify, or "back-up" student understanding. Student responses should allow able teachers the opportunity to identify those students who have not yet fully grasped the concept. Teachers must possess a repertoire of teaching skills and strategies that can be

brought to bear during initial intervention episodes. This repertoire should include listening and questioning skills, knowledge of and facility with alternative algorithms, and an ability to present concepts and ideas in formats that will address learners possessing various learning-style strengths and motivational levels. As the instructional dialogue continues, the teacher has ample opportunity to elicit both formal and informal feedback from students.

At some point in this process, the teacher must decide how many (as well as which) of the learners understand the lesson ideas well enough to go on to independent work. Additional work must be done with students who are not ready to consolidate ideas and proceed independently. This additional work is usually accomplished with individuals or small groups, but is almost always teacher directed. It is important that teachers assume the role of diagnostician in order to determine the source of student learning problems. Understanding why students are having difficulty is critical to overcoming that difficulty. If prerequisite skills and understandings are determined to be a major factor, the base for understanding these necessary ideas and concepts under discussion must be built.

When these "gaps in understanding" have been addressed, the instructional activities related to learning the initial concepts may be resumed. Since most teachers routinely do what they consider their "best" or "most-effective" lesson as their initial presentation to a group, they are sometimes hard put to come up with and present concepts using alternative approaches. Yet development of a set of alternative strategies is an important part of professional growth, and it is essential to meeting the intervention needs of students.

The second pattern also involves initial instruction in a whole-group setting. Good instruction in this model is as dependent on two-way interaction between teacher and student as the instruction in the first model. The difference occurs after this initial whole-group instruction. Rather than making determinations about the appropriateness of intervention based only on teacher judgment, this model depends on students working in groups to "solidify" concepts. Typically after teacher-centered instruction, the students would be grouped to continue discussion and work on exercises and problems. At its core, this model assumes that students will have learned at different rates and to different degrees of understanding. It also assumes that students can effectively communicate with, and indeed, teach one another. Group problem-solving strategies usually present students with problems to solve, or projects to complete, or tasks to perform prior to their receiving any direct instruction prior to initial efforts on their part.

At the core of classroom intervention is effective instruction aimed at **each** student individually. Many individuals equate great teaching with great oration. If this was ever true, it is no longer the case. Teachers must become facilitators of learning and not dispensers of knowledge. In general, this requires that instruction be focused as much upon the process of learning as upon

what is learned. Instruction must be paced to take into account the fact that students learn at different rates. It must be structured so that drill and practice follow rather than precede concept development and so that all diagnoses and remediation of error take place as soon as possible in order to avoid the more-formal intervention options available at the building and district levels.

## Building-Level Intervention

When the intervention possibilities provided in the classroom are not sufficient to meet the need of an individual student, it is sometimes necessary to provide other instructional alternatives. These would include interclass grouping (a strategy that has been employed in reading in elementary schools for years), the establishment of a resource or intervention room (appropriately staffed), tutorial programs, and a formal intervention assistance team established at the building level.

Interclass grouping might be done at either the elementary school or the high school. This option is most appropriate when groups are formed for short periods of time with highly fluid structures and membership. Since intervention is best handled at the classroom level, this alternative should be used only after the classroom teacher and/or the intervention assistance team has decided that the options for intervention within the classroom cannot meet the needs of the student. Groups should be formed when students are having difficulty with the same or related performance objectives **and** when there is strong evidence that the reasons for the difficulty are somewhat the same across the group. It may not make sense to group two students just because neither can compute with fractions, if the reasons they cannot compute are different. Suppose one student does not understand the concept of fraction and the other student does not know basic arithmetic facts. The "illness" may look the same to the untrained eye, but the treatment should be quite different. This is one reason diagnostic procedures are an important component of intervention programs.

The establishment of an intervention or resource room is also an alternative which may be valuable for all school buildings. These centers may be places where students can receive valuable one-on-one attention.

It is absolutely essential that the person or persons staffing a center such as this be knowledgeable about content, methods, and materials necessary for the development of a remedial program, and in addition, be able to implement such a program. Occasionally, there is a temptation to assign "less-competent" faculty members to such a center in order to remove the need for such individuals to deal with large groups or with whole-group instruction. This will ultimately prove to be counterproductive, as students who are unable to learn effectively and efficiently in a traditional classroom setting are the very ones in need of the most-talented teachers if they are to understand and be able to apply mathematical concepts.

Tutorial programs offer proven and practical ways to help students who are having difficulty in mathematics. Tutorial programs offer a way for students to get much-needed personal attention. Again, it is important that all persons acting as tutors, whether they be volunteers, other students, or classroom teachers, receive special training in terms of methods and content appropriate for students who have not obtained specified objectives. Well intentioned individuals who do not understand that the treating the symptoms instead of the underlying cause of those symptoms can be as dangerous in education as it is in medicine may actually do students who are having difficulty with mathematics more harm than good.

