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ABSTRACT

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LEARNING TO ATTEND TO STUDENTS' MATHEMATICAL THINKING: CASE STUDY OF A COLLABORATION

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Center for the Learning and Teaching of Elementary Subjects

Institute for Research on Teaching

College of Education

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The work is designed to unfold in three phases, beginning with literature review and interview studies designed to elicit and synthesize the points of view of various stakeholders (representatives of the underlying academic disciplines, intellectual leaders and organizations concerned with curriculum and instruction in school subjects, classroom teachers, state- and district-level policymakers) concerning ideal curriculum, instruction, and evaluation practices in these five content areas at the elementary level. Phase II involves interview and observation methods designed to describe current practice, and in particular, best practice as observed in the classrooms of teachers believed to be outstanding. Phase II also involves analysis of curricula (both widely used curriculum series and distinctive curricula developed with special emphasis on conceptual understanding and higher order applications), as another approach to gathering information about current practices. In Phase III, models of ideal practice will be developed, based on what has been learned and synthesized from the first two phases, and will be tested through classroom intervention studies.

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Abstract

Recent calls for reform in mathematics education suggest students must learn to recognize mathematical elements in situations, flexibly apply appropriate mathematical tools, and engage in mathematical reasoning. These things suggest teachers must make their students' thinking a prominent part of their classroom instruction; they must make their students thinking public. In this report the authors describe the efforts of two university researchers and one teacher to make students' thinking public in a combination fourth- and fifth-grade classroom. Through a series of student interviews, classroom activities carried out by all three participants, and an instructional unit on fractions taught by the teacher, they explored ways in which teachers could make their students' thinking public. From their work in this classroom they found students were accustomed to a way of doing school mathematics that included specific norms of interaction. The students expected these norms to characterize classroom discussions and struggled with the changes the participants tried to make. In addition, other issues became important during the participants' time in this classroom. The open-ended discussions they encouraged took a large amount of time and raised questions for the teacher about the importance of developing understanding over covering the content. The changes advocated involved fundamental rethinking of assumptions and beliefs about mathematics teaching and learning. Finally, their work in this classroom caused all three to struggle with their roles as collaborators, researchers, and change agents.

LEARNING TO ATTEND TO STUDENTS' MATHEMATICAL THINKING: CASE STUDY OF A COLLABORATION

Ralph T. Putnam and James W. Reineke[1]

Mathematics educators and researchers are calling for radical revisions in how mathematics is taught in elementary school classrooms (National Council of Teachers of Mathematics, 1989, 1991; Romberg & Carpenter, 1986). Reformers argue that, rather than learning isolated computational skills, students should learn to recognize the mathematical elements in situations, flexibly apply appropriate mathematical tools, and engage in mathematical reasoning such as conjecturing and justifying. All these goals suggest the importance of making students' mathematical thinking more prominent in instruction--making students' thinking public. Students need opportunities to communicate, either orally or through writing, their thoughts about particular mathematical situations or problems and develop a willingness to reflect upon and discuss their own thinking and that of others.

This report documents the efforts of two university researchers and one teacher to make students' mathematical thinking more public in a fourth/fifth-grade classroom in a Professional Development School[2] (PDS). We first describe briefly the setting for this work and sketch out

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[2]Professional Development Schools is a name given to selected schools by the Michigan Partnership for New Education. Professional Development Schools are based on the Holmes Group (1986, 1990) recommendations that teacher education should be more school based, schools should learn to make more contextual use of research, and research should be done in schools. Teachers, staff, and administration of Professional Development Schools work with university faculty in planning and conducting research they believe will be useful for their school.

our main activities over the year. We then explore what emerged as a central focus of our work together--the role played by classroom norms and students' expectations when trying to make changes of discourse in classrooms. Finally we discuss three issues or themes from this work that seem especially salient for thinking about what it takes for a teacher to make meaningful change in his or her mathematics teaching.

A Collaborative Project

Getting Started

Our collaboration with Alice Smith[3] began with a project of the Center for the Learning and Teaching of Elementary Subjects. Having completed a number of surveys and case studies to document and better understand current practice, our goal was to collaborate closely with a small number of teachers to support them in making meaningful changes in their teaching.

