

## DOCUMENT RESUME

ED 356 973

SE 053 513

AUTHOR Birenbaum, Menucha; And Others  
 TITLE Towards a Stable Diagnostic Representation of Students' Errors in Algebra.  
 INSTITUTION Educational Testing Service, Princeton, N.J.  
 SPONS AGENCY Office of Naval Research, Arlington, VA. Cognitive and Neural Sciences Div.  
 REPORT NO ETS-RR-92-58-ONR  
 PUB DATE Oct 92  
 CONTRACT C-N00014-90-J-1307; R&T-4421559  
 NOTE 29p.  
 AVAILABLE FROM Educational Testing Service, Rosedale Road, Princeton, NJ 08541.  
 PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC02 Plus Postage.  
 DESCRIPTORS \*Algebra; \*Cognitive Structures; \*Computer Assisted Instruction; \*Educational Diagnosis; Equations (Mathematics); \*Error Patterns; Foreign Countries; Junior High Schools; Junior High School Students; Knowledge Level; Mathematics Education; Mathematics Instruction; Mathematics Skills; Misconceptions; Models; Remedial Instruction; Remedial Mathematics  
 IDENTIFIERS \*Error Analysis (Mathematics); Israel (Tel Aviv); Solution Methods (Mathematics)

## ABSTRACT

Diagnoses of students' performance on procedural mathematical tasks need to display a certain level of stability and robustness if they are to be used as the basis for remediation, particularly with computer-delivered instruction. The purpose of this study was to compare two diagnostic approaches for describing students' (n=231) errors in algebra with the goal of investigating the relative stability of the diagnoses derived from these approaches. The two approaches utilized were bug analysis and rule-spaced analysis. Bug analysis compares students' answers with entries in a bug matrix constructed from applying students' incorrect procedures (mal-rules) to the test items. Rule-space analysis creates an attribute matrix of solutions strategies for solving test items and compares students' responses on parallel sets of items. Consistent with the findings of recent studies, a relatively large number of bugs were found to be unstable; stable bugs tended to be infrequent. In contrast, the results of the rule-space analysis yielded relatively more stable diagnoses. Four advantages of attribute rule-space analyses over bug analyses are presented: (1) deficient subskills as attributes are known mathematical entities and recognizable to teachers; (2) identified attributes are integral subcomponents of the task enabling failure to be traced to one or more deficiencies in subskills; (3) remedial scripts for subskill deficiencies can be prepared as a consequence of the identified advantages; and (4) teachers and researchers avoid extensive efforts to find mal-rules that are unreliable. A list of 28 references is included. (Author/MDH)

EDRS

SE

RR-92-58-ONR

ED356973

**TOWARD A STABLE DIAGNOSTIC REPRESENTATION OF STUDENTS' ERRORS IN ALGEBRA**

**Menucha Birenbaum  
Anthony E. Kelly  
Kikumi K. Tatsuoka**

This research was sponsored in part by the  
Cognitive Science Program  
Cognitive and Neural Sciences Division  
Office of Naval Research, under  
Contract No. N00014-90-J-1307  
R&T 4421559

Kikumi K. Tatsuoka, Principal Investigator



Educational Testing Service  
Princeton, New Jersey

Reproduction in whole or in part is permitted  
for any purpose of the United States Government

Approved for public release; distribution unlimited.

U.S. DEPARTMENT OF EDUCATION  
Office of Educational Research and Improvement  
EDUCATIONAL RESOURCES INFORMATION  
CENTER (ERIC)

This document has been reproduced as  
received from the person or organization  
originating it.  
 Minor changes have been made to improve  
reproduction quality.

\* Points of view or opinions stated in this docu-  
ment do not necessarily represent Official  
OERI position or policy.

2

**BEST COPY AVAILABLE**

513  
5052

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 10/1/92	3. REPORT TYPE AND DATES COVERED Interim, April 1989 - August 1992		
4. TITLE AND SUBTITLE Toward a Stable Diagnostic Representation of Students' Error in Algebra		5. FUNDING NUMBERS C-N00014-90-J-1307 61153 N RR 04204-01 R&T 4421559		
6. AUTHOR(S) Menucha Birenbaum, Anthony E. Kelly and Kikumi K. Tatsuoka		8. PERFORMING ORGANIZATION REPORT NUMBER ETS RR-92-58-ONR		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Educational Testing Service Rosedale Road Princeton, NJ 08541		9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Cognitive Science Program (1142GS) Office of Naval Research 800 N. Quincy Street Arlington, VA 22217-5000		
11. SUPPLEMENTARY NOTES		10. SPONSORING/MONITORING AGENCY REPORT NUMBER		
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release Distribution unlimited		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words)  Diagnoses of students' performance on procedural mathematical tasks need to display a certain level of stability and robustness if they are to be used as the basis for remediation, particularly with computer-delivered instruction. The purpose of this study was to compare two diagnostic approaches for describing students' errors in algebra--a bug analysis and a rule-space analysis--with the goal of investigating the relative stability of the diagnoses derived from these approaches. Consistent with the findings of recent studies, a relatively large number of bugs were unstable; stable bugs tended to be infrequent. In contrast, the results of the rule-space analysis yielded relatively more stable diagnoses. The results were discussed in light of their consequences for designing remediation.				
14. SUBJECT TERMS Cognitive diagnosis, Classification, IRT, Stability of Errors Algebra			15. NUMBER OF PAGES 21	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	

**Stability of Error Models**

**Toward a Stable Diagnostic Representation of  
Students' Errors in Algebra**

**Menucha Birenbaum**

**Tel Aviv University, Israel**

**Anthony E. Kelly**

**Rutgers University**

**Kikumi K. Tatsuoka**

**Educational Testing Service**

**Yaffa Gutvirtz**

**Ironi Daled High School , Tel Aviv, Israel**

**Running head: Stability of Error Models**

EDRS

Copyright © 1992. Educational Testing Service. All rights reserved.

### Abstract

Diagnoses of students' performance on procedural mathematical tasks need to display a certain level of stability and robustness if they are to be used as the basis for remediation, particularly with computer-delivered instruction. The purpose of this study was to compare two diagnostic approaches for describing students' errors in algebra -- a bug analysis and a rule-space analysis -- with the goal of investigating the relative stability of the diagnoses derived from these approaches. Consistent with the findings of recent studies, a relatively large number of bugs were unstable; stable bugs tended to be infrequent. In contrast, the results of the rule-space analysis yielded relatively more stable diagnoses. The results were discussed in light of their consequences for designing remediation.

### Toward a Stable Diagnostic Representation of Students' Errors in Algebra

Cognitive scientists have proposed and investigated several computational mechanisms for explaining students' procedural errors in mathematics, including Repair theory (Brown & Burton 1978; Brown & VanLehn, 1980; VanLehn, 1990), misgeneralization (Sleeman, 1984a, 1984b), deletion (Young & O'Shea, 1981), and the competing-rules model (Payne & Squibb, 1990). Regardless of the adequacy of the proposed mechanism for accounting for how errors are generated (whether in response to an impasse or as the result of misgeneralizing a learned rule), a persistent concern about existing models of errors is their instability (VanLehn, 1982; Sleeman, Kelly, Martinak, Ward & Moore, 1989; Payne & Squibb, 1989).

In order to investigate the stability of the diagnoses produced by mal-rules, researchers have observed the recurrence of mal-rules within a test (Payne & Squibb, 1990; Blando, Kelly, Schneider & Sleeman, 1989; Tatsuoka, Birenbaum & Arnold, 1989) or across tests (Payne & Squibb, 1990; Sleeman, Kelly, Martinak, Word & Moore, 1989; VanLehn, 1982; Bricken, 1987). Both within and across testings, a large number of mal-rules have been found to be unstable, and the stable ones tend to be very infrequent. Consequently, doubts have arisen regarding the potential usefulness of mal-rules for remedial purposes (Sleeman, et al., 1989).