## District-Level Intervention

Students who continue to have difficulties after classroom- and building-level intervention programs have been implemented need to be placed in district programs. These might include a highly individualized summer school program, a before- or after-school program during the regular school year, and in the case of secondary school students, a required remedial academic course. These programs represent the most "serious" (and costly) intervention, and it is important that true "alternative" instruction be given. Often, students in these programs have a long history of difficulty in mathematics. They have not achieved expected performance levels through regular classroom instruction. "More of the same" will not be successful . These efforts should rely heavily on the use of manipulatives and technology regardless of the grade level or age of the student. These programs should begin with diagnostic analyses that will reveal areas of fundamental difficulty and misunderstanding. Students should then have the opportunity to redevelop basic mathematical concepts. Connections need to be made from what is known to what is to be learned. All students should receive instruction in problem-solving techniques and methods of applying known mathematics ideas to a variety of situations. It should be noted that for this and all other intervention programs and practices, extensive practice in computational skills is usually counterproductive. It takes time away from appropriate concept development and, indeed, often masks lack of understanding by substituting memorized procedures.

Specific intervention services and instructional strategies are provided in the model competency-based education section. In additio to a matrix which illustrates the relationship of the most-important components and suggestions for documenting intervention services, several thoughtful examples have been provided. These examples reflect the very best of what intervention should be. Meeting the needs of an increasingly diverse student population while society's need for a well-educated citizenry continues to escalate presents educators with new challenges and opportunities. The intervention services and strategies should be considered in light of their appropriateness for situation, content, and student. Teachers have always been adept at adapting general ideas and suggestions to specific situations. This talent is nowhere more important than in the selection and implementation of intervention strategies.

What follows are examples of intervention episodes. In each episode, the intervention is tied to an assessment. All of the examples begin with an assessment and illustrate the thought process in which a teacher might engage when making intervention decisions. Several different types of intervention strategies and services are discussed.

## An Intervention Episode

Objective under consideration:

Given an application involving percentages, the student will write an explanation of the process used to find the required solution and receive a score of__as determined by the district rubric.

The assessment for this objective:

Sue and four of her friends plan to order school jackets from the Schoolhouse Catalogue Company. The cost of each jacket is $60.00, but if 4 or more are purchased, a saving of 1/3 the cost of each jacket is realized. If the sales tax is 5.5 % and shipping is 8%, how much would each jacket cost? Explain in words the process you used in getting your answer and validate your answer.

Four of the responses received by Mr. Hill are listed below.

Barbara's paper:

| | | | |
|---|---|---|---|
| 1/3 X 60 = 20 | 60 - 20 = 40 | 40 X .055 = 2.20 | 40 X .08 = 3.20 |

40.00
2.20
+ 3.20
45.40

Suzi's paper:

First you got to decide how much the jacket costs because more than four jackets are being ordered. Then you have to add the shipping and sales tax to get 13.5%. Then multiply the discounted cost of the jacket by 1.135 and you will get the right answer.

John's Paper

Because 1/3 is off, 2/3 is left so the jacket costs 40. To find the tax multiply 40 times 5.5 which is 22.0 and to find the shipping charge multiply the 40 times 8% which is 32.0 therefore, the cost will be 92.

Bobby's Paper:

> The jacket costs 60-1/3 because there are 4 girls buying them. That is 59 2/3. The extra taxes and shipping totals 13.5. Since 59 2/3 is the same as 59.67 (rounded off) the cost will be 13.5 plus 59.67 (be sure to line up the decimal pts). So,
>
> 13.50
> + 59.60
> 73.10

Who needs intervention? What kind of intervention should be provided? What should Mr. Hill do?

One of the things that ought to be noted is that it is artificial to make decisions about intervention based on responses to one assessment item or situation. It is also clear that it would be hard to discuss intervention for these four students outside the context of what the rest of the class is doing. The rubric to be used in scoring is a district decision and will not be discussed here. Even without discussion of a scoring rubric, it is clear that none of the four students did exactly what Mr. Hill hoped they would do. It is equally clear that the "intervention" for each student would differ markedly.

Mr. Hill might meet with each student individually and discuss that student's misconceptions. Those discussions and the activities which would follow constitute the intervention process. The quality and success would depend largely on Mr. Hill's competence as a teacher. He would need to diagnose and refine his assessments during these individual discussions. Finding out why a student did not achieve and then seeing that achievement occurs is an iterative and interactive process.

The process begins in the mind of the teacher as he or she considers the results of assessment. In this case, Mr. Hill is thinking about four of his students who did not respond as he had hoped.

> *Suzi is such a bright student, actually in some ways the brightest in this class. A lot of the students didn't catch that multiplying 40 by 1.135 would yield the correct answer. As they become more proficient with mental mathematics and calculator thinking, I think more of them will see the problem that way. On the other hand, she didn't really do any of the computation. I know in her case it wasn't because she couldn't. I wonder why she wouldn't. Barbara, on the other hand, seems to love to do arithmetic. Some kids are like that; they just seem to like the way numbers look on the paper. But she hasn't explained anything in words. Sometimes I wish I could turn Barbara and Suzi into one student. They are both good students, but their approaches to problems and the way they work with numbers are really different.*

The next day, Suzi and Barbara got the following note from Mr. Hill paper clipped to copies of their problem solutions:

Barbara and Suzanne:

I want you to meet together in the back of the class today. I want you to do four things.

