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AUTHOR Leidig, Julie; And Others
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ABSTRACT

This student edition of learning modules in refresher mathematics is one of the five books developed during a 21-month cooperative project to develop instructional materials that enhance skills in a workplace context. Partners in the project included the University of Texas at Austin, consultants, the Austin/Travis County Private Industry Council, Hart Graphics, IBM, and Texas Instruments. The guide contains two modules that cover the following topics for a suggested 20-week course in refresher mathematics: decimals, fractions, word problems, estimating and averages, ratios and proportions, percents, standard measurement, signed numbers, algebraic equations and inequalities, solving for unknowns, working with formulas, and graphing ordered pairs. The modules are divided into a total of 23 lessons that use a functional workplace context, discovered through observing how workers use skills, conducting job task analyses, and interviewing workers, supervisors, and training managers. Lessons in this student edition contain some or all the following elements of a lesson plan: learning objective; introduction; vocabulary; practice problems; developing understanding; skill building problems, activities, and exercises; self-check; reviews; and answer keys. (KC)

# Sharpening Your Skills in the Workplace





STUDENT EDITION

# The Competitive Edge: Sharpening Your Skills in the Workplace

## MATHEMATICS

### Student Edition


**By**

Julie Leidig
Sally McCarty
Holly O'Rear
Mary Hartzell

**Coordinated by**

Elaine Shelton



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*Editor* / Barbara Sutherland
*Artists* / Elinor Gamache and Debra Dionisio
*Publishing Technician* / Kathryn N. Roberts
*Production Assistant* / Debbie Hajji
*Production Coordinator* / Barbara Sutherland

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Extension Instruction and Materials Center
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Austin, TX 78713-7218
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# Contents

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**The University of Texas at Austin, Extension Instruction and Materials Center, Educational Resources, Skills Building for Tomorrow Project**
Dr. Kathryn Allen, Director, Extension Instruction and Materials Center
Dr. Barbara Selke-Kern, Director, Educational Resources
Elaine Shelton, Director, Skills Building for Tomorrow Project (SBT)
Julie Leidig, SBT Instructor Coordinator/Curriculum Developer/Assistant Editor
Sally McCarty, SBT Instructor/Curriculum Developer
Holly O'Rear, SBT Instructor/Curiculum Developer
Mary Hartzell, SBT Instructor/Curriculum Developer
Cathy Crandall, Coordinator of Customer Service

**Consultants**
Sue Lagerquist, Instructional Consultant
Kurt von Quintus, Technical Consultant

**Austin/Travis County Private Industry Council**
Bill Demestihas, Executive Director

**Hart Graphics**
Mary Astor
Lola Barkley
Cheryl Bingham
Orlando Boleda
David Camp
Donald Castilleja
Daniel Castillo
Domisco Castillo
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Shakir Doctor
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Dois Ripley
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Dario Solis
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Scott Thomas
James Waddington
Daniel Walker
Dennis Weisser
Sandra Washington

**IBM**
Melinda Acevedo
Bertha O. Alvarez
Jesse Ancira
Joyce Arnold
A. C. Berry
Bill Black
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Greg Sharp
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**Texas Instruments**
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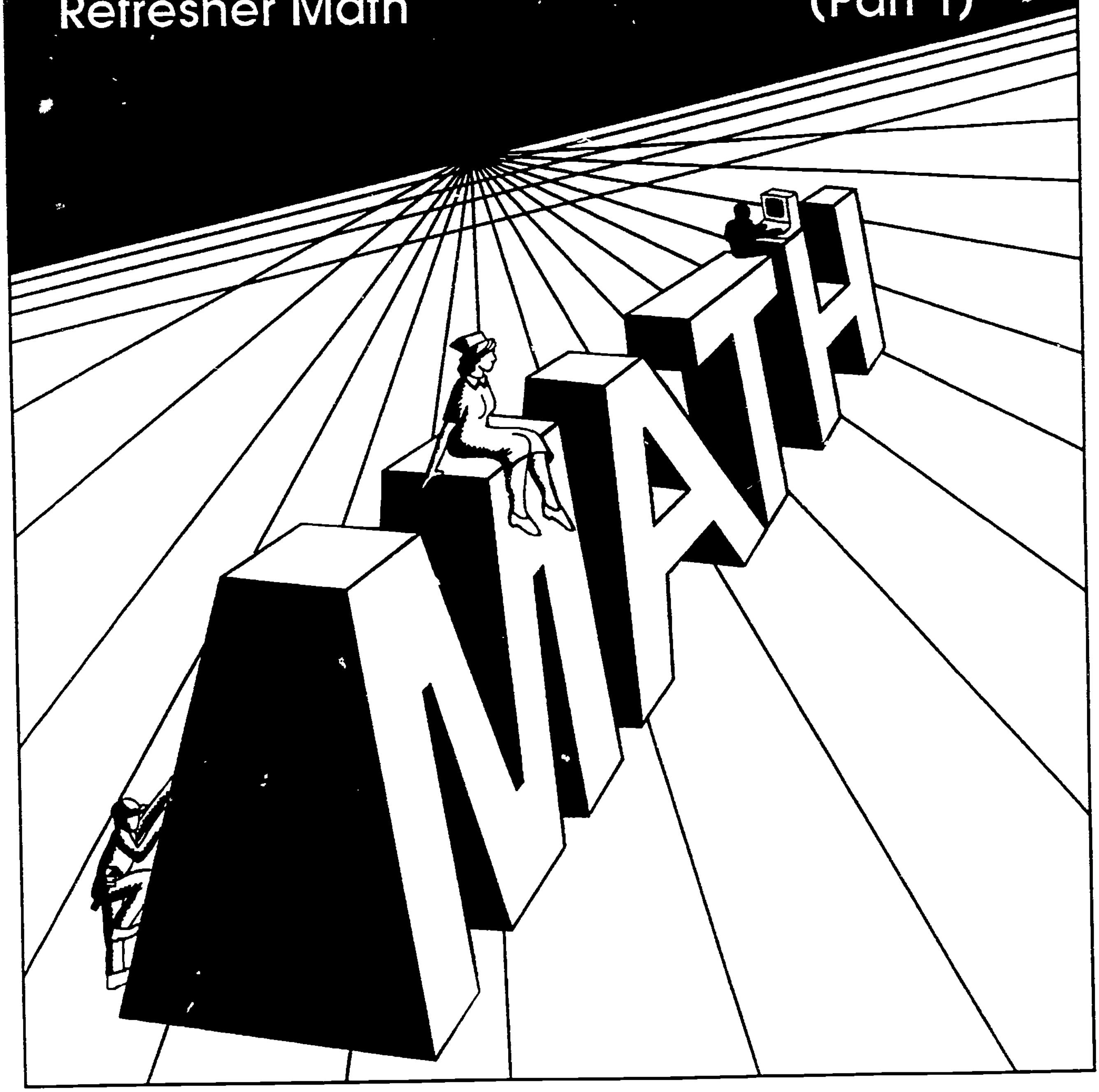
Howard Swathwood
Rungston Titapas
Warida Trejo
Pat Watson
Freddie West
Bob Wright
Sook Yoo

# Module 1

## Refresher Math (Part 1)



# Lesson 1.1
# Adding and Subtracting Decimals

## Learning Objectives

*To master the basic vocabulary of decimals*
*To read and compare decimal values*
*To practice rounding off decimal values*
*To add and subtract decimals*

Decimals are used every day in the workplace. For example, every time you use amounts of money, you are using decimals. Therefore, it is important to understand what decimals mean and know how to use them. This lesson will review what a number with a decimal means, what the places to the right of a decimal point stand for, and how to add and subtract numbers that have decimals.

Can you think of examples of how you use decimals at work or in daily life?

## Vocabulary

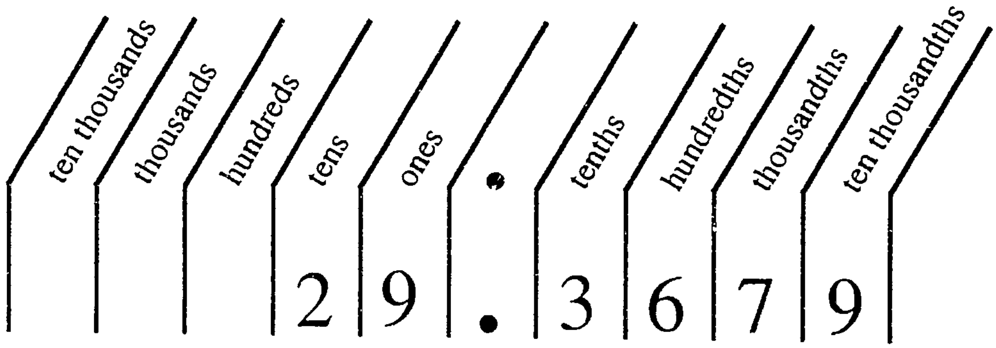
*decimal*—a number that includes a decimal point and place values to the right of the decimal point (5.2, 65.75, etc.)
*tenths*—one place value to the right of a decimal point
*hundredths*—two place values to the right of a decimal point
*thousandths*—three place values to the right of a decimal point
*ten thousandths*—four place values to the right of a decimal point

## Develop Your Understanding

### *Decimal Values*

What is the value of each number in 29.3679?

| | | | | | |
|---|---|---|---|---|---|
| The 2 is in the tens place. | 2 | × | 10 | = | 20 |
| The 9 is in the ones place. | 9 | × | 1 | = | 9 |
| The 3 is in the tenths place. | 3 | × | 0.1 | = | 0.3 |
| The 6 is in the hundredths place. | 6 | × | 0.01 | = | 0.06 |
| The 7 is in the thousandths place. | 7 | × | 0.001 | = | 0.007 |
| The 9 is in the ten thousandths place. | 9 | × | 0.0001 | = | 0.0009 |




How would you read the number aloud: 29.3679?

Read the 29 as you would read a regular whole number. Next, say the word *and* to show the presence of a decimal point. Then read the numbers to the right of the decimal point as if they were a whole number ending with the value place name of the last digit.

For example, 29.3679 would be read: twenty-nine *and* three thousand six hundred seventy-nine ten thousandths. It ends with the words *ten thousandths* because the final number is in the ten thousandths place.

Which is greater, 0.074 or 0.74?

To determine which decimal is greater, line up the decimal points and compare the place values from left to right as you would for whole numbers.

↓
0.074
0.74

The 7 is greater than the 0, so 0.74 is greater than 0.074.

Which is greater, 12.367 or 12.379?

Line up the numbers:

↓
12.367
12.379

Compare the two numbers, moving from left to right, until you find a difference:

12.3**6**7
12.3**7**9

There is no difference until the hundredths place. But since 7 is larger than 6, the number 12.379 is greater than the number 12.367.

*Once you have found a difference in values, there is no need to compare values that are farther to the right.*

**Now Try These**

a. What is the value of the "6" in the number 5.162?

b. Circle the larger number: 0.396 or 0.485?

### *Rounding*

How would you round off the number 15.57?

*Whenever you round numbers, you must first decide how far to go; you may want to round off to the nearest hundredth, tenth, or to the nearest whole number.*

Rules for Rounding Off Numbers:

1. Look one place to the right of your chosen place value.
2. For values of 4 or below, round *down* to 0. Drop all digits to the right.
3. For values of 5 or above, add 1 to the next place value to the left. Drop all digits to the right.

* Remember that an original number is more accurate than a rounded number. Do not round off numbers when exact precision is important!

*Example 1*

To round 15.57 to the nearest *tenth*, look at the hundredths place. Since 7 is between 5 and 9, round up and add a 1 to the value in the tenths place:

$$15.57 \rightarrow 15.6$$

*Example 2*

To round 14.294 to the nearest hundredth, look at the thousandths place. Since 4 is less than 5, round this value down to 0:

$$14.294 \rightarrow 14.290 \rightarrow 14.29$$

When the farthest value to the right of the decimal is a zero, that value can be dropped.

**Now Try These**

a. Round off 9.212 to the nearest *tenth*.

b. Round off 30.49 to the nearest *whole number*.

c. Round off 2.106 to the nearest *hundredth*.

### *Adding and Subtracting*

How would you add the following?

$$2.679 + 0.384 = ?$$
$$24.3774 + 23.699 = ?$$

When adding decimals, be sure that you line up the decimal points so that digits from the same value place are being added.

$$\begin{array}{r} 2.679 \\ \underline{+\ .384} \\ 3.063 \end{array} \qquad \begin{array}{r} 24.3774 \\ \underline{+\ 23.699\phantom{0}} \\ 48.0764 \end{array}$$

The process of adding decimals is exactly the same as the process of adding whole numbers. Be sure that you put the decimal point in the correct place in the solution.

How would you subtract the following?

$$1.322 - 0.977 = ?$$
$$2 - 0.378 = ?$$

Remember to line up the decimal points. Notice how a whole number can be written when using it with decimals.

$$\begin{array}{r} 1.322 \\ \underline{-\ 0.977} \\ 0.345 \end{array} \qquad \begin{array}{r} 2.000 \\ \underline{-\ 0.378} \\ 1.622 \end{array}$$

The process of subtracting decimals is also the same as the process of subtracting whole numbers. You may borrow from place values across the decimal point. Be sure that a decimal point is included in the correct place in the solution.

**Now Try These**

a. 12.494 + 0.36 =

b. 5.573 - 0.689 =

**Skill Builders**

Read each of these numbers aloud:

1. 14.3
2. 15.67
3. 22.9984

Circle the greater number in each pair:

4. 15 or 15.2
5. 2.9 or 2.93
6. 3.678 or 3.54
7. 1.5 or 1.4

Round these decimals off to the nearest whole number:

8. 15.5 ____________
9. 16.25 ____________
10. 3.4 ____________
11. 0.2 ____________
12. 4.9 ____________
13. 1.489 ____________

Add:

14. 3 + 0.27 + 19.4 =

15. 12.79 + 15.97 =

16. 1.36 + 12.32 + 0.9 =

17. 4.4 + 1.13 + 0.9987 =

Subtract:

18. $\begin{array}{r} 49.5 \\ \underline{-\ 3.2} \end{array}$

19. $\begin{array}{r} 50 \\ \underline{-\ \ \ 0.989} \end{array}$

20. $\begin{array}{r} 50.058 \\ \underline{-\ 43.264} \end{array}$

21. $\begin{array}{r} 19 \\ \underline{-\ 7.83} \end{array}$

**Check Yourself**

1. Read 49.3678 aloud.

2. Round off to the nearest tenth: 13.499

3. Circle the greater number: 17.864 or 17.834

4. Add: 0.49 + 12 + 174.9832 =

5. Subtract: 17.008 – 14.679 =

# Lesson 1.2
# Multiplying Decimals

## Learning Objectives

*To learn to multiply decimals*

In this lesson you will have a chance to practice multiplying decimals. Multiplication of decimals is something you may use daily as you figure your income and expenses involving dollars and cents, or if you use measurements at work. Dollars and cents are designated by decimal points, as are measurements like inches, gallons, liters, and centimeters.

## Vocabulary

*product*—the answer to a multiplication problem

$$\begin{array}{r} 12 \\ \underline{\times\ 6} \\ 72 \end{array} \quad \leftarrow \quad \text{product}$$

## Develop Your Understanding

The process of multiplying decimals is the same as the process for multiplying whole numbers. However, the product of any multiplication problem with decimals must have the same number of decimal places as the sum of decimal places in the problem.

### *Steps for Multiplying Decimals*

1. Multiply as you usually do with whole numbers. Disregard the decimal point until you have found the product.

2. Count up all the decimal places in the original 2 (or more) numbers.
3. Beginning with the far right place value in the product, count this number of places to the left, and place the decimal point there.

*Example 1*

*Multiply 3.4 by 1.2.*

Write the problem vertically:

$$\begin{array}{r} 3.4 \\ \underline{\times\ 1.2} \end{array}$$

Multiply as whole numbers, disregarding the decimal points:

$$\begin{array}{r} 34 \\ \underline{\times\ 12} \\ 68 \\ \underline{34\phantom{0}} \\ 408 \end{array}$$

The number of decimal places in the product must match the total number of places in the original numbers. Start on the right and count left:

$$\begin{array}{rl} 3.4 & \quad 1 \text{ decimal place} \\ \underline{\times\ 1.2} & \quad +\underline{1 \text{ decimal place}} \\ 68 & \\ \underline{34\phantom{0}} & \\ 4.08 & \quad 2 \text{ decimal places in the product} \end{array}$$

move 2 places left from here

place decimal here

**Now Try These**

a. $\begin{array}{r} 3.2 \\ \underline{\times\ 0.1} \end{array}$

b. $\begin{array}{r} 2.5 \\ \underline{\times\ 0.75} \end{array}$

*Example 2*

Do not drop the final zero until *after* the decimal point is placed:

| | |
|---|---|
| 5.08 | 2 decimal places |
| × 2.5 | +1 decimal place |
| 12.700 | 3 decimal places in the product |

The number "12.700" may then be written "12.7." Place values of zero on the right of the decimal point may be dropped when there are no other numbers farther to the right.

**Now Try These**

Calculate the products, then rewrite each product, dropping zeros where possible:

a. 3.6<br>× 3.5

b. 2.25<br>× 1.04

*Example 3*

A zero can act as a place holder when there are fewer numbers in the product than decimal values in the original numbers:

| | |
|---|---|
| 0.2 | 1 decimal place |
| × 0.3 | +1 decimal place |
| 0.06 | 2 decimal places in the product |

When you write a decimal without a whole number, you may choose to use a zero as a place holder for the whole number. However, it is not necessary that you do so:

0.06 = .06

## Now Try These

a. $\begin{array}{r} 3.68 \\ \times\ .02 \\ \hline \end{array}$

b. $\begin{array}{r} .045 \\ \times\ .01 \\ \hline \end{array}$

## Skill Builders

Multiply:

1. $\begin{array}{r} 17.1 \\ \times\ 4.9 \\ \hline \end{array}$

2. $\begin{array}{r} 6.01 \\ \times\ .07 \\ \hline \end{array}$

3. $\begin{array}{r} 11.5 \\ \times\ 2.4 \\ \hline \end{array}$

4. $\begin{array}{r} 0.002 \\ \times\ 5 \\ \hline \end{array}$

Multiply and round each product to the nearest *hundredth* if possible.

5. $\begin{array}{r} 3.6 \\ \times\ 25 \\ \hline \end{array}$

6. $\begin{array}{r} 71.28 \\ \times\ .18 \\ \hline \end{array}$

7. $\begin{array}{r} 3.765 \\ \underline{\times\ 9} \end{array}$

8. $\begin{array}{r} 45.01 \\ \underline{\times\ 4.09} \end{array}$

9. $\begin{array}{r} 437.3 \\ \underline{\times\ 22} \end{array}$

10. $\begin{array}{r} 16 \\ \underline{\times\ 41.56} \end{array}$

## Check Yourself

Use your *multiplication* skills to solve these problems.

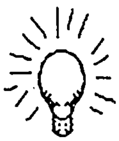
1. If 6 people each contribute $4.25 to a birthday party fund, how much money will be in the fund?

2. Larry works 7.5 hours every day from Monday through Friday. How many hours does he work in a week?

# Lesson 1.3
# Dividing with Decimals

## Learning Objectives

*To practice dividing combinations of decimals and whole numbers*
*To learn to divide numbers by larger numbers*



The ability to divide numbers that include decimals is an important skill both at work and outside of work. At work, you may need to divide something into equal parts or use very exact measurements. Outside of work, you may need to divide a bill equally between several people or calculate how much money you can afford to spend each day. In this lesson, you will learn division using different combinations of whole numbers and decimals. You will also learn how to divide a number by a larger number, an important step toward learning to calculate percentages.

## Vocabulary

*dividend*—a number to be divided
*divisor*—a number to divide the dividend by
*quotient*—the answer to a division problem

$$\begin{array}{rrl} & 5 & \leftarrow \textit{quotient} \\ \textit{divisor} \rightarrow 6 & \overline{)30} & \leftarrow \textit{dividend} \\ & \underline{-30} & \\ & 0 & \end{array}$$

## Develop Your Understanding

All division problems have a dividend and a divisor. The dividend is the number that will be "cut into pieces"; the divisor tells you how many pieces to cut.

In horizontal form, the dividend comes first:

$$\underset{\text{dividend}}{12} \div \underset{\text{divisor}}{3} = \underset{\text{quotient}}{4}$$

In vertical form, the dividend is on the inside of the division symbol:

$$\begin{array}{rl} & \phantom{)}4 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow & 3\overline{)12} \leftarrow \text{dividend} \end{array}$$

***Steps for Dividing with Decimals***

1. If the divisor has a decimal point in it, move the decimal point to the right until it becomes a whole number.
2. Move the decimal point in the dividend to the right an equal number of places.
3. Insert the decimal point in the place where the quotient will go.
4. Perform division.

***Changing the Divisor***

Division should not be performed while the divisor has a decimal point in it. When the divisor has a decimal point, move the decimal point to the right as many places as necessary to make the divisor a whole number. Next, move the decimal point in the dividend to the right an equal number of places. YOU MUST change the dividend and divisor in exactly the same way. It is all right if the dividend still contains a decimal.

*Example 1*

$$18.2\overline{)36.4}$$

Move the decimal point in the divisor one place to the right. This changes 18.2 to 182:

$$18.2 \rightarrow 182.$$

Now, move the decimal point in the dividend one place to the right. This changes 36.4 to 364:

$$36.4 \rightarrow 364.$$

The problem now looks like this:

$$\begin{array}{r} 2.\phantom{0} \\ 182\overline{)364.} \\ \underline{-364}\phantom{.} \\ 0\phantom{.} \end{array}$$

The quotient is 2.

**Now Try These**

a. $2.56\overline{)7.68}$

b. $2.04\overline{)8.16}$

**Decimals in the Quotient**

*Example 2*

$$0.25\overline{)31.6}$$

In this case, the divisor contains two decimal places, but the dividend contains only one.

| |
|---|
| *Adding zeros to the far right of a decimal does not change the decimal's value. For example, 1.25 = 1.25000.* |

Move the decimal point two places over in both the divisor and the dividend:

0.25 → 25.        31.6 → 3,160.

Add zeros on the right as place holders when you move the decimal point more times than there are place values.

Before you start to divide, put a decimal point directly above the decimal point in the dividend, in the place where you will fill in the quotient:

$25 \overline{)3160.}$

Begin dividing. Once you are on the right side of the decimal point, you can continue to add zeros to the right of the remainder when needed.

$$
\begin{array}{r}
126.4 \\
25 \overline{)3160.} \\
\underline{-25}\phantom{00.} \\
66\phantom{0.} \\
\underline{-50}\phantom{0.} \\
160\phantom{.} \\
\underline{-150}\phantom{.} \\
100 \\
\underline{-100} \\
0
\end{array}
$$

**Now Try These**

Round to the nearest *thousandth.*

a. $31 \overline{)622}$

b. $19 \overline{)42}$

*Example 3:*

Using decimals, you can divide a number by a greater number.

$$
\begin{array}{r}
0 \\
130 \overline{)65}
\end{array}
$$

When the divisor is greater than the dividend, begin by inserting a decimal point into the quotient. You may then add a zero to the dividend.

$$
\begin{array}{r}
0.\phantom{0} \\
130 \overline{)65.0}
\end{array}
$$

Next, proceed to divide as usual, inserting the quotient to the right of the decimal point.

$$
\begin{array}{r}
0.5 \\
130 \overline{)65.0} \\
\underline{-650} \\
0
\end{array}
$$

The quotient is 0.5.

**Now Try These**

a. $12 \overline{)3}$

b. $6 \overline{)2.4}$

**Skill Builders**

Calculate the following problems. Round all quotients off to *two* decimal places.

1. $2 \overline{)8.24}$

2. $27 \overline{)168.75}$

3. $2.5 \overline{)7.5}$

4. $3.62 \overline{)5.068}$

5. $1.008\overline{)6}$

6. $51\overline{)143}$

7. $3\overline{)56}$

8. $0.03\overline{)162}$

9. $42\overline{)21}$

10. $250\overline{)10}$

**Check Yourself**

Use your *division* skills to solve these problems.

1. The parts department received an order for a part that costs $1.47 each. The customer forgot to fill in how many parts she wanted on the order form, but enclosed a check for $5.88. How many parts will this pay for?

2. Eleven people on a manufacturing line won a $1,000 reward for a cost-saving suggestion and split the money equally. About how much money did each person get?

# Lesson 1.4
# Using Fractions: Adding, Subtracting, and Reducing to Lowest Terms

## Learning Objectives

*To develop an understanding of fractional terms and their meanings*
*To recognize different types of fractions*
*To practice adding and subtracting fractions that have the same denominator*
*To practice reducing fractions to their lowest terms*

The ability to understand and use fractions is becoming more important as modern workplaces shift to emphasize technical skills. You will need to review the vocabulary of fractions, and to quickly recognize the three types of fractions. In this lesson, you will learn to change improper fractions to mixed numbers and mixed numbers to improper fractions. You will also practice adding and subtracting fractions with the same denominator, and learn to reduce fractions to lowest terms.

Give three examples of how fractions are used in your job or in other jobs in your area.

1.________________________________________

2.________________________________________

3.________________________________________

## Vocabulary

*fraction*—part of a whole. In math, this is expressed in the form: $\frac{\text{part}}{\text{whole}}$

*bar*—the short line separating a numerator from a denominator; also a symbol for division

*denominator*—the bottom number in a fraction

*numerator*—the top number in a fraction

*numerator*—the top number in a fraction
*terms*—the numerator and denominator comprise the terms of a fraction
*proper fraction*—a fraction with a smaller numerator than denominator
*improper fraction*—a fraction with a numerator equal to or greater than the denominator
*mixed number*—the sum of a whole number and a proper fraction
*greatest common factor (GCF)*—the largest number that will divide evenly into both the numerator and denominator of a fraction

## Develop Your Understanding

### *Types of Fractions*

> **INSTRUCTIONS**
>
> To prepare a bath for plating:
>
> 1. Start with $\frac{1}{2}$ gallon of water at 60° to 80°F.
> 2. Add $\frac{1}{8}$ gallon of solution A.
> 3. With rapid agitation, add $\frac{3}{16}$ gallon of solution B.
> 4. Add water to bring volume to $1\frac{1}{2}$ gallons.
> 5. Add five $\frac{1}{4}$-gallon portions of solution C, or $\frac{5}{4}$ gallons.

There are three types of fractions in this work order. They may be classified as follows:

1. $\frac{1}{2}$ is a proper fraction
2. $\frac{1}{8}$ is a proper fraction
3. $\frac{3}{16}$ is a proper fraction
4. $1\frac{1}{2}$ is a mixed number
5. $\frac{5}{4}$ is an improper fraction

A *proper fraction* shows a quantity less than 1. The numerator of a proper fraction is always smaller than the denominator.

An *improper fraction* shows a quantity equal to or greater than 1. The numerator of an improper fraction is the same as or greater than the denominator.

A *mixed number* is another way to write a fraction that is greater than 1. A mixed number is a whole number plus a proper fraction.

### *Changing Improper Fractions and Mixed Numbers*

Steps for changing an improper fraction to a mixed number:

1. Determine how many times the denominator will divide evenly into the numerator. This will be the whole number part.
2. Subtract the amount taken out for the whole number from the numerator. The remainder will be the new numerator.
3. Keep the same denominator.

*Example 1*

For the improper fraction $\frac{5}{4}$:

Step 1: 4 goes into 5 one time, so the whole number will be 1.
Step 2: After subtracting 4 from the numerator, the remainder is 1, so the numerator will be 1.
Step 3: The denominator will still be 4.

$\frac{5}{4}$ $\qquad$ $5 \div 4 = 1$ with remainder of 1 $\qquad$ $\frac{5}{4} = 1\frac{1}{4}$

Steps for changing a mixed number to an improper fraction:

1. Multiply the denominator of the fraction by the whole number.
2. Add the numerator of the fraction to the product.
3. Write the result over the original denominator.

*Example 2*

For the mixed number $1\frac{5}{8}$:

Step 1: The product of the whole number and the denominator is 8.
Step 2: 8 plus the numerator (5) is 13. This is the new numerator.
Step 3: The denominator will still be 8.

$1\frac{5}{8}$ $\qquad$ $8 \times 1 = 8$ $\qquad$ $8 + 5 = 13$ $\qquad$ $1\frac{5}{8} = \frac{13}{8}$

## Now Try These

a. Change $\frac{8}{5}$ to a mixed number. ________________

b. Change $5\frac{1}{2}$ to an improper fraction. ________________

### *Adding Fractions*

When fractions have the same denominator, add only the numerators, leaving the denominators as they are.

*Example 3*

$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

## Now Try These

a. $\frac{2}{7} + \frac{3}{7} =$ ____________________

b. $\frac{1}{15} + \frac{7}{15} =$ ____________________

### *Subtracting Fractions*

When subtracting fractions with the same denominator, subtract only the numerators and leave the denominators as they are.

*Example 4*

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$

**Now Try These**

a. $\frac{6}{7} - \frac{1}{7} =$ ________________

b. $\frac{2}{3} - \frac{1}{3} =$ ________________

### *Reducing Fractions to Their Lowest Terms*

When working with fractions, it is important to simplify them or, in other words, reduce them to their lowest terms. To do this you need to find the *greatest common factor* (GCF) for the numerator and the denominator. This is the greatest number that will divide into both the numerator and the denominator. When working with an improper fraction, remember to reduce it by making it a mixed number and then reducing the remainder by the GCF.

*Example 5*

$\frac{40}{60}$ will reduce to $\frac{2}{3}$ by dividing both the numerator and the denominator by 20, which is the greatest common factor.

$40 \div 20 = 2$

$60 \div 20 = 3$

$$\frac{40}{60} = \frac{2}{3}$$

*Example 6*

$\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$ $\qquad$ $2 \div 2 = 1$ $\qquad$ $\frac{2}{4} = \frac{1}{2}$

$4 \div 2 = 2$

$\frac{6}{4}$ is reduced by making it a mixed number and then reducing the remainder by dividing it by 2 (the GCF). The whole number does not change when the fraction is reduced to lowest terms.

## Now Try These

Reduce each fraction to lowest terms, using the GCF.

a. $\frac{4}{8}$ = ______________

b. $\frac{10}{15}$ = ______________

c. $\frac{9}{3}$ = ______________

## Skill Builders

Circle the proper fraction in each pair.

1. $\frac{1}{2}$ or $\frac{4}{3}$

2. $\frac{6}{7}$ or $\frac{7}{6}$

3. $\frac{3}{2}$ or $\frac{5}{7}$

Change each improper fraction to a mixed numeral.

4. $\frac{7}{3}$ = ______________

5. $\frac{16}{5}$ = ______________

6. $\frac{18}{7}$ = ______________

7. $\frac{40}{9}$ = ______________

Change each mixed numeral to an improper fraction.

8. $1\frac{1}{3}$ = ____________________

9. $6\frac{7}{8}$ = ____________________

10. $8\frac{1}{4}$ = ____________________

11. $13\frac{2}{3}$ = ____________________

Add the fractions.

12. $\frac{1}{5} + \frac{2}{5}$ = ______________

13. $\frac{2}{3} + \frac{2}{3}$ = ______________

14. $\frac{7}{8} + \frac{2}{8}$ = ______________

15. $\frac{11}{12} + \frac{1}{12}$ = ____________

Subtract the fractions.

16. $\frac{3}{6} - \frac{2}{6}$ = ______________

17. $\frac{7}{13} - \frac{3}{13}$ = ____________

18. $\frac{8}{10} - \frac{5}{10}$ = ____________

19. $\frac{8}{15} - \frac{6}{15}$ = ____________

Reduce each fraction to lowest terms.

20. $\frac{8}{10} =$ __________ 21. $\frac{30}{60} =$ __________

22. $\frac{3}{12} =$ __________ 23. $\frac{16}{12} =$ __________

24. $\frac{20}{15} =$ __________

## Check Yourself

*Problem:* A new mix of chemicals has been developed that will improve the quality of the plating operation. Suppose that you are mixing it by hand, using this formula:

$\frac{1}{8}$ gallon of substance A

$\frac{3}{8}$ gallon of substance B

$\frac{6}{8}$ gallon of substance C

$\frac{5}{8}$ gallon of substance D

1. If you mix the chemicals and check the total measurement, what is the total amount of the mixture you have so far? __________

2. You need to pour off some of the mixture so that you will only have $1\frac{1}{2}$ gallons. How much should you pour off? __________

# Lesson 1.5
# Finding Common Denominators

## Learning Objectives

*To learn to add and subtract fractions with like and unlike denominators, and mixed numbers*

In Lesson 1.4, you reviewed the basics of fractions and practiced adding and subtracting fractions with the same denominator. However, it is often necessary to add or subtract fractions with different denominators. For example, when adding hours worked on a time sheet, you may need to add fractions of an hour such as $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{5}$. You must convert such fractions to fractions with the same denominator before adding or subtracting.

## Vocabulary

*multiple*—the product of a whole number and another whole number
*least common multiple (LCM)*—the lowest number that two or more numbers can be multiplied into evenly
*least common denominator (LCD)*—the LCM of two or more denominators
*equivalent fractions*—fractions that may look different but are equal in value

## Develop Your Understanding

### *Adding and Subtracting Fractions with Unlike Denominators*

In order to add or subtract fractions with unlike denominators, they must first be converted to fractions with the same denominators.

Steps for Adding or Subtracting Fractions with Unlike Denominators:

1. Find the least common multiple (LCM) of the denominators. This number is the least common denominator (LCD).
2. Convert each fraction to an equivalent fraction, each having the least common denominator (LCD).
3. Add or subtract the numerators only, keeping the same denominator.

**Step 1: Finding the Least Common Multiple**

The *multiples* of a number are the products of that number and other whole numbers, including 1.

*Example 1*

The first 5 multiples of the number 2 are 2, 4, 6, 8, and 10:

$$2 \times 1 = 2$$
$$2 \times 2 = 4$$
$$2 \times 3 = 6$$
$$2 \times 4 = 8$$
$$2 \times 5 = 10$$

**Now Try These**

List the first 5 multiples of each number:

a. 3: ______, ______, ______, ______, ______.

b. 7: ______, ______, ______, ______, ______.

