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ABSTRACT

This report presents a model and an approach that institutions of higher education can use to analyze and project the impact of endogenous and exogenous factors on both net and gross tuition in the context of the increasing practice of tuition discounting. It is noted that two key questions drive this effort: (1) what influence does gross tuition pricing have on institutionally funded financial aid, and hence, net tuition? and (2) how much does a college or university need to increase its stated tuition rate to realize a desired level of net tuition revenue growth? To answer these questions, this report first examines relationships among tuition, student need, institutionally funded financial aid, and other institutional factors from a broad education industry perspective. The next section depicts the interaction of these elements by applying the discounting model developed to a fictitious, representative college. The detailed derivation of the model, and the formulae used, are presented in the appendix. (GLR)

Tuition *Discounting:*

THE IMPACT OF INSTITUTIONALLY FUNDED FINANCIAL AID

NATIONAL ASSOCIATION OF COLLEGE AND UNIVERSITY BUSINESS OFFICERS

Tuition Discounting

The Impact of Institutionally Funded Financial Aid

Loren Loomis Hubbell

National Association of College and University Business Officers

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I

Introduction

In recent years, rising tuition rates at both public and private institutions have caused consternation among payers and policy makers alike, as rapidly growing tuition exceeded most indicators of economic growth, inflation, and familial ability to pay, reversing trends of more proportionate growth in the previous decade.[1]

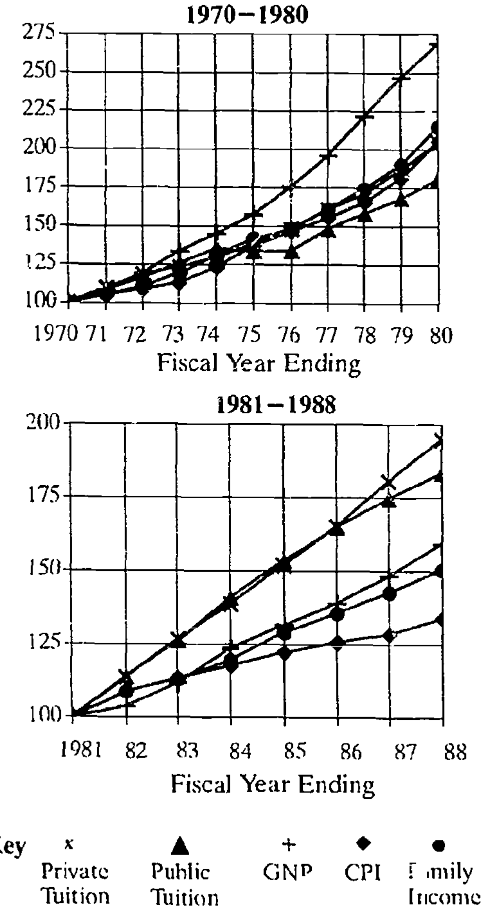
Figure 1 illustrates the trends from 1970 to 1988. The graphs present—in indexed form — the gross national product (GNP), the consumer price index (CPI), median family income, and average tuition rates, in order to compare the growth in each. From 1970 to 1980, public and private tuition rates grew, on average, at a pace commensurate with the growth in inflation and family income; the economy, as measured by the GNP, grew at a much faster tempo. Beginning with a surge in tuition rates in 1981, however, average tuition costs grew rapidly, surpassing the growth of the economy, inflation, and familial ability to pay.

Today there is a great deal of concern that the stated price of attending college is beyond levels affordable to students and their families. As a result, there has been a widespread clamor for restraint of tuition growth. The feelings of families and concerned groups have been voiced by the

[1]See the NACUBO Executive Briefing Series, "Tuition Policy and Pricing," for a discussion of tuition trends and pricing policies.

FIGURE 1
Student Resources Distribution

Indices of Growth:
Tuition, the Economy, Inflation, and Family Income



Source: CPI, median family income and average tuition: Kent Halstead, *Higher Education Tuition* (Washington, D.C.: Research Associates of Washington, April 1989), table 4. GNP, *Digest of Education Statistics 1989* (Washington, D.C.: National Center for Education Statistics, December 1989), table 33.

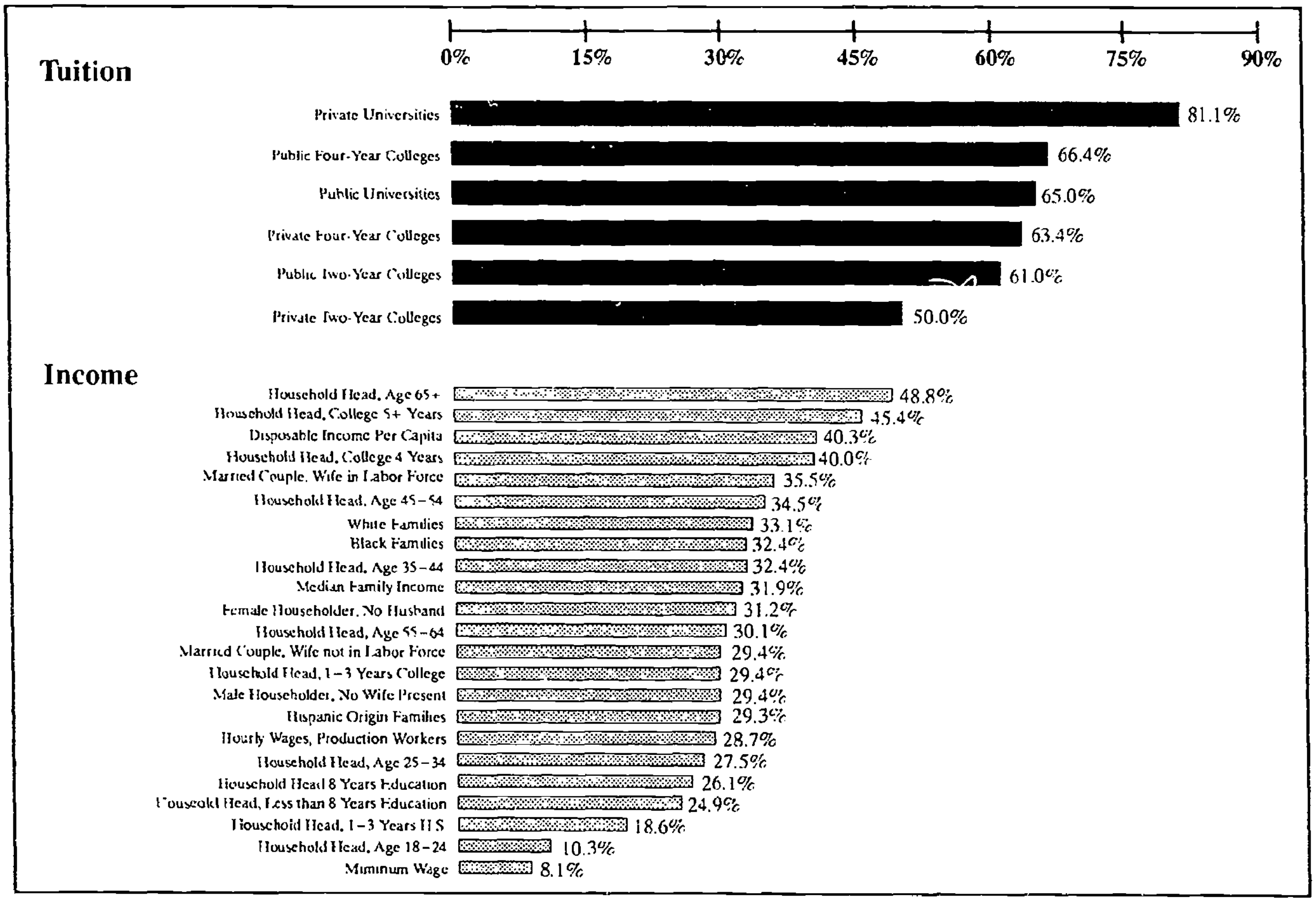
media, the Department of Education, politicians, and countless industry analysts and commentators. A slowdown, in 1991, in the growth of tuition at many institutions has been credited to the pressures brought to bear by increasingly vocal policy makers and payers.

With the tremendous, myriad cost pressures on colleges and universities and the constriction of state and local revenue (and hence funding for education), the sustainability of these lower growth rates is seriously in question. Many states with severe budgetary problems are considering — or have enacted — deep cuts in the funding of education and, in turn, sharp increases in tuition rates. At the same time, other nontuition institutional revenue sources have been adversely affected by the worsening economy. Yet cost pressures remain high, giving public and private institutions little respite from the need to increase receipts. The combination of these factors leads to a potentially unstable tuition market and an uncertain industry pricing structure. In the current analysis, the persistence of more moderate tuition growth remains up in the air as the market factors affecting institutional budgets are running counter to sustained lower tuition growth and many institutions are still raising the cost of tuition rapidly. For these and other reasons, tuition growth remains a prevailing issue in higher education.

One issue of primary concern is that, because of the disparate growth of tuition and income, many families find it difficult to afford stated tuition prices. In analyzing familial ability to pay, it is important to realize that no single measure of family income captures the breadth of economic factors affecting today's students and their respective families. For example, median family income with both spouses in the work force is a good average measure for some families, whereas median income with a female householder and no husband more accurately describes the economic circumstances of others. The average disposable income per capita correctly represents the situation of some independent students, while the growth in minimum wage is more appropriate for others. Therefore, a wide range of economic measures needs to be considered when evaluating the impact of tuition growth on families.

Figure 2 compares tuition growth in public and private universities, four-year colleges, and two-year colleges to growth in standard

FIGURE 2
Growth in Tuition Rates Compared with Family Resources, 1980–1985

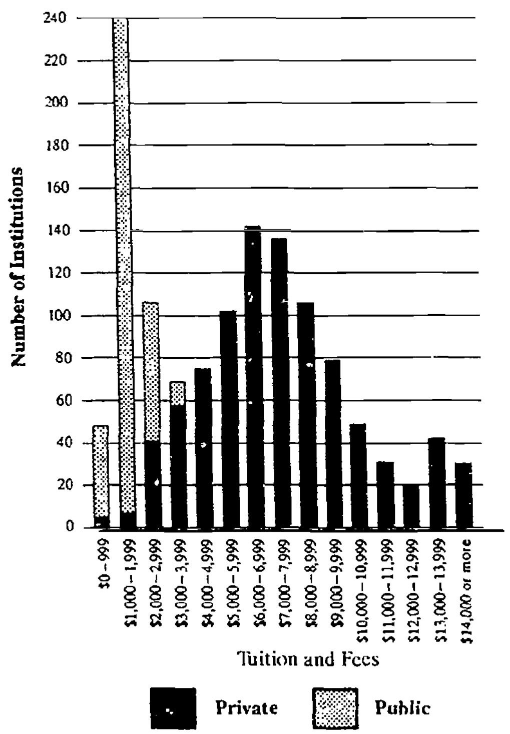


Source: Carol Frances, *Key Indicators for Higher Education*, March 1990.

measures of average family income. The clear result of this comparison is that no measure of family income grew as fast as tuition prices in the period from 1980 to 1985. Even without looking at the individual percentage growth numbers, the shape of the bar chart demonstrates a widening gap between family income and the stated price of attending college. Stated differently, tuition costs have represented a growing portion of family income. The growth in tuition can be particularly problematic when compared to the income status of lower- and middle-income students.

One reason for the outcry against tuition rates is that, despite wide diversity in college and university prices, the public is more aware of the high rates of a few institutions than the range of available educational options. In the debate over rising tuition costs, public attention has largely focused on a small, select group of expensive institutions, and the tremendous variety of pricing among colleges and universities has been overlooked. Figure 3 demonstrates the diversity of tuition costs in the higher education market. Public institutions have historically represented the less costly end of the education

FIGURE 3
The Number of Four-Year Institutions Ranked by Tuition and Fees, 1989 – 1990



Source: *The Chronicle of Higher Education Almanac*, September 5, 1990, p. 25. The data is from The College Board and includes only those 923 private and 352 public institutions that provided or estimated 1980–90 tuition and fee charges by June 21, 1989.

market, and private institutions have spanned the mid-to-upper end of the spectrum; taking these two groups together, there is clearly a broad range of educational price options.

Not only is there an assortment of educational price options, there is also an enrollment emphasis towards the lower cost offerings In 1987, approximately 20 percent of all students attended institutions where tuition was more than $2,000 and the total cost of attendance was more than $5,700. Turning that statistic around, roughly 80 percent of all students attended colleges where the total cost of attendance was less than $5,700 per year.[2] Families in general, however, do not perceive the average cost of attendance to be at this level.

A recent survey of 13- to 21-year-olds found that these individuals perceive the average prices of public and private education, at all levels, to be more expensive than the reality. For example, the survey respondents estimated that the average price for tuition, books, and supplies at a public four-year college or university was $6,841 in fall 1988. This figure is nearly $5,000 more than the actual figure of $1,977, as reported by The College Board. Respondents further believed that the average cost of attending a private four-year institution was $10,843 as compared with the actual figure of $8,120. Finally, respondents approximated the price of attending a two-year public college at $3,519 — more than double the actual cost of $1,158.[3]

The combination of proportionally more expensive educational alternatives and public misperception of the more affordable alternatives leads to real questions regarding the accessibility of higher education.

As tuition rates climb in proportion to average measures of family income and as prospective students overestimate the costs of attending college, will fewer middle- and lower-income students attend college? In recent decades, higher education has actively sought the advantages of a diverse student body. Yet perceived price growth may increasingly discourage middle- and lower-income students from considering application at the same time that rising stated tuition costs increasingly discourage those who do. One question that some experts are asking is whether the breadth of higher education opportunity will become, to a large extent, the province of the rich and the well-advised poor.

To offset the increasing burden of educational costs on families, and to promote student recruitment or retention, colleges and universi-

[2] Halstead, *Higher Education Tuition*, p. 7 and Table 11.
[3] Rita J. Kirshstein, Valentina K. Tikoff, Charles Masten, and Edward P. St. John, *Trends in Institutional Costs* (Washington, DC: Pelavin Associates, 1990), p. ii.

ties have offered increasingly generous financial aid packages to students. These packages have been designed, to a large extent, to meet student need, particularly as tuition growth has outpaced the growth in federally funded programs. Additionally, institutions have increasingly turned to non-need-based aid to promote student recruitment and retention.

In the past, student need has been met by a combination of internally and externally funded financial aid; currently, colleges and universities are offering more help than ever before. The factors that determine student need are both the familial ability to pay and the total student budget — this latter item including tuition, room and board for residential students, living expenses for commuting students, books, supplies, travel, and other related costs. The gap between the appropriate student budget and the calculated amount the family can afford to pay equals student need.

Historically, student need has been fulfilled, to a large extent, by "outside" resources. These external sources include a number of grant and loan programs funded by the federal government. The largest of these are the Perkins and Stafford loan programs, the Pell and Supplemental Educational Opportunity Grant (SEOG) programs, and the College Work-Study (CWS) program. A number of states also provide financial aid for needy students who meet specific criteria. For example, New York State administers the tax-dollar supported Tuition Assistance Program (TAP), which awards grants to needy native students who wish to attend college instate; Alabama funds a student program which provides flat, non-need-based tuition equalization grants.[4] In addition, a number of foundations and local civic groups provide some scholarship or loan funding assistance to students, either on the basis of need or other criteria.

Institutions also help students afford the cost of attending college with grants and loans funded by internal resources. Most colleges, however, cannot meet the entire amount of residual student need after externally funded financial aid has been applied. The remaining student need is then funded by whatever resources the students' families can muster.

A few institutions can still afford "need-blind" admissions. This is a policy whereby the institution promises to meet the complete amount of need for the students it admits. This policy is executed by applying the externally funded financial aid to the full extent available, and then meeting the remaining student need with institutionally funded grant and loan programs. A need-blind policy also means that admissions decisions are made without any bias regarding the prospective student's ability to afford to attend the college or university. In effect, this policy guarantees economic access to the students who are, in the eyes of the institution, academically and otherwise qualified. The cost of need-blind admissions can be extremely high, however, and in recent years a number of colleges have had to discontinue such policies because of the expense involved.

As tuition growth has surpassed the growth in externally funded financial aid, institutions have stepped into the breach with growing amounts of institutionally funded financial aid. In the eighties, average tuition rates on university and college (four-year) campuses grew slightly faster than federal loan programs, and radically faster than the declining grant programs. As tuition growth transcended both the growth in average family income and externally funded financial aid, the result was an increase in the residual amount of student need. As this student need has grown, the source of financial assistance that has grown the most to meet it has been institutionally funded financial aid. For example, institutionally funded grants from independent colleges and universities now dominate the scholarship funds awarded on their campuses each year. In 1988, independent colleges and universities awarded nearly $2 billion in scholarship grants to their students, as com-

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[4]For a state-by-state summary of state programs that impact the students' cost of attending independent institutions, see John Ashcroft and Clark Kerr, Chairmen, *The Preservation of Excellence in American Higher Education: The Essential Role of Public Colleges and Universities*, Report of the ECS Task Force on State Policy and Independent Higher Education (Denver Co.: Education Commission of the States, 1990) Appendix C.

pared with the $1.1 billion in federal grants awarded. As figure 4 demonstrates, the dominance of institutionally funded grants over those which are federally funded is in sharp contrast to the relationship between the funding levels in the mid- and late-seventies.

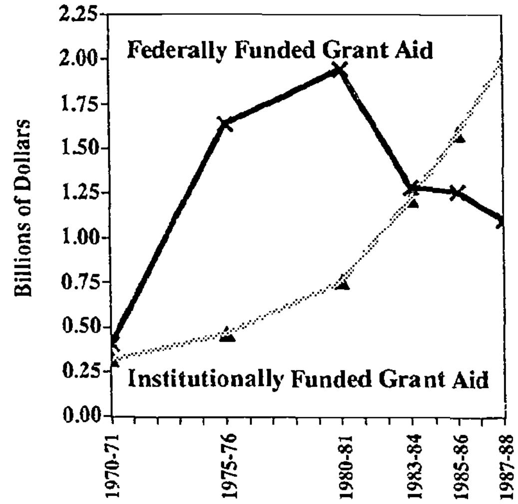
Across the higher education industry, institutionally funded financial aid has grown faster than programs funded at either the state or national level. Figure 5 compares the nominal and real growth in funding for private and public institutions from the 1980–1981 to the 1989–1990 academic years. The ten-year growth of 202 percent in institutionally funded aid exceeds growth in state aid, federal loan programs, and the federally funded CWS program.

Institutions have also increasingly turned to non-need-based aid to promote student recruitment and retention. As the number of students in the 18- to 24-year-old category has declined, institutions have utilized this form of aid to recruit and retain the mix of students that they believe will enhance the reputation of the school, as well as the educational experience of those attending. These non-need-based aid programs provide complete or partial scholarships on the basis of academic merit or special skills (e.g., academic and athletic scholarships). Although the data cited in this section relate to independent colleges and universities, anecdotal evidence suggests that the increased funding of non-need-based aid is an industrywide trend.

Notwithstanding the fact that need-based aid far exceeds non-need-based aid at independent colleges and universities, the growth in non-need-based aid has been striking.[5] In aggregate, the amount spent on non-need-based aid grew from $107.3 million to $724 million between the 1971 and 1988 fiscal years.[6] The level of this increase, by Carnegie classification of institutions, is shown in figure 6.[7]

Figure 7 compares the growth in need-based and non-need-based aid over the same period. In all categories, save Liberal Arts Colleges II, non-

FIGURE 4
Sources of Grant Funding at Independent Colleges and Universities



Source: Derived from NIICU, *A Commitment to Access: Undergraduate Financial Assistance Provided Directly by Independent Colleges and Universities* (Washington, D.C.: The National Institute of Independent Colleges and Universities, 1990), figure 2, table B-7.

Note: The institutionally funded grant aid does include non-need-based aid which slightly distorts the picture. See also figures 6 and 7.

[5] Need-based aid represented 69 percent of the aid awarded at independent colleges and universities in the 1988 fiscal year. The remaining aid was split between non-need-based at 24 percent and other, 7 percent. The 69 percent of aid spent on need-based scholarships represents a proportional decline from the 70 percent spent in 1980–81, and the 73 percent in 1970–71. Despite the proportional decline, there has been a large increase in the absolute dollars spent on need-based aid. In the analysis presented in this monograph, "non-need-based/merit" and "other" have been combined under the heading of "non-need-based aid."

[6] Derived from NIICU, *A Commitment to Access*, Tables 3, 4, and B-7.

[7] The Carnegie classification divides the higher education industry into institutional groups according to the scope of programs provided and the degrees offered. The NIICU study from which this data was taken did not provide information on two-year colleges.

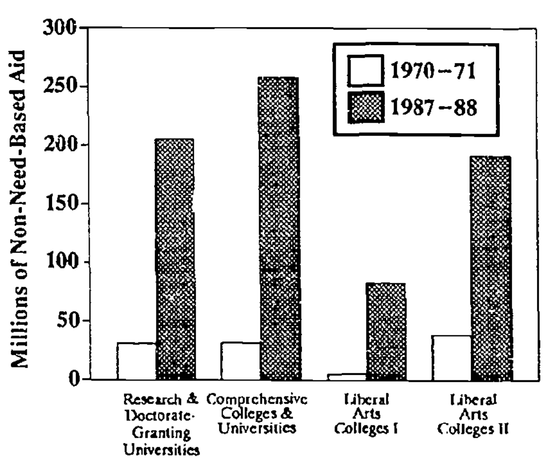
FIGURE 5
Growth in Financial Aid, 1980–81 to 1990–91

	Nominal Change	Real Growth
Federal Grants	(2%)	(37%)
Federal Work Study	25%	(20%)
Federal Loans	96%	26%
State Grants	133%	51%
Institutionally Funded and Other Grants	202%	95%

Source: Derived from The College Board, "Trends in Student Aid: 1981–1991" *Update* (New York, NY: The College Board, August 1991), tables 1 and A, and p. 14.

