DOCUMENT RESUME

ED 358 140 TM 019 927

AUTHOR Edgerton, Richard T.

TITLE The Emerging Implementer of the NCTM "Standards": Concepts Searching for Expression.

PUB DATE Apr 93

NOTE 22p.; Paper presented at the Annual Meeting of the American Educational Research Association (Atlanta, GA, April 12-16, 1993).

PUB TYPE Reports - Descriptive (141) -- Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.

DESCRIPTORS *Academic Standards; *Educational Innovation; Educational Practices; High Schools; Interviews; Males; *Mathematics Instruction; Models; Nontraditional Education; Program Implementation; Seconuary School Mathematics; *Secondary School Teachers; Student Evaluation; *Teacher Role; *Teaching Methods; Theory Practice Relationship

IDENTIFIERS *NCTM Curriculum and Evaluation Standards; NCTM Professional Teaching Standards; Reform Efforts

ABSTRACT

A high school teacher was studied who was known to have begun implementation of a modern paradigm of mathematics education, as described in the "Curriculum and Evaluation Standards for School Mathematics" and the "Professional Standards for Teaching Mathematics," both by the National Council of Teachers of Mathematics. His instructional and assessment practices are described, and his teaching practices are profiled according to the recommendations of the standards document as he struggled toward more complete implementation. Interviews, observations, and collections of teacher-used materials provided the information for the study. The subject had been teaching for 20 years, during which he worked on a master's degree in mathematics education. Classes in problem-solving and educational psychology in his studies led him to question his teaching practices. He attempted changes in his teaching practice that were implemented in a spotty fashion and were difficult to maintain. This subject found implementation of the "Standards" to be a long-term project, which required time for reading, discussion, and reflection, as well as support from many sources. (SLD)

THE EMERGING IMPLEMENTER OF THE NCTM STANDARDS: CONCEPTS SEARCHING FOR EXPRESSION

by

RICHARD T. EDGERTON

Presented at the Annual Meeting of the
American Educational Research Association
Atlanta, Georgia.
April 16, 1993

ABSTRACT

A teacher was studied who was known to have begun implementation of a modern paradigm of mathematics education, as described in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and the Professional Standards for Teaching Mathematics (NCTM, 1991). The purpose of the study was to describe his instructional and assessment practices, profile his teaching practices according to the recommendations of the Standards Documents, and to describe his struggles toward more complete implementation. This case grew out of a larger study (Edgerton, 1992) and was pursued because of the teacher's issues, problems, conflicts, and resolutions in attempting to implement his understanding of the recommendations of the Standards Documents. Interviews, observations, and collections of teacher–used materials provided information for this study.

Teachers who participated in the original study were nominated by local-area school district mathematics coordinators and the state mathematics supervisor. The coordinators listed and rank-ordered the names of ten teachers according to the coordinators knowledge of the teachers' level of implementation of the Standards Documents in their teaching practices. The coordinators were guided by selected sections Standards Documents. The participants were studied for one academic year, with the subject of this paper being studied for two years.

This paper's subject taught for twenty years, the last few of which he worked on a masters degree in mathematics education. For his program of study, he took classes in problem-solving and educational psychology which made him question his teaching practices. His questioning led him to conflicts in the content of the texts he was using, the traditional expository style of instruction, and his methods of collecting assessment information. He attempted changes in his teaching practices which were implemented in a spotty fashion and difficult to maintain. This subject found implementation of Standards to be a long-term project and required time for reading, discussion, and reflection as well as support from many sources.

The National Council of Teachers of Mathematics (NCTM) recently released two documents which propose to dramatically change the content, instruction, and assessment of school mathematics and the preparation of teachers of mathematics. The documents are the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and the Professional Standards for Teaching Mathematics (NCTM, 1991). The documents were developed through an unprecedented level of collaboration between mathematicians, mathematics teachers, mathematics teacher educators, and representatives from business, industry, and other academic disciplines (Crosswhite, Dossey, & Frye, 1989). The recommendations made in the Standards Documents have resulted from the recognition that our understanding of how students learn mathematics has departed from current practices in mathematics education. Conventional forms of instruction and assessment do not address current emphases on understanding and problem-solving.