When you have two or more fractions with unlike denominators to add or subtract, begin by comparing the denominators and listing multiples of each denominator. The smallest number that is a multiple of all the denominators is called the *least common multiple (LCM).*

*Example 2*

$$\frac{1}{3} + \frac{1}{4}$$

*Find the LCM of 3 and 4.*

Multiples of 3 are: 3, 6, 9, 12, 15, 18.......

Multiples of 4 are: 4, 8, 12, 16, 20, 24.....

*12 is the LCM of 3 and 4.*

Sometimes one of the denominators will also be the LCM.

*Example 3*

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

*Find the LCM of 2, 4, and 8.*

Multiples of 2 are: 2, 4, 6, 8, 10, 12...

Multiples of 4 are: 4, 8, 12, 16, 20, 24...

Multiples of 8 are: 8, 16, 24, 32, 40, 48...

*8 is the LCM of 2, 4, and 8.*

**Now Try These**

Find the LCM of each of the following:

a. 5 and 10

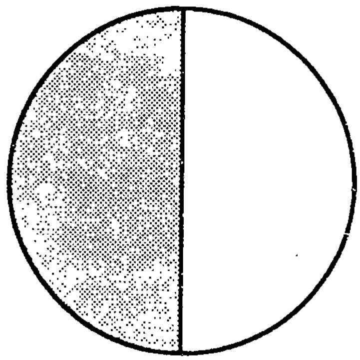
b. 3, 6, and 8

**Step 2: Converting to Equivalent Fractions**

Fractions that have the same value are sometimes called by different names. Fractions that look different but are equal in value are called *equivalent fractions.*

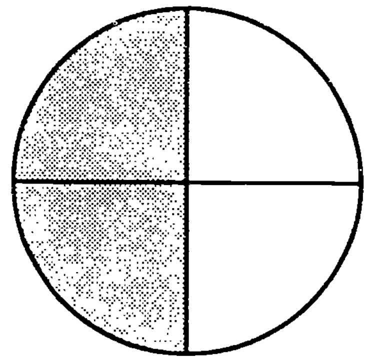
Circle A is divided into 2 parts. The shaded portion of Circle A is $\frac{1}{2}$ of the circle. Circle B, on the other hand, is divided into 4 parts. The shaded portion of Circle B is the same size as the shaded portion of Circle A. It is also $\frac{2}{4}$ of the circle.

$\frac{1}{2}$



Circle A

$\frac{2}{4}$



Circle B

Since $\frac{1}{2}$ of a circle is equivalent to $\frac{2}{4}$ of the same circle, $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions:

$$\frac{1}{2} = \frac{2}{4}$$

You can compute equivalent fractions without using circles.

| |
|---|
| *If you multiply both the numerator and the denominator of any fraction by the same number, the result will be a fraction equivalent to the original one.* |

The above rule is true because any fraction that has the same numerator and denominator is equal to 1; and any number multiplied by 1 is equal to itself.

*Example 4*

*Find 2 fractions that are equivalent to $\frac{2}{3}$.*

$$\frac{2 \times 2 = 4}{3 \times 2 = 6} \qquad \frac{2 \times 3 = 6}{3 \times 3 = 9}$$

Therefore, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$.

**Now Try These**

Find two equivalent fractions for each example:

a. $\frac{3}{4}$  b. $\frac{1}{7}$

### *Least Common Denominator*

Suppose you want to add two fractions with unlike denominators:

$$\frac{1}{6} + \frac{1}{9} = ?$$

Checking the multiples of 6 and 9, you find that the LCM is 18:

Multiples of 6: 6, 12, 18, 24....

Multiples of 9: 9, 18, 27, 36....

The next step is to convert both fractions to equivalent fractions with the same denominator. The LCM is the number you will use for the denominator. This number is also called the *LCD* or *least common denominator:*

$$\frac{1}{6} = \frac{?}{18} \qquad \frac{1}{9} = \frac{?}{18}$$

To compute equivalent fractions using the LCD, work backwards. Find the number that the original denominator must be multiplied by to result in the LCD. Then, multiply the numerator of the fraction by the same number:

$\frac{1}{6} = \frac{?}{18}$  6 must be multiplied by <u>**3**</u> to get 18.

$\frac{1 \times 3 = 3}{6 \times 3 = 18}$  Multiply the numerator by <u>**3**</u>, too.

$\frac{1}{6}$ is converted to $\frac{3}{18}$.

Next, repeat the same process for the fraction $\frac{1}{9}$:

$\frac{1}{9} = \frac{?}{18}$ 9 must be multiplied by **2** to get 18.

$\frac{1 \times 2 = 2}{9 \times 2 = 18}$ Multiply the numerator by **2** as well.

$\frac{1}{9}$ is converted to $\frac{2}{18}$.

The problem can now be solved:

$$\frac{1}{6} + \frac{1}{9} = \frac{3}{18} + \frac{2}{18} = \mathbf{\frac{5}{18}}$$

**Now Try These**

Use this problem to answer each item:

$$\mathbf{\frac{5}{6} + \frac{1}{3} =}$$

a. Find the LCM:

b. Convert to equivalent fractions containing the LCD:

c. Add and reduce to lowest terms:

***Working with Mixed Numbers***

When adding mixed numbers, group the whole numbers separately from the fractions, add, and combine the results:

*Example 5*

$$3\frac{1}{2} + 6\frac{1}{4} =$$

| | |
|---|---|
| $3 + 6 + \frac{1}{2} + \frac{1}{4}$ | *Separate whole numbers from fractions.* |
| $3 + 6 = 9$ | *Add the whole numbers.* |
| $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ | *Add the fractions.* |
| $9 + \frac{3}{4} = \mathbf{9\frac{3}{4}}$ | *Combine the two sums; be sure to put the result in lowest terms.* |

**Now Try These**

a. $12\frac{1}{3} + 4\frac{1}{2} =$

b. $2\frac{1}{14} + 6\frac{3}{7} =$

When subtracting mixed numbers, you may need to "borrow" a whole number to subtract the fractions:

*Example 6*

| | |
|---|---|
| $5\frac{1}{5} - 2\frac{3}{10} =$ | *Convert the fractions with unlike denominators to fractions with the same denominator.* |
| $5\frac{2}{10} - 2\frac{3}{10}$ | *Since you cannot subtract* $\frac{3}{10}$ *from* $\frac{2}{10}$, *borrow* $\frac{10}{10}$ *from the "5."* |

$4\frac{12}{10} - 2\frac{3}{10}$ *Separate the whole numbers and fractions.*

$4 - 2 = 2$ *Subtract the whole numbers.*

$\frac{12}{10} - \frac{3}{10} = \frac{9}{10}$ *Subtract the fractions.*

$5\frac{2}{10} - 2\frac{3}{10} = \mathbf{2\frac{9}{10}}$ *Combine the results.*

**Now Try These**

a. $8\frac{1}{8} - 7\frac{3}{4} =$

b. $4\frac{3}{5} - 1\frac{1}{2} =$

**Skill Builders**

Add or subtract, and reduce to lowest terms.

1. $\frac{1}{5} + \frac{2}{10} =$

2. $\frac{1}{3} + \frac{3}{4} =$

3. $\frac{2}{4} + \frac{5}{8} =$

4. $\frac{2}{3} + \frac{4}{5} =$

5. $\frac{5}{6} + \frac{1}{4} =$

6. $\frac{3}{7} + \frac{1}{4} =$

7. $\frac{3}{8} - \frac{1}{6} =$

8. $\frac{5}{8} - \frac{1}{2} =$

9. $\frac{8}{9} - \frac{5}{6} =$

10. $\frac{1}{2} - \frac{2}{7} =$

11. $8\frac{1}{2} + 10\frac{2}{3} =$

12. $1\frac{3}{8} + 2\frac{3}{4} =$

13. $9\frac{2}{3} - 4\frac{2}{6} =$

14. $14\frac{7}{10} - 3\frac{4}{15} =$

## Check Yourself

*Problem 1:*

What is the total process time for all job tasks?

| Job Task | Process Time |
|---|---|
| black oxide | $1\frac{4}{6}$ hr |
| layup | $\frac{2}{3}$ hr |
| lamination | $4\frac{1}{2}$ hr |

Total process time = ____________________

*Problem 2:*

Job: Cutter
Task: Program cut measurements into machine.

Cut 1: $8\frac{1}{2}$ inches

Cut 2: $\frac{3}{16}$ inch

Cut 3: $\frac{3}{8}$ inch

Total inches cut = _______

# Lesson 1.6
# Multiplying and Dividing Fractions

## Learning Objectives

*To practice multiplying fractions and mixed numbers*
*To practice dividing fractions and mixed numbers*

Now that you have practiced adding and subtracting fractions with both like and unlike denominators, you can enhance your skill with fractions by learning to multiply and divide them. You may have done this before in the following way:

A pamphlet is $8\frac{1}{2}$ inches long. If you want to make a pamphlet that is half the size of this one, how long would it be?____________________

## Vocabulary

*reciprocal*—the reciprocal of a fraction is that fraction with its numerator and denominator inverted.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$: $\left(\frac{2}{3}\right)$ inverted = $\frac{3}{2}$

## Develop Your Understanding

### *Multiplying Fractions*

If you know how to multiply whole numbers, multiplying fractions is not difficult.

> *To multiply two fractions, multiply the numerators and multiply the denominators.*

*Example 1*

Suppose you want to find out what $\frac{1}{2}$ of the fraction $\frac{3}{7}$ is. You can set this up as a multiplication problem:

$$\frac{3}{7} \times \frac{1}{2} =$$

To get the answer, multiply the two numerators, and follow the same procedure with the two denominators.

$$\frac{3}{7} \times \frac{1}{2} = \frac{3 \times 1}{7 \times 2} = \frac{3}{14}$$

The product is $\frac{3}{14}$. We can also say that " $\frac{3}{14}$ is $\frac{3}{7}$ of the fraction $\frac{1}{2}$."

## Now Try These

*Be sure to reduce all answers to lowest terms if applicable.*

a. $\frac{2}{3} \times \frac{4}{5} =$

b. $\frac{1}{8} \times \frac{1}{8} =$

### *Whole Numbers*

The product of a fraction and a whole number tells the value of that part of the whole. For example, if you wanted to know what a fifth of the number 3 is, you could multiply $\frac{1}{5}$ by 3. In order to solve the problem, 3 can be rewritten as a fraction.

> *Any whole number can be expressed as a fraction by putting the whole number over a denominator of "1."*

*Example 2*

To multiply $\frac{1}{5}$ by 3, change 3 to the fraction $\frac{3}{1}$:

$$\frac{1}{5} \times 3 = \frac{1}{5} \times \frac{3}{1} = \frac{1 \times 3}{5 \times 1} = \frac{3}{5}$$

The product means that $\frac{1}{5}$ of the number 3 is $\frac{3}{5}$.

### Now Try These

Multiply and reduce answers to lowest terms.

a. $\frac{5}{6} \times 2 =$

b. $8 \times \frac{9}{10} =$

### *Mixed Numbers*

Convert mixed numbers to improper fractions before you multiply.

*Example 3*

$$3\frac{1}{2} \times 1\frac{1}{3} = \frac{7}{2} \times \frac{4}{3} = \mathbf{\frac{28}{6} = 4\frac{2}{3}}$$

**Now Try These**

a. $2\frac{1}{4} \times 1\frac{1}{9} =$

b. $1\frac{1}{5} \times 1\frac{3}{8} =$

### *Dividing Fractions*

When dividing fractions, you must use the *reciprocal* of the divisor, or second fraction in the problem. To find the reciprocal of a fraction, reverse the numerator and the denominator:

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

The reciprocal of a whole number has a numerator of 1 over a denominator of the whole number: The reciprocal of 3 is $\frac{1}{3}$.

> *To divide two fractions,* ***multiply*** *the first fraction by the reciprocal of the second fraction.*

*Example 4*

$$\frac{1}{2} \div \frac{1}{4} =$$

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$$

You can now solve the problem using multiplication:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{1 \times 4}{2 \times 1} = \frac{4}{2} = 2$$

> *Notice that when you divide a fraction by a fraction, the quotient may be greater than the number you started out with.*

## Now Try These

a. $\frac{2}{5} \div \frac{8}{9} =$

b. $\frac{1}{2} \div \frac{3}{4} =$

### *Mixed Numbers*

Convert mixed numbers to improper fractions when you divide. Then, take the reciprocal of the second improper fraction and multiply.

*Example 5*

$$1\frac{5}{6} \div \frac{2}{3} = \frac{11}{6} \div \frac{2}{3} = \frac{11}{6} \times \frac{3}{2} = \frac{11 \times 3}{6 \times 2} = \frac{33}{12} = 2\frac{9}{12} = 2\frac{3}{4}$$

## Now Try These

a. $2\frac{1}{2} \div 3\frac{3}{4} =$

b. $1\frac{5}{8} \div 4\frac{1}{5} =$

## Skill Builders

Multiply and convert to lowest terms:

1. $\frac{1}{15} \times \frac{2}{3} =$

2. $\frac{3}{20} \times \frac{1}{4} =$

3. $\frac{4}{11} \times 11 =$

4. $\frac{2}{5} \times \frac{5}{6} =$

5. $\frac{1}{3} \times 24 =$

6. $18 \times \frac{1}{6} =$

7. $\frac{7}{2} \times \frac{2}{14} =$

8. $\frac{3}{27} \times \frac{9}{4} =$

9. $1\frac{3}{10} \times \frac{4}{5} =$

10. $4\frac{1}{2} \times \frac{3}{4} =$

Divide and convert to lowest terms:

11. $\frac{1}{3} \div \frac{1}{3} =$

12. $\frac{1}{12} \div \frac{3}{4} =$

13. $\frac{7}{16} \div \frac{1}{4} =$

14. $\frac{10}{17} \div \frac{2}{3} =$

15. $\frac{7}{8} \div 2 =$

16. $32 \div \frac{1}{2} =$

17. $3\frac{1}{4} \div \frac{1}{4} =$

18. $5\frac{3}{5} \div 2\frac{1}{3} =$

19. $1\frac{15}{16} \div 2\frac{4}{5} =$

20. $1 \div \frac{4}{5} =$

## Check Yourself

TRY A SHORTCUT: You can use a shortcut to reduce products and quotients to lowest terms. You can reduce as you go along by dividing *opposite* numerators and denominators by their GCF:

$\frac{5}{4} \times \frac{8}{25}$ compare 5 with 25; both can be divided by **5**

compare 4 with 8; both can be divided by **4**

Rewrite the problem:

$$\frac{5}{4} \times \frac{8}{25} = \frac{\cancel{5}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{25}_{5}} = \frac{1}{1} \times \frac{2}{5} = \frac{2}{5}$$

This shortcut can help you solve problems quickly without having to multiply large numbers.

Use the shortcut to solve these problems:

1. $\frac{2}{3} \times \frac{3}{2} =$

2. $\frac{14}{3} \times \frac{9}{7} =$

3. $\frac{6}{8} \div \frac{3}{4} =$

4. $1\frac{4}{5} \div \frac{1}{10} =$

# Lesson 1.7
# Fractions and Decimals

## Learning Objectives

*To understand relationships between fractions and decimals*
*To practice converting fractions to decimals and decimals to fractions*
*To learn some common fraction/decimal conversions*

You have already practiced basic techniques for adding, subtracting, multiplying, and dividing decimals and fractions. Since both decimals and fractions are used to express values that are less than one, it is possible to express many of the same values in fraction or decimal form. Since both forms may be used for many purposes at work, you will need to know how to convert from one form to another in order to compare values.

## Develop Your Understanding

### *Converting Fractions to Decimals*

To convert a fraction to a decimal, simply divide the numerator by the denominator. (The bar in a fraction is like a division symbol.)

*Example 1* — *Convert $\frac{4}{5}$ to decimal form.*

$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ \underline{-40} \\ 0 \end{array}$$

*Dividing the numerator (4) by the denominator (5) gives a quotient of 0.8.*

$\frac{4}{5} = 0.8$

## Now Try These

Convert each fraction to decimal form. Round off to the nearest *hundredth* if applicable.

a. $\frac{2}{3}$

b. $\frac{4}{7}$

### *Converting Decimals to Fractions*

To convert a decimal to a fraction, rewrite the decimal as a numerator without the decimal point; the denominator will be a multiple of 10.

To determine the correct denominator, count the original number of decimal places. The denominator will be a "1" followed by that number of zeros. Note that if the decimal value farthest to the right is in tenths, the denominator will be 10; if that value is in hundredths, the denominator will be 100; if that value is in thousandths, the denominator will be 1000.

*Example 2*

*Convert 0.71 to fraction form.*

| | |
|---|---|
| $\underline{71}$ | *Write 0.71 as a numerator, dropping the decimal point.* |
| $\frac{\phantom{71}}{100}$ | *There are 2 decimal places, meaning 71 hundredths, so there will be 2 zeros in the denominator.* |
| $0.71 = \frac{71}{100}$ | *The decimal "71 hundredths" is the fraction "71 over 100."* |

After converting a decimal to fraction form, be sure to reduce the fraction to lowest terms.

*Example 3*

*Convert 0.65 to fraction form, and reduce to lowest terms.*

| | |
|---|---|
| $\underline{65}$ | *Write 65 as the numerator, dropping the decimal point.* |
| $\frac{65}{100}$ | *Since there are 2 decimal places, the denominator is 100.* |
| $\frac{65 \div 5}{100 \div 5} = \frac{13}{20}$ | *Divide both the numerator and denominator by 5.* |
| $0.65 = \frac{13}{20}$ | |

**Now Try These**

Convert each decimal to a fraction. Reduce to lowest terms if applicable.

a. 0.225

b. 0.3

***Common Conversions***

These fractions and decimals are very often substituted for each other. You may want to memorize these:

| **Fraction** | **Decimal Equivalent** |
|---|---|
| $\frac{1}{10}$ | 0.1 |
| $\frac{1}{5}$ | 0.2 |
| $\frac{1}{4}$ | 0.25 |
| $\frac{1}{2}$ | 0.5 |
| $\frac{3}{4}$ | 0.75 |

## Skill Builders

Convert these fractions to decimal form. Round off to the nearest *thousandth* if applicable.

1. $\frac{5}{6}$

2. $\frac{3}{5}$

3. $\frac{1}{8}$

4. $\frac{1}{3}$

5. $\frac{9}{10}$

6. $\frac{3}{20}$

Convert these decimals to fractions, then express in lowest terms.

7. 0.625

8. 0.72

9. 0.144

10. 0.4

11. 0.0045

12. 0.02

**Check Yourself**

Read the following. Then solve George's problem.

George's supervisor said, "Someone on the first shift marked these packages differently. I want all the weights marked in fractions."

Package 1 = $\frac{3}{8}$ lb

Package 2 = 0.5 lb

Package 3 = 0.25 lb

Package 4 = $\frac{3}{4}$ lb

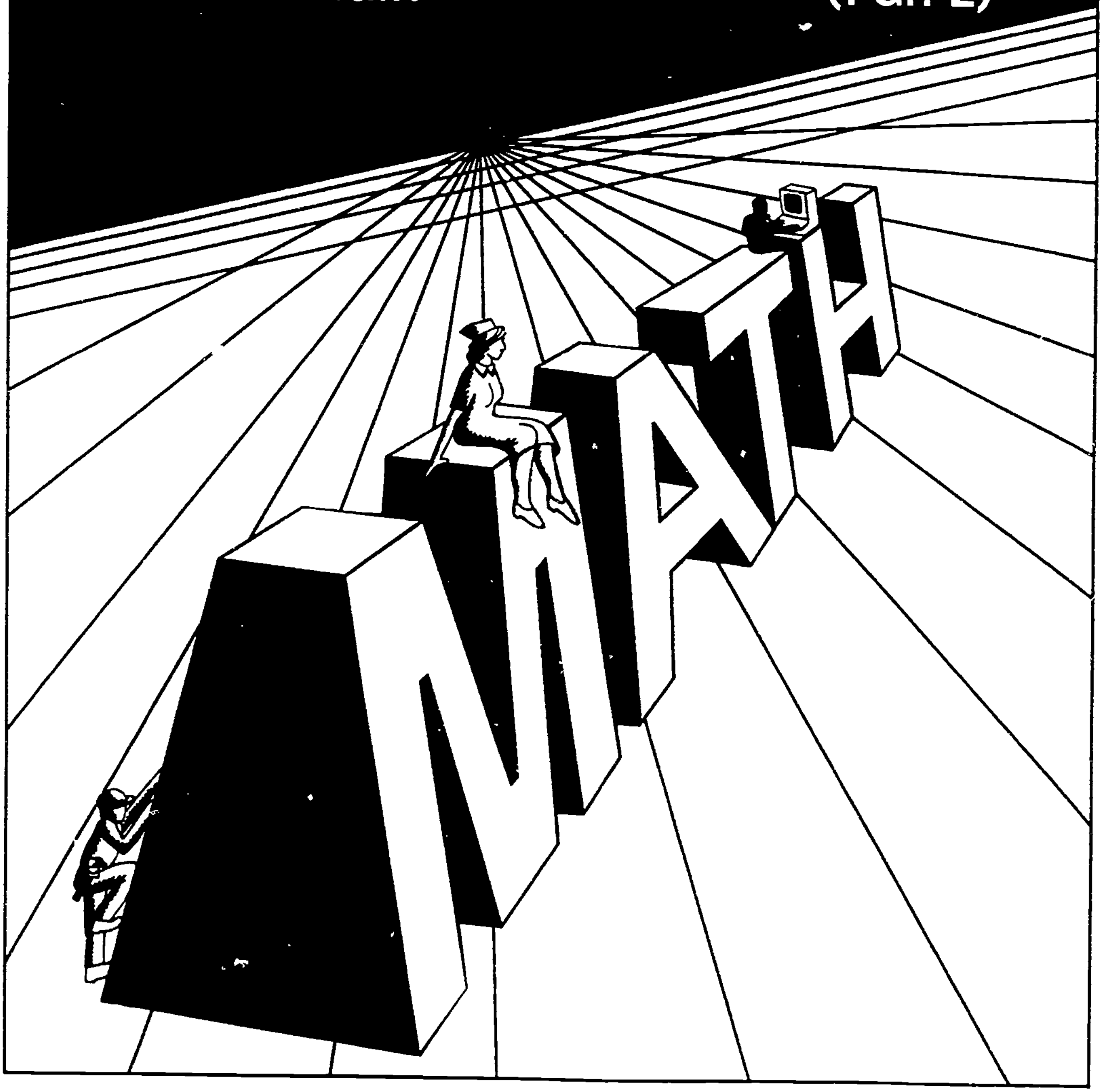
How would George rewrite the weights of Package 2 and Package 3?

1. Package 2 = _______ lb

2. Package 3 = _______ lb

# Module 2

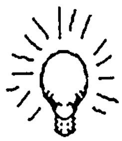
## Refresher Math (Part 2)



# Lesson 2.1
# Solving Word Problems

## Learning Objectives

*To develop a strategy for solving word problems and/or problems that arise in the workplace*



Just the mention of word problems is enough to make some people uncomfortable. In this lesson, we are going to give new meaning to this task by teaching a strategy that will give you confidence in solving these puzzles, mysteries, and challenges.

## Develop Your Understanding

Word problems are similar to instructions in that they must be read very carefully. Most people usually focus on the numbers instead of the "story." You must learn to *visualize* what is described in the problem, and then apply this four-step strategy.

1. Define the question.
2. Determine key information.
3. Determine mathematical operation or operations to be used.
4. After solving, check to see if your answer is reasonable.

### *Defining the Question*

Before you can define the question, you have to find it! In other words, make sure of what the problem is asking you to do before you try to solve it. What type of unit should be attached to the answer? Should it be in hours, ounces, number of people, or some other measure?

Most questions can be found at the end of each word problem, but sometimes the question does not have a question mark. For example, the question may be stated, "Find how much..."

### *Determining Key Information*

After finding the question, the next step involves separating the *key information* from unnecessary information. Some problems state more information than you need to solve the problem. Focus only on the information that is needed. The key information consists of the numbers and the words or symbols that are with the numbers. For example, if the shipping department ordered 12 boxes of tape at $3.00 a box and you were asked to find the total cost, you might analyze the problem as follows:

| | |
|---|---|
| QUESTION: | What is the cost of 12 boxes of tape? |
| KEY INFORMATION: | 12 boxes and $3.00 a box |
| ANSWER IN (UNITS): | dollars and cents |

### *Determining Mathematical Operation or Operations to Be Used*

The third step in solving a word problem is determining the mathematical operation or operations to be used. In order to do this effectively, you can find the key words, make diagrams, or do both. Listed below are some important key words for the four fundamental mathematical operations.

**ADDITION**

total
plus
add
sum
more
increase
altogether
both
combined
in all
additional

**SUBTRACTION**

difference
fewer
decreased
minus
reduce
left
remainder
any type of *-er* comparison

**MULTIPLICATION**

of
times
at
by
as much
product
per
twice

**DIVISION**

quotient
divided evenly
split
shared
out of
every

These key words can help you decide which operation to use. Sometimes, however, a key word may indicate the *opposite* operation. For example, in order to find the *increase* in something, you may have to *subtract*. Remember:

> Addition and subtraction are opposites.
> Multiplication and division are opposites.

### *Multiple-Step Operations*

In certain situations, you will need to use more than one operation. For instance, you may be asked to combine amounts and find the difference. This requires you to add and then subtract. Once again, you must visualize the problem and ask yourself, "How can I get there from here?"

### *Checking the Answer*

Once you arrive at an answer, check to see if it looks reasonable. In order to get an exact check, reverse the operations that led you to the answer and try to get the original number back. For example, if you used division and then addition to solve a problem, you would use subtraction and then multiplication to check it.

*Example 1*

Judy is reading a 324-page training manual. She has already read 112 pages. If she reads 53 pages a day, how many days will it take to finish reading the manual?

| | |
|---|---|
| QUESTION: | *How many days until Judy finishes?* |
| KEY INFORMATION: | *324 pages total; 112 pages already read; reading rate is 53 pages per day* |
| ANSWER IN (UNITS): | *days* |
| MATHEMATICAL OPERATIONS: | *subtraction, then division* |
| CHECK: | *multiplication, then addition* |

After filling out the chart, you might say to yourself: The number of pages in the book minus the number already read is the number of pages left to read. I need to know how many days it will take to read the remaining pages if Judy reads 53 pages per day. So, I need to **set up the operation** as follows:

| | |
|---|---|
| *Subtract:* | *Then divide:* |
| 324 − 112 = 212 | 212 ÷ 53 = 4 days |

**Check:**

*Multiply:*
53 pages × 4 (days) = 212

*Then add:*
212 + 112 = 324

**Now Try These**

a. Irene assembled parts for a special display at the HAK Computer Show. One part comes in four sections. Section A is 4.862 centimeters, Section B is 7.899 centimeters, Section C is 3.2 centimeters, and Section D is 0.3 centimeter. What is the total length when all four sections are combined?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

ANSWER: ____________________

b. Three-fifths $(\frac{3}{5})$ of the employees at XI signed up to contribute to United Help. How many employees out of 1,200 contributed?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

ANSWER: ____________________

c. The ABC Computer Co. has 892,000 employees working in 50 branches throughout the United States. If 21,000 employees are laid off, how many will remain?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

ANSWER:

d. 300 computer manuals were packed in 50 boxes. Joyce was told to divide the manuals evenly. How many manuals were put in each box?

QUESTION:

KEY INFORMATION:

ANSWER IN (UNITS):

OPERATION:

CHECK:

ANSWER:

## Skill Builders

*For each problem, look for the* ***key words.*** *Then, solve the problem and check it. Be sure to read* ***carefully.***

1. Lance works 6 hours for XI. On weekends he earns an extra $200.00 working on race cars. What other information do you need to find his total income?

2. The delivery truck driver recorded the following trips:

| | |
|---|---|
| Dripping Springs | 7.1 miles |
| Blanco | 16.4 miles |
| Johnson City | 15.8 miles |
| Wimberley | 12.9 miles |

What was the total mileage for the four trips?

3. T.J. uses 30 rolls of tape per week, and he must fill a requisition order for 6 weeks. Rolls of tape come in packages of 3. How many packages does he need to order?

4. Peggy planned to spend $6\frac{1}{2}$ hours organizing the kitting stockroom. She has worked $1\frac{3}{4}$ hours. How many hours does she have left to work?

5. In a memo, Joe was told to stack boards that were $\frac{1}{2}$" thick and 11' long. The stack should be 30" high. How many boards share the 30" stack?

6. UR2 Graphics gave a $250,000.00 bonus to their employees. The amount was shared evenly among 750 employees. Find the amount each employee received.

7. In 1990, the ABC Computers had sales of $2,143,021. For 1991, their sales totaled $4,000,021. What is the increase in sales?

8. The personnel office reported 60 employees absent from work on Friday and 90 on Monday. There are 1,200 employees. Which day had less than 0.06 of the total number of employees absent? Show all work.

9. The trimmer at UR2 Graphics was told to trim the Moreland Manual $\frac{1}{2}$". He was later told to take another $\frac{1}{4}$" off. The inspector was then told that another $\frac{1}{8}$" should be taken off. How much had been trimmed altogether *before* the inspector was told of the change?

10. Shakir was told to cut a 62" tape into three 18 $\frac{1}{3}$" strips. How much tape will be left?

11. Three-eighths ($\frac{3}{8}$) of the people on the Orange Line worked on Saturday. One-fourth ($\frac{1}{4}$) of the people on the Orange Line worked on Sunday. *What further information would you need* to find the total number of employees not working on these days?

12. UR2 Graphics gives an employment test that takes 2 hours. The math section takes $\frac{1}{5}$ of the time. How long is the math test?

13. John's supervisor told him to cut $1\frac{1}{16}$" off the flyer he was preparing for "Taco to Munch." The flyer was originally $10\frac{1}{4}$" long. How many inches were cut off? *(Caution: Read carefully.)*

14. In a union poll, $\frac{3}{5}$ of the 500 members agreed with Charles, and $\frac{2}{5}$ agreed with John. Find the number of people who agreed with John.

15. Laverne is ordering paper for the copying machine. Paper comes 500 sheets to a ream and 12 reams to a case. If a case costs $90.00, how much does the paper cost per sheet? (Be sure to round your answer to the nearest cent.)

16. Total prize money for the XI Big Bash Tennis Tournament was $93,950.00. Smash Wilson won first place and collected $33,725.00. The remaining money was divided equally among 5 other players who had the best records of the 214 entrants. How much did each of these other players receive?

17. Edwin was told to find the cost of replacing 18 windows at XI. If 3 windows can be installed in one hour, and each window costs $45.37, how much will it cost the company to replace the windows if the cost of labor is $9.00 per hour?

18. XI has a wall made up of 564 panels. It takes a window washer 5 minutes to clean each panel. How many days will it take 2 washers working 8-hour days to wash all the panels? Round off to the nearest whole number of days.

## Check Yourself

1. Make a diagram, picture, or mnemonic device to help you remember the four-step strategy for solving word problems.

2. Before each key word, write the symbol of the mathematical operation usually used when solving word problems. The first one is done for you.

| | | | |
|---|---|---|---|
| __+__ | total | ______ | split |
| ______ | of | ______ | combined |
| ______ | by | ______ | product |
| ______ | difference | ______ | shared |
| ______ | left | ______ | altogether |
| ______ | every | ______ | sum |

# Lesson 2.2
# Estimating and Averaging

## Learning Objectives

*To practice estimating answers using various operations*
*To practice the skill of averaging*

There are many times in the workplace when you add or subtract numbers or make some other calculation. It is always wise to estimate the answer, even when using a calculator. If a number were accidentally entered incorrectly, an estimate of the answer would quickly point out the problem and allow you to correct your figures before you acted on an incorrect answer. Averaging is another useful skill and is often used to track defect or scrap rates, production rates, or defects.

## Vocabulary

*averaging*—to measure the central tendency of a group of numbers; i.e., to find out what the midpoint is for a group of numbers.
*estimating*—to figure the general answer to mathematical operations by rounding numbers off to the nearest whole, tenths, or hundredths place, etc.

## Develop Your Understanding

Any time you are making a mathematical calculation, whether you are using a calculator or not, it is wise to estimate the answer. Numbers are often entered incorrectly into a calculator, or batteries may be running down, and an answer on the calculator can be wrong. Estimating is also useful in everyday life when buying something or expecting change back.

### *Estimating*

When estimating, you must first determine the place value. For example, if you are dealing with money, you may decide to round off to the nearest whole dollar.

Remember, to round a number, you look at the digit to the right of the place you wish to round to. Then:

1. For values of 4 or below, round *down* to 0. Drop all digits to the right.
2. For values of 5 or above, add 1 to the next place value to the left. Drop all digits to the right.

*Example 1*

Suppose the supply totals for each week of the last month were as follows:

$9.24
$10.75
$4.50
$5.49

Round each number to the nearest whole dollar in this manner:

$9.24
↑

Look at the digit to the right of the decimal. It is a 2, so according to the rule, leave the number as it is and simply drop the 2 and the 4. The number would then become $9.00.

For $10.75, look at the digit next to the decimal. It is a 7. Following the rule, add 1 to the whole number and drop the end digits. The number is now $11.00.

$4.50 becomes $5.00, and $5.49 becomes $5.00.

Take the estimates and add them:

$$\begin{array}{rr} & \$\ \ 9.00 \\ & \$\ 11.00 \\ & \$\ \ 5.00 \\ + & \$\ \ \underline{5.00} \\ & \$\ 30.00 \end{array}$$

*The total estimate is $30.00.*

The actual answer is:

$$\begin{array}{r} \$9.24 \\ 10.75 \\ 4.50 \\ +\ 5.49 \\ \hline \$29.98 \end{array}$$

*The estimate was only off by 2 cents.*

If you needed to estimate the combined width of panel layers produced on Monday's first run, you might have the following figures:

$$\begin{array}{r} 1.237 \\ 0.987 \\ 2.119 \\ 1.005 \end{array}$$

First, decide what place to round them to. In this case, it would be best to choose the tenths place. Next:

1. Look at the digit to the right of the tenths place.
2. If it is below 5, leave the number as it is and simply drop the end digits.
3. If it is a 5 or above, add 1 to the number and drop the end digits.

↓

For example, with the number 1.237, we look at the digit to the right of the 2 (which is in the tenths place). This number is a 3, so leave the number as it is and simply drop the digits to the right: drop the 37, and 1.237 becomes 1.2.