The kernel of the problem posed by unstable mal-rules as cognitive models of error was articulated by VanLehn (1982, p. 46): "[Lack of stability] challenges us to change our image of a bug as something that necessarily exists over time as part of the child's long term beliefs. . ." In other words, for the purposes of remediation we cannot be confident that a buggy analysis of a student's performance in a mathematics task necessarily produces a stable student model. In order for human or machine-delivered remediation to proceed on a reliable basis, a stable diagnosis is a necessary, if not sufficient, prerequisite.

An alternative approach to error diagnosis is to refocus attention to the source of the impasse that causes buggy behavior (stable or unstable) on the part of the student, rather than attempting to model the cognitive response to the impasse. For example, a number of mal-rules have been identified when students are confronted with an equation in the form  $ax = b$ , including  $x = b$  (Sleeman et al., 1989),  $x = b - a$  (Sleeman et al., 1989; Payne & Squibb, 1990),  $x = -(a + b)$  (Gutvitz, 1989),  $x = a - b$  (Gutvitz, 1989), and  $x = a + b$  (Gutvitz, 1989; Payne & Squibb, 1990). What each of these bugs has in common is that each is a response to the students' nonmastery of the subskill of dividing across by the coefficient of  $x$ . The cause of the impasse is the nonmastered subskill.

As noted by VanLehn (1982), it is extremely difficult to tease out of a set of items the presence or absence of subskills using the pattern of right and wrong answers. The rule space technique, developed by Tatsuoka, was designed to handle this problem (e.g., Tatsuoka, 1983, 1985, 1990, 1991; Tatsuoka & Tatsuoka, 1987). The rule-space classifies students into knowledge states that consist of response patterns that are described in terms of mastery or nonmastery of predetermined task attributes. The analysis collapses across items, and classifies students according to factors (subskills in this case) that are identified to be integral to the successful completion of an item or subsets of items. In this paper we report on the results of a rule space analysis of students' performance on linear equations in one unknown in which the "attributes" were described at the level of the source of the student's errors (e.g., "has not mastered the distributive law").

More technically, rule-space is a probabilistic approach whose purpose is to identify the examinee's state of knowledge, based on an analysis of the task's cognitive requirements. The following is a brief presentation of the rule-space approach:

First the task's cognitive requirements (also called attributes) are specified. From these, an item  $x$  attribute incidence matrix,  $Q$ , is constructed. This matrix is binary and of order  $K \times m$  (the number of attributes  $\times$  the number of items). If  $q_{kj}$  is the  $(k,j)$  element of this matrix (where  $k$  indicates an attribute and  $j$  indicates an item) then,  $q_{kj}=1$  if item  $j$



involves attribute  $k$ , and  $q_{kj}=0$  otherwise. Concepts represented by unobservable variables that can be derived from the incidence matrix  $Q$  are called cognitive states (or attribute patterns). Boolean Description Functions are used systematically to determine those cognitive states and map them into observable item-score patterns (called ideal item-score patterns) (see Tatsuoka, 1991; Varadi & Tatsuoka, 1989). Once the ideal item-score patterns are obtained, the actual data are considered.

The rule space then maps the actual item-score patterns of the examinees onto the cognitive states in order to find the ideal item-score pattern closest to a given student's actual response pattern. This pattern classification problem is handled by the rule-space model. Item Response Theory (IRT) is utilized for formulating the classification space, which is a Cartesian product space of IRT ability/proficiency,  $\theta$ , and variable(s),  $\zeta$ , which measure the unusualness of item-score patterns (Tatsuoka, 1984; Tatsuoka & Linn, 1983). Bayes' decision rules are used for the classification of an examinee into the cognitive states. Once this classification has been carried out, one can indicate which attributes a given examinee is likely to have mastered or failed to master.

The present study examined the stability of the diagnostic models produced by rule space and those produced by a bug analysis. Rule space and buggy analyses were applied to two sets of algebra items that were designed to be parallel in terms of their attributes (task requirements).

## Methodology

### Subjects

The sample consisted of 231 8th and 9th graders (ages 14-15) from an integrated junior high school in Tel Aviv. Fifty-seven percent of the subjects were girls. The students studied mathematics in high and low achievement groupings (106 in the former and 125 in the latter).

### Instruments and procedures

A 32-item diagnostic test in linear algebraic equations in one unknown was developed by Gutvitz (1989) based on a detailed task analysis including a procedural network and a mapping sentence (e.g. Birenbaum & Shaw, 1985). The test was developed for the purpose of identifying students' bugs in solving those equations. All items were open-ended and the students were asked to show all solution steps. The present study used a subset of those items which consisted of two sets of nine parallel items attribute-wise: in set 1 (items 1, 2, 3, 6, 8, 10, 11, 12, 13); in set 2 (items 25, 24, 27, 23, 18, 19, 20, 22, 30). (The 18 items appear in Appendix A).

The correlation coefficient between the scores on the two sets was 0.85. The item difficulty indices (percent correct) in set 1 (items 1, 2, 3, 6, 8, 10, 11, 12, 13) ranged from 0.63 to 0.93 with an average of 0.78. In set 2 (items 25, 24, 27, 23, 18, 19, 20, 22, 30) the range was from 0.53 to 0.91 with an average of 0.76. The item discrimination indices (item-total correlations) in set 1 ranged from 0.49 to 0.75, with an average of 0.61. In set 2 the range was from 0.51 to 0.73, with an average of 0.61. The correlation coefficients between the two sets with respect to item difficulties and item discrimination indices were 0.93 and 0.82, respectively.

### The bug analysis:

On the basis of a detailed examination of the procedures followed by the students in solving the test items, 34 mal-rules (bugs) were identified (see Gutvitz, 1989 for a listing of the bugs). A bug X item matrix was then constructed. The entries of this matrix were the answers to the test items produced by applying the mal-rules. The students' actual answers were then matched to the entries in the bug matrix and coded accordingly. Of the actual responses, 94.6% were matched to identified bugs or to the correct rule, the rest were either unidentified bugs or clerical errors. Of the 231 subjects, 50 answered all 18 items correctly, and were therefore excluded from subsequent analysis. The coded responses included 38 different codes: one indicating the correct answer, one indicating unidentified errors, one indicating clerical errors, one indicating omissions, and the rest indicating the various identified bugs. The codes for parallel items were then compared.

Matches and mismatches were counted across the nine pairs of parallel items for each of the 181 examinees, and classified according to the following primary categories: (a) matched correct (1,1); (b) one correct and one error (1,0; 0,1); (c) matched bug; and (d) nonmatched errors (nonmatched bugs or unidentified errors).

**The rule-space analysis:**

1. Determining the attributes: A set of 11 attributes was specified for a solution strategy for solving the items (see Table 1) and used to produce an incidence matrix (see Appendix A). For example, the following attributes are appropriate for item 10 (note that "evaluating" means that the student decides from the outset not to rewrite the equation in standard form until the final step -- thereby avoiding a negative x-term):

$4(2x + 3) = 10x$	("evaluating" the equation and applying the distributive law)
$8x + 12 = 10x$	(subtracting a term from both sides)
$12 = 10x - 8x$	(adding or subtracting variable terms)
$12 = 2x$	(dividing across by the coefficient of x, when a<b)
$6 = x$	(applying the symmetry law)
$x = 6$	

See the operations denoted for item 10 in Appendix A, and the attribute list in Table 1.

-----  
 Insert Table 1 about here  
 -----

2. Testing the adequacy of the attribute matrix: A multiple regression with item difficulties as the dependent variable and the 11 attribute vectors of the Q matrix as the independent variables was performed. The set of attributes accounted for 94% of the variance ( $R^2=.94$ ;  $R^2_{adj}=.89$ ).

3. The BILOG program (Mislevy & Bock, 1983) was used for estimating the item parameters ( $a$ 's and  $b$ 's) of the IRT two-parameter logistic model. The  $b$  values for the first subtest correlated 0.90 with the  $b$  values for the second subtest. The correlation for the  $a$  values of the two subsets was 0.75. The  $b$  values of the first and second subtests ranged

from -2.12 to -.26 and from -1.90 to .04, respectively. The  $g$  values of the first and second subtests ranged from .68 to 1.52 and from .72 to 1.55, respectively.