Four teachers who were known to have begun implementation of the instructional practices suggested by the NCTM Standards Documents were studied earlier (Edgerton, 1992) so their instructional and assessment practices could be profiled. Interviews, observations, and collections of teacher–used materials provided information concerning the two types of practices. This case grew out of that study and was pursued because of the teacher's issues, struggles, and resolutions to the problems of attempting to implement his understanding of the recommendations of the Standards Documents. The subject of this study, Tom, was seen as an "emerging implementer" of the Standards Documents because he was presently struggling with his first few years of teaching in a non-traditional way even though he taught for over twenty years.

Both studies were performed in the Puget Sound area of Washington state. School district mathematics coordinators and the state mathematics supervisor compiled a list of nominees for the initial study. The coordinators listed and rank-ordered the names of ten teachers according to the coordinators' knowledge of the teachers' level of implementation of the Standards Documents in their teaching practices. The coordinators used sections selected by the investigator from the Standards Documents (listed below) to guide their nominations and rank-ordering. The list was used to contact nominees to see if they were interested in being a part of the study. Only five calls were needed to secure the four participants. At the conclusion of the original study, Tom was again contacted to see if he would agree to be the subject of a continued study about his process of implementing the Standards Documents. His agreement to the continued study lead to a second year of observations and interviews and are described below.

Tom's instructional practices were profiled according to the aspects that are to receive "Increased Attention" and "Decreased Attention", as summarized in the Curriculum and Evaluation Standards (NCTM, 1989, p. 129). The major changes in patterns of instruction proposed for grades 9–12 can be summarized with:

INCREASED ATTENTION to—	DECREASED ATTENTION to—
•The active involvement of students in constructing and applying mathematical ideas	•Teacher and text as exclusive sources of knowledge
•Problem solving as a means as well as a goal of instruction	•Rote memorization of facts and procedures
•Effective questioning techniques that promote student interaction	•Extended periods of individual seatwork practicing routine tasks
•The use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole-class discussions, project work)	•Instruction by teacher exposition
•The use of calculators and computers as tools for learning and doing mathematics	•Paper-and-pencil manipulative skill work
•Student communication of mathematical ideas orally and in writing	

•The establishment and application of the interrelatedness of mathematical topics	
•The systematic maintenance of student learnings and embedding review in the context of new topics and problem situations	
•The assessment of learning as an integral part of instruction	•The relegation of testing to an adjunct role with the sole purpose of assigning grades

Teachers' assessment practices were profiled according to the aspects that are to receive "Increased Attention" and "Decreased Attention", as summarized in the Curriculum and Evaluation Standards (NCTM, 1989, p. 191). The major changes in patterns of assessment that the document proposes can be summarized with:

INCREASED ATTENTION to—	**DECREASED ATTENTION to—**
•Assessing what students know and how they think about mathematics	•Assessing what students do not know.
•Having assessment be an integral part of teaching (Note: this is the same as "The assessment of learning as an integral part of instruction" above.)	•Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades
•Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics	•Focusing on a large number of specific and isolated skills organized by a content-behavior matrix
•Developing problem situations that require the applications of a number of mathematical ideas	•Using exercises or word problems that require only one or two skills
•Using multiple assessment techniques, including written, oral, and demonstration formats	•Using only written tests
•Using calculators, computers, and manipulatives in assessment	•Excluding calculators, computers, and manipulatives from the assessment process

Tom's Instructional Style

Tom has been a classroom teacher for twenty years. I observed Tom teach numerous class sessions over a two-year period. Tom taught in a large three-year suburban high school. The students were predominantly middle-class and upper-middle-class white, with some Asian and a few Hispanic and African-American students. His room was visually busy with two bookcases, murals painted on the walls, and posters in the remaining open places. Student desks were arranged in groups of three so that groups of six could be formed easily. The grouped desks faced the front of the classroom. At the back corner of the room was Tom's desk. Adjoining his desk was a table that held a Macintosh computer and a manual for Geometer's Sketchpad. At the front of the room was a chalkboard which spanned the width of the room with another chalkboard on the left side of the room. School notices and class announcements were posted at the rear of the room on a decorated cork bulletin board.

Over the approximately two years I observed Tom for the two studies I observed him teach Geometry, Algebra/Trigonometry, and Precalculus. His classes usually had about thirty students and met each weekday for fifty-five minutes. Tom used texts provided by the school as the primary source of exercises, but supplemented with materials of his own invention and questions from various sources.