With 0.987, look at the number to the right of 0.9; it is an 8. Therefore, add 1 to the number 9 and drop the digits to the right:

$$\begin{array}{rl} 0.9 & \text{drop the 87} \\ +\ 1 & \\ \hline 1.0 & \text{So 0.987 becomes 1.00.} \end{array}$$

2.119 becomes 2.1. And 1.005 becomes 1.0.

Add your estimates as follows:

$$\begin{array}{rr} & 1.2 \\ & 1.0 \\ & 2.1 \\ & +1.0 \\ \hline \textit{The total estimate is:} & 5.3 \end{array}$$

The actual width for the combined layers is:

$$\begin{array}{r} 1.237 \\ 0.987 \\ 2.119 \\ \underline{+1.005} \\ \mathit{5.348} \end{array}$$

**Now Try These**

Estimate the following costs of plating chemicals for one day. Round to the nearest whole dollar.

| | | | | | |
|---|---|---|---|---|---|
| a. | $10.24 | b. | $9.23 | c. | $10.68 |
| | 15.79 | | 11.89 | | 9.34 |
| | 12.36 | | 12.45 | | 11.02 |
| | 15.05 | | 8.57 | | 9.99 |

The actual costs were:
a. $53.44
b. $42.14
c. $41.03

How close were your estimates?

### *Averaging*

Averaging is another important skill in the workplace. It is useful when looking at overall patterns of cost, production, temperature, or other measures. It gives you the midpoint of the numbers you are averaging. To average:

1. Add to get the total of the figures you are considering.
2. Then divide that total by the number of figures that you added.

*Example 2*

Suppose you want to know the average temperature of the plating bath on Monday morning. Add the temperatures taken each hou· and then divide by the number of temperatures.

| TIME | TEMPERATURE |
|---|---|
| 7:00 a.m. | 99° |
| 8:00 a.m. | 100° |
| 9:00 a.m. | 103° |
| 10:00 a.m. | 98° |
| 11:00 a.m. | 101° |

Add 99° + 100° + 103° + 98° + 101°, to get 501°.

Since you added 5 *temperatures* to get that total, *divide the total by 5:*

$$501° \div 5 = 100.2°$$

*100.2° was the average temperature* for the plating bath on Monday morning.

**Now Try These**

Find the average daily cost of supplies for the lamination area for one week.

| | |
|---|---|
| Monday: | $100.25 |
| Tuesday: | $57.50 |
| Wednesday: | $75.00 |
| Thursday: | $105.75 |
| Friday: | $200.50 |

If you estimated the average, what would you estimate it to be?

## Skill Builders

1. Estimate the total cost for these items: Round to the nearest whole dollar.

   $2.25
   10.99
   3.49
   5.59

2. Estimate the total production numbers by rounding to the nearest hundred. The following are the totals for each day: 127, 228, 109, 179, 188.

   Then calculate the actual total production, and compare with your estimate.

3. Your supervisor is concerned that the cost of supplies in your area has increased greatly in the last month. Previous monthly costs were about $200.00. Given the following supply costs for each week, what would you estimate is your current monthly supply spending? Round to the nearest whole dollar.

   | | |
   |---|---|
   | week 1 | $175.23 |
   | week 2 | 25.15 |
   | week 3 | 50.99 |
   | week 4 | 26.75 |

4. Your supervisor takes everyone in your area out to lunch. The cost for each person is as follows. Estimate the total bill *not including* tax and tip. (Round to the nearest dollar.)

$4.99 $3.50 $2.99 $3.25 $5.75

5. What is the average of the following numbers? Round *your answer* to the nearest hundredth.

23, 19, 18, 15, 20, 26

6. What is the average of these cost figures?

$11.12, $12.28, $10.60, $12.25, $13.75

7. What is the average number of pencils being used weekly if the numbers used for each of the last four weeks were 19, 9, 10, and 12?

8. The air temperature for the yellow room must be kept within a certain range. Lately, it may be going too far out of acceptable limits. Find the average temperature for the room for the past five days. Daily temperatures were: 69°, 72°, 67°, 73°, 74°.

9. What is the average daily production rate for your area for this week if the number of layers produced each day is as follows?
150, 25, 200, 78, 129

10. Mary went to the company store to purchase several items. They cost $29.99, $9.99, $4.49, $3.29, and $12.79. She handed $70.00 to the clerk. Estimate the total cost of the items, and then estimate how much change she should receive in return.

11. You enter the following numbers into a calculator and get the answer shown:

```
 104
 302
 459
 398
----
4012
```

What would you estimate the answer to be (to the nearest hundred)?

Were the numbers entered in the calculator correctly?

12. There is a daily average of 13.5 people in Area B each week, and they need two sets of safety gloves each day for each person. Estimate how many sets of safety gloves they need each week.

13. ABC Computers uses about 15.2 lb of dry chemicals and 10.7 lb of liquid chemicals each week on the plating line. Estimate the total amount of chemicals used on the line over a 4-week period.

14. If George was absent for the following number of days for each of the last 6 months, what is his average absentee rate for those 6 months?

    0, 4, 2, 8, 1, 3

15. The following temperature readings were taken from the machine in the lamination area during one week. Estimate the average temperature of the machine for this period to the nearest whole degree.

    200.51°, 175.25°, 224.90°, 208.62°, 100.45°

# Lesson 2.3
# Working with Ratios

## Learning Objectives

*To write ratios based on verbal or written instructions*
*To practice computing equivalent ratios*
*To solve word problems using ratios*

Ratios are methods of making numerical comparisons. They are useful in a variety of ways, from expanding the amount of liquid yielded by mixing chemicals to making accurate price comparisons. It is important to have an understanding of ratios before you move on to working with percents.

## Vocabulary

*ratio*—the ratio of one number to another is the first number divided by the second. A ratio is also a way of comparing two numbers or showing relationships between them.
*equivalent*—equal in value

## Develop Your Understanding

### *Writing Ratios*

A *ratio* is one number or amount divided by a second number or amount. Ratios can be written in three ways. For example, to show the ratio of 6 to 12, you can:

1. use a colon $6:12$
2. use a division symbol $6 \div 12$
3. use a fraction $\frac{6}{12}$

Often, when working with ratios, writing them as fractions is preferred. A ratio that is written as a fraction may be treated as any other fraction. Using a colon is also a very common way of writing ratios. Using a division symbol is seen less frequently.

*Example 1: What is the ratio of 8 to 10?*

Write the ratio as a fraction, with the first number given as the numerator (on the top), and the second number given as the denominator (on the bottom).

$\frac{8}{10}$ is the ratio of 8 to 10.

When a ratio shows a relationship using a unit of measurement such as miles, pounds, or gallons, it is best if both the numerator and the denominator are expressed in the same type of unit. This makes it easier to compare amounts.

*Example 2: Show the ratio of 3 inches to 1 foot.*

Here, you can convert feet to inches so that the unit of both numbers will be the same.

1 foot = 12 inches

The ratio of 3 inches to 1 foot is the same as the ratio of 3 inches to 12 inches, which is:

$$\frac{3}{12}$$

**Now Try These**

Write each of these ratios as a *fraction*. Both numbers in the ratio must be in the same type of unit.

a. 3 pounds to 17 pounds

b. 2 inches to 5 inches

c. 180 miles to 55 miles

d. 45 minutes to 1 hour

e. 2 pounds to 9 ounces (*Note:* 1 pound = 16 ounces)

### *Putting Ratios in Lowest Terms*

Fractions are most properly expressed in lowest terms. The same is true of ratios.

*Example 3: Reduce the ratio* $\frac{12}{16}$ *to lowest terms.*

First, you must find the greatest common factor (GCF) of 12 and 16. The GCF is the largest number that can be divided evenly into both parts of the fraction.

The greatest number that can be divided into both 12 and 16 is 4.

Next, divide both the numerator and the denominator of the fraction by 4 to obtain an *equivalent fraction:*

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

The fraction $\frac{3}{4}$ is the same as the fraction $\frac{12}{16}$ expressed in lowest terms. Therefore, we can say that $\frac{12}{16}$ and $\frac{3}{4}$ are *equivalent ratios.*

> *Ratios that are written as improper fractions, where the numerator is larger than the denominator, are NOT usually rewritten as mixed numbers.*

**Now Try These**

Put each of these ratios in lowest terms:

a. 28:14

b. $\frac{6}{30}$

c. $10 \div 25$

### *Calculating Equivalent Ratios*

In the section above, you learned how to find one kind of equivalent ratio by putting a ratio into lowest terms. Each ratio has many possible equivalent ratios. All that is needed to find an equivalent ratio is to use multiplication or division to perform the same operation on both the numerator and denominator.

**Caution: DO NOT use addition or subtraction to calculate equivalent ratios!**

*Example 4: Calculate five equivalent ratios for the ratio* $\frac{4}{6}$.

1. First, put the $\frac{4}{6}$ into lowest terms by dividing each part of the ratio by 2:

   a. $\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

2. Next, use multiplication to calculate 4 more equivalent ratios:

   b. $\frac{4 \times 2}{6 \times 2} = \frac{8}{12}$

   c. $\frac{4 \times 3}{6 \times 3} = \frac{12}{18}$

   d. $\frac{4 \times 4}{6 \times 4} = \frac{16}{24}$

   e. $\frac{4 \times 5}{6 \times 5} = \frac{20}{30}$

Finally, we have 6 ratios: $\frac{4}{6}$ and 5 equivalent ones:

$$\frac{2}{3} = \mathbf{\frac{4}{6}} = \frac{8}{12} = \frac{12}{18} = \frac{16}{24} = \frac{20}{30}$$

## Now Try These

Calculate any *two* equivalent ratios for each ratio given.

a. $\frac{5}{6}$

b. $\frac{10}{8}$

c. $\frac{3}{6}$

### *Solving Problems with Ratios*

You can solve word problems using ratios. Following these steps:

1. Write the ratio *in words* first. *Do not omit this step!*
2. Find numbers to substitute for the words in the ratio. Sometimes, you will have to do calculations to find the numbers.
3. Calculate an equivalent ratio that solves the problem.
4. Put the answer in lowest terms.

*Example 5*

*Employees working on an assembly line produced 800 panels last week. Of the total number of panels produced, 24 were defective. What is the ratio of defective to nondefective panels produced?*

Begin by asking yourself: *What ratio do I need?* Then write the ratio in words:

$$\frac{\textit{defective panels}}{\textit{nondefective panels}}$$

In order to write the ratio in numbers, you will need to know how many of the 800 panels produced were NOT defective. You can find this number by subtracting the number of defective panels from the total number produced:

800 total panels − 24 defective panels = 776 nondefective panels

Now you have both of the numbers you need. The ratio of defective to nondefective panels is 24:776 or $\frac{24}{776}$:

$$\frac{\textit{defective panels}}{\textit{nondefective panels}} = \frac{24}{776}$$

Since both 24 and 776 may be divided by 8, this ratio may be most simply expressed as follows:

$$\frac{24 \div 8}{776 \div 8} = \frac{3}{97}$$

*Example 6*

*In order to make 2 servings of oatmeal, you must use 1 cup of oats and 2 cups of water. How much of each ingredient would you need to make 8 servings of oatmeal?*

First, write the ratio of oats to water in the recipe for 2 servings:

$$\frac{\textit{oats}}{\textit{water}} = \frac{1}{2}$$

Next, calculate an equivalent ratio that will supply enough oatmeal for 8 servings. Since the first ratio gives enough oatmeal for 2 servings, divide 8 by 2 to get the number of 2-serving recipes you would need to make 8 servings:

8 ÷ 2 = 4. Therefore, you will need *4 recipes of oatmeal to get 8 servings.*

Multiply both parts of the ratio $\frac{1}{2}$ by 4 in order to obtain an equivalent ratio.

$$\frac{\textit{oats}}{\textit{water}} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

You will need *4 cups of oats* and *8 cups of water* to make 8 servings of oatmeal.

*Example 7*

When more than two ingredients or components are involved, ratios may still be used.

*In order to mix a batch of chemicals, use 1 part Chemical A to 3 parts Chemical B to 2 parts Chemical C. If you are using 6 gallons of Chemical B, how many gallons of Chemical C should you use?*

The relationship of the three chemicals can be shown as a three-way ratio:

The ratio of Chemical A to Chemical B to Chemical C is 1:3:2.

Here, we are only concerned with Chemicals B and C. The ratio of B to C is 3:2. Write this ratio as a fraction:

$$\frac{\textit{Chemical B} =}{\textit{Chemical C} =} \quad \frac{3}{2}$$

Next, you must find an equivalent ratio where the amount of Chemical B is equal to 6.

$$\frac{3}{2} = \frac{6}{?}$$

Find the number that 3 must be multiplied by to get 6. You can do this by dividing 6 by 3:

$$6 \div 3 = 2$$

Next, multiply the denominator, 2, by this number to get the number of gallons that will give you an equivalent ratio:

$$\frac{3 \times 2}{2 \times 2} = \frac{6}{4}$$

$$\frac{\textit{Chemical B} =}{\textit{Chemical C} =} \quad \frac{6}{4}$$

Therefore, you will need *4 gallons of Chemical C.*

## Skill Builders

### *Skill Practice*

*Write each ratio in three ways, using a colon, division symbol, and fraction.*

1. You have 5 apples and 3 oranges. Write the ratio of *oranges to apples.*

2. There is one secretary at the plant for every 18 production workers. Write the ratio of *secretaries to production workers.*

3. Yesterday, Fred worked on 25 panels that were good and 3 that were defective. Give the ratio of *defective panels to total panels worked.*

*Write the following ratios as fractions. Use the same units for both parts of the fractions.*

4. 46 to 1

5. 8 days to 1 week

6. 2 pounds to 32 ounces

7. 2.5 inches to 5 inches

8. Reduce each of these ratios to lowest terms.

   a. $\frac{6}{9}$　　b. $\frac{24}{8}$　　c. $\frac{15}{36}$

9. Replace each question mark to complete an equivalent ratio for each ratio shown:

a. $\frac{2}{6} = \frac{?}{3}$

b. $\frac{7}{21} = \frac{1}{?}$

c. $\frac{30}{?} = \frac{15}{4}$

10. Compute 5 equivalent ratios for the ratio 3:1.

***Word Problems***

*Write each ratio in words; then, solve the problems. Reduce all ratios to lowest terms in your answers.*

11. A certain plant has 1,230 employees, *including* 210 managers. What is the ratio of *managers to other employees* at the plant?

*Use the chart below to answer questions 12 -14.*

The following chart shows how much of each ingredient is required in order to make a batch of 12 pancakes:

| Pancake Mix | Milk | Eggs | Yield |
|---|---|---|---|
| 1 cup | $\frac{1}{2}$ cup | 1 | 12 pancakes |

12. How many eggs would you use if 2 cups of milk were used?

13. What is the ratio of *pancake mix to milk?* Be sure that both parts of the ratio are whole numbers.

14. What is the ratio of *pancakes to eggs* in the recipe?

15. A 50-pound bag of powder costs $15, while a 75-pound bag of the same powder costs $25. What is the ratio of *cost to weight* in each case? Are the two ratios equivalent?

16. Out of a total of 2,250 employees at Gulpo Drink Company's Florida plant, 25 are currently on leaves of absence (LOA). What is the ratio of *total employees to employees on LOA?*

17. ABC Computers has a goal of achieving a ratio of *6 defects for every 1,000 computer chips produced.* Out of a sample of 500 chips inspected, 4 were defective. Did ABC achieve its goal?

18. A new printing company had expenses of $68,000 in its first month and income of $204,000. Find the ratio of *expenses to income* for the company's first month.

19. A mixture of paint has a ratio of *two parts of yellow paint to 4 parts of blue.* If a particular batch of this paint has 8 gallons of yellow paint, how much blue paint is in the batch?

20. One pound of laundry soap sells for $2.40. Find the *cost per ounce* and write this as a ratio with cost as the numerator and ounces as the denominator. (*Hint:* 1 pound = 16 ounces.)

## Check Yourself

Plink Industries has a rule that each employee should have at least *1 day of training for every 6 months worked.* Which of these employees has not had enough training, according to this rule? Use equivalent ratios to solve this problem.

**Clarabelle** has worked for the company for 12 years and has had 25 days of training.

**Roderick** has worked for the company for 6 years and 3 months and has had 8 days of training.

**Grace** has worked for the company for 2 years and 2 months and has had 5 days of training.

# Lesson 2.4
# Working with Proportions

## Learning Objectives

*To practice solving simple proportions*
*To set up and solve word problems using the concept of proportions*
*To practice calculating unit cost*

In the previous lesson, you learned how to set up ratios and use them to solve problems. In this lesson, you will learn how to use two ratios to compare costs or amounts.

## Vocabulary

*proportion*—a statement that two ratios are equivalent.
*extremes*—the two outside numbers in a proportion. In the proportion a:b = c:d, *a* and *d* are the extremes.
*means*—the two inside numbers in a proportion. In the proportion a:b = c:d, *b* and *c* are the means.
*cross-product*—in a proportion, the cross-product is the product of the extremes or the product of the means.

## Develop Your Understanding

### *Proportions*

Any statement that two ratios are equal or equivalent is called a *proportion*. When you found equivalent ratios in the last lesson, you were creating proportions.

$$\frac{2}{5} = \frac{4}{10}$$

Since the two ratios are equivalent, the above statement is a proportion.

> *An important property of proportions is that the* **cross-products** *of any proportion will always be equal.*

To find the cross-products in a proportion where the ratios are written as fractions:

1. Multiply the *numerator* of the first ratio by the *denominator* of the second ratio.
2. Multiply the *denominator* of the first ratio by the *numerator* of the second ratio.

*Example 1*

$$\frac{2}{5} = \frac{4}{10} \qquad \frac{2}{5} \quad \frac{4}{10}$$

The first cross-product is the numerator of the first ratio (2) times the denominator of the second ratio (10).

$$2 \times 10 = 20$$

The second cross-product is the denominator of the first ratio (5) times the numerator of the second ratio (4).

$$5 \times 4 = 20$$

Note that the two cross-products are equal.

$$20 = 20$$

Another way to find the cross-products is to write the proportion using colons.

*Example 2*

The proportion $\frac{2}{5} = \frac{4}{10}$ can also be written:

$$2:5 = 4:10.$$

Here, the two numbers on the outside (2 and 10) are called the *extremes*, while the two numbers on the inside (5 and 4) are called the *means*. To find the cross-products:

1. Find the product of the two extremes.
2. Find the product of the two means.

Extremes: $2 \times 10 = 20$

Means: $4 \times 5 = 20$

Once again, the two cross-products are equal.

$20 = 20$

**Now Try These**

Find the cross-products for each proportion:

a. $\frac{1}{6} = \frac{2}{12}$

b. $\frac{3}{11} = \frac{6}{22}$

c. $\frac{7}{6} = \frac{21}{18}$

d. 3:1 = 9:3

e. 4:2 = 16:8

### *Solving Proportions*

Knowing that the cross-products of any proportion will be equal, you can find a missing number in a pr' portion. As long as you have one ratio and half of the other ratio, you can use cross-products to complete the proportion.

*To solve a proportion:*

1. Set the first ratio equal to the second.
2. Use a letter such as "N" (for "number") to take the place of an unknown number.
3. Find the cross-products of the ratios and set them up as an equation. To multiply a number by "N," simply write the number next to N. $(4 \times N = 4N)$
4. Divide both cross products by the number next to "N." You will have the value of "N" by itself. $(4N \div 4 = N)$

*Example 3*

*Find an equivalent ratio for* $\frac{2}{8}$ *that will have a denominator of 16.*

**Steps 1 and 2:** Set up the first ratio as equal to the second. Use N to represent the unknown number.

$$\frac{2}{8} = \frac{N}{16}$$

**Step 3:** Find the two cross-products:

*(Multiply the first numerator by the second denominator.)*

$2 \times 16 = 32$

*(Multiply the first denominator by the second numerator.)*

$8 \times N = 8N$

*Write the product of 8 and N as "8N," which is a shorter way of writing "8 × N."*

Now, write the two cross-products as equal:

$32 = 8N$

**Step 4:** Divide both of the cross-products by 8 to get the answer:

$32 = 8N$

$\frac{32}{8} = \frac{\cancel{8}N}{\cancel{8}}$ *The two 8s on the right cancel each other out.*

$4 = N$ *Substitute "4" for "N":* $\frac{2}{8} = \frac{4}{16}$

To check your answer, find the cross-products and make sure they are equal.

## Now Try These

Use cross-products to solve for the unknown value in each proportion.

a. $\frac{7}{2} = \frac{N}{6}$

b. 1:1 = 32:N

c. $\frac{15}{N} = \frac{30}{50}$

d. N:18 = 1:9

### *Word Problems with Proportions*

Proportions can be used to compare amounts.

*Example 4*

*Suppose your car gets 50 miles to 2 gallons of gas. You would like to know how many miles you could drive with 8 gallons of gas.*

**Steps 1 and 2:** You know that your basic ratio of miles per gallon is 50 miles: 2 gallons. You would write this as the first ratio in your proportion:

50:2 =

Next, you would like to know how many miles you could drive with 8 gallons of gas. *Put the number of gallons in the same position in both ratios.* Since the number of gallons is the second number in the ratio you have already written, it belongs in the same place in the new ratio:

50:2 = N:8 (where N stands for the number of miles)

**Step 3:** Now, find the two cross-products by multiplying extremes and means.

Extremes: $50 \times 8 = 400$
Means: $2 \times N = 2N$

Set the first cross-products equal to the second:

$$400 = 2N$$

**Step 4:** Divide both cross-products by 2 to find the value of N:

$$400 \div 2 = 2N \div 2$$

$$200 = N$$

The value of N is 200. Therefore, *you could drive 200 miles on 8 gallons of gas.*

**Now Try These**

a. You need 120 square feet of office space for every 4 employees. How many square feet would you need for 5 employees?

b. Last Tuesday at the Perfect Printing Company, the ratio of perfect books produced to flawed books produced was 15:1. If the total number of flawed books was 7, how many perfect books did Perfect Printing produce last Tuesday?

### *Finding the Best Buy*

Have you noticed that many stores now show "unit prices" that help you compare the price per ounce (or pound, gallon, etc.) of certain products? You can use ratios and proportions to calculate price comparisons on your own:

1. Write ratios with price on the top and units on the bottom.
2. Divide the price by the number of units for each ratio.
3. The ratio with the smallest quotient is the one with the cheapest unit cost.

*Example 5*

*You need to order more pencils for your department. You can get a box of 250 Zappo pencils for $12.75 or a box of 150 EZ Rite pencils for $7.05. Which box is a better buy?*

**Step 1:** In each case, the "unit cost" you will need to compare is the price of *one* pencil. In order to do this, write two ratios of cost to number of pencils:

Zappos: $\dfrac{\textit{total cost}}{\textit{\# of pencils}} = \dfrac{\$12.75}{250}$

EZ Rites: $\dfrac{\textit{total cost}}{\textit{\# of pencils}} = \dfrac{\$\,7.05}{150}$

**Step 2:** Solve each ratio by dividing the total number of pencils into the total cost:

Zappos: $12.75 ÷ 250 = $0.051

*One of the Zappo pencils is 5.1 cents.*

EZ Rites: $7.05 ÷ 150 = $0.047

*One of the EZ Rite pencils is 4.7 cents.*

*Since one EZ Rite pencil is cheaper than one Zappo, EZ Rites are the better buy.*

## Skill Builders

### *Skill Practice*

1. The cross-products of proportions must be equal. Mark an "X" next to any of the following proportions that are *incorrect*:

   a. $\dfrac{1}{2} = \dfrac{85}{164}$

b. $\frac{6}{8} = \frac{18}{24}$

c. $\frac{3}{1} = \frac{225}{75}$

d. 12:2 = 96:8

e. 1:17 = 34:2

f. 17:1 = 34:2

*Find the cross-products of the proportions in problems 2 through 5:*

2. 25:1 = 250:10

3. $\frac{6}{9} = \frac{2}{3}$

4. 1.5:3 = 5:10

5. $\frac{11}{3} = \frac{33}{9}$

*Solve for the missing value in problems 6 through 10:*

6. 18:3 = 36:N

7. $\frac{3}{13} = \frac{12}{N}$

8. $2.5:1 = 50:N$

9. $\frac{12}{144} = \frac{N}{12}$

10. $\frac{8}{N} = \frac{40}{25}$

***Word Problems***

11. It takes you 3 days to complete a project. How many projects can you complete in 9 days?

12. You need 5 gallons of paint to paint two rooms. How many gallons of paint will you need to paint 13 rooms of the same size?

13. If 2 inches on a map represent 150 miles, how many inches will represent 500 miles?

14. Denise needs to hire 7 temporary employees for each special project in her department. Right now, she has 53 temporary employees, and she needs to hire 3 more. How many special projects does Denise's department have?

15. Blotto brand typing paper costs $1.80 for 75 sheets.

    a. What is the cost of one sheet?

    b. Typo brand costs $2.34 for 90 sheets. Is it a better buy than Blotto?

16. Out of every $100 spent on employee benefits at Plink Industries, $54 goes toward health care. If Plink spent a total of $70,000 on benefits last year, how much of this total went toward health care?

17. The Spendit Store has a special on canned yams. Normally, you can purchase a one-pound can of yams for $1.20. This week, you can buy a can of yams that weighs 2 pounds, 4 ounces for only $2.70. What is the difference in unit price? (16 ounces = 1 pound)

# Lesson 2.5
# Converting Fractions and Decimals into Percents

## Learning Objectives

*To practice changing fractions and decimals into percents and back again*

Solving problems with percents has many everyday applications in business, the workplace, and at home. To solve percent problems, you will need to convert fractions and decimals into percents and back again.

## Vocabulary

*percent*—percent means "parts of a hundred."

For example, 15% is 15 parts out of 100 parts. A percent can be written in fraction, ratio, and decimal form. A percent is usually expressed by writing a percent sign (%). You can write:

$$15\% = 15 \text{ hundredths} = \frac{15}{100} = 0.15$$

## Develop Your Understanding

### *Converting Percents to Decimals*

When solving percent application problems, you will often need to convert a percent into a workable form. One of the easiest ways to do this is to change a percent to a decimal.

To convert a percent to a decimal, drop the percent sign and move the decimal point two places to the left. Remember that a whole number has an invisible decimal point on its right side.

*Example 1*

45% = 45 = 0.45

*Example 2*

0.2% = 002 = 0.002

*Example 3*

100% = 1 00 = 1.

Note that the zeros after the decimal point are no longer needed after converting 100% to one.

**Now Try These**

Convert these percents to decimals.

a. 28% b. 0.6% c. 200%

***Converting Decimals to Percents***

To convert a decimal back to a percent, move the decimal point two places to the right and add a percent sign.

*Example 4*

0.56 = 5 6 = 56%

*Example 5*

0.05 = 0 5 = 5%

*Example 6*

1.2 = 1 2 0 = 120%

Note that you must add a zero to show place value when changing 1.2 to 120%.

## Now Try These

Convert these decimals to percents.

a. 0.36 = b. 0.09 = c. 4.3 =

### *Converting Fractions to Percents*

You will sometimes convert fractions to percents when solving percent problems. You can do this by converting the fraction to a decimal and then changing the decimal to a percent.

To convert a fraction to a decimal, divide the numerator by the denominator. Then convert the decimal quotient to a percent by moving the decimal point two places to the right and adding a percent sign.

*Example 7*

Convert $\frac{1}{4}$ to a percent by dividing the numerator by the denominator. You read the fraction as "one divided by four."

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{-8\phantom{0}} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Next, convert the decimal to a percent.

$$0.25 = 25 = 25\%$$

## Now Try These

Convert these fractions to percents.

a. $\frac{1}{5} =$ b. $\frac{3}{4} =$ c. $\frac{3}{8} =$

### *Converting Percents to Fractions*

To convert a percent to a fraction, start by writing the percent as a decimal. Then convert the decimal to a fraction. Remember you can reduce the fraction to lowest terms by dividing the numerator and the denominator by the greatest common factor (GCF).

*Example 8*

$$72\% = 0.72 = \frac{72}{100} = \frac{18}{25}$$

In example 8 you would say, "seventy-two percent is equal to seventy-two hundredths (decimal form) is equal to seventy-two hundredths (fraction form) is equal to eighteen twenty-fifths (reduced fraction form)."

*Example 9*

$$4\% = 0.04 = \frac{4}{100} = \frac{1}{25}$$

*Example 10*

$$125\% = 1.25 = \frac{125}{100} = 1\,\frac{25}{100} = 1\,\frac{1}{4}$$

*Example 11*

$$2.5\% = 0.025 = \frac{25}{1000} = \frac{1}{40}$$

Note that the decimal 0.025 is read "25 thousandths," and the fraction must be written with a denominator of 1,000.

*The number of zeros in the denominator of the fraction matches the number of decimal places in the decimal version of the number.*

**Now Try These**

Convert these percents to fractions.

a. 95% = b. 8% = c. 215% = d. 3.4% =

## Skill Builders

### *Skill Practice*

*Convert these percents to decimals.*

1. 35% =
2. 7% =
3. 105% =
4. 0.9% =
5. 1.5% =

*Convert these decimals to percents.*

6. 0.46 =
7. 0.02 =
8. 1.29 =
9. 0.375 =
10. 0.085 =

*Convert these fractions to percents using the two-step process you have learned.*

11. $\frac{1}{2}$ =
12. $\frac{4}{25}$ =
13. $\frac{7}{10}$ =
14. $\frac{1}{8}$ =
15. $\frac{7}{8}$ =

*Convert these percents to fractions using the two-step process you have learned.*

16. 20% =

17. 8% =

18. 175% =

19. 9.8% =

20. 10.9% =

### *Word Problems*

*Solve these word problems by converting fractions, decimals, and percents. Convert fractions to lowest terms in your answers.*

21. Della trains at the company gym 3 out of every 5 working days. What percent of her workweek does she train?

22. A papercutter's work order required that he add $\frac{3}{16}$ inch to job specifications. Change $\frac{3}{16}$ of an inch to a decimal measurement.

23. The label on Rodney's shirt says it is 80% cotton. What fraction of the shirt is cotton?

24. If a microchip production plant inspects 1 out of every 3 chips during final inspection, what percent is inspected?

25. Eighty-eight percent (88%) of the employees at a plant had taken their vacation leave before December. What fraction of the employees had taken leave before December?

26. A city's sales tax is $0.06 for every dollar. Write that tax rate as a percent.

27. In a recent graphics order, 5 out of every 200 forms were found to be defective. What percent was defective?

28. A company found that 0.42 of its employees had been with the company ten years or longer. What fraction of the employees had worked there ten years or more?

# Lesson 2.6
# Solving Percent Problems

## Learning Objectives

*To practice finding a percentage of a number*
*To practice finding the percentage one number is of another number*
*To practice finding a number when a percentage of it is given*

Solving percent problems is one of the most valuable math tools you can develop. You can calculate interest on loans and bank accounts, and figure taxes and discounts. You can apply percentage calculations to manufacturing production and quality control measurements. In this lesson, you will solve the three kinds of percent problems.

## Develop Your Understanding

In the previous lesson, you practiced converting fractions and decimals into percents and back again. Now you will apply these conversion skills to solve percent problems.

### *How to Look at Percent Problems*

Any percent problem has three elements:

the *whole*, the *part*, and the *percent*.

For example, in the following statement

*Twenty percent of 200 people is 40 people:*

200 people is the *whole*, or the total amount;
40 people is the *part*, or the portion involved; and
20% is the *percent*.

Whenever you solve a percent problem, you will be given two of these elements, and will have to find the third. You will learn in detail how to solve problems in this lesson. The chart below is a handy reference you can keep as a reminder:

| **When you know...** | **and...** | **In order to find...** | **You should:** |
|---|---|---|---|
| the whole | the percent | the part | *multiply* the whole by the percent (in decimal form) |
| the whole | the part | the percent | *divide* the part by the whole: $\frac{\text{part}}{\text{whole}}$ |
| the part | the percent | the whole | *divide* the part by the percent (in decimal form): $\frac{\text{part}}{\text{percent}}$ |

This chart may be confusing at first. If so, don't try to understand it now. Refer back to the chart after you finish each major section of the lesson, and it will make more sense.

### *Finding the Percentage of a Number*

Some problems give you a number and ask you to find a certain percentage of that number. To find the percentage of a number

1. follow the steps for changing a percent to a decimal;
2. multiply the decimal by the number.

*Note: You will know the **whole** and the **percent**; you are asked to find the **part**.*

*Example 1: What is 15% of 60?*

**Step 1:** Convert 15% to a decimal:

15% = 15. = 0.15

**Step 2:** Multiply 60 by the decimal:

$$\begin{array}{r} 60 \\ \underline{\times\ 0.15} \\ 300 \\ \underline{60\phantom{0}} \\ 9.00 \end{array}$$

*The product is 9.*

**Conclusion:** 15% of 60 is 9. In this problem, 60 is the *whole*, 15% is the *percent*, and 9 is the *part*.

Note that "15% of 60" means 15% multiplied by 60. When you see the words *"percent of,"* it means that you should multiply.

**Now Try These**

a. What is 25% of 84?

b. What is 54% of 16?

### *Finding the Percentage One Number Is of Another Number*

Other types of percent problems give you two numbers and ask you to find what percent the first number is of the second. To find the percent one number is of another

1. make a fraction with the *part* over the *whole*; write the *part* as the numerator and the *whole* as the denominator;
2. convert the fraction to a decimal by dividing;
3. change the decimal quotient to a percent.

*Note: You will know the* **whole** *and the* **part**; *you are asked to find the* ***percent***.

*Example 2: 3 is what percent of 4?*

**Step 1:** Make a fraction with the *part* over the whole.

$$\frac{3}{4}$$

**Step 2:** Convert the fraction to a decimal by dividing.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{-28\phantom{0}} \\ 20 \\ \underline{-20} \\ 00 \end{array}$$

**Step 3:** Change the decimal to a percent.

0.75 = 75%

**Conclusion:** Thus, 3 is 75% of 4. In this problem, 4 is the *whole*, 75% is the *percent*, and 3 is the *part*.

**Now Try These**

a. 4 is what percent of 5?

b. 15 is what percent of 75?

### *Finding a Number When a Percent of It Is Given*

In the third type of percent problem, you are given the portion that represents a certain percentage of a number and asked to find the original number. To find a number when a percent of it is given

1. convert the percent to a decimal;
2. divide the given number by the decimal.

