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ABSTRACT

This study evaluated the generally recommended concrete-to-abstract hierarchy for presenting a new skill, with three students with learning disabilities in grades 1, 2, and 4. The three subjects enrolled in the Multidisciplinary Diagnostic and Training Program's classroom housed on the University of Florida campus in Gainesville. Following collection of baseline data, place value concepts and skills were taught using a concrete, semiconcrete, and abstract teaching sequence in a direct instruction model. Instruction was limited to 15 minutes a day for 9 to 15 days. Student progress was monitored using Student Behavior charts, and posttest results indicated significant gains by all three subjects, with retention demonstrated 3 weeks later in a different classroom setting. Among findings were that, for all three students, the transition to abstract understanding occurred suddenly and conclusively but at varying points within the concrete-to-abstract sequence. (Contains 16 references.) (DB)

Validating the Concrete to Abstract Instructional Sequence for Teaching Place Value to Learning Disabled Students[1]

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MULTIDISCIPLINARY
DIAGNOSTIC AND
TRAINING
PROGRAM

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THE UNIVERSITY OF FLORIDA
MULTIDISCIPLINARY DIAGNOSTIC AND TRAINING PROGRAM (MDTP)

The MDTP is administered through a joint effort by Shands Teaching Hospital and the Department of Special Education at the University of Florida. The MDTP staff is composed of professionals from the fields of pediatric neurology, education, school psychology, and speech and language pathology. The MDTP has specified elementary school students with diverse medical, learning, and/or behavorial problems as its primary population. Major responsibilities of the MDTP are to use all appropriate disciplines to provide diagnostic and intervention services to school systems referring students, train education and health professionals at the preservice and inservice level, and assist parents of students experiencing difficulty in school.

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Abstract

Student achievement in mathematical skills depends on quality instruction. The generally accepted conceptual hierarchy for presenting a new skill follows a concrete to abstract continuum. However empirical studies supporting this are lacking. This study pinpoints place value skills and investigates the effectiveness of concrete to abstract teaching. Three subjects meeting the pretest criterion were chosen. Baseline data were collected and interventions applied subsquent to acceptable performance criteria. All teaching activities were presented using a direct instruction model. Student progress was monitored using Standard Behavior Charts. Posttest results indicated significant gains by all three subjects. Retention scores substantiated that skills were maintained. A concrete to abstract teaching sequence for place value concepts produced positive celerations with little teaching time invested.

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Trafton (1984) notes that public concern about the quality of basic skills instruction has resulted in an increased attention on the teaching of mathematics. General agreement exists regarding the relationship between quality mathematics instruction and student achievement. Isolating the teaching components that constitute quality, however, has been a challenge. Position papers generated by the membership of the National Council of Teachers of Mathematics (NCTM) stress the importance of continuing the search for quality instruction. Improving the effectiveness and efficiency of mathematics teaching are stressed (NCTM, 1980). Moreover, issues involving mathematics skill hierarchies, concept learning, retention, and generalization of arithmetic skills still need to be addressed.

The current educational emphasis on acquisition of basic skills, minimum competency testing, and progression through basal math programs has affected mathematics instruction. Many teachers feel compelled to cover specified amounts of material at a rapid pace. Correct computational answers are the primary measure of student success in mathematics and are frequently used to determine mastery and subsequent readiness for teaching a new skill.

Davis (1983) cautions against an overemphasis on computation at the expense of teaching important mathematical concepts. He notes that the teaching of concepts is especially important to students with learning problems. According to Ashlock (1986) students experiencing mathematical difficulties frequently do not have the prerequisite understanding and skills needed to learn new ideas and procedures. He states that teachers who introduce paper-and pencil procedures to students who still need to work problems with concrete aids are encouraging students to memorize a complex sequence of mechanical acts. Such memorization lends itself to faulty algorithms and frustration for the student and teacher.

Reisman (1982) concurs with Ashlock and further suggests that students must develop a basic understanding of mathematical relationships before they can succeed at arithmetic computation. Gaps in mathematical foundations occur when relations underlying computational algorithms are not fully understood. Davis (1983) reports the extent of a student's conceptual knowledge is probably a strong indicator of performance in class, on tests, and in applying mathematics to solve problems. Thus, teaching fundamental mathematical concepts appears critical to successful instructional programs.