Tom used a lecture/discussion format and various questioning techniques to facilitate student understanding. He posed questions during discussions to elicit student participation and to gauge the level of student understanding. About 30% of Tom's students actively participated in class discussions, about 40% watched or listened, and the remainder were apparently inattentive. His presentations usually provided some way for the students to visualize the concept under discussion while he adapted his instruction to the students. He typically lectured, questioned, explained, assigned work, and assessed his students' understanding each class period.

Tom used activities to involve his students in the construction of their own understanding. Students explored the concept of "sliding" graphs by using the "Easy Grapher Template," a simple device showing all basic mathematical curve forms. The template was a reasonable substitute for a graphing calculator as Tom used it and was available at the school and in local book stores for less than one dollar. Tom's students used the template to draw the forms of various curves that would result from the manipulation of selected parts of the functions. They generated their own set of rules for when the curves flipped with respect to an axis, slid left or right, and rotated about a line. After three days of this activity, Tom had his students display their graphs on the chalkboards. Each group was responsible for the presentation of one of the graphs. Students used the templates both in drawing their graphs on the chalkboard and checking the other groups' graphs. During the discussion of the first graph, one student remarked that the graph had been slid the wrong direction. A discussion ensued about the "rules" for sliding the graph $y = \sqrt{2-x}$. Micki thought a change of one of the graphs on the board should not have been made because the original answer fit the rules and the change did not.

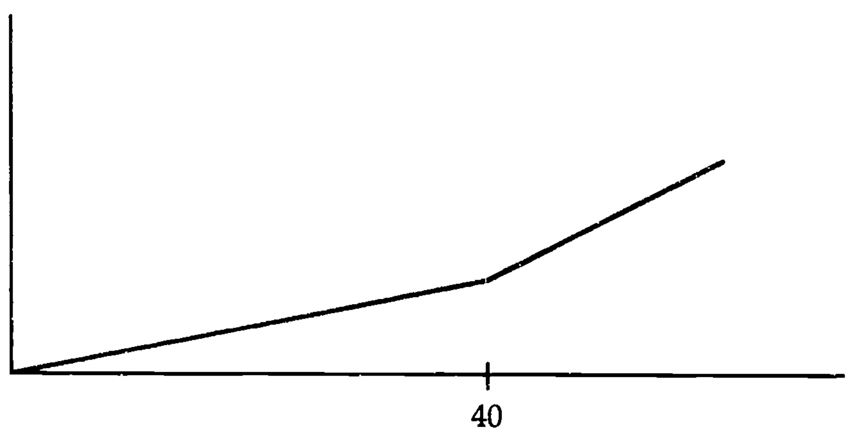
Tom: Are you in for the way it is or the way it was?
Micki: The way it was.
Tom: The way it was.
Micki: OK. If you put in $^{-}2$ here and you have 4, this point, (0, 4), doesn't match up with the coordinates. You have to have to have a (0, 2) in order for this to work.
Tom: You lost me there Micki. What are you substituting for x?
Micki: If you're going to substitute x with $^{-}2$, the coordinates would be $y = 0$, with (0, 4), but up there you have (0, 2). The coordinates don't match up.
Tom: Wait a minute. You're saying (0, 2) or (2, 0)? What are you substituting for x?
Micki: I was going to substitute $^{-}2$, (could not hear part of this) but if you follow the rules on the rule sheet, you always move the opposite direction of the sign.
Tom: Now wait, you said "the rules on the rule sheet." Did I give you a rule sheet?
Micki: No, I wrote them down when you talked about them.
Tom: So these are your rules, not mine.
Micki: No, they're your rules, I wrote them down when you talked about them.
Tom: OH. Now what is the rule that you're referring to?
Micki: OK, when the negative is hooked to the y then reflection is over the x-axis, when the negative is hooked to the x then reflection is over the y-axis, is happening. So, that's the negative x.
Tom: That negative is attached to the x, so reflected through the y-axis?
Micki: And when the 2 is not, is in the equation with the x like with parentheses, or in this case with square root, it is hooked on the x.

Tom: So we're saying that 2 is attached to the x? The 2 is attached to the input?
Micki: It's in the opposite direction. That answer is not right the other way with a $^{-}2$.
Tom: I'm going to claim that I didn't write that rule down but I did say that I know that if the 2 is attached to the x it will cause a horizontal translation.

