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ABSTRACT

College algebra is currently being taught using graphing calculators such as the TI-81. This document presents exploratory and problem solving activities that have resulted from classroom experiences with the TI-81. The activities have been designed to enhance the learning of some standard college algebra topics through the easy access to visualization provided by the TI-81. The activities incorporate the following topics: (1) interpretations of the graphs of functions modeling real world problems; (2) shifts and translations of basic functions; (3) intersections of curves; (4) solutions of equations and inequalities; (5) graphs of rational functions; (6) graphs and applications of exponential functions; (7) graphs and applications of logarithmic functions; and (8) zeros of polynomials. A list of selected answers to problems posed in the activities is provided. (MDH)

EXCURSIONS with the TI - 81 GRAPHICS CALCULATOR in COLLEGE ALGEBRA

AMATYC NATIONAL CONFERENCE

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Excursions with the TI-81 in College Algebra

Abstract

Our classroom experiences in College Algebra with the TI-81 have resulted in the development of several exploratory and problem solving activities. These activities have been designed to enhance the learning of some standard college algebra topics through the easy access to visualization provided by the TI-81.

These activities incorporate the following topics:

1. Interpretations of the graphs of functions modeling real world problems
2. Shifts and translations of basic functions
3. Intersections of curves
4. Solutions of equations/inequalities
5. Graphs of rational functions
6. Graphs and applications of exponential functions
7. Graphs and applications of logarithmic functions
8. Zeros of polynomials

Each of these topics will be explored through a demonstration, followed by participants engaging in a student exploratory or minilab. Discussions of how these activities could be incorporated into the classroom to enhance and affect student learning will be encouraged.

As experienced mathematics educators, our excursions into incorporating the TI-81 into our classes have caused us to realize how dramatically different mathematics education can be. Easy access to visualization as provided by the calculator allows student ownership of concepts. Emphasis shifts from algorithmic techniques to understanding of concepts and applications. We hope the participants experiences in this minicourse will excite them about the opportunities this technology provides for mathematics education.

Excursions with the TI-81 Graphics Calculator in College Algebra

AMATYC National Conference - Indianapolis
November 5, 1992 - 1:00pm-5:00pm

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Interpreting Functions Using Graphs

Problem: A ball shot straight up from the top of a building 100 feet tall with an initial velocity of 32 feet per second follows a path given by $s(t) = -16t^2 + 32t + 100$ where $s(t)$ represents the height of the object (in feet) at t seconds.

1. Find a complete graph of this function on the window $[-4.8, 4.7]_x$ by $[-20, 120]_y$.

2. What portion of the graph applies to the path of the ball (the problem situation)?

3. What x values on this graph are meaningful for the ball?

4. Interpret the meaning of the y-intercept.

5. What is the height after 1/2 second?

6. How long before the ball reaches 100 feet again?

7. How long before the object reaches its maximum height and what is its maximum height?

8. What part of the graph represents hitting the ground? Approximately when does the ball hit the ground?

Interpreting Functions Activity

Problem: The function $f(t) = .04t^3 - 1.84t^2 + 21.12t$ is an algebraic representation of the temperature (Fahrenheit) in a certain town for a 24-hour period. Assume $t = 0$ is 6 a.m.

1. Use your calculator to graph on a $[-10, 37.5]_x$ by $[-20, 85]_y$ window with scales of 5 to see the complete graph of the function.

2. What x values are meaningful for this application? Sketch the portion of the graph that applies to the problem situation.

3. What do x and y represent in this function?

4. What is the temperature at 3:00 pm.?

5. Approximately when will the temperature be 45° F?

6. Approximately when will the temperature be 80° F?

7. What are the x intercepts of this graph and what do they represent in this problem?

8. What is the highest temperature (approximately) and when is it achieved?

9. What is the lowest temperature (approximately) and when is it achieved?

Exploring Shifts and Translations

Exploration 1: Graph each of the following using the calculator, adding one equation at a time. Use the friendly window changing the Ymax to 6, ie. $[-4.8, 4.7]_x$ by $[-3.2, 6]_y$

- **a.** $y = |x|$
- **b.** $y = |x| + 1$
- **c.** $y = |x| + 3$
- **d.** $y = |x| - 2$

- **e.** Each equation is of the form $y = |x| + k$. What effect does the k have on the graph?