1. Grade the other person's percentage application problem on a scale of 1 to 5.

2. Add to what the other person has done in ways that will improve the paper.

3. Discuss the changes that were made on both papers. Decide

   a. who did the best job originally
   b. who did the best "editing" job
   c. write **one** answer and put both your names on it

4. Individually, do the following:

   Mary is saving money for a special stereo set she wants to buy. The stereo set costs $623.98. Mary has $400 in her savings account. She gets 5 dollars a week allowance and puts 60 percent of that in her piggy bank. Her savings account earns 8% annually. If Mary's savings earn interest for one year and she uses her piggy bank money, will she have enough money to buy the stereo? Explain in words the process you used in getting your answer and validate your answer.

After making up the assignment sheet for Barbara and Suzi, Mr. Hill went on to John's paper.

*I don't know what John is thinking about. How could he think $94.00 is a reasonable answer when the original cost was $60.00. I wonder if he really doesn't understand that 22 is a ridiculous answer for 5 and a half percent of 40. He surely knows that 20 is fifty percent of 40. He does a good job of memorizing algorithms and rules, but I am just not sure he* ***understands.*** *I need to talk to him, but I think he is going on that field trip tomorrow. I will write a note and leave it with his homeroom teacher and have him come in before school the day after tomorrow. I ought to be able to tell in 10 or 15 minutes if he understands the concept of percent. I wonder if he ties the concept of percent back to decimals, and I wonder if he sees decimals as special case fractions. I guess I could start by asking him if he could explain the relationship between fractions, decimals, and percents. Then I might check his estimation and number sense with questions like a third of a dozen and 50 percent of 1,500. I will try to see if he understands the underlying concepts involved. If he doesn't, he will have to come in Friday during his study hall and work through the multibase block decimal activities. If he is just being careless, I suppose I can talk to him about the importance of exactness in mathematics, and also about the reasonableness of answers.*

Mr. Hall has put Bobby's paper off until last.

*I don't know whether to feel sorry for this kid or be glad he didn't do well. He never listens in class. I have the feeling he is so lost I might as well be speaking another language. I guess as far as he is concerned it is a foreign language. I don't think he understands the underlying concepts. He doesn't understand fractions or percents. I wonder if he had to find one-third of a number like 12 or 6 if he could do it. There is just no point in his going on in this class without extra help. I need to check with the guidance counselor and see if his schedule would permit being assigned to the third period math intervention class in addition to this class. If not, I suppose I could get a math club tutor assigned. I think I will talk with Mr. Hodgin about the Intervention Assistance Team program we are setting up. Bobby may be a real candidate.*

## A Second Episode

Objective under consideration:

Given a set of whole number addition and subtraction problems, the student will add and subtract them as directed with an accuracy of 95%.

The assessment for this objective: (third grade)

1. 125 + 37= S
   What is the value of S?

2. 125 - 37=___

3.
$$\begin{array}{r} 22 \\ 33 \\ +\ 55 \\ \hline \end{array}$$

4. 23-7-5=___

5.
$$\begin{array}{r} 327 \\ +\ 723 \\ \hline \end{array}$$

6. 1 + 543 + 16=___

7. 10,000-9,999=M
   What is the value of M?

8. 25 + 25 + 55=___

9.
$$\begin{array}{r} 803 \\ -\ 204 \\ \hline \end{array}$$

10. 1+2+3+4+5+6+7+8+9+10=C
    What is the value of C?

11. 250 - 157= F
    What is the value of F?

12. 250 - 167=___

13. 250 - 147=___

14.
$$\begin{array}{r} 123 \\ +\ 333 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 85 \\ -\ 17 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 157 \\ +\ 151 \\ \hline \end{array}$$