4. The BUGLIB program (Varadi & Tatsuoka, 1989) was used for deriving the ideal score patterns corresponding to the attribute mastery patterns that constituted the groups into which the students' actual response patterns were classified. As a result, 78 groups (knowledge states) were generated. The same program was also used for the classification. The classification was applied to each subset of items separately; that is, each student was classified twice, once according to his or her responses to set 1, and once according to the responses to the parallel set, set 2.

5. The results of the classifications (i.e., the students' attributes patterns on the two sets of 11 attributes) were then compared. Of the 231 subjects, 50 answered all 18 items correctly, and 4 answered all items incorrectly; thus 54 subjects were therefore excluded from subsequent analysis. Matches and mismatches were counted across the 11 pairs of attributes for each of the 177 examinees and classified according to the following primary categories: (a) matched mastery (1,1); (b) mastery/nonmastery (1,0; 0,1); and (c) matched nonmastery (0,0).

## Results

### Mal-rule stability

Before presenting the results at the group level, two examples of the bug analysis for the two parallel sets of items for two students are presented in Table 2. A comparison of the two row-vectors for the first student (No. 13) indicated that he consistently answered correctly one pair of parallel items and consistently applied incorrect rules on five pairs of items. On the remaining two pairs of items he inconsistently applied different mal-rules, and on one pair he omitted the response to one item. Thus the percentage of matched correct responses for this student was 11.11%, the percentage of matched bugs was 55.56%, the percentage of non-matched errors was 33.33%.

The second student (No. 82) also correctly answered one pair of parallel items (11.11%), she consistently applied the same bug to four pairs (44.44%), and the percentage of unmatched

errors was 44.44%. In no case did either of the students get one of the items in a pair correct and the other item incorrect.

---

Insert Table 2 about here

---

It should be noted that although the two students had the same pattern of correct/incorrect answers, their bugs differed in type and frequency. While the first student was consistently applying three mal-rules [A:  $a + x \Rightarrow ax$ ; B:  $ax + a \Rightarrow (a + a)x$ ; and C:  $ax = b \Rightarrow x = a/b$  (when  $a > b$ )], the second student consistently applied only one mal-rule [F:  $ax \cdot b = c \Rightarrow ax = c @ b$ ; when  $\cdot$  is "+" then  $@$  is "-" and vice versa].

Evaluated at the group level, 64.58% of the total matched responses across the 9 pairs of items were matched correct answers. A further 18.97% included one correct and one incorrect response, and 6.38% were nonmatched errors (including nonmatched bugs and unidentified errors). The remaining 10.07% of the total matched responses were matched bugs. To better understand this final percentage, note that for the right/wrong scoring the overall match of correct (1,1) and incorrect (0,0) responses was 81.03%, (64.58% matched correct and 16.45% matched incorrect). Thus, of the incorrect pairs (0,0), 61% consisted of matched bugs. Greater insight into the percentage of matched bugs may be gained by inspecting Table 3. This table presents the frequency of stable bugs for each pair of parallel items. As can be seen, the thirty-four stable bugs are sparsely distributed across the nine item-pairs.

---

Insert Table 3 about here

---

**Attribute stability.**

Before presenting the results at the group level, the following is an example of the rule-space analysis at the individual level. The example is based on the responses given by the two students whose bug analyses were presented above. Since both answered correctly the same pair of items, (No. 4 in each subset) and erred on all the other items, their attribute mastery pattern is identical. The two vectors of 11 attributes for these students, as derived from their responses to the two parallel subsets, are presented in Table 4. A comparison of the two row-vectors indicates that they are identical; i.e., they reflect the same knowledge state. Thus, for both students, the percentage of matched mastery attributes (1,1) is 18.18%, the percentage of matched nonmastery is 81.82% and that of one mastery and one nonmastery is 0.00%. The students' response pattern to the test items perfectly matched the knowledge state indicating mastery of only two attributes (9 and 11, see Table 1), and nonmastery of all the rest.

-----  
Insert Table 4 about here  
-----

At the group level the percentage of matched and nonmatched responses across the 11 pairs of attributes are as follows: 80.18% of the responses yielded a match [63.38% of the responses for mastery and 16.80% for nonmastery (0,0)]. The percentage of nonmatched attributes [mastery/non mastery or (1,0), (0,1) patterns] was 19.82%. The correlation coefficient between the mastery scores derived from the two subsets in the total sample, which is an index of the reliability of these scores, was 0.79. Note that at the item level (0/1 scores) that coefficient was 0.85. The percentage of mastery for each attribute may be found in Appendix A.

### Discussion

The results of the present study showed that a rule space analysis of attributes defined in terms of the subskill components of a procedural task produced a relatively stable within-test student model. On the bug-level, although our analysis found more stable bugs than were previously reported during a single testing session (see data on School 3 in Payne and Squibb, 1989), many bugs had very low frequencies. While an unmastered skill is likely to remain unmastered (without intervening tutoring), the impasse that results from it may trigger many buggy responses (some stable and infrequent, and many unstable). For the same reason, a measure of mastery/nonmastery of a subskill is likely to demonstrate stability across testings (and be more stable than a corresponding buggy analysis), but this prediction needs to be tested empirically.

#### Advantages of Attribute Analyses over Bug Analyses

1. A clear advantage of focusing on the deficient subskills (as attributes) is that they are known mathematical entities. Consequently, remedial prescriptions for the teacher are in terms that are immediately meaningful for them (see Putnam, 1987). Bugs, on the other hand, are often a mystery both to the researcher and the teacher because, "many bugs have conditions and actions that simply do not appear in any arithmetic algorithm . . ." (VanLehn, 1990, p. 6, original emphasis).

2. The identified attributes are integral subcomponents of the task; thus if a student fails the task, the failure, at least at the procedural level, must be traceable to one or more deficiencies in these subskills (if the subskill analysis was exhaustive). The generative nature of bugs, on the other hand, means that a given catalog of bugs may explain errors for the data reported in one study, but not in another and, within the same study, bugs applicable in one school may not be applicable in a different school (Payne & Squibb, 1989). The capriciousness of bugs can lead to inaccurate diagnoses of mathematical errors (Sleeman et al., 1989; VanLehn, 1990).

3. As a consequence of the above advantages of attributes, remedial scripts for subskill deficiencies can be prepared beforehand. These scripts may be based on the recommendations of experienced teachers, culled from published studies, or stem from the tutors' "best guesses" about successful remedial strategies. A study using rule space as the basis for remediation has produced positive results (Tatsuoka & Tatsuoka, 1992). Since bugs may be produced capriciously, it is a daunting, if not impossible, task to prescribe remediation.

4. Finally, it is very labor intensive for teachers and researchers to identify, catalog, and diagnose mal-rules [VanLehn (1982) notes that three or four thousand hours were given to hand analyses of protocols]. And even with this expensive input there is no guarantee that all of the possible mal-rules will be found (Sleeman et al., 1989; Payne & Squibb, 1989; VanLehn, 1982). VanLehn (1982, p. 46) noted that even with "excellent tests, an improved DEBUGGY, and a dedicated staff of experienced diagnosticians," 34% of the population of students could not be diagnosed in terms of bugs and slips. VanLehn further noted that the remedial consequences of poor diagnosis for remediation purposes is that the computer system has then, "nothing informative to tell the teacher about the student" (p. 37, original emphasis).

While we are pleased with the within-test stability results for the rule-space analysis, future studies should investigate the stability of the rule-space results over time. In addition, cognitive models for algebra other than the subskill model described here should also be investigated.