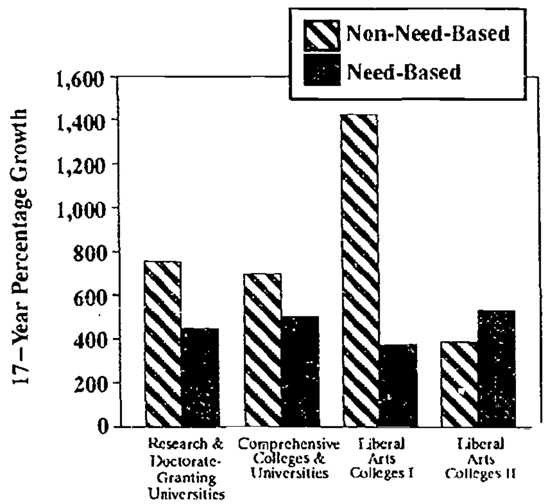
Note that the institutionally funded category includes scholarships, fellowships, and trainee stipends from those government and private programs (except SEOG, SSIG, and CWS) that allow the institution to select the recipient. This category does not capture institutionally funded loans or institutionally funded work-study programs.

FIGURE 6
Non-Need-Based Aid at Independent Institutions, 1970–71 and 1987–88



Source: Derived from NIICU, *A Commitment to Access*, tables 3, 4, and B-7.

FIGURE 7
Growth in Need-Based and Non-Need-Based Aid at Independent Institutions, 1970–71 to 1987–88



Source: Derived from NIICU, *A Commitment to Access*, tables A-1, A-5, A-9, and A-13.

need-based aid at independent colleges and universities grew faster than need-based aid over the period shown. This suggests that institutions are increasingly using financial aid awards to attract and retain students with special talents or skills. This also documents further the growing competition among institutions for specially qualified students whom the institutions believe will enhance the competitive stature of their respective schools.

Although this monograph deals with the effects that tuition rates, familial ability to pay, and financial aid have on institutions, it is important to understand at the outset that institutionally funded financial aid is, and has been, an important tool in promoting educational access. Institutions have used financial aid to create diverse student bodies; opportunities have been offered to many students who would not otherwise have been able to attend the college. Need-based aid has been, in part, responsible for the increased participation in higher education of previously underserved populations — in particular, minority groups. The continued underrepresentation of many minorities in higher education is testimony to the ongoing need to find ways to increase means of access to colleges and universities.

Although many of the issues surrounding financial aid are complex, the idea behind financial aid itself is relatively simple: such aid is fundamentally a discount on the stated tuition price.

From the perspective of the student, all scholarship aid is a reduction in the cost of attending college. Loans help make the residual cost more affordable by extending the period of time over which educational expenses are paid. Loans, however, do nothing to stop the growing cost of a higher education.

From the economic perspective of the institution, external scholarships and loans are resources the student contributes to the cost of his or her education. (Although, in the case of most externally funded financial aid programs, the college is instrumental in helping the external awarding agency administer the funds for the students on their campus.) Institutionally funded financial scholarship aid is, however, a discount on the stated tuition rate.[8] Whereas in accounting terms scholarship aid is recorded as an expense, in economic terms it is revenue which the institution agrees to forego. The discount is made, according to institutional policy and on a confidential and case-by-case basis, to individual students. The bottom line to the institution is that the amount actually charged to each student varies based upon that student's ability to pay and, in the case of non-need-based aid, the special talents the student brings to the college or university.

In many ways, institutionally funded financial aid is analogous to the contractual allowances recorded in the financial statements of health care providers. The health care industry, like the higher education industry, provides services to and receives payment from a mosaic of payers; all of these payers contribute a different amount, based on numerous factors. For both health care and higher education, the stated price of their services pertains to a declining portion of the served population.[9]

Hospitals, for example, agree to accept insurance payments that are less than their stated charges as payment in full for the health care treatment provided. The unpaid balance between the amount the hospital charges, and the amount the insurance company agrees to pay, is potential patient service revenue which the hospital contractually agrees to forego. In hospital financial statements this amount, called a contractual allowance, is offset against gross patient revenue to produce net, or real, patient service revenue. The net patient service revenue represents the amount that the hospital has actually "charged" and collected for the services rendered.

This is similar to what happens in the sphere of higher education. When an institution makes a scholarship award, it agrees to accept less than its stated price as payment in full for the educational services rendered. A major difference is that college and university financial statements do not display the net tuition revenue that would represent the real amount charged for education. By accounting convention, the "educational contractual allowance" is buried in the

[8] ALTHOUGH FINANCIAL AID IS AWARDED, AS PREVIOUSLY DESCRIBED, ON the basis of the total cost of attendance and familial ability to pay, the revenues foregone through institutionally funded scholarship aid are not proportionately assessed against both the educational and general budget and auxiliaries. This is because the auxiliaries, such as housing and food service, are treated as nearly stand-alone businesses providing needed services, much like an outside contractor, to the student body. The steps that the college takes to attract or retain its students are central to the operations of the educational institution and are not properly offsets to the profitability of the independent operations within the broad education enterprise. Therefore, financial aid is economically an offset against tuition.

[9] AS MORE AND MORE STUDENTS RECEIVE INSTITUTIONALLY FUNDED FINANCial aid, the stated price of tuition is paid by a declining percentage of the student population. Similarly in health care, as the number of "self-pay" patients had declined and increasing numbers of third-party payers move to diagnosis-related payment scales, the percent of the population actually paying hospital rates has declined.

expense section, as a portion of the financial aid expense line.

Because institutionally funded financial aid is reported as an expense item, instead of an offset against revenue, very few colleges actually measure or track their real "bottom line" revenue from students. Nationally, net tuition revenue (gross tuition less institutionally funded financial aid) is between 82 and 86 percent of the gross tuition number for private institutions and is only slightly higher for publics.[10]

The accounting perspective of institutionally funded financial aid as a cost leads one to focus on aid as an expenditure problem, and neglects the revenue viewpoint. For example, a recent major study of financial aid examined the question purely from a perspective of cost, studying the impact of institutionally funded financial aid on the educational and general expenditures (E&G) budget. Gross tuition revenue was not included as a data item in the data collection. Yet institutionally funded financial aid is fundamentally a revenue issue.

Colleges and universities need to monitor their tuition discounting because it is the net tuition, and not the gross, that represents real institutional revenue. Net tuition revenue is the amount that the institution actually receives for the educational services it renders. It is this number, in conjunction with the nontuition educational revenue, that determines the college or university's ability to meet its needs. Net tuition revenue is the real or "bottom line" source for the streams that fund instructional programs, student services, and administrative costs. Therefore, institutions need to budget for, as well as monitor, net tuition dollars at least as closely (if not more so) than gross tuition revenue.

The suggestion that discounting should be closely watched does not mean that it should be feared or abandoned. As has been previously discussed, tuition discounting is useful both in promoting educational access and in creating a diverse student body at private and public colleges and universities. Both of these outcomes are considered desirable by most institutions and policy makers. Tuition discounting, used correctly, can be a valuable instrument in the administrator's toolbox.

Looking at tuition discounting from one perspective, it increases the marginal revenue of an institution in that it helps to bring in and retain students who contribute partial monies and who would not otherwise be able to afford to attend. Because many educational costs are fundamentally fixed over the near term (largely due to personnel commitments and facilities expenditures), the marginal cost of each additional student is low. Consequently, it can be argued that discounting benefits the institution

[10]THE DISCOUNT FACTOR IS DERIVED USING RESTRICTED AND UNRESTRICTED institutionally funded financial aid. The logic for including restricted financial aid is this: most institutions follow an implicit practice of budgetary substitution with most restricted funds. That means that if money is received which is restricted to the chemistry department, for example, the dollars received replace unrestricted dollars budgeted for that department. Very few donors make gifts which are actually incremental to the funds designated for a given purpose. The donated funds replace unrestricted funds allowing the institution to use the unrestricted funds for other purposes. By analogy, restricted financial aid funds, except in very special situations, will also act as substitutes for unrestricted budget dollars and will seldom change the overall level of financial aid awarded.

Substitution will not occur in the special case where the donated funds enable or encourage the institution to spend money in a way in which the institution would not have chosen, absent the donated funds. Substitution will also not occur when an institution chooses not to avail itself of accessible restricted funds.

In general, however, when the institution embarks on a capital funds drive to raise scholarship endowment funds, in most cases, the institution has tapped its pool of available support to generate dollars for scholarship funds because the institution *as a whole* needed additional support, and because scholarship funds were easier to market to the donor pool than other campus needs. Therefore, the money raised for, and spent on, financial aid is not usually incremental to the amount that the institution *discretionarily* chooses to spend on financial aid, or would choose to spend in the future if the institution were to have additional unrestricted funds. The bottom line is that at most colleges and universities, management decides on financial aid levels and then funds that aid level with restricted or unrestricted dollars as appropriate and available. The level of restricted funding has little to no impact on the aggregate amount of financial aid awarded and hence the tuition revenue the institution agrees to forego.

Because, at most institutions, the amount spent on institutionally funded financial aid is discretionary, based on student need and management's recruitment and retention objectives, and *not* based upon the level of restricted funding, both restricted and unrestricted institutionally funded financial aid are considered when calculating the discount the institution offers to its students.

financially by increasing the revenue stream which funds the fixed expenses.[11]

This argument begins to fall apart, however, as the price of education rises above the payment capability of increasing portions of the population. Discounting growth is, in this case, a sign of the deterioration of the purchasing power of education consumers. In this light, discounting represents marginal revenue losses from the institutions' core markets.

It is under the latter scenario that the economic pressures giving rise to the discounting phenomenon have profound implications for the management of institutions. These pressures influence not only those institutions that offer internally funded financial aid, but also those which are contemplating large tuition increases.[12] Many colleges and universities are not cognizant of the factors driving their net tuition, and have difficulty reconciling their expense budget growth with the net revenue growth after discounting. Robust gross tuition rate growth often results in net revenue growth that is lower than expected, and, consequently, unanticipated budget pressure.

The impact of the rapidly growing practice — and extent — of tuition discounting is little understood and potentially significant. Yet because of the economic impact of discounting, colleges and universities need to understand and manage it on their campuses. Issues that should be considered include both the financial matters addressed in this monograph, and policy and operations questions such as:

- Is discounting necessary to fill the institution with the desired number of students?
- Is the discounting achieving the goal of promoting educational access?
- Is the financial commitment to access supported in other ways throughout the institution, in order to ensure greater success from the investment in diversity?
- Is the financial burden placed on "full-pay" student families too big? Is it appropriate?

This monograph presents a model and an approach that institutions can use to analyze and project the impact of endogenous and exogenous factors on both net and gross tuition. Two key questions drive the model of, and the approach to, net and gross tuition:

- What influence does gross tuition pricing have on institutionally funded financial aid and, hence, net tuition?
- How much does a college or university need to increase its stated tuition rate to realize a desired level of net tuition revenue growth?

To answer these questions, this monograph first examines the relationship between tuition, student need, institutionally funded financial aid, and other institutional factors from a broad education industry perspective. The next section depicts the interaction of these elements by applying the discounting model developed for this monograph to a fictitious, representative college. The detailed derivation of the model, and the formulae used, are presented in the Appendix.

[11]See Clotfelter, Ehrenberg, Getz, and Siegfried, *Economic Challenges in Higher Education* (University of Chicago Press, 1991), chapter 13, for a discussion concerning fixed vs. variable costs in higher education.

[12]"Large" here means greater than the growth in the resources of the student population served.

II

The Interrelationship of Tuition and Financial Aid Policies

This section explores the broad dynamics and interaction of tuition pricing, tuition rates, familial resources, financial aid, and net tuition in the higher education industry. The perspective is one that blends industry data and individual institutional examples.

To begin with, it is important to realize that tuition pricing is influenced by a number of different forces besides financial aid, including rising costs, shortfalls in other revenue sources, competitive pressure, and opportunity.

For example, the growth in educational and general costs has been driven both by inflation and by institutional choices. The inflationary pressure can be seen in the growth of the Higher Education Price Index (HEPI), while the choice to spend more is reflected in expenditure growth in excess of inflation rates.

The HEPI is a device that measures the rate of inflation on the goods and services purchased by colleges and universities. It has increased faster than the consumer price index (CPI) each year since 1980–81. This is in marked contrast to the seventies where, from 1973–74 through

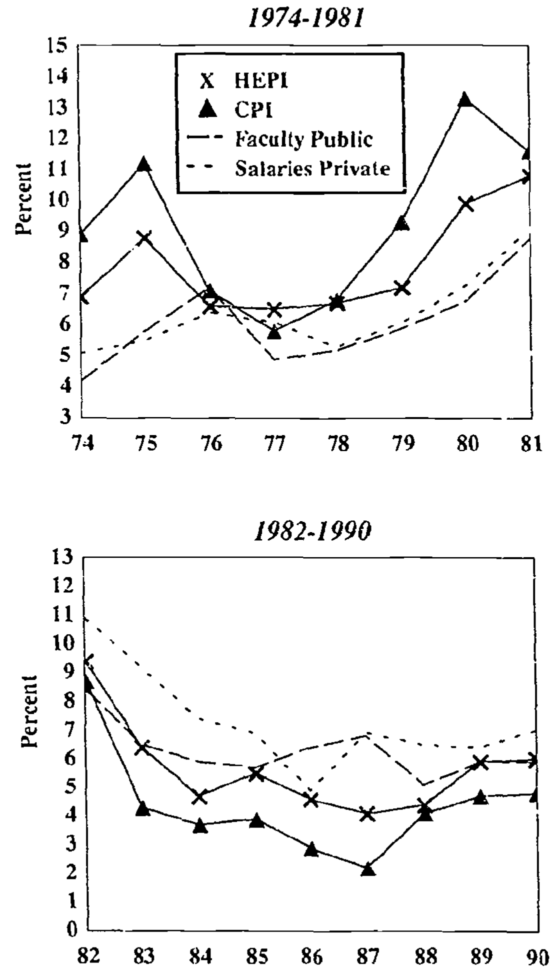
1980–81, the CPI led the HEPI (except during 1976–77). The high growth rate in the HEPI represents strong inflationary pressure on institutional costs.

Much of the difference between the growth in the HEPI and the CPI is in faculty salaries, which make up approximately a large portion of the HEPI and are not part of the CPI. Figure 8 displays the average annual price increases in the salaries of full professors at public and private institutions and compares them with the CPI and HEPI. The graph reveals that, by and large, the comparatively moderate growth in the HEPI between 1974 and 1981 was achieved through salary increases that were below the annual increase in the cost of living (as measured by the CPI). Similarly, the more aggressive growth of the HEPI from 1982 through 1990 reflects, in large part, salary increases that have been greater than the CPI.

Colleges and universities have also chosen to increase their expenditures by buying more and undertaking more, the cost of which is not reflected in a price measure such as the HEPI. One of the most frequently cited examples of this relates to computers: in recent years, although the cost of equipment has dropped dramatically, colleges and universities have purchased more computers and consequently their costs have risen. A second example that comes to mind is the field of student services. Institutions compete, in part, on the basis of student support, gracious living accommodations, and the proper learning environment; in recent years, the fulfillment of these goals has been increasingly expensive. Figure 9 charts the average growth in educational expenditures per full-time equivalent (FTE) student between 1974 and 1990 (estimated) and compares it with the growth in HEPI for the same period. The bar chart demonstrates that education expenditures per FTE student have risen between 32 and 137 percent faster than the HEPI since 1974.

Accordingly, inflationary pressures and discretionary choices have led to institutional cost increases in excess of the CPI. These cost increases require commensurate revenue increases and, in turn, place strong upward pressure on tuition rates.

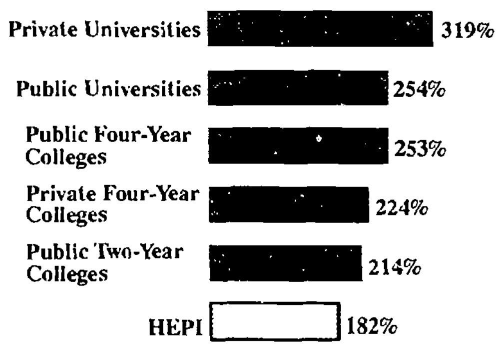
FIGURE 8
Growth in the HEPI, CPI, and Faculty Salaries, 1974–1990



Source: Research Associates of Washington, *Higher Education Price Indexes, 1990 Update* (Washington, D.C.: Research Associates of Washington, 1990), tables B and C.

Shortfalls in other revenue sources, in particular government funding, have also led institutions to increase their tuition charges faster than the growth in institutional costs. Between 1980 and 1985, tuition and fee revenue grew more rapidly than educational and general

FIGURE 9
Growth in Educational Spending per FTE Student Compared with Growth in the HEPI, 1974–1990



Source: Research Associates of Washington, *Higher Education Price Indexes, 1990 Update* (Washington, D.C.: Research Associates of Washington, 1990), tables B and C.

Note: Educational expenditures include instruction, library, student services, administration, and plant operations and maintenance. Data on two-year private colleges was not available.

(E&G) expenses at both public and private institutions. Over the same five-year period, nontuition, as well as educational and general revenues grew more slowly than did expenditures. In the previous five years (1975 to 1980), the same phenomenon occurred at private institutions, although at their public counterparts tuition and nontuition revenue both grew at the same rate as E&G expenses. This information suggests that one component of the tuition increase in the eighties has been the need to make up for shortfalls in other revenue sources.

Figure 10 illustrates this trend. The graph compares the actual tuition and nontuition revenues with their theoretical levels had they grown at the same rate as E&G expenditures. Focusing on the 1985 bars, the graph shows tuition and fee revenue growth in excess of the theoretical levels assuming E&G expenditure growth rates. This is matched with nontuition growth, which is less than the theoretical level. These highlighted differences in the 1985 bars demonstrate the extent to which tuition revenue has grown to compensate for other revenue shortfalls.

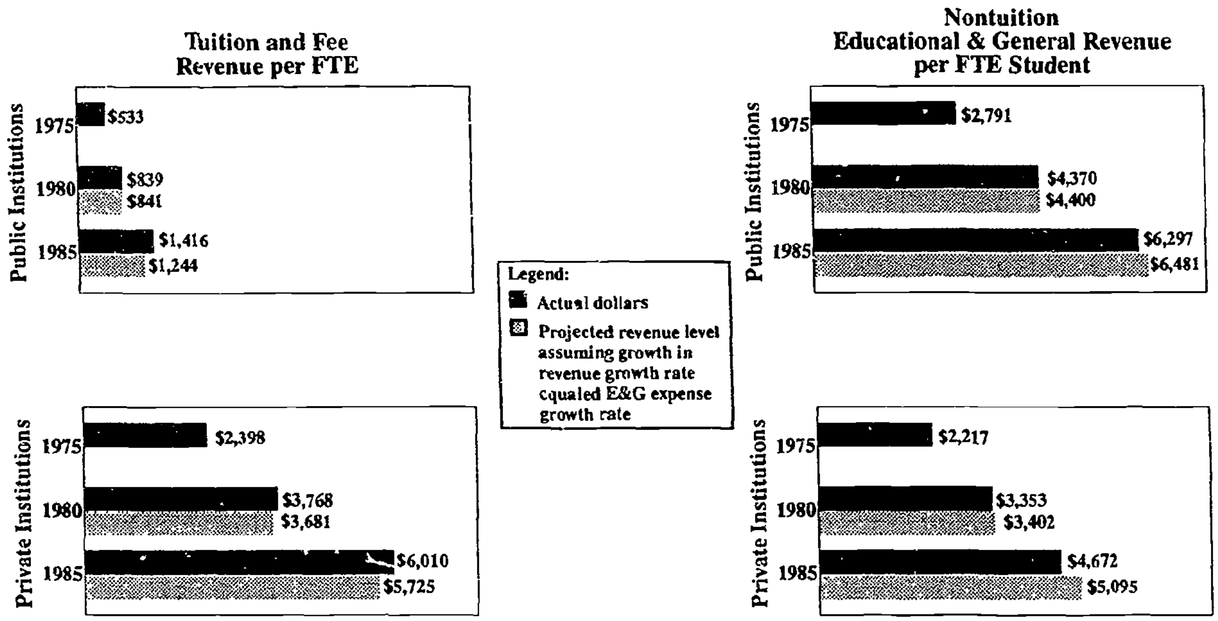
Tuition rates have also risen, in part, because of opportunity to increase. Over the past decade institutions have competed largely on the basis of doing and offering more — more programs, more student services, more research, etc. — rather than on the basis of price. Because students viewed education as an investment that provided good returns (prestige, higher paying jobs, etc.), they were therefore willing to pay the increasing tuition prices. Thus, student perception and industry price leadership enabled colleges and universities to expand. In fact, the combination of these competitive pressures meant that institutions that did not expand were often viewed as less desirable.

The highly publicized decisions by both Columbia and Washington University to close undersubscribed and expensive programs, and the announcements by other well-known institutions that they were examining the cost effectiveness of their programs, represent a sharp reversal of the expansionary trends of the eighties. Although the hard evidence has yet to arrive, many experts believe that institutions are increasingly concentrating on the delivery of core academic programs and student services and, in addition, that students are less able and/or less willing to pay rising tuition rates. Consequently, the expansionary cost pressures and tuition growth opportunities that characterized the eighties should diminish, to some extent, in the nineties.

Tuition costs have risen faster than familial ability to pay, and student need (the gap between student resources and the cost of attending college) has increased. As a result of this ever-growing student need, students have required (and will continue to require) expanded amounts of financial assistance in order to afford higher education.