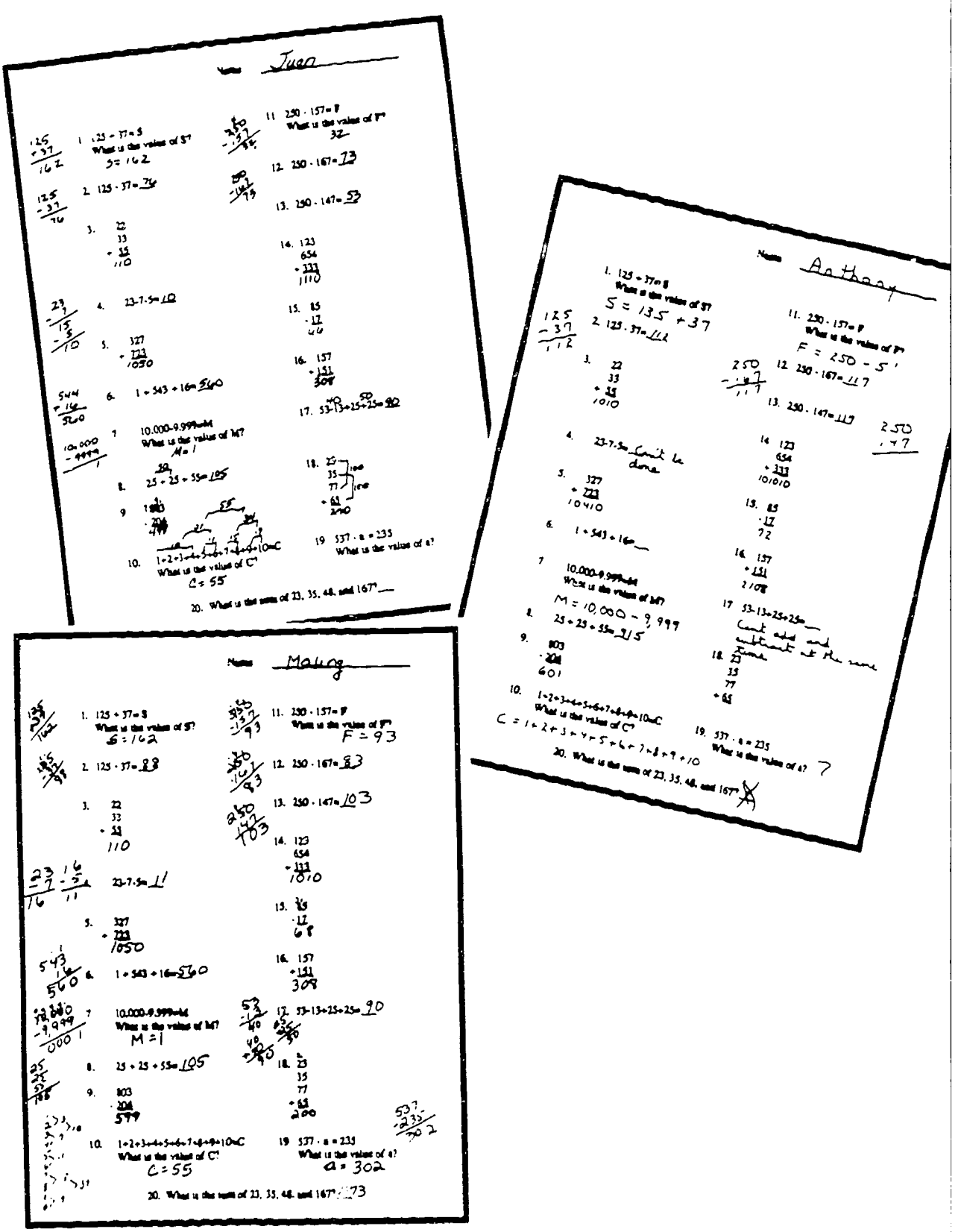
17. 53-13+25+25=___

18.
$$\begin{array}{r} 23 \\ 35 \\ 77 \\ +\ 65 \\ \hline \end{array}$$

19. 537 - a = 235
    What is the value of a?

20. What is the sum of 23, 35, 48, and 167?___

Three of the responses received by Mr. Nidy are included.



Name Juan

1. 125 + 37 = S
What is the value of S?
S = 162

2. 125 - 37 = 76

3. 22
33
+ 55
110

4. 23-7-5= 10

5. 327
+ 723
1050

6. 1 + 543 + 16= 560

7. 10,000-9,999=M
What is the value of M?
M = 1

8. 25 + 25 + 55= 105

9. 803
- 204
499

10. 1+2+3+4+5+6+7+8+9+10=C
What is the value of C?
C = 55

11. 250 - 157= F
What is the value of F?
32

12. 250 - 167= 73

13. 250 - 147= 53

14. 123
654
+ 333
1110

15. 85
- 17
68

16. 157
+ 151
308

17. 53+13+25+25= 80

18. 25
35
77
+ 65
200

19. 537 - a = 235
What is the value of a?

20. What is the sum of 23, 35, 48, and 167? ___

Name Anthony

1. 125 + 37= S
What is the value of S?
S = 135 + 37

2. 125 - 37= 112

3. 22
33
+ 55
1010

4. 23-7-5= Can't be done

5. 327
+ 723
10410

6. 1 + 543 + 16= ___

7. 10,000-9,999=M
What is the value of M?
M = 10,000 - 9,999

8. 25 + 25 + 55= 105

9. 803
- 204
601

10. 1+2+3+4+5+6+7+8+9+10=C
What is the value of C?
C = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10

11. 250 - 157= F
What is the value of F?
F = 250 - 5

12. 250 - 167= 117

13. 250 - 147= 117

14. 123
654
+ 333
1010

15. 85
- 17
72

16. 157
+ 151
308

17. 53+13+25+25= Can't add and subtract at the same time

18. 25
35
77
+ 65
200

19. 537 - a = 235
What is the value of a? 7

20. What is the sum of 23, 35, 48, and 167? X

Name Malory

1. 125 + 37= S
What is the value of S?
S = 162

2. 125 - 37= 88

3. 22
33
+ 55
110

4. 23-7-5= 11

5. 327
+ 723
1050

6. 1 + 543 + 16= 560

7. 10,000-9,999=M
What is the value of M?
M = 1

8. 25 + 25 + 55= 105

9. 803
- 204
599

10. 1+2+3+4+5+6+7+8+9+10=C
What is the value of C?
C = 55

11. 250 - 157= F
What is the value of F?
F = 93

12. 250 - 167= 83

13. 250 - 147= 103

14. 123
654
+ 333
1010

15. 85
- 17
68

16. 157
+ 151
309

17. 53-13+25+25= 90

18. 25
35
77
+ 65
200

19. 537 - a = 235
What is the value of a?
a = 302

20. What is the sum of 23, 35, 48, and 167? 273

Mr. Nidy considers the results of the assessment of an important performance objective.