## References

- Blando, J. A., Kelly, A. E., Schneider, B. R., & Sleeman, D. (1989). Analyzing and modeling arithmetic errors. Journal for Research in Mathematics Education, 3(20), 301-308.
- Bricken, W. M. (1987). Analyzing errors in elementary mathematics. Unpublished doctoral dissertation, School of Education, Stanford University, Stanford, CA.
- Brown, J. S. , & Burton. R. B. (1978). Diagnostic models for procedural bugs in basic mathematical skills. Cognitive Science, 2, 155-192.
- Brown, J. S., & VanLehn, K. (1986). Repair theory: A generative theory of bugs in procedural skills. Cognitive Science, 4, 379-426.
- Burton, R. B. (1982). Diagnosing bugs in simple procedural skills. In D. H. Sleeman & J. S. Brown (Eds.), Intelligent tutoring systems. (pp. 157-183). New York: Academic Press.
- Matz, M. (1982). Towards a process model for high school algebra errors. In D. Sleeman & J. S. Brown (Eds.), Intelligent tutoring systems. New York: Academic Press.
- Payne, S. J., & Squibb, H. R. (1990). Algebra mal-rules and cognitive accounts of error. Cognitive Science, 14, 445-481.
- Putnam, R. T. (1987). Structuring and adjusting content for students: A study of live and simulated tutoring of addition. American Educational Research Journal, 24, 13-48.
- Resnick, L. B. (1982). Syntax and semantics in learning to subtract. In T. Carpenter, J. Moser, & T. Romberg (Eds.), A cognitive perspective. Hillsdale, NJ: Erlbaum.
- Resnick, L. B. , Cauznille-Marmeche, E., & Mathieu, J. (1987). Understanding algebra. In J. Sloboda & D. Rogers (Eds.), Cognitive processes in mathematics. Oxford, England: Clarendon.

- Sheehan, K., Tatsuoka, K. K., & Lewis, C. (1991). Using the rule-space model to diagnose document processing errors. Paper presented at the ONR contractors meeting. Princeton NJ.
- Sleeman, D. (1984a). Mis-generalization: An explanation of observed mal-rules (Technical Report). Stanford, CA: Stanford University, Heuristic Programming Project.
- Sleeman, D. (1984b). An attempt to understand students' understanding of basic algebra. Cognitive Science, 8, 387-412.
- Sleeman, D. H. (1984b). Basic algebra revisited: A study with 14-year olds. International Journal of Man-Machine studies, 22, 127-150.
- Sleeman, D, Kelly, A. E., Martinak, R., Ward, R. D., & Moore, J. L. (1989). Studies of diagnosis and remediation with high school algebra students. Cognitive Science, 13, 551-568.
- Tatsuoka, K. K. (1983). Rule-space: An approach for dealing with misconceptions based on item response theory. Journal of Educational Measurement, 20, 34-38.
- Tatsuoka, K. K. (1984). Caution indices based on item response theory. Psychometrika, 49, 95-110.
- Tatsuoka, K. K. (1985). A probabilistic model for diagnosing misconceptions by the pattern classification approach. Journal of Educational Statistics, 50 55-73.
- Tatsuoka, K. K. (1990). Toward an integration of item response theory and cognitive analysis. In: N. Fredriksen, R. Glaser, A. Lesgold & M. C. Shafto (Eds. ), Diagnostic monitoring of skill and knowledge acquisition. (pp. 543-488). Hillsdale NJ: Lawrence Erlbaum Associates.
- Tatsuoka, K. K. (1991). Boolean Algebra applied to determination of universal set of knowledge states. Research Report ONR-1. Princeton NJ: Educational Testing Service.

- Tatsuoka, K. K., Birenbaum, M., & Arnold, J. (1989). On the stability of students rules of operation for solving arithmetic problems. Journal of Educational Measurement, 26, 351-361.
- Tatsuoka, K. K., & Linn, R. L. (1983). Indices for detecting unusual patterns: Links between two general approaches and potential applications. Applied Psychological Measurement, 7, 81-96.
- Tatsuoka, K. K., & Tatsuoka, M. M. (1987). Bug distribution and pattern classification. Psychometrika, 52, 193-206.
- Tatsuoka, K. K., & Tatsuoka, M. M. (1992). A psychometrically sound cognitive diagnostic model: Effect of remediation as empirical validity. Research Report. Princeton, NJ: Educational Testing Service.
- VanLehn, K. (1982). Bugs are not enough: Empirical studies of bugs, impasses and repairs in procedural skills. The Journal of Mathematics Behavior, 3, 3-71.
- VanLehn, K. (1990). Mind bugs. The origins of procedural misconceptions. Cambridge MA: The MIT Press.
- Varadi, F., & Tatsuoka, K. K. (1989). BUGLIB, Unpublished computer program. Trenton, New Jersey.
- Young, R. M., & O'Shea, T. (1981). Errors in children's subtraction. Cognitive Science, 5, 153-177.

Table 1

Attributes Used in the Q Matrix.

No.	Description
1	Adding a term to both sides of the equation
2	Subtracting a term from both sides of the equation
3	Applying arithmetic order of operations
4	Applying the distributive law
5	Adding or subtracting variable terms
6	Dividing across by the coefficient of $x$ , [resulting in $x=b/a$ when $a=b$ ]
7	Dividing across by the coefficient of $x$ , [resulting in $x=b/a$ when $a<b$ ]
8	Dividing across by the coefficient of $x$ , [resulting in $x=b/a$ when $a>b$ ]
9	Applying symmetry law
10	Evaluating the equation to determine the simplest solution path
11	Applying symmetry law and evaluating the equation to determine the simplest solution path

Table 2

Examples of two Students Bug Patterns for the Nine Parallel Item-Pairs

Item sets	Item-Pairs								
	1	2	3	4	5	6	7	8	9
Student # 13									
First set	A	B	C	+	B	Ui	D	B	Ui
Second set	A	B	C	+	B	Om	E	B	Cl
Student # 82									
First set	Cl	F	Ui	+	F	Cl	F	F	F
Second set	Ui	F	C	+	F	G	F	F	Ui

**Note.**

+ = Correct response

Mal-rules:

A:  $a + x \Rightarrow ax$

B:  $ax + a \Rightarrow (a + a) x$

C:  $ax = b \Rightarrow x = a/b$  (when  $a > b$ )

D:  $ax + b + x \Rightarrow (a + b + 1) x$

E:  $ax + b \Rightarrow (a + b) x$

F:  $ax \cdot b = c \Rightarrow ax = c @ b$ ; when  $\cdot$  is "+" then @ is "-" and vice versa.

G:  $ax \cdot bx = cx \Rightarrow a = cx @ bx$ ; when  $\cdot$  is "+" then @ is "-" and vice versa.

Other errors:

Cl: Clerical error

Ui: Unidentified

Om: Omitted

Table 3

Frequency of Stable Bugs by Item-Pairs

Bug No.	Item-Pairs								
	1&25	2&24	3&27	6&23	8&18	10&19	11&20	12&22	13&30
2	8								
3									1
4		1			4			4	
7							2		
9			30						
10			2				1		
14		1							
18							1		
19			10						
20		4							
21		1							
24		1			2		1		
26		2			1		2	1	6
28		3	6	4	3		3	3	
30				2					
32			1				10	1	
33						3			
34	12								10
46						2			
48		1	2						
51		1							
52							1		
59	1	1	1	1	1		1	1	
63							1		
75						1			
98						1			
102	1								2
104							1		
106						1			
116			1	1					
117		1			1		1	1	
121				1					
130			1	1					
131	1								1
No. of different bugs	5	11	9	6	6	5	12	6	5
Frequency	23	17	54	10	12	8	25	11	20

Table 4

Attribute Mastery Patterns for Students 13 and 82.

Attribute	1	2	3	4	5	6	7	8	9	10	11	Knowledge State	$D^2$
Subset 1	0	0	0	0	0	0	0	0	1	0	1	74	0.0
Subset 2	0	0	0	0	0	0	0	0	1	0	1	74	0.0

Note: The distance,  $D^2$ , is the Mahalanobis Distance from the student's point to the centroid of the closest group on the  $\theta$  and  $\zeta$  axes.