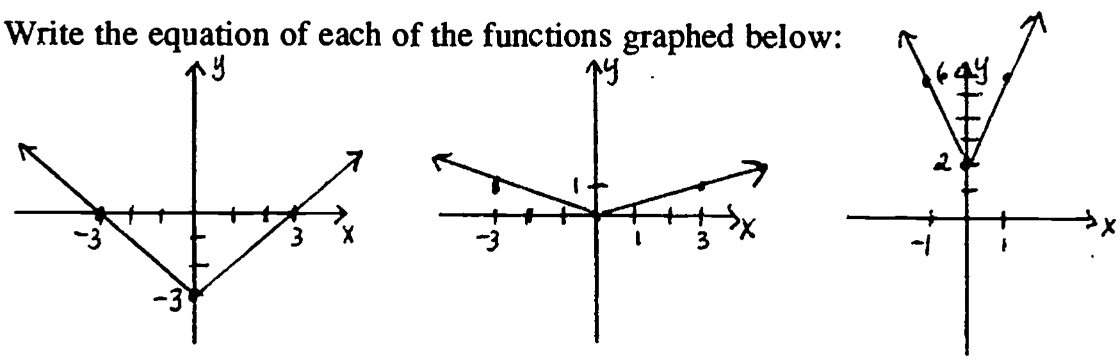
- **f.** What does the $|x|$ part of the equation cause in the graph?

Exploration 2: Graph each of the following using the calculator, adding one equation at a time. Use the same range values as #1 above.

- **a.** $y = |x|$
- **b.** $y = 2|x|$
- **c.** $y = 3|x|$
- **d.** $y = \frac{1}{2}|x|$

- **e.** Each equation is of the form $y = a|x|$ for $a > 0$. What effect does the positive a have on the graph?

Exercise: Write the equation of each of the functions graphed below:



Exploring Shifts and Translations Activity

Exploration 1: **a.** Use the friendly window, ie. $[-4.8, 4.7]_x$ by $[-3.2, 3.1]_y$, to graph the following functions. Enter and graph the first as y1, then repeat this process with the second as y2, the third as y3, etc without turning off any of the preceeding graphs. At the end you should see all four functions graphed. Sketch your final window labeling each graph with its equation.

i. $y = \sqrt{x}$

ii. $y = \sqrt{x+2}$

iii. $y = \sqrt{x-1}$

iv. $y = \sqrt{x+4}$

b. Without using your calculator, draw a sketch of $y = \sqrt{x} - 2$. Check your result by graphing on your calculator.

c. Each equation can be written in the form $y = \sqrt{x-h}$. What effect does the h have on the graph?

d. What does the $\sqrt{x}$ part of the equation cause in the graph?

Exploration 2: **a.** Use the friendly window, ie. $[-4.8, 4.7]_x$ by $[-3.2, 3.1]_y$, to graph the following functions. Enter and graph the first as y1 and the second as y2. Sketch your result labeling each graph with its equation. Turn these "off" and enter and graph the remaining two functions as y3 and y4. Again, sketch your result labeling each graph with its equation.

i. $y = x^2$

ii. $y = -x^2$

iii. $y = |x|$

iv. $y = -|x|$

b. Without using your calculator, draw a sketch of $y = -\sqrt{x}$. Check your result by graphing on your calculator.

c. For an equation of the form $y = -f(x)$, what effect does the $-$ have on the graph of $y = f(x)$?