*I might as well start with Anthony's paper. I think I will need to spend a lot of time trying to figure out what he did. Sometimes when he answers questions the way he did number 1, I can't tell if he doesn't know what to do or if he is just being a pain. The second problem would indicate that he still subtracts by taking the smallest digit from the largest digit, whether or not that is the procedure indicated in the problem. Yes, he did the same thing on 12 and 13. He does know his basic subtraction facts, but either does not understand the procedure, or it is not important enough to him to go to the trouble of "borrowing." The results on number 3 indicate that he doesn't "carry" either. He just adds in columns, and puts the number down at the bottom of the column. He did the same thing on 14. I wonder why he thinks 4 "can't be done"? He didn't even try 18 or 20 or 6. Interestingly, he put a question mark by 19. That means he must have read it. Anthony is in deep trouble in mathematics, but he does know his basic addition and subtraction facts, he correctly "lines up" the problems that are presented in horizontal format when he begins working, and he tried most of the problems. At least I have some place to start working with him. I will need to have him work with materials where he does some "trading." One of his big problems seems to be with the place value concept. I will need to talk to him tomorrow during station time. Maybe I can use money as a model. I am sure he knows there are ten pennies in a dime and ten dimes in a dollar. Then again, maybe the decimal point would be confusing. I guess the best way might be to have him play some trading games with the multibase blocks or bundled sticks. Maybe Tom and he could make a subtraction poster using sticks. They play together, and Tom understands place value and subtraction. After I am sure he grasps the place value ideas, we can use the materials to do addition and subtraction.*

I need a break, I think I will look at MaLing's paper next...

*She got every answer right, but she sure used a sledgehammer. Even on number 7 she wrote it out and did all that "borrowing." At least on 10 she grouped the addends a little bit, but she sure didn't see that she had five pairs of 11. Her computational skills are fine, but she either does not have much facility with mental math or she is uncomfortable using it. Maybe she thinks this is what I want. She is so bright. What kinds of things should I be doing to help her become more "friendly" with numbers? I think I will call the math supervisor tomorrow and see if she has any ideas.*

Mr. Nidy considers Juan's paper next.

*Good, Juan seems to have addition down pat. On problem 6 he at least combined the 1 and the 543 before he added the 26, and he did a nice job on problem 8 too. He didn't pick up the pattern I had hoped he would see in 14 or 10. But he used some kind of a combining sequence on 10 and he saw the compatible numbers on 18. I don't see any pencil scratches indicative of carrying on 16 so maybe he used mental mathematics there, and he did some real thinking on problem 17. But the subtraction doesn't look nearly as good. He set up problem 2 correctly, but he seems to think that 15 minus 7 is 6. Similar thing on problem 4, he said 13-7=5. Well, at least he knows 5 minus 5 is 0. He got problem 7 but seems to have used mental arithmetic. He set up the borrowing correctly on 9, but subtracted incorrectly. The kid seems to understand the process of subtraction, even multidigit subtraction, but he just has not memorized his subtraction facts. He is so good on his addition facts, I am amazed that he does not seem to understand the family of facts idea, if 3+4=7, then 7-3=4 and 7-4=3. Maybe I just need to point that out to him. More likely he will need some practice though. I will have Mrs. Brown use the flash cards to play Poppers with him.*

The next day Mr. Nidy's parent volunteer got the following note:

Mrs. Brown,

Please get the subtraction fact flash cards out of the filing cabinet. You may have to dig deep. I don't believe in using them often, but Juan could use a little practice with a game called Poppers. As you go through the cards with him, he should put the cards in three piles.

1. Poppers - the answer just "pops" into his head
2. Thinkers - he has to think about the answer before he can say it
3. Not Yet - the answer doesn't come to him without using counters

After he goes through the cards one time, make sure he counts the number of cards in each pile. Keep a record so he can see he is getting more cards into the Popper pile.

I will talk to him about subtraction and the family of facts before you get here to make sure he understands the concept of subtraction.

Thanks.