For example, consider a family with two children born four years apart. Both children desire to attend College A, and both are

FIGURE 10
Growth in Tuition and Other Revenues per FTE Compared with E&G Expenditure Growth Rates



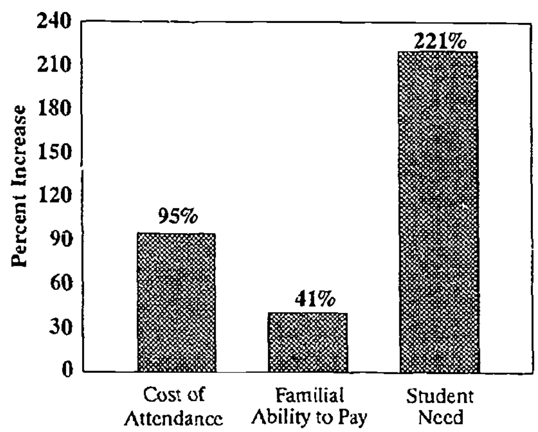
Source: Rita J. Kirshstein, Valentina K. Tikoff, Charles Masten, and Edward P. St. John, *Trends in Institutional Costs* (Washington, D.C.: Pelavin Associates, 1990), pp. 54–60.

accepted by the institution. College A is a four-year college, and tuition and room and board come to $9,400 annually. The financial aid office uses a total student budget of $10,000, which includes — in addition to the $9,400 — an additional $600 for books and other expenses. When the first child applies for financial aid, the financial aid evaluation determines that the family can theoretically afford to pay $7,000. Over the ensuing eight years, both children attend the college. Annually the total cost of attendance at the college increases by 10 percent, but the family's resources to fund education only increase by 5 percent. Figure 11 illustrates the change in cost of attendance, familial ability to pay, and student need from the freshman year of the first child to the senior year of the second. Over this period, student need grew more than twice as much as cost of attendance, which in turn grew more than twice as fast as familial ability to pay. The student need of $3,000 in the first year grew to $9,637 by the second child's senior year, increasing from 30 to 49 percent of the cost of attendance.

Because tuition charges have risen faster than federally funded financial aid, there is a greater burden on state programs and schools to meet the increased need. As discussed earlier in the monograph, federally funded financial aid has not kept pace with the growth in tuition. Therefore, even though the federal government still provides the majority of financial assistance, the federal share of student aid has dropped sharply at many institutions in recent years.[13] This has put pressure on state and institutional programs to increase their funding of

[13]Federally funded financial aid has dropped from a high of 85.6 percent of all aid in 1981 to 76 percent in 1991. This is the lowest

FIGURE 11
An Example of the Growth in Student Need over Eight Years



Assumptions

Cost of attendance, year 1, $10,000 with 10 percent annual increase

Family resources to fund education, year 1, $7,000 with 5 percent annual increase

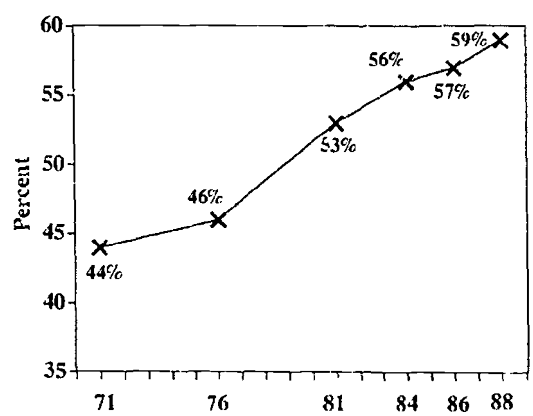
The analysis was performed for two students attending sequentially. Percentage growth is measured as the change in each category from the freshman year of the first to the senior year of the second.

financial aid, and both groups have done so (institutions more than states).

What would happen if institutions did not increase their financial assistance programs? If the families of students at a particular college were unwilling or unable to meet the additional burden caused by the shortfall in external funding, the students concerned would be powerless to attend that college. Under this scenario, higher priced colleges and universities would quickly become inaccessible to large segments of the population. To make sense of this scenario, it is important to realize the pervasiveness of student need. In 1971, 44 percent of the undergraduates at independent colleges and universities received some form of institutionally funded financial aid. By 1988, this figure had grown to 59 percent of the student population (see figure 12).

Institutionally funded financial aid is a major component of E&G expenditures at many private colleges. A recent study of institutionally funded financial aid at private institutions revealed that institutionally funded financial aid makes up 20 percent or more of the E&G expenditures at nearly one-fifth of all private colleges and universities. For another one-fourth of the private institutions, institutionally funded aid is between 15 and 19 percent of E&G expenditures (see figure 13). This means that for

FIGURE 12
Percent of Undergraduates at Independent Institutions Receiving Institutionally Funded Financial Aid

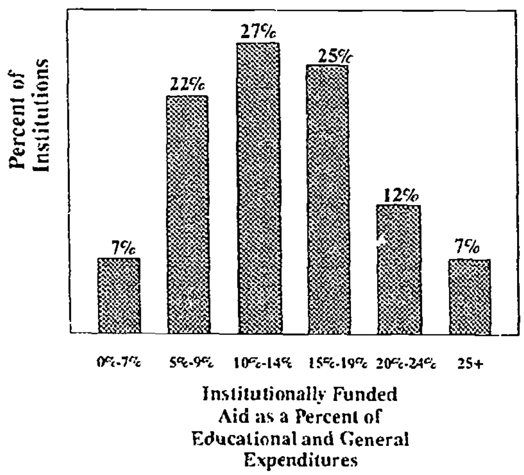


Source: NIICU, *A Commitment to Access*, figure 1.

ratio for federal funds since 1971 when the ratio was also 76 percent. Over the same period, institutionally funded financial aid has changed in a near mirror image to federal funds: institutionally funded aid was 18.9 percent of all aid in 1971; it dropped to 9.7 percent as federal funds rose and then grew to 17.6 percent as federal funds declined as a percentage of all aid granted. The balance between the two is the share held by state-funded grants programs which have hovered between 4.7 percent and 6.9 percent of all aid.

The College Board, *Update*, tables 1 and A.

FIGURE 13
Distribution of Private Colleges and Universities by Level of Current Fund Financial Aid



Source: NIICU, *A Commitment to Access*, figure 4.

many colleges and universities, a significant portion of the "cost" their revenue must cover is financial aid.

In addition, an accurate overview of discounting must take into account the fact that colleges of all sizes, affiliations, and reputations are affected by the growing demand for financial aid. A popular misconception is that this growing burden of institutionally funded financial aid is really only an issue for small or relatively unheralded colleges and universities. Yet financial aid at one prestigious women's liberal arts college grew from almost nothing to over 5 percent of tuition revenue in just five years. In other examples of an increasingly common practice, a nationally known and respected liberal arts college (coed), an Ivy League institution, and a Catholic college with a national student body all devote over 20 percent of their tuition revenue to financial aid. Financial aid is not a phenomenon affecting isolated segments of the higher education industry. It is a problem which can be categorized throughout the industry by price, but not one which can be divided by size, quality, or reputation.[14]

By focusing on gross tuition revenue instead of the net, colleges and universities can be led to decisions which are not necessarily in the best interests of the institution. The following examples illustrate this point.

One institution attempted to stem a disastrous decline in enrollment from 1979 through 1983 with increasing amounts of financial aid. In 1984, the school gave out more financial aid than ever before, and for the first time in five years enrollment stabilized. Attempting to continue this success, in 1985 the administration awarded even more financial aid; enrollment dropped somewhat, but there was only a slight decline in gross tuition income. The next few years, enrollment continued to decline and the college poured tremendous resources into financial aid. Finally, in 1988, gross tuition receipts turned up again. The college breathed a collective sigh of relief — administrators believed that the end of the financial crisis was in sight. What the board and management had not realized was that their net tuition revenues were continuing to erode, even as the gross amount went up. Appearances were deceiving— the college was still earning less and less each year (see figure 14).

---

[14]THE RECENT NIICU STUDY PROVIDES EVIDENCE FOR THE INDEPENDENT sector of the pervasiveness of the rapid financial aid growth phenomenon across institutional type. While their data suggest more rapid growth in financial aid at comprehensive colleges and universities, the rate of growth found in other institutional categories is remarkably undifferentiated.

Average Annual Rate of Growth in Financial Aid
(Inflation adjusted growth in parentheses)

	1971–1938	1981–1988
Research and Doctorate-Granting Institutions	28.0%(5.7%)	20.5%(11.7%)
Comprehensive Colleges and Universities	33.3%(7.5%)	24.2%(14.5%)
Liberal Arts Colleges I	26.8%(5.0%)	20.6%(11.8%)
Liberal Arts Colleges II	27.1%(5.4%)	19.9%(11.3%)

Derived from:
NIICU, *A Commitment to Access*, tables A-1, A-2, A-5, A-6, A-9, A-10, A-13, A-14

A second college increased tuition rates 6 percent in 1990. However, because of burgeoning financial aid, net tuition rose only 2 percent. Unfortunately, expenses other than financial aid grew at 6 percent and the largely tuition-dependent college suffered a substantial operating deficit for the year.

In another instance, a major research university offered scholarships to 38 percent of its incoming freshmen, although the school had only budgeted assistance to 32 percent. The result was a $5 million shortfall.[15]

In these three cases, the institutions in question courted disaster by overlooking the importance of net tuition. A final example illustrates the control and benefits that come with net tuition awareness.

A president at a financially troubled college cut financial aid by over 25 percent in one year in an attempt to bring the college back to profitable operations. He simultaneously put through an 8 percent increase in tuition rates, made a number of improvements on campus, and spent an enormous amount of time making the case for the renewal of the college to concerned parents, students, and alumni. A nervous board watched gross tuition revenue drop from $9 million to $8.91 million and FTE students drop from 900 to 825. In this instance the president was able to demonstrate real financial gains from this admittedly risky move by pointing the board away from gross tuition and towards "free" versus "paying" heads.[16] As depicted in figure 15, the number of paying heads increased by ten FTE students while the number of free heads dropped by 85 FTE students. The ratio of free to paying heads dropped from 3:7 to a little over 2:7, and the bottom line was more real net revenue despite the decline in gross tuition.

FIGURE 14
Gross and Net Tuitions at ABC College

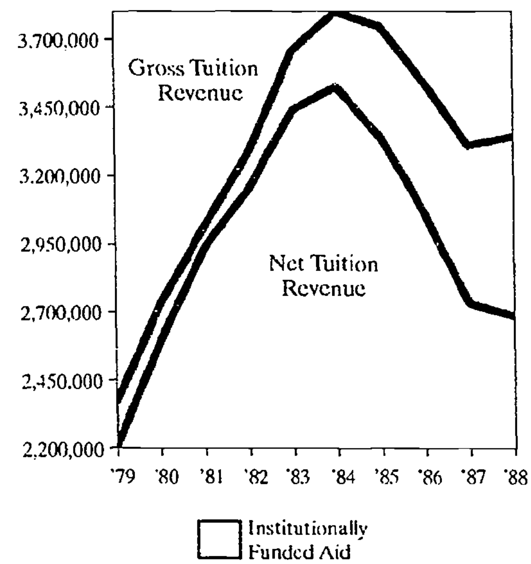


FIGURE 15
Free and Paying Heads

	1987	1988
Gross Tuition	**9,000,000**	**8,910,000**
Institutionally Funded Financial Aid	2,700,000	2,000,000
Net Tuition	6,300,000	6,910,000
Paying Heads	630	640
Free Heads	270	185
Total FTE Students	**900**	**825**

These four examples illustrate the power of the interrelationship of tuition, financial aid, and the real discretionary resources earned by the institution. Without understanding the role of net tuition, as well as the forces that drive it, institutions cannot accurately gauge the real revenue available to fund educational programming.

Nationally, because the growth in institutionally funded financial aid has exceeded the growth in tuition, net revenue for the funding of

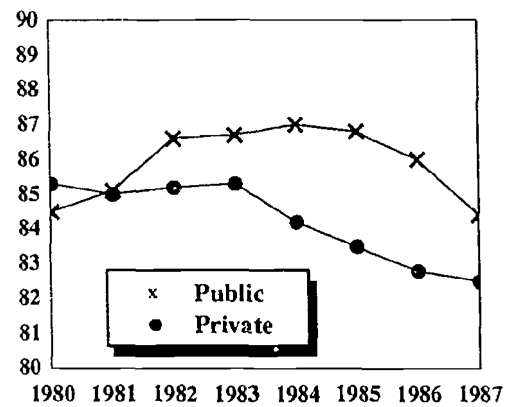
[15] "The Frantic Rush to Fill the Freshman Class," Fortune, Sept. 24, 1990, p. 108.

[16] "Paying heads" are calculated as net tuition divided by the full-time student tuition rate. They are, in essence, the number of full-paying, full-time students on campus. "Free heads" are the total fees on campus less the number of paying heads. Free heads are the number of full-scholarship, FTE students on campus.

education has not increased as quickly as gross tuition receipts.

As portrayed in figure 16, both public and private institutions retained less of each tuition dollar in the late-eighties than they did in the mid-eighties.[17] Anecdotal evidence suggests that tuition retention, at least at private institutions, has continued to decline since 1987, the final year on the graph. Although the decline represented in this graph is hardly "budget-shattering," it does represent a downward trend for the industry, and, for some institutions, a slippery slope of declining revenue.

FIGURE 16
Estimated Percent of Tuition Retained (Tuition and Fees less Institutionally Funded Financial Aid)



These tuition retention ratios are derived from HEGIS data. Since the release of the first year of IPEDS data substantial discrepancies have been discovered in the HEGIS numbers which caused an overstatement in the reporting of institutionally funded aid. While the private/public split is not readily available for the IPEDS numbers, the aggregate discount in 1987 is now estimated to be approximately 86 percent. Notwithstanding the adjustment in the overall level of institutionally funded aid, the author believes that the shape of these lines is indicative of the trends in public and private tuition retention between 1980 and 1987.

Source: Derived from *Digest 1989*, tables 266, 270, 271; The College Board, *1990 Update*, tables 1, 7, and A.

[17]RETENTION ON EACH TUITION DOLLAR IS ANOTHER WAY OF EXPRESSING the net tuition issue. Retention is net tuition divided by gross tuition. It is the percent of each gross tuition dollar that the institution actually earns as spendable revenue.

III

An Institution-Specific Tuition Discounting Model

This section discusses the tuition discounting model, utilizing the fictional prototype Snow College. Snow's student and pricing data are manipulated to illustrate the specific interrelationships of gross and net tuition, need, financial aid, and other institutional financial factors (costs and nontuition revenue). This section focuses on answering the two key questions asked in the introduction:

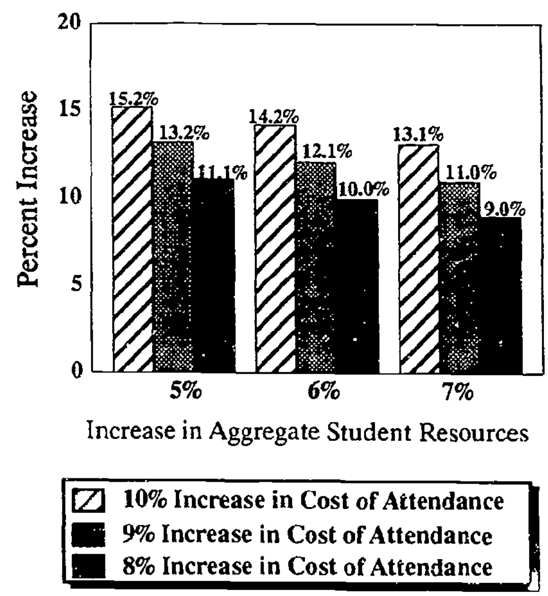
- What impact does gross tuition pricing have on institutionally funded financial aid and hence, net tuition revenue?
- By how much does the college need to increase its stated tuition rate to realize a desired level of net tuition revenue growth?

The background information on the college necessary to run the model is as follows. Snow College is a small institution with a stable enrollment of 4,000 undergraduate students. The tuition rate in 1991 was $7,000. The nontuition charges in the average student budget for the year were $3,000 for room, board, books, supplies, and other related costs.

In 1991, the college was 75 percent tuition dependent. At that time, Snow College had $300,000 in SEOG, CWS, and Perkins funds available for distribution as grants and loans to needy students. Pell and Stafford aid was awarded as well, also on the basis of need. Snow College has studied the distribution of the expected family contribution (EFC) towards the cost of attending the college. It has found that the EFCs for its students are distributed uniformly from $0 to $25,000.[18] That means that there are the same number of students who have an ability to pay $0 for their education as there are who have the ability to pay $25,000 for their education, and every point in between. Twenty-five students received non-need-based grants for the complete cost of attending Snow College in 1991, an amount which did not change during the period projected by the model.

Figure 17 illustrates the combined impact of the different growth rates in the cost of attendance and familial ability to pay on student need at Snow College.[19] In general, aggregate student need rises sharply as the growth in the cost of attendance outdistances the growth in familial resources. If, for example, the college chooses to increase the package of tuition, room, board, books, etc., by 10 percent for 1992, but the aggregate familial resources to fund the education of the student body increases by only 5.0 percent, then student need for financial aid will increase by 15.2 percent. If, on the other hand, the college decides to increase the cost of attendance by 8 percent and familial resources increase by 7 percent, then student need for financial assistance will only increase by 9 percent.

FIGURE 17
Impact of Cost of Attendance Increases and Student Resource Growth on Aggregate Student Need



The accelerating growth in student need as the cost of attendance rises faster than familial resources places pressure on the institution to increase financial aid for the students. The main issue for the college is whether the burden of this need will be carried by external funding sources, by the institution, by the family, or by some combination of these agents.

Because federally funded aid will not rise commensurately with student need at Snow College, the college may need to increase financial aid dramatically over the next ten years to avoid placing additional financial burdens on its student populations. Indeed, federal financial aid is expected to grow quite slowly over the next ten years for students at Snow College. Pell funds will actually decline as the growth in income moves families out of the narrow EFC range in which Pell awards are made. SEOG,

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[18] THE CONTINUOUS UNIFORM DISTRIBUTION WAS CHOSEN FOR THE MODEL for illustrative purposes only. It is an easy distribution with which to work, making the approach and the formulae outlined in the appendix more accessible. Each institution will have a unique distribution of resources which may or may not resemble uniform distribution. therefore, the specific results shown for the hypothetical Snow College will not correlate to specific results an any individual institution. However, the authors believe that the illustration of the dynamic relationships between tuition, familial resources, externally funded aid, and other factors can provide some insight to institutions struggling with tuition and financial aid issues.

[19] THE FORMULAE USED TO CALCULATE THE RESULTS AND THE APPROACH taken in their development are presented in detail in the Appendix.

CWS, and Perkins funds are projected to grow at 1 percent annually. The model's ten-year growth in the externally funded category is driven by the use of Stafford loans, which are used by a greater percent of students each year. More students are projected to take out the education loans because the tuition rate growth in excess of the rate of family income drives up the number of needy students.

Figure 18 compares the ten-year increase in student need to the associated growth in externally funded financial aid at various levels of annual growth in student need.[20] For example, if student need were to grow at between 9.0 and 9.02 percent per year, the ten-year growth in student need would be 137.1 percent but externally funded aid would grow only 22.0 percent. The gap widens as student need grows faster, so an annual increase in need of between 15.0 percent and 15.02 percent results in ten-year growth in student need of 304.9 percent, but the growth of externally funded financial assistance is only 58.2 percent.

Because of the projected shortfall in federally funded financial aid, Snow College will need to increase its financial aid funding approximately twice as fast as student need in order to maintain financial assistance at a constant percent of need. Due to the widening gap between the growing student need and the federal financial aid funds, the residual need after external aid has been awarded increases more rapidly than need over the ten-year period. Therefore, if Snow maintains a policy of always meeting a set percentage of residual need, then institutionally funded financial aid will grow as rapidly as the residual need amount.

For example, an 11.0 percent annual growth in need results in a 184.0 percent cumulative increase in student need and a 501.7 percent growth in institutionally funded financial aid. Figure 18 reveals that all need points modeled, the ten-year increase in need-based institutionally funded financial aid is more than two and one-half times the growth in family need.

FIGURE 18
Ten-Year Increase in Externally Funded and Institutionally Funded Financial Aid by Student Need Category, 1991-2001

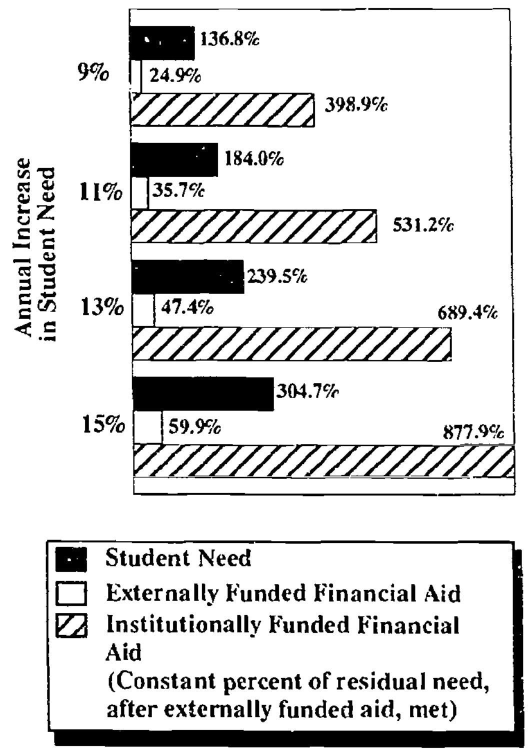


Figure 19 looks at the composition of tuition for families of Snow College students at all resource levels. The white and shaded areas indicate the relative amounts which are paid out of familial resources, Stafford loans, and other federal financial aid programs.[21] The darkly

[20]FIGURE 18 IS CALCULATED USING A GROWTH IN FAMILY RESOURCES OF 5 percent annually. The annual growth on the cost of attendance is chosen to produce the desired growth in student need. Need-based aid is the only form of institutionally funded aid included.

[21]OTHER FEDERAL FINANCIAL AID PROGRAMS ARE GROUPED TOGETHER and their aggregate contribution represented by the area of the white triangle. The Appendix presents the detailed calculation of Pell awards by EFC and the aggregate funding from other programs such as SEOG, CWS, and Perkins. The individual Pell awards by EFC are used in the model calculations.

hatched section in the upper left is institutionally funded financial aid — the tuition discount for need-based aid (the picture assumes that Snow College meets 100 percent of student need.)