We chose to carry out this project in a Professional Development School because teachers would have time to meet and reflect with us and there was a general expectation at the school that teachers would be working to examine and make changes in their teaching. Smith was one of four teachers who responded when we asked teachers to form a working group to think about mathematics teaching and learning. Our original intent was to form a group of four or five teachers and four Michigan State University faculty and graduate students to work on these issues together. Although the specific activities and focus of the group was to be negotiated among the teachers and the researchers, our starting

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[3]Teacher name is a pseudonym.

assumption was that a central theme would be working together to consider the role of students' mathematical thinking in instruction and how to make this thinking a more public and prominent part of instruction (Putnam, in press). After two meetings with such a group, three of the teachers decided not to participate because of other PDS commitments and personal reasons: Some teachers were already committed to working with a home literacy project that they felt would take significant amounts of their time and attention; one teacher was beginning a co-teaching arrangement with a MSU graduate student; one teacher had medical problems that made additional time commitments difficult.

So our group became a small one: Putnam, Reineke, and Smith working together to reflect on the mathematics teaching and learning in her classroom and think about ways to enhance them. A hallmark of our collaboration was its flexibility: We all learned and changed as a result of our work together.

What We Brought to the Collaboration

We came to this collaborative project thinking about the relationship between researchers and classroom teachers in new ways. We were skeptical of the assumption in some previous research that the role of researchers should be one of presenting to teachers instructional prescriptions abstracted from other studies, leaving teachers on their own to figure out how the techniques, procedures, or behaviors should be adapted for particular classrooms. Rather, we believed with the Holmes Group (1986, 1990) that teachers and researchers should, at least at times, conduct research together in schools, working to keep the emerging knowledge closely grounded in classroom practice. Collaborating in this way is like entering into a conversation where participants' roles are constructed and reconstructed throughout the collaboration (Heaton, Reineke, & Frese, 1991). The roles of participants at any given time reflect the assumptions, goals, and beliefs of the individuals and the group. In light of this conception of collaborative research, we believed it was important to make explicit the framework we brought to the project and that it meshed or connected with what Smith hoped to accomplish by joining the conversation. In other words, we believed that the direction our project took must be shaped by the interests of all the participants, not just those of the researchers who initiated the effort.

We came to this collaboration with multiple goals. We wanted to work together with Smith to improve her mathematics teaching and the ways she thought about it. Based in part on what we had been learning from studying other teachers and students, we were committed to the idea that students ought to have opportunities to make their mathematical thinking public and that good mathematics teaching would make this thinking a central part of instruction (Putnam, in press). We hoped to work together with Smith to explore ways of making students' mathematical thinking a more public and prominent part of the mathematics classroom. We thought that attention to students' mathematical thinking could serve as an important theme or organizer to help keep our attention focused throughout the year, without prescribing particular procedures, activities, or materials for change.

The calls for collaborative research suggested by the Holmes Group, however, are constructed by educational researchers and scholars, not by teachers. They envision new ways of conducting educational research that are, for the most part, brought to teachers and schools by researchers praising their benefits for teachers. As a teacher, Smith was not quick to accept this particular vision of collaborative research. Rather, she was interested in learning from us, as MSU researchers, ways to improve her mathematics teaching. Although she was interested in talking and thinking about new ways of teaching mathematics, Smith was concerned about district tests her students were required to take in the spring. The tests focused on computational accuracy and Smith thought her students, the fifth-graders in particular, did not know their basic facts and computational skills. She was also concerned that her students had a great deal of trouble with story problems and that they did not seem willing to think hard and take their work seriously.

Smith described herself as teaching her combination fourth/fifth-grade classroom traditionally, from the book. Although Smith believed that her teaching, like most elementary school mathematics instruction, focused too heavily on computational skills and not enough on students' understanding, she did not have clear images for how to go about making significant changes in her classroom practice. What she talked about wanting from our collaboration was learning from us, the experts, about materials and activities she could use to enhance the teaching toward her existing instructional goals.

In short, whereas Smith came into this collaboration with a desire to learn new instructional techniques, we (Putnam and Reineke) came with a hope to facilitate Smith's thinking differently in two arenas. First, we wanted her to think in new ways about mathematics--reconsidering what it might mean to teach and learn mathematics in meaningful ways. Second, we wanted to involve her in a new conception of educational research.