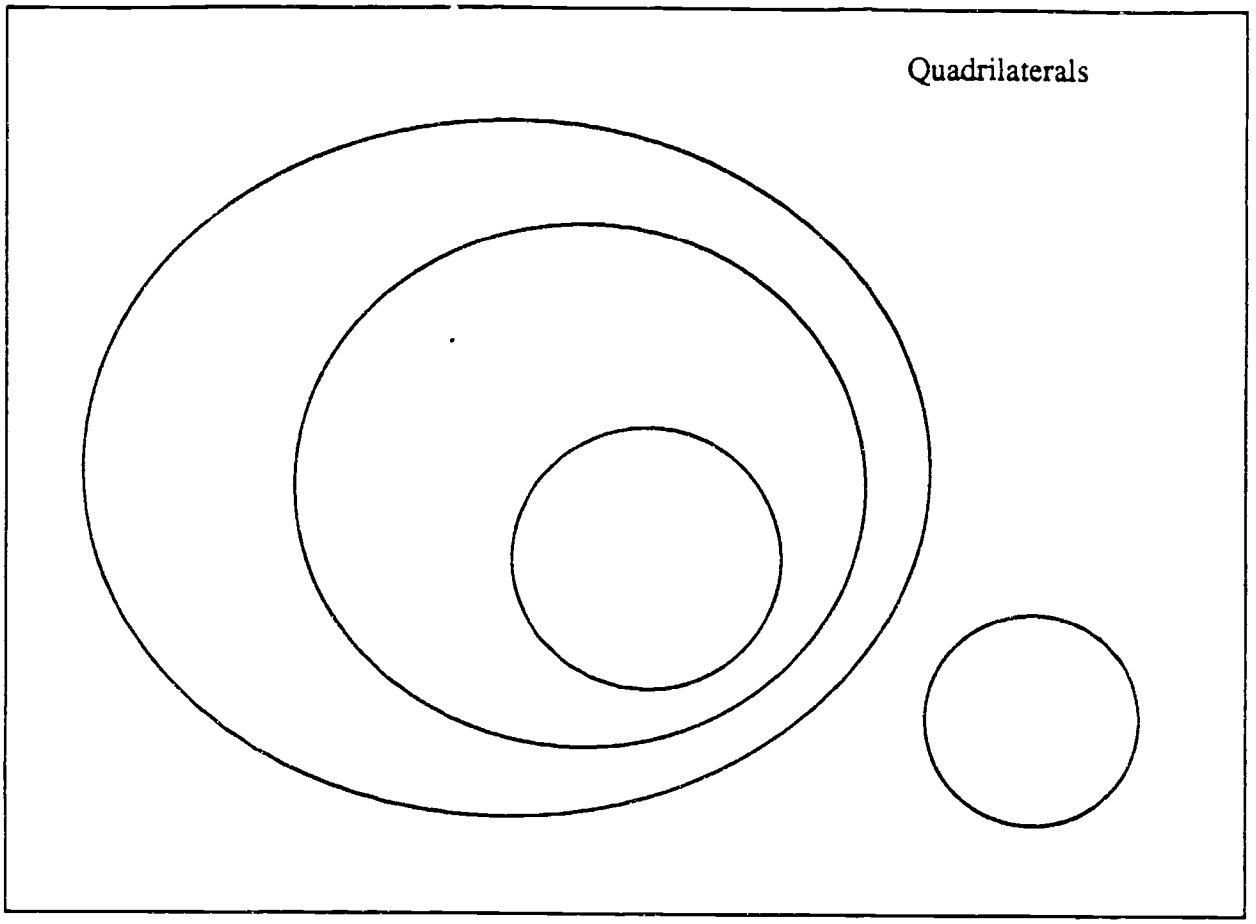
Mr. Nidy was concerned about the results of the assessment of at least three of his students, MaLing, Juan, and Anthony. MaLing met the specified criteria and intervention was not required in her case, However, because Mr. Nidy is a knowledgeable and concerned teacher, he will no doubt be dealing with what he considers a problem in the way MaLing understands and uses mathematics. This will be an intervention. Neither Juan nor Anthony met the criteria. Mr. Nidy feels that the problem in Juan's understanding is relatively minor and the planned intervention is a discussion and then at least one practice session with a parent volunteer. Anthony's problems go much deeper and the intervention will be much more "intrusive." Because the intervention is more intrusive, documentation of the intervention is more important and should be more detailed.

## A Third Episode

Objective under consideration:

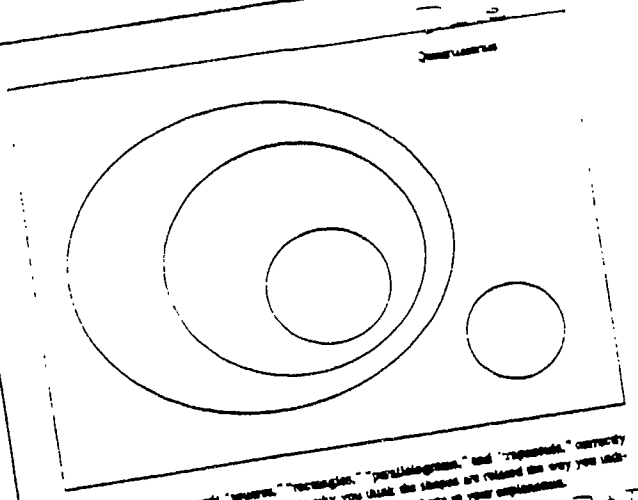
The student will demonstrate understanding of simple two-dimensional figures by describing the relationship between squares and rectangles, rectangles and parallelograms, and parallelograms and trapezoids.

The assessment for this objective:




Using the four words "squares," "rectangles," "parallelograms," and "trapezoids," correctly label the diagram and explain in words why you think the shapes are related the way you indicated in the diagram. Consider the characteristics of the shape in your explanation.

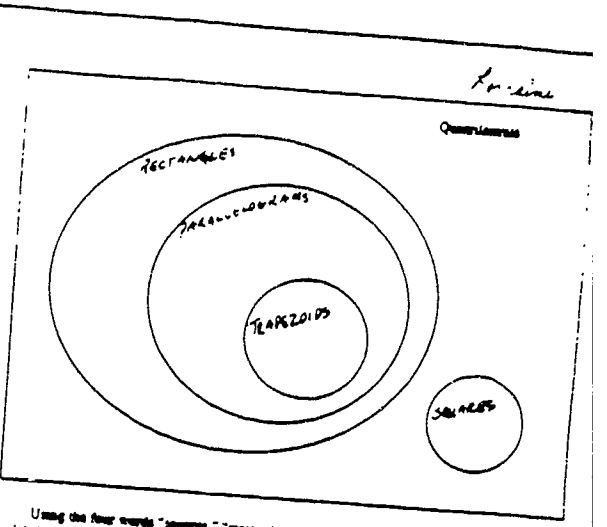
The top three papers in the stack to be graded belong to Kathy, Lorraine, and Dale. Mrs. Doerr thinks maybe she can get them done before the faculty meeting.