Challenge 3: a. Each of the following combines different types of shifts and translations. Without using your calculator or plotting points, draw a sketch of each of the following functions. Check each using your calculator.

i. $y = (x + 1)^2 - 2$

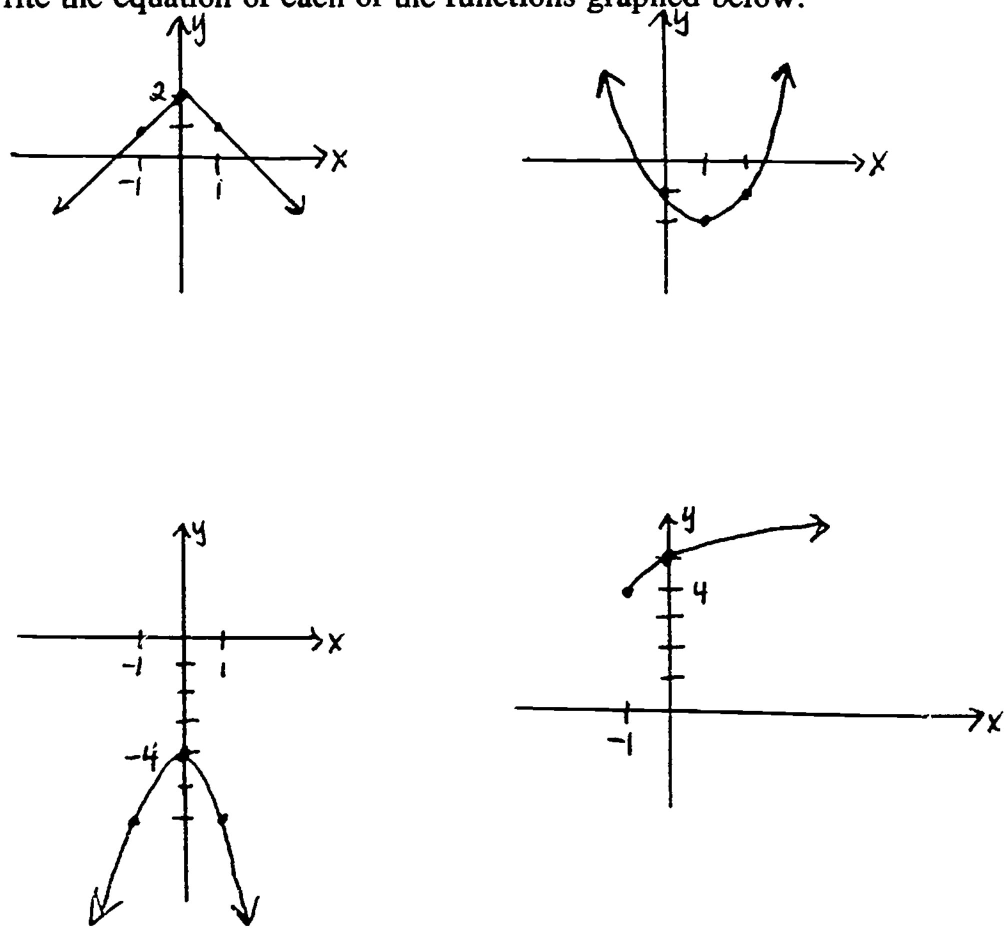
ii. $y = -x^2 + 1$

iii. $y = 3\,|x - 1|$

iv. $y = \sqrt{x - 2} + 2$

b. Describe in words how to obtain the graph of $y = -\sqrt{x + 1} - 4$ from the graph of $y = \sqrt{x}$.

c. Write the equation of each of the functions graphed below:



Solutions to Equations

Problem: Solve $\sqrt{3x+1} = 2 - \sqrt{x}$ graphically to the nearest hundredth.

1. Enter the left side of the equation as Y1 in your calculator. Enter the right side of the equation as Y2. Find an appropriate range and graph. What part of the graph represents the solution to the equation?

2. An efficient technique for finding the solution to a specified accuracy uses the Xscale and/or Yscale. For this problem, set the Xscl = .01 and graph. Use the ZOOM IN feature to repeatedly zoom in on the solution (point of intersection) until only two tick marks appear on the bottom of the screen. Read your solution to the nearest hundredth.

3. An alternate method for finding the solution to an equation is to rewrite the equation with 0 on the right side and find the zeros of the function on the left side. In this case, $\sqrt{3x+1} + \sqrt{x} - 2 = 0$. For this problem, we can simply enter Y1 – Y2 as Y3 and show all three graphs concurrently.