Mathematical skills and concepts are typically taught sequentially. Initial instruction involves simple number relationships and then progresses to more complex tasks. Underhill, Uprichard, and Heddens (1980) suggest that as students progress sequentially through arrangements of mathematical tasks they transfer the learning of skills from one level to the next higher level. Acquisition of new skills is, therefore, dependent upon previous learning.

Hierarchical approaches to mathematics are used in most classrooms. Some teachers also use a mathematical concept sequence for teaching specific skills in the hierarchies. It is generally agreed that the concept sequence progresses from concrete to semiconcrete to abstract learning experiences (Beattie, 1986). Instruction using the concept sequence begins with the manipulation of objects, then uses pictures or tallies, and finally involves the abstract symbols (i.e., numbers) in isolation.

The idea that mathematical learning progresses through three levels of abstraction has evolved since the beginning of the nineteenth century (Burton, 1984; Suydam & Higgins, 1977). Pestalozzi is credited with emphasizing the need for object-teaching. Moreover, he suggests that children must experiment in the concrete before applying abstract rules or exercises (Cajori,1896, pp. 211-212). By the 1930's, many mathematicians accepted the premise that students should progress through three stages involving objects first, then pictures, and finally symbols.

Bruner (1966) and Piaget and Inhelder (1958) identify these stages as enactive, iconic, and symbolic. These authors contend that students first learn mathematics through manipulating concrete objects in an enactive sense. When this is completed, the students can learn through pictures or representations of objects in an iconic manner and finally through manipulating abstract symbols (Suydam & Higgins, 1977).

Underhill et al. (1980) identify the sequence components as concrete, semiconcrete, and abstract. These investigators report that during initial skill learning the student needs to concentrate on both the manipulated objects and the symbolic processes that describe the manipulations. Moreover, Underhill et al. suggest the goal of mathematical curricular activities is to build step-wise associations through practice so that the learner will not always need concrete manipulatives, but will eventually be able to <u>think</u> about the set of experiences (iconic imagery). Because the symbols used relate to real-world experiences, mathematical computation will be meaningful to the learner.

Although the manipulation of objects, uses of pictures or tallies, and the use of abstract symbols teaching sequence has widespread theoretical acceptance, little research exists to document its effectiveness. Professional opinion rather than empirical studies supports the use of the concrete to abstract continuum.

Several members of the MDTP Staff have selected the concrete to abstract teaching sequence as an area for programmatic research. This monograph presents the first study in a series of investigations designed to examine this mathematical intervention.

The purpose of this study was to investigate the effectiveness of teaching learning disabled students an initial place value skill (i.e., identifying tens and/or ones in a double digit number) using the concrete to abstract teaching sequence.

Precision teaching was used to monitor the intervention's effectiveness on a daily basis. Additionally, a teacher-made pre- and posttest was administered. Student performance was evaluated in terms of skill acquisition, celeration, retention and generalization to a new setting.

Subjects

The three subjects involved in this study were enrolled in the Multidisciplinary Diagnostic and Training Program's (MDTP) classroom housed in Norman Hall on the University of Florida campus in Gainesville, Florida. Each subject attended the MDTP classroom for five weeks and then returned to their home schools (i.e., Metcalfe and High Springs Elementary schools in Alachua County and Melrose Elementary in Columbia County). Learning characteristics for each student are summarized in Appendix A.

Design

A multiple baseline (Baer, Wolf, & Risley, 1968) single subject design was used. Single subject designs provide within subject control and are appropriate for documenting the effectiveness of an academic intervention used with several individual students.