Using the four words "squares," "rectangles," "parallelograms," and "trapezoids," correctly label the diagram and explain in words why you think the shapes are related the way you indicated in the diagram. Consider the characteristics of the shape in your explanations.

I don't understand this diagram But I know how squares, rectangles, parallelograms and Trapezoids are related All four of these shapes are quadrilaterals because they have 4 sides Trapezoids and parallelograms are different because parallelograms have 2 pairs of parallel sides ≠ but trapezoids have only one pair of parallel sides ≠. Rectangles are parallelograms (2 pairs of parallel sides with 4 90° angles Squares are rectangles (2 pairs of parallel sides and 4 90° angles but also all sides are of equal length

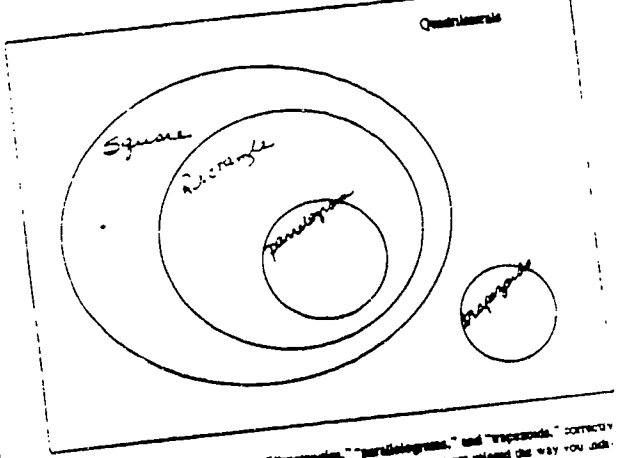
Kristin




Using the four words "squares," "rectangles," "parallelograms," and "trapezoids," correctly label the diagram and explain in words why you think the shapes are related the way you indicated in the diagram. Consider the characteristics of the shape in your explanations.

There are more rectangles than any other shape so it should be the biggest circle Squares are the most special shape so they are on themselves Since rectangles have parallel sides, it put parallelograms in the rectangles Trapezoids were the ones that were left

Kristy




Using the four words "squares," "rectangles," "parallelograms," and "trapezoids," correctly label the diagram and explain in words why you think the shapes are related the way you indicated in the diagram. Consider the characteristics of the shape in your explanations.

It's kind of like parallelogram like squares are built around being parallelograms and squares are just very special rectangles - which are very special parallelograms because they

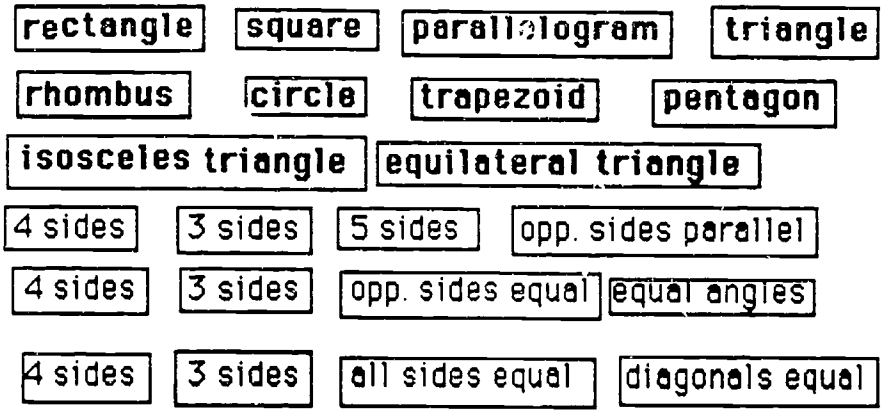
*Lorraine hasn't demonstrated that she knows anything about the relationships between the shapes. I wonder why she thinks that there are more rectangles than any other shape. If she had just explained why she thinks squares are the "most special shape," maybe I could have given her some credit. I don't know exactly where to start with her. I think that I will have her work with the elementary gifted program with the property cards. I'll write her a note right now and explain and then have her come in after school before she works with the elementary students.*

Lorraine,

I would like you to make a presentation for the elementary gifted group. I was hoping you could explain the property cards. I don't know if you have worked with them or not. They were among the materials available to be checked out as we went through the last unit. You will need to check them out and go through the exercises in the kit, and then you and I can discuss how you want to present the ideas to the elementary students. I think you need to make up some kind of game or activity the students could use. Take the cards today and come in before school tomorrow so I can see if you have any questions.

Mrs. Doerr

(The next day when Lorraine spread out part of the property cards, she immediately began to think of activities.)