In this example, students examined the rules *they* created through previous exercises. Micki's "rule sheet" was entirely her own creation. Tom continued the discussion with more challenges to the students' rule-system without formally quoting rules himself.

As Tom had his students work on exercises, he attempted to connect the exercises with situations that could occur in the real world. When students became more proficient with graphical concepts, Tom expanded the topic by introducing "piece functions." He drew the accompanying graph and asked what it could represent. Tom introduced the graph by saying:

> This is a particular equation that is defined differently over different pieces of its domain. It's defined one way from 0 to 40 and another way after 40. I have one particular way in mind, that I want you to think of. (Tom receives a few voluntary suggestions on the meaning of the graph from students.) I was asked to give you a hint. Do any of you work outside of school? That's a function that's defined differently over different pieces of its domain. (The discussion continues below.)



Many novel comments followed, beginning with a few about age, such as how much faster a person "goes down-hill" after they turn forty. When one student offered "income, including overtime," the other students showed their satisfaction for that remark with approving nods and positive-sounding whispers. The graph connected with their present knowledge and was meaningful as a representation of something that really occurs. The subsequent discussion which involved algebraic representations for income was followed with interest. Tom explained:

> Let's take \$5 per hour. With this particular function it would be $y = 5x$. x is the number of hours and y is the number of dollars you make/earn. If you get time and half, what's the equation of this function? (He pauses until a student responds with $7.5x$) $7.5x$? So, lets define one way from 0 to 40 and another way from 40. Now if we're going to write this, we can write it one way: $y = 5x$ when x is less than or equal to 40. And $y = 7.5x$ when x is greater than 40. This is an example of a piece function. They call it a piece function because it is called differently over different pieces of its domain. There are a lot of piece functions. Some people in sales make a certain commission of the first \$5,000 they sell and a different percentage after that amount. There are a lot of piece functions out there in the economic world.

Using the graph to stimulate students' interest resulted in the students being able to see a real-world application of their studies and connect it to their existing knowledge. Tom said he used problems like the one above so that students could see how mathematics was related to topics outside of the discipline and how different topics within mathematics were related.

Tom used questions to extend students' thinking of the ideas under discussion. During a session on the translation of graphs, each group was given a graph from the previous day's assignment to write on the board. Tom asked for students' ideas and let the class provide the rationale for a conclusion on the direction of a translation:

> (Answering a student's question) Why does it go to the left? Because....I don't know! Is there anyone who translated that to the right instead of the left? (There is a show of hands.) Quite a few folks did. Do you think it goes to the left? (More hands.) About half of you felt it translated to the left and a few felt it went to the right. Jennifer? (I could not hear what she said.) So you're saying that when the input is $^{-}2$ then the output is 0. So, how do the rest of you feel about that? (Some responses.) You said that with conviction, as if you really knew what you were saying. Some of you think it should start on the opposite side? $^{+}2$? I know some of you still feel the other way. Let's go on – we'll come back and go over (question) fourteen. Number eighteen. Let's refocus now. What's that? (Pointing at the graph.) How many think it should be transferred to the other side? (There is a show of hands.) We need to decide on fourteen and eighteen what patterns we see. Let's do this. How many agree with this graph for number fourteen? (There is a show of hands.) OK. How many disagree? (More hands.) On number eighteen, how many agree with this graph? (More hands.) Disagree? (More hands.) OK, the only problem that I see is that 3 people at one table that all disagree and I was wanting to have there be some disagreement at each table. So, we can discuss it – you want to come over here and disagree with them? (Splitting a group so that people who disagree could talk with each other.) Let's take five minutes in these groups you're in now and see if we can reach agreement on fourteen and eighteen. I haven't told you what *I* think yet. So, these are either correct or incorrect. In groups, see what you think. (Group discussion for

> five minutes.) Number eighteen: how many groups agree with graph as it is? (Almost everyone raises his/her hand.) Disagree? (No hands.) At this point, we all agree with these as they are? I still want to say a few words about these. Some folks have ideas about rules, about translations about rules and these seem to disagree with some of the rules.