*Note: You will know the* ***part*** *and the* ***percent****. You are asked to find the* ***whole****.*

*Example 3: 34 is 40% of what number?*

**Step 1:** Convert the percent to a decimal.

40% = 0.40 = 0.4

**Step 2:** Divide the given number by the decimal.

$0.4\overline{)34.0}$ is equivalent to

$$\begin{array}{r} 85 \\ 4\overline{)340} \\ \underline{-32} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

**Conclusion:** Thus, 34 is 40% of 85. In this problem, 85 is the *whole*, 40% is the *percent*, and 34 is the *part*.

Note that in this problem, you knew that 34 was a part of an unknown number. Using division, you found that it was a part of 85.

## Now Try These

a. 120 is 80% of what number?

b. 6 is 10% of what number?

*Review the chart at the beginning of this lesson before you do the Skill Builders section.*

## Skill Builders

### *Skill Practice*

*Find the percent asked for in each problem.*

1. What is 25% of 80?

2. What is 75% of 40?

3. What is 2.5% of 12?

*Find the percent one number is of another number in each of these problems.*

4. 3 is what percent of 5?

5. 28 is what percent of 40?

6. 1 is what percent of 3?

*Find a number when a percent of it is given for each problem.*

7. 4 is 10% of what number?

8. 200 is 80% of what number?

9. 18 is 40% of what number?

Use your skills in calculating percents to solve the following manufacturing production problems. Units can represent any product, such as books or computer parts.

*Find the number of units that passed inspection in each case below.*

10. 98% of 50 units = _____ good units

11. 85% of 980 units = _____ good units

12. 99.9% of 1,000 units = _____ good units

*Find the percentage of good units in each ratio below. The numerator is the number of good units, and the denominator is the total number of units.*

13. $\frac{36}{48}$ = _____% good units

14. $\frac{750}{800}$ = _____% good units

15. $\frac{1950}{2000}$ = _____% good units

*Find the total number of units in each case below.*

16. 24 is 60% of _____ total units

17. 100 is 80% of _____total units

18. 14 is 87.5% of _____ total units

### *Word Problems*

19. A $10,000 car is purchased with a 12% down payment. How much is the down payment?

20. You and a friend spend $22.20 dining at a restaurant. How much will you tip the waiter if you tip 15%?

21. Linda's take home pay after deductions is $1,230 per month. Her net income is 75% of her gross income. What is her total monthly income before taxes?

22. A company spends 2.5% of its $1,000,000 dollar budget on rent. How much does it spend on rent?

23. The job specifications tell you to cut the furnace temperature by 14%. The furnace is set at 959°F. How much will the temperature decrease? What will the new temperature be?

# Module 2 Review #1
# Lessons 2.1-2.6

Before answering the following questions, you may wish to review the key concepts in Lessons 2.1 - 2.6. This may be done by reading over the lessons, following a review done by your instructor, or working in groups to discuss the key facts of each lesson.

After this, try to solve each of the following without looking back at the lessons.

1. List three key things you should do first when solving word problems.

2. If $\frac{2}{5}$ of 5,000 employees at XYI Company contribute to an IRA, how many total employees contribute to an IRA?

3. The current workforce at the Dallas, Texas, plant numbers 6,000. It is projected to grow by 15% in the coming year. How many new employees will be hired or will transfer to the Dallas plant?

4. The scrap rate for the Blue Line in January was 17%. The total number of computer boards produced was 15,690. How many boards were defective and had to be scrapped?

5. Estimate the following department costs, rounding to the nearest dollar:

$52.99
$23.40
$12.50
$72.35
$29.19

6. If the scrap rates were as follows for last week, what was the average?

| | |
|---|---|
| Monday: | 12 |
| Tuesday: | 32 |
| Wednesday: | 8 |
| Thursday: | 9 |
| Friday: | 11 |

7. Estimate the total of the following bill:

$12.29
22.29
31.89
16.38
15.99

8. What is the average downtime (in days) for the drill machine if the following totals were turned in:

Week 1: down 2 out of 5 days
Week 2: down 0 out of 5 days
Week 3: down 3 out of 5 days
Week 4: down 1 out of 5 days

9. A recipe calls for $\frac{3}{4}$ cup of water to make $\frac{1}{2}$ jar of paste. How much water would you use to make 2 jars of paste?

10. $\frac{15}{30} = \frac{3}{B}$ What is the value of B?

11. A chemical add formula calls for:

3 parts chemical A
2 parts chemical B
4 parts chemical C

If you must add 4 parts chemical B, how many parts of chemical A and chemical C do you add to maintain your chemical balance?

12. If you get 12 gallons of gas for $18, how much would you get for $15?

13. $\frac{N}{24} = \frac{3}{8}$ What is the value of N?

14. $\frac{6}{x} = \frac{20}{30}$ What is the value of x?

15. 8% of 280 is how many?

16. $\frac{3}{20}$ = what percent?

17. Of 1,625 boards, 4% had defects. How many had defects?

18. If your department has a total budget of $8,000, and $1,200 is spent on supplies, what percent is spent on supplies?

19. Convert each of the following to a percent:

$\frac{1}{2}$

$\frac{4}{5}$

20

0.20

0.02

20. Convert the following to decimals:

40%
4%
400%
37.5%

21. Convert the following to fractions:

2%
35%
0.50
0.24

# Lesson 2.7
# Standard Measurement (Part 1)

## Learning Objectives

*To become familiar with standard units of measure and their uses*
*To practice adding and subtracting measurements*
*To practice converting between different units of measurement*

Many types of measurement are used in the workplace. Different units are used to measure length, weight, volume or capacity (dry or liquid), temperature, time, and so on. Understanding these units of measure and their relationship to each other can enhance your ability to carry out your job, troubleshoot possible problems, and create solutions.

## Vocabulary

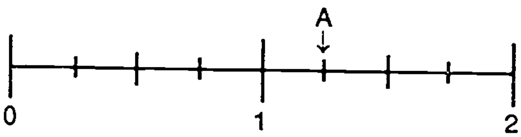
*standard measurement system*—the system of measurement used in the United States; includes feet, pounds, gallons, etc.
*conversion*—changing from one unit of measure to another; for example, changing from feet to inches.
*conversion factor*—the number or formula used to convert from one unit of measure to another.

## Develop Your Understanding

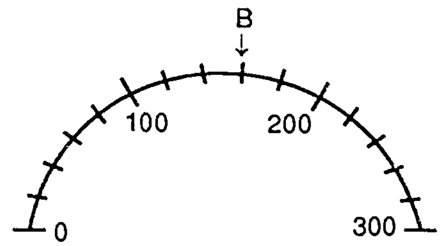
### *Reading Scales*

Any type of measurement is done on a *scale*. A scale is a series of marks at certain distances from each other. Most scales are plotted along lines, but other shapes may also be used. You can learn to read any type of scale by following a few simple rules.

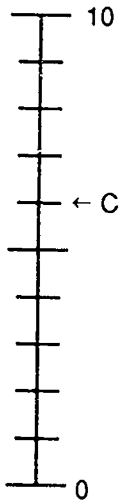
*Examples of Scales*




Scale 1




Scale 2




Scale 3

Each of the scales above has large markings that are labeled with numbers, as well as smaller markings in between that are not labeled. It is easy to read measurements at points that are labeled, but how about the ones in between?

**Rules for Reading Scales**

1. Find two labeled markings that are close together.
2. Subtract the lesser measurement from the greater one to find the distance between them.
3. Count the number of lines it takes to get from the first marking to the second.
4. Divide the distance between the labeled markings by the number of lines between them. This number shows how many units are represented by each line.
5. Add or subtract from the nearest labeled marking to find the measurement.

*Example 1*

*Look at Scale 1, and find the measurement at point A.*

**Step 1:** The two labeled markings nearest to point A are "1" and "2."
**Step 2:** Subtract: $2 - 1 = 1$. Therefore, 1 is the distance between the two markings.
**Step 3:** It takes 4 lines to get from marking 1 to marking 2.
**Step 4:** $1 \div 4 = \frac{1}{4}$. This means that every line represents $\frac{1}{4}$ of a unit.
**Step 5:** The nearest labeled marking is "1." Point A is one line past 1, so add:

$$1 + \frac{1}{4} = 1\frac{1}{4}.$$

*The measurement at point A is* $1\frac{1}{4}$ *units.*

**Now Try These**

Using the diagram "Examples of Scales," measure the number of units at:

a. point B on Scale 2:

b. point C on Scale 3:

### *Measurement Tables*

The table on the next page shows standard units of measure and their relationships to one another. These relationships contain *conversion factors* you can use to go from one unit of measure to another. You may want to make a copy of this table, since you will often refer to it to solve measurement problems.

## STANDARD MEASUREMENTS

### *Measure of Length*

1 foot (ft.) = 12 inches (in.)
1 yard (yd.) = 3 feet = 36 inches
1 mile (m.) = 5,280 feet = 1,760 yards

*Note:* The symbol for inches is 5" = 5 inches
The symbol for feet is 5' = 5 feet

### *Liquid Capacity Measure*

1 cup (c.) = 8 ounces (oz.)
1 pint (pt.) = 2 cups
1 quart (qt.) = 2 pints
1 gallon (gal.) = 4 quarts

### *Weight Measure*

1 pound (lb.) = 16 ounces (oz.)
1 ton = 2,000 pounds

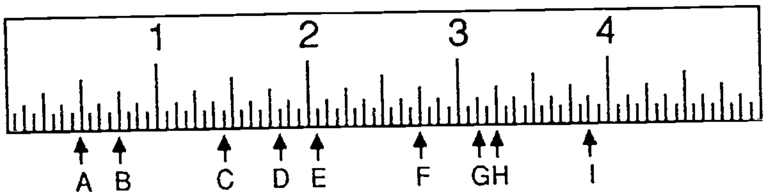
### *Reading a Standard Ruler*

The standard ruler is one of the most basic types of scales used in the United States. The basic unit on a standard ruler is the inch. Inches are divided into sixteen fractional parts. These parts are called:

- half inches — *1 inch = 2 half inches*
- quarter inches — *1 inch = 4 quarter inches*
- eighths of an inch — *1 inch = 8 eighths of an inch*
- sixteenths of an inch — *1 inch = 16 sixteenths of an inch*

When reading a ruler, always reduce fractions of an inch to lowest terms, as you would do for any other fraction. For example, a measurement of $2\frac{8}{16}$ inches should be reduced to $2\frac{1}{2}$ inches.

Consider the following standard ruler:




What is the measurement marked at:

Point A?
Point B?
Point C?
Point D?

Follow the rules for reading scales to answer the above questions. Remember that half inches are marked by the longest lines in between inches; quarter inches are the next longest; and sixteenths of an inch are the shortest lines.

### *Finding the Distance Between Two Points*

You can use two methods to find the distance between two points on a standard ruler:

**Method 1:** Count the number of units between the first point and the second.

For example, Point A lies on the $\frac{1}{2}$" mark. Point B is on the $\frac{6}{8}$" or $\frac{3}{4}$" mark. To find the distance in between them, look at the ruler and count the number of $\frac{1}{8}''$ lines required to move from one to the other. There are two, so your answer would be $\frac{2}{8}''$ or $\frac{1}{4}''$ (reduced) between Point A and Point B.

**Method 2:** Use your knowledge of fractions and subtract.

$$\frac{6}{8}-\frac{1}{2}=\frac{6}{8}-\frac{4}{8}=\frac{2}{8}'' \text{ or } \frac{1}{4}''$$

*Try both ways to find the distance between Point B and Point C.*

**Now Try These**

Look back at the diagram of the standard ruler on the previous page, and answer these questions.

What measurement is marked at:

a. Point G?

b. Point E?

c. Point F?

d. Point I?

What is the distance between:

e. Point B and Point D?

f. Point E and Point F?

g. Point D and Point F?

h. Point H and Point E?

***Adding Measurements***

Sometimes two measurements must be added to find a combined length. If you are adding inches, simply proceed as you would with any type of fractions. Find a common denominator, add, and reduce to lowest terms:

$$\frac{1}{2}+\frac{6}{8}=\frac{4}{8}+\frac{6}{8}=\frac{10}{8}=1\frac{2}{8}=1\frac{1}{4}$$

When adding feet and inches, however, it is best to use a *matrix*. Use rows and columns to keep different kinds of measurement separated:

| Feet | Inches |
|---|---|
| 2 ft. | 10 in. |
| +3 ft. | 5 in. |
| 5 ft. | 15 in. |

As a rule, measurements should always be converted to the highest or largest type of unit possible: feet rather than inches, gallons rather than cups. In the matrix, adding 2 ft. 10 in. to 3 ft. 5 in. gives a sum of 5 ft. 15 in. Once you have a result, move any inches in excess of 12 to the "Feet" column. In this case, there are 15 inches:

15 in. - 12 in. = 3 in.
12 in. = 1 ft.

*Therefore, 5 ft. 15 in. = 6 ft. 3 in., which is the final answer.*

**Now Try These**

1. 3 ft. 6 in.
   +6 ft. 11 in.

2. 5 ft. 5 in.
   + 2 ft. 10 in.

3. 9 ft. 4 in.
   +3 ft. 6 in.

***Subtracting Measurements***

Subtraction is done in a similar manner. Simply remember the basic scale of measurement. To subtract the following:

| Feet | Inches |
|---|---|
| 6 ft. | 3 in. |
| -2 ft. | 10 in. |

Begin with the Inches column. Since 10 cannot be subtracted from 3, you will need to borrow one unit from the Feet column. BE CAREFUL!

> *Subtracting measurements is different from ordinary subtraction. In order to borrow from the next column, you must know how many of the smaller units are in one of the larger units.*

In this case, you know that *1 foot = 12 inches.* Therefore, when you borrow one from the Feet column, you must add **12** units to the Inches column:

| Feet | Inches | = | Feet | Inches |
|---|---|---|---|---|
| 6 ft. | 3 in. | = | 5 ft. | 15 in. |
| -2 ft. | 10 in. | = | -2 ft. | 10 in. |
| | | | 3 ft. | 5 in. |

The final answer is 3 ft. 5 in.

**Now Try These**

a. 5 ft. 5 in.
− 2 ft. 11 in.

b. 12 ft. 1 in.
− 5 ft. 6 in.

c. 4 ft. 7 in.
− 2 ft. 10 in.

### *Multiplying or Dividing Measurements*

Sometimes you may need to multiply a length to determine how much is needed to cover a larger distance, or to divide something into equal parts. When multiplying or dividing feet and inches, the rules are simple:

1. Multiply or divide the inches by the number.
2. Multiply or divide the feet by the number.
3. Change inches into feet if the answer has more than 12 inches.

*Example 2: Multiply 2 ft. 2 in. by 7.*

2 ft. × 7 = 14 ft.
2 in. × 7 = 14 in. 14 in. = 1 ft. 2 in.

The answer is 15 ft. 2 in.

*Example 3: Divide 6 ft. 4 in. by 2.*

6 ft. ÷ 2 = 3 ft.
4 in. ÷ 2 = 2 in.

The answer is 3 ft. 2 in.

## Now Try These

a. 6 ft. 3 in.
$\times$ 5

b. 3 ft. 10 in.
$\times$ 2

c. 2 ft. 6 in.
$\times$ 6

d. 4 ft. 3 in. $\div$ 3

e. 7 ft. 4 in. $\div$ 2

### *Conversion of Units*

These rules for adding, subtracting, multiplying, and dividing units of measure hold true regardless of what measurement unit you are using. The critical factor to know is the *conversion factor.* With this number you can convert to larger and smaller units of measure by multiplying or dividing.

Use the Standard Measurements conversion tables to convert measured amounts into different units. For example:

*Liquid Capacity Measure*

1 cup = 8 ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

*How many ounces are in 2 pints?*

You will need to work backwards in order to answer this question. The table shows the following information:

1 pint = 2 cups
1 cup = 8 ounces

If 1 pint contains 2 cups, then 2 pints contain 2 × 2 or 4 cups.
If 1 cup contains 8 ounces, then 4 cups contain 4 × 8 or 32 ounces.
*Therefore, there are 32 ounces in 2 pints.*

## Now Try These

Use the measurement tables to answer these questions.

a. How many inches are in 7 feet?

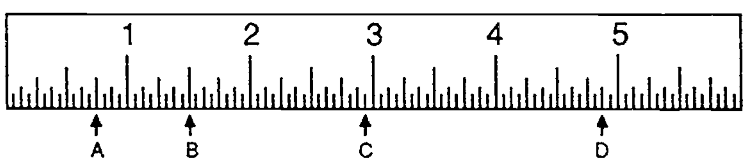
b. How many pints are in 3 gallons?

c. How many ounces are in 5 lb? (weight measure)

## Skill Builders

### *Skill Practice*

Use the following standard ruler to answer questions 1-5.




1. What is the measurement marked at Point A?
2. What is the measurement marked at Point B?
3. What is the distance between Point C and Point D?
4. What is the distance between Point A and Point D?
5. Read the measurement at point B and the measurement at Point D. Add these measurements together.

Solve the following.

6. 12 ft. 10 in. (*Convert the answer to yards, feet, and inches.*)
    <u>+ 8 ft. 9 in.</u>

7. 16 ft. 3 in. (*Convert the answer to yards, feet, and inches.*)
    <u>−12 ft. 11 in.</u>

8. Multiply:
    6 ft. 3 in.
    <u>× 4</u>

9. Divide: 15 ft. 6 in. ÷ 2

10. 4 gallons 3 quarts
    <u>+ 2 gallons 3 quarts</u>

11. 14 pounds 6 ounces
<u>+ 8 pounds 10 ounces</u>

12. 3 quarts 2 pints
<u>− 2 quarts 3 pints</u>

***Word Problems***

13. Gerald needs to add some chemicals to the plating bath. He only has a 3-gallon container in which to mix them. If he mixes the following, will it fit into the container? *What is the total amount of the mixture?*

    5 quarts Chemical A
    3 pints Chemical B
    6 cups Chemical C
    2 quarts Chemical D
    <u>9 cups Chemical E</u>

14. The new crates will only hold 25 pounds each. If they are packed for shipment with the following items, will there be any weight limit problems?

50 ounces of shipping insulation
12 pounds of Kit XYZ
36 ounces paperwork
17 ounces plastic wrapping
68 ounces of Kit ABC

# Lesson 2.8
# Standard Measurement (Part 2)

## Learning Objectives

*To practice measuring perimeter, area, liquid capacity, and time*

In Lesson 2.7, "Standard Measurement (1)," you examined the mathematical logic necessary to work with measurement systems, and you also examined some of the measurement units of the standard system. In this lesson, you will examine other types of measurement, such as perimeter, area, liquid volume or capacity, and time.

## Vocabulary

*perimeter*—the distance around the outside of an object
*area*—the amount of surface space inside an object, e.g., floor space in a room
*volume or capacity*—the total amount of space inside something; usually refers to the amount of something in a measuring container, such as one cup of water

## Develop Your Understanding

### *Perimeter*

An important measurement to be familiar with is *perimeter*, which is the distance around the outside of an object. Whether you are figuring the perimeter of circuit panels, buildings, or property, the formula is always the same: simply add the measurement of each side together. For example, if you have a six-sided property that you wish to put a fence around, you would simply add the length of each of the six sides to get the total perimeter.

Sometimes, you can take a shortcut to find the perimeter. For example:

*A square, by definition, has 4 equal sides. Find the perimeter of any square by multiplying the length of one of the sides by 4.*

*Opposite sides of a rectangle are also equal. Find the perimeter of any rectangle by multiplying the length by 2, multiplying the width by 2, and adding the products together.*

**Now Try These**

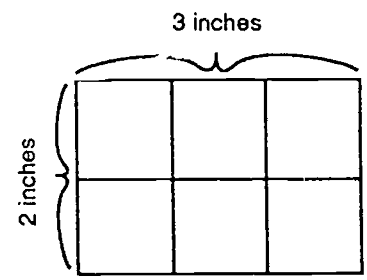
a. What would be the perimeter of a 5-sided building with measurements of 31 ft., 28 ft., 40 ft., 29 ft., and 30 ft.?

b. What would be the perimeter of a square in which one side is 5 inches?

c. What would be the perimeter of a rectangular panel that is 12 inches long and 11 inches wide?

***Area***

Another important measurement is *area*. Area is the amount of flat surface space within an object. Measurements of area are expressed in *square units*, or units that have two dimensions: length and width. A square inch is a square that is 1 inch long and 1 inch wide. A square mile is an area that measures 1 mile on all four sides.



*To find the area of any square or rectangle, multiply the length by the width. The product is expressed in square units: square inches (sq. in.), square feet (sq. ft.), etc. This formula ONLY works for squares and rectangles.*

*Example 1*

*Find the area of the rectangular part shown on the previous page.*

The *length* of the part is 2 inches.
The *width* of the part is 3 inches.

2 in. x 3 in. = 6 sq. in.

The area of the part is 6 square inches.

**Now Try These**

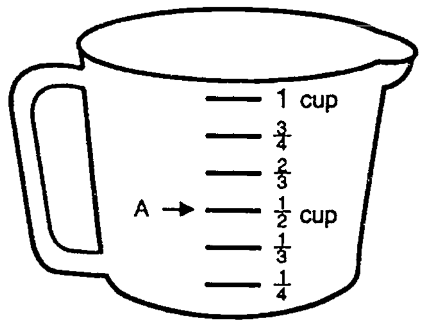
a. What is the area of a panel that is 10 inches long and 9 inches wide?

b. What is the area of a house that is 64 feet long and 25 feet wide?

***Liquid Measurement***

Forms of liquid measurement are often used at work. Sometimes this involves the standard system of cups, pints, quarts, etc., and sometimes metric measurement of volume or capacity is used. It is important to be familiar with both systems and to be comfortable with their use. You will have a chance to practice working with the metric system in the next lesson.

The standard system is the one most often used in cooking and other activities in the United States. For example, if you have the following measuring cup filled with water to the point marked by *A*, how much water would you have?




To find this measurement, read the marks on the measuring cup, and note that the water line comes to the $\frac{1}{2}$ cup mark. Therefore, there is $\frac{1}{2}$ cup of water in the cup. In the standard system, we measure whole units such as cups, pints, quarts, etc., or we measure some fraction of that whole.

***Time***

Depending on the system of time your place of employment uses, you may or may not be familiar with the 24-hour clock. This is the system commonly used by the military and many countries outside the United States.

Before working with the 24-hour clock, review how to make conversions and conduct mathematical operations with the 12-hour clock.

*Example 2*

*How would you add the following times together to check your total time spent on a given project?*

2 hours 20 minutes
3 hours 45 minutes
5 hours 35 minutes

Once again, you can use a matrix approach to solving this problem. To convert minutes to hours, use a conversion factor of 60 minutes to 1 hour.

| Hours | Minutes |
|---|---|
| 2 hrs. | 20 min. |
| 3 hrs. | 45 min. |
| + 5 hrs. | + 35 min. |
| 10 hrs. | 100 min. |

After adding the hours and minutes separately, check to see whether some of the minutes can be moved to the hours column. In this case, 100 minutes becomes 1 hour and 40 minutes:

| Hours | Minutes |
|---|---|
| 2 hrs. | 20 min. |
| 3 hrs. | 45 min. |
| + 5 hrs. | + 35 min. |
| 10 hrs. | 100 min. |
| + 1 hr. | - 60 min. |
| 11 hrs. | 40 min. |

The total is 11 hours, 40 minutes.

### *The 24-hour Clock*

The 24-hour clock labels the A.M. or morning hours with the numbers 1 through 12. Then, instead of starting over with 1 again, it continues to count upwards. This way 1 P.M. through 12 P.M. become the 13th through the 24th hours. Time on the 24-hour clock is expressed with four digits. The first two stand for the hour, and the second two stand for the minutes. For example, 0630 would be 6:30 A.M.

*To calculate what a P.M. time would be on the 24-hour clock:* add its hour number to 12. For example, 6 P.M. would be 12 + 6 = 18, or in 24-hour clock time, 1800 hours.

*To calculate what a time on the 24-hour clock would be on the 12 hour clock:* subtract 12 from the hour if it is greater than 12, insert a colon and add the letters "P.M." If the hour is 12 or less, insert a colon and add the letters "A.M."

*Example 3*

*Express the following times in 12-hour clock time:*

1520:
1230:
2400:

The first two digits represent the hour.

1520: The hour is greater than 12. 15 -12 = 3.
1230: The hour is not greater than 12. The numbers stay the same.
2400: The hour is greater than 12. 24 - 12 = 12.

*Answers:*

| | |
|---|---|
| *1520:* | *3:20 P.M.* |
| *1230:* | *12:30 P.M.* |
| *2400:* | *12:00 midnight* |

*Note:* On the 24-hour clock, 1200 is noon and 2400 is midnight.

**Now Try These**

a. Add the following times together, and find out the total length of time involved:

| | | |
|---|---|---|
| 6 days | 7 hours | |
| 3 days | 15 hours | |
| | 12 hours | 45 minutes |
| 1 day | 8 hours | 50 minutes |

b. Convert the following times to the 12-hour clock system (remember to designate A.M. or P.M.):

0525:
1730:
2045:

**Skill Builders**

1. Find the perimeter of the following:

   a. A multisided building with these dimensions:

      Side 1: 65 ft.
      Side 2: 53 ft.
      Side 3: 48 ft.
      Side 4: 51 ft.
      Side 5: 62 ft.

b. A rectangular panel whose length is 12.3 inches and whose width is 11.5 inches.

c. A square part with one side 8.6 inches long.

2. Cindy needs to order boxes to ship some new panels. The panels are square-shaped with one side measuring 11 inches. The packing material will add 2 inches all the way around the panels. The best price on the boxes is for square ones that have an interior perimeter of 53 inches.

   a. What is the total perimeter needed for the panels and packing material?

   b. Are these boxes big enough for the panels?

3. Find the area of the following:

   a. A square part with one side measuring 5 inches.

b. A rectangular building whose length is 25.3 ft. and whose width is 18.6 ft.

4. ABC Computers has just gotten a contract for a new series of parts. The engineers have asked Linda to check their design to see if it meets with the needs of the company's machinery. The design is for a circuit panel that will have four parts on it. The panel is a rectangle that measures 11 inches by 12 inches. Each of the four parts is the same size. Each one is square-shaped with one side equal to 4.5 inches. There must be at least 2 free inches on the panel around each of the parts and between each of the parts to provide enough space for the machine clamps to hold each panel. *(Hint: Drawing a diagram will help you solve this problem.)*

   What is the total area of the panel?

5. Add the following time spent on Project GHJ by Department Blue. What is the total time spent on it? (Convert to highest scale.)

| | | |
|---|---|---|
| 8 days | 12 hours | |
| 3 days | 9 hours | 45 minutes |
| 6 days | 11 hours | 55 minutes |
| 13 days | 7 hours | 34 minutes |

6. Convert from 12-hour clock to 24-hour clock:

   a. 9:30 A.M. =
   b. 5:30 P.M. =
   c. 3:15 P.M. =
   d. 11:45 A.M. =

Convert from 24-hour clock to 12-hour clock (designate A.M. or P.M.):

e. 0520 =
f. 1025 =
g. 2230 =
h. 1720 =

**Check Yourself**

1. John spent the following times working on Project ABC:

| | |
|---|---|
| Monday: | 9:30 A.M. – 2:30 P.M. |
| Tuesday: | 8:45 A.M. – 11:15 A.M. |
| Wednesday: | 11:20 A.M. – 3:08 P.M. |

a. How much total time did John spend on this project?

b. John must record his work time for each project by hand on his time sheet. However, his time sheet calls for 24-hour clock time. Record John's times as they should be recorded on his time sheet:

Monday:

Tuesday:

Wednesday:

2. John records the following times on his time sheet for time spent on Project DEF:

Monday: 0930 - 1345
Tuesday: 1025 - 1250
Wednesday: 0840 - 1410

a. How much time did John actually spend on Project DEF?

b. Convert to 12-hour clock time, and show the hours that John spent each day on Project DEF.

Monday:

Tuesday:

Wednesday:

# Lesson 2.9
# Metric Measurement

## Learning Objectives

*To learn the basic principles of metric measurement*
*To practice converting between standard and metric measurements*

Although the standard system is still the principal method of measurement used in the United States, much of the rest of the world uses the metric system. As international trade increases, it will become very important to have a working knowledge of the metric system, and to be able to convert from one system to the other. Experience with the metric system will also be a great help to you if you ever travel outside the United States.

## Vocabulary

*metric measurement system*—a system of measurement based on a scale of 10, used outside the United States and throughout most of the world; includes grams, meters, liters, etc.

## Develop Your Understanding

### *The Metric System*

Many people groan at the thought of learning a system of measurement other than the one they have used all their lives. You may feel that a new system would be more difficult than the one we have now. However, it could be argued that metric measurement as a system is more logical and easier to use than our standard system.

The metric system was developed by a group of scientists about 200 years ago. All of the measurement units in the metric system are based on a scale of 10, just like our number system. In other words, whenever you have *ten* of some kind of unit in the metric system, this is equal to *one* of the next larger unit. This is not true of the standard system, which has many different conversion factors for different types of measurement.

Each type of metric unit has a *prefix* and a *suffix*. The suffix tells what type of measurement it is: length, capacity, weight, etc. The prefix tells the size of the unit. Each type of measurement in the metric system has a basic unit which has no prefix.

**METRIC SYSTEM TERMS**

| **Prefix** | **Meaning** |
|---|---|
| kilo- | basic unit times 1,000 |
| hecto- | basic unit times 100 |
| deka- | basic unit times 10 |
| deci- | one-tenth (0.1) of basic unit |
| centi- | one-hundredth (0.01) of basic unit |
| milli- | one-thousandth (0.001) of basic unit |

| **Suffix/Basic Unit** | **Meaning** |
|---|---|
| meter | measure of length |
| gram | measure of weight |
| liter | measure of liquid capacity |

Units of measurement are formed by combining a prefix with a suffix.

*Example 1*

*A meter is the basic unit of length. What is a* ***dekameter****?*

Look on the chart and find the prefix "deka-." The meaning is "basic unit times 10."

A dekameter is 10 meters.

**Now Try These**

a. A liter is the basic unit of liquid capacity. What is a *milliliter?*

b. A gram is the basic unit of weight. What is a *kilogram?*

### *Conversions in the Metric System*

Use the following tables to convert between units in the metric system. The tables also show abbreviations for each type of unit.

**Measure of Length**
**Basic Unit: the meter (m)**

10 millimeters (mm) = 1 centimeter (cm)
10 centimeters = 1 decimeter (dm)
10 decimeters = 1 meter (m)
10 meters = 1 dekameter (dam)
10 dekameters = 1 hectometer (hm)
10 hectometers = 1 kilometer (km)
1 kilometer = 1,000 meters (m)

**Measure of Weight**
**Basic Unit: the gram (g)**

10 milligrams (mg) = 1 centigram (cg)
10 centigrams = 1 decigram (dg)
10 decigrams = 1 gram (g)
10 grams = 1 dekagram (dag)
10 dekagrams = 1 hectogram (hg)
10 hectograms = 1 kilogram (kg)
1,000 kilograms = 1 metric ton (t)

**Measure of Capacity**
**Basic Unit: the liter (l)**

10 milliliters (ml) = 1 centiliter (cl)
10 centiliters = 1 deciliters (dl)
10 deciliters = 1 liter (l)
10 liters = 1 dekaliter (dal)
10 dekaliters = 1 hectoliter (hl)
10 hectoliters = 1 kiloliter (kl)
1,000 liters = 1 kiloliter

Because the metric system is based on 10, you can use a matrix approach to converting between units.

Looking at measures of length, for example, you might construct a matrix like this one:

| kilometers km | hectometers hm | dekameters dam | meters m | decimeters dm | centimeters cm | millimeters mm |
|---|---|---|---|---|---|---|
| | | | | | | |

Fill in the matrix as follows:

Starting with the unit attached to a measurement, fill in one digit at a time, moving from *right to left*.

*Example 2*

*Put these three measurements into the matrix: 91 mm, 312 cm, 423 m*

After following the directions, your matrix should look like this:

| kilometers km | hectometers hm | dekameters dam | meters m | decimeters dm | centimeters cm | millimeters mm |
|---|---|---|---|---|---|---|
| | | | | | 9 | 1 |
| | | | 3 | 1 | 2 | |
| | 4 | 2 | 3 | | | |

The matrix is a very convenient way of converting units. A quick look at the matrix will tell you that:

*91 mm is the same as 9 cm, 1 mm*
*312 cm is the same as 3 m, 1 dm, 2 cm*
*423 m is the same as 4 hm, 2 dam, 3 m*

## Now Try These

Use the matrix to express each measurement below in highest units.

| kilometers km | hectometers hm | dekameters dam | meters m | decimeters dm | centimeters cm | millimeters mm |
|---|---|---|---|---|---|---|
| | | | | | | |

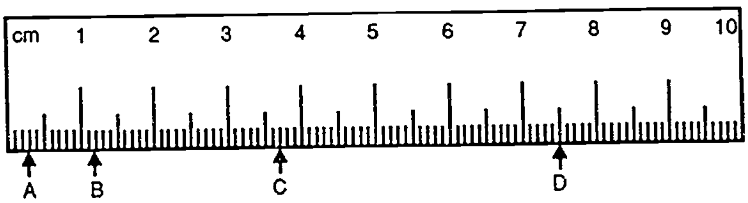
a. 221,400 millimeters =

b. 35 decimeters =

c. 6.5 centimeters =
*(Remember that each unit is* ***one tenth*** *of the unit to its left.)*

### *Using a Metric Ruler*

The ruler shown below is a metric ruler. The shortest lines represent millimeters, and the tallest ones are centimeters. Note that "cm" is printed on the ruler to indicate that the lines labeled with numbers are centimeters. This ruler is one decimeter long. Ten decimeters equal one meter. The meter stick is a bit longer than our yardstick.




*Example 3*

*How many millimeters are there between 12 mm and 2 mm?*

To find the answer, all you have to do is subtract: 12 - 2 = 10.

There are 10 mm between 12 mm and 2 mm.

## Now Try These

a. What measurement is marked on the ruler by: Point A?
(answer in cm and mm) Point B?
Point C?