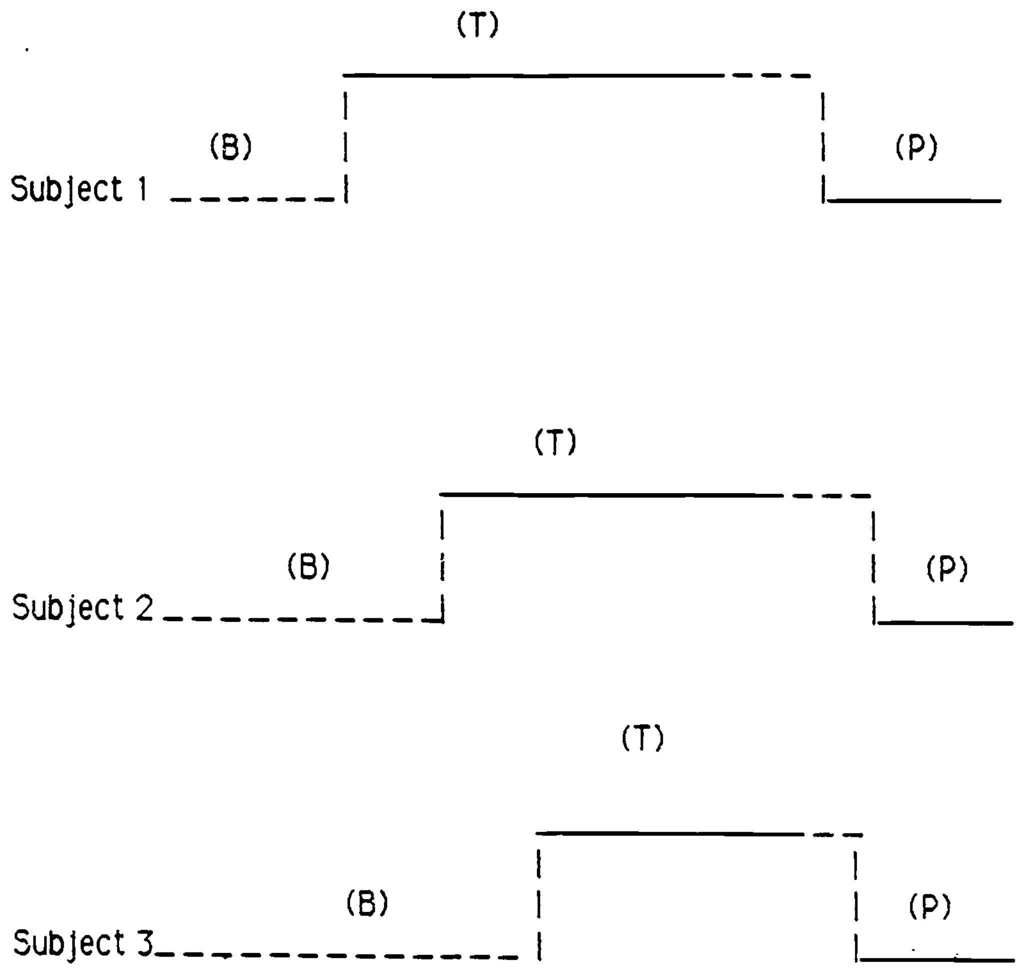
Specific to this research, the multiple baseline across three individuals receiving concrete to abstract instruction on place value occurred. The subjects were determined eligible for the study based on a place value pretest (see Appendix B). Criterion for study participation was a score of 70% or less. Baseline data were collected simultaneously across all three subjects. When the baseline data exhibited acceptable stability in trend, the concrete to abstract intervention was applied to the first subject. When criterion-level performance was attained the intervention was applied to the second subject. The same procedure was used for application of the intervention to the third subject.

Procedure

Three phases were included in the study: a baseline phase, a treatment phase, and a posttreatment phase (see Figure 1). Baseline and treatment phases took place in the University of Florida's MDTP classroom. The posttreatment phase took place in the students' home schools.

Figure 1

Diagram of Experimental Conditions



Note: ____ = Independent variable in contact with subject.
--- = Independent variable not in contact with subject.
| = Beginning of new phase.
B = Baseline phase.
T = Treatment phase.
P = Posttreatment phase.

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Baseline Phase

During the baseline phase one minute precision teaching timings were administered to each subject on a daily basis. The instructor met individually with each student during the same work period each day. The same teacher dialogue was used to present the probe on each occasion (i.e., "I want you to tell me how many ones or tens the underlined number represents.") Appendix C illustrates the precision teaching probe used throughout the study. Teacher feedback regarding student performance was withheld. The student's performance data were entered on AC-4 Standard Behavior Charts (Berquam, 1979). The charts, however, were not discussed with the students. Baseline data were gathered for a minimum of three days (Tawney & Gast, 1984).