What We Did: Overview of the Year

The three of us engaged in a variety of activities over the 1990-1991 school year, all oriented toward helping us think together about the teaching and learning in Smith's classroom and how it might change. In the fall Reineke and Putnam observed Smith's mathematics teaching, seeking to document and understand her current approach to teaching mathematics. We then embarked on a series of agreed-upon activities to push our thinking and to begin making changes in the classroom. First, we each interviewed two students about multiplication and division, a topic that Smith was currently teaching, while they solved word problems. This served as an important site for the three of us to think and talk about students' thinking--better understanding difficulties and resources they might bring to the learning about multiplication and division in the classroom.

We then developed some relatively complex problems that offered considerable room for multiple solution strategies by students. On three different days, one of the researchers presented such a problem to the entire class of fourth- and fifth graders and asked students to work individually on the problem for a few minutes, drawing pictures or giving written justifications for their solutions. We then divided the class into small groups for discussion, each led by one of the three adults, with the goal of getting students to discuss their alternative solution strategies and explore the mathematical ideas involved in the problems. These sessions served both as models for Smith for what discussion around students' mathematical thinking might look like and helped begin to change the norms for interaction in the classrooms.

In the spring, we jointly planned an instructional unit on fractions that Smith taught to her fourth- and fifth graders. Our original intent was that Smith would take the lead in planning a unit with Reineke and Putnam serving in a facilitative support role. But Smith was not comfortable taking the leadership for planning the unit; she felt she did not know enough about alternative ways to think about fractions or about how classroom activities might be structured. So Putnam and Reineke planned a series of lessons for Smith to teach. The lessons used paper folding as the primary representation for thinking about fractions. They were structured around small-group tasks--problems involving the paper folding--followed by whole-group teacher-led discussion of the various ways students had solved the problems in their groups. We designed the activities to emphasize students' alternative solutions and ways of thinking about the mathematics.

Student Norms and Expectations

One thing that became immediately and strikingly salient to us as we started to try to make changes in this classroom was the importance and power of students' norms, beliefs, and expectations in shaping what goes on in the classroom. Changing the classroom discourse is not simply a matter of the teacher deciding to ask different sorts of questions or of arranging students in different ways, such as small groups, to foster rich discussion. Students, too, have to learn new ways of thinking and interacting, often in the face of long-held beliefs and expectations for what it means to participate in mathematics lessons. For students bring to school well-developed motivational sets (Dweck, 1989), norms of interaction (Heath, 1982), and ideas about what problems are worth solving and how to solve them (Goodnow, 1990). They also grow accustomed to certain patterns of interaction in school. The students in Smith's classroom had learned how to act appropriately in it; they had learned the "rules" for getting along and for what constituted appropriate participation in mathematics lessons. And these expectations were in many ways at odds with the sort of classroom discourse we were hoping to foster.

Existing Classroom Norms

During our early visits to this classroom, the norms of interaction we observed fit with traditional views of classroom instruction; that is, the content of the lesson was presented by the teacher at the front chalkboard and the students worked quietly at their desks. During the presentation Smith asked "teacher questions" (Edwards & Mercer, 1987) and her students responded with what they believed to be the right answer. If, by chance, their answer was not correct the teacher would inform them of its incorrectness and tell them what they had done wrong. The students would repeat the problem at their desks until they solved it correctly. Once the right answer was announced the other students would look to see if they had computed the problem correctly. Following the presentation, the students would be given an assignment which often included many problems of the same type. This usually occurred twice during each lesson--once for the fifth-grade students and once for the fourth-grade students. While the teacher was addressing students in one of the two grade levels, the other students would work independently at their desks.

Smith began one lesson by drawing a series of examples on the overhead projector at the front of the room. Each drawing consisted of a row of 10 boxes with some of the boxes shaded in to represent a specific decimal number. For example three tenths was drawn:

For each drawing Smith asked her students how the number being represented was written and spoken. For three tenths, one student suggested that it should be written .3 and spoken "three tenths." Smith responded "good' and tried to go on. Another student, however, thought he knew another way to write that number. He suggested it could be written $\frac{3}{10}$. Smith responded that the class was talking about decimal numbers, not fractions, so $\frac{3}{10}$ would not be correct--at least not in this situation.