It is interesting to compare this chart with figure 20. In figure 20, the tuition discount model is adjusted to reflect what these components would look like if Snow had had a cost of attendance of $13,000 instead of $10,000 in 1991. First, the trigger for financial aid moves upward from an EFC of $10,000 to an EFC of $13,000. Now, instead of 40 percent of the student body receiving some form of financial assistance, 52 percent of the students are eligible and receiving financial aid. The amount of Stafford loan funding increases to meet the higher demand for aid. What doesn't change is the contribution from other federal aid programs. The change in cost of attendance has no impact on Pell, SEOG, CWS, or Perkins loan funding. Finally, the amount of institutionally funded aid required is sharply higher in the $13,000 example than it is in the $10,000 example. This means that the tuition in the $13,000 example is more deeply discounted than in the lower cost model. However, despite the discounting, the net tuition revenue is higher at Snow when it has a $13,000 cost of attendance. These findings are summarized in figure 21.

Another way to look at the composition of tuition payment funding sources is to consider the impact of institutionally funded aid on the cost of attendance for full-pay students. Figure 22 demonstrates how the tuition discount received by needy students can be seen as a charitable contribution from their more affluent peers. If all students were full-pay, or if all student needs were met by external funds, then the cost of attendance would decrease. This gap between the actual cost of attendance and the cost of attendance if all students were full-pay (or fully funded by external financial assistance), is the amount that some families contribute to support those students who cannot pay. Families of full-paying students end up making a sizable "charitable donation" to Snow College to fund the education of students who could not

FIGURE 19
Funding Sources for the Cost of Attendance by Family Resource Level: Cost of Attendance = $10,000

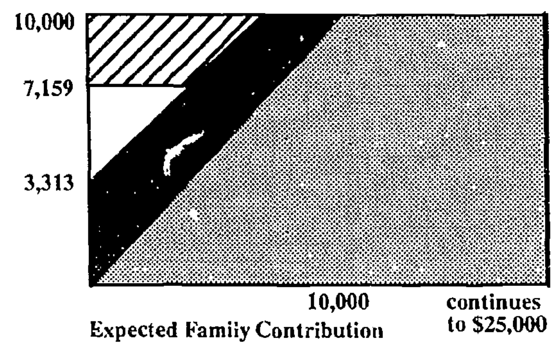
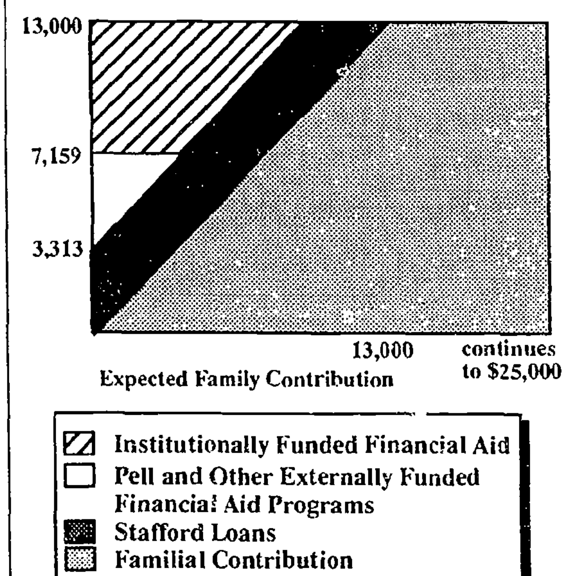


FIGURE 20
Funding Sources for the Cost of Attendance by Family Resource Level: Cost of Attendance = $13,000



otherwise afford to attend. As the practice of tuition discounting grows, so does the level of this "Robin Hood" transfer of wealth.

It is important to note that the concept of wealth transfer in support of education is not new. The federal government's scholarship

FIGURE 21
Impact of Different Costs of Attendance on Tuition Revenues and Financial Aid

	Example A	Example B
Cost of Attendance	$10,000	$13,000
Gross Tuition Revenue	28,000,000	40,000,000
Student Need	8,000,000	13,520,000
Externally Funded Financial Aid:		
Stafford Loans	4,422,722	6,012,962
Other Financial Assistance	1,183,133	1,183,133
Need-Based Internally Funded Financial Aid*	2,394,145	6,323,905
Net Tuition Revenue	25,605,855	33,676,095
Tuition Discount	8.6%	15.8%

* *This example assumes that 100% of need is met and that the institutionally funded aid is in the form of grants. No non-need-based grants are included.*

policies and (to a lesser extent) loan programs are wealth transfer vehicles. What is new is that the recent growth in student need and the shortfall in government assistance has led institutions to more aggressively participate in this discounting process.

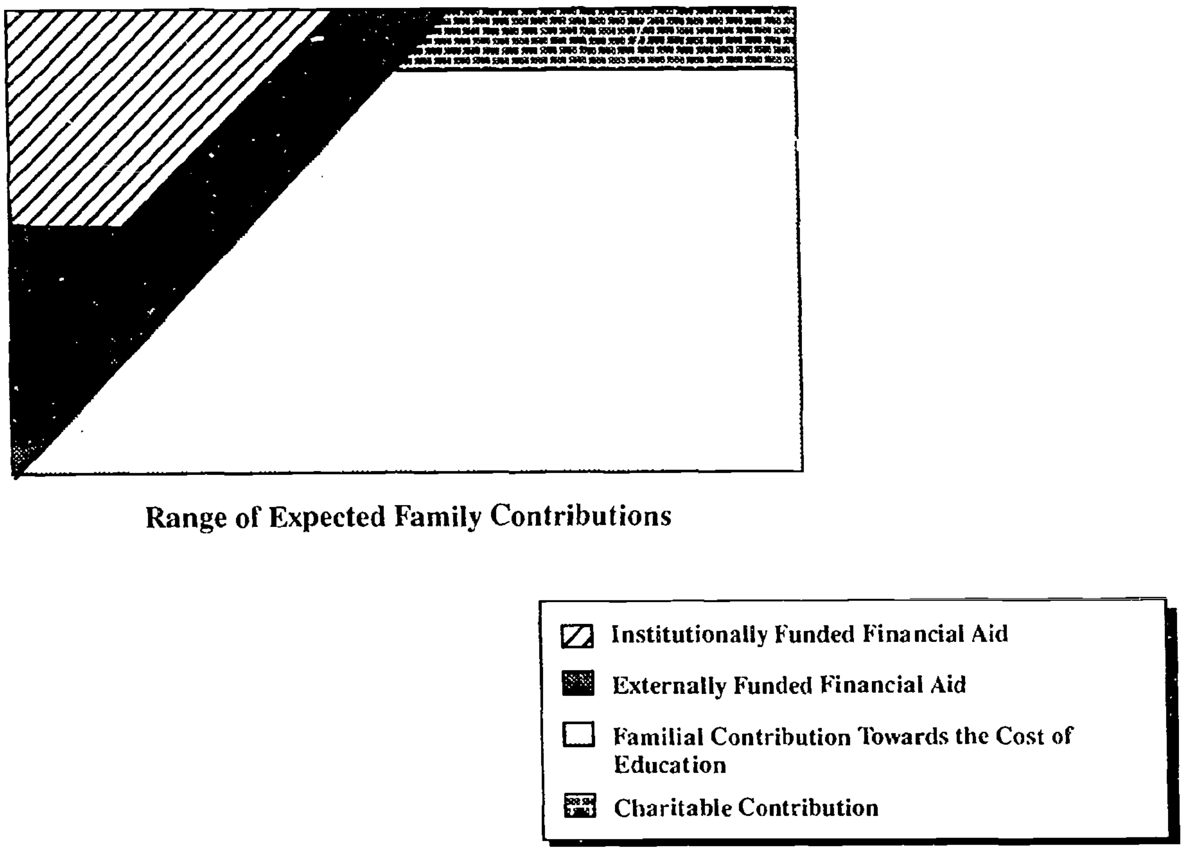
Net tuition revenue grows at a predictably slower pace when gross tuition increases faster than familial ability to pay, which means that Snow College must maintain its expenditure budget at a lower level of growth than tuition rates. If Snow College continues to fund a fixed percent of the residual student need after federally funded financial assistance, or if Snow College increases its funding percentage, then the net tuition earned will grow at a slower pace than gross tuition. This is characterized in figure 23. On this graph, net tuition rates are calculated for a range of family income growth scenarios. The graph depicts the spread in net tuition revenue caused by the hyper-sensitivity of institutionally funded financial aid to the gap in growth rates between tuition and family income. It also highlights the need for institutions to understand the relationships between gross tuition, familial resources, and net income.

Figure 24 looks at the tuition discount from a different perspective, presenting the percent of each tuition dollar that is real, incremental money received by the college. The graph demonstrates the erosion of tuition as higher and higher amounts of institutionally funded financial aid are required. The construct of net tuition (vs. gross tuition) can have a powerful impact on how institutions perceive and manage their costs. The graphs suggest, for example, that budgeting against gross tuition growth could cause an institution to spend more than their revenue stream will support.

Knowing the change in familial resources, and the growth in costs and nontuition revenue, Snow College can project the level of net and

FIGURE 22

Funding Sources for the Cost of Attendance by Family Resource Level: Charitable Giving from the Self-Pay Student Families



Range of Expected Family Contributions

gross tuition increase required. Figure 25 illustrates the relationship of net and gross tuition. The graph depicts, for different levels of net tuition growth desired by the college, the increase required in tuition rates. For example, to achieve a 10 percent increase in net tuition revenue when familial resources increase only 5 percent and the nontuition portion of the student budget grows at 7 percent, Snow College must raise their gross tuition rate from $7,000 to $7,911 — a 13 percent increase. If, on the other hand, the desired net tuition growth is only 6.0 percent, the required gross tuition rate increase is more moderate — a 7.9 percent increase to $7,553 will meet the college's needs.[22]

[22]These calculations do not include the impact of gross tuition increases on student demand. In basic economic theory, as the price increases, demand decreases. The obstacle in adding such a feedback loop to the model is that the shape of function describing the relationship between price and demand in higher education is not known. Without knowing the elasticity of demand any attempt to quantify it could well muddy the resulting calculations. Clearly, this is an area where additional research, which is beyond the scope of this monograph, could add significant value.

FIGURE 23
Net and Gross Tuition Under Varying Levels of Student Resource Growth

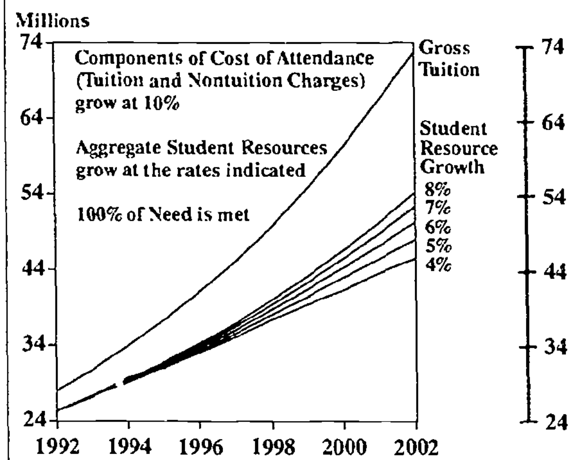
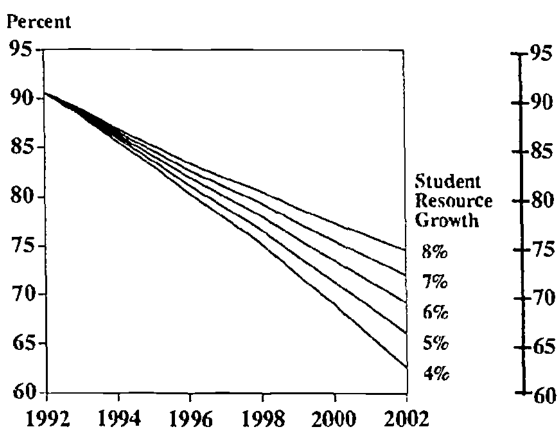
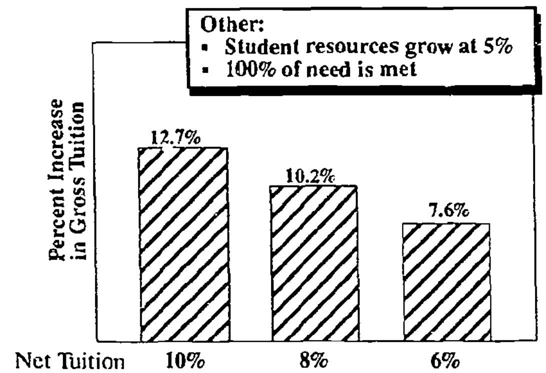


FIGURE 24
Percent of Tuition Retained Under Varying Levels of Net Tuition Growth, Shown Above



The final piece in the tuition puzzle is the impact of other institutional factors on net and gross tuition. If tuition is priced on a "cost basis," that is, at a level which is sufficient to cover institutional costs, then the net tuition revenue must be sensitive to a myriad of institutional factors. Simplifying the calculation, net tuition

FIGURE 25
Gross Tuition Growth Rates Required to Meet Net Tuition Revenue Requirements



growth must be set based upon cost and nontuition revenue growth.[23]

Figure 26 shows the relationship of cost and nontuition revenue growth to net and gross tuition. For example, if Snow College's costs grow 10.0 percent while nontuition revenue grows at 8.0 percent, then net tuition must grow at 10.7 percent, slightly faster than expenses. The excess of the net tuition growth over expense growth is determined by the shortfall in nontuition revenue. Given, then, net tuition growth of 10.7 percent in a year when familial resources grow 5.0 percent, tuition rates must increase 14.0 percent.

[23] "COST" IN THIS CALCULATION IS ALL CURRENT FUND EXPENSES LESS ALL financial aid. Cost should include mandatory transfers and budgeted nonmandatory transfers and/or current fund surplus. "Nontuition revenue" includes all current fund revenue less tuition and less restricted revenues received by the college to fund current federally supported financial aid programs.

IV

Implications for the 1990s

Tuition discounting is a growing phenomenon that is somewhat egalitarian in its scope. It affects institutions both small and large, both prestigious and less celebrated. Tuition discounting is stratified more by cost of attendance and familial resources than by any other criterion normally used to segment the education industry.

As gross tuition rates grow faster than familial ability to pay, rapidly increasing student need will predictably outpace growth in federally funded programs. This will create a growing demand for institutionally funded financial aid. To the extent that institutions meet the rising demand for assistance with scholarship aid, tuition discounting grows and net tuition growth becomes significantly less robust than its gross tuition counterpart. If institutions do not meet the rising need or meet it through institutionally funded loans, the burden of the unmet need falls back on the students. The risk to the institution in not meeting need is that students faced with sharply rising educational costs may well seek less expensive alternatives.

A question arises: why not just forego tuition increases and make the nineties the decade of zero tuition growth? At least one college is trying that strategy in 1992, hoping to achieve a higher increase in its net tuition revenue than had been realized in 1991 with moderate tuition hikes but rapid financial aid growth. However, most schools will not try this strategy. Despite growing discounting, most institutions still earn marginal net revenue on each additional gross tuition dollar. There is incentive, therefore, to keep increasing the gross tuition rate.

One result of the growth in tuition discounting is a further blurring of the price that students (and their families) pay. What did students pay for an education at Snow College? The amount ranged from $0 to $10,000, with 41 percent of the students paying less than $10,000. With only 59 percent of the population paying the stated rate, the stated cost of attendance becomes an essentially meaningless figure for the per student price for the educational services rendered. The perceived cost of attendance remains, however, equal to the institution's stated rate. This increasingly artificial stated rate is the price that is conveyed to prospective students, quite possibly deterring those who are less affluent from attending Snow.

As the range of actual payment broadens and the published tuition rates and costs of attendance become increasingly less meaningful, institutions will need to include net tuition, or "real revenue," in their financial modeling, budgeting, and monitoring. Without focusing on net tuition, institutions will be unable to understand or manage the impact of discounting on their financial health.

Notwithstanding the potentially dangerous erosion of net tuition revenue, many, if not most, institutions will need to keep increasing tuition rates and chasing the marginal revenue dollars as:

- cost growth continues to put pressure on the bottom line;
- federally funded aid lags behind tuition growth;
- an uncertain economy threatens familial income; and
- student need rises.

In this complex environment, it will be a competitive advantage to understand the full financial implications of choosing one pricing strategy over another. Ultimately, however, knowledge of financial issues is not enough. Institutions will need to make substantive changes in the way educational services are structured and delivered in order to stabilize tuition discounting by bringing the economic pressures on tuition in line with the price increases families can afford to pay.

APPENDIX

The Tuition Discounting Model: Detail Description and Formulae Derivation

This appendix presents, in detail, the derivation of the formulae used in the tuition discounting model. A written overview is provided and then the model is outlined in geometric and algebraic form, whenever practicable, in order to make the design as accessible as possible to the widest range of users. By going through these calculations with the appropriate input assumptions and logarithm modifications for his or her institution, the reader can use this model to understand the pressures on, and interaction of, tuition, financial aid, and other institutional factors.[1] The approach used in the development of this model and outlined in the following sections can also be used to develop an institution-specific model. For example, a college or university may want to use a different distribution of student resources or relax the assumption of a fixed student body size. Changing these two assumptions will clearly result in different formulae, but the approach to deriving them will not change, as the model is unaffected by the assumptions used.

Clearly, however, the formula derivations are not for the mathematically "faint of heart." It is suggested that those who are less numerically dexterous read the assumptions carefully and then focus on the topic sentences presented in bold face in each section.

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[1] SEE ALSO DAVID S. P. HOPKINS AND WILLIAM F. MASSY, *PLANNING MODELS FOR COLLEGES AND UNIVERSITIES* (Stanford CA: Stanford University Press, 1981), appendix 3 for a comparable approach to modeling financial aid, presented in summary format.

This appendix is divided into eight sections:

- Model Inputs
- Cost of Attendance
- Familial Ability to Pay
- Externally Funded Financial Aid
- Institutionally Funded Financial Aid
- Net Tuition
- Other Institutional Factors
- Conclusion

Each section of the appendix (with the exception of the conclusion) presents the sequential development of pieces of the tuition discounting model. Figure 1 quickly outlines the results of each section.

Model Inputs

The tuition discounting model is driven by 17 input assumptions relating to institutional factors, familial resources, and externally funded financial aid.

In this section each assumption or input variable is defined and its use in the model described.

T (1) This variable represents the tuition rate charged to full-time students in the first year of the model.

t (k) This is the rate of increase in tuition for each year, k, after the first year. For example, if the tuition model is run for ten years, then t (k) must be input for years k = 2 through k = 10. For instance, if the institution doing the modeling had a $5,000 tuition rate in year one and anticipated a $5,500 rate in year two, then t (2) would equal 10 percent. By inputting t(k) the user can model the impact of predefined tuition rates on institutionally funded financial aid and net tuition. Alternatively, the user can use the model to calculate t(k) and the associated gross tuition rates by assuming certain levels of cost and other revenue growth, and defining the distribution of student resources and externally funded financial aid.

N is the full-time equivalent (FTE) number of students. The number of students is presumed to be constant throughout the modeled period.

RB(1) This variable is the dollar amount, in year one of the model, of the nontuition portion of the average student's budget. This variable is predominantly the room and board charge but may also include other elements of the student budget such as books and transportation. The sum of RB(1) and T(1) should equal the average total budget for the average FTE student attending the college in the first year of the model.

rb This variable is the rate of growth in the nontuition portion of the student budget. The rate of growth in these charges is assumed to be constant in the model, and so only one rate must be input. Alternatively rb can vary by year.

Z The tuition model uses the EFC developed under the general needs calculations as the basis for familial ability to pay. In the iteration of the model used in the monograph the students' EFCs were assumed to be distributed uniformly from $0 to $25,000 in the first year. The variable "Z" is the maximum EFC in the first year of the model. The variable Z is critical to modeling the uniform distribution. If, however, a school were to choose a different distribution to describe familial ability to pay, then the variable Z would be replaced by whatever parameter(s) are critical to the new distribution.

w The variable "w" is the rate of growth in the aggregate amount that families of students attending the college can afford to pay for their children's/spouse's/own education. Because

FIGURE 1
Model Development Structure

Appendix Section	Key Results
Model Inputs	The model inputs are the characteristics of the institution which drive the model. The input variables defined here are used throughout the model.
Cost of Attendance	From the input variables, the model derives the projected cost of attendance for each year.
Familial Ability to Pay	The model uses the distribution of student resources (input) and the cost of attendance (derived) to determine how much of the student's budget can be funded by the family. Familial ability to pay and the cost of attendance then drive determination of aggregate student need.
Externally Funded Financial Aid	Once the aggregate student need is calculated, the institution needs to know how much of the need will be met by external funds. This section presents the derivation of formulas for the federally funded aid available to meet student need.
Institutionally Funded Financial Aid	After the application of externally funded aid to student need there is usually a gap of unmet need. This gap is filled to varying degrees with institutionally funded aid. This section presents formulas for institutionally funded, need-based, and non-need-based aid.
Net Tuition	This section takes the calculated institutionally funded aid and develops formulas for the tuition discount and net tuition. This section answers the question: What is the impact of tuition policy choices (input) on the net real revenues (derived)?
Other Institutional Factors	This section uses all of the previously derived results to answer a different question: given the parameters on cost and (nontuition) revenue (input) what is the net real revenue the institution must earn and what is the gross tuition level which will achieve this net revenue level (derived)?

ability to pay is measured by EFCs, w is the rate of growth in the total EFCs of all students at the institution.

Pmax(1) This variable is for the maximum Pell award in the first year of the model.

pmax "pmax" is the average annual growth in Pmax(k), the maximum Pell award in each year k.

B is the aggregate amount of SEOG, CWS, and Perkins funds that the college or university can award to students in the first year of the model. The Perkins amount includes both the new capital contribution to the institution's loan fund and the capital made available by student loan repayments of earlier NDSL and Perkins loans.

b The variable "b" is the annual growth rate in the SEOG, CWS, and Perkins funds that the college can award to students.