Treatment Phase

During the treatment phase the three subjects were taught place value using a concrete, semiconcrete, and abstract teaching sequence. Individual activities were presented to each subject for concrete, semiconcrete, and abstract instruction. All teaching activities were presented using a direct instruction model. Steps included in the model were:

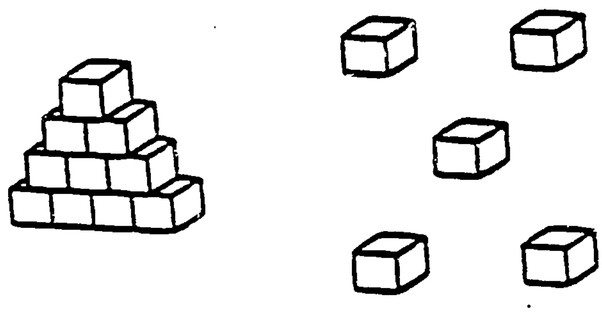
1. Provide an advance organizer.
2. Demonstrate and model the skill.
3. Provide guided practice.
4. Provide independent practice.

Materials used for concrete instructional activities included one inch plastic cubes produced by Developmental Learning Materials, teacher-made place value strips, and teacher-made place value cards (see Figure 2).

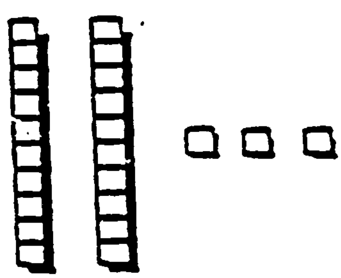
Figure 2

Materials Used For Concrete Instruction

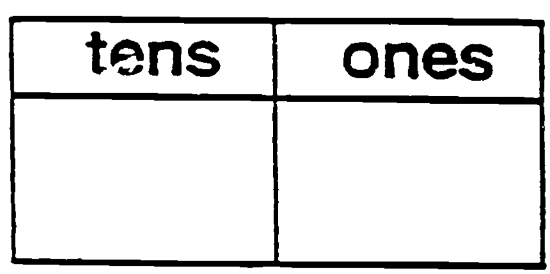
Place Value Cubes



Place Value Strips



Place Value Cards



worksheets with pictures of place value sticks and cubes were used for semiconcrete instruction. Abstract level teaching was performed using worksheets without pictorial representations. Instruction during the treatment phase was structured the same for all subjects. A teacher script was used and instructional time was limited to 15 minutes each day. At the completion of each 15 minute instructional period, a one minute precision teaching timing was administered to the student using the same baseline probe sheet. Correct and incorrect responses were charted on the AC-4 Standard Behavior Charts (Berquam, 1979).

Students progressed through the place value activities at their own rate. Criterion checks were made to ensure student comprehension before moving to a new activity. The duration of the treatment phase ranged from 9 to 15 days for the three subjects. The variability in number of instructional days resulted from individual differences in ability to meet the criterion set for acceptable performance on the designated tasks.

Posttreatment Phase

Immediately following the last instructional activity each student was given a posttest identical to the pretest. Direct instruction on the mathematical skill was discontinued and the students returned to their home schools. The precision teaching probe used during the baseline and treatment phase was administrated periodically for two weeks to serve as a maintenance check for the subjects after completing the treatment intervention. Then, maintenance checks were discontinued for a week. Finally, the primary investigator administered an alternate form of the pretest (See Appendix D) and the one minute timing to check retention.

Results

All three students made significant gains on the criterion-referenced posttest measures. Table 1 displays the percentage scores for the three measures. The subjects' percentage gain scores were 80%, 90%, and 40%. These increases represent the difference between receiving an F and a B on the place value test for two students and an F and an A for one student (i.e., assuming 40% is an F, 80% is a B, and 90% is an A).

Table 1

Place Value Percentage Scores

Subject	Pretest	Posttest	Retention
1	0%	80%	80%
2	0%	90%	80%
3	40%	80%	80%

Retention scores, obtained 3 weeks after instruction in a different classroom setting, further substantiate the benefit of the concrete to abstract intervention. All three students earned 80% on the retention measure. Closer analysis of the retention errors revealed an interesting phenomenon. All three students missed the same two questions. Both questions asked how many tens there were in single digit numbers (i.e., 6 and 4). The instructional lessons presented to the students only provided practice with double digit numbers. The discrepancy between what was taught and what was measured accounts for the problems missed on the posttest and retention measures. Thus, one can assume the few difficulties demonstrated were a result of the testing instrument rather than lack of skill acquisition or retention.