## Appendix A

The Incidence Matrix, O, for the 18 items, the Item Difficulties and Discrimination Indices, and the Percentage of Mastery for Each Attribute

Items	Attribute	% Correct	IRT	
			b	a
	11			
	1 2 3 4 5 6 7 8 9 0 1			
1	$3+x=6+3*2$	74	.71	-1.00
25	$4+x=6+2*3$	73	.72	-.94
2	$7x+7=14$	81	1.00	-1.18
24	$12x+12=24$	81	1.12	-1.08
3	$16x=4$	63	1.28	-.26
27	$28x=7$	54	1.13	.04
6	$35=7x$	93	1.20	-2.12
23	$24=6x$	92	1.29	-1.90
8	$3+6x=18$	77	1.17	-.85
18	$8+4x=26$	85	1.30	-1.25
10	$4(2x+3)=10x$	83	1.52	-1.05
19	$6(x+3)=12x$	81	1.04	-1.13
11	$6+4x+x=22$	77	1.38	-.78
20	$5+3x+x=16$	76	1.35	-.74
12	$98=7+7x$	83	1.39	-1.07
22	$75=5+5x$	84	1.55	-1.07
13	$x-4=4+2*4$	73	.68	-.98
30	$x-6=3+5*3$	67	.74	-.61
% Mastered	6966 9595987 4449 5961597			



TATLOR-ORAT-FCI  
4 MAR 92  
From ALL AREA, COG, DRAG, MEASUREMENT

## Distributive List

Dr. Terry Anderson  
Educational Psychology  
200 Education Bldg.  
University of Illinois  
Champaign, IL 61801

Dr. Terry Arent  
Code 11-003  
Office of Naval Research  
800 N. Quince Street  
Arlington, VA 22217-3000

Dr. Henry Allen  
Educational Testing Service  
Princeton, NJ 08541

Dr. Nancy S. Anderson  
Department of Psychology  
University of Maryland  
College Park, MD 20742

Dr. Stephen J. Andriole, Chairman  
College of Information Studies  
Drexel University  
Philadelphia, PA 19104

Dr. Gregory Anstg  
Educational Testing Service  
Princeton, NJ 08541

Dr. Philip Anobile  
Ortiz School of Management  
Rutgers University  
92 New Street  
Newark, NJ 07102-1205

Edward Atkins  
13703 Lakeswood Ct.  
Rockville, MD 20850

Dr. William M. Bert  
University of Minnesota  
Dept. of Educ. Psychology  
330 Burton Hall  
178 Pillsbury Dr., S.E.  
Minneapolis, MN 55455

Dr. Isaac J. Brjer  
Law School Admissions  
Service  
Box 60  
Heronston, PA 18040-0040

Leo Birenboim  
United States Nuclear  
Regulatory Commission  
Washington DC 20515

Dr. William O. Berry  
Director of Life and  
Environmental Sciences  
AFOSR/NL, HL, Bldg. 410  
Bolling AFB, DC 20312-6448

Dr. Thomas G. Bever  
Department of Psychology  
University of Rochester  
River Station  
Rochester, NY 14627

Dr. Menucha Birenboim  
Educational Testing  
Service  
Princeton, NJ 08541

Dr. Werner F. Birkh  
Fernstudienamt der Bundeswehr  
Kaiser Strasse 262  
D-3000 Eseln 90  
FEDERAL REPUBLIC OF GERMANY

Dr. Ronn Boman  
Defense Manpower Data Center  
99 Pacific St.  
Suite 135A  
Moseley, CA 90043-3231

Dr. Kenneth R. Boff  
AL/CFH

Wright-Patterson AFB  
OH 45433-6375

Dr. Guyworth Bando  
Educational Testing Service  
Princeton, NJ 08541

Dr. Richard L. Bannitt  
HQ, USMPCOM/ASPECT  
2301 Greya Bay Road  
North Chicago, IL 60064

Dr. Robert Berman  
Code 253  
Naval Training Systems Center  
Orlando, FL 32834-3224

Dr. Robert Berman  
Annapolis College Testing  
Program  
P. O. Box 100  
Leon City, IA 52243

Dr. Ann Brown  
Graduate School of Education  
University of California  
EMBT-433 Tolman Hall  
Berkeley, CA 94720

Dr. David V. Budson  
Department of Psychology  
University of Haifa  
Mount Carmel, Haifa 31999  
ISRAEL

Dr. Gregory Candell  
CTR/McMillan/McGraw-Hill  
2300 Garden Road  
Monterey, CA 90940

Dr. Pat Carpenter  
Carnegie-Mellon University  
Department of Psychology  
Pittsburgh, PA 15213

Dr. Eduardo Casarini  
Educational Testing Service  
Kensdale Road  
Princeton, NJ 08541

Dr. Paul R. Chatterler  
Perceptics  
1911 North Ft. Myer Dr.  
Suite 800  
Arlington, VA 22209

Dr. Michael Chl  
Learning R & D Center  
University of Pittsburgh  
3609 O'Hara Street  
Pittsburgh, PA 15260

Dr. Susan Chipman  
Cognitive Sciences Program  
Office of Naval Research  
800 North Quince St.  
Arlington, VA 22217-3000

Dr. Raymond S. Christal  
UBS LAMP Science Advisor  
AL/HRMIL  
Brooks AFB, TX 78235

Dr. Deborah Claman  
National Institute for Aging  
Bldg. 31, Room 3C-35  
9000 Rockville Pike  
Bethesda, MD 20892

Dr. Norman Cliff  
Department of Psychology  
Univ. of So. California  
Los Angeles, CA 90089-1601

Dr. Paul Cobb  
Purdue University  
Education Building  
W. Lafayette, IN 47907

Dr. Rodney Colking  
NIMH, Bala Beheler and  
Capitola Edman Research  
3600 Fishers Lane, Rm 11C-10  
Parkman Building  
Rockville, MD 20857

Office of Naval Research  
Code 11-0  
800 N. Quince Street  
Arlington, VA 22217-3000

Director  
Testing Systems Department  
Code 13  
Naval Personnel R&D Center  
San Diego, CA 92133-6800

Director  
Testing Systems Department  
Code 15A  
Naval Personnel R&D Center  
San Diego, CA 92133-6800

Library, Code 231  
Naval Personnel R&D Center  
San Diego, CA 92133-6400

R&D Coordinator, After: Joe Hart  
Office of the DCNO, MPT, Op-1121  
Department of the Navy, AA-0817  
Washington, DC 20376-2008

Commanding Officer  
Naval Research Laboratory  
Code 4827  
Washington, DC 20375-3000

Dr. Albert T. Corbett  
Department of Psychology  
Covington-Mellon University  
Pittsburgh, PA 15213

Dr. John M. Corwell  
Department of Psychology  
MO Psychology Program  
Tulane University  
New Orleans, LA 70118

Dr. William Crooe  
Department of Psychology  
Texas A&M University  
College Station, TX 77843

Dr. Kenneth R. Cross  
Access Sciences, Inc.  
P.O. Box 519  
San Barbara, CA 93102

Dr. Linda Curran  
Defense Manpower Data Center  
Suite 400  
1400 Wilson Blvd  
Reston, VA 22099

Dr. Timothy Dewey  
American College Testing Program  
P.O. Box 168  
Iowa City, IA 52243

Dr. Charles E. Davis  
Educational Testing Service  
Mail Stop 22-T  
Princeton, NJ 08541

Dr. Ralph J. DeAyala  
Measurement, Statistics,  
and Evaluation  
Benjamin Bldg., Rm. 1234F  
University of Maryland  
College Park, MD 20742

Dr. Geary Delaney  
Exploratorium  
3601 Lyce Street  
San Francisco, CA  
94123

Dr. Sharon Derry  
Florida State University

Department of Psychology  
Tallahassee, FL 32306

Hai-Ki Deng  
Editor  
6 Cooperate Pl.  
Rm. 17A-1K207  
P.O. Box 1120  
Hannover, NJ 08551-1120