M(1) This variable is the number of full-time equivalent, full-cost-of-attendance grants made to students on a basis other than need, in the first year of the model. This includes merit scholarship students, athletic scholarship students, etc. As a simplifying assumption, M(1) is assumed to be aid which is incremental to the school's need-based scholarship program.

m "m" is the variable for the annual growth in the institution's non-need-based aid program. This variable is the rate of growth in the number of non-need-based, full-cost-of-attendance grants.

r(1) "r(1)" is the variable for the rate of tuition dependency in year 1. This input variable is derived by dividing gross tuition revenue in year 1 by the total educational and general gains for the year. Tuition dependency is an important factor in the determination of the pressure that other institutional factors will place on the institution's pricing policy.

c "c" is the annual rate of growth in adjusted costs during the modeled years. Adjusted costs include all E&G costs including mandatory transfers, less all institutionally funded financial aid, plus the budgeted E&G operating surplus.

e This is the variable for the annual rate of growth in current fund educational and general income excluding tuition revenue. For example e captures the growth in charitable giving, endowment income, and sponsored research but is unaffected by the growth in revenues from tuition, auxiliaries, and other independent operations.

Seventeen major assumptions on institutional characteristics, familial ability to pay, and externally funded financial aid (the 16 input variables defined above and the assumptions of the uniform distribution of familial resources) drive the tuition discounting model.[2] The following sections of the appendix build the structure of the model from these inputs, developing formulae to describe the interrelationships of factors affecting tuition rates and institutionally funded financial aid.

Gross Tuition and Cost of Attendance

The objective of this section is to project the average total cost of attendance for each year. As a by-product of the cost of attendance calculations, the section also concludes with the projection of gross tuition revenues for the institution.

Because need-based financial aid is awarded on the basis of each student's need compared to the cost of attendance, the model first calculates tuition rates, nontuition charges, and then the total cost of attendance for each modeled year.

The cost of attendance is a function of the gross tuition and nontuition charges which make up the student budget. This total student budget, developed each year by the financial aid office, is used as the basis for creating each student's financial aid package. The budget includes tuition, fees, room and board (for resident students), books, transportation, etc.

In reality, the institution develops different budgets for each category of student: resident, commuter, dependent, independent, etc. The model, however, assumes that the school can quantify an "average" budget for its student body.

To calculate the average cost of attendance and the gross tuition revenues for each year, the model first projects the stated rate and the nontuition charges of the average student budget:

[2] THE CHOICE OF THE UNIFORM DISTRIBUTION IS MADE FOR EXPOSITIONAL clarity. As noted above, an institution may use its unique distribution of students and follow the approach outlined in this appendix to derive an institution-specific model.

T(k) T(k) is the stated tuition rate in year k. Because the first year tuition rate, T(1), was input, as were the tuition growth rates t(k), calculating the tuition rate for each year is quite straightforward.

$$T(k) = T(k-1) \times [1 + t(k)] \qquad \text{for } k > 1$$

RB(k) Less flexibility is assumed in the nontuition charges and a single growth variable determines RB(k), the nontuition charges in year k. RB(1), the nontuition charges in year 1, was input as was the growth rate, rb. RB(k) is then calculated as follows:

$$RB(k) = RB(1) \times (1 + rb)^{k-1} \qquad \text{for } k > 1$$

If different growth rates in the nontuition portion of the average cost of attendance, for each modeled year, are desired, the model can be generalized with the additional input of individual growth rates for each year after the first: rb(k) for k greater than 1. The nontuition charges are then calculated in the same manner as tuition charges:

$$RB(k) = RB(k-1) \times [1 + rb(k)] \qquad \text{for } k > 1$$

With the stated tuition rate and the nontuition charges projected for each year, the model can now project total average cost of attendance and aggregate gross tuition revenues:

COA(k) This is the variable for the total budgeted cost of attendance for the modeled, "average," FTE student.

$$COA(k) = T(k) + RB(k) \qquad \text{for all } k$$

GT(k) The gross tuition revenue in year k is calculated as

$$GT(k) = T(k) \times N \qquad \text{for all } k$$

Familial Ability to Pay

The calculation of familial ability to pay is fundamentally directed towards understanding the individual needs of specific students and the aggregate need of the student body. Toward that end this section is divided into two parts: first, understanding the implications of the input assumptions of family resource distribution, and second, calculating the aggregate student need.

The individual and aggregate student resources for education are functions of the range of expected family contributions, the distribution of students across that range, and the annual growth in familial wealth.

The base assumption of familial resources used in the model is that the variable used to measure familial ability to pay (the general needs calculation of EFC) is distributed uniformly in the first year of the model from an EFC of $0 to a maximum EFC of $Z. Using the continuous uniform distribution, the base assumption can be presented graphically as a rectangle with the base, or x-axis, being the range of EFCs, and the height, or y-axis, being the number of students who have the particular EFC on the axis below.

Because the base of the rectangle is EFC categories and the height, the number of students per EFC category, the area of the rectangle will be N, the number of students. Pushing the argument further, because the base of the rectangle is Z, and the product of Z and the number of students per EFC (the height of the rectangle) is N, the height must be N/Z. This is illustrated in figure 2. Therefore, having assumed the uniform distribution of student resources and inputting the maximum student resource level, Z, we can now tell the distribution of students across resource levels in the first year of the projection.

This distribution of students is critical in developing the aggregate need of the students for financial aid. From the distribution of students, the model moves to a calculation of aggregate student resources and from there to the

FIGURE 2
Student Resources Distribution

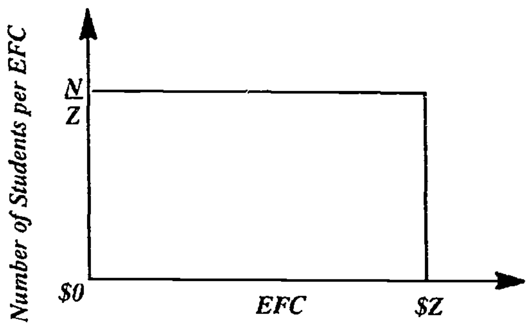
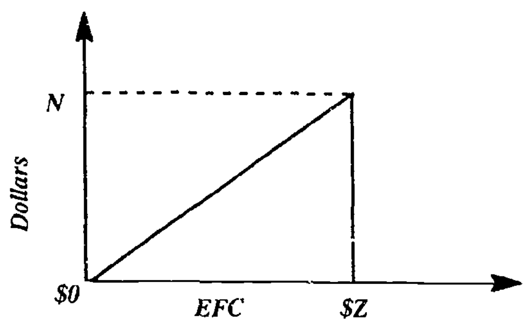


FIGURE 3
Aggregate Student Resources



calculation of aggregate need for financial assistance.

To translate the student resources distribution into aggregate student resources you need to multiply the total resources available at each EFC by the number of students at each EFC. For example, there are N/Z students with an EFC of Z. therefore, the aggregate resources available from students with EFC resources of Z equals Z × N/Z, or just N. Calculating the aggregate resources at each EFC level using this simple multiplication produces the triangle illustrated in figure 3. The area of the resulting triangle is the aggregate student resources held by the student body. Solving geometrically for the area of the triangle (area = 1/2 × base × height), you can calculate the area of the triangle as NZ/2. This says that the aggregate student resources are equal to the number of students (N) times the average EFC (Z/2), which is an intuitively logical result.

Formulaically, what you've done is to integrate the product of the student distribution function per EFC and the EFC across the range of EFCs. This solution for aggregate student resources, depicted in the integral calculation below, is the approach an institution would take if a student distribution other than the continuous uniform distribution were to be used. In that case, the N/Z would be replaced by whatever distribution function makes sense for the college or university.

$$\int_0^z \frac{N}{Z} \times x \, dx = \frac{N}{Z} \times \frac{X^2}{2} \Big|_o^z = \frac{NZ}{2}$$

The reason for calculating aggregate student resources is that in order to be useful, the model needs to be able to project the impact of changing student resources on the aggregate need for financial assistance. One of the input assumptions to the model was the growth rate in student resources. In this next section the growth in student resources is combined with the aggregate first year result calculated above to determine total resource levels for each year. Knowing the aggregate resources in each year and the assumed student distribution throughout the model (in this case the uniform distribution), you can then determine the precise parameters for the distribution of students in each modeled year. The model then uses this number of students at each resource level to calculate the total need in each year.

The input assumption, which is important at this juncture, is the annual growth rate of aggregate student resources, w. Knowing the aggregate student resources in the first year and the growth rate in resources you can calculate the aggregate student resources in any year as follows:

W(k) The aggregate student resources, or wealth, in each year k, is a function of the first year wealth, NZ/2, and the growth rate, w. The calculations in the previous section are the derivation of aggregate student resource in year 1, or W(1).

$$W(k) = (1+w)^{k-1} \times \frac{N \times Z}{2}$$

Combining the knowledge of the aggregate wealth in each year with the general form of the aggregate resource formula, defined above, determines the parameters for the distribution of student resources in each year of the model:

- Let j be the maximum EFC in year k.
- Then the N students at the institution are distributed uniformly from \$o to \$j in year k with N/j students in each category.

What you need to know is the value of j for each year of the model. Knowing j allows you to calculate the number of students at each resource level as well as the range of EFCs. To solve for j, set the formula for the aggregate wealth in each year, W(k), to the general integral formal defined above. The solution for j then proceeds as follows:

$$W(k) = \int_o^j \frac{N}{j} \times x \, dx$$

$$\frac{(1+w)^{k-1}NZ}{2} = \frac{N}{j} \times \frac{X^2}{2} \Big|_o^j$$

$$\frac{(1+w)^{k-1}NZ}{2} = \frac{Nj}{2}$$

$$j = (1+w)^{k-1} \times Z$$

Therefore, for each year k, student resources (EFCs) are distributed uniformly from 0 to $Z \times (1+w)^{k-1}$ with $\frac{N}{Z\,(1+w)^{k-1}}$ students in each EFC category.

It is interesting to pause for a moment and think about what these formulae say about the distribution of students and their resources to support educational expenses. The increase of the maximum EFC from Z in year 1, to $(1+w) \times Z$ in year 2, and $(1+w)^2 \times Z$ in year 3, means that the range of financial ability is expanding each year. As the range of financial ability spreads, there are fewer students in any *particular* range of EFCs. This is seen in the factor for the number of students at each resource level which drops from $\frac{N}{Z}$ in year 1, to $\frac{N}{Z\,(1+w)}$ in year 2, and then to $\frac{N}{Z\,(1+w)^2}$ in year 3.

This description of the impact of the assumed annual increase in wealth on the distribution of individual and aggregate student resources is presented, in exaggerated scale, in figures 4 and 5. Figure 4 is the multiyear analog to figure 2, while figure 5 is the multiyear analog to figure 3.

Note that in figure 4 the area of the rectangles in each year equals the number of full-time equivalent students, N, which is assumed to be constant throughout the modeled period. Also note that area of the triangles in figure 5 always equal the aggregate student resources, W(k), for each year.

Having developed the model for the distribution of students and the families' available resources to support education, the next step is to use these results to calculate the need for each student and then the aggregate need of the student body.

Individual student need is simply the gap between the cost of attendance and the EFC calculated for the family.

G (i, k) is the need of a student with an EFC of i, in year k. The need of a student with a general needs calculation ability to pay (EFC) of i, in year k, is the difference between the cost of attendance and the EFC:

$$G(i,k) = COA(k) - i \qquad \text{for } i \le COA(k)$$
$$G(i,k) = 0 \qquad \text{for } i > COA(k)$$

FIGURE 4
Student Resource Distribution
Years 1, 2, and 3

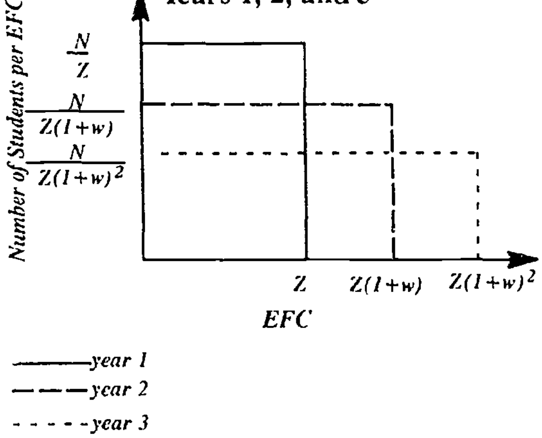
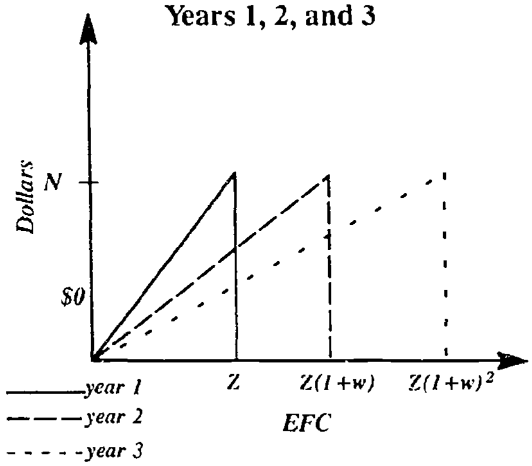


FIGURE 5
Aggregate Student Resources
Years 1, 2, and 3



Graphically G(i,k) is presented in the shaded portion of figure 6. The graph depicts need decreasing steadily from the full cost of attendance when the student's EFC is $0, to no need when the student's EFC equals or exceeds the cost of attendance.

Logically then, the aggregate need of the student body is equal to the individual student need at each EFC times the number of students at that EFC level.

AG(k) The aggregate need of the student body is a function of the cost of attendance and the distribution of student resources.

Thinking about aggregate student need in geometric terms, the aggregate cost of attendance at each EFC level will always be the cost of attendance multiplied by number of students at that EFC level. Because we have $\frac{N}{Z\,(1\,+\,w)^{k-1}}$ students at each EFC level, each of whom has an average student budget of COA(k), the top of the rectangle in figure 7 is a horizontal line of $\frac{COA\,(k) \times N}{Z\,(1\,+\,w)^{k-1}}$ The diagonal line running from the origin to the aggregate cost of attendance is the amount of each student can pay (the EFC) multiplied by the number of students who can pay it, $\frac{N}{Z\,(1\,+\,w)^{k-1}}$.

Just as *individual* student need is presented in the shaded portion of figure 6, aggregate student need is described in the shaded area of figure 7.

What is of critical importance to the college using this model is just how big this shaded area is. Continuing the geometric derivation, the aggregate financial need of the student body of the area of the shaded triangle in figure 7. Specifically, AG(k), the aggregate need in year k, will be

$$AG(k) = \frac{1}{2}\left[\frac{COA\ (k) \times N}{Z\ (1+w)^{k-1}} \times COA(k)\right]$$

$$AG(k) = \frac{N \times [COA(k)]^2}{2Z(1+w)^{k-1}}$$

The same result may be derived by analyzing the question of need formulaically. Aggregate student need, in the general case, equals the product of the individual need and student

FIGURE 6
Individual Student Need by EFC Level

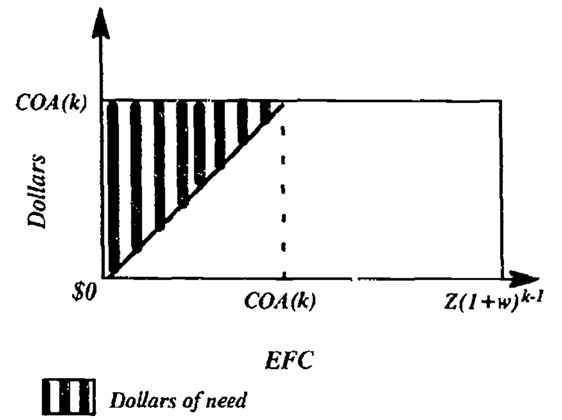
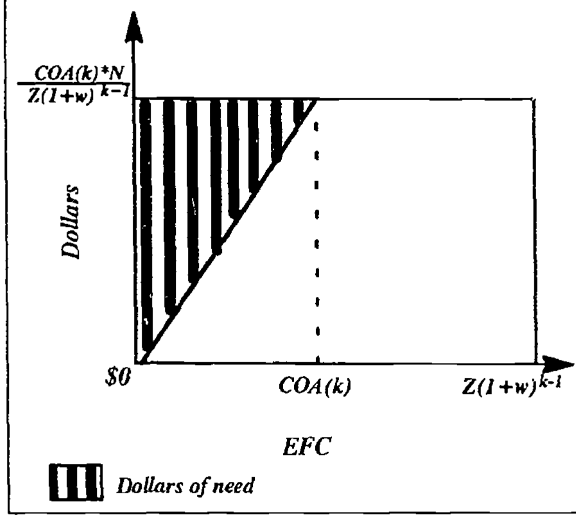


FIGURE 7
Aggregate Student Need by EFC Level



distribution functions, integrated areas all possible EFCs. In this case the equations work as follows:

$$AG(k)=\int_0^{Z(1+w)^{k-1}} G(x,k)\frac{N}{Z(1+w)^{k-1}}dx$$

$$=\frac{N}{Z(1+w)^{k-1}}\left[\int_0^{COA(k)}[COA(k)-x]dx\right]$$

$$=\frac{N}{Z(1+w)^{k-1}}\left[X\times COA(k)-\frac{X^2}{2}\Big|_0^{COA(k)}\right]$$

$$=\frac{N}{Z(1+w)^{k-1}}\left[\frac{COA(k)^2}{2}\right]$$

$$AG(k)=\frac{N\times COA(k)^2}{2Z(1+w)^{k-1}}$$

Note: The integral is only evaluated to COA(k) because G(x,k)=0 when xCOA(k)

Clearly this is the same result for AG(k) as was derived geometrically above.

The geometric and formulaic derivations of AG(k) point to an interesting result: if an institution's students' abilities to pay can be modeled by the uniform distribution, then the aggregate need for financial aid is a simple function of the number of students, the cost of attendance and the maximum EFC.

Once the formula for aggregate need is developed, the institution can determine the combined impact of changes in cost of attendance and familial resources on the demand for financial aid.

Knowing AG(k) is an extremely powerful result because it allows the institution to relate changes in familial resources and the stated tuition rates to their combined impact on student need. For example, assume a college decides to raise its cost of attendance by 10 percent in a year when aggregate familial resources only grow 5 percent. By how much does aggregate need grow?

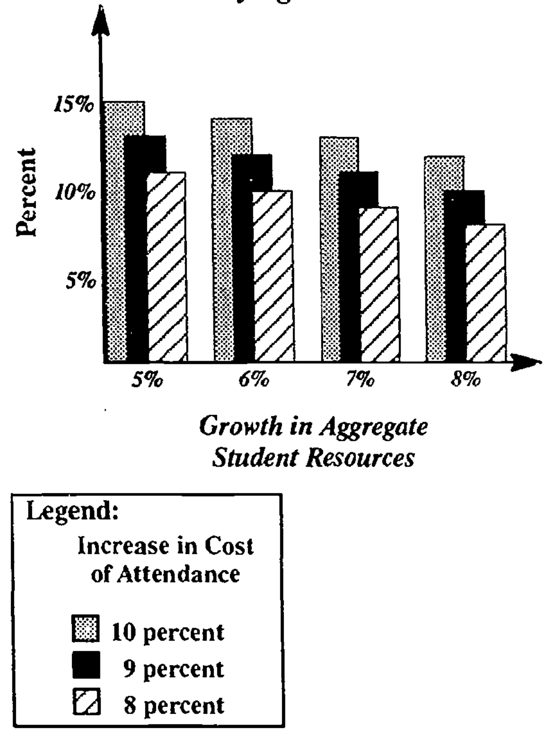
$$\frac{AG(k+1)}{AG(k)}=\frac{\dfrac{N\times COA(k+1)^2}{2Z(1+w)^{(k+1)-1}}}{\dfrac{N\times COA(k)^2}{2Z(1+w)^{k-1}}}=\frac{COA(k+1)^2}{COA(k)^2(1+w)}$$

$$=\frac{[COA(k)\times(1.1)]^2\star}{COA(k)^2(1.05)}=\frac{1.21}{1.05}\approx 1.152$$

★*Because COA(K + 1) = COA(k) times 1.1 in this example*

So, a 10 percent increase in cost of attendance in a year in which familial resources only grew at 5 percent yields a 15.2 percent increase in the aggregate need of the student body. Using different inputs the college can see that a 9 percent increase in cost of attendance instead of 10 percent results in a 13.2 percent in need. An 8 percent increase would yield a still lower 11.1 percent growth in aggregate student need. Figure 8 explores this relationship at varying levels of growth in familial resources and cost of attendance. *The unsurprising result is that when growth in college charges and related student costs outstrip growth in familial resources, the demand for financial aid increases dramatically.*

FIGURE 8
Growth in Aggregate Student Need Under Varying Scenarios



Externally Funded Financial Aid

Having developed formulae for the aggregate need of the student body, the institution needs to know how much of that need will be met by externally funded financial aid, and how much either will be left for the institution to fund or will fall back to the student as unmet financial need. This section of the appendix focuses on federally funded financial aid, developing algorithms to model Pell and SEOG grants, Stafford and Perkins aid, and CWS program funds. Because of the wide variation in state programs, they are not included, although the outlined approach to modeling federally funded aid can guide the user in adding state aid to an institution-specific model.

Pell awards are made to eligible students based upon student need, as measured by the Pell expected family contribution, the Pell cost of attendance calculation, and the maximum Pell award.

Perhaps the most complex aspect of the tuition discounting model is relating the *general needs calculation* of expected family contribution to the *Pell calculation* of expected family contribution (the Pell index). The general need calculation EFC is the basis used to model student resource distributions throughout the discounting model. However, because Pell grants are awarded on the basis of a different assessment of need, a translation from the one needs analysis system to the other must be modeled. In this section of the appendix, the derivation of the formula for estimating aggregate Pell award dollars is approached in four steps:

- determining the percent of students at each EFC level who are eligible for Pell;
- determining, for eligible students, the Pell index at each general need calculation EFC level;
- calculating the Pell award at each Pell index level; and
- consolidating the results of the calculations in the first three steps with the distribution of students by general need calculation EFC, to determine the aggregate amount of Pell awarded.

The basis for the Pell estimation model is data gathered from five institutions on EFC, Pell index, and Pell award.[3] The data used included all financial aid application filers from each school — a population which included students eligible for both Pell and other federal programs, those filers eligible for neither, students ineligible for Pell but eligible for other federal programs, and those eligible for Pell but ineligible for other federal programs. This study population yielded 11,617 students across a wide range of EFCs.