Tom mentioned in a post-observation interview that he felt the students' responses suggested poor understanding of graph-sliding. He elected to have the students wrestle with the concept of graph-sliding within their groups rather than providing the correct answers himself. Tom also said that he wanted to challenge the generalizations the students had been building over the past few assignments: "I wanted them to check their intuitions and then try to make sense of those times when there were contradictions, when things didn't make sense." Tom made a pedagogical decision to force his students to explore and correct themselves rather than tell them what they should know. He used this method because it developed student understanding more effectively although it was less efficient.

Tom's questioning opened the way to varying the tasks which he had students perform. Tom employed a large-group format for lectures and used small groups for generating discussions during the working of exercises and the production of projects. During a discussion on three dimensional graphing, Tom realized students were confused on the appearance of such a graph. His two-dimensional representation on the chalkboard of three dimensions seemed to confuse them further, so he paused for about a minute, then said "I don't really know how you are going to do this, but I want you to make a model for this function." He then wrote the following on the side board:

$$f(x,y) = 2x + y$$
$$\text{for } x \geq 0;$$
$$y \geq 0;$$
$$3x + y \leq 7.$$

Students then worked in their triads discussing how they would make such a model and what each would bring the next day to construct it. Tom circulated to each group to make sure they all understood what was being asked of them. The class was then dismissed with Tom seemingly unconcerned that the intended assignment was not given, rather, he was more interested in having students understand the concept of three-dimensional graphing. Tom's next class was also Algebra-Trig, giving Tom the opportunity to demonstrate his adaptive strategies by beginning the class with a kinesthetic demonstration of three dimensions. He set up a grid system of months running across the front of the room and days (to thirty-one) along the side wall and had each student stand on each "point" according to their date of birth. He explained that the third dimension would be represented by the top of each person's head and one could "map" the function of their birth days by using those points. It was just then that the connection between the class' physical model and the definition of function must have come to him because he began to ask what would happen if two persons had the same birth date and how a function would be effected by that occurrence. The students then returned to their seats to receive the same assignment given to the students in the previous class on the construction of a physical model for $f(x, y) = 2x + y$. The students spent the next two days constructing their models, which Tom summarized as "crude but interesting. In all, I was very pleased with them." The models were made from a wide variety of materials, such as Popsicle sticks, toothpicks, foam, and skewers. Each group completed a model which represented a three-dimensional solution.

Tom's use of questions and problems to facilitate instruction provided the students a forum for communication. Students' ideas were welcomed and validated as answers were discussed, projects were contemplated, and exercises were worked. Students were allowed to write proofs in paragraph form and were occasionally asked to write a paragraph on their feelings about a topic. At one time, Tom found out students were feeling uncomfortable with the added responsibilities that came with unconventional instruction. This information prompted Tom to lead a class discussion about expectations, feelings, and responsibilities.

Review was an integral part of Tom's daily class routine. Tom usually searched for exercises which employed techniques, vocabulary, or procedures from past lessons. One area from which Tom asked review questions was quadratics. Review was done "as we go" and "as we need things."

Tom's students used their calculators in working exercises, although there was no provision yet for computer use in the class. The classes did visit a computer lab occasionally, but scheduling and coordinating the lab experiences was difficult. Tom was a computer user himself, as were many of his students. The projects and questions that Tom posed for his students were facilitated by the use of a calculator or graphing template, but did not require the use of these implements nor that of a computer. Some concept formation was developed in the computer labs, but visits were infrequent.

The information that Tom gathered from students' understanding was from multiple sources. Tom got information about what students knew from their assignment papers, what they said in class during discussions, from questionnaires, and from their test papers. He looked at assignment papers daily to check for completion of exercises and rubber stamped the papers which were completed satisfactorily. Checking for correctness was done by students' posting answers on the chalkboards and Tom's subsequent checking for consensus. Tom rarely read answers to exercises. Tom modeled student self-assessment by showing them how to check their graphs by showing them how to "pick a test point, crank out the value for *y*, and see if it lands on your graph." Papers were bundled and handed in at the end of a unit so that students had the benefit of retaining a set of related work. Tom gave a test at the end of a unit of study. Tom attempted to grade tests "fairly, giving credit for working in a constructive manner that demonstrates a logical and reasonable plan."