Dr. Neil Dumas  
Educational Testing Service  
Princeton, NJ 08541

Dr. Peter Draper  
University of Illinois  
Dept. of Psychology  
605 S. Daniel St.  
Champaign, IL 61820

Defense Technical  
Information Center  
DTIC/DDA-2  
Cameron Station, Bldg 3  
Alexandria, VA 22304  
(4 Copies)

Mr. David DeBak  
Personnel Decisions Research  
Institute  
43 Main Street, SE  
Riversham, Suite 405  
Minneapolis, MN 55414

Dr. Richard Duce  
Graduate School of Education  
University of California  
Santa Barbara, CA 93106

Dr. Nancy Edinger  
College of Education  
Division of Special Education  
The University of Arizona  
Tucson, AZ 85721

Dr. John Ellis  
Naval Personnel R&D Center  
Code 13  
San Diego, CA 92132-6000

Dr. Susan Emberson  
University of Kansas  
Psychology Department  
426 Freer  
Lawrence, KS 66043

Dr. George Engelhard, Jr.  
Division of Educational Studies  
Emory University  
210 Fieldstone Bldg.  
Atlanta, GA 30322

ERIC Facility Acquisitions  
1301 Fessenden Drive, Suite 300  
Rockville, MD 20850-0205

D. E. Anderson  
University of Colorado  
Department of Psychology  
Campus Box 343  
Boulder, CO 80509-0343

Dr. Martha Evans  
Dept. of Computer Science  
Illinois Institute of Technology  
10 West 31st Street  
Chicago, IL 60606

Dr. Lorraine D. Eyle  
US Office of Personnel Management  
Office of Personnel Research and  
Development Support  
1920 H St., NW  
Washington, DC 20415

Dr. Franco Fazio  
Director General LEVADIFE  
Piazzale E. Adorno, 3

0014 ROMA EUR  
ITALY

Dr. Beatrice J. Fair  
Army Research Institute  
PERL-IC  
3281 Eisenhower Avenue  
Alexandria, VA 22304

Dr. Marshall J. Fair  
Fair-Sight Co.  
2528 North Vassar Street  
Arlington, VA 22207

Dr. Leonard Fahn  
Landscape Center  
for Measurement  
University of Iowa  
Iowa City, IA 52242

Dr. Michael L. Ferguson  
American College Testing  
P.O. Box 148  
Iowa City, IA 52243

Dr. Gerhard Fischer  
Liebiggasse 5  
A 1020 Vienna  
AUSTRIA

Dr. Myron Finkel  
U.S. Army Headquarters  
DAFIS-TR  
The Pentagon  
Washington, DC 20316-0200

Mr. Paul Foley  
Naval Personnel R&D Center  
San Diego, CA 92132-6000

Dr. Norman Fredericks  
Educational Testing Service  
(05-R)  
Princeton, NJ 08541

Dr. Alfred R. Frey  
AFOSR/ML, Bldg. 416  
Bolling AFB, DC 20332-6448

Chair, Department of  
Computer Science  
George Mason University  
Fairfax, VA 22000

Dr. Alan S. Givins  
EBO Systems Laboratory  
31 Federal Street, Suite 401  
San Francisco, CA 94107

Dr. Robert D. Gibbons  
University of Illinois at Chicago  
NPI 908A, MC 913  
812 South Wood Street  
Chicago, IL 60622

Dr. Janice Gillford  
University of Massachusetts  
School of Education  
Amherst, MA 01003

Dr. Helen Gligley  
Naval Research Lab., Code 2530  
4555 Overlook Avenue, S.W.  
Washington, DC 20375-5000

Dr. Herbert Ginsburg  
Box 104  
Yeshiva College  
Columbia University  
525 West 121st Street  
New York, NY 10027

Dr. Drew Glusker  
Educational Testing Service  
Princeton, NJ 08541

Dr. Robert Gliner

Learning Research  
A Development Center  
University of Pittsburgh  
6099 O'Hara Street  
Pittsburgh, PA 15260

Dr. Susan R. Goldsaw  
Fiskville College, Box 45  
Vanderbilt University  
Nashville, TN 37203

Dr. Timothy Goldenish  
Department of Psychology  
University of New Mexico  
Albuquerque, NM 87131

Dr. Sherris Goss  
AFHRL/AGM2  
Brooks AFB, TX 78215-5601

Dr. Wayne Gray  
Graduate School of Education  
Fordham University  
113 West 60th Street  
New York, NY 10023

Dr. Bert Green  
Johns Hopkins University  
Department of Psychology  
Chase & 34th Street  
Baltimore, MD 21218

Prof. Edward Haerel  
School of Education  
Stanford University  
Stanford, CA 94305-3066

Dr. Henry M. Hall  
Hall Research, Inc.  
4913 Elm Road, North  
Arlington, VA 22207

Dr. Ronald E. Hamilton  
University of Massachusetts  
Laboratory of Psychometric  
and Evaluation Research  
Hale South, Room 133  
Amherst, MA 01003

Dr. Deloys Harmsch  
University of Illinois  
31 Coody Drive  
Champaign, IL 61820

Dr. Patrick N. Harrison  
Computer Science Department  
U.S. Naval Academy  
Annapolis, MD 21402-5002

Ms. Rebecca Hester  
Naval Personnel R&D Center  
Code 13  
San Diego, CA 92132-6000

Dr. Thomas M. Hines  
ACT  
P.O. Box 168  
Iowa City, IA 52243

Dr. Paul W. Holland  
Educational Testing Service, 21-T  
Rosedale Road  
Princeton, NJ 08541

Prof. Lutz F. Horak  
Institut für Psychologie  
RWTH Aachen  
Ingenieurstr. 17/19  
D-5100 Aachen  
WEST GERMANY

Ms. John S. Hough  
Cambridge University Press  
40 West 20th Street  
New York, NY 10011

Dr. William Howell  
Chief Scientist

**AFHRL/CA**  
Brooks AFB, TX 78235-5401

**Dr. Ben Holliba**  
BBN Laboratories  
10 Moulton Street  
Cambridge, MA 02238

**Dr. Paul Hunt**  
Dept. of Psychology, W-25  
University of Washington  
Seattle, WA 98195

**Dr. Nayak Nayak**  
College of Education  
Univ. of South Carolina  
Columbia, SC 29208

**Dr. Maria J. Lloyd**  
Center for the Study of  
Education and Instruction  
Leiden University  
P. O. Box 9533  
2300 RB Leiden  
THE NETHERLANDS

**Dr. Robert Janssen**  
Elec. and Computer Eng. Dept.  
University of South Carolina  
Columbia, SC 29208

**Dr. Kumar Jag-dar**  
University of Illinois  
Department of Statistics  
128 Third Hall  
725 South Wright Street  
Champaign, IL 61820

**Dr. Peter Johnson**  
Department of Psychology  
University of New Mexico  
Albuquerque, NM 87131

**Professor Douglas M. Joon**  
Graduate School of Management  
Rutgers, The State University  
of New Jersey  
Newark, NJ 07102

**Dr. John Justice**  
Department of Psychology  
University of Michigan  
Ann Arbor, MI 48104

**Dr. Brian Justice**  
Carnegie-Mellon University  
Department of Statistics  
Pittsburgh, PA 15213

**Dr. Marcell Just**  
Carnegie-Mellon University  
Department of Psychology  
Schery Park  
Pittsburgh, PA 15213

**Dr. J. L. Kaind**  
Code 442/8K  
Naval Ocean Systems Center  
San Diego, CA 92132-5000

**Dr. Michael Kaplan**  
Office of Basic Research  
U.S. Army Research Institute  
5001 Eisenhower Avenue  
Aberdeen, VA 22033-5400

**Dr. Jeremy Kipshid**  
Department of  
Mathematics Education  
105 Adelphi Hall  
University of Georgia  
Athens, GA 30602