4. The same ZOOM IN technique that we used above can be used to find the zero of this function but it is more efficient to turn off the other two graphs before beginning.

Solutions to Equations Activity

Problem: Solve $3x^{2/5} = 3x^{1/5} + 2$ graphically to the nearest hundredth.

1. Enter the left side of the equation as Y1 ($3x^2$ ^ (1/5)) in your calculator. Enter the right side of the equation as Y2. Graph on the standard window (ZOOM 6). What part of the graph represents the solutions to the equation?

2. Set the Xscl = .01 and ZOOM IN on each intersection point to find the solutions to the nearest hundredth.

3. Use the alternate method for finding the solutions by setting Y3 = Y1 – Y2.

4. Compare and contrast the advantages of the two methods of finding solutions to equations graphically.

5. Support your result by solving this equation algebraically.

Intersections of Curves

Problem: The following two functions represent demand and supply functions. The demand function gives the price of an item as a function of the quantity of the item that is demanded by the consumer. The supply function gives the price of an item as function of the quantity of the item supplied by the producer.

$p_1(x) = \sqrt{28 - 0.1x}$ (demand); $p_2(x) = \sqrt{9 + 0.1x} - 2$ (supply)

1. Economists call the point where the supply equals demand the equilibrium point. Write the equation that represents finding the equilibrium point in this situation.

2. Enter the left side of the equation as Y1 in your calculator. Enter the right side of the equation as Y2. Find a complete graph. What part of the graph represents the solution to the equation?

3. Find the solution using the ZOOM IN technique from page 6 with both the Xscl and Yscl. For this problem, set the Xscl = 1 and the Yscl = .01 and graph. Use the ZOOM IN feature to repeatedly zoom in on the point of intersection until only two tick marks appear on the bottom and left of the screen. Read your solution to the nearest integer for x and the nearest hundredth for y.

Intersections of Curves Activity

Problem: Quick Manufacturing Company produces T-shirts and has fixed annual costs of \$202,000. The cost of materials to produce one T-shirt (i.e., variable cost) is \$1.50 and they charge \$4 for each T-shirt sold.

1. Write the total annual cost C(*x*) of producing T-shirts as a function of the number of T-shirts, *x*, produced that year.

2. Use your calculator to draw a graph of this function using [0, 142,500] by [– 100,000, 540,000] for your window with Xscl = 5000 and Yscl = 25000. Sketch your graph on paper.

3. Write the total revenue R(*x*) as a function of the number of T-shirts sold.

4. Use your calculator to draw the graph of the revenue function on the same axes as the cost function in part **b** above. Sketch your graph on paper.

5. Use your graph and the ZOOM IN process previously illustrated to find the break even point.

6. For what values of *x* does Quick Manufacturing Company lose money? For what values of *x* does Quick Manufacturing Company make money?

7. Write the function *P*(*x*) to represent the total profit as a function of the number of T-shirts sold.

8. Use your calculator to graph the profit function on the same axes as above. Sketch your graph on paper. What is the profit at the *x* value where the Revenue and Cost graphs intersect?

An Application of an Exponential Function

Problem: In calculus we develop a model that describes the spread of rumors. In a population of 100 people, where p is the number of people who have heard the rumor at time t in days, the function is given by:

$$\ln(p) - \ln(100 - p) = 0.48t - \ln 99$$

1. To use the calculator to graph this relation, it must be expressed as a function. Rewrite this equation so that the number of people who have heard the rumor, p, is a function of the time, t, i.e. $p(t) = \dfrac{100e^{.48t}}{99 + e^{.48t}}$

2. Graph the function found in #1. Use $[0, 28.5]_x$ by $[-10, 120]_y$ as your window.

3. How many people know the rumor on the beginning of the first day? ($t = 0$)

4. How long before 10 people hear the rumor?

5. How long before half the population knows the rumor?

6. State the horizontal asymptote of this graph. Explain why this is an asymptote in terms of the problem situation.

7. Describe how the steepness of the shape of the graph relates to the speed of the spread of the rumor.

Discovering Graphs of Exponential Functions Activity

Exploration 1: **a.** Graph $y = 2^x$ by hand by completing a table of values and plotting. Check by graphing on your calculator using the window $[-4.8, 4.7]_x$ by $[-2, 10]_y$.