Tom tried a variety of testing situations. He tried having students take tests in pairs or quiz in groups, turning in only one paper per group. He offered "retests" which were shorter and were usually made up of one application problem. Tom allowed students to schedule a retest only if they demonstrated outside effort by coming to the "tutoring lab."

Tom gathered information about students' feelings about what they were learning and their level of understanding by administering questionnaires. He would ask questions like "How is math class going so far?" Among the responses he got would be critical comments like "You aren't teaching us how to do it." Tom viewed such comments as indicators of students' reluctance to change from a traditional model of instruction to one where students were given more responsibility for their own learning. He also assessed students' conceptual weaknesses this was by asking questions like "*Why* must we have common denominators to add fractions?" Students usually responded with rules which confirmed Tom's suspicion that they had weak understanding of the concepts underlying the procedures they performed.

Tom mentioned in an interview that he had a great number of things to change in his teaching. He felt that "no one will ever arrive" at the kind of teaching proposed in the Standards Documents, but many have gone a long way already. He continued to use a lecture/discussion format in his classes, but was making changes by asking more project questions, having very large group (about 100 students) discussions in the school's theater with two other teacher's classes, and forcing the students to rely on each other more. He already had students working collaboratively but was considering some changes in their responsibilities. Tom said: "I know I have only just begun – I want that community of learners to happen." Tom articulated a desire to shift the emphasis of classroom instruction from him lecturing and explaining to "having the students figure things out" by the use of more student-centered activities. Even in the second year of this study Tom felt he was primarily "traditional" and the only visible change was he allowed his students work in groups. Tom said he had a "sick feeling" about his teaching because what the students *can* do is superficial even though they can do a lot. He outlines his goals as including a more definite move from transmission of information.

Tom's move toward the recommendations in the Standards Documents was facilitated by course work in mathematics education and educational psychology. Tom took a course titled Problem Solving which helped him to understand the "constructivist" view of learning. The educational psychology course was on learning theory, which was very different from his last experience, which was grounded behaviorism. One of the books he read for the psychology class was How Children Fail by Holt, which "spoke" to him in a much different way than when he first read it in the 1970's. The book helped him see "how students get their teachers to do their work for them." Because of his experiences and his other readings, he became very uncomfortable with many of the things he did in class. The two classes combined with some reflection on his practices caused him to conclude he was spending too much time in front of his students. Tom took these classes the Summer before the initial year of this study and completed a Masters degree in mathematics education during the Summer before the second year of the study.

Tom thought the class texts provided some good problems for the students, but led them through the process too much so that the students got only one way to view a solution. He found the texts' approaches much more frustrating than ever before. Consequently Tom used the text less and chose to rewrite many of the same problems in ways that would encourage more divergent thought.

Tom said the area where he has probably made the least progress is in his "testing, my evaluation processes." This was a continual problem for Tom because it made him feel "inadequate" to continue with his present practices but he did not see alternative models being used around him. He tried testing in pairs, which provided interesting conversations among the students as they worked on the problems and allowed another way for students to learn about concepts. Tom's goal in evaluation was to write tests that stimulated thought, were fun, and gave students a chance to apply what they learned.

Tom was able to articulate some barriers to his progress. He felt his teaching schedule did not allow him time to reflect on the changes he was trying to make in class nor to prepare activities for the students. Tom also felt that he would benefit from more opportunities to talk with colleagues, to do more reading, and to observe more teachers. Feedback from students also made him wary of changes. His students reported he was "not teaching as much" although he suspected he was "still teaching a lot" even though he recognizes he was not "teaching" as much as before. Tom concluded, after reflection, that "teaching a lot was not my goal, it is how much they learn or how much they leave with. That's my goal."

Tom was frequently unhappy with his teaching practices throughout my observations and interviews and began searching for methods to express his vision of mathematics education. He began to make more frequent observations of his colleagues, occasionally with those who used very traditional methods, and reflected on their style compared with the traditional methods he used. After observing one "very traditional" teacher he concluded "She does things so well – the kids are all on task, they are successful, and they feel good about what they are doing. Maybe she *is* doing a great job." He began to reach out to teachers in other departments to propose "team teaching" classes in physics or art. Tom also expressed an interest in developing workshops for teachers in his school district to help them implement the Standards.