In addition to the criterion-reference measures, daily 1-minute timings provided data for analysis (see Figure 3). Special education teachers must continually concern themselves with the rate of student learning. Teaching efficiency and how quickly skill fluency is obtained becomes very important when teaching students who are typically 2 or 3 years behind their peers. In this study, the students celerations (i.e., rate of learning) were X 2.35, X 1.25, and X 1.67 respectively. These rates meet the generally accepted X 1.25 criterion (White & Haring, 1980). The student rates were maintained after instruction ceased.

The daily 1-minute timings revealed another interesting learning pattern. Each student made a significant gain at one particular point in the instructional sequence. One student demonstrated this gain during concrete instruction, while the other two students demonstrated similar gains during semiconcrete instruction. Thus, the transition to understanding an abstract level probe based on concrete or semiconcrete instruction seemed to occur suddenly and conclusively, but at varying points within the concrete to abstract sequence. Once the student realized the abstract representations (i.e.,numbers) were directly related to the manipulatives and/or pictures presented during the lessons, understanding was achieved immediately and maintained. This finding supports the recommendation (Underhill et al., 1980) to pair manipulatives with the symbolic processes that describe them to promote conceptual understanding.

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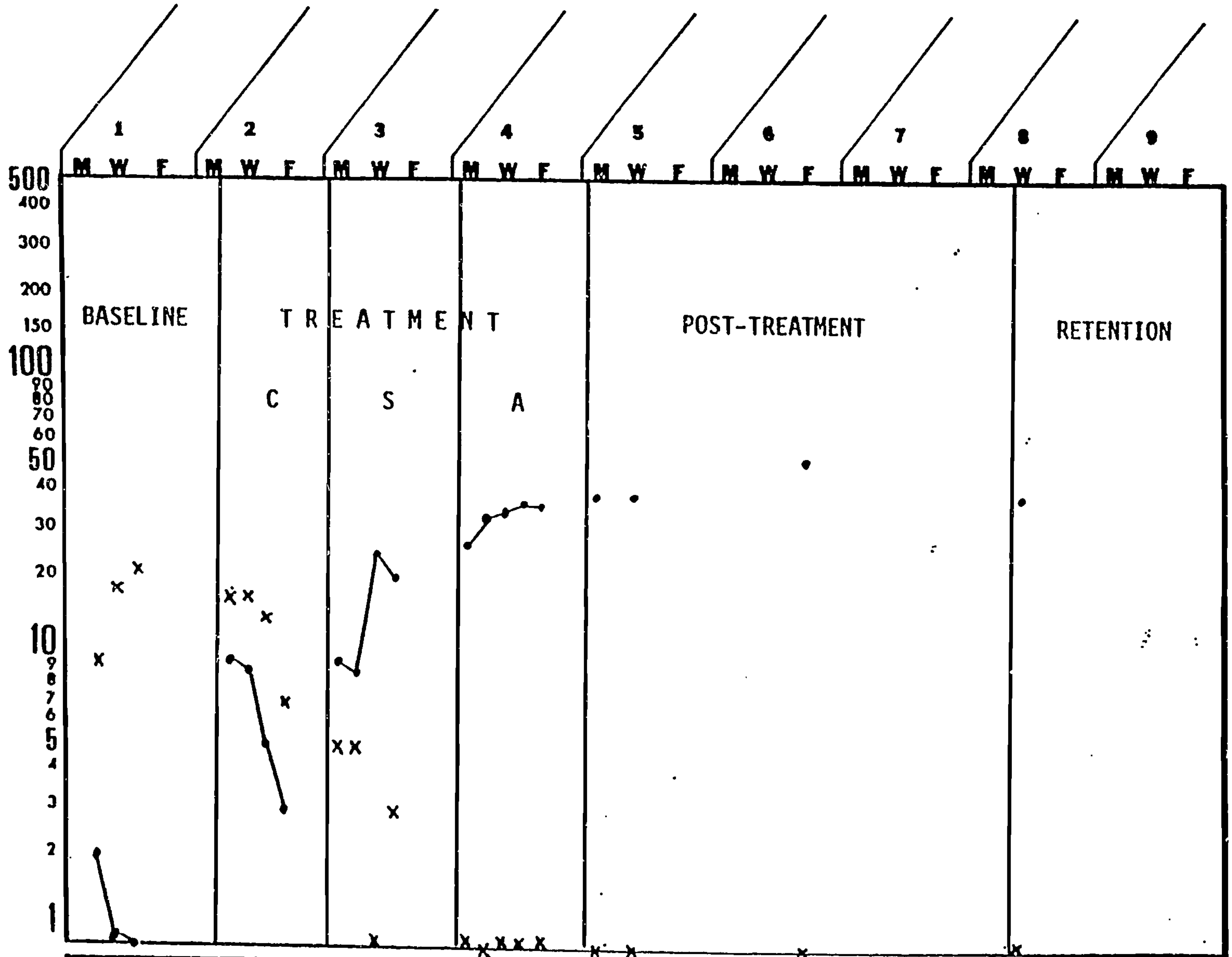