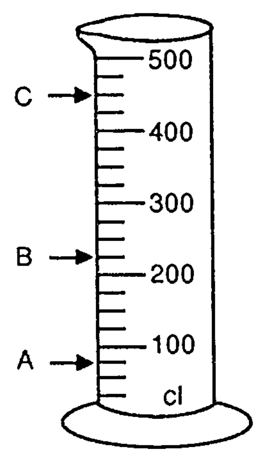
b. How far is it from Point A to Point D?

c. 7 mm + 10 mm = ____ mm = (convert) ____ cm _____ mm

***Metric Capacity***

The metric system uses liters as the basic measurement of volume or capacity. Since a liter is a fairly large measurement, roughly equivalent to one quart, most instruments for measuring capacity use milliliters, centiliters or deciliters as a basic unit. Remember to look for an abbreviation on the instrument to determine which units are labeled.

Study the metric cylinder below. Note that the labeled markings are *centiliters (cl)*.




*If the cylinder is filled with water up to Point A, how much water would you have?*

(If you are not sure how to find the answer, go back to Lesson 2.7 on measurement).

*The answer to the question on the previous page is 75 cl.*
Since the distance between the labeled marks is 100 centiliters and it takes a jump of 4 lines to go from 100 to 200 cl, divide 100 by 4:

> $100 \div 4 = 25$. Thus, each small line represents 25 cl.
> Point A is three lines from the bottom: $25 \times 3 = 75$

**Now Try These**

a. What level is marked by Point B?

b. What level is marked by Point C?

c. How far is it between Point A and Point C?

### *Converting Between Metric and Standard Systems*

Do you remember that a *conversion factor* is the number that expresses the relationship between larger and smaller scales of measurement? Conversion factors can also be used to convert measurements from one system to another. In order to convert, you will need a table of conversion factors. These factors are easy to use, once you understand them. Because many of the factors are rounded, they are not always exact.

**Metric to Standard Conversion Factors**

| **Multiply:** | **by:** | **to get:** |
|---|---|---|
| meters | 39.37 | inches |
| | 3.28 | feet |
| | 1.09 | yards |
| centimeters | 0.39 | inches |
| millimeters | 0.04 | inches |
| kilometers | 0.62 | miles |
| liters | 1.06 | liquid quarts |
| grams | 0.04 | ounces (weight) |
| kilograms | 2.2 | pounds |
| metric tons | 2,204.62 | pounds |

**Standard to Metric Conversion Factors**

| Multiply: | by: | to get: |
|---|---|---|
| inches | 25.4 | millimeters |
| feet | 0.3 | meters |
| yards | 0.91 | meters |
| miles | 1.61 | kilometers |
| liquid quarts | 0.95 | liters |
| dry quarts | 1.1 | [illegible]ers |
| ounces (weight) | 28.35 | [illegible]ams |
| pounds | 0.45 | [illegible]ms |
| tons | 0.91 | m[illegible]ons |

In order to use the conversion tables, you must first decide whether to convert from metric to standard or from standard to metric. This requires careful thinking.

*Example 4*

*How many yards = 1 meter?*

First, ask yourself: Should my *answer* be standard or metric? In this case, you must answer in *yards*, a standard measurement. Refer to this chart:

| Answer in | Use |
|---|---|
| Standard units | Metric to Standard |
| Metric units | Standard to Metric |

The chart tells you that, since you want an answer in standard units, you should use a metric to standard conversion:

multiply meters (1) by 1.09 to get the number of yards.

*Answer: There are 1.09 yards in 1 meter.*

How many kilometers = 1 mile?

How many ounces = 1 gram?

## Now Try These

Convert each of the following:

a. 50 kilometers = ? miles

b. 100 meters = ? yards

c. 10 pounds = ? kilograms

d. 25 grams = ? ounces

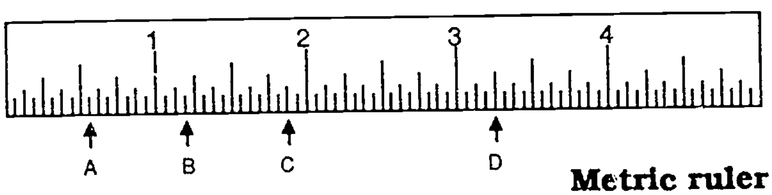
e. 16 ounces = ? grams

f. 3 liquid quarts = ? liters

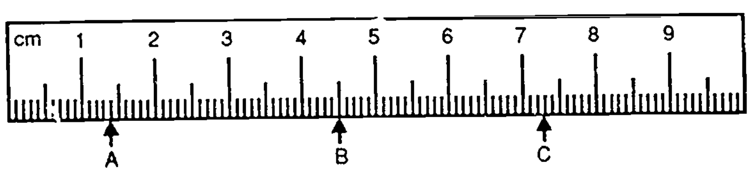
## Skill Builders

Use the following illustrations to answer questions 1 and 2.

**Standard ruler**




**Metric ruler**



1. Find the ruler based on the standard system of measurement. What measurement is marked by:

   a. Point A?

   b. Point B?

   c. Point C?

   d. How far is it from Point A to Point C?

2. Find the metric ruler.

   What point is marked by :

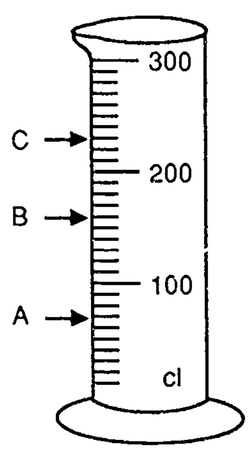
   a. Point A?

   b. Point B?

   c. Point C?

   d. How far is it from Point A to Point B?

3. Study the metric cylinder below, and answer the questions that follow.




   a. What level of measurement is marked by Point A?
   Point B?
   Point C?

   b. How much liquid is contained between Point A and Point C?

4. A roll of tape purchased in England has 2,000 meters of tape. How many *yards* of tape are in the roll?

5. Plink Industries packs items from its plant in Germany together with items made locally. The items from Germany come ready to be placed in the shipping crates. The contents of one crate are shown below. What is the combined weight of the items *in pounds?*

Item 1: 30 kilograms
Item 2: 3.5 pounds
Item 3: 200 hectograms
Item 4: 20 kilograms
Item 5: 15 pounds
Item 6: 105 ounces

# Lesson 2.10
# Adding and Subtracting Signed Numbers

## Learning Objectives

*To understand the concept of signed numbers*
*To develop skill in adding and subtracting signed numbers*

Algebra is a powerful tool that simplifies problem solving, especially in technical fields. It is very useful for finding a missing piece of information when other pieces are known. When you begin to study algebra, you must learn to look at numbers in a new way. Before you begin this study, most of the numbers you use express values that are easy to see. You can show the meaning of the number "4," for example, by counting 4 *apples*. Some numbers, on the other hand, are *abstract* numbers that are difficult to show. Understanding how to use these abstract numbers in mathematical operations will enable you to solve new kinds of problems easily.

## Vocabulary

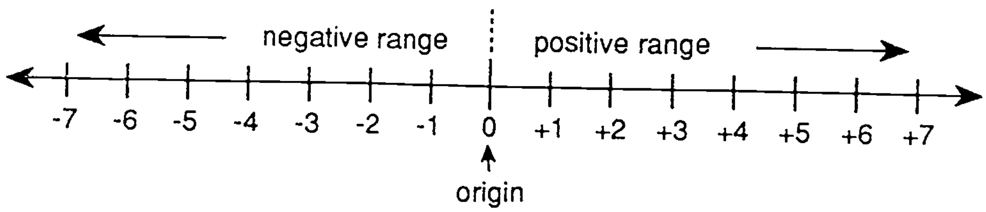
*signed number*—any number that carries a positive or negative sign.
*integer*—integers consist of whole numbers, their opposites, and zero.
*absolute value*—the distance an integer is from zero.

## Develop Your Understanding

### *Integers and Signed Numbers*

When you first studied math, you were probably taught that there is "no such thing as less than zero." But in real life, there are numbers that *are* "less than zero." For example, winter temperatures in northern climates often fall *below zero.* When a checking account is overdrawn, it is said to have a "*negative balance*"—this means a balance of less than zero.

The study of algebra begins with the study of *integers*. Integers include both *positive* and *negative* whole numbers. You are already familiar with positive whole numbers. Negative whole numbers are the opposites of positive whole numbers. The number line below shows some integers. Note that zero is also an integer.




*The arrows at each end of the number line show that the line can go on forever in each direction. The point "0" on the line is called the origin.*

All numbers to the **right** of the origin are called **positive numbers.** These numbers may be written *either* with a plus sign (+) in front or with no sign.

All numbers to the **left** of the origin are called **negative numbers.** These numbers must be written with a minus (-) sign in front.

The number line also represents many numbers that are not integers. Fractions and decimals as well as integers can be called **signed numbers.**

*Example 1*

The number (+7) can be read "seven" or "positive seven."
The number (-7) can be read "negative seven" or "the opposite of seven."

| |
|---|
| *A number value with a sign is said to be the* ***opposite*** *of the same number value with the opposite sign. For example, "(+3)" is the opposite of "(-3)."* |

### Absolute Value

The distance a signed number is from zero is called its *absolute value.* Distance can be measured from the origin to either the left or the right. The absolute value of zero is zero.

*Example 2*

The absolute value of both (-2) and (+2) is 2.
The absolute value of both (-100) and (+100) is 100.

## Now Try These

*Find the absolute value for problems a, b, and c.*

a. The absolute value of (-9) is ____ .

b. The absolute value of (+14) is ____.

c. The absolute value of ($-\frac{1}{2}$ ) is ____.

d. What *two* signed numbers have an absolute value of 4? ____ and ____.

### *Adding Signed Numbers*

Signed numbers can be added. Be sure to keep operational symbols (the "+" for addition) separate from the sign of a number by using parentheses. There are only two rules to remember for adding signed numbers. A different rule is used to add numbers with the same sign than to add numbers with different signs.

**To Add Numbers with the Same Signs:**
1. Find the absolute value of each number.
2. Add the absolute values together.
3. Keep the **original sign,** whether positive or negative.

*Example 3*

*Add two positive numbers:*

(+5) + (+7) =

1. The absolute values of (+5) and (+7) are 5 and 7.

2. Adding absolute values: 5 + 7 = 12

3. The original sign of both numbers is (+). Therefore,

(+5) + (+7) = (+12)

*Example 4*

*Add two negative numbers:*

(-5) + (-7) =

1. The absolute values of (-5) and (-7) are 5 and 7.

2. Adding absolute values: 5 + 7 = 12

3. The original sign of both numbers is (-). Therefore,

   (-5) + (-7) = (-12)

**Now Try These**

a. (+2) + (+6) =

b. (-4) + (-5) =

c. (-3) + (-5) + (-200) =

| |
|---|
| *When adding two or more numbers, the order in which the numbers are added may be changed without changing the result. For example: 1 + 3 = 3 + 1.* |

Sometimes you will add numbers with different signs. Remember this important rule:

**To Add Numbers with Different Signs:**

1. Find the absolute value of each number.
2. Subtract the smaller absolute value from the larger one.
3. Keep the sign of the number with the larger absolute value.

*Example 5*

*Add a positive number to a negative number:*

(+10) + (-4) =

1. The absolute values of (+10) and (-4) are 10 and 4.

2. Subtract 4, the smaller absolute value, from 10, the larger one.

   10 - 4 = 6

3. (+10) has the larger absolute value; its sign is (+). The answer will be positive:

   (+10) + (-4) = (+6)

*Example 6*

*Add a negative number to a positive number:*

(-9) + (+2) =

1. The absolute values of (-9) and (+2) are 9 and 2.

2. Subtract 2, the smaller absolute value, from 9, the larger one.

   9 - 2 = 7

3. (-9) has the larger absolute value; its sign is (-). The answer will be negative:

   (-9) + (+2) = (-7)

**To Add a Series of Numbers:**

1. Combine all positive values into one positive number.
2. Combine all negative values into one negative number.
3. Follow the rules for adding numbers with different signs.

*Example 7*

*Add six numbers with different signs:*

(+4) + (-3) + (-8) + (+3) + (-4) + (+8) =

1. Combine all positive numbers:

   (+4) + (+3) + (+8) = (+15)

2. Combine all negative numbers:

   (-3) + (-8) + (-4) = (-15)

3. Follow the rules for adding numbers with different signs:

   (+15) + (-15) =

Subtract absolute values: 15 - 15 = 0    (0 has no sign)

Therefore, (+4) + (-3) + (-8) + (+3) + (-4) + (+8) = 0

| *Adding any signed number to its opposite will result in an answer of zero.* |
|---|

**Now Try These**

a. 4 + 5 =

b. (+7) + (-8) =

c. (9) + (5) =

d. (+6) + (-32) =

e. (80) + (-21) + (7) + (-41) + (-1) =

### *Subtracting Signed Numbers*

Signed numbers can also be subtracted. There is only one rule for subtraction.

**To Subtract One Signed Number from Another:**

*Add the opposite* of the number you are subtracting.

*Example 8*

*Subtract a positive number from a negative number:*

(-5) - (+3) =

Instead of subtracting (+3), you will add the opposite of (+3), which is (-3):

(-5) - (+3) = (-5) + (-3)

Now follow the steps for adding numbers with the same sign:

(-5) + (-3) = (-8) *The answer is (-8).*

*Example 9*

*Subtract a negative number from a negative number:*

(-10) - (-12) =

Instead of subtracting (-12), you will add the opposite of (-12), which is (+12):

(-10) - (-12) = (-10) + (+12) =

Now follow the steps for adding numbers with different signs:

(-10) + (+12) = (+2) *The answer is (+2).*

**Now Try These**

a. 5 - (-6) =

b. 14 - (+5) =

c. 45 - (+2) - (-7) =

**Adding and Subtracting Signed Numbers: Summary**

1. A positive number plus a positive number equals a positive number.
   (+) + (+) = (+)
2. A negative number plus a negative number equals a negative number.
   (-) + (-) = (-)
3. A positive number plus a negative number equals the difference between the two numbers, using the sign of the larger number.
   (+7) + (-2) = 7 - 2 = (+5)
4. To add several signed numbers, combine all positive and all negative numbers, and apply rule 3.
   (+2) + (-5) + (-7) + (+3) = (+5) + (-12) = (-7)
5. To subtract one signed number from another, add the opposite of the number to be subtracted.
   (-3) - (-2) = (-3) + (+2) = (-1)

## Skill Builders

### *Skill Practice*

*Add* (+5) to each of the following:

1. -8
2. +6
3. -5
4. -2

*Add* (-34) to each of the following:

5. -17
6. +16
7. +23
8. +50

*Add* these integers:

9. (-21) + (-16) =

10. (+38) + (-19) =

11. (-55) + (-14) + (+28) + (+30) =

12. (-102) + (+38) =

*Subtract* (+7) from each of the following:

13. -64

14. +5

15. -31

16. +3425

*Subtract* (-31) from each of the following:

17. +45

18. +2

19. -701

20. +1

*Subtract* these integers:

21. -6 -(-34) =

22. 4,567 - (-763) =

### *Word Problems*

23. Sue went to check the temperature of Solution A. It dropped to -12°F. How many degrees below freezing did the temperature fall? Remember, freezing = 32°F.

**Check Yourself**

Team A—DAILY PRODUCTION CHART—UR2 GRAPHICS

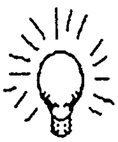
| | *Completed* | *Pulled* |
|---|---|---|
| S. JONES | 32 | 4 |
| T. WELLS | 29 | 2 |
| J. SMITH | 41 | 1 |

The above chart reflects the number of kits each employee completed and the number of kits Quality Control pulled. Team A's supervisor needs to turn in a report as to how many kits are left to be shipped after Quality Control pulled the damaged kits. Set this up as a mathematical problem with signed numbers, and solve it. *(Hint: Use a positive integer for each kit completed and a negative integer for each kit pulled.)*

# Lesson 2.11
# Multiplying and Dividing Signed Numbers

## Learning Objectives

*To develop skills in multiplying and dividing signed numbers*



This lesson will focus on learning and applying the rules for multiplying and dividing signed numbers. These rules must be mastered before solving algebraic equations in the workplace.

## Develop Your Understanding

### *Multiplying Signed Numbers*

To multiply two signed numbers, follow these rules:

same signs = positive answer    different signs = negative answer

*Example 1*

| | |
|---|---|
| $4 \times 3 = 12$ | positive × positive = positive |
| $-4 \times -3 = 12$ | negative × negative = positive |
| $-4 \times 3 = -12$ | negative × positive = negative |
| $4 \times -3 = -12$ | positive × negative = negative |

*Note: A positive number can be written without the "+" sign.*

**Now Try These**

*Multiply the following, using the rules for multiplying signed numbers:*

a. (-4) × (-8) =

b. (-10) × (3) =

c. (4) × (-9) =

### *Dividing Signed Numbers*

The basic rules for dividing signed numbers are the same as the rules for multiplying them:

same signs = positive answer          different signs = negative answer

*Example 2*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 12 | ÷ | 4 | = | 3 | positive | ÷ | positive | = | positive |
| -12 | ÷ | -4 | = | 3 | negative | ÷ | negative | = | positive |
| 12 | ÷ | -4 | = | -3 | positive | ÷ | negative | = | negative |
| -12 | ÷ | 4 | = | -3 | negative | ÷ | positive | = | negative |

**Now Try These**

*Divide the following, using the rules for dividing signed numbers:*

a. (-49) ÷ (-7) =

b. 68 ÷ 2 =

c. (-40) ÷ 5 =

### *Multiplying Several Signed Numbers*

To multiply several signed numbers, multiply two numbers at a time, and follow the rules for multiplication.

*Example 3*

$(-8) \times (-9) \times (-3) = -216$
$(-8) \times (-9) = 72 \times (-3) = -216$

**Now Try These**

a. $(-4) \times (-8) \times (-6) \times =$

b. $12 \times 5 \times (-80) =$

### *Working with Other Signed Numbers*

Fractions and decimals can be signed numbers, too. The same rules for multiplication and division apply to these numbers as to whole numbers. Be careful, however; remember that fractions and decimals have their own special rules for multiplication and division. Review those rules before working with signed fractions and decimals.

*Example 4*

$$\frac{-1}{2} \times \frac{1}{3} = \frac{-1}{6}$$

*Example 5*

$(-12.6) \div (-2.1) = (+6)$

$$-2.1\,\overline{)\,-12.6}$$

$$\begin{array}{r} +6 \\ -21\,\overline{)\,-126} \\ \underline{-126} \\ 0 \end{array}$$

**Now Try These**

a. $\frac{4}{5} \div \frac{-2}{3} =$

b. $(-5.2) \times (-10.4) =$

**Skill Builders**

1. $(+6) \times (+100) =$

2. $(-50) \times (-80) =$

3. $(-2) \times (-3) \times (+2) =$

4. $(+2) \times (+1) \times (+6) \times (-1) =$

5. $\frac{-2}{3} \times \frac{5}{8} =$

6. $7\frac{1}{3} \times (-6\frac{1}{3}) =$

7. $(-3) \times \frac{5}{6} =$

8. $$\begin{array}{r} -0.861 \\ \underline{\times\ -8.5} \end{array}$$

9. $$\begin{array}{r} 2.1 \\ \underline{\times\ 0.02} \end{array}$$

10. $(\frac{-5}{7}) \times (\frac{-5}{8}) =$

11. $(+10) \div (+100) =$

12. $(+99) \div (-3) =$

13. $(-56) \div (-2) =$

14. $19.28 \div (-0.8) =$

15. $2.95 \div (-10) =$

16. $\frac{-1}{8} \div 1\frac{1}{8} =$

17. $-114 \div 6 =$

# Module 2 Review #2
# Lessons 2.7-2.11

After reviewing the key concepts in each lesson, solve each of the following problems. You may work all, or some, of the problems in each section. You will need extra paper.

**Lessons 2.7, 2.8, and 2.9**

1. Add the following, and convert to highest measurement:

   a. 3 yards 10 inches + 14 feet 6 inches + 22 inches

   b. 5 gallons 25 ounces + 5 quarts 7 pints + 8 pints 12 cups

2. Subtract the following, and convert.

   a. 12 feet – 16 inches

   b. 3 quarts – 18 ounces

   c. 1 meter – 12 millimeters

   d. 18 decimeters – 18 centimeters

   e. 1 deciliter – 10 milliliters

3. Convert as instructed:

   a. 12 kilometers = ? miles

   b. 12 miles = ? kilometers

   c. 12 centimeters = ? inches

   d. 50 meters = ? yards

   e. 100 pounds = ? kilograms

   f. 16 ounces = ? grams

4. 16 feet 8 inches × 4 = ?

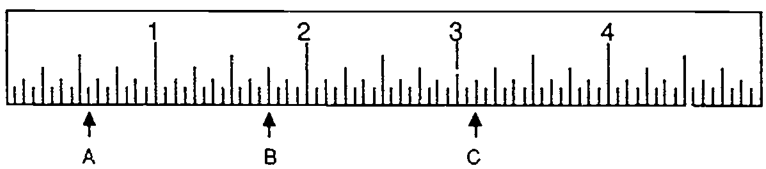
5. 3 yards 3 feet 6 inches × 5 = ?

6. 11 feet 8 inches ÷ 4 = ?

7. 5 yards 4 feet 6 inches ÷ 2 = ?

8. On the following ruler, what measurement is marked by Point A? ________
Point B? ________ Point C? ________

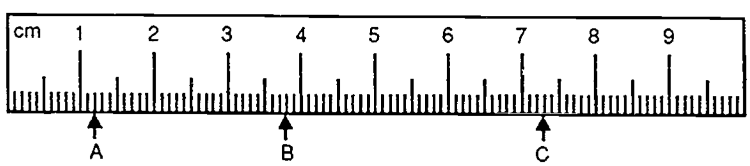
How far is it from Point A to Point B?
from Point B to Point C?
from Point A to Point C?




9. On the following ruler:

What measurement is marked by Point A? ________ Point B? ________
Point C? ________

How far is it from Point A to Point B?
from Point B to Point C?



10. What is the perimeter of:

a. a square with one side equal to 8 inches?

b. a rectangle with a height of 4 inches and a length of 12 inches?

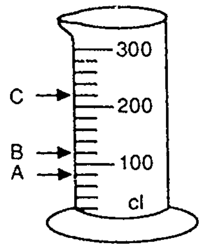
11. What is the area of the above two boxes?

12. What measurement is marked on the following cylinder by:

Point A?

Point B?

Point C?




13. Add the following:

3 days 22 hours + 5 days 16 hours 124 minutes

14. Convert the following:

0520: 1639: 2323:

12:30 A.M.:
4:15 P.M.:
6:20 P.M.:

**Lessons 2.10 and 2.11**

15. Draw a number line that goes from −10 to +10. Draw a + and a − sign to show the direction for adding and subtracting positive numbers.

16. How would you write:

    a. positive 10 plus positive 4?

    b. negative 10 plus negative 4?

    c. positive 10 minus positive 4?

    d. negative 10 minus negative 4?

    e. positive 10 plus negative 4?

    f. negative 10 plus positive 4?

17. After writing out each of the above, solve them.

18. Solve each of the following:

    a. $3 + 9 =$

    b. $(-4) + (-5) =$

    c. $2 + (-12) =$

    d. $15 + (-5) =$

    e. $8 - 4 =$

    f. $(-6) + (-5) =$

    g. $4 - (-2) =$

    h. $(-8) - (-6) =$

19. Solve each of the following:

a. $3 \times 4 =$

b. $(-3) \times 4 =$

c. $(-3) \times (-4) =$

d. $3 \times (-4) =$

e. $3 \times 3 =$

f. $(-3) \times (-3) =$

g. $9 \div 3 =$

h. $(-9) \div (-3) =$

i. $(-9) \div 3 =$

j. $9 \div (-3) =$

k. $15 \div (-5) =$

l. $(-20) \div (-2) =$

m. $25 \div 5 =$

n. $(-30) \div 6 =$

# Lesson 2.12
# Algebraic Equations and Inequalities

## Learning Objectives

*To become familiar with some of the basic symbols used in algebraic equations and inequalities*
*To memorize and apply the Order of Operations Rule for solving equations*
*To practice setting up equations and inequalities based on the information given in word problems*

The ability to solve equations and inequalities is one of the basic skills you will need in order to work algebra problems. First, you will need to review what equations and inequalities are, and how they can be used to find the answers to problems that you might encounter at work.

## Vocabulary

*equation*—a mathematical sentence containing an "=" sign which states that two expressions are equal.
*variable*—a symbol, usually a letter, that holds the place of a number when the value of the number is unknown. Variables can be either capital letters (A, B, C, X, Y, etc.) or lowercase letters (a, b, c, x, y, etc.).
*inequality*—a mathematical sentence that states that two expressions are *not* equal

## Develop Your Understanding

### *Equations*

An equation is a mathematical statement containing the symbol "=." When you see the "=" symbol, it means that the numbers or expressions on one side of the symbol have the same value as those on the other side, even if they do not look the same.

Some examples of equations are:

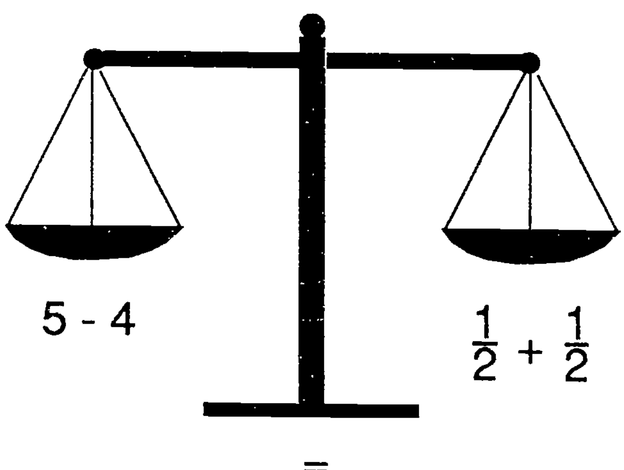
$2 + 3 = 5$

$5 - 4 = \frac{1}{2} + \frac{1}{2}$

$3 \times 6 = 9 \times 2$

$-4 \times -2 = 8 \div 1$

An equation is like a scale that is perfectly balanced:




You can check an equation to make sure it is true. For example:

$5 - 4 = \frac{1}{2} + \frac{1}{2}$

$5 - 4 = 1$

$\frac{1}{2} + \frac{1}{2} = 1$

Since 1 = 1, this equation is true.

In order for an equation to remain in balance, anything that is done to the left side must also be done to the right side. If, for example, 200 is added to both sides of an equation, it will remain true:

$$5 - 4 = \frac{1}{2} + \frac{1}{2}$$

$$200 + 5 - 4 = \frac{1}{2} + \frac{1}{2} + 200$$

**Now Try These**

Check each equation to make sure it is true.

a. $1 - \frac{1}{2} = 0.25 + 0.25$

b. $3 \times 9 = 100 - 75$

c. $45 \div 5 = -6 + 15$

### *Solving Equations: The Order of Operations*

You are probably used to seeing open-ended equations where you are given the left side of the equation only and must solve it to find the right side. This is simple when there is only one operation involved:

$46 - 28 =$

To solve this equation, you would simply subtract 28 from 46 to get 18.

What happens when there is more than one operation in an equation? How would you go about solving it? There is a simple rule for this, called the *Order of Operations Rule.*

**Order of Operations Rule**

1. First, do anything contained inside parentheses.
2. Perform multiplication and division operations, moving from left to right.
3. Finally, add and subtract, moving from left to right.

You will have a chance to work with parentheses in the next section. For the following example, apply only rules 2 and 3.

*Example 1*

*Solve the equation:*

$12 - 3 \times 2 =$

Applying the Order of Operations Rule, you would first multiply or divide from left to right:

$3 \times 2 = 6$

The equation now looks like this:

$12 - 6 =$

Now, add or subtract from left to right:

$12 - 6 = 6$

Therefore, $12 - 3 \times 2 = 6$

**Now Try These**

a. $6 - 7 \times 3 + 15 =$

b. $-10 \div 5 \times 6 + 4 =$

c. $85 - \frac{2}{3} \times 3 =$

### *The Uses of Parentheses*

Some equations contain parentheses. Parentheses are used in a variety of ways:

- Mathematical expressions (including operations) that are inside parentheses should be simplified *first* when solving an equation:

  $9 \times (3 - 6) = 9 \times (-3) = -27$

- When a number appears immediately in front of a set of parentheses, the expression inside the parentheses is multiplied by that number after it is simplified:

  $9(2 + 4) = 9(6) = 9 \times 6 = 54$

- When two sets of parentheses appear side by side, the expression inside the first one should be multiplied by the expression in the second one after both are simplified:

  $(-2)(-3) = -2 \times -3 = 6$

  $(7 - 12)(4 \div 2) = (-5)(2) = -10$

| |
|---|
| **Remember, always simplify all expressions contained in parentheses before you go on to solve the equation.** |

*Example 2*

How would you solve the following equation? Would the answer be the same if you moved from left to right without following the Order of Operations Rule?

$4 - 2(6 - 2) \div 2 =$

First, to solve the equation properly, simplify everything inside parentheses:

$4 - 2(6 - 2) \div 2 = 4 - 2(4) \div 2$

Next, multiply and divide from left to right:

$4 - 2(4) \div 2 = 4 - 8 \div 2 = 4 - 4$

Finally, add and subtract from left to right:

$4 - 4 = 0$

The solution is 0. Would you get the same answer if you just did the problem from left to right without worrying about parentheses or the type of operation? Try just moving from left to right, ignoring parentheses and the Order of Operations Rule. Your answer will be something like 5. Be sure to follow the order of operations to arrive at the correct answer.

**Now Try These**

Solve each equation. Be sure to write down all the steps.

a. $2(10 - 12) =$

b. $3 - (-8 - 6) =$

c. $3(-4 \times 3) + 2(6 - 7) =$

d. $100 - 7(-3 - 4 + 2) =$

### *Variables*

Sometimes an equation contains a *variable*. Variables are letters of the alphabet, used to take the place of a number when we do not know its value. Some examples of equations containing variables are:

| | |
|---|---|
| $2 + a = 7$ | "a" is the variable |
| $R(12) = 120$ | "R" is the variable |
| $\frac{3 - n}{2} = 5$ | "n" is the variable |
| $6(b) = b + 10$ | "b" is the variable |

*(A variable may appear more than once in the same equation.)*

When a variable is written next to a number, this means it is multiplied by that number:

5r means "5 times r"

16j means "16 times j"

6q = 18 means "6 times q equals 18"

Once you know the value of a variable, you can substitute the value into the equation to replace the variable.

*Example 3*

Solve the equation $7z = ?$ where $z = 4$. Since you know that $z = 4$, replace z with 4:

$7(4) = ?$

*When a number replaces a variable in this way, use parentheses to show the difference between 7(4) and 74.*

$7(4) = 28$

*Example 4*

When you have an *open-ended* equation with a variable to solve, the variable can be any number you choose:

$3 - x = ?$

For the value of x, let's select 3, 15, and -290.

where $x = 3$, $3 - x = 3 - 3 = 0$

where $x = 15$, $3 - x = 3 - 15 = -12$

where $x = -290$, $3 - x = 3 - (-290) = 3 + 290 = 293$

*Example 5*

When an equation with one variable is *closed* (both sides of the equation are shown), there will be only one solution.

$16T - 25 = 7$

$3v - 18 = v$

In each of these cases, there is only one possible value for T or v. You will learn how to solve problems of this type in Lesson 2.13.

**Now Try This**

For each open-ended equation below, solve for the values of each variable indicated.

a. $2d - 1 = ?$ where $d = -5$ and $d = 30$

b. $3g = ?$ where $g = -1$ and $g = 3$

c. $22 - 2f = ?$ where $f = 15$ and $f = -11$

### *Inequalities*

We have explored *equations*, which are mathematical statements that two expressions have the same value. An *inequality* is a statement that shows that two expressions have different values and compares them.

The symbols ">" and "<" are used in inequalities in place of the "=" sign.

*Example 6*

When the closed end of the inequality sign points to the right, the sign is read "is greater than."

> When you see:
> $5 > 3$
> Read:
> "Five is greater than three."

When the closed end of the inequality sign points to the left, the sign is read "is less than."

> When you see:
> $8 < 10$
> Read:
> "Eight is less than ten."

| |
|---|
| *Remember, the open end of the inequality sign points to the larger value; the closed end points to the smaller value.* |

**Now Try These**

Insert the symbol "<", ">" or "=" in the space between each set of values:

a. 24 ______ 30 b. 1.254 ______ 1.2 c. $\frac{1}{2}$ __________ 2

d. $\frac{1}{2}$ __________ 0.5 e. 300 ______ 296 f. 3 ______ $\frac{19}{6}$

***Inequalities with Variables***

Sometimes a variable may appear in an inequality.

$x > 4$ may be read "x is greater than 4"

We don't know the exact value of x, but we know it could be any number larger than 4, including a fraction or decimal value.

Sometimes, an inequality can show a value that is between two other numbers:

$3 < x < 17$ may be read "x is greater than 3 and less than 17"

$55 > y > 22$ may be read "y is less than 55 and greater than 22"

If we know that the value of y is between 55 and 22, then $y = 20$ could not possibly be a true statement.

**Now Try These**

For each example, look at the inequality. Then write "possible" or "impossible" for each value of the variable given. The first one is done for you:

| Inequality | Value | Is the value possible? |
|---|---|---|
| a. $13 < d$ | $d = 11$ | *impossible (11 is not greater than 13)* |
| b. $45 > M$ | $M = 0$ | |
| c. $-3 < p$ | $p = -1$ | |
| d. $100 < z < 101$ | $z = 100.734$ | |
| e. $7 < L < 9$ | $L = 6$ | |

### *Converting Words to Equations and Inequalities*

When you solve word problems, the most challenging task is translating what is told to you in words into mathematical equations or inequalities that you can then solve. Below are several examples that should help you.

*Example 7*

How would you set up an equation to solve this problem?

*Two-thirds of the panels produced on Thursday needed to be rewired. The number of panels needing rewiring was 512. How many panels were produced on Thursday in all?*

You know how many panels needed rewiring, but you don't know how many were produced.

First, *assign a variable to the unknown number:*

let N = total number of panels produced

Now, use the information you already have to set up a relationship between the variable and known values:

Two-thirds of the panels is equal to 512.

$$\frac{2}{3} N = 512$$

You could then go on to solve the equation mathematically and find the answer to the problem. You will study this procedure in the lessons that follow.