Profiles of Tom's Teaching Practices

Tom's teaching practices were conventional in the way he organized class discussions. Tom addressed many of the recommendations in the Standards Documents by attempting to develop projects and kinesthetic experiences which would enhance students' understanding. Tom used whole-class discussions as the primary method of information transferal, although he said "I know I am still talking too much." His assessments were consistent with his instruction: conventional exercises for which students demonstrated knowledge of procedures and questions which addressed student understanding of concepts.

Tom used collaborative work groups with his students throughout most of his teaching career, but said he always retained control of the discussions. In this respect, Tom made a decision before the release of the Standards Documents to teach differently than he had learned. Tom decided to make further changes in his teaching practices after taking a mathematics problem-solving class at a local university. He was challenged mathematically by problems and had the opportunity to reflect on the experience of being a learner. He was immersed in situations which were designed to apply the Standards Documents and began to wonder whether students learned anything in conventional settings. A subsequent class in learning theory increased Tom's doubt about the effectiveness of his past teaching methods. Tom's course work began the summer before the first of the two studies, making his *recognition* of the paradigm of instruction promoted by the Standards Documents recent with regards to his career.

Tom's instructional practices allowed his students to formulate some concepts themselves and to be checked on their understanding in the forum of the classroom. He had students draw graphs on their own, then use a graphing template to help with graph translations (sliding). During their work, Tom's students formulated their own rules for how to perform translations, which were challenged during class discussions and questioning. Tom's use of this method, occasionally enhanced with computer investigations, provided problems that promoted investigation. He chose to use class discussions to check for students' understanding of the concepts and to summarize the situations. Tom had students discuss their understanding first, then confirmed the rule so that they could build the concept themselves rather than borrow his interpretation of it.

Tom adapted his instruction to how he perceived students' understanding, based upon students' responses during class discussions. When he realized students did not understand his drawing of three dimensions, Tom abandoned his lesson plan and asked students to create a model. He carried his perception over to his next class by having students stand in specific places of the room to represent three dimensions before assigning the project to them.

Tom's assessment methods were very much like his teaching methods. The questions he asked on tests and quizzes contained questions from the text materials which asked for replication of procedures, and questions which tested for student understanding. Sometimes Tom's students questioned his use of such problems, claiming he had never taught them "how to do it." Tom sympathized with their frustration and was frustrated himself about his students' desire to only be held accountable for procedures: "they just want to know how to get the answer, not what it means." Tom articulated a desire to alter his evaluation procedures but found no good models from which to adapt. Tom's performance as a teacher is effected by his *image* of teaching, built by his experiences as a learner (Ball, 1988). His image of teaching was challenged recently through his own experiences as a student, prompting a revision of his methods.

Tom asked some questions which required thoughtful reflection, some that were answered simplistically, and others that he was not sure of how they were going to solve. Tom still dominated the conversation in his classroom, but made a conscious effort to promote more student discussion and to invent problems for his students. Tom expressed an overwhelming desire to make advances in his implementation of the Standards Documents and discussed plans to look for courses, discussions, and trial-and-error in his own teaching to help him. Tom experienced a cognitive dissonance as a student which forced him to evaluate his instructional and assessment methods. He is striving for change but did "not know what the product would look like."

Tom's implementation of the Standards Documents

Tom implemented many of the "increased emphasis" portions of the Standards Documents at the complete level. He demonstrated the use of a variety of instructional approaches by using small-group and large-group formats for instruction. Project questions were occasionally given which prompted student discussions in their groups. Tom relied most heavily on a conventional lecture/discussion format for instruction.

Tom established the interrelatedness of mathematical topics by actively facilitating connections within mathematics and those between mathematics and other disciplines. Numerous opportunities were given students to develop different representations for mathematical concepts and to check their own conceptual understanding. The relationships between the representations the students created were facilitated and discussed.

Tom integrated assessment and instruction by using multiple assessment methods such as oral questioning and checking students' daily work. Assessments were frequently used to alter Tom's instruction as he adapted his teaching to the students level of understanding. Assessments achieved a clearer picture of the students' capabilities.

Tom implemented the remaining "increased emphasis" portions at a partial level. To involve his students in active construction and application of mathematical ideas, Tom assigned his students projects which enhanced their understanding of mathematical concepts, however, he identified the key relationships within the concepts. Tom retained control of what students learned as he actively demonstrated and explained the concept to be learned before or during the students' investigations.