	M	T	W	Th	F
1		2/9	1/18	0/21	
2		10/17	9/17	5/15	3/7
3	10/5	9/5	25/1	20/3	
4	27/1	32/0	33/1	37/1	36/1
5	39/0		39/0		
6					51/0
7	NO	INSTRUCTION			
8			39/1		
9					

NAME Subject 1

GRADE 2nd

BEHAVIOR See/Say Place Value

GOAL

Note. C= concrete
S= semiconcrete
A= abstract

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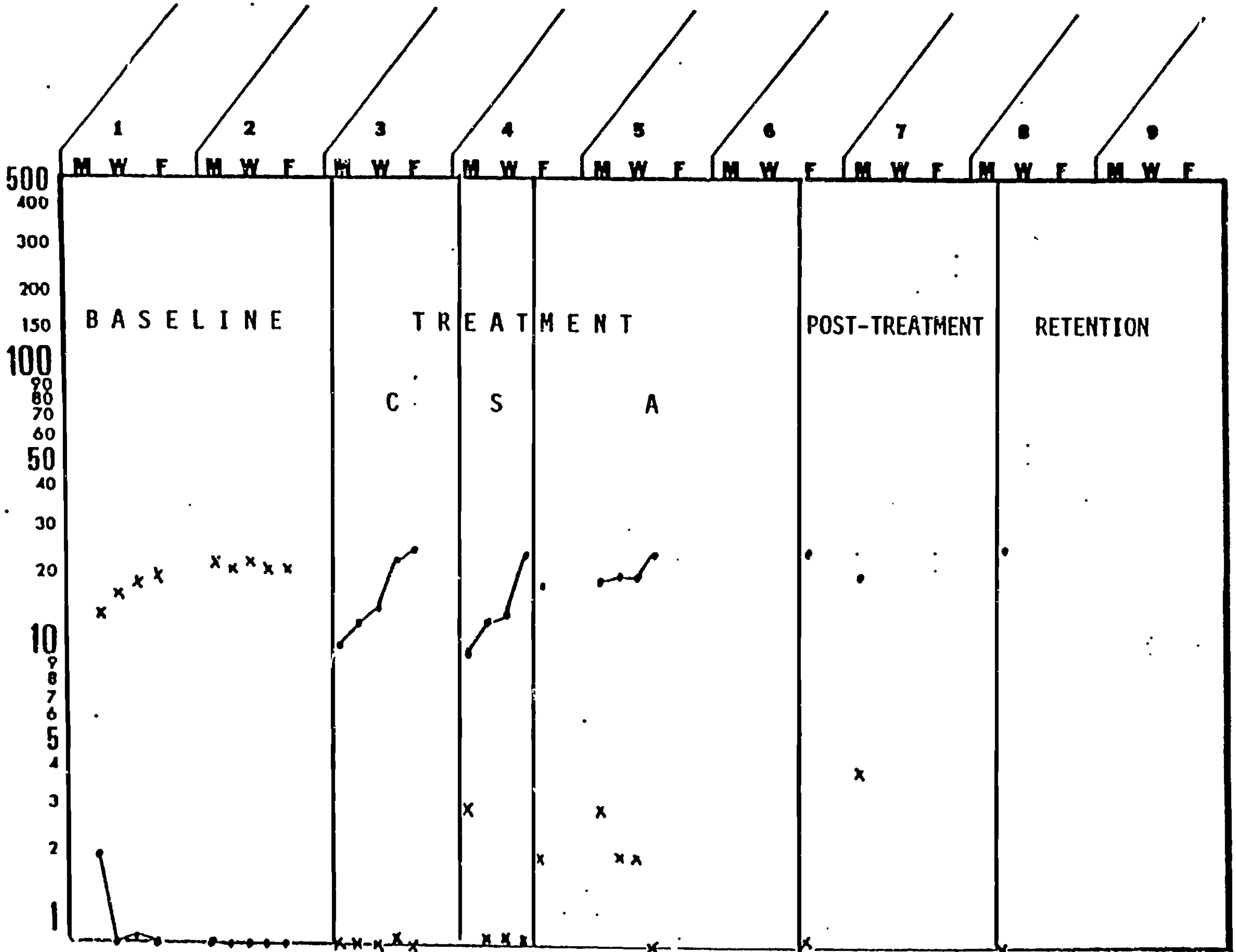