*Example 8*

*When the 25 employees at a small Plink Industries distribution office gave a staff party, they all shared the cost of the food. The organizers of the party promised that no one would have to spend more than $6.00. Fifteen of the employees, "The High Rollers," each spent twice as much as the other ten ("The Budgeteers"). Each High Roller spent the same amount as every other High Roller; each Budgeteer spent the same amount as every other Budgeteer. The total spent came to $100. How much did each of the* ***Budgeteers*** *spend? Set up the equation(s) or inequalities you would need to solve this problem.*

First, what is the value you need to know? It is the amount of money spent by each *Budgeteer*. Let's call this "B":

B = amount (in dollars) spent by each Budgeteer.

You know that nobody spent *more than* $6.00; therefore, B must be *less than* $6.00:

$$B < 6$$

Each High Roller spent twice as much as each Budgeteer. Therefore, the amount spent by a High Roller is the amount spent by a Budgeteer multiplied by 2. Since the amount spent by a Budgeteer is "B," then:

$2B$ = amount (in dollars) spent by each High Roller.

The total spent was exactly $100. This is the total of ten times "B," or the amount each Budgeteer spent, plus fifteen times "2B," or the amount each High Roller spent:

$$10B + 15(2B) = 100$$

AND, $B < 6$

You now have all the information in the problem in mathematical form. In the next two lessons, you will solve problems of this type.

**Now Try These**

Write the information given in each problem in the form of equations or inequalities, choosing your own variables.

a. Molly ordered three times as many pairs of safety glasses for her area in January as she did in February. The total number of pairs ordered for the two months was 24. How many pairs did she order in February?

b. Donald has boxed and shipped a total of 1,569 books in the last three weeks. How many books has he averaged per week?

## Skill Builders

1. Is this equation true? $30 - 18 = 24 \times \frac{1}{2}$

2. Is this equation true? $1{,}800 \times 0.01 = 3 \times 5$

3. $144 - (2 \times 8) \div 4 + 18 =$

4. $25 - (45 \times \frac{1}{5}) + (10 \div 2) =$

5. $(-5) \times (-3) - (14) =$

6. $(2)(4)(\frac{1}{8}) =$

7. $3 - 2(-6 + 4) \div 2 =$

8. $-(12 - 6)(3 \times 4) =$

9. $88 + (-5 - 11) \times (-3 + 5) =$

For problems 10-12, substitute each value given for the variable into the equation and solve it.

10. $3x + 17 = ?$

    a. where $x = 3$

    b. where $x = -3$

11. $y \div 2 + 114 = ?$

    a. where $y = 100$

    b. where $y = 1$

12. $6A - A = ?$

    a. where $A = 11$

    b. where $A = -1$

13. If $x < 25$, is it possible that $x = 0.002$?

14. Fill in ">" or "<" for each blank.

a. $21$ ______ $3(8)$

b. $4x$ ______ $92$ if $x = 26$

c. $(\frac{1}{2})(\frac{1}{4})$ ______ $\frac{1}{4}$

d. $0.925$ ______ $0.9205$

15. If $1 < w < 2$, what is one possible value for $w$?

For problems 16-20, use the information given to set up equations or inequalities using variables. DO NOT try to solve the problems!

16. Frieda bought and ate a candy bar on Monday. On Tuesday, she bought two candy bars and ate them both. On Wednesday, she bought three candy bars and only ate two; on Thursday she ate the one that was left. On Friday, Frieda bought and ate four candy bars. Her total spent on candy bars for the week was $6.40. If each of the bars was the same price, what is the price of one candy bar?

17. Zelda drank more than thirty 6-packs of Rhubarb Delight soda pop last year. Is it possible that she drank only 174 cans of this delicious beverage?

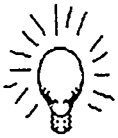
18. Phil sees a lot of movies. He saw an average of 4 per week during a certain period of time. During that time, he saw a total of 52 movies. How many weeks were included in that period?

19. When the budget for Plink Industries was decided last year, the Marketing Department received five times as much money as the Special Projects Department. The manager of the Special Projects Department complained, and got an additional $80,000 added to her annual budget. As a result, the two departments wound up with the same amount of money. How much did Special Projects get before the additional money was given?

# Lesson 2.13
# Addition and Subtraction, Solving for an Unknown

## Learning Objectives

*To understand and practice the method used to find the value of an unknown number using addition and subtraction*



In everyday life and in the workplace, there may be times when you will need to know the value of an unknown number. Many times you may figure it out by reasoning through the problem or situation. In this lesson, you will examine a very simple method to use whenever you need to find the value of an unknown number.

## Vocabulary

*A, B, X, Y or any letter used in a mathematical problem*—the letter simply stands for an unknown value, another way of saying "I don't know." In the previous lesson, these letters were called *variables*.

*substitution*—replacing a variable with the number that it stands for.

## Develop Your Understanding

In the last lesson, you learned about algebraic equations, what they stand for, and basically how to work with them. In this lesson you will apply this knowledge as you find the value of unknown variables in addition and subtraction problems.

Remember that any time you see a variable in a math problem, the letter simply means "I don't know" and stands for an unknown number. Variables are useful because they remind you that there is a particular number that belongs there. As soon as you know what the number is, you can substitute it for the variable.

As you learned in the last lesson, an algebraic equation is simply a mathematical sentence with an equal sign in the middle. It means that one side equals the other side. Because the two sides are equal, you must keep them that way when working with the problem. Therefore, whatever you do to one side, you must also do to the other side.

### *Addition*

In order to solve an algebraic equation, you will need to isolate the variable on one side of the equal sign. When the variable is by itself on one side of the equal sign, the other side of the equation will tell you the value of the variable.

*Keep in mind that equations must remain balanced. Therefore, you must do the same thing to* ***both*** *sides of the equation.*

**Steps in Solving for an Unknown: Using Addition**

1. Look at the equation and find what operation is used.

$$b - 3 = 5$$

In the above equation, 3 is subtracted from some number to get 5. You could probably work this out by reasoning through it, but it is useful to practice the steps to solve equations with an easy problem like this.

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

$$b - 3 + 3 = 5 + 3$$

The opposite of subtraction is addition. Therefore, **add** 3 to **both** sides of the equation. When (+3) is added to (-3), the opposites cancel each other out to zero:

$$b - 3 + 3 = 5 + 3 =$$
$$b - 0 = 5 + 3$$

3. Simplify both sides of the equation.

$$b = 5 + 3 \qquad b = 8$$

Zero subtracted from any number leaves that number. The value of the variable "b" is 8.

4. Check your answer by substituting it into the original equation and solving.

$$8 - 3 = 5$$

Yes, the equation is true. "b = 8" is the correct solution.

**Now Try These**

Use the steps demonstrated above to find the value of the following equations. Remember to get the letter by itself by doing the opposite mathematical operation of that done on the letter side, and applying this to both sides of the equation.

a. $x - 22 = 14$

b. $V - 5 = 95$

c. $d - 12 = 18$

### *Subtraction*

In the above examples, you solved equations by addition. The same rules hold for subtraction. If you examine an algebraic equation and find that you are adding a number to the letter side, perform the opposite operation, which is subtraction, to get the letter by itself.

**Steps in Solving for an Unknown: Using Subtraction**

1. Look at the equation and find what operation is used.

$$F + 6 = 20$$

In the above equation, 6 is added to some number to get 20.

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

   To find out what that number is, simply subtract 6 from **both** sides of the equation.

   $F + 6 - 6 = 20 - 6 \qquad F + 0 = 20 - 6$

3. Simplify both sides of the equation.

   $F + 0 = 20 - 6 \qquad F = 14$

4. Check your answer by substituting it into the original equation and solving.

   $14 + 6 = 20$

   Therefore, 14 is the correct value for "F."

**Now Try These**

Solve the following equations.

a. $J + 10 = 75$

b. $m + 8 = 23$

c. $P + 4 = 48$

***Word Problems***

In everyday life, equations to find an unknown value do not compose themselves. You must apply the facts of a situation and set up an equation on your own, and then solve for the unknown number.

To do this, apply the skills you have learned in earlier lessons. First, identify what the key information is and then what mathematical operation should be applied.

*Example*

The Blue Team finished a certain number of panels during week 1, 82 panels in week 2, and 28 panels in week 3. They also had to scrap 29 panels. This means that the total yield was 125 panels. How many panels did they produce in week 1? Solve this by setting up an equation as follows:

Let X = the number of panels finished during week 1

$$X + 82 + 28 - 29 = 125$$

Begin by performing the mathematical operations as shown and combining 82, 28, and -29 into one number. After that, you want to get the X by itself, so you will do the opposite operation to any numbers on the letter side of the equation.

$$X + 82 + 28 - 29 = X + 81$$

Now, substitute this simplified number into the equation:

$$X + 81 = 125$$

Isolate the variable by performing the opposite operation:

$$X + 81 - 81 = 125 - 81$$

$$X = 44$$

Put 44 into the original equation to check the answer:

$$44 + 82 + 28 - 29 = 125$$

Therefore, they produced 44 panels during week 1.

**Now Try These**

Our production this week included 78 layers for one job, 32 for another, and an unknown number for a third. We do know that our final numbers included 12 scrapped (discarded) layers and a final figure of 140 layers produced after the 12 were scrapped. How many layers were produced in the third job? Set up your equation and solve for the unknown.

## Skill Builders

Solve for the unknown variable.

1. $a - 10 = 25$

2. $B + 12 = 12$

3. $n - 25 = 60$

4. $K + 15 = 75$

5. $j - 23 = 35$

6. $V + 47 = 22$

7. $p - 23 = -12$

8. $C - 5 + 6 = 15$

9. $d + 12 - 8 = 22$

10. $E - 6 - 7 = 225$

11. $f + 18 + 23 - 14 = 62$

12. $R + 22 - 7 + 4 = 10$

13. $c + 105 - 23 = -40$

14. $T + 19 + 58 = -20$

15. $y + 23 - 40 - 9 = 0$

## Check Yourself

Set up and solve each of the following equations.

1. We ordered the following number of safety smocks during November: 23, 46, 12, 74. When we finished the month, 48 had been discarded, and some replacements ordered. Our inventory showed 125 on hand at the end of the month. How many replacement safety smocks were ordered?

2. Our production numbers for January were 5% higher than those in December. Our December production was 595 layers completed. If we had to discard a certain number for defects and our final January production was 590, how many defective layers were produced? (Round your answer to the nearest whole number.)

# Lesson 2.14
# Multiplication and Division, Solving for an Unknown

## Learning Objectives

*To understand and practice the method used to find the value of an unknown number using multiplication and division*

When an equation contains an unknown or variable, algebra is used to find the value of that unknown. In this lesson, you will develop skill in solving for an unknown in equations using multiplication and division, and then apply that skill to some common problems found in the workplace.

## Develop Your Understanding

### *Multiplication*

In the previous lesson, you practiced solving for an unknown in addition and subtraction equations by using the opposite operation. Multiplication and division are opposites in the same way that addition and subtraction oppose each other. To solve for an unknown in a multiplication equation, use division.

### Steps in Solving for an Unknown: Using Division

1. Look at the equation and find what operation is being done.

   $3x = 12$

   Ask yourself, "what has been done to the variable?" Here, it has been multiplied by 3.

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

   The opposite of multiplication is division.

   $\frac{3x}{3} = \frac{12}{3}$ Divide each side by 3.

   $\frac{\cancel{3}}{\cancel{3}}x = \frac{12}{3}$ The 2 "3's" in the fraction on the left cancel each other out.

3. Simplify both sides of the equation.

   $x = \frac{12}{3}$ $12 \div 3 = 4$

   $x = 4$

   The value of the variable "x" is 4.

4. Check your answer by substituting it into the original equation and solving.

   $3(4) = 12$

   The equation is true, so $x = 4$ is the correct answer.

**Now Try These**

Solve for the value of each variable.

a. $4b = 8$

b. $7c = 63$

*Division*

To solve for an unknown in a division equation, use multiplication.

**Steps in Solving for an Unknown: Using Multiplication**

1. Look at the equation and find what operation is used.

   $\frac{x}{2} = 6$

   Ask yourself, "what is being done to the variable?" Here, the variable "x" is divided by 2. Remember that the fraction bar indicates division. In other words, $\frac{x}{2}$ can be read "x divided by 2."

2. Isolate the variable by applying the **opposite** of whatever math operation is being done.

   The opposite of division is multiplication.

   $2 \times \frac{x}{2} = 6 \times 2$ Multiply both sides by 2.

   $\frac{\cancel{2}x}{\cancel{2}} = 6 \times 2$ $6 \times 2 = 12$

3. Simplify both sides of the equation.

   $\frac{\cancel{2}}{\cancel{2}}x = 12$ The 2s in the fraction on the left cancel each other out.

   $x = 12$

   The value of the variable "x" is 12.

4. Check your answer by substituting into the original equation and solving.

   $\frac{12}{2} = 6$

   The equation is true, so $x = 12$ is the correct answer.

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## Now Try These

Solve for the value of each variable.

a. $G \div 6 = 5$

b. $\frac{m}{9} = 3$

### *Special Types of Problems*

*Proportions*

In an earlier lesson, you learned that an equation in which two ratios are equal is called a *proportion*. You can cross-multiply and solve for the unknown as follows:

$\frac{7}{5} = \frac{91}{g}$ Find the cross-products, and set them as equal.

$\frac{7}{5} \quad \times \quad \frac{91}{g}$

$7g = 5\ (91)$

$7g = 455$

$\frac{\cancel{7}g}{\cancel{7}} = \frac{455}{7}$ Divide both sides by 7 to isolate the *g*.

$g = \frac{455}{7}$ Solve for *g*.

$g = 65$

*Fractions of a Variable*

Just as a variable can be *part* of a fraction, a variable can also be multiplied by a fraction. In this case, treat the variable as a whole number. Any whole number can be written as a fraction using a denominator of 1.

$\frac{1}{3}y$ means "$\frac{1}{3}$ times the variable y"

$$\frac{1}{3}y = \frac{1}{3} \times \frac{y}{1} = \frac{1 \times y}{3 \times 1} = \frac{y}{3}$$

To solve an equation containing a fraction of a variable, simplify first:

$$\frac{1}{3}y = 5$$

$$\frac{1}{3}y = \frac{y}{3}$$

$$\frac{y}{3} = 5$$

Now solve as a proportion:

$$\frac{y}{3} = \frac{5}{1} \qquad y \times 1 = 3 \times 5$$

$$y = 15$$

The value of the variable "y" is 15. Substitute into the original equation to check:

$$\frac{1}{3}(15) = 5 \qquad \frac{1}{3} \times \frac{15}{1} = \frac{1 \times 15}{3 \times 1} = \frac{15}{3} = 5$$

The equation is true.

## Now Try These

Solve for the value of each variable.

a. $\frac{8}{x} = 4$

b. $\frac{1}{3}z = 9$

## Skill Builders

Solve and check each multiplication equation.

1. $9y = 126$

2. $4a = 52$

3. $\frac{1}{2}y = 10$

4. $91 = 13x$

5. $\frac{1}{5}x = 2$

6. $5x = 15.5$

7. $2w = 32$

Solve and check each division equation.

8. $\frac{a}{8} = 16$

9. $\frac{w}{7} = 21$

10. $\frac{x}{2} = 10$

11. $\frac{d}{25} = 5$

12. $\frac{x}{9} = -9$

13. $\frac{c}{16} = \frac{1}{2}$

14. $\frac{k}{112} = 2$

Use all problem-solving skills, and solve the following:

15. Sandra spends 22% of her gross salary to buy mutual funds through her company. This 22% equals $440.00 a month. What is Sandra's monthly gross salary?

16. The artist at Goodway Graphics was told to draw a map for their delivery truck driver using 2 inches to represent 150 miles. How many inches will represent 500 miles?

17. If 5 times a number is equal to 25, what is the number?

18. A thin metal disc is 3 millimeters thick. If 120 of these discs are to be stacked in the yellow room at XI, how many centimeters thick will the stack be?

19. If a number is divided by 7, and the quotient is 7, what is the number?

20. Patrick was told to find the voltage of an electrical current which has 12 amperes of electrical current in the circuit. Let V = the number of volts. Using $V = \frac{10}{3} \times 12$, solve for V.

## Check Yourself

Solve the following equations:

1. $0.2x = 10$

2. $2x + 4 = 10$

3. $2x - 0.4 = 10$

4. $\frac{x}{2} = 10$

5. $\frac{1}{2}x = 10$

6. $x + \frac{1}{2} = 10$

# Lesson 2.15
# Working with Formulas

## Learning Objectives

*To solve math problems using formulas*

A formula is a general guideline for solving similar types of problems. As you work with formulas, you will need to apply most of your math skills. The key to using formulas successfully is substituting accurate numbers for variables.

## Vocabulary

*formula*—an equation that can include letters (variables), symbols, and numbers. A formula is a shorthand set of instructions that states the solution to a problem.

## Develop Your Understanding

Look at these examples of formulas:

| | |
|---|---|
| $A = L \times W$ | Area = Length × Width |
| $I = prt$ | Interest = principal × rate × time |
| $D = r \times t$ | Distance = rate × time |

When you work with formulas, you will be given the formulas and the definitions of those formulas. You will also be given the values (numbers) of all of the letters (variables) except one. You must solve the problem to find the unknown value.

*Example 1*

*Find the area of a floor when the length is 20 feet and the width is 15 feet.*

| | |
|---|---|
| $A = L \times W$ | Write the formula. |
| $A = 20$ feet $\times$ 15 feet | Substitute the values. |
| $\begin{array}{r} 20 \\ \times \underline{15} \\ 100 \\ \underline{20\phantom{0}} \\ 300 \end{array}$ | Solve for the unknown. |

A = 300 square feet

You were given the values for length and width. You solved the problem to find the unknown value. You found that the area was 300 square feet. When solving problems using formulas, follow these steps.

1. Write the formula. *(Do not skip this step!)*
2. Substitute the values for the letters.
3. Solve for the unknown value.

**Now Try These**

a. Find the area when the length is 6 feet and the width is 3 feet.

$A = L \times W$

b. Find the gas mileage (in miles per gallon) when you traveled 300 miles and used 20 gallons of gasoline.

$G = \frac{m}{g}$

$$\text{Miles per gallon} = \frac{\text{miles you drove}}{\text{gallons of gas you used}}$$

You can even make your own formula.

*Example 2*

Write a formula for total earnings for the week. Kim's regular earnings are $400 per week. His overtime earnings gave him an additional $60.

Total earnings = regular earnings + overtime

The formula can be written using letters to stand for the definitions.

| | |
|---|---|
| T = E + O | T = total earnings |
| | E = regular earnings |
| | O = overtime |

Solve the equation by plugging in numbers for the letters.

T = $400 + $60

T = $460

You solved for the unknown value and found that the total earnings are $460.

**Now Try These**

a. Write a formula for the number of seconds in any number of hours.

b. Write a formula for the number of days one employee works every year.

Some formulas require several math operations. You will need to know the order of operations to solve these more complex formulas.

***Order of Operations***

1. Do everything inside the parentheses first.
2. Do all multiplication and division in order going from left to right.
3. Do all addition and subtraction in order from left to right.

*Example 3*

Verna is joining a bowling league. Her average score is 120. Use the formula below to find Verna's bowling handicap.

Handicap = 0.8 (200 – average)

| | |
|---|---|
| H = 0.8(200 – a) | Write the formula. |
| H = 0.8(200 – 120) | Substitute the values. |
| | Solve for the unknown by using the order of operations |
| H = 0.8(80) | Parentheses first |
| $\begin{array}{r} 80 \\ \times\, 0.8 \\ \hline 64.0 \end{array}$ | Multiply |
| H = 64 | |

Verna's handicap is 64 points.

**Now Try These**

a. Find Frank's bowling handicap if his average is 150.
Handicap = 0.8 (200 – average)

b. Find the Celsius temperature reading when the Fahrenheit temperature reading is 41°. Use the formula below where °C = Celsius temperature and °F = Fahrenheit temperature.

$$°C = \frac{5}{9}\ (°F - 32)$$

**Skill Builders**

1. Use the formula to find the distances traveled.

   Distance = Rate × Time
   D = R × T

   a. rate = 60 miles per hour
      time = 3 hours

   b. rate = 25 miles per hour
      time = 0.5 hour

2. Find the etch rate for removing a metallic layer in each situation below.

   $$\text{Etch Rate} = \frac{\text{thickness before - thickness after}}{\text{etch time}}$$

| *Etch time* | *Thickness before etch* | *Thickness after etch* | *Etch rate* |
|---|---|---|---|
| a. 10 minutes | 16.4 mm | 10.2 mm | _____per min. |
| b. 4 minutes | 7.8 mm | 6.8 mm | _____per min. |
| c. 30 minutes | 48.86 mm | 33.26 mm | _____per min. |

3. Find the interest to be paid on the following loans.

Simple Interest = Principal × Rate × Time
I = PRT

| *Amount of Interest* | *Principal* | *Rate* | *Time* |
|---|---|---|---|
| a. $__________ | $10,000.00 | 8% | 4 years |
| b. $__________ | $600.00 | 18% | 1 year |

4. Find the pe~centage of computer parts that passed final inspection in each case below.

Yield = $\frac{\text{quantity out}}{\text{quantity in}}$ Change this to % form to get *percent yield.* *(round to the nearest tenth of a percent)*

| *Quantity Out* | *Quantit · In* | *% Yield* |
|---|---|---|
| a. 420 | 448 | _____ |
| b. 1,086 | 1,200 | _____ |

5. Compare the productivity of two employees working as stockkeepers. Use the productivity formula for nonselling employees to find which employee was more productive during a two-week period.

$$\text{Units processed per \$1} = \frac{\text{units of work}}{\text{total pay to do work}}$$

| *Employee* | *Processed Cases of Stock* | *Total Pay* |
|---|---|---|
| Keith | 3,488 | $640.00 |
| Glenn | 4,044 | $800.00 |

6. A car is purchased for $9,450. The credit union requires a 15% down payment. How much is the down payment, and how much will be financed?

Down Payment = Rate(%) × Whole

7. Find the mortgage on a home when the purchase price is $65,000 and the down payment is 20% of the purchase price. Use both formulas given.

   a. Down payment = Rate(%) × Whole

   b. Purchase price − down payment = mortgage

# Lesson 2.16
# Graphing Ordered Pairs

## Learning Objectives

*To understand the concept of a coordinate system grid*
*To determine the coordinates of points on a grid*
*To graph points on a grid using coordinates*

In the *Communications* book, Lesson 1.7, you will study three basic types of graphs that are frequently used in business. In this lesson, you will work with one additional type of graph, called *a coordinate system grid.* Coordinate system grids are used in drafting and mapping and also on some manufacturing machinery. Becoming familiar with coordinate system grids will assist you in reading blueprints, maps, and schematics more effectively.

## Vocabulary

*grid*—a system of evenly spaced horizontal and vertical lines that represent units of measurement
*coordinate system*—a set of points located on a grid
*x-axis*—the horizontal base line of a grid
*y-axis*—the vertical base line of a grid
*origin*—the point where the x-axis intersects the y-axis
*coordinates*—the two numbers that show the location of a point on a grid
*ordered pair*—a set of coordinates in the form (x,y)
*quadrant*—any of the four parts of a coordinate system grid

## Develop Your Understanding

### *Grids and Coordinate Systems*

Technical diagrams, blueprints, and schematics must be very precise. Because of the need for precise measurement, these visual aids are usually made using a grid. A *grid* is a system of evenly spaced horizontal and vertical lines that represent units of measurement. Grids are used to locate or plot points. A set of points on a grid is called a *coordinate system*. An example of a grid is shown below.

Coordinate System Grid

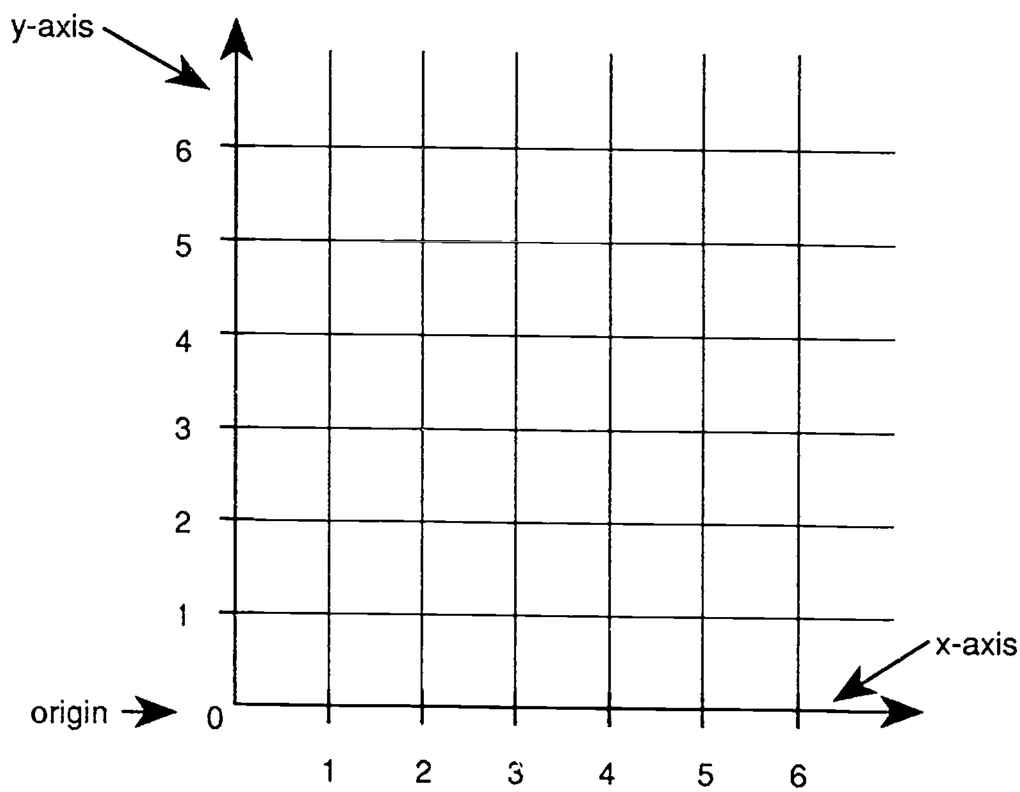



*Figure 1*

The grid in figure 1 shows only positive values.

The horizontal line at the bottom is called the *x-axis*. The vertical line at the far left is called the *y-axis*. The arrows at the ends of the x-axis and the y-axis show that, in theory, these lines can extend infinitely. In practice, you can make a grid as large or as small as needed for your purpose.

The point where the x-axis meets the y-axis is called the *origin*.

To locate a point on a grid, you would need to know the point's *coordinates*. The coordinates of a point are the two numbers that show the point's horizontal and vertical locations. The horizontal coordinate, or $x$-value, is always given first. The vertical coordinate, or $y$-value, is always given second. The two coordinates together are called an *ordered pair*. Ordered pairs are written in the form (x,y), with numbers plugged in to replace the letters. For example, the coordinates of the origin are (0,0).

*Example 1*

*Find the point indicated by the ordered pair (3,2).*

In the ordered pair (3,2), 3 is the x-coordinate, and 2 is the y-coordinate.

1. Start at the origin, and count 3 places to the right along the x-axis. This is the location of the $x$-value 3.
2. Count 2 units up from the $x$-axis point 3.
3. You should now be at the intersection of the lines at coordinate 3 on the $x$-axis and coordinate 2 on the $y$-axis, which is the point (3,2).

The point (3,2) is marked on the grid in figure 2 with the letter "A."

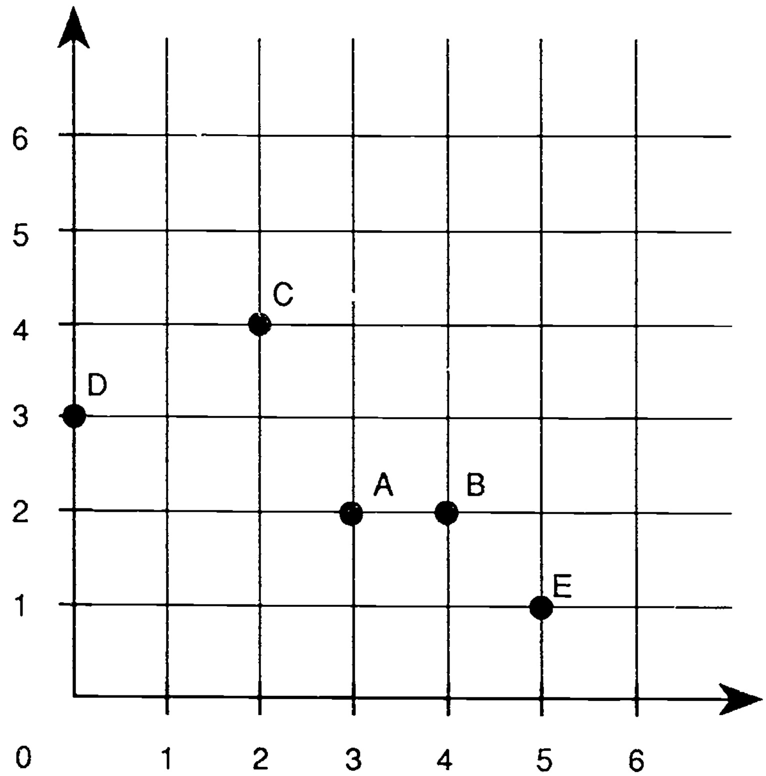



*Figure 2*

**Now Try These**

Look at figure 2, and fill in the blanks with the letters that mark these points:

a. (2,4) ________________

b. (4,2) ________________

c. (5,1) ________________

d. (0,3) ________________

### *Four-Quadrant Coordinate Systems*

Many coordinate system grids use both positive and negative numbers. This type of grid, like the ones you have already used, includes an $x$-axis and a $y$-axis that cross at the origin. This time, however, there are arrows at both ends of each axis, to show that they can be extended infinitely in both directions.

The $x$-axis in this type of system looks like a horizontal number line:

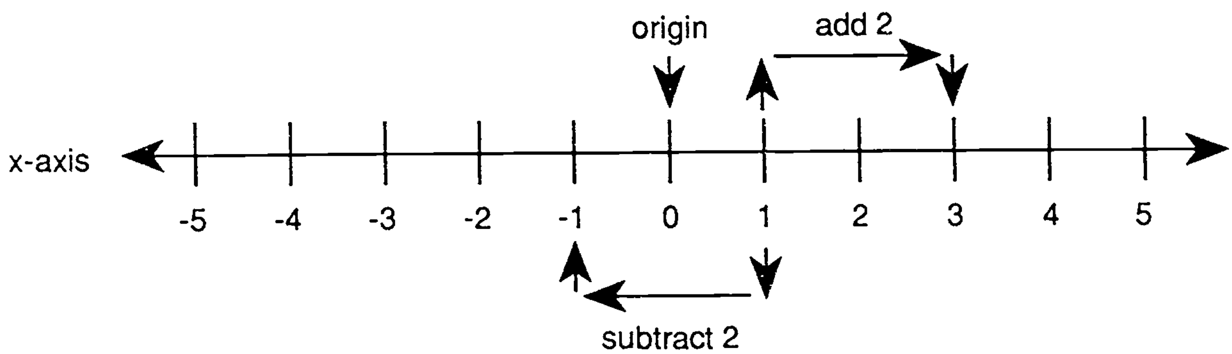



*Figure 3*

Note that integers are evenly spaced along the $x$-axis. Positive numbers are always on the right of the origin (the point 0), while negative numbers are always on the left. Moving along the axis to the right is like adding. For example, if you start at the $x$-value 1 and move 2 units to the right, you will arrive at the $x$-value 3.

This is the same as adding:

$$1 + 2 = 3$$

On the other hand, moving along the axis to the left is like subtracting. If you start at the $x$-value 1 and move 2 units to the left, you will arrive at the $x$-value -1.

This is the same as subtracting:

$$1 - 2 = -1$$

The $y$-axis works in the same way, except that this number line is vertical.

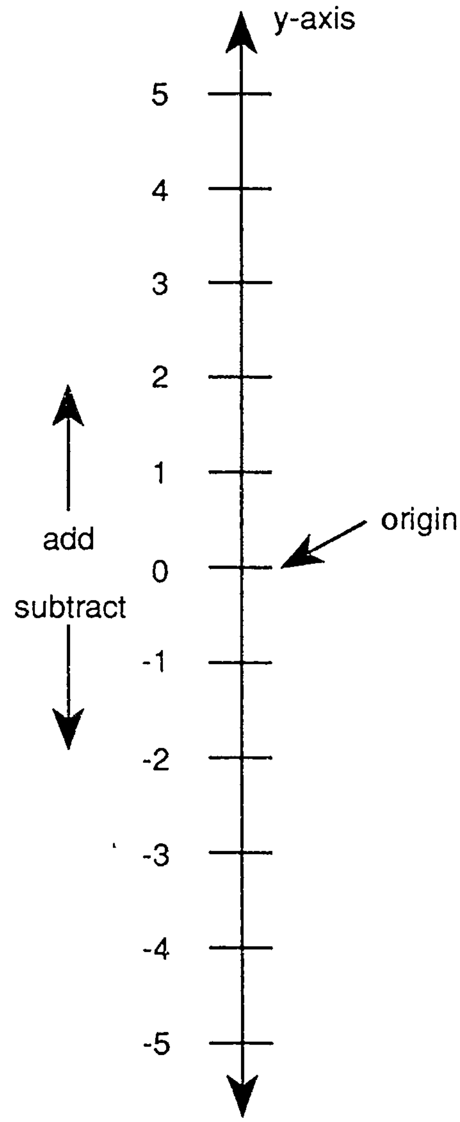



*Figure 4*

On the $y$-axis, values above the origin are positive, and values below the origin are negative. Therefore, moving up is the same as adding, and moving down is the same as subtracting.

An example of a complete coordinate system grid is shown in figure 5.