The percent of students in each EFC category eligible for Pell is directly related to both the EFC category and to the Pell maximum. It is intuitively logical that while the needs assessment methodologies differ, both methods will tend to agree, at least broadly, that certain groups of students are more needy and others less needy. Thus one would expect, and the data bear this out, that at the lowest EFCs, where the general methodology points to the highest level of need, most students are eligible for Pell. Similarly, at the highest EFCs, few students are eligible for Pell.

The role of the Pell maximum in determining the eligibility of students in each EFC level for Pell awards is slightly less intuitive. One of the Pell award formulae calculates the award as the difference between the Pell maximum and the Pell index with no award given when the difference is less than $200. Because of this formula, as the maximum Pell award increases, the range of Pell indices that produce awards increases. Because the EFC and Pell index are at least broadly correlated, this means that students in a wider range of EFCs will be eligible to receive Pell awards.

Figure 9 shows the percent of students at each EFC level who were eligible for Pell at four different levels of Pell maximum — $2,400; $3,200; $4,000; and $4,800. The graph demonstrates that as the maximum increases the percent of students eligible for Pell also increases. The graph also illustrates a high level of participation in Pell at lower EFCs with a radical drop in the percent of students eligible at higher EFCs. This is an intuitively good result because, a priori, it was expected that need would decrease as EFCs increased. Finally, the graph shows that the shape of the "backward s-curve" changes with the Pell maximum: as the maximum award increases, the "backwards s" becomes more angular, with the drop in the percent of students eligible for Pell sharper as one transitions between high and low participation levels.

To model the "backwards s-curves" at each Pell maximum shown in the graph and at each possible maximum between those illustrated, functions were required that were sensitive to the change in shape as the Pell maximum increased, and that could be sized to fit the data range of percents (from 0 to 100 percent participation). Ultimately, two curves were used to model the shape: an arctangent from 0 EFC to a break point, and 1/x curve from the break point onward. Although the formulae for these curves are fairly complex, the resulting fit is good. (See figure 10 for the raw data and fitted curve at a Pell maximum award of $3,200.)

The resulting formula for estimating the percent of students at each level who are eligible for Pell is as follows:

Pmax(k) pmax is the maximum Pell award in year k. The maximum is calculated using the maximum Pell award in the first year and a growth factor, ***pmax***.

$$Pmax(k) = Pmax(1) \times [(1 + pmax)^{(k-1)}]$$

for all k > 0

Ptp(efc,pmax(k)) Ptp(efc) is the percent of students at each EFC who are eligible for Pell at a given Pell maximum award level

[3] Thanks to Boston College, Saint Lawrence College, Siena College, Tulane University, and Villanova University. These schools aided research by making available anonymous extracts of selected fields from their financial aid databases.

FIGURE 9
Percent of Students Eligible to Receive Pell Awards

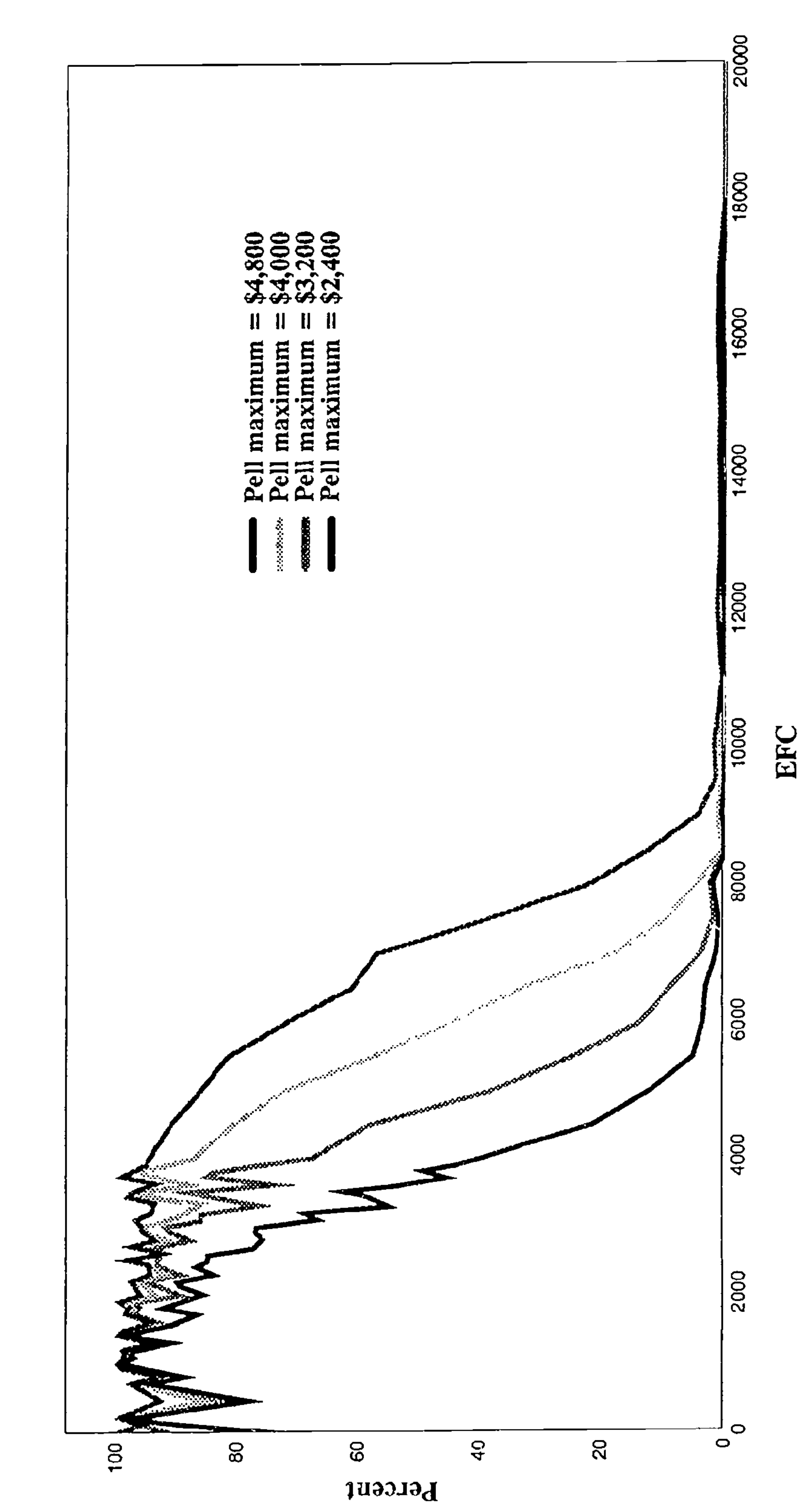
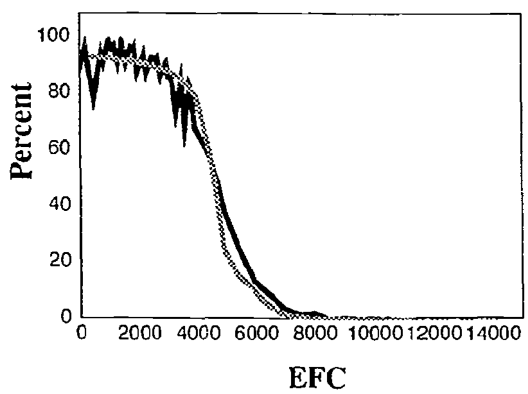


FIGURE 10
Actual and Modeled Pell Participation (Pell Maximum = $3,200



$$\textit{Inflection} = -1.627\,604 \times 10^{-7} \times Pmax(k)^3 + 1.875\,000 \times 10^{-3} \times Pmax(k)^2 - 5.520\,833 \times Pmax(k) + 8{,}400$$

$$\textit{break} = 3.255\,208 \times 10^{-7} \times Pmax(k)^3 - 3.515\,625 \times 10^{-3} \times Pmax(k)^2 + 13.854\,166 \times Pmax(k) - 13.000$$

$$\textit{sizing} = -9.765\,625 \times 10^{-9} \times Pmax(l)^3 + 1.093\,750 \times 10^{-4} \times Pmax(k)^2 + -4.062\,500 \times 10^{-1} \times Pmax(k) + 530$$

$$Ptp(efc, pmax(k)) = \arctan \left[\frac{sign \times (|efc - inflection|)^{.765}}{100} \right] \times \frac{-1}{2 \times 1.570796} + .5$$

for efc ≤ break

$$= \frac{sizing}{efc - break}$$

for efc > break

*Where **sign** equals –1 when the EFC is less than the inflection point and 1 when the EFC is greater than the inflection point and |**function**| refers to the absolute value of the function in brackets.*

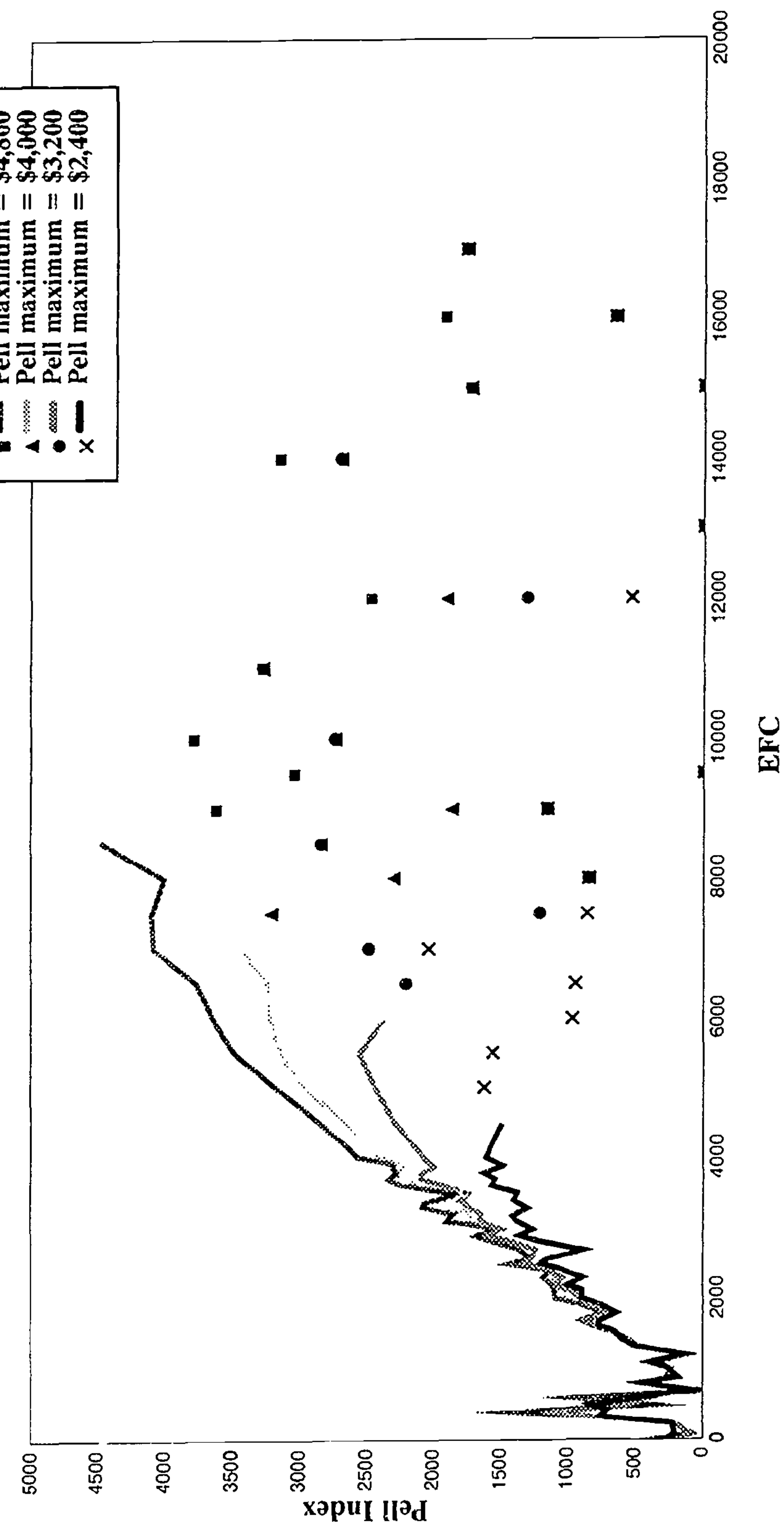
Knowing what proportion of students are eligible to participate in Pell at each EFC level, the next step is to create a bridge that allows us to go from the EFC to the Pell index, for the eligible students, at each EFC level.

The Pell indices that result in actual awards are also a function of the Pell maximum: as the Pell maximum increases, higher and higher indices will fall into the award range. To develop the bridge from EFCs to the Pell indices, the author looked at the average Pell index, for those students receiving Pell awards at each EFC for four levels of Pell maximum. Those Pell indices are displayed graphically in figure 11. The lines show where there are significant numbers of students in the average. As the EFC increases there are fewer students who have Pell indices that are low enough to result in actual awards. With each Pell maximum there was a strong break point in the data, beyond which the number of students in the average indices graphed was quite small. These results are presented as symbols only. This drop is consistent with the lower participation levels at higher EFCs and the a priori expectation that few students would have a high ability to pay and little need under one assessment methodology and the inverse under another.

Although there appears to be a great deal of confusion below an EFC of 700, above 700 and below the break point, a clearer and more linear pattern emerges. With regression analysis, a formula was developed for each line in the form of $m \times x + b$. To link the factors for the four lines, with the Pell maximum that drives their magnitude, a cubic formula was developed to fit the data. To simplify the formulae, and because the fit was good, the intercept was held constant in the regression analysis. The resulting formulae are as follows:

Pefc(x) Pefc(x) is the Pell index at each general needs assessment EFC level, x.

FIGURE 11
Average Pell Indices for Eligible Students at Different Pell Maximums



For $700 \leq x \leq$ break point,

$$Pefc(x) = [4.583\ 333 \times 10^{-13} \times pmax(k)^3 - 2.517\ 187 \times 10^{-8} \times pmax(k)^2 + 2.277\ 154 \times 10^{-4} \times pmax(k) - 1.745\ 000 \times 10^{-3}] \times x + 35$$

where the break point is calculated as follows:

$$break\ point = 3.255\ 208 \times 10^{-7} \times pmax(k)^3 - 3.515\ 625 \times 10^{-3} \times pmax(k)^2 + 13.854\ 166 \times pmax - 13{,}000$$

Below 700 and above the break points the indices were simply averaged at the four maximum Pell levels. Linking formulae were developed to allow the formulae to be used at maximum Pell levels between those specifically analyzed. The resulting formulae are presented in table 1.

Having calculated the Pell index using the four formulae developed above, the next step is to calculate the amount of the Pell award to be given.

P(x) The Pell award given a Pell index of x(Pefc(efc)=x) is calculated as the lowest of the following:

- the maximum Pell award (Pmax)-Pell index;
- Pell cost of attendance — Pell index;
- 60 percent of the Pell cost of attendance; or
- $0, when the lowest of the above 3 is less than $200.

The resulting Pell awards for two institutions — one with a Pell cost of attendance of $3,000 and the other with a Pell cost of $10,000 — and assuming a Pell maximum of $2,400, are shown in the shaded areas of figure 12.

To simplify the calculations, and because of constraints found elsewhere in the model, the tuition discounting model is limited to institutions with average costs of attendance greater than or equal to $6,400. The model further assumes that the Pell cost of attendance[4] exceeds $4,000.[5] Under these assumptions, the Pell calculation rules that apply to the modeled institution are the first and last above: the Pell award equals the Pell maximum award minus the Pell index, and when the calculated Pell award is less than $200, no award is made.

The rationale behind the decision to use the first and last rules is demonstrated in the graph of the alternative formulae shown as figure 13. Figure 13 was developed using a cost of attendance of $6,400, a Pell cost of attendance of $4,500, and a maximum Pell award of $2,400. In this graph, one can see that the lowest of the three award rules will be Pmax-P(efc) for all Pell indices so that only one calculation rule is required.

Having simplified the Pell calculation and linked the two methodologies for determining familial contribution, the next step is to determine the amount of Pell awards given the modeled distribution of students and their resources.

To calculate the aggregate Pell awarded, in theory one would integrate the product of the participation index, **Ptp(efc, Pmax (k))**, the Pell

[4]THE PELL COST OF ATTENDANCE DIFFERS FROM THE GENERAL NEED analysis cost of attendance in its treatment of the nontuition charges. In the Pell analysis amounts for room and board, books, transportation, child care costs, etc., are capped by certain ceilings. The general need analysis does not use ceilings and, in fact, goes the other way with floors for many nontuition charges in the independent student budget. The difference between the two determinations of the cost of attendance appears to be, on average, about $2,000 dollars higher for the general need analysis than for the Pell.

See Frederick J. Fisher, "Toward the Use of a Single Need Analysis (or Virtue May Be More Than Its Own Reward)" *Journal of Student Aid*, vol. 20, no. 2 (Washington, D.C.: National Association of Student Financial Aid Administrators, 1990) and table 3.

[5]THE ASSUMPTION THAT THE PELL COST OF ATTENDANCE EXCEEDS $4,000 IS an important simplifying assumption. At a Pell cost of attendance of $4,000, and a Pell EFC of 0, the 60 percent times cost of attendance award rule produces the same result as the maximum award less the Pell index when the maximum equals $2,400. Therefore, when the Pell cost of attendance exceeds $4,000 and the growth in the cost of attendance exceeds the Pell award maximum growth assumption, it is established a priori that the award rule that will govern the amount granted will be the Pell maximum award — Pell index. Because this formula is independent of the precise Pell cost of attendance, the model does not have to track a variable for this alternative student budget.

EFC Range	Maximum Pell Award	Average Pell Index	Formula
$0 \le efc < 300$	2,400 3,200 4,000 4,800	250 250 288 288	for pmax $\le$ 3,600 $Pefc(efc) = 250$ for pmax > 3,600 $Pefc(efc) = 288$
$300 \le efc < 700$	2,400 3,200 4,000 4,800	642 679 773 924	$Pefc(efc) = 5.859\,375 \times 10^{-9} \times pmax(k)^3$ $-1.171\,875 \times 10^{-5} \times pmax(k)^2$ $-2.687\,500 \times 10^{-2} \times pmax(k)$ $+693$
above break point	2,400 3,200 4,000 4,800	1,080 1,412 1,547 2,039	$Pefc(efc) = -1.803\,385 \times 10^{-7} \times pmax(k)^3$ $+1.885\,156 \times 10^{-3} \times pmax(k)^2$ $-5.701\,458 \times pmax(k)$ $+6{,}638$

Table 1

award, P(Pefc(efc)), and the distribution of students, $\frac{N}{Z\,(1 + w)^{k-1}}$, across all EFCs. In practice, because of the intractable function for the participation index, it is easier to turn to discrete summations and tables of participation indices. Figure 14 shows, for a Pell maximum award of \$2,400, the participation index, the average Pell index, the Pell award (assuming a Pell cost of attendance of more than \$4,000), and the resulting Pell awards by category and in aggregate. Note that the uneven pattern of the Pell index matches the uneven pattern found in the actual data and graphed in figure 11. This uneven pattern is the direct result of using two different and uncorrelated methodologies for needs analysis.

Stepping back from the details of the Pell calculation, what generalizations can be drawn about the relationship of Pell to student need? First, because the Pell award is independent of the actual cost of attendance[6], the aggregate Pell will not increase as educational prices increase. Therefore, to the extent that student need rises as a function of the rising cost of attendance, Pell will not grow to meet that need (figure 15 illustrates this point). Second, as family resources grow, aggregate Pell awards decrease, assuming no growth in the maximum Pell award. This second observation, shown graphically in figure 16, makes intuitive sense because, as resources increase, need decreases. Third, Pell maximum awards have grown at a sluggish average of 2.52 percent since 1980, though since 1987 the maximum has grown at the slightly higher rate of 2.74 percent. (See figure 17 for historical Pell maximums and annual growth.) These averages are well below the growth in the cost of higher education, causing Pell to meet less and less of students' needs each year.

Stafford loans are made on the basis of student need and are often the second funding source used to meet need after the Pell grant has been determined.

The Stafford program makes loans of up to \$2,625 to freshmen and sophomores and up to \$4,000 to juniors and seniors, with demonstrated need. The amount of the loan made is

[6]THIS CONTINUES THE ASSUMPTION THAT THE PELL COST OF ATTENDANCE exceeds the Pell maximum divided by 60 percent, or in the case of a Pell maximum of \$2,400, exceeds \$4,000.