	M	T	W	Th	F
1		2 / 15	0 / 18	1 / 19	0 / 20
2	0 / 22	0 / 21	0 / 22	0 / 21	0 / 2
3	12 / 0	14 / 0	16 / 0	23 / 1	25 / 0
4	11 / 3	14 / 1	15 / 1	24 / 1	18 / 2
5	19 / 3	20 / 2	20 / 0	25 / 0	
6					25 / 1
7	20 / 4				
8	NO INSTRUCTION				
9		26 / 0			

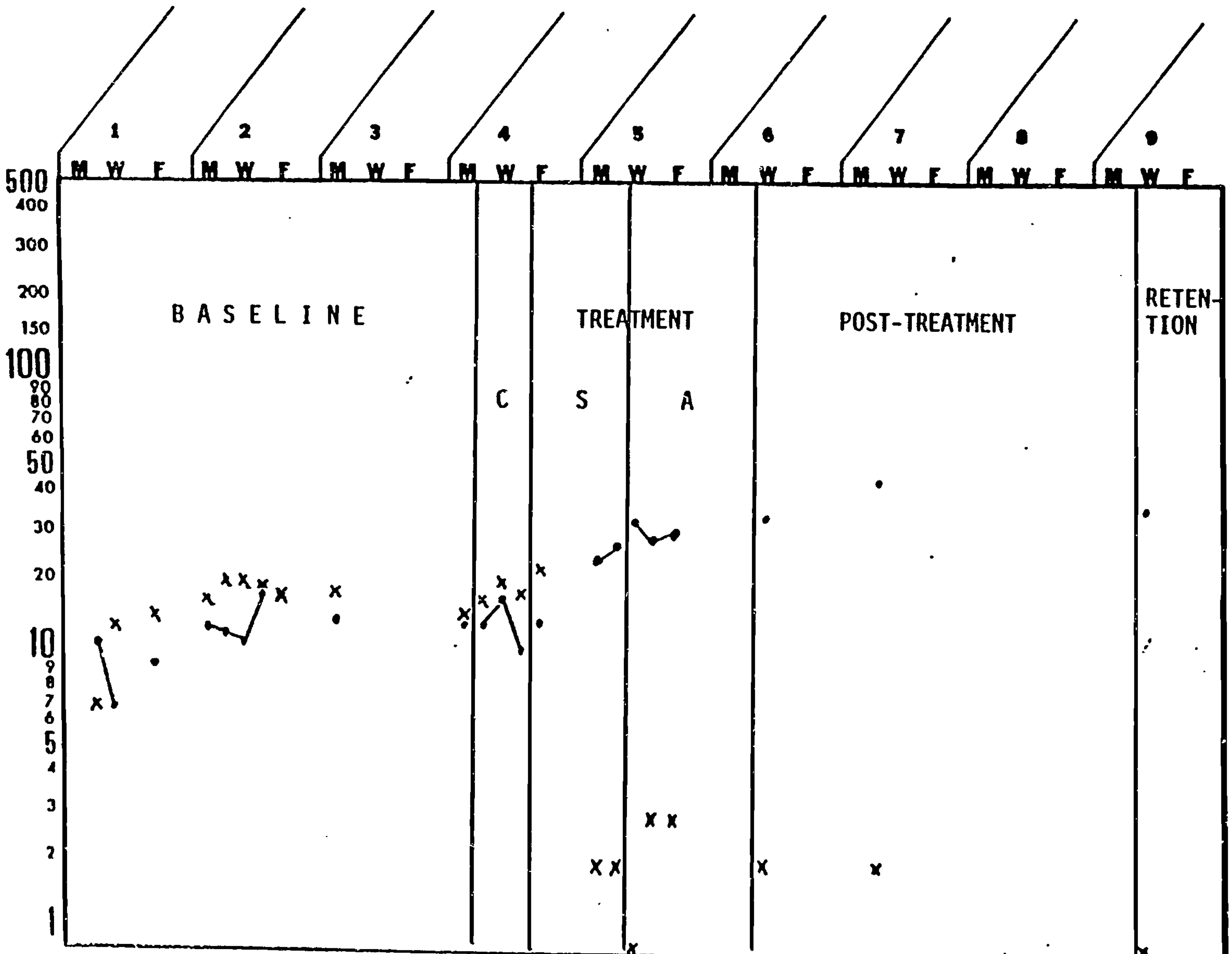
NAME Subject 2

GRADE 1st

BEHAVIOR See/Say Place Value

GOAL

Note. C= concrete
S= semiconcrete
A= abstract



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	M	T	W	Th	F
1		12/7	7/14		10/16
2	14/17	13/20	12/20	18/19	18/18
3	15/18				
4	14/15	14/17	17/20	12/18	15/22
5	25/2	28/2	32/1	29/3	30/3
6			34/2		
7		46/2			
8	NO INSTRUCTION				
9			37/1		

NAME Subject 3 **GRADE** 4th

BEHAVIOR See/Say Place Value **GOAL** ____

Note. C= concrete
S= semiconcrete
A= abstract

Discussion

The data suggest that students can be taught to identify and understand the concept of ones and tens in double digit numbers using the concrete to abstract teaching sequence in a short amount of time. Only 15 minutes per day for 9 to 14 days were required for all three students to reach criterion on the scripted lessons. The findings also suggest that the instructional sequence promotes skill retention, a frequent problem among learning disabled students, and skill generalization to a new setting. The latter finding is particularly relevant to learning disabled students who are mainstreamed to regular settings.

In summary, the concrete to abstract teaching strategy was easy to implement, yet significant in its effect. Due to the success obtained in this study additional research validating this procedure is recommended. Replication across skills and subjects would add to the existing data base.

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2C

Appendix A

Learning Characteristics of Subjects

Subject One

Subject one was a second grade learning disabled student. He had been retained twice; once in kindergarten and again in first grade. His cumulative mathematics average for the previous school year was a "D". A standardized test measure using the Kaufman Test of Educational Achievement (K-TEA) revealed below average performance in mathematics (i.e., standard score of 76; 5th percentile; 2nd stanine).