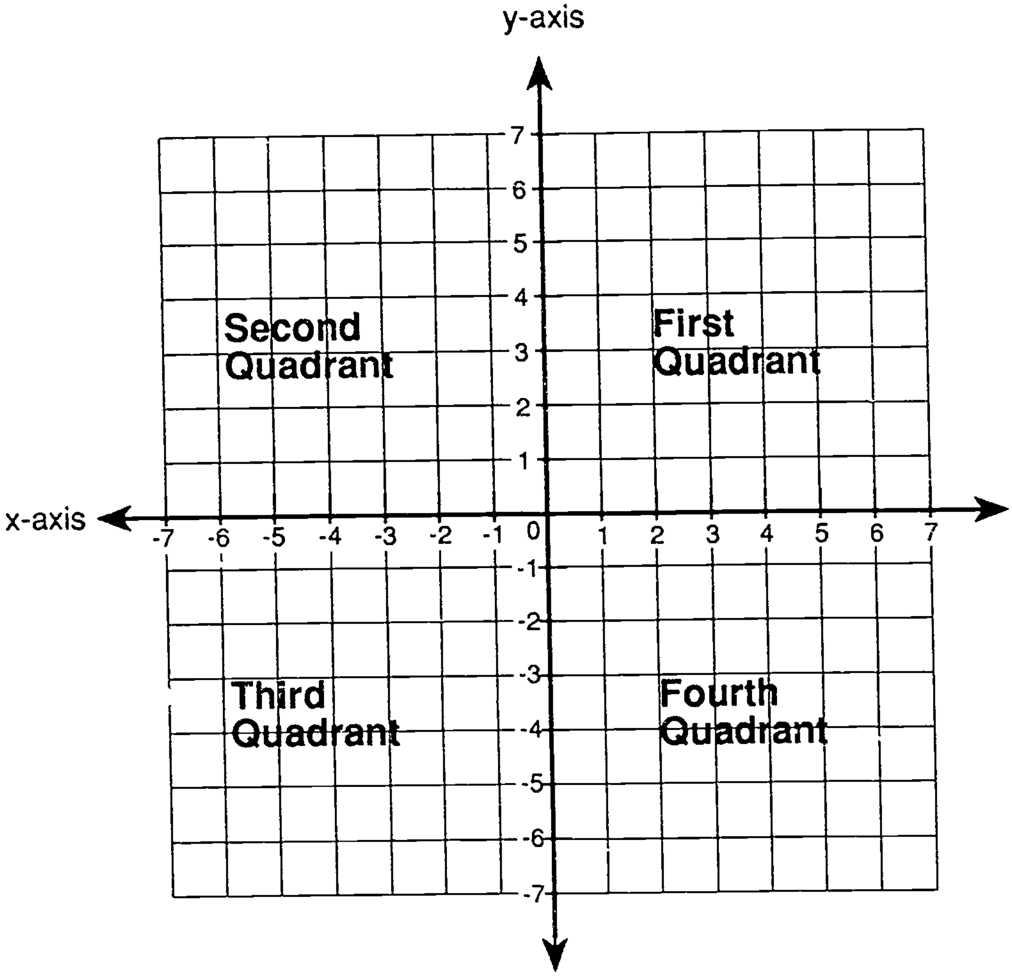



*Figure 5*

Note that this grid is divided into four quadrants:

| | |
|---|---|
| First Quadrant: | Positive $x$-values and positive $y$-values |
| Second Quadrant: | Negative $x$-values and positive $y$-values |
| Third Quadrant: | Negative $x$-values and negative $y$-values |
| Fourth Quadrant: | Positive $x$-values and negative $y$-values |

The procedure for locating points on this type of grid is the same as the procedure you practiced in the previous section.

*Example 2*

*Find the point indicated by the ordered pair (-1,4).*

In the ordered pair (-1,4), -1 is the $x$-coordinate, and 4 is the $y$-coordinate.

1. Starting at the origin, find the point -1 on the $x$-axis by counting one unit to the left.
2. From that point, count 4 units straight up.
3. You should now be at the point where the lines extending from the $x$-value -1 and the $y$-value 4 cross. This is the point (-1, 4)

The point (-1, 4) is marked on the grid in figure 6 with the letter "F."

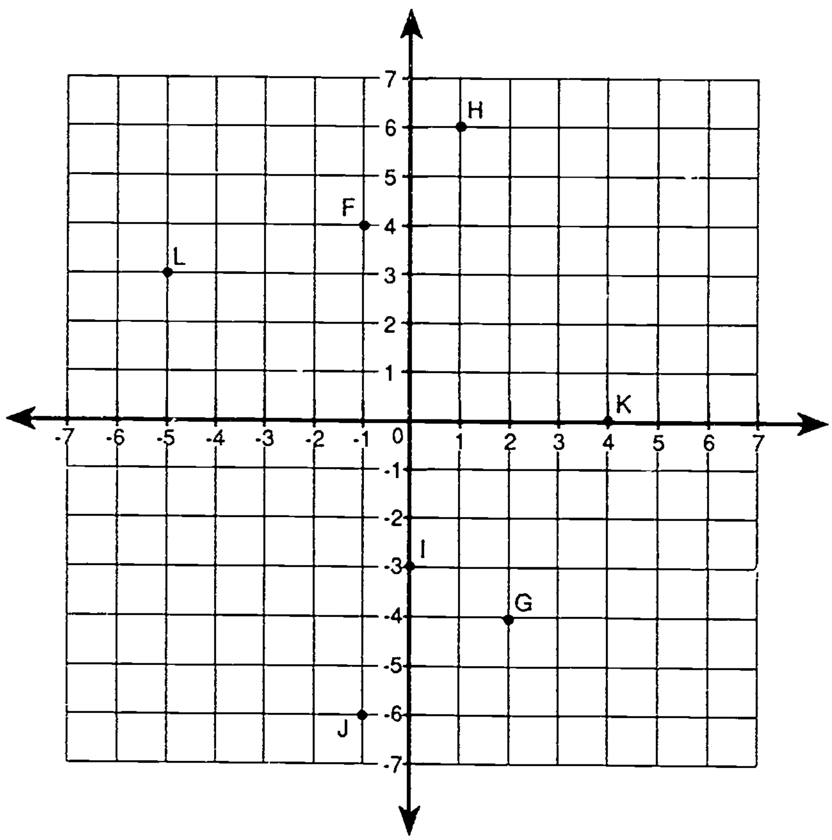



*Figure 6*

**Now Try These**

Look at figure 6, and fill in the blanks with the letters that mark these points:

a. (1,6) ________ b. (-1,-6) ________ c. (2,-4) ________

d. (-5,3) ________ e. (0,-3) ________ f. (4,0) ________

### *Plotting Points on a Grid*

You can use coordinates to plot points on a grid or coordinate system.

*Example 3*

*Plot the point (-2,-6) on the grid in figure 7.*

In the ordered pair (-2,-6), -2 is the $x$-coordinate, and -6 is the $y$-coordinate.

1. Start at the origin. Count 2 units to the left on the x-axis. This point has the $x$-value of -2.
2. Count down six units from this point. You should be at the place where the lines coming from the $x$-value -2 and the $y$-value -6 intersect.
3. Mark this point with a dot, and label it (-2,-6).

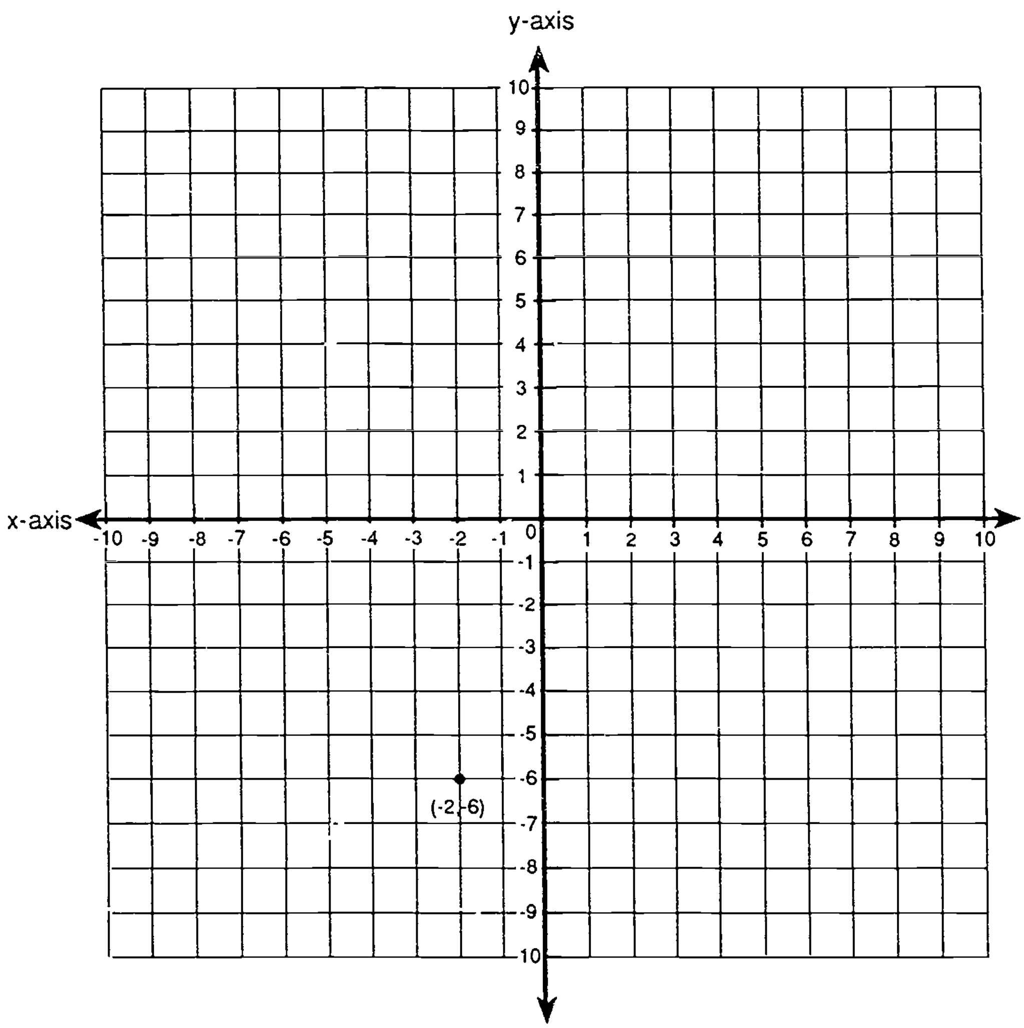



*Figure 7*

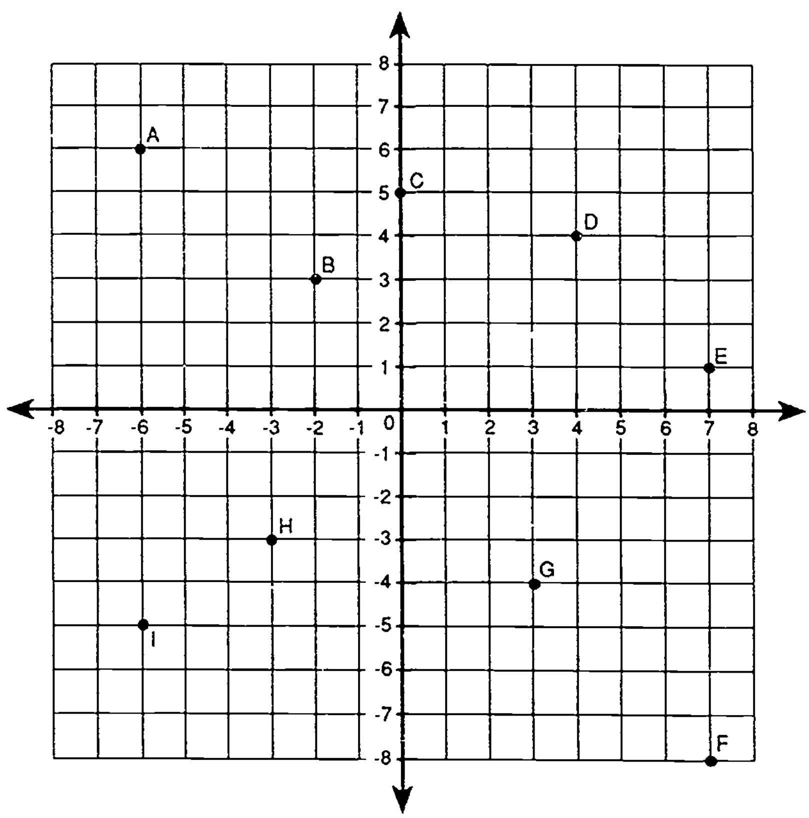
## Now Try These

Find and plot each of these points on the grid in figure 7.

a. (-4,1)   b. (3,3)   c. (-10,-8)   d. (6,-5)

## Skill Builders

Using the grid below, find the coordinates of each point, and write them next to the letter corresponding to the point.




1. A ____________ 2. B ________________ 3. C ________________

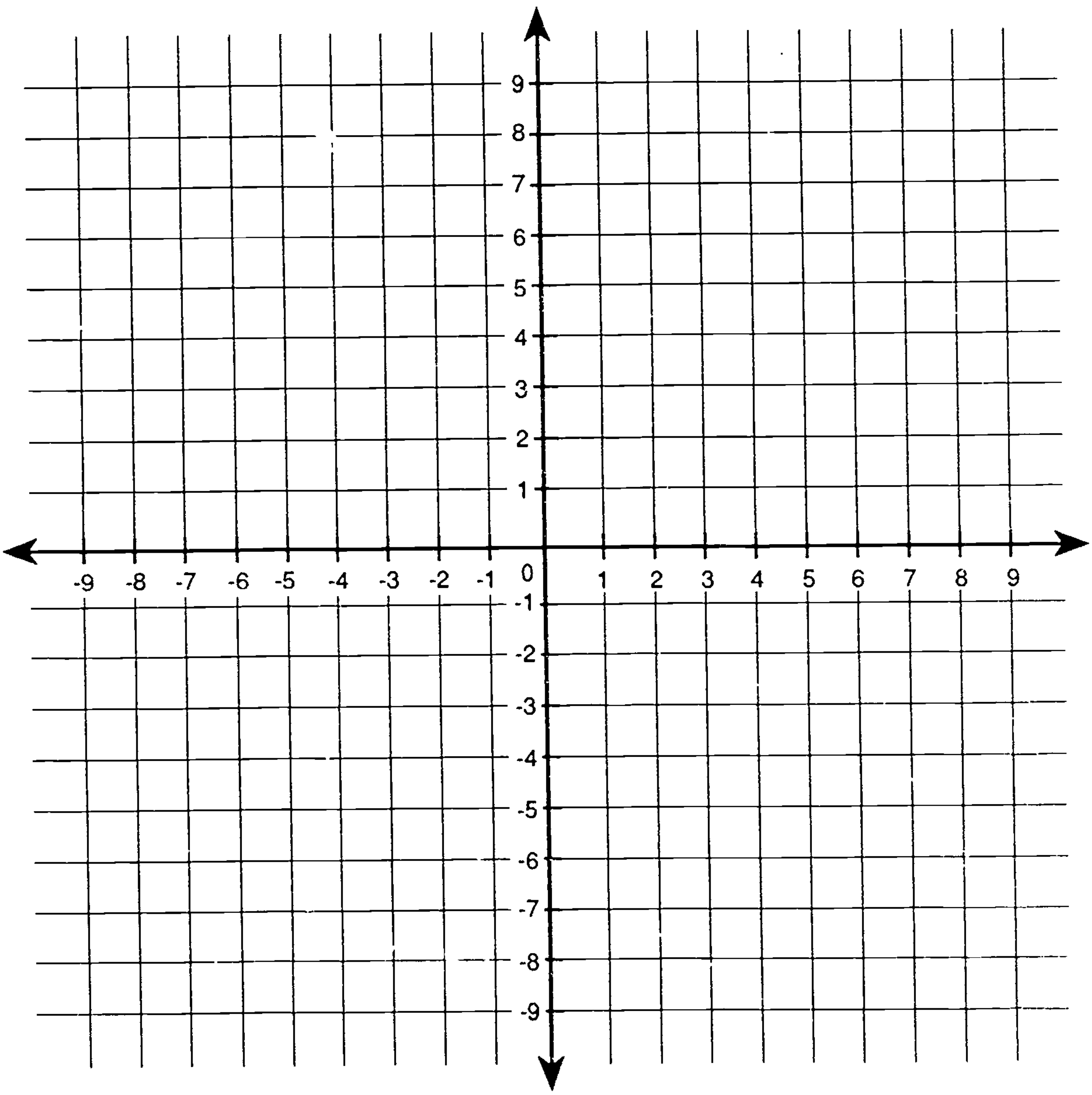
4. D ____________ 5. E ________________ 6. F ________________

7. G ____________ 8. H ________________ 9. I ________________

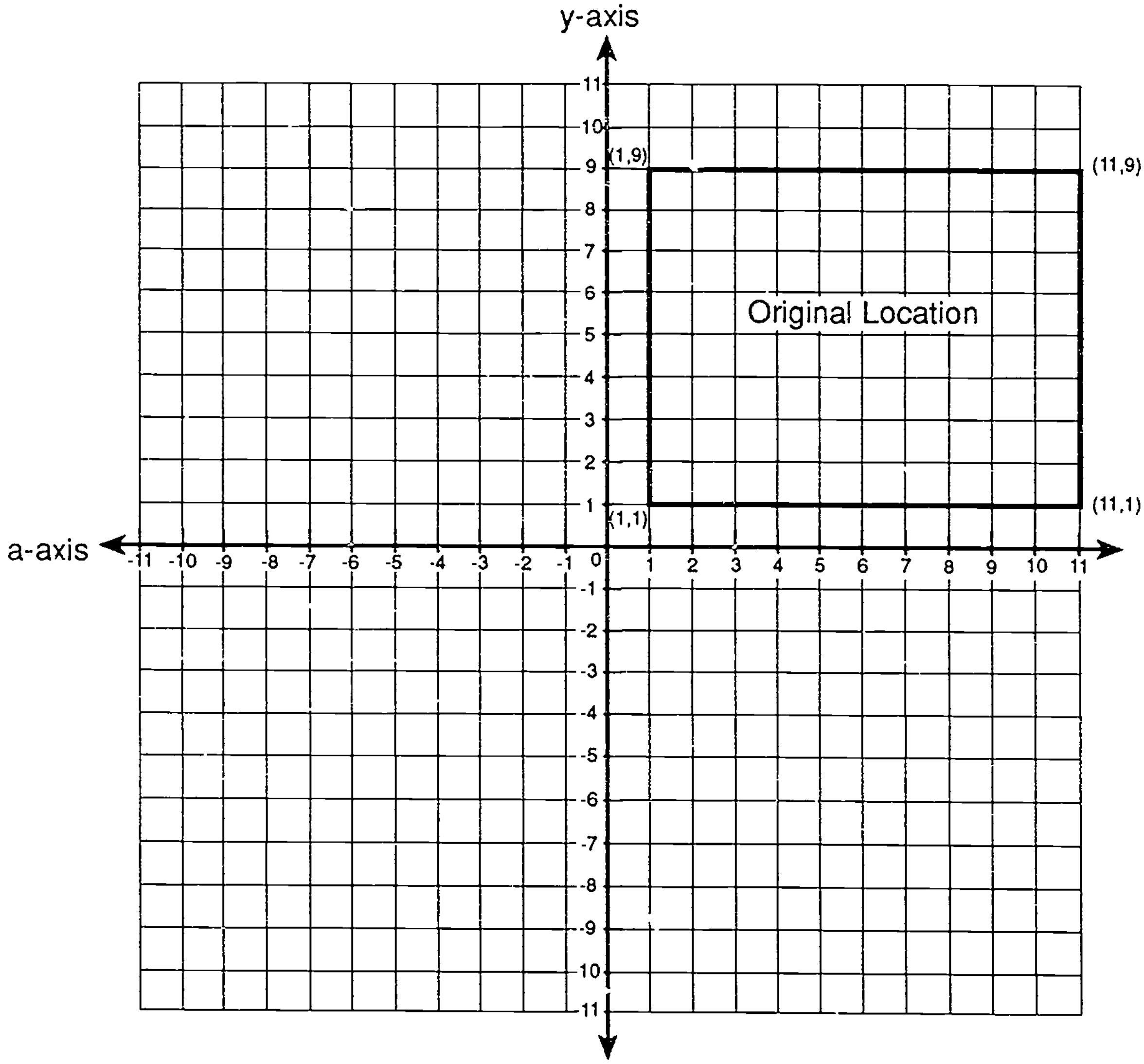
10. Plot each of the following points on the grid below. Then, connect the points in order to make a drawing.

Point 1: (-8,0)
Point 2: (-4,4)
Point 3: (0,8)
Point 4: (4,4)
Point 5: (8,0)
Point 6: (4,-4)
Point 7: (0,-8)
Point 8: (-4,-4)

(After connecting all the points in order, connect point 8 back to point 1.)



11. The diagram below shows the layout of a rectangular figure on a grid. Each unit on the grid represents one inch.




The original coordinates of the 4 corners of this figure are:

Lower left corner: (1,1)

Upper left corner: (1,9)

Lower right corner: (11,1)

Upper right corner: (11,9)

After the original drawing was made, the rectangle was moved 3 inches to the left and 2 inches down. What are the new coordinates of:

a. the lower left corner?

b. the upper left corner?

c. the lower right corner?

d. the upper right corner?

*(Hint: To solve this problem, you can either subtract 3 from each x-value and 2 from each y-value or use the grid to count out the adjustments for each ordered pair.)*

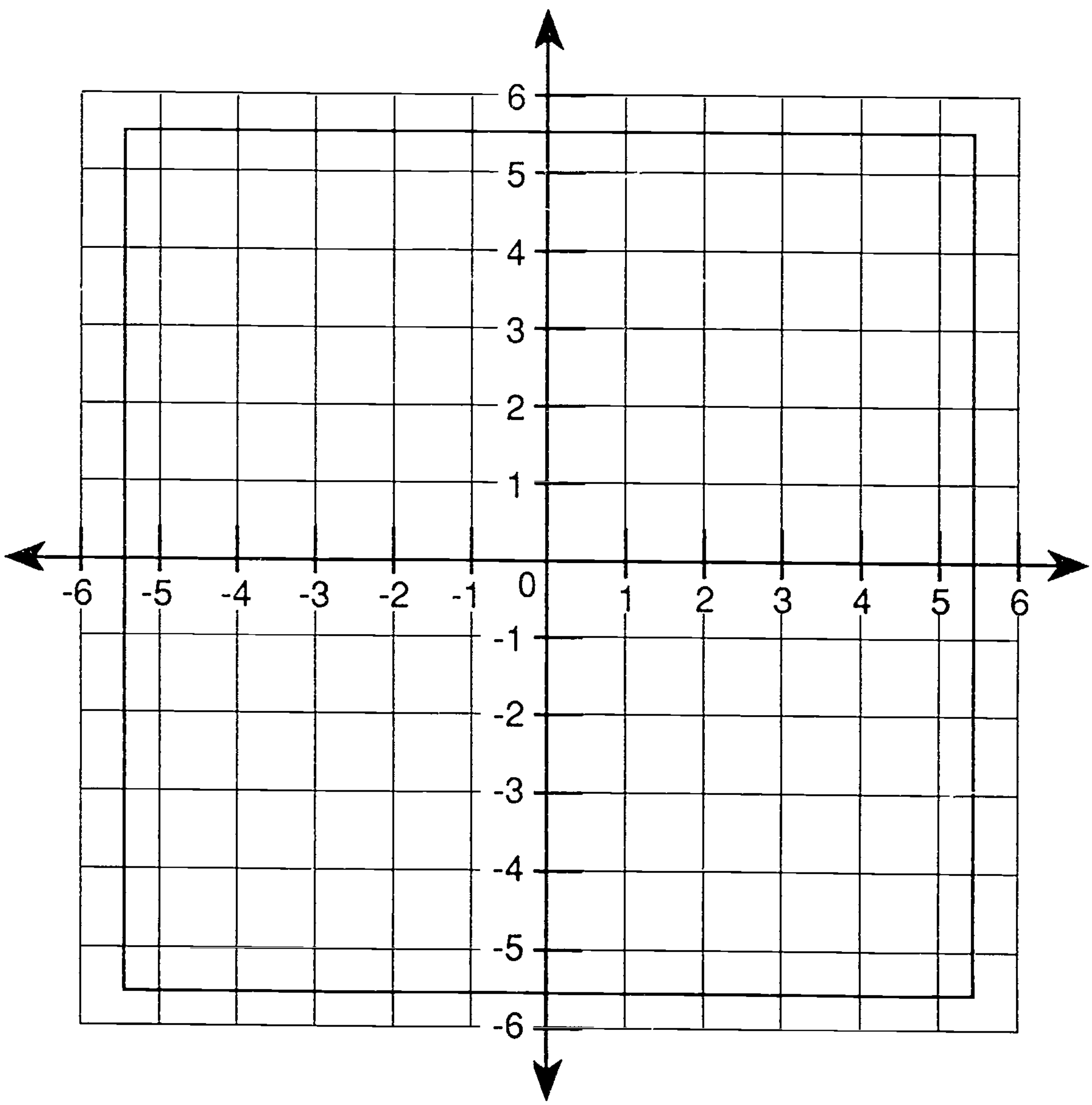
12. The diagram below shows a computer wiring board on a grid. The lines in the grid are 1 millimeter apart. You must drill three sets of holes in the board according to the following instructions:

   a. Drill Hole A at (-2,3) and Hole B exactly 3 mm to the right of Hole A.

   b. Drill Hole C at (1,4) and Hole D exactly 2 mm to the left and 1 mm below Hole C.

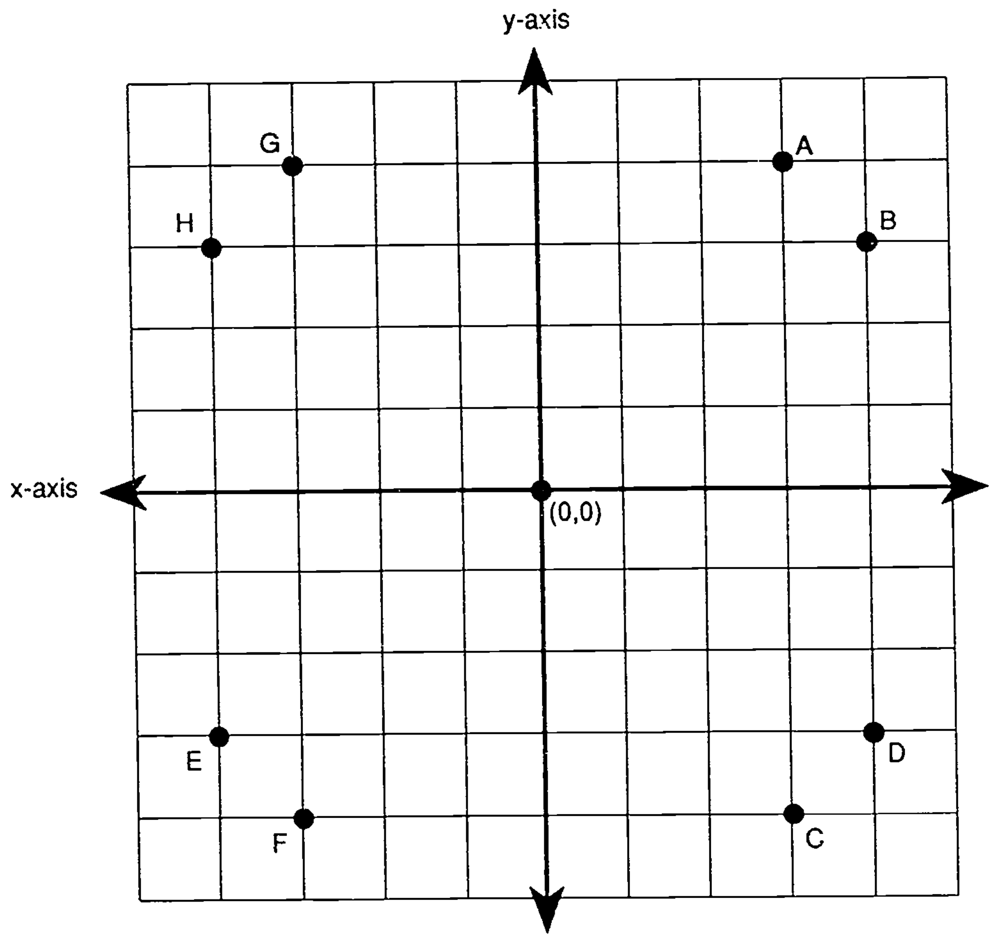
   c. Drill Hole E at (-1,-5) and Hole F exactly 1 mm above Hole E.

Make a dot at the location of each hole in the board, and label the dots with the correct letters according to the instructions above.



## Check Yourself

Which of the points shown below have positive *x*-values?



# Module 2 Review #3
# Lessons 2.12-2.16

You may wish to review the key concepts in Lessons 2.12 through 2.16 before completing these review exercises. However, if you feel comfortable with what you have learned, go ahead and attempt them. You may keep your lessons at hand to use as a reference source, but try to solve each problem without looking back at the lessons.

Solve each of the following:

1. $3 + 2 \times 4 - 5 =$

2. $4 + (3 \times 5) - 8 =$

3. Last month Company A produced 10 times as many widgets as it did gizmos. If the total produced was 110, how many gizmos and how many widgets were produced?

Show all work for the following equations:

4. $4x + 5 = ?$ where $x = 3$

5. $6 + N = 28$

6. $B - 7 = 3$

7. $8X = 16$

8. $9C = 45$

9. $\frac{9}{X} = 3$

10. $\frac{8}{X} = 2$

11. $\frac{1}{5} X = 6$

12. $\frac{4}{5} X = 8$

13. $\frac{X}{3} = 9$

14. $\frac{X}{5} = 20$

15. $X + 12 = 20$

16. $14 + X = 3$

17. $5 = R - 10$

18. $12 = N - 12$

19. John produced four times as many panels last week as Bob. If the total they produced was 70, how many did John produce and how many did Bob produce?

20. If a certain number is divided by 5 and gives an answer of 5, what is the number?

21. Use this formula to figure the following:

I = PRT where I = interest

P = principal

R = rate of interest

T = time of loan

John takes out a car loan at 12% for 5 years for a total of $9,000. How much total interest will he pay?