FIGURE 12
Pell Awards
Pell Cost of Attendance = $3,000

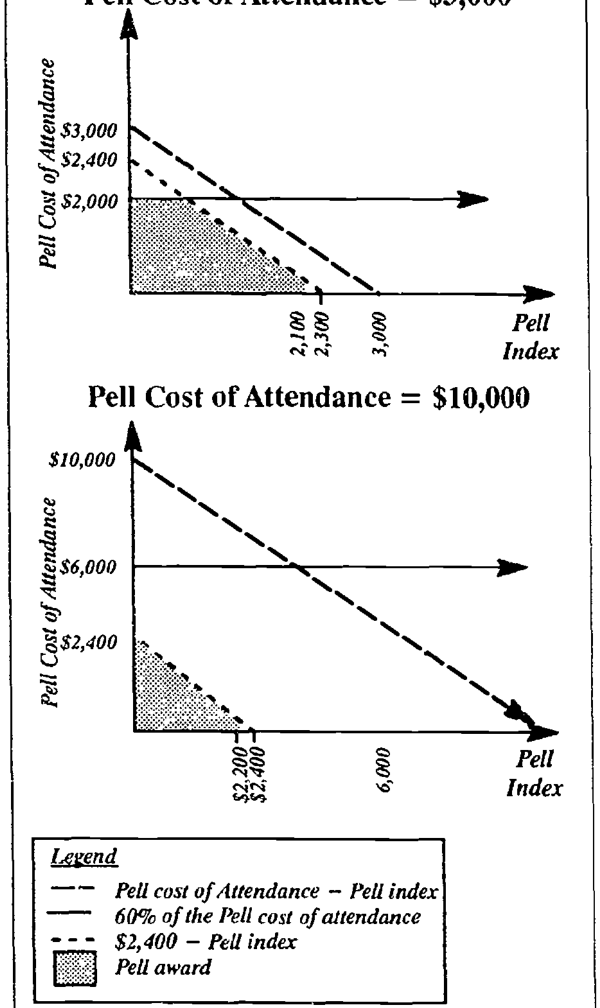
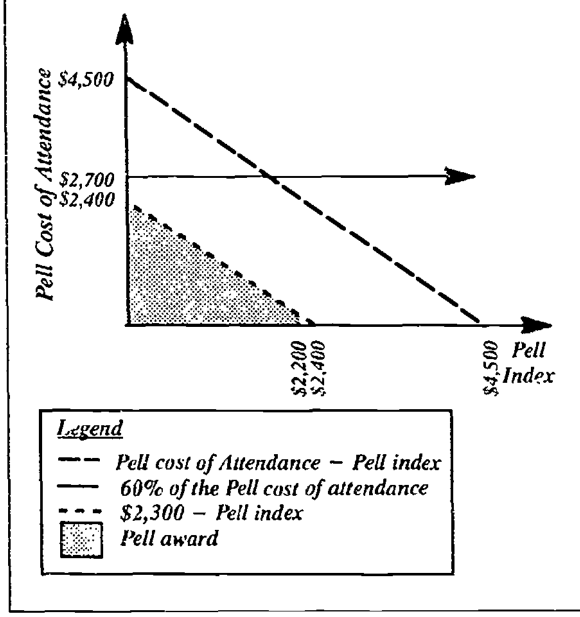


FIGURE 13
Pell Awards



capped by the lower of the cost of attendance less the EFC or the applicable loan maximum. It is therefore extremely straightforward to look at the Stafford loan amount at each EFC and the aggregate Stafford loan funding.

The first step is to calculate the average loan maximum for which students are eligible. Assuming equal numbers of lower- and upperclassmen this average maximum is $3,313.

Figure 18 shows the average Stafford loan funding available at each EFC level. The graph shows a constant loan amount until the cost of attendance minus the EFC is less than $3,313. The loan amount then decreases to equal COA(k) – EFC.

To expand the graph of individual loan amounts to aggregate Stafford funding is a straightforward process. Because of the assumption of the uniform distribution of students, the aggregate Stafford amount equals the number of students at each EFC level times the amount of the Stafford loan — in essence, $\dfrac{N}{Z\,(1 + w)^{k-1}}$ times the areas of the shaded parallelogram and shaded triangle graphed in figure 15.

AS(k) The aggregate Stafford loan funding, in year k, is denoted as AS(k).

$$AS(k) = \frac{N}{Z(1+w)^{k-1}} \times 3{,}313\ [COA(k) - 3{,}313] + \frac{N}{Z(1+w)^{k-1}} \times \frac{3{,}313^2}{2}$$

$$AS(k) = \frac{N}{Z(1+w)^{k-1}} \times [3{,}313\ COA(k) - 5{,}487{,}985]$$

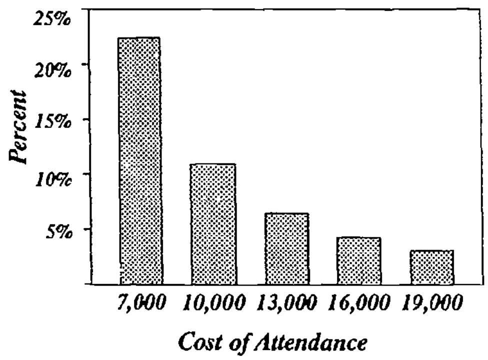
FIGURE 14

Sample Pell Calculation

Average EFC	Ptp(efc, 2400)	Pell Index	Pell Award	Number of Students	Aggregate Pell* Assuming N=400 and $Z(1+w)^{k-1}$=25,000
0	94.14%	250	2,150	$\frac{N}{Z(1+w)^{k-1}}$ *50	16,191
100	94.01	250	2,150	$\frac{N}{Z(1+w)^{k-1}}$ *100	32,341
200	93.89	250	2,150	$\frac{N}{Z(1+w)^{k-1}}$ *100	32,297
300	93.75	642	1,758	Same as Above	26,371
400	93.61	642	1,758	Same as Above	26,331
500	93.46	642	1,758	Same as Above	26,290
600	93.31	642	1,758	Same as Above	26,246
700	93.14	319	2,081	Same as Above	31,013
800	92.97	360	2,040	Same as Above	30,345
900	92.78	401	1,999	Same as Above	29,676
1,000	92.59	441	1,959	Same as Above	29,021
1,100	92.38	482	1,918	Same as Above	28,349
1,200	92.16	522	1,878	Same as Above	27,691
1,300	91.92	563	1,837	Same as Above	27,017
1,400	91.66	604	1,796	Same as Above	26,340
1,500	91.39	644	1,756	Same as Above	25,676
etc.	etc.	etc.	etc.	etc.	etc.
				total=	883,133

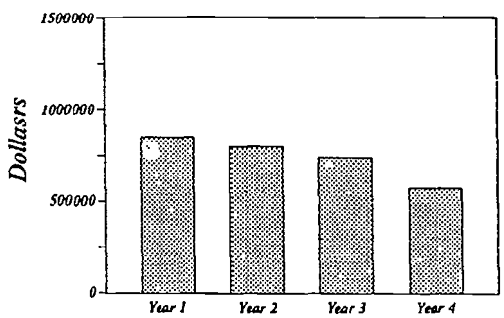
*This column will not tie exactly to the product of Ptp(efc,2400) * Pell Award * Number of Students because of rounding in the Ptp(efc,2400) column.

FIGURE 15
Percent of Student Need Met by Pell*



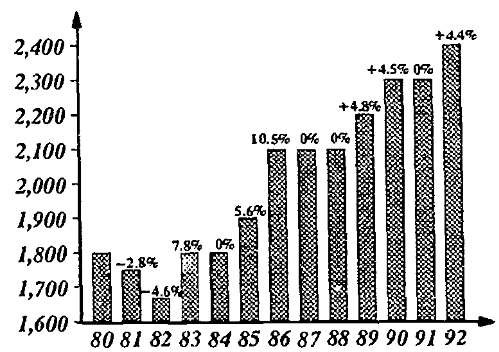
**The calculation assumes students are evenly distributed between an EFC of $0 and $25,000.*

FIGURE 16
Aggregate Pell Awards‡



‡Assumes: 1. 4,000 students evenly distributed between $0 and $25,000 efc. 2. Pell cost of attendance exceeds $4,000. 3. Familial resources grow at 5% per year. 4. No growth in the Pell maximum award. 5. No growth in cost of attendance.

FIGURE 17
Maximum Pell Awards and Percentage Increases by Year



Source: The College Board, *Update*, Table 8.

Because the Stafford loan funding is in part a function of the cost of attendance, as the cost of attendance increases, the aggregate amount of Stafford loan funding increases (as compared with Pell grants, which do not increase with a change in the cost of attendance). The amount of the increase in funding caused by the growth in cost of attendance is, however, offset to some extent by the increase in familial resources (notice the "1 + w" in the denominator raised to a power of "k – 1"). This offset is intuitively logical as the increase in funding should be linked to the growth in costs, adjusted downward for the increase in familial ability to pay.

The relationship between Stafford funding and aggregate need is simple to calculate. The ratio of the two shows that as the cost of attendance increases, the percent of need met by Stafford loans decreases.

$$\frac{AS(k)}{AG(k)} = \frac{\dfrac{N}{Z(1+w)^{k-1}} \times [3{,}313\ \ COA(k) - 5{,}487{,}985]}{\dfrac{N}{Z(1+w)^{k-1}} \times \dfrac{[COA(k)]^2}{2}}$$

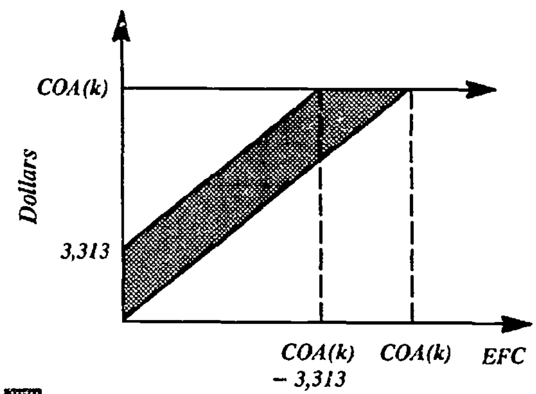
$$= \frac{6{,}626}{COA(k)} - \frac{10{,}975{,}969}{[COA(k)]^2}$$

This equation states that when the cost of attendance is $6,400, 77 percent of student need is met by Stafford loans. However, if the cost attendance were to grow to $10,000 the percent of need met by Stafford loans would drop to 55 percent.

Perkins, SEOG, and CWS awards are made to students on the basis of individual need and funding availability.

Because these remaining Title IV programs are awarded on the basis of need, but only to the

FIGURE 18
Individual Student Stafford Loan Funding at Each EFC Level



extent that the funds are made available to the institution, modeling the impact of these programs is quite easy:

B(k) B(k) is the variable for the amount of Perkins, SEOG, and CWS funds available to the institution for redistribution to its students in year k. For Perkins loans, B(k) includes both the new capital made available to the institution and the loan repayment funds from earlier NDSL and Perkins loans.

$$B(k) = B(1) \times (1 + b)^{(k-1)} \qquad \text{for } k > 1$$

The model assumes that there has been some residual need for financial assistance after externally funded aid has been awarded and that the institution maximizes the use of available funds by packaging financial aid with Pell grants and Stafford loans before adding in Perkins, SEOG, and/or CWS. If this is not the case at a particular institution[7], then the formulae for externally funded financial aid need to be adjusted to reflect the underutilization of the program funds.

Having calculated the aggregate value of funding from Pell, Stafford, and other Title IV programs, the total amount of student need met by external sources is the sum of these three separately calculated pieces.

EFFA(k) The total externally funded financial aid projected by the model is the sum of the separately calculated components: Pell, AP(k); Stafford, AS(k); and the block grant programs — SEOG, CWS, and Perkins — B(k).

$$EFFA(k) = AP(k) + AS(k) + B(k) \text{ for } k > 1$$

An example calculated for two institutions for three years helps to tie these pieces together. If two colleges, one with a tuition rate of $3,400, the second with a tuition rate of $7,000, were to aggressively increase the cost of attendance at 10 percent, while family incomes grew at 5 percent, what would aggregate student need be and how much would be met by externally funded financial aid? To answer the questions, a few more pieces of data are required: assume that familial resources to fund education are distributed uniformly in year 1, from $0 to $25,000; that nontuition student budgets at both institutions average $3,000 in the first year; that the $60,000 the institution receives in Perkins, SEOG, and CWS capital grows 1 percent annually; that the institution maintains a steady enrollment of 4,000 students throughout the modeled period; and that a constant FTE of 25

[7] For example, an institution that packages a student who has an identified need of $4,000 with a $1,000 Pell grant and $1,000 in Perkins, SEOG, and CWS funds, will only include $2,000 in Stafford loans instead of the available Stafford maximum of $2,650 or $4,000. Because the institution cannot reallocate the foregone Stafford dollars to another student but can discretionarily allocate the Perkins, SEOG, and CWS among needy students, the financial aid package represents foregone externally funded financial aid. Of course, in the rare cases when available funds exceed student need, an institution should and will package aid in the manner most advantageous to its students, "underutilizing" those program funds that are most costly to students.

This abundance of external financial aid funding is rare and is not considered in the formula model. Modifying the existing formulae with the addition of a packaging style or content constraints for externally funded financial aid would address this case and would be a straightforward modification of the existing formula structure.

non-need-based scholarships are awarded annually. The input variables from this discussion are shown in table 2.

Using these inputs the calculations of aggregate need and externally funded financial aid proceed as follows:

$$
\begin{aligned}
T(2) &= T(1) \times [1+t(1)] \\
&= 3{,}740 \textit{ for College } 1 \\
&= \textit{and } 7{,}700 \textit{ for College } 2
\end{aligned}
$$

$$
\begin{aligned}
T(3) &= T(2) \times [1+t(2)] \\
&= 4{,}114 \textit{ for College } 1 \\
&\quad \textit{and } 8{,}470 \textit{ for College } 2
\end{aligned}
$$

$$
\begin{aligned}
RB(2) &= RB(1) \times (1+rb)^{2-1} \\
&= 3{,}300 \textit{ for both institutions}
\end{aligned}
$$

$$
\begin{aligned}
RB(3) &= RB(1) \times (1+rb)^{3-1} \\
&= 3{,}630 \textit{ for both institutions}
\end{aligned}
$$

$$
\begin{aligned}
COA(1) &= T(1)+RB(1) \\
&= 6{,}400 \textit{ for College } 1 \\
&\quad \textit{and } 10{,}000 \textit{ for College } 2
\end{aligned}
$$

$$
\begin{aligned}
COA(2) &= T(2)+RB(2) \\
&= 7{,}040 \textit{ for College } 1 \\
&\quad \textit{and } 11{,}000 \textit{ for College } 2
\end{aligned}
$$

$$
\begin{aligned}
COA(3) &= T(3)+RB(3) \\
&= 7{,}744 \textit{ for College } 1 \\
&\quad \textit{and } 12{,}100 \textit{ for College } 2
\end{aligned}
$$

Therefore, aggregate need equals:

$$
\begin{aligned}
AG(1) &= \frac{N \times [COA(1)]^2}{2Z(1+w)^{1-1}} \\
&= 3{,}276{,}800 \textit{ for College } 1 \\
&\quad \textit{and } 8{,}000{,}000 \textit{ for College } 2
\end{aligned}
$$

$$
\begin{aligned}
AG(2) &= \frac{N \times [COA(2)]^2}{2Z(1+w)^{2-1}} \\
&= 3{,}776{,}122 \textit{ for College } 1 \\
&\quad \textit{and } 9{,}219{,}048 \textit{ for College } 2
\end{aligned}
$$

$$
\begin{aligned}
AG(3) &= \frac{N \times [COA(3)]^2}{2Z(1+w)^{3-1}} \\
&= 4{,}351{,}531 \textit{ for College } 1 \\
&\quad \textit{and } 10{,}623{,}854 \textit{ for College } 2
\end{aligned}
$$

The calculations show a 33 percent increase in student need over the three years at College 1 which matches the 33 percent increase in need at College 2. Expressed as a percent of gross tuition, however, student need has grown from 24.1 percent to 26.4 percent at College 1 and a higher 28.6 percent to 31.4 percent at College 2.

The next step is to calculate the funding of the Pell, Stafford, and remaining Title IV programs.

The aggregate Pell award is a function of the Pell maximum and the distribution of student EFCs.

$$Pmax(1) = \$2{,}400 \ (\textit{input})$$

$$
\begin{aligned}
Pmax(2) &= Pmax(1) \times (1+pmax)^{2-1} \\
&= 2{,}460 \\
&\quad \textit{for both Colleges}
\end{aligned}
$$

$$
\begin{aligned}
Pmax(3) &= Pmax(1) \times (1+pmax)^{3-1} \\
&= 2{,}522 \\
&\quad \textit{for both Colleges}
\end{aligned}
$$

Using the Pell maximums and applying them as appropriate to the various segments of the Pell formula, results in the following parameters:

$$
\begin{aligned}
\textit{Inflection point (year 1)} &= -1.627\,604 \times 10^{-7} \times Pmax(1)^3 \\
&\quad +1.875\,000 \times 10^{-3} \times Pmax(1)^2 \\
&\quad -5.520\,833 \times Pmax(1) \\
&\quad +8{,}400 \\
&= 3{,}700 \\
&\quad \textit{for both Colleges}
\end{aligned}
$$

$$\textit{Inflection point (year 2)} = 3{,}742 \quad \textit{for both Colleges}$$

$$\textit{Inflection point (year 3)} = 3{,}792 \quad \textit{for both Colleges}$$

$$
\begin{aligned}
\textit{Breakpoint (year 1)} &= 3.255\,208 \times 10^{-7} \times Pmax(1)^3 \\
&\quad -3.515\,625 \times 10^{-3} \times Pmax(1)^2 \\
&\quad +13.854\,166 \times Pmax(1) \\
&\quad -13{,}000 \\
&= 4{,}500 \\
&\quad \textit{for both Colleges}
\end{aligned}
$$

Factor	College 1	College 2
N	4,000	4,000
T(1)	3,400	7,000
RB(1)	3,000	3,000
t(1), t(2), t(3), rb	10%	10%
Z	25,000	25,000
w	5%	5%
Pmax(1)	$2,400	$2,400
pmax	2.5%	2.5%
B(1)	60,000	60,000
b	1%	1%
M(1)	25	25
m	0	0

Table 2

Breakpoint = 4,652
(*year* 2) *for both Colleges*

Breakpoint = 4,801
(*year* 3) *for both Colleges*

$$\begin{aligned} \textit{sizing} &= -9.765\,625 \times 10^{-9} \times Pmax(1)^3 \\ (\textit{year } 1) &\quad +1.093\,750 \times 10^{-4} \times Pmax(1)^2 \\ &\quad -4.062\,500 \times 10^{-1} \times Pmax(1) \\ &\quad +530 \end{aligned}$$

$$\begin{aligned} &= -9.765\,625 \times 10^{-9} \times 2{,}400^3 \\ &\quad +1.093\,750 \times 10^{-4} \times 2{,}400^2 \\ &\quad -4.062\,500 \times 10^{-1} \times 2{,}400 \\ &\quad +530 \end{aligned}$$

= **50**
for both Colleges

sizing = 47
(*year* 2) *for both Colleges*

sizing = 44
(*year* 3) *for both Colleges*

$$Ptp(efc,k) = \arctan\left[sign \times \frac{(|efc - inflection|)^{.765}}{100} \right] \times \frac{-1}{1.570796 \times 2} + .5$$

for efc ≤ breakpoint
sign = −1 when efc ≤ inflection
= 1 when efc > inflection

or

$$= \frac{sizing}{efc - breakpoint}$$

for efc > breakpoint

For example,

$$Ptp(200{,}1) = \arctan\left[\frac{-1 \times (|200-3{,}700|)^{.765}}{100}\right] \times \frac{-1}{1.570796 \times 2} + .5$$

$$= .938870 \textit{ for both Colleges}$$

$$Ptp(15{,}000;1) = \frac{50}{15{,}000-4{,}500}$$

$$= .004\ 762 \textit{ for both Colleges}$$

Next calculating the Pell Index:

$$\begin{aligned} Pefc(efc) = {} & 250 \textit{ if } efc < 300 \textit{ and } Pmax(k) \le 3{,}600 \\ & \textit{or } 288 \textit{ if } efc < 300 \textit{ and } Pmax(k) > 3{,}600 \\ & \underline{\textit{or}} \\ & 5.8959\ 375 \times 10^{-9} \times Pmax(k)^3 \\ & -1.171\ 875 \times 10^{-5} \times Pmax(k)^2 \\ & -2.687\ 500 \times 10^{-2} \times Pmax(k) \\ & +693 \\ & \textit{if } 300 \le efc < 700 \\ & \underline{\textit{or}} \\ & 35 + efc \times [4.583\ 333 \times 10^{-13} \times Pmax(k)^3 \\ & -2.517\ 187 \times 10^{-8} \times Pmax(k)^2 \\ & +2.277\ 154 \times 10^{-4} \times Pmax(k) \\ & -1.745\ 000 \times 10^{-3}] \\ & \textit{if } 700 \le efc \le \textit{breakpoint} \\ & \underline{\textit{or}} \\ & -1.803\ 385 \times 10^{-7} \times Pmax(k)^3 \\ & +1.885\ 156 \times 10^{-8} \times Pmax(k)^2 \\ & -5.701\ 458 \times Pmax(k) \\ & +6{,}638 \\ & \textit{if breakpoint} < efc \end{aligned}$$

For example, in year 1,

$$Pefc(200) = 250 \textit{ for both Colleges}$$

Next, calculating the Pell award:

$$\begin{aligned} Pefc(efc) &= Pmax(k) - Pefc(efc) \\ & \quad \textit{or} \\ & \quad 0 \textit{ if } P(efc) < 200 \end{aligned}$$

For example, in year 1,

$$\begin{aligned} P(200) &= Pmax(1) - Pefc(200) \\ &= 2{,}400 - 250 \\ &= 2{,}150 \\ & \quad \textit{for both Colleges} \end{aligned}$$

$$\begin{aligned} P(10{,}000) &= Pmax(1) - Pefc(10{,}000) \\ &= 2{,}400 - 1{,}320 \\ &= 1{,}080 \\ & \quad \textit{for both Colleges} \end{aligned}$$

Finally, putting it all together, in year 1,

$$AP = \sum_{\substack{all \\ efc}} P(efc) \times Ptp(efc) \times N(efc)$$

For example, at EFC = 10,000 the function summed is:

$$\begin{aligned} & P(10{,}000) \times Ptp(10{,}000) \times N(10{,}000) \\ &= 1{,}080 \times .009\ 091 \times N(10{,}000) \\ &= 10 \times N(10{,}000) \end{aligned}$$

Using these formulae,

$$\begin{aligned} AP(1) &= 883{,}133 \\ & \quad \textit{for both Colleges} \\ AP(2) &= 921{,}563 \\ & \quad \textit{for both Colleges} \\ AP(3) &= 878{,}335 \\ & \quad \textit{for both Colleges} \end{aligned}$$

The calculation of Stafford loan funding involves the same student factor $\frac{N}{Z\ (1\ +\ w)^{k-1}}$ but also takes into account the cost of attendance. Therefore, the aggregate funding from Stafford loans will be different for the two colleges in the example.

$$AS(1) = \frac{N}{Z(1+w)^{1-1}} \times [3{,}313\ COA(1) - 5{,}487{,}985]$$
$= 2{,}514{,}434$ *for College* 1 *and* $4{,}422{,}722$ *for College* 2

$$AS(2) = \frac{N}{Z(1+w)^{2-1}} \times [3{,}313\ COA(2) - 5{,}487{,}985]$$
$= 2{,}717{,}796$ *for College* 1 *and* $4{,}716{,}955$ *for College* 2

$$AS(3) = \frac{N}{Z(1+w)^{3-1}} \times [3{,}313\ COA(3) - 5{,}487{,}985]$$
$= 2{,}926{,}859$ *for College* 1 *and* $5{,}021{,}216$ *for College* 2

The remaining Title IV program funds are projected using the base funding level and an annual funding growth factor. The first year, B(1), was an input factor and equals $100,000 for both colleges.

$$B(2) = B(1) \times (1+b)^{2-1}$$
$= 60{,}600$ *for both colleges*

$$B(3) = B(1) \times (1+b)^{3-1}$$
$= 61{,}206$ *for both colleges*

The total amount of externally funded financial aid received each year is then the sum of the Pell, Stafford, and other programs.

$$EFFA(1) = AP(1) + AS(1) + B(1)$$
$= 3{,}276{,}800$ *for College* 1 (*note: Stafford cut back* 180,767 *so as not to exceed need*) *and* $5{,}365{,}855$ *for College* 2

$$EFFA(2) = AP(2) + AS(2) + B(2)$$
$= 3{,}752{,}967$ *for College* 1 *and* $5{,}699{,}118$ *for College* 2

$$EFFA(3) = AP(3) + AS(3) + B(3)$$
$= 3{,}980{,}349$ *for College* 1 *and* $5{,}960{,}757$ *for College* 2

Having calculated student need and externally funded financial aid for the two colleges, the results are summarized in figure 19.