Subject Two

Subject two was a first grade learning disabled student. His most recent mathematics reportcard grade was a "D" for "developmental" indicating a need for basic math instruction. The K-TEA revealed math performance only slightly below average (i.e., standard score of 99; 47th percentile; 5th stanine).

Subject Three

Subject three was a fourth grade learning disabled student. He had been retained once in second grade. His most recent mathematics reportcard grade was an "N" for "needs improvement." The K-TEA revealed below average performance in mathematics (i.e., standard score of 73; 4th percentile; 2nd stanine).

In sum, all three students were learning disabled elementary level males experiencing varying degrees of difficulty with mathematics skills. Reportcard grades and K-TEA scores revealed less than optimum performance.

Appendix B

Pretest

1.	43	How many ones are in this number?
2.	37	How many ones are in this number?
3.	23	How many tens are in this number?
4.	13	How many tens are in this number?
5.	50	How many ones are in this number?
6.	18	How many ones are in this number?
7.	10	How many tens are in this number?
8.	7	How many tens are in this number?
9.	49	How many ones are in this number?
10.	2	How many tens are in this number?

Appendix C

See-Say Place Value Probe

41	26	64	31	61	85	27	11
48	23	52	58	43	71	90	66
87	32	15	19	15	82	39	17
64	65	37	92	16	84	43	46
12	93	81	57	68	99	92	17
18	43	46	12	93	81	18	91
24	68	13	47	71	29	56	28
17	69	14	24	30	50	31	68
45	17	91	33	54	98	47	51

Appendix D

Retention Check

1.	53	How many ones are in this number?
2.	38	How many ones are in this number?
3.	18	How many tens are in this number?
4.	45	How many tens are in this number?
5.	40	How many ones are in this number?
6.	48	How many ones are in this number?
7.	10	How many tens are in this number?
8.	6	How many tens are in this number?
9.	35	How many ones are in this number?
10.	4	How many tens are in this number?

3J