The next example, five tenths, was drawn on the overhead and the teacher asked a student to come to the front and write and say the number. The student wrote 51.0 and the class objected to what the student had written. Smith stopped the class from commenting on the student's work saying "Just be quiet please. This is a learning experience. . . everybody gets a chance to show how they are understanding and if you don't understand, that's quite all right." The student told Smith that the numeral he had written should be read "fifty-one and zero tenths." Smith asked if he had shown 51 wholes in the drawing and the student said "No." Smith told the student he was reading it right when he said "zero tenths" and used that as a way to help the student with the problem. She wrote ___ . ___ ___ on the overhead and asked the student to fill in the blanks. She asked him what place in the drawing represented the tenths place. When the student had difficulty identifying the tenths place, Smith turned back to the numeral the student had written earlier and said "you said this was 'zero tenths' so how could you write 'five tenths' in these spaces?" After a short discussion the student wrote "5" in the blank just to the right of the decimal point and Smith summarized saying "Good, whatever number is just to the right of the decimal point is the number of tenths."

The lesson continued with a series of these examples. For each example Smith drew a picture to represent a specified decimal number. For each picture she called on a student to write and say the number. After the student responded, she would evaluate the student's response by either praising them or by walking the student through the problem until he or she could answer the problem.

Students in Smith's classroom, as students in many classrooms, are faced with a difficult task. Along with trying to make sense of the content being presented, students need to determine what actions the teacher deems appropriate in specific situations (Leinhardt & Putnam, 1987). These "rules of conduct" then become the norms of interaction in the classroom. But knowing what behaviors are appropriate covers only part of what needs to be considered. Students need to understand acceptable ways of interacting among themselves and with the teacher. In the example presented above, the students in Smith's class easily participated in the discussions she initiated. They seemed to understand when it was appropriate to speak and when they should listen. When her students spoke at an inappropriate time, Smith reminded them of what are and are not acceptable ways of talking.

Edwards and Mercer (1987) have suggested that classroom conversation is "an instance of talk in general" (p. 42). As such, classroom interaction is framed by local versions or instantiations of the co-operative principle (Grice, 1975). This principle holds that people involved in a conversation will (a) contribute only what they have evidence for and believe to be true, (b) provide only the amount of information that is necessary, (c) make their contribution relevant to the conversation, and (d) make their contribution intelligible. What each of these maxims actually mean in practice is dependent on the particular social situation in which they are used; that is, what these maxims look like in a given classroom emerges through participation in classroom discourse.

The norms of interaction Smith and her students had constructed in her classroom reflected the *I-R-E* (initiation, response, evaluation) pattern identified by educational researchers (Cazden, 1988; Edwards & Mercer, 1987; Mehan, 1979). In this pattern the teacher presents the class with a problem and elicits a response from one or more students. Following the student's response, the teacher evaluates what they have said, either praising them for being correct or pointing out a mistake and working to correct the error. After learning this pattern of interaction, the students, it would seem, would construct an instantiation of the co-operative principle that reflects the pattern and anyone attempting to restructure the norms of interaction would be seen as violating this principle.

Changing Classroom Discourse

Interrupting the students' patterns of interaction was exactly what we intended to do. All three of us brought to this project the goal of getting elementary students talking and thinking about mathematics. The students had grown accustomed to interacting in specific ways and we were asking them to change those ways. The existing instantiation of the co-operative principle had students providing only a numerical answer for which they did not need to provide evidence. Whether a response made sense to other students was not really an issue. But now we were asking students to tell us how they had solved the problem and why they thought their solution worked; we wanted students to convince their classmates that their solution worked. We hoped that the new instantiaton of the co-operative principle that the class constructed would be informed by the discipline of mathematics; that is, students would develop an intuitive understanding of the problems we posed, make conjectures about the mathematics involved in the problems, and attempt to refute the solutions presented by the group members. Trying to develop norms of interaction where students were actively engaged in assessing mathematical situations and possible solution strategies, however, proved difficult.

When we began changing the mathematical tasks in this classroom, the students seemed to expect similar interaction when working on the problems we developed. They had difficulty attending to what was being said by other members of their group. They did not see this as a necessary part of doing mathematics for a couple of reasons. First, in the past the teacher had decided which response was correct and there was only "one right way." They were not familiar with the responsibility of assessing a solution for its value in solving the problem at hand. Second, they were not used to talking among themselves. During their previous classroom instruction, interaction occurred between the teacher and the student responding to the problem posed. The only interaction between students was surreptitious discussions of things not associated with the mathematics being discussed. Furthermore, some students were rather unwilling to think about problems in a way that was different than what they had done in the past. Students who could compute mathematical algorithms faciiely saw little utility in drawing a picture or deriving a way of convincing other people in their group that their solution worked.