x	-4	-3	-2	-1	0	1	2	3	4
y									

b. Use your calculator to graph each of the following one at a time (use a different Y for each and turn off the previous graph before doing the new graph) using the window $[-4.8, 4.7]_x$ by $[-2, 10]_y$. Draw your sketch below and label the ordered pairs where $x = -1, 0, 1$ on each. After doing each one individually, show them all on the screen at once.

i. $y = 1.7^x$ *ii.* $y = 3^x$ *iii.* $y = 4^x$ *iv.* $y = 5^x$

c. Using your results from **a** and **b** above,

i. Predict the general shape of $y = a^x$ where $a > 1$.

ii. State a point that all the graphs have in common.

iii. What is the y value for $x = 1$ on the graph of $y = a^x$?

iv. Explore what happens to the y values on graphs ***i*** and ***ii*** from **b** as $x \to -\infty$ by completing the following table of values. State your conclusion.

x	-10	-50	-100
y1			
y2			

d. State the domain and range for each function in part **b**.

Exploration 2: a. Graph $y = \left(\frac{1}{2}\right)^x$ by hand by completing a table of values and plotting. Check by graphing on your calculator.

x	-4	-3	-2	-1	0	1	2	3	4
y									

b. Use your calculator to graph each of the following one at a time (use a different Y for each) using the window [-9.6,4.65] by [-2,8] and label the ordered pairs where $x = -1,0,1$ on each. After doing each one individually, show them all on the screen at once.

i. $y = \left(\frac{1}{2}\right)^x$ *ii.* $y = \left(\frac{2}{3}\right)^x$ *iii.* $y = \left(\frac{2}{5}\right)^x$ *iv.* $y = .1^x$

c. Using your results from **a** and **b** above,

i. Predict the general shape of $y = a^x$ where $a < 1$.

ii. State a point that all the graphs have in common.

iii. What is the y value for $x = 1$ on the graph of $y = a^x$?

iv. Explore what happens to the y values on graphs *i* and *ii* from **b** as $x \rightarrow \infty$ by completing the following table of values. State your conclusion.

x	10	50	100
y1			
y2			

d. State the domain and range for each function in part **b**.

Zeros of Polynomials

Problem: To find zeros of polynomial functions using the rational root theorem aided by the graphics calculator.

1. Find the zeros of $f(x) = 3x^4 - 4x^3 - 50x^2 + 77x - 20$.

   a. Use the rational root theorem to list all the possible rational zeros of $f(x)$.

   b. Sketch the graph on the window $[-4.8, 4.7]_x$ by $[-400, 500]_y$.

   c. Use the trace feature of the calculator to list the values or approximate values of all the zeros.

   d. Do any of the zeros appear to be rational? Confirm using the evaluate function capability of the calculator.

   To evaluate a function:

   a) Enter the function in Y1, Y2, Y3, or Y4 (We'll assume Y1)
   b) Store your x value in x, i.e., to store 4 in x, press:
   4 [STO] [XIT] [ENTER] or 4 [STO] [STO] [ENTER]
   c) Evaluate the function by pressing: [2nd] [Y-VARS] 1 [ENTER]
   (Note: [2nd] [Y-VARS] 2 would evaluate functions in Y2, etc.)

   e. Use synthetic division with the zeros found in part **d** to factor the polynomial.

   f. Use algebraic techniques to find the exact value of the irrational zeros. Confirm that these do appear on your graph.

2. Find the zeros of $f(x) = x^3 + x^2 - x + 3$.

a. Use the rational root theorem to list all the possible rational zeros of $f(x)$.

b. Sketch the graph on the window $[-4.8, 4.7]_x$ by $[-10, 10]_y$.

c. Use the trace feature of the calculator to list the values or approximate values of all the zeros. Are any of the zeros rational?

d. Use a ZOOM-IN technique to approximate the real zero to the nearest hundredth.

e. How many zeros (over the complex numbers) should this polynomial have? Explain.