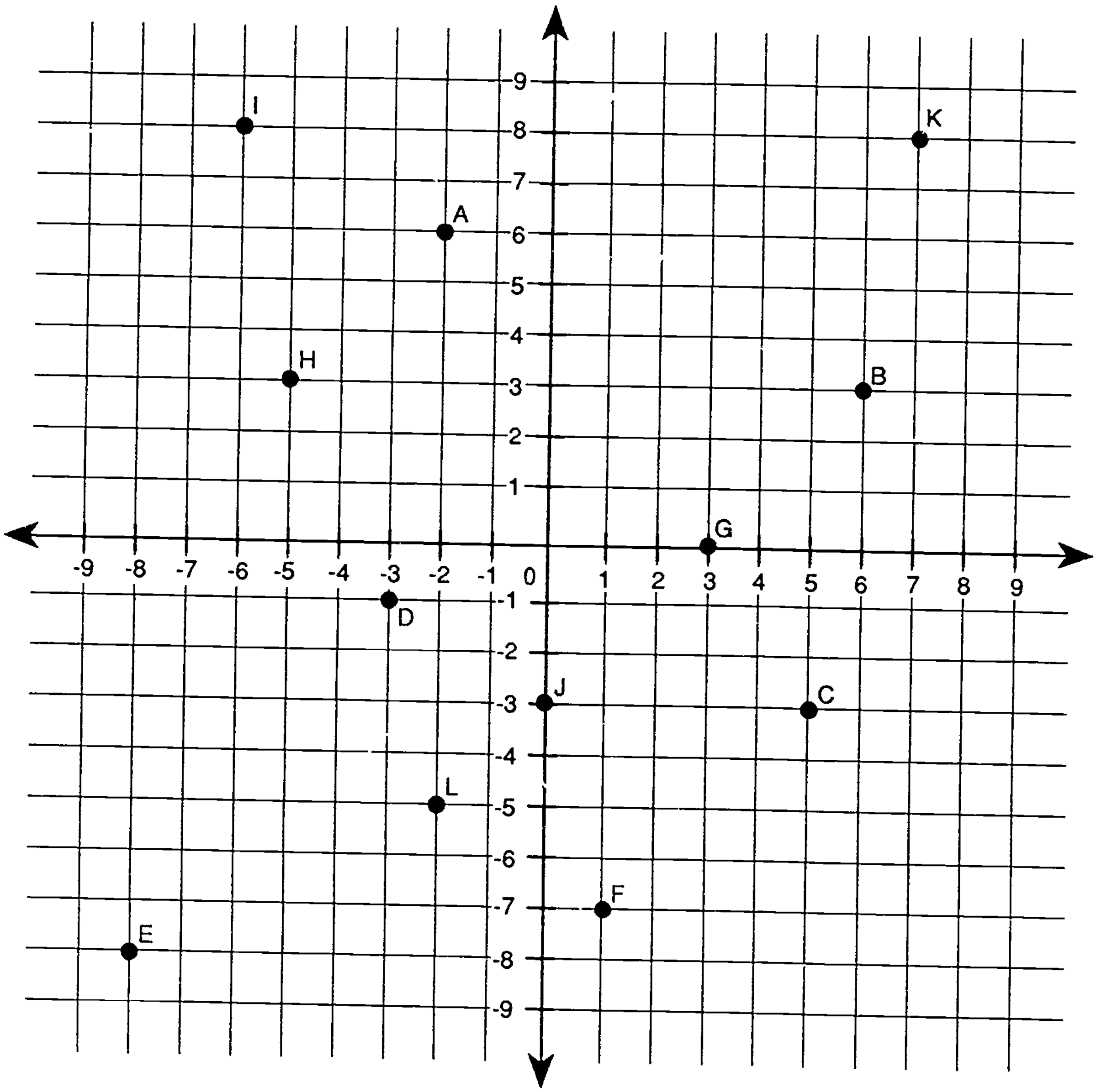
22. Find the distances traveled if:

Distance = Rate × Time

A. Rate = 65 miles per hour     Time = 3.5 hours

B. Rate = 30 miles per hour     Time = 7.4 hours

23. Refer to the following grid, and identify the coordinates of the points marked on it.




Point A:

B:

C:

D:

E:

F:

G:

H:

I:

J:

K:

L:



2·2·J

# Answer Key

# Lesson 1.1

**Now Try These**

*Decimal Values*

a. six hundredths

b. 0.485

*Rounding*

a. 9.2

b. 30

c. 2.11

*Adding and Subtracting*

a. 12.854

b. 4.884

**Skill Builders**

1. Fourteen and three tenths
2. Fifteen and sixty-seven hundredths
3. Twenty-two and nine thousand nine hundred eighty-four ten thousandths
4. 15.2
5. 2.93
6. 3.678
7. 1.5
8. 16
9. 16
10. 3
11. 0
12. 5
13. 1
14. 22.67
15. 28.76
16. 14.58
17. 6.5287
18. 46.3
19. 49.011
20. 6.794
21. 11.17

**Check Yourself**

1. Forty-nine and three thousand six hundred seventy-eight ten thousandths
2. 13.5
3. 17.864
4. 187.4732
5. 2.329

## Lesson 1.2

**Now Try These**

*Example 1*

a. 0.32
b. 1.875

*Example 2*

a. 12.6
b. 2.34

*Example 3*

a. 0.0736
b. 0.00045

**Skill Builders**

1. 83.79
2. 0.4207
3. 27.6
4. 0.01
5. 90
6. 12.8304; 12.83 rounded
7. 33.885; 33.89 rounded
8. 184.0909; 184.09 rounded
9. 9,620.6
10. 664.96

**Check Yourself**

1. $25.50
2. 37.5 hours

## Lesson 1.3

**Now Try These**

*Example 1*

a. 3
b. 4

*Example 2*

a. 20.065
b. 2.211

*Example 3*

a. 0.25
b. 0.4

**Skill Builders**

1. 4.12
2. 6.25
3. 3
4. 1.4
5. 5.95
6. 2.8
7. 18.67
8. 5,400
9. 0.5
10. 0.04

**Check Yourself**

1. 4 parts
2. about \$90.91

## Lesson 1.4

**Develop Your Understanding**

Example 2: a. $1\frac{3}{5}$ b. $\frac{11}{2}$

Example 3: a. $\frac{5}{7}$ b. $\frac{8}{15}$

Example 4: a. $\frac{5}{7}$ b. $\frac{1}{3}$

Example 6: a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. 3

**Skill Builders**

1. $\frac{1}{2}$
2. $\frac{6}{7}$
3. $\frac{5}{7}$
4. $2\frac{1}{3}$
5. $3\frac{1}{5}$
6. $2\frac{4}{7}$
7. $4\frac{4}{9}$
8. $\frac{4}{3}$
9. $\frac{55}{8}$
10. $\frac{33}{4}$
11. $\frac{41}{3}$
12. $\frac{3}{5}$
13. $\frac{4}{3}$ or $1\frac{1}{3}$
14. $1\frac{1}{8}$
15. 1
16. $\frac{1}{6}$
17. $\frac{4}{13}$
18. $\frac{3}{10}$
19. $\frac{2}{15}$
20. $\frac{4}{5}$
21. $\frac{1}{2}$
22. $\frac{1}{4}$
23. $1\frac{1}{3}$
24. $1\frac{1}{3}$

**Check Yourself**

1. $\frac{15}{8}$ or $1\frac{7}{8}$ gallons

2. $\frac{3}{8}$ gallon

# Lesson 1.5

**Now Try These**

*Example 1*

a. 3, 6, 9, 12, 15 b. 7, 14, 21, 28, 35

*Example 3*

a. 10 b. 24

*Example 4*

Answers will vary.

a. $\frac{6}{8}, \frac{9}{12}$ b. $\frac{2}{14}, \frac{3}{21}$

*Least Common Denominator*

a. LCM: 6 b. $\frac{5}{6} + \frac{2}{6}$ c. $\frac{7}{6} = 1\frac{1}{6}$

*Example 5*

a. $16\frac{5}{6}$ b. $8\frac{1}{2}$

*Example 6*

a. $\frac{3}{8}$ b. $3\frac{1}{10}$

**Skill Builders**

1. $\frac{2}{5}$
2. $1\frac{1}{12}$
3. $1\frac{1}{8}$
4. $1\frac{7}{15}$
5. $1\frac{1}{12}$
6. $\frac{19}{28}$
7. $\frac{5}{24}$
8. $\frac{1}{8}$
9. $\frac{1}{18}$
10. $\frac{3}{14}$
11. $19\frac{1}{6}$
12. $4\frac{1}{8}$
13. $5\frac{1}{3}$
14. $11\frac{13}{30}$

**Check Yourself**

1. $6\frac{5}{6}$ hours

2. $9\frac{1}{16}$ inches

## Lesson 1.6

**Introductory Paragraph**

recipe: $\frac{1}{8}$ cup

pamphlet: $4\frac{1}{4}$ inches

**Develop Your Understanding**

*Multiplying Fractions*

a. $\frac{8}{15}$

b. $\frac{1}{64}$

*Whole Numbers*

a. $\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

b. $\frac{72}{10} = 7\frac{1}{5}$

*Mixed Numbers (Multiplication)*

a. $2\frac{1}{2}$

b. $1\frac{13}{20}$

*Dividing Fractions*

a. $\frac{18}{40} = \frac{9}{20}$

b. $\frac{4}{6} = \frac{2}{3}$

*Mixed Numbers (Division)*

a. $\frac{2}{3}$

b. $\frac{65}{168}$

**Skill Builders**

1. $\frac{2}{45}$

2. $\frac{3}{80}$

3. 4

4. $\frac{1}{3}$

5. 8

6. 3

7. $\frac{1}{2}$

8. $\frac{1}{4}$

9. $1\frac{1}{25}$

10. $3\frac{3}{8}$

11. 1

12. $\frac{1}{9}$

13. $1\frac{3}{4}$

14. $\frac{15}{17}$

15. $\frac{7}{16}$

16. 64

17. 13

18. $2\frac{2}{5}$

19. $\frac{155}{224}$

20. $1\frac{1}{4}$

**Check Yourself**

1. 1
2. 6
3. 1
4. 18

## Lesson 1.7

**Now Try These**

*Example 1*

a. 0.67

b. 0.57

*Example 3*

a. $\frac{9}{40}$

b. $\frac{3}{10}$

**Skill Builders**

1. 0.833
2. 0.6
3. 0.125
4. 0.333
5. 0.9
6. 0.15
7. $\frac{5}{8}$
8. $\frac{18}{25}$
9. $\frac{18}{125}$
10. $\frac{2}{5}$
11. $\frac{9}{2000}$

12. $\frac{1}{50}$

**Check Yourself**

1. $\frac{1}{2}$ lb
2. $\frac{1}{4}$ lb

## Lesson 2.1

**Now Try These**

a. 16.261
b. 720
c. 871,000
d. 6

**Skill Builders**

1. Lance's hourly wage
2. 52.2 miles
3. 60 packages
4. $4\frac{3}{4}$ hours
5. 60
6. $333.33
7. $1,857,000
8. Friday
9. $\frac{3}{4}$"
10. 7"
11. number of employees; also, whether some of the same people worked both Sat. and Sun.
12. 24 minutes
13. $1\frac{1}{16}$"
14. 200
15. 2¢
16. $12,045.00
17. $870.66 ($54.00 for labor and $816.66 for windows)
18. 3 days

**Check Yourself**

1. Answers will vary.
2.

| | |
|---|---|
| + | + |
| x | + |
| x | x |
| - | + |
| - | + |
| + | + |

# Lesson 2.2

**Now Try These**

*Example 1*
a. $53.00 b. $42.00 c. $41.00

*Example 2*
The estimated average should be about $108.00 (the average 107.80).

**Skill Builders**

1. $22.00
2. 800 (actual total was 831)
3. $278.00
4. $21.00
5. 20.17
6. $12.00
7. 12.5
8. 71°
9. 116.4
10. Total estimated costs should be $60.00. Change due should be about $10.00.
11. 1300
    No, they were entered incorrectly.
12. 140
13. 104 lb
14. 3 days
15. 182 estimated

# Lesson 2.3

**Now Try These**

***Writing Ratios***

a. $\frac{3}{17}$

b. $\frac{2}{5}$

c. $\frac{180}{55}$

d. $\frac{45}{60}$

e. $\frac{32}{9}$

### *Putting Ratios in Lowest Terms*

a. $\frac{2}{1}$ or 2:1

b. $\frac{1}{5}$

c. $2 \div 5$

### *Calculating Equivalent Ratios*

Answers will vary. Some possibilities:

a. $\frac{10}{12}, \frac{15}{18}$

b. $\frac{20}{16}, \frac{5}{4}$

c. $\frac{1}{2}, \frac{2}{4}$

### Skill Builders

1. 3:5, $3 \div 5$, $\frac{3}{5}$

2. 1:18, $1 \div 18$, $\frac{1}{18}$

3. 3:25, $3 \div 25$, $\frac{3}{25}$

4. $\frac{46}{1}$

5. $\frac{8}{7}$

6. $\frac{32}{32}$

7. $\frac{2.5}{5}$

8. a. $\frac{2}{3}$

   b. $\frac{3}{1}$

   c. $\frac{5}{12}$

9. a. 1
   b. 3
   c. 8

10. Answers will vary; some possibilities:
    6:2, 9:3, 12:4, 15:5, 18:6

11. 7:34

12. 4 eggs

13. 2:1 (the ratio is $1:\frac{1}{2}$; 2:1 is an equivalent ratio)
14. 12:1
15. For the 50-pound bag, the ratio is $15:50 lb. For the 75-pound bag, the ratio is $25:75 lb.
16. 90:1
17. No; the actual ratio was equivalent to 8 out of 1,000.
18. 1:3
19. 16 gallons
20. 15 cents per ounce; $0.15:1 ounce

**Check Yourself**

Only Roderick has not had enough training. (Clarabelle should have had at least 24 days of training. Grace should have had 4. Roderick should have had at least 12.)

## Lesson 2.4

---

**Now Try These**

***Proportions***

a. 12
b. 66
c. 126
d. 9
e. 32

***Solving Proportions***

a. N = 21
b. N = 32
c. N = 25
d. N = 2

***Word Problems with Proportions***

a. 150 square feet
b. 105 perfect books

**Skill Builders**

1. a, d, and e should be marked with an "X"
2. 250
3. 18
4. 15
5. 99
6. N = 6
7. N = 52
8. N = 20
9. N = 1
10. N = 5

11. 3 projects
12. 32.5 gallons
13. 6.67 inches or $6\frac{2}{3}$ inches
14. 8 projects
15. a. 2.4 cents
    b. no (2.6 cents per sheet)
16. $37,800
17. There is no difference in unit price.

## Lesson 2.5

***Converting Percents to Decimals***

a. 0.28
b. 0.006
c. 2.0

***Converting Decimals to Percents***

a. 36%
b. 9%
c. 430%

***Converting Fractions to Decimals***

a. 20%
b. 75%
c. 37.5%

***Converting Percents to Fractions***

a. $\frac{95}{100}$ $(\frac{19}{20})$

b. $\frac{8}{100}$ $(\frac{2}{25})$

c. $2\frac{15}{100}$ $(2\frac{3}{20})$

d. $\frac{34}{1000}$ $(\frac{17}{500})$

**Skill Builders**

1. 0.35
2. 0.07
3. 1.05
4. 0.009
5. 0.015
6. 46%
7. 2%

8. 129%
9. 37.5%
10. 8.5%
11. 50%
12. 16%
13. 70%
14. 12.5%
15. 87.5%
16. $\frac{20}{100}$ $(\frac{1}{5})$
17. $\frac{8}{100}$ $(\frac{2}{25})$
18. $1\frac{75}{100}$ $(1\frac{3}{4})$
19. $\frac{98}{1000}$ $(\frac{49}{500})$
20. $\frac{109}{1000}$
21. 60%
22. 0.1875 inch
23. $\frac{4}{5}$
24. 33% (rounded)
25. $\frac{88}{100}$ $(\frac{22}{25})$
26. 6%
27. 2.5%
28. $\frac{42}{100}$ $(\frac{21}{50})$

## Lesson 2.6

---

**Now Try These**

*Example 1*

a. 21

b. 8.64

*Example 2*

a. 80%
b. 20%

*Example 3*

a. 150
b. 60

**Skill Builders**

1. 20
2. 30
3. 0.3
4. 60%
5. 70%
6. 33.3%
7. 40
8. 250
9. 45
10. 49
11. 833
12. 999
13. 75%
14. 93.75%
15. 97.5%
16. 40
17. 125
18. 16
19. $1,200
20. $3.33
21. $1,640
22. $25,000
23. 134.26°F decrease
    824.74°F new temperature

## Module 2 Review #1
## Lessons 2.1-2.6

---

1. a. define the question
   b. determine key information
   c. determine what mathematical operation to use
2. 2,000
3. 900
4. 2,667
5. $190
6. 14

7. \$98
8. 0.3
9. 3
10. 6
11. A: 6 parts
    C: 8 parts
12. 10 gallons
13. 9
14. 9
15. 22.4
16. 15%
17. 65
18. 15%
19. $\frac{1}{2} = 50\%$
    $\frac{4}{5} = 80\%$
    $20 = 2000\%$
    $0.20 = 20\%$
    $0.02 = 2\%$
20. $40\% = 0.4$
    $4\% = 0.04$
    $400\% = 4$
    $37.5\% = 0.375$
21. $2\% = \frac{1}{50}$
    $35\% = \frac{7}{20}$
    $0.50 = \frac{1}{2}$
    $0.24 = \frac{6}{25}$

## Lesson 2.7

***Reading Scales***

a. 160 units
b. 6 units

***Reading a Standard Ruler***

Point A: $\frac{1}{2}''$

Point B: $\frac{6}{8}''$ or $\frac{3}{4}''$

Point C: $1\frac{7}{16}''$

Point D: $1\frac{13}{16}''$

Distance between Point A and Point B: $\frac{1}{4}''$

Point B and Point C: $\frac{11}{16}''$

***Finding the Distance Between Two Points***

a. $3\frac{1}{8}''$

e. $1\frac{1}{16}''$

b. $2\frac{1}{16}''$

f. $\frac{11}{16}''$

c. $2\frac{6}{8}''$ or $2\frac{3}{4}''$

g. $\frac{15}{16}''$

d. $3\frac{7}{8}''$

h. $1\frac{3}{16}''$

***Adding Measurements***

a. 10 ft. 5 in. b. 8 ft. 3 in. c. 12 ft. 10 in.

***Subtracting Measurements***

a. 2 ft. 6 in. b. 6 ft. 7 in. c. 1 ft. 9 in.

***Multiplying and Dividing Measurements***

a. 31 ft. 3 in. b. 7 ft. 8 in. c. 15 ft.
d. 1 ft. 5 in. e. 3 ft. 8 in.

***Conversion of Units***

a. 84 inches b. 24 pints
c. 80 ounces

**Skill Builders**

1. $\frac{3}{4}''$
2. $1\frac{1}{2}''$
3. $1\frac{15}{16}''$
4. $4\frac{1}{8}''$
5. $6\frac{3}{8}''$
6. 7 yd. 7 "
7. 1 yd. 4 "
8. 25 ft.
9. 7 ft. 9 in.
10. 7 gallons 2 quarts

11. 23 lb
12. 1 pint
13. No, we have 1 cup too much.
    3 gallons 1 cup total
14. No, the total shipment per crate is 22 pounds, 11 ounces.

## Lesson 2.8

**Now Try These**

***Perimeter***

a. 158 feet
b. 20 inches
c. 46 inches

***Area***

a. 90 square inches
b. 1600 square feet

***The 24-hour Clock***

a. 11 days 19 hours 35 minutes
b. 0525: 5:25 A.M.
   1730: 5:30 P.M.
   2045: 8:45 P.M.

**Skill Builders**

1. a. 279 feet
   b. 47.6 inches
   c. 34.4 inches
2. a. The total perimeter needed is 52 inches.
   b. Yes, the boxes are big enough.
3. a. 25 sq. in.
   b. 470.58 sq. ft.
4. The total area of the panel is 132 sq. in.
5. 4 weeks 3 days 17 hours 14 minutes
6. a. 0930
   b. 1730
   c. 1515
   d. 1145
   e. 5:20 A.M.
   f. 10:25 A.M.
   g. 10:30 P.M.
   h. 5:20 P.M.

**Check Yourself**

1. a. 11 hours 18 minutes
   b. Monday: 0930 – 1430
      Tuesday: 0845 – 1115
      Wednesday: 1120 – 1508
2. a. 12 hours 10 minutes
   b. Monday: 9:30 A.M. – 1:45 P.M.
      Tuesday: 10:25 A.M. – 12:50 P.M.
      Wednesday: 8:40 A.M. – 2:10 P.M.

## Lesson 2.9

---

**Now Try These**

***The Metric System***

a. one-thousandth of a liter
b. 1,000 grams

***Conversions in the Metric System***

a. 2 hm, 2 dam, 1 m, 4 dm
b. 3 m, 5 dm
c. 6 cm, 5 mm

***Using a Metric Ruler***

a. Point A: 3 mm
   Point B: 1 cm, 2 mm
   Point C: 3 cm, 7 mm
b. 7 cm 2 mm
c. 17 mm = 1 cm, 7 mm

***Metric Capacity***

a. 225 cl
b. 450 cl
c. 375 cl

***Converting Between the Standard and Metric Systems***

*Example 4*
1.61 km = 1 mile
0.04 ounce = 1 gram

**Now Try These**

a. 31.06 miles  b. 109 yards  c. 4.5 kg  d. 1 ounce
e. 453.6 grams  f. 2.85 liters

**Skill Builders**

1. a. $\frac{9}{16}$"
   b. $1\frac{3}{16}$"
   c. $1\frac{7}{8}$"
   d. $1\frac{5}{16}$"
2. a. 1 cm, 4 mm
   b. 4 cm, 5 mm
   c. 7 cm, 3 mm
   d. 3 cm, 1 mm
3. a. Point A: 70 cl
   Point B: 160 cl
   Point C: 230 cl
   b. 160 cl
4. 2,180 yards
5. 179 pounds, 1 ounce

## Lesson 2.10

**Now Try These**

*Example 2*

a. 9
b. 14
c. $\frac{1}{2}$
d. +4 and -4

*Example 4*

a. +8
b. -9
c. -208

*Example 7*

a. 9
b. -1
c. 14
d. -26
e. 24

*Example 9*

a. 11
b. 9
c. 50

**Skill Builders**

1. -3
2. 11
3. 0
4. 3
5. -51
6. -18
7. -11
8. 16
9. -37

10. 19
11. -11
12. -64
13. -71
14. -2
15. -38
16. 3418
17. 76
18. 33
19. -670
20. 32
21. 28
22. 5330
23. 44°

**Check Yourself**

32 + 29 + 41 +(-4) + (-2) + (-1) = 95

## Lesson 2.11

**Now Try These**

***Multiplying Signed Numbers***

a. 32
b. -30
c. -36

***Dividing Signed Numbers***

a 7
b. 34
c. -8

***Multiplying Several Signed Numbers***

a. -192
b. -4800

***Working with Other Signed Numbers***

a. $-1\frac{1}{5}$
b. 54.08

**Skill Builders**

1. 600
2. 4,000
3. 12
4. -12

5. $-\frac{5}{12}$

6. $-46\frac{4}{9}$

7. $-2\frac{1}{2}$

8. 7.3185

9. 0.042

10. $\frac{25}{56}$

11. $\frac{1}{10}$ or 0.1

12. -33
13. 28
14. -24.1
15. -0.295
16. $-\frac{1}{9}$
17. -19

## Module 2, Review #2
## Lessons 2.7-2.11

---

| | | | | |
|---|---|---|---|---|
| 1. | a. | 8 yd. | 2 ft. | 2 in. |
| | b. | 9 gallons | 1 cup | 1 oz. |
| 2. | a. | 10 ft. | 8 in. | |
| | b. | 2 qt. | 1 cup | 6 oz. |
| | c. | 9 dm | 8 cm | 8 mm |
| | d. | 16 dm | 2 cm | |
| | e. | 9 cl | | |
| 3. | a. | 7.44 miles | | |
| | b. | 19.32 kilometers | | |
| | c. | 4.68 inches | | |
| | d. | 54.5 yards | | |
| | e. | 45 kilograms | | |
| | f. | 453.6 grams | | |

All measurement problems are converted to the highest measurement.

| | | | |
|---|---|---|---|
| 4. | 22 yards | 8 inches | |
| 5. | 20 yards | 2 feet | 6 inches |
| 6. | 2 feet | 11 inches | |

7. 2 yards 3 feet 9 inches

8. Point A: $\frac{9}{16}$ in.

Point B: $1\frac{3}{4}$ in.

Point C: $3\frac{1}{8}$ in.

A to B: $1\frac{3}{16}$ in.

B to C: $1\frac{3}{8}$ in.

A to C: $2\frac{9}{16}$ in.

9. Point A: 1.2 cm

Point B: 3.8 cm

Point C: 7.3 cm

A to B: 2.6 cm

B to C: 3.5 cm

10. a. 32 in.
    b. 32 in.

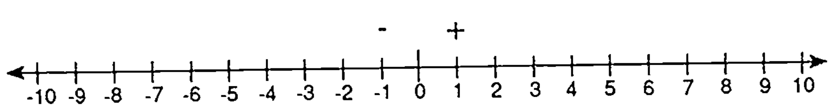
11. a. 64 sq. in.
    b. 48 sq. in.

12. Point A. 80 cl
    Point B. 120 cl
    Point C. 220 cl

13. 1 week 2 days 16 hours 4 minutes

14. 0520 = 5:20 A.M.
    1639 = 4:39 P.M.
    2323 = 11:23 P.M.
    12:30 A.M. = 2430 or 0030
    4:15 P.M. = 1615
    6:20 P.M. = 1820

15.




16. & 17.
10 + 4 = 14
-10 + (-4) = -14
10 - 4 = 6
-10 - (-4) = -6
10 + (-4) = 6
-10 + 4 = -6

18. a. 12
    b. -9
    c. -10
    d. 10
    e. 4
    f. -11
    g. 6
    h. -2

19. a. 12
    b. -12
    c. 12
    d. -12
    e. 9
    f. 9
    g. 3
    h. 3
    i. -3
    j. -3
    k. -3
    l. 10
    m. 5
    n. -5

## Lesson 2.12

**Now Try These**

***Equations***

a. true
b. not true
c. true

***Order of Operations***

a. 0
b. -8
c. 83

***Uses of Parentheses***

a. -4
b. 17
c. -38
d. 135

***Variables***

a. if d = -5, answer is -11; if d = 30, answer is 59
b. if g = -1, answer is -3; if g = 3, answer is 9
c. if f = 15, answer is -8; if f = -11, answer is 44

***Inequalities***

a. < b. > c. < d. = e. > f. <

***Inequalities with Variables***

b. possible
c. possible
d. possible
e. impossible

*Converting Words to Equations and Inequalities*

a. let s = the number of pairs ordered in February

$3s + s = 24$

b. let A = the average number per week

$3A = 1,569$

**Skill Builders**

1. yes
2. no
3. 158
4. 21
5. 1
6. 1
7. 5
8. -72
9. 56
10. a. 26

    b. 8
11. a. 164

    b. 114.5 or $114\frac{1}{2}$
12. a. 55

    b. -5
13. yes
14. a. < b. > c. < d. >
15. Answers will vary; must be a fraction or decimal between 1 and 2.
16. let p = price of one candy bar

    $p + 2p + 3p + 4p = \$6.40$
17. let c = number of cans Zelda drank

    if $c > 6(30)$, could $c = 174$?
18. let w = the number of weeks

    $4w = 52$
19. let m = amount of money that Special Projects got

    $5m = m + \$80,000$

## Lesson 2.13

**Now Try These**

***Addition***

a. 36

b. 100

c. 30

*Subtraction*

a. 65

b. 15

c. 44

*Word Problems*

78 + 32 + X - 12 = 140

X = 42

**Skill Builders**

1. 35
2. 0
3. 85
4. 60
5. 58
6. -25
7. 11
8. 14
9. 18
10. 238
11. 35
12. -9
13. -122
14. -97
15. 26

**Check Yourself**

1. 23 + 46 + 12 + 74 - 48 + X = 125
   X = 18
2. 595 + (595 (5%)) - X = 590
   X = 35

## Lesson 2.14

**Now Try These**

*Multiplication*

a. b = 2

b. c = 9

*Division*

a. G = 30

b. m = 27

***Special Types of Problems***

a. x = 2

b. z = 27

**Skill Builders**

1. y = 14
2. a = 13
3. y = 20
4. x = 7
5. x = 10
6. x = 3.1
7. w = 16
8. a = 128
9. w = 147
10. x = 20
11. d = 125
12. x = -81
13. c = 8
14. k = 224
15. x = $2,000.00
16. $x = 6\frac{2"}{3}$ or 6.67"
17. x = 5
18. 36 cm.
19. x = 49
20. 40 volts

**Check Yourself**

1. x = 50
2. x = 3
3. x = 5.2
4. x = 20
5. x = 20
6. $x = 9\frac{1}{2}$

## Lesson 2.15

**Now Try These**

*Example 1*

a. A = 18 square feet

b. G = 15 miles per gallon

*Example 2*

a. Seconds = # of hours × 60 (min. per hour) × 60 (sec. per minute)
   S = H × 3600 (in most simplified form)

b. Work days = [52 weeks × 5 days + overtime days] – vacation and sick leave

*Example 3*

a. H = 40 points

b. C = 5 degrees

**Skill Builders**

1. a. D = 180 miles
   b. D = 12.5 miles
2. a. R = 0.62 millimeter per minute
   b. R = 0.25 millimeter per minute
   c. R = 0.52 millimeter per minute
3. a. I = $3,200.00
   b. I = $108.00
4. a. % yield = 93.8%
   b. % yield = 90.5%
5. Keith processed 5.45 units per dollar. Glenn processed 5.06 units per dollar. Therefore, Keith was more productive.
6. Part = $1,417.50 down payment
   Amount financed = $8,032.50
7. a. Part = $13,000.00
   b. Mortgage = $52,000.00

## Lesson 2.16

---

**Now Try These**

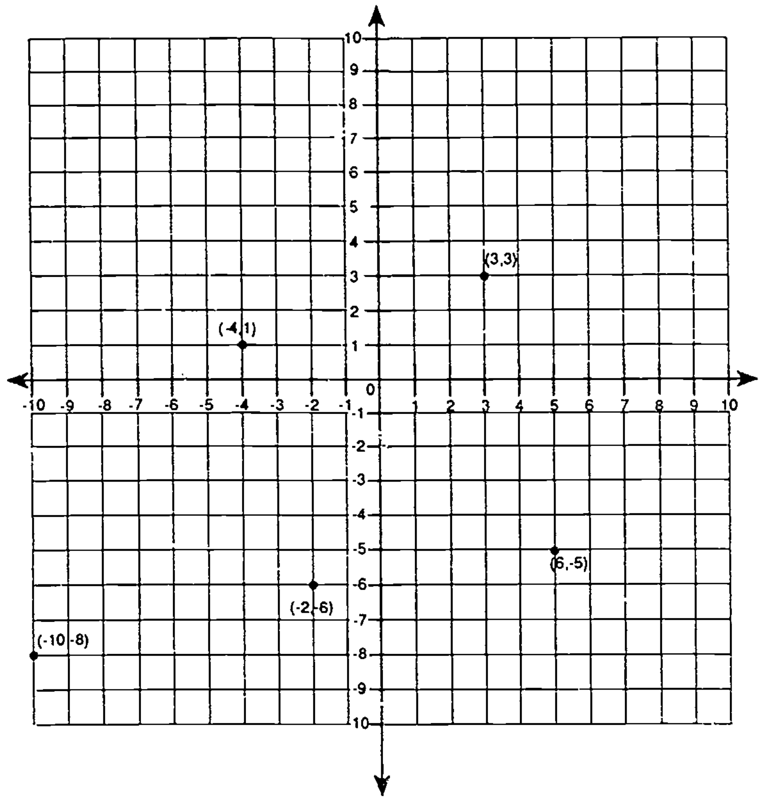
*Example 1*

a. C
b. B
c. E
d. D

*Example 2*

a. H
b. J
c. G
d. L
e. I
f. K

*Example 3*



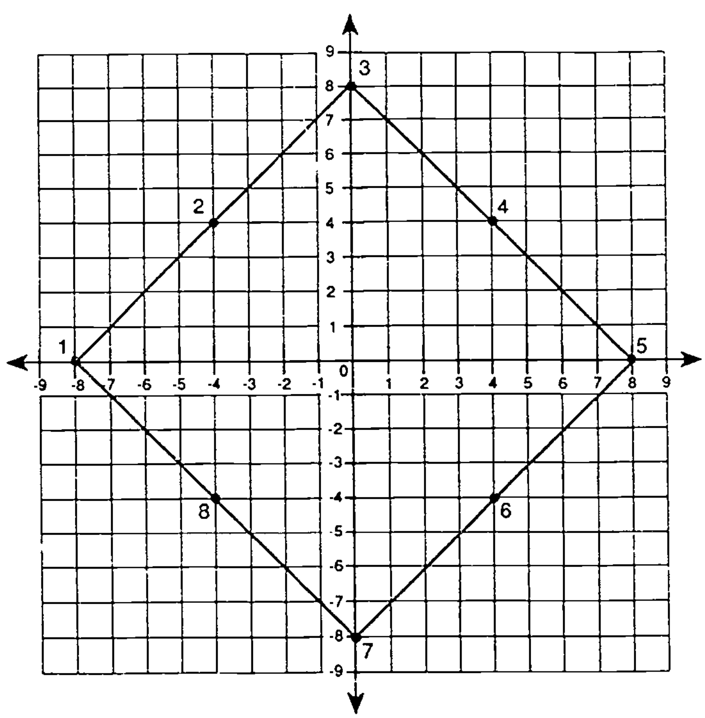

**Skill Builders**

1. (-6,6) 2. (-2,3) 3. (0,5)

4. (4,4) 5. (7,1) 6. (7,-8)

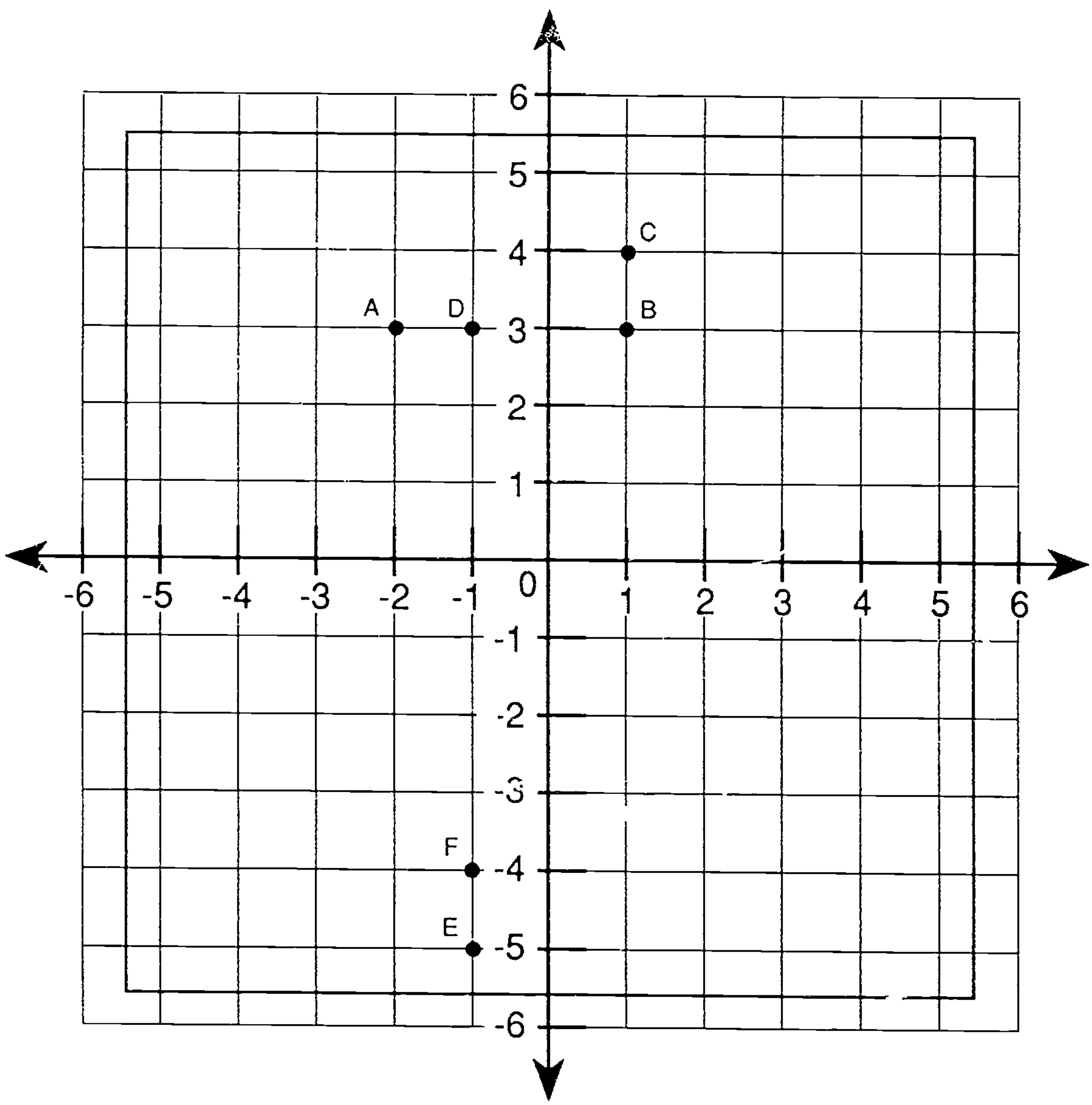
7. (3,-4) 8. (-3,-3) 9. (-6,-5)

10.



11. a. (-2,-1)
    b. (-2,7)
    c. (8,-1)
    d. (8,7)

12.




**Check Yourself**

A, B, C, and D

# Module 2 Review #3
## Lessons 2.12-2.16

1. 6
2. 11
3. 10 gizmos and 100 widgets
4. 17
5. 22
6. 10
7. 2
8. 5
9. x = 3
10. x = 4
11. x = 30
12. x = 10
13. x = 27
14. x = 100
15. x = 8
16. x = -11
17. R = 15
18. N = 24
19. Bob produced 14, and John produced 56
20. 25
21. $5,400
22. a. 227.5
    b. 222
23. A: (-2, 6)
    B: (6, 3)
    C: (5, -3)
    D: (-3, -1)
    E: (-8, -8)
    F: (1, -7)
    G: (3, 0)
    H: (-5, 3)
    I: (-6, 8)
    J: (0, -3)
    K: (7, 8)
    L: (-2, -5)

SN 150 S