*Dale understands the geometric concepts. I don't know why he would claim not to know how to use Venn diagrams. Clearly he meets the criteria for the objective. But equally clearly I need to talk to him about diagrams, and also about trying to figure things out. Given the clarity of his explanation, I am amazed that the correct labeling didn't just jump out at him.*

*Kathy seems to understand that there is a relationship, but she had labeled the diagram "backwards." Her explanation is fuzzy. I think I will have her go through a set of geoboard task cards.*

*Just looking through these papers, it doesn't look as if anyone did much better that these first three. It is clear that even when the students understood all or part of the relationship between the geometric figures, they weren't able to use the Venn diagram to illustrate the relationships. I need to do another lesson for the whole group on Venn diagrams. I think they are very helpful when trying to understand geometric interrelationships. Well, I am already late for the faculty meeting....*

These intervention episodes are intended to illustrate the various levels of analysis necessary to make sound diagnoses about individual student learning and the kinds of prescriptive instructional responses selected and/or developed to address the specific needs of students. The relationship between assessment (making judgments about student achievement) and intervention (the range of alternative instructional behaviors designed to redress specific learning deficiencies) must always be maintained. The work is difficult, the challenge enormous, and the reward priceless. This is the very best of what teaching is about.

# Intervention Services Model

| Level | Resources | Records | Activities |
|---|---|---|---|
| Classroom<br>Intraclass grouping<br>Alternative instruction | Courses of Study<br>Pupil Performance Objectives<br>Appropriate Instructional Materials<br>(always understood to include calculators, computers, and manipulatives) | a. Student Folder<br>(Folder should contain records of pupil performance objectives mastered and allow for documentation of intervention provided) | 1. Use of skill grouping<br>2. Modification of Material<br>3. Adjustment of instruction to learning style<br>4. Personalization of instruction<br>5. Use of corrective instruction<br>6. Use of self-instruction packages<br>7. Use of learning contracts<br>8. Use of diagnostic/prescriptive teaching<br>9. Conduct student conferences<br>10. Provide time in resource room<br>11. Develop instructional plan with student<br>12. Provide independent activities coded to specific objectives<br>13. Provide skill practice<br>14. Use interclass grouping<br>15. Provide Tutoring:<br>a. Peer tutoring<br>b. Volunteer tutoring<br>c. Parent tutoring<br>d. Cross-age tutoring<br>e. Cross-grade tutoring<br>16. Remedial instruction<br>17. Use outside resource personnel |
| Building<br>Interclass grouping | Student Folder, documentation of interclass grouping, course of study, PPOs<br>Appropriate Instructional Materials | Continue to update student folder | |
| Resource/Intervention Room | Student Folder, documentation of Resource/Intervention effort<br>course of study, PPOs<br>Appropriate Instructional Materials | Continue to update student folder<br><br>Complete record of intervention effort given to professional overseeing tutoring for recording in student folder | |
| Tutorial | Pupil Performance Objectives, Appropriate Instructional Materials | Update student folders and document intervention effort | |
| Intervention Assistance Team | | | |
| District<br>Summer School | Courses of Study, student folders, Pupil Performance Objectives, Appropriate Instructional Materials, documentation of intervention effort | Provide list of PPOs mastered to professional responsible for recording student progress | |
| In-Term Extra Hours Program (with Teacher) | PPOs not mastered, Appropriate Instructional Materials , documentation of intervention effort | Student Folder | |
| Required Remedial Academic Course | Courses of Study, student folder, Pupil Performance Objectives, Appropriate Instructional Materials | (Folder should contain records of pupil performance objectives mastered and allow for documentation of intervention provided) | |

## STANDARDIZED TESTS

The competency-based education program in mathematics must provide for a standardized test at each grade level and for at least three levels districtwide. A standardized test is one which is administered to a specified population under the same conditions at a particular point in time. Such a test may be purchased or developed locally.

Districts may meet the grade level testing requirement by—

using an appropriate form of the same achievement test as that selected to meet the ability/achievement testing requirement in grades 4, 6, 8, and 10 in some or all other grade levels. The ninth grade proficiency test would meet this requirement at that grade level. The use of appropriate forms of the same achievement test will provide longitudinal information, but on a narrow set of objectives. The majority of such tests, in spite of subscales designated as conceptual or problem solving in nature, consist of items that are at the knowledge and skill level.

or

using the achievement test administered at grades 4, 6, 8, and 10 to meet the requirement at those grade levels, and in all other grades (with the exception of grade nine if the ninth grade proficiency test is used to meet the requirement at that grade level) developing a local instrument that reflects knowledge/ skills, conceptual understanding, and problem solving. Since the activity of solving problems requires all three of these levels, it is suggested that locally developed instruments use the holistically scored problem-solving/application performance objectives identified for each grade. This has the advantage of broadening the assessment base as well as giving greater emphasis to problem solving.

The competency-based education requirement for districtwide tests, to be administered in mathematics once in grades one-four, once in grades five-eight, and once in grades nine-eleven, will be automatically met through administration of the prescribed achievement tests at grades four, six and/or eight, and ten. The ninth grade proficiency test might be substituted for or used in addition to the achievement test at grade ten to meet the high school testing requirement. Items on the tests should be correlated with the district-selected pupil performance objectives. An aggregated item analysis can then be used for programmatic assessment. Scores on standardized, norm-referenced tests do not give sufficient information for making decisions about individual students.