During one of the small-group sessions the students were discussing the number of sundaes a store owner could make with a specified number of ice cream flavors and toppings. In the first part of the problem, the store owners had four flavors of ice cream and three toppings. The students were asked to find out how many different types of sundaes the store owners could make (using one flavor of ice cream and one topping) and to formulate a way of convincing their fellow group members. Many students immediately said the store owners could make 12 different types of sundaes because 4 x 3 = 12. Once the students agreed that 12 was the correct response, some of them no longer attended to what was being discussed. Providing a justification for their interpretation of the problem was not part of the normal interaction routines developed in this class; it violated the existing co-operative principle.

But the quality of students' participation did change. During each of the subsequent small-group problem-solving sessions, more students drew pictures or other representations of solutions that did not involve simply applying a standard algorithm. Students were increasingly willing to talk about and explain how they had thought of the problem and to listen to other students. As Smith taught the unit on fractions, she and her students managed to work out interaction patterns that provided room for student discussion of the mathematics being learned.

Difficulties associated with mathematical discussion in the classroom are not limited to students. The traditional forms of teaching that characterized Smith's classroom have recently been characterized as being authoritarian and impoverished (Putnam, in press; Romberg & Carpenter, 1986). Traditional teaching methods, it is argued, overly emphasize isolated computational skills. Getting students to discuss mathematical ideas in the classroom, it is hoped, will provide a more thorough understanding of mathematical concepts. The recent calls for reform suggest that teachers need to transfer the authority for assessing what is right or wrong to the students. But, like other teachers we have talked with (Peterson, Putnam, Vredevoogd, & Reineke, 1992), Smith was concerned about the importance of covering the curriculum. She felt that getting students involved in discussions of mathematical concepts might hurt the algorithmic competence they would need for the district wide mathematics test that was administered each fall. Smith felt that she could ensure the students familiarity with the algorithms if she continued to stress computational skill systematically during her mathematics instruction. As a consequence of this belief, Smith, at times, reverted to direct instruction of algorithms. At other times, however, Smith followed the ideas brought out by students. Her reaction to these conversations was mixed. On one hand, Smith expressed interest in what her students were thinking and, consequently, enjoyed these discussions. On the other hand, Smith was often concerned that the conversations were wasting valuable instructional time. Indeed, the conversations we, as researchers, found exciting, Smith often found problematic.

Issues Concerning Teacher Change

As we worked together over the school year, several issues in addition to the importance of attending to classroom norms and student expectations emerged as especially important in our efforts to foster changes in this mathematics classroom. First, the richer, more open-ended student discussions that Smith was trying to foster took considerably more time than her traditional approach. This raised tensions for Smith between spending the time for students to develop understanding and covering the amount of mathematics content, particularly computational procedures, that she felt students should learn. Second, the sorts of meaningful changes we were trying to foster in this classroom cannot be reduced to a set of activities, techniques, or materials to be introduced and implemented by the teacher. Rather, they involved more fundamental rethinking by the teacher of assumptions and beliefs about mathematics, about teaching, and about learning. Finally, as researchers in this classroom, we struggled with our roles as collaborators, researchers, and change agents. Like teachers striving to foster student independence of thought and self-directed learning, we faced the dilemma of creating a balance between our goals and vision for what good mathematics teaching might be like and the need for meaningful changes to come from teachers themselves (see Cohen & Ball, 1990).

Struggles With Time and Coverage

We hear again and again in talk about teaching for understanding that teachers should emphasize depth over breadth--that understanding takes time and you cannot expect to cover as much material if you teach in ways that foster rich understanding. This was an important issue for Smith throughout the year, but it did not start with us.

Prior to our collaboration, Smith felt pressed for time: There was so much to cover and so little time to cover it. This belief seemed to stem from at least two sources. First, Smith's school district provided teachers with a list of grade-level objectives to cover each year. The objectives emphasized computational skills listed as discrete mathematical topics. On this list, for example, one-digit multiplication, two-digit multiplication, and three-digit multiplication are listed as separate topics. The second source of tension was two tests Smith's students were required to take in the spring. School administrators used scores from both tests to place students in appropriate educational settings. District teachers administered the Stanford Achievement Test to place students in enrichment or Chapter 1 classes. Teachers gave the other test a district-wide mathematics assessment, to assist middle-school administrators in placing students in appropriate math classes. Thus, these were relatively high-stakes tests for students and teachers: Smith justifiably believed it was important to cover and help students master the material they tested.