To address problem-solving as a goal of instruction, problems appeared, but many lacked connection with the real world. Tom assigned exercises from the text that contained contrived situations rather than having students refine the question and collect their own data. Exercises dominated the curriculum rather than problems. Tom did attempt to invent problems which were not routine and required higher-level thinking by the students.

Tom's questioning techniques involved asking key questions then letting the class interaction work at the solution as the students conjectured, judged the validity of arguments, and analyzed the appropriateness of the answer. Tom used appropriate wait-time to encourage thoughtful responses. Tom's questioning strategies facilitated interaction among the students, but the resulting discourse required his direction to maintain its intended outcome. Students learned from their listening and talking, but reliance on Tom was still present. Questions tended to be of a leading nature with only a few responses requiring higher-level thinking.

The use of calculators and computers was present in the classroom, but they were used almost exclusively for a way to find numerical answers. Tom also used the "Easy Grapher Template" to substitute for graphics calculators in concept building. Computer use increased late in the study but they are not yet integrated into Tom's instructional practices.

Tom provided for the communication of mathematical ideas by allowing students opportunities to discuss concepts and feelings, but the discussions were orienting rather than providing a means for deeper conceptual understanding. Rules were discussed and challenged, with Tom retaining control of the discussion pattern. Mathematics was treated as a structured system, but the bulk of the understanding was not created by students. Review was accomplished by assigning exercises from previously used parts of the text and by giving questions that wove together topics from geometry and algebra.

Tom assessed students' mathematical knowledge primarily for checking procedures rather than understanding even though he frequently checked for more than correctness of answers. Answers and how they were derived were reviewed by Tom, but the information was mainly used for correction of rule systems and as a score by which grades were determined. Problem situations occurred infrequently and connected a very limited number of mathematical ideas. The problems were commonly an algorithmic replication of recently presented material. Some problems had been formulated such that Tom was uncertain how students would answer them. The variety of assessment sources Tom used were limited but still gave some picture of student understanding. He questioned while lecturing, asked for oral "proofs," encouraged pictorial representations of the students' ideas, and asked for their candid feelings. There was no apparent systematic organization to the bulk of information Tom gathered except for that which was used for grading.

Discussion

For people to *want* to change, there must be some reason. Their present way of doing things must be shown to have weaknesses. Tom's experience with the classes on problem-solving and learning theory forced him to conclude what he was doing instructionally required modification. He was open to alternatives once he saw the weakness of his practices and had experienced a new paradigm of instruction. It is important to note that Tom was naturally reflective and willing to change when reason dictated it. I doubt if everyone will accept change if their present paradigm is shaken, particularly if they are not reflective individuals.

Implementation of the Standards Documents will proceed differently for each teacher. Not only will teachers choose different portions of the documents to implement first, but they will also attempt their implementation at different times. Implementation will appear spotty instead of uniform as teachers learn from their experiences. Tom grouped is students but controlled most of the conversation. He attempted implementation of the Communication standard but was not consistent with the use of groups for collaborative problem solving. Tom wanted to increase students' involvement and responsibilities but needed more time and support in accomplishing the shift as he experimented with other instructional techniques. His practices and intents suggest supporting implementation by providing a variety of experiences, expecting a variety of outcomes, and establishing a cycle of assessment and feedback.

There must be many levels of support for change because of the time necessary for its enactment. Teachers need time to process what they are learning and to adapt it to their situations. Teachers have busy lives: the curriculum is crowded, classrooms are being populated by increasingly diverse students, teachers are taking more duties in and around their schools, and regular teaching duties take a lot of time. Discussion and reflection are the necessary tools for those attempting change and without them, progress stalls. Tom found few teachers open to the kind of change he sought and felt the need to travel outside his school district for help in sustaining his transformation.

Teachers need time away from school and the responsibilities that it imposes for change to develop. Limiting ourselves to evenings, weekends, and summers for the kind of work we want teachers to do can only make the process more difficult for them. They need extended periods of time to work on mathematics in problem situations, talk with their colleagues, observe other teachers at work, and try out their ideas with ample opportunities for reflection, feedback, and revision. In Tom's words, "I need more time to play with the mathematics and work though the ideas with other people." One reasonable way to accomplish this would be to permanently shorten every teacher's workday to provide the time necessary for professional growth.

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