Institutionally Funded Financial Aid

Institutionally funded financial aid can be either need-based or non-need-based. This section explores the relationship between institutionally funded financial aid and student need and presents formulae for calculating aggregate levels of institutionally funded financial assistance.

Need-based, institutionally funded financial aid is a function of institutional policy and residual student need.

Residual student need is the amount of student need which is not funded by externally provided financial assistance. It is therefore simply, for each year k, the difference between aggregate student need as measured by AG(k), and the total financial assistance provided by external funds, EFFA(k).

Need-based financial assistance is, therefore, a function of this residual student need and the institution's policy on meeting it. If the institution meets 100 percent of student need, then institutionally funded aid will equal the residual need. If the institution meets 80 percent of residual need, then institutionally funded aid will be 80 percent of the gap. In general, the level of need-based, institutionally funded financial aid required by the cost of attendance, student resources level, and availability of externally funded financial aid can be calculated as follows.

FIGURE 19
Aggregate Student Need and Externally Funded Financial Aid — Two Examples *
(Numbers in Thousands)

	College 1			College 2		
	Year 1	Year 2	Year 3	Year 1	Year 2	Year 3
Gross Tuition Revenues	13,600.0	14,960.0	16,456.0	28,000.0	30,800.0	33,800.0
Student Need	3,276.8	3,776.1	4,351.5	8,000.0	9,219.0	10,623.9
Student Need as a % of Tuition Revenues	24%	25%	26%	29%	30%	31%
Pell	883.1	921.6	878.3	883.1	921.6	878.3
Stafford	2,514.4#	2,717.8	2,926.9	4,422.7	4,717.0	5,021.2
Other Title IV	60.0	60.6	61.2	60.0	60.6	61.2
Total Federally Funded Financial Assistance	3,276.3	3,700.0	3,866.4	5,365.9	5,699.1	5,960.8
Aid as a % of Need	100%	98%	89%	67%	62%	56%

*Totals may not sum because of rounding.
#The Stafford was cut back by 180,767 so as not to exceed need.

F(h,k) The amount of institutionally funded, need-based financial aid in year k, assuming h percent of residual need is met, is calculated as follows:

$$F(h,k) = h \times [AG(k) - EFFA(k)]$$
for $0 < h \leq 100\%$, and for all k

This formula shows that need-based financial aid rises proportionately with the residual need for financial assistance as measured by the mathematical expression AG(k) − EFFA(k) — an intuitively logical result.

Non-need-based financial aid is a function of the cost of attendance and institutional commitments to students.

Non-need-based financial aid is offered to a variety of students at many institutions. Examples of these financial aid programs include college merit awards and athletic scholarships. The "cost" of these programs is captured in the model in the variables AM(k), M(k). M(1) and m are input. The model calculates M(k) for all years after the first, and AM(k) for all years.

M(k) This variable is for the number of FTE scholarships which are awarded in year k. These scholarships are incremental to need-based aid funded by other internal or external scholarship programs.

$$M(k) = M(1) \times (1 + m)^{k-1} \qquad \text{for } k > 1$$

AM(k) This variable is for the aggregate amount of financial aid — above need-based awards — that is granted in non-need-based programs.

$$AM(k) + M(k) \times COA(k) \qquad \text{for all k}$$

Net Tuition

Net tuition equals gross tuition less institutionally funded financial aid.

The net tuition is the amount the student brings to the college or university to fund educational costs. Viewed differently, it is the amount which the college charges for educational services (tuition) less the revenue amount which the college agrees to forego (institutionally funded financial aid). The foregone revenue is the tuition discount.[8]

D(h,k) The aggregate amount of institutionally funded financial aid in year k, given that h percent of the students' residual need after externally funded aid is met by need-based institutional scholarships, is the tuition discount, D(h,k).

$$D(h,k) = F(h,k) + AM(k) \qquad \text{for all k} \quad 0 < h \leq 1$$

NT(h,k) This variable is for the net tuition earned by the institution in year k, given that h percent of the students' residual need — after externally funded aid — is met by need-based institutional scholarships.

$$NT(h,k) = GT(k) - D(h,k) \qquad \text{for all k} \quad 0 < h \leq 1$$

The formulae developed in this and the preceding sections project the impact of tuition pricing policy, student resources, externally funded aid, and internal financial aid policy on net tuition revenues.

This section continues the development of the modeled answer to the question asked in the introduction to the monograph: What impact does gross tuition pricing have on institutionally funded financial aid and, hence, net tuition? The relationship among gross tuition pricing, familial resources, and externally funded financial aid determines the need of the student body for institutionally funded aid. The policy of the individual college or university in meeting that need and in awarding non-need-based scholarships then determines the amount of institutionally funded aid granted and the net amount of tuition the institution will receive.

To illustrate this, the section below continues with the example, begun earlier, of the two colleges priced today at \$3,400 and \$7,000 for tuition. The calculations in this section show the development and projection of institutionally funded aid (both need-based and non-need-based), the tuition discount, and net tuition, assuming that 90 percent of the residual student need is met by institutional discounts.

Need-based aid, assuming that 90 percent of residual need is to be met, is calculated as follows:

$$\begin{aligned} F(90,1) &= .90 \times [AG(1) - EFFA(1)] \\ &= 0 \textit{ for College } 1 \\ &\quad \textit{and } 2{,}370{,}731 \textit{ for College } 2 \end{aligned}$$

$$\begin{aligned} F(90,2) &= .90 \times [AG(2) - EFFA(2)] \\ &= 68{,}547 \textit{ for College } 1 \\ &\quad \textit{and } 3{,}167{,}937 \textit{ for College } 2 \end{aligned}$$

$$\begin{aligned} F(90,3) &= .90 \times [AG(3) - EFFA(3)] \\ &= 436{,}618 \textit{ for College } 1 \\ &\quad \textit{and } 4{,}196{,}788 \textit{ for College } 2 \end{aligned}$$

Non-need-based aid is a function of the cost of attendance and the FTE number of aid recipients. The variables for this calculation are M(k), m, and AM(k). M(k) will equal 25 for both col-

[8] Technically, the amount of the discount is the financial aid provided by current funds plus the capital contribution made by the institution to the loan fund to support financial aid loans made in the current year. This includes all institutionally funded financial aid grants, the required institutional match for CWS and Perkins aid programs, and new loan fund capital provided by the institution.

To simplify the tuition discounting model, the creators have ignored the loan fund and required cost sharing except to the extent that Perkins and NDSL funds are available to support externally funded financial aid. An institution with a significant institutionally funded loan fund needs to modify the formulae presented in the monograph in order to take this into account.

leges because the number of non-need-based aid recipients in the first year was 25 with no growth assumed (m = 0).

$$AM(1) = M(1) \times COA(1) = 160{,}000 \textit{ for College } 1 \textit{ and } 250{,}000 \textit{ for College } 2$$

$$AM(2) = M(2) \times COA(2) = 176{,}000 \textit{ for College } 1 \textit{ and } 275{,}000 \textit{ for College } 2$$

$$AM(3) = M(3) \times COA(3) = 193{,}600 \textit{ for College } 1 \textit{ and } 302{,}500 \textit{ for College } 2$$

The tuition discount is then the sum of these two institutionally funded financial aid factors.

$$D(90,1) = F(90,1) + AM(1) = 160{,}000 \textit{ for College } 1 \textit{ and } 2{,}620{,}731 \textit{ for College } 2$$

$$D(90,2) = F(90,2) + AM(2) = 244{,}547 \textit{ for College } 1 \textit{ and } 3{,}442{,}937 \textit{ for College } 2$$

$$D(90,3) = F(90,3) + AM(3) = 630{,}218 \textit{ for College } 1 \textit{ and } 4{,}499{,}288 \textit{ for College } 2$$

Finally, the net tuition is the difference between the gross tuition and the tuition discount.

$$NT(90,1) = GT(1) - D(90,1) = 13{,}440{,}000 \textit{ for College } 1 \textit{ and } 25{,}379{,}269 \textit{ for College } 2$$

$$NT(90,2) = GT(2) - D(90,2) = 14{,}715{,}453 \textit{ for College } 1 \textit{ and } 27{,}357{,}063 \textit{ for College } 2$$

$$NT(90,3) = GT(3) - D(90,3) = 15{,}825{,}782 \textit{ for College } 1 \textit{ and } 29{,}380{,}712 \textit{ for College } 2$$

These results are summarized in figure 20.

Other Institutional Factors

The preceding sections show the derivation of net tuition from the factors that directly impact this source of revenue — tuition pricing policy, student resources, externally funded financial aid levels, and internally funded financial aid. The tuition question can, and should, be turned around, however, to ask: "What tuition price should be set to meet aggregate institutional needs?" To answer this question the institution must first determine what its net tuition revenue requirements are and, second, calculate the gross tuition levels which will result in those needs being met. This section takes the reader through that two-step process.

Required net tuition revenue levels are a function of tuition dependency, and nontuition revenue and cost growth.

The institutional pressure on net tuition comes from two sources: cost[9] increases and the performance of other revenue sources. The relative magnitude of pressure which each exerts is governed by the level of dependence on tuition revenues. Intuitively this makes sense. If an institution that is highly tuition dependent has rising costs and stagnant nontuition revenue growth, net tuition will need to increase only slightly faster than costs. Here the cost factor exerts the most pressure. If the same cost growth had occurred at an institution which was highly dependent on revenue sources other than tuition — for example, a public university — then stagnant growth in other revenues would exert enormous upward pressure on net tuition to fill the gap between revenues and expenses.

This calculation of net tuition growth uses the input factors c, e, and r(1) and the rate of tuition discount calculated for the first year of the model. The formula is derived as follows.

[9] "COST" AS DEFINED FOR THIS MODEL INCLUDES ALL EDUCATION AND general costs except institutionally funded financial aid, which is categorized as a revenue offset instead of a cost. "Cost" also includes the surplus that the institution budgets for nonmandatory transfers to plant fund reserves and the quasi endowment.

FIGURE 20
Institutionally Funded Financial Aid and Net Tuition – Two Examples*

	College 1			College 2		
	Year 1	Year 2	Year 3	Year 1	Year 2	Year 3
Gross Tuition Revenues	13,600.0	14,960.0	16,456.0	28,000.0	30,800.0	33,800.0
Student Need	3,276.8	3,776.1	4,351.5	8,000.0	9,219.0	10,623.9
Externally Funded Financial Aid	3,276.8	3,700.0	3,866.4	5,365.9	5,699.1	5,960.8
Institutionally Funded Financial Aid:						
Need-based	0.0	68.5	436.6	2,370.7	3,167.9	4,196.8
Non-need-based	160.0	176.0	193.6	250.0	275.0	302.5
Tuition Discount	160.0	244.5	630.2	2,620.7	3,442.9	4,499.3
Net Tuition	13,440.0	14,715.5	15,825.8	25,379.3	27,357.1	29,380.7
Unmet Need	0.0	7.6	48.5	263.4	352.0	466.3
Tuition Retention Rate	99%	98%	96%	91%	89%	87%

*Totals may not sum because of rounding.

d(h,k) The rate of tuition discount is the ratio of the tuition discount to gross tuition revenues.

$$d(h,k) = \frac{D(h,k)}{GT(k)} \qquad \text{for all k, } 0 < h \leq 1$$

nt(h,k) The rate of growth in net tuition revenue, given that h percent of residual need, after externally funded financial aid, is met by institutional grants in year k, is calculated as a ratio of net tuition:

$$nt(h,k) = \frac{NT(h,k)}{NT(h,k-1)} \qquad \text{for all k, } 0 < h \leq 1$$

The rate of growth in net tuition revenue, given that h percent of residual need is met in year k, can alternately be derived from other institutional factors.

$$\text{Total E\&G Cost including Financial Aid and Budgeted Surplus} = \text{Total E\&G Cost including Financial Aid and Budgeted Surplus}$$

$$\begin{aligned} &= [\textit{Total E\&G Cost...}] \times [r(1)+1-r(1)] \\ 1 &= r(1)+[1-r(1)] \\ 1-r(1) \times d(h,1) &= r(1)-r(1) \times d(h,1)+[1-r(1)] \end{aligned}$$

In words, the mathematical expression above says that the rate of cost, net of financial aid, equals the rate of discounted tuition (net tuition) plus the rate of nontuition. The left side of the equation is now of the form that matches the basis of the cost growth factor, c. Similarly, the rate of nontuition revenues, $[1-r(1)]$, has the same basis as the input growth rate, e.

Applying those factors, the user can solve for the required net tuition growth to meet the gap left between cost and nontuition revenue increases (table 3).

Knowing the desired net tuition levels, the college or university can calculate the gross tuition revenue necessary to meet the institution's net funding requirements.

Once it is established what the net tuition must be to meet institutional requirements and economic circumstances, it is possible to determine the level of gross tuition which produces the desired result. Theoretically, the formula developed thus far could be turned around to drive the derivation of gross tuition. This calculation, however, would be complex and the results would contain no more significant digits than a quick trial and error estimation. To do this estimation, choose a trial rate for the growth in gross tuition for the second year, t(2), run it through the calculation to produce a derived net tuition, and compare the net tuition calculated for an estimated t(2), to that derived from other institutional factors. If the calculation based upon t(2) produces a net tuition which is in excess of institutional requirements, then try a lower estimate of t(2); if the inverse is true, increase the estimate of t(2). With a few quick runs of the tuition model the user can develop a good estimate of t(2) and hence the gross tuition rate and revenues in the second year, T(2) and GT(2).

Completing the example of the two colleges with different pricing policies, it is now possible to calculate the pressure placed on net and gross tuition by other institutional factors. In addition to the input assumptions made earlier, the user must now specify the tuition dependency in the first year, r(1), and the growth in nontuition revenues and adjusted costs, e and c, respectively (table 4). By holding these three variables constant between the two schools, the effect of their tuition levels against familial ability to pay is highlighted.

Note that because this calculation derives the required net and gross tuition growth rates, nt(h,k) and t(k), the values assumed for t(k) at the beginning of the example are no longer used.

The first step is to calculate the actual discount rate from the first year, d(90,1), to use as part of the projection base.

$$\bar{d}(90,1) = \frac{D(90,1)}{GT(1)}$$
$$= 1.2\% \textit{ for College } 1,$$
$$\textit{and } 9.9\% \textit{ for College } 2$$

The second step is to calculate nt(h,k) at the 90 percent level of meeting residual need. The 90 percent level is chosen to remain consistent with the earlier example (table 5).

Knowing the net tuition growth rate, net tuition revenue can now be calculated.[10]

$NT(90,1)$ = 13,440,000 *for College* 1
and 25,379,269 *for College* 2
(Calculated in the earlier example and used as the starting point here.)

$NT(90,2) = NT(90,1 \times [1+nt(90,2)]$
= 14,851,200 *for College* 1
and 28,044,092 *for College* 2

$NT(90,3) = NT(90,2) \times [1+nt(90,3)]$
= 16,410,576 *for College* 1
and 31,016,766 *for College* 2

With the required net tuition revenue, the distribution of familial ability to pay, the amount of — and growth in — room and board

[10]THE CALCULATIONS SHOWN ARE ROUNDED; THE MODEL, IN THIS CALCULATION, is not rounded until the very end. Therefore, the calculations above do not match exactly to the modeled results.

In year 2, then, net tuition growth, nt(h,2), must be as follows:

$$[1-r(1) \times d(h,1)] \times (1+c) = [r(1)-r(1) \times d(h,1)] \times [1+nt(h,2)]+[1-r(1)] \times (1+e)$$

Solving for nt(h,2),

$$nt(h,2) = \left[\frac{[1-r(1) \times d(h,1)] \times (1+c) - [1-r(1)] \times (1+e)}{r(1) - r(1) \times d(h,1)}\right] - 1$$

Projecting this expression for net tuition into the future, and arbitrarily setting nt(h,1) to zero, the expression for net tuition in year k, looks like this:

$$\begin{aligned}[1-r(1) \times d(h,1)] \times (1+c)^{k-1} &= [1-r(1)] \times (1+e)^{k-1}+ \\ &\quad [r(1)-r(1) \times d(h,1)] \times [1+nt(h,2)] \times [1+nt(h,3)] \\ &\quad ...[1+nt(h,k-1)] \times [1+nt(h,k)] \\ &= [1-r(1)] \times (1+e)^{k-1}+[r(1)-r(1) \times d(h,1)] \times \prod_{y=1}^{k} [1+nt(h,y)]\end{aligned}$$

Solving this equation for nt(h,k):

$$nt(h,k) = \left[\frac{[1-r(1) \times d(h,1)] \times (1+c)^{k-1}-[1-r(1)] \times (1+e)^{k}}{[r(1)-r(1) \times d(h,1)] \times \prod_{y=1}^{k} [1+nt(h,y)]}\right] - 1$$

for $k > 1$, $0 < h \leq 1$, $nt(h,1)=0$

Putting the pieces together, the net tuition, NT(h,k), can be determined from NT(h,k-1) and the net tuition growth, nt(h,k), derived above from other institutional factors.

$$NT(h,k)=NT(h,k-1) \times [1+nt(h,k)] \qquad \text{for } k > 1, 0 < h \leq 1$$

Because the formula for nt(h,k) is a function of institutional cost and nontuition revenue growth, this formula for net tuition revenue reflects the institutional requirements for net tuition in order to meet the forecast revenue demand.

Table 3

Factor	College 1	College 2
r(1)	80%	80%
c	10%	10%
e	8%	8%

Table 4

charges, the number of non-need-based scholarship recipients, and the availability of federally funded aid (beyond Pell and Stafford), the gross tuition revenue and required level of institutionally funded aid can be calculated. The results of the calculations are presented in figure 21.

Conclusion

The tuition discounting model introduced here is a first step toward understanding the phenomenon of tuition discounting. As a result, it is more illustrative of trends and relationships among variables than it is predictive of any single institution's budget behavior. This is in part because of the simplifying assumptions used.

For example, the externally funded financial aid considered in the model includes only the major federal programs. Though many states have financial grants and/or loan programs, their idiosyncratic nature does not permit their inclusior in a generalized model of tuition discounting. Another example is the assumption that institutionally funded financial assistance consists entirely of grant aid. In fact, many colleges and universities have loan programs which they use to meet a portion of student need.

Therefore, the tuition model presented in this paper is not *predictive* of any specific college. It is, however, *descriptive* of the interrelationships of pricing policy, familial ability to pay, institutional revenue requirements, externally funded aid, and tuition discounting. Furthermore, the methodologies outlined in this appendix allow an institution to tailor the model to meet the particular characteristics of their operations.

$$nt(90,1) = 0 \text{ (by definition)}$$

$$nt(90,2) = \left[\frac{[1-r(1) \times d(90,1)] \times (1+c)^{2-1} - [1+r(1)] \times (1+e)^{2-1}}{[r(1)-r(1) \times d(90,1)] \times [1+nt(90,1)]}\right] - 1$$

$$= 10.5\% \text{ for College 1 and } 10.6\% \text{ for College 2}$$

$$nt(90,3) = \left[\frac{[1-r(1) \times d(90,1)] \times (1+c)^{3-1} - [1+r(1)] \times (1+e)^{3-1}}{[r(1)-r(1) \times d(90,1)] \times [1+nt(90,1)] \times [1+nt(90,2)]}\right] - 1$$

$$= 10.5\% \text{ for College 1 and } 10.5\% \text{ for College 2}$$

Table 5

FIGURE 21

Impact of Other Institutional Factors on Net and Gross Tuition: Two Examples

	College 1			College 2		
	Year 1	Year 2	Year 3	Year 1	Year 2	Year 3
Net Tuition Required to Meet Institutional Needs	13,440.0	14,850.4	16,408.0	25,379.3	28,057.0	31,015.1
Growth in Net Tuition	-----	10.5%	10.5%	---	10.6%	10.5%
Stated Tuition Rate Necessary to Generate the Net Tuition Levels Above	3,400	3,779	4,286	7,000	7,941	9,056
Gross Tuition Revenue	13,600.0	15,116.0	17,144.0	28,000.0	31,764.0	36,224.0
Growth in Gross Tuition Revenue	---	11.2%	13.4%	---	13.4%	14.0%
Tuition Discount	160.0	265.6	736.0	2,620.7	3,707.0	5,208.9
Growth in Tuition Discount	---	66.0%	177.1%	---	41.4%	40.5%
Percent of Tuition Retained	98.8%	98.2%	95.7%	90.6%	88.3%	85.6%

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