This felt pressure was at least partly responsible for Smith's belief that she needed to teach her fourth- and fifth-grade students in separate mathematics lessons, in spite of believing that the fourth graders were capable of doing fifth-grade work. Smith felt that there were expectations from other teachers as well that certain content should be covered in certain grades. In her initial interview with us, Smith talked about this tension about what to cover:

> Right now, I have kind of a quandary with these particular fourth graders, because I feel that some of them really should be . . . accelerated. Some of them are really capable of actually being in the fifth-grade math book right now, without great difficulty. And that's a real philosophical problem because of the fact that, or it may be a practical problem, I guess, because if I move them into fifth-grade

content, then when they go into fifth-grade next, the teacher in that grade will complain they've already had the material in the book and they won't want to take them into a sixth-grade book.

Thus, at the same time she felt limited by the district objectives and expectations of other teachers, Smith felt pushed by them. This pressure led to a feeling that extraneous content should not get in the way of teaching computational skills. For example, in talking during an early interview about the textbook's presentation of story problems, Smith mentioned she had a general feeling of dissatisfaction with the story problems, stemming partly from how they were written, but also from their placement in the text. The text had pages of story problems scattered throughout the book. For Smith, having students work on these story problems as they came up in the book meant interrupting students' work on the operations being taught and wasted precious time better spent on computation. Smith argued that it might be better if she could pull some of these pages and do a whole unit on story problems.

Limiting content, emphasizing computation, and adhering to a predetermined list of objectives flew in the face of the conception of mathematics education we (Putnam and Reineke) brought to this project. Throughout our collaboration we encouraged Smith to follow her students' thinking, all the while stressing the depth of learning over the breadth of coverage. We hoped that discussions that grew out of the small-group problems and the unit on fractions would lead to mathematical areas other than those apparent at a first glance at the problems, usually the choice of which operation should be performed. These discussions interested Smith. Her interest grew with the interviews she conducted with a few of her students earlier in the year and with the three small-group projects we tried together. Smith had not, however, given up her concern for covering the content as listed in the district objectives and tested in the spring. Throughout our planning of the unit on fractions, Smith stressed the importance of covering the expected computational procedures with fractions. She felt the unit needed to include, among other things, least common denominators and addition and subtraction of fractions with like and unlike denominators.

Smith's concern for covering the content continued as she taught the unit on fractions. We initially planned activities we thought would take one week, but, as the students began talking about fractions and other mathematics topics, the weeklong plans took two weeks to complete and Smith's concern grew. She thought she was spending entirely too much time on fractions. We tried to point out that within the unit Smith and her students were working on many different mathematical skills. She agreed to continue as long as we agreed to make sure that computational procedures were addressed in the unit.

Changing Beliefs About Mathematics, Teaching, and Learning

Smith began this collaboration with a desire to learn about activities and materials that she might use to improve her mathematics teaching. She was especially concerned that her students were not mastering their basic facts and computational skills and had special difficulty with story problems. She was essentially looking for activities or techniques to reach her existing instructional goals better. Putnam and Reineke, in contrast, came to the collaboration with a belief that making meaningful changes in mathematics teaching involves more than additional activities, materials, or techniques. If changes are to be more than superficial or "proceduralized," they must entail more fundamental rethinking of the nature of mathematics we want students to learn in schools and how mathematics is best taught and learned (Putnam, Heaton, Prawat, & Remillard, in press).

Smith's beliefs about mathematics, learners, and teaching did change over the course of the year. For example, at the beginning of the year, when Smith talked to us about what she wanted her fourth- and fifth-grade students to learn about mathematics, the first instructional goal she emphasized was computational skill: "I guess one thing I hope [is] that they learn to compute basic problems with more accuracy." She saw this learning as important and straightforward: "Well I think . . . math basically, for <u>basic</u> computation, is about as simple as you can get, as long as they know their facts. Smith also talked about the nonproblematic and certain nature of mathematics:

> I think math is an area that your mind can get, you know, really kind of <u>excited</u> about, because it makes sense! And there's so little in life that makes sense. And to me it's kind of reassuring to find <u>one thing</u> that, I mean there <u>is</u> an absolute answer in most, you know, particular problems at this level. I know . . . there are different ways of <u>doing</u> it, but you can still get <u>an</u> answer.