**Mr. Hae-Rim Kim**  
University of Illinois  
Department of Statistics

**106 Third Hall**  
725 South Wright St.  
Champaign, IL 61820

**Dr. Joo-Ann Kim**  
Department of Psychology  
Middle Tennessee State  
University  
Murfreesboro, TN 37132

**Dr. Sung-Hwan Kim**  
KEDI  
92-4 Unyong-Dong  
Seochu-Cu  
Seoul  
SOUTH KOREA

**Dr. G. Greg Klingberg**  
Portland Public Schools  
Research and Evaluation Department  
301 North Dixon Street  
P. O. Box 3107  
Portland, OR 97208-3107

**Dr. William Koch**  
Box 7246, Mesa, and Biol. Ctr.  
University of Texas-Austin  
Austin, TX 78708

**Dr. Kenneth Kozlowski**  
Department of Psychology  
Carnegie-Mellon University  
5005 Forbes Avenue  
Pittsburgh, PA 15213

**Dr. Richard J. Kuebel**  
School of Industrial  
Engineering  
Orlison Hall  
Purdue University  
West Lafayette, IN 47907

**Dr. James Kratoch**  
Computer-based Education  
Research Laboratory  
University of Illinois  
Urbana, IL 61801

**Dr. Patrick Kybaess**  
AFHRL/MOEL  
Brooks AFB, TX 78235

**Mr. Carolyn Laery**  
1515 Spencerville Rd  
Spencerville, MD 20686

**Dr. Marcy Lambert**  
University of North Carolina  
Dept. of Computer Science  
CB #3173  
Chapel Hill, NC 27599

**Richard Lasterman**  
Commandant (G-FWT)  
US Coast Guard  
2100 Second St., SW  
Washington, DC 20393-0001

**Dr. Michael Levin**  
Educational Psychology  
210 Edward's Bldg.  
1510 South Sixth Street  
University of IL at  
Urbana-Champaign  
Champaign, IL 61820-0000

**Dr. Charles Levin**  
Educational Testing Service  
Princeton, NJ 08541-0001

**Mr. Hain-bong Li**  
University of Illinois  
Department of Statistics  
106 Third Hall  
725 South Wright St.  
Champaign, IL 61820

**Dr. Maria C. Lina**  
Graduate School  
of Education, EMST  
Tolman Hall  
University of California  
Berkeley, CA 94720

**Dr. Robert L. Lina**  
Campus Box 249  
University of Colorado  
Boulder, CO 80502-0249

**Logan Inc. (Ann Library)**  
Tutorial and Training Systems  
Division  
P.O. Box 45150  
San Diego, CA 92161-5150

**Prof. David P. Lohman**  
College of Education  
University of Iowa  
Iowa City, IA  
52242

**Dr. Richard Looch**  
ACT  
P. O. Box 168  
Iowa City, IA 52242

**Dr. George R. Marandy**  
Department of Measurement  
Statistics & Evaluation  
College of Education  
University of Maryland  
College Park, MD 20742

**Vern M. Mohr**  
NPRDC, Code 142  
San Diego, CA 92132-6800

**Dr. Bruce Mosden**  
George Mason University  
4400 University Drive  
Fairfax, VA 22030

**Dr. Sandra P. Marshall**  
Dept. of Psychology  
San Diego State University  
San Diego, CA 92182

**Dr. Elizabeth Martin**  
ALHRA, Stop 44  
Wiltons AFB  
AZ 85240

**Dr. Nadine Martin**  
Department of Neurology  
Center for Cognitive Neuroscience  
Temple University School of Medicine  
3401 North Broad Street  
Philadelphia, PA 19140

**Dr. Paul Mayberry**  
Center for Naval Analysis  
4401 Ford Avenue  
P.O. Box 14268  
Alexandria, VA 22304-0268

**Dr. James R. McBride**  
HansERO  
6000 Elmhurst Drive  
San Diego, CA 92130

**Mr. Christopher McCutcher**  
University of Illinois  
Department of Psychology  
608 E. Daniel St.  
Champaign, IL 61820

**Dr. Robert McKinley**  
Educational Testing Service  
Princeton, NJ 08541

**Dr. Joseph McLaughlin**  
Naval Personnel Research  
and Development Center  
Code 14  
San Diego, CA 92132-6800

Alan Mend  
c/o Dr. Michael Levin  
Educational Psychology  
210 Education Bldg.  
University of Illinois  
Champaign, IL 61824

Dr. Vittorio Messori  
CNR-Isiteste Tecnologia Didattica  
Via Alfegem 7/a II  
GENOVA-ITALIA 16143

Dr. Timothy Miller  
ACT  
P. O. Box 168  
Iowa City, IA 52243

Dr. Robert Mistry  
Educational Testing Service  
Princeton, NJ 08541

Dr. Ivo Molenar  
Fysiciteit Sociale Wetenschappen  
Rijksuniversiteit Groningen  
Oude Kruisstraat 2/1  
9712 TS Groningen  
The NETHERLANDS

Dr. Allan Monroe  
Behavioral Technology  
Laboratory - USC  
230 N. Harbor Dr., Suite 300  
Redondo Beach, CA 90277

Dr. E. Muroki  
Educational Testing Service  
Roseland Road  
Princeton, NJ 08541

Dr. Roma Neudecker  
Educational Studies  
Wilford Hall, Room 218  
University of Delaware  
Newark, DE 19716

Academic Prog. & Research Branch  
Naval Technical Training Command  
Code N-42  
NAS Memphis (75)  
Millington, TN 38854

Dr. W. Alvin Norwood  
University of Oklahoma  
Department of Psychology  
Norman, OK 73071

Head, Personnel Systems Department  
NPRDC (Code 12)  
San Diego, CA 92132-6800

Director  
Training Systems Department  
NPRDC (Code 14)  
San Diego, CA 92132-6800

Library, NPRDC  
Code 941  
San Diego, CA 92132-6800

Literata  
Naval Center for Applied Research  
in Artificial Intelligence  
Naval Research Laboratory  
Code 5510  
Washington, DC 20375-5000

Office of Naval Research,  
Code 110C3  
800 N. Chazy Street  
Arlington, VA 22217-5000  
(6 Copies)

Special Assistant for Research  
Management  
Chief of Naval Personnel (PERA-OUT)  
Department of the Navy  
Washington, DC 20350-2000

Dr. Judith Overman  
Mail Stop 228-1  
NASA-Ames Research Center  
Moffett Field, CA 94035

Dr. Everett Palmer  
Mail Stop 262-4  
NASA-Ames Research Center  
Moffett Field, CA 94035

Dr. Peter J. Pashley  
Educational Testing Service  
Roseland Road  
Princeton, NJ 08541

Wayne M. Peltason  
American Council on Education  
GED Testing Service, Suite 20  
One Dupont Circle, NW  
Washington, DC 20036

Dr. Ray Pen  
Institute for the  
Learning Sciences  
Northwestern University  
1600 Maple Avenue  
Evanston, IL 60201

G. Peltomaki  
Rue Vrijs Tonnesteel 47  
Oostdarmic NSP  
1050 Brussels  
BELGIUM

Dr. Ray S. Pevet  
ARI (PERI-1)  
3000 Eisenhower Avenue  
Alexandria, VA 22303

C.V. (MD) Dr. Asseolo Peri  
Capitolo ITMCC  
Milepieri U.D.G. 7 Set  
MINISTERO DIFESA - MARINA  
00100 ROMA - ITALY

CDR Frank C. Petto  
Naval Postgraduate  
School  
Code 01PB  
Monterey, CA 95943

Dept. of Administrative Sciences  
Code 54  
Naval Postgraduate School  
Monterey, CA 95943-5026

Dr. Peter Pinell  
School of Education  
University of California  
Berkeley, CA 94720

Dr. Martha Polson  
Department of Psychology  
University of Colorado  
Boulder, CO 80508-0544

Dr. Peter Polson  
University of Colorado  
Department of Psychology  
Boulder, CO 80508-0544

Dr. Joseph Poetha  
ATTN: PERI-1C  
Army Research Institute  
3000 Eisenhower Ave.  
Alexandria, VA 22303-5600

Psy Info - CD and M  
Agencies Psychological Assoc.  
1200 Ohio Street  
Arlington, VA 22201

Dr. Mark D. Rothman  
ACT  
P. O. Box 168

Iowa City, IA 52243

Dr. J. Wesley Regino  
AFHRL/ED1  
Brooks AFB, TX 78235

Mr. Steve Reiss  
Department of Psychology  
University of California  
Riverside, CA 92521

Dr. Brian Reiser  
Cognitive Science Lab  
221 Nansen Street  
Princeton University  
Princeton, NJ 08542

Dr. Lenora Resnick  
Learning R & D Center  
University of Pittsburgh  
3929 O'Hara Street  
Pittsburgh, PA 15261

Dr. Gilbert Rissard  
Mail Stop K28-14  
Ordnance Aircraft Systems  
Bethpage, NY 11714

Mr. W. A. Rizzo  
Head, Human Factors Division  
Naval Training Systems Center  
Code 26  
12330 Research Parkway  
Orlando, FL 32826-3224

Dr. Linda G. Roberts  
Science, Education, and  
Transportation Program  
Office of Technology Assessment  
Congress of the United States  
Washington, DC 20510

Mr. Louis Roman  
University of Illinois  
Department of Statistics  
NSI Bldg Hall  
725 South Wright St.  
Champaign, IL 61820

Dr. Donald Rubin  
Statistics Department  
Science Center, Room 604  
1 Oxford Street  
Harvard University  
Cambridge, MA 02138

Dr. Pamela Samsarian  
Department of Psychology  
University of Tennessee  
3108 Austin Ferry Bldg.  
Knoxville, TN 37946-0800

Dr. Walter Scharfiter  
Learning RAD Center  
University of Pittsburgh  
3929 O'Hara Street  
Pittsburgh, PA 15260

Dr. Mary Schertz  
4100 Parkside  
Carlsbad, CA 92008

Dr. Myrna F. Schwartz  
Director  
Neuropsychology Research Lab  
Mass Rehabilitation Hospital  
1200 West Tabor Road  
Philadelphia, PA 19141

Dr. Robert I. Seidel  
US Army Research Institute  
3000 Eisenhower Ave.  
Alexandria, VA 22303

Mr. Robert Semmes  
N218 Elliot Hall  
Department of Psychology  
University of Minnesota

Minneapolis, MN 55455-0244

Dr. Valerie L. Shalin  
Department of Industrial  
Engineering  
State University of New York  
342 Lawrence D. Bell Hall  
Baldwin, NY 11510

Mr. Richard J. Shover  
Graduate School of Education  
University of California  
Santa Barbara, CA 93106

Ms. Kathleen Shuman  
Educational Testing Service  
Princeton, NJ 08541

Dr. Kazuo Shigenaga  
7-8-24 Kagurano-Kojin  
Fujisawa 251  
JAPAN

Dr. Roodolf Shonaker  
Naval Research Laboratory  
Code 5303  
4333 Overlook Avenue, S.W.  
Washington, DC 20375-5000

Dr. Zita M. Simola  
Director, Manpower & Personnel  
Research Laboratory  
US Army Research Institute  
500 Eisenhower Avenue  
Alexandria, VA 22304-5400

Dr. Derek Smeets  
Computing Sciences Department  
The University  
Aberdeen AB9 2FX  
Scotland  
UNITED KINGDOM

Dr. Robert Smith  
Naval Ocean Systems Center  
Code 443  
San Diego, CA 92132-3000

Dr. Richard E. Snow  
School of Education  
Stanford University  
Stanford, CA 94305

Dr. Judy Spry  
ACT  
P.O. Box 168  
Iowa City, IA 52243

Dr. Bruce D. Steinberg  
Corry College  
Milton, MA 02186

Dr. Martha Steeking  
Educational Testing Service  
Princeton, NJ 08541

Dr. William Stout  
University of Illinois  
Department of Statistics  
101 Illini Hall  
725 South Wright St.  
Champaign, IL 61820

Dr. Edward Stricker  
Educational Testing Service  
Mail Stop 65-T  
Princeton, NJ 08541

Dr. David Thissen  
Psychometric Laboratory  
CB# 327A, Davis Hall  
University of North Carolina  
Chapel Hill, NC 27598-3270

Mr. Thomas J. Thomas  
Federal Express Corporation  
Human Resources Development  
3886 Director Row, Suite 301  
Memphis, TN 38131

Dr. Gary Thompson  
Delaware Manpower Data  
Center  
96 Pacific Street  
Suite 151A  
Menlo Park, CA 94040

Chick, Department of Psychology  
University of Maryland,  
Baltimore County  
Baltimore, MD 21228

Dr. Kurt VanLan  
Learning Research  
& Development Ctr.  
University of Pittsburgh  
3609 O'Hara Street  
Pittsburgh, PA 15260

Dr. Frank L. Vance  
Navy Personnel R&D Center  
San Diego, CA 92132-6000

Dr. Jerry Vogt  
Department of Psychology  
St. Norbert College  
De Pere, WI 54115-2000

Dr. Jacques Voncken  
University of Geneva  
Department of Psychology  
Geneva  
SWITZERLAND 1204

Dr. Howard Weiner  
Educational Testing Service  
Princeton, NJ 08541

Elizabeth Wild  
Office of Naval Technology  
Code 227  
800 North Colony Street  
Arlington, VA 22217-5000

Dr. Michael T. Wolf  
University of  
Wisconsin-Milwaukee  
Educational Psychology Dept.  
Box 413  
Milwaukee, WI 53201

Dr. Ming-Mei Wong  
Educational Testing Service  
Mail Stop 65-T  
Princeton, NJ 08541

Dr. Thomas A. Worn  
FAA Academy  
P.O. Box 21082  
Oklahoma City, OK 73121

Dr. David J. Wain  
1640 Elliot Hall  
University of Minnesota  
75 E. River Road  
Minneapolis, MN 55455-0344

Dr. Douglas Wetzel  
Code 13  
Navy Personnel R&D Center  
San Diego, CA 92132-6000

Dr. Barbara White  
School of Education  
Telman Hall, EBST  
University of California  
Berkeley, CA 94720

German Military  
Representative  
Furnishment  
Kocher Str. 261  
D-5000 Köln 99  
WEST GERMANY

Dr. David Wiley

School of Education  
and Social Policy  
Northwestern University  
Evanston, IL 60208

Dr. David C. Wilkins  
University of Illinois  
Department of Computer Science  
405 North Mathews Avenue  
Urbana, IL 61801

Dr. Bruce Williams  
Department of Educational  
Psychology  
University of Illinois  
Urbana, IL 61801

Dr. Mark Wilson  
School of Education  
University of California  
Berkeley, CA 94720

Dr. Eugene Winograd  
Department of Psychology  
Emory University  
Atlanta, GA 30322

Dr. Robert A. Wislar  
U.S. Army Institute for the  
Behavioral and Social Sciences  
500 Eisenhower Avenue  
Alexandria, VA 22304-5600

Dr. Martin P. Wislaff  
PERSEREC  
96 Pacific St., Suite 436  
Menlo Park, CA 94040

Dr. Martin C. Wittrock  
Graduate School of Education  
UCLA, Los Angeles  
Los Angeles, CA 90024

Mr. John H. Wolfe  
Navy Personnel R&D Center  
San Diego, CA 92132-6000

Dr. Kentaro Yamamoto  
03-07  
Educational Testing Service  
Roseland Road  
Princeton, NJ 08541

Mr. Donald Yen  
Educational Testing Service  
Princeton, NJ 08541

Dr. Wendy Yen  
CTR/McGraw Hill  
Del Monte Research Park  
Menlo Park, CA 94040

Dr. Joseph L. Young  
National Science Foundation  
Room 320  
1800 G Street, N.W.  
Washington, DC 20550