In addition to these statements by Smith about her goals and views of mathematics and how it is learned, two incidents highlighted for us differences between Smith and us in beliefs and mathematics and learning. First, after the initial session in which we posed the ice cream sundae problem to the students to work on individually and in groups, the three of us looked over some of the students' written work toward solutions to the problem. Putnam and Reineke were struck that Deveda, whom Smith described as one of her weakest math students, had been one of the few students to draw pictures to come to a sensible nonalgorithmic solution to the problem. We were excited that Deveda had shown some good mathematical thinking that might serve as an important example for other students and providing a starting point for rich discussion and learning about multiplication. Smith, in contrast, found Deveda's pictures to be a clumsy attempt to make do in absence of knowing what operations (multiplication) to apply. The second incident took place during one of the first days of Smith's teaching of the fractions unit we had developed together. Reineke observed the lesson, during which he was pleased to see students presenting different solution strategies and ways of thinking of the task--engaging in some interesting mathematical discussion and thinking.

After the lesson, Smith apologized to Reineke for letting the discussion get so off track. What Reineke had seen as an exciting opening up of the mathematical thinking and discussion, Smith had viewed as tangential and off task. This incident was particularly important because it served as an important site for dialogue: Talking through these instances together helped us to better understand the tension Smith was feeling about not getting through the content fast enough and for Smith to better understand what we meant when we talked about wanting students to have opportunities to explore ideas and reason through them together.

By the end of the year, Smith was talking differently about students and the mathematics they were learning. She was, for example, more attentive to their mathematical thinking and willing to think of mathematics less convergently:

> I did find it really interesting to observe how they would approach things when they didn't have the same kind of

structure or the same formatting to go into a task with, when they had to use more of their own reasoning and apply principles that they knew somewhat to new tasks but not really know exactly how to go about accomplishing something, when they didn't have any rules necessarily that apply to what they were doing.

Smith was thinking of mathematics as less clear cut and nonproblematic than she had at the beginning of the year. As Reineke was putting away the tape recorder after observing a lesson late in the year, Smith said to him, "Jim, you know what you're doing don't you? You're taking the one area that I always thought was clear cut and you're making it fuzzy."[4]

Negotiating Roles as Collaborators

We went into this collaboration simultaneously with beliefs and images about what good mathematics teaching might be like. But we also went with a commitment to collaboration and to valuing the teacher's goals and perspectives.

There is a tension or dilemma here that is similar to the dilemma teachers face if they want to encourage students to be self-directed voracious learners and thinkers. On the one hand, we wanted Smith to change in a particular direction--we wanted her to incorporate more reflective discussion into her mathematics teaching, to have instruction focus more on conceptual understanding of the mathematics being learned rather than on simply learning computational techniques. Just as teachers have goals for their students, we have goals and images for what we would like to see classrooms and teachers be like. But, also like teachers

[4]This is a paraphrase, reconstructed from memory, since the tape recorder was turned off.

who want to empower their students, we want changes to come from teachers themselves, realizing that we cannot simply "tell" teachers to do things in new ways and expect meaningful change to take place.

It is overly simplistic to say we just have to empower teachers. Teachers' perspectives and voices on what goes on in classrooms are obviously critical and have been underrepresented in much research and policy discussions. But researchers, too, have important perspectives to bring to thinking about and fostering change in classrooms. What made this collaboration successful was that we brought goals, but were flexible and expected that how they played out and the goals themselves would be open to negotiation. At times this meant us being more directive than we had wanted to be, but we did so very much in the spirit of "scaffolding"--and then turning more and more over to Smith (Wood, Bruner, & Ross, 1976).

Conclusion

Our collaboration with Smith was successful in that it helped all of us better understand what goes on in mathematics classrooms and how to foster changes. By the end of the year, Smith was feeling more comfortable with the richer images of mathematics teaching that we been trying to foster, and some significant changes had taken place in her classroom. Students' mathematical thinking had come to play a more prominent role in the discourse of this classroom.

Smith had become confident enough about her mathematics teaching to be willing to teach mathematics to all the fourth- and fifth-grade classes at the school when the teachers decided over the following summer to departmentalize instruction by subjects. During the next school year she joined a mathematics study group with other teachers and a MSU graduate student to continue working at improving and rethinking their mathematics teaching. Reineke continued to observe in Smith's class and having conversations with her periodically over the 1991-1992 school year and has seen Smith continue to work toward ways to make student discussion a more central part of her instruction. Smith continues to struggle with the "depth versus breadth" issue, but in-depth exploration has been winning out: She spent a large part of the year exploring fractions with her fourth and fifth graders.

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