Zeros of Polynomials Activity

Problem: To find zeros of polynomial functions using the rational root theorem aided by the graphics calculator.

1. Find the zeros of $f(x) = 6x^4 - 25x^3 + 26x^2 + 4x - 8$.

   a. Use the rational root theorem to list all the possible rational zeros of $f(x)$.

   b. Sketch the graph on the window $[-4.8, 4.7]_x$ by $[-10, 10]_y$.

   c. Use the trace feature of the calculator to list the values or approximate values of all the zeros.

   d. Do any of the zeros appear to be rational? Confirm using the evaluate function capability of the calculator.

   e. Use synthetic division with the zeros found in part **d** to factor the polynomial.

   f. How many zeros should this polynomial have? Explain the discrepancy between this and the number of x-intercepts on your graph.

2. Find the zeros of $f(x) = x^3 + 2x - 1$.

   a. Use the rational root theorem to list all the possible rational zeros of $f(x)$.

   b. Sketch the graph on the window $[-4.8, 4.7]_x$ by $[-10, 10]_y$.

   c. Use the trace feature of the calculator to list the values or approximate values of all the zeros. Are any of the zeros rational?

   d. Use a ZOOM-IN technique to approximate the zero to the nearest hundredth.

3. Find the zeros of $f(x) = x^4 + x + 2$.

End Behavior of Rational Functions

Problem: To examine end behavior of rational functions visually and reinforce the need for algebraic long division.

Graph each of the following functions using three windows. Begin with the indicated friendly window. Then set the ZOOM factors as specified and use the ZOOM OUT feature twice. On the last window, observe the overall shape of the graph and explore the y values as x approaches $\pm \infty$.

1. $y = \dfrac{6x+5}{2x-4}$; $[-9.6, 9.4]_x$ by $[-6.4, 6.2]_y$; Set X Fact = 8, Y Fact = 1

2. $y = \dfrac{x^3-5}{x^3+8}$; $[-4.8, 4.7]_x$ by $[-3.2, 3.1]_y$; Set X Fact = 5, Y Fact = 1

3. $y = \dfrac{5x-1}{x^2+1}$; $[-4.8, 4.7]_x$ by $[-3.2, 3.1]_y$; Set X Fact = 5, Y Fact = 1

4. $y = \dfrac{x^2-5}{x-1}$; $[-9.6, 9.4]_x$ by $[-6.4, 6.2]_y$; Set X Fact = 4, Y Fact = 4

5. $y = \dfrac{x^3+3}{5x-10}$; Use ZOOM 6; Set X Fact = 3, Y Fact = 9

Discovering Vertical Asymptotes of Rational Functions

Problem: Use your calculator to sketch the graph of each of the following functions and record your sketch on paper. Use a friendly window ($[-4.8,4.7]_x$ by $[-3.2,3.1]_y$)

1. $y = \dfrac{4}{x-1}$

   a. What happens on the graph at $x = 1$?
   b. What is the domain of the function?
   c. Complete the following table of values for this function: (Use evaluate function routine from page 13)

x y	1.1	1.05	1.01	1.005	1.001	1.0001
x y	.9	.95	.99	.995	.999	.9999

   d. What is happening to y as x approaches 1 but is bigger than 1?
   e. What is happening to y as x approaches 1 but is smaller than 1?

2. $y = \dfrac{20x+10}{(5x-6)(x+4)}$

   a. What happens on the graph at $x = 6/5$ and $x = -4$?
   b. What is the domain of the function?
   c. Complete the following table of values for this function:

x y	− 3.9	− 3.95	− 3.99	− 3.995	− 3.999	− 3.9999
x y	− 4.1	− 4.05	− 4.01	− 4.005	− 4.001	− 4.0001

   d. What is happening to y as x approaches − 4 but is bigger than − 4?
   e. What is happening to y as x approaches − 4 but is smaller than − 4?
   f. Complete the following table of values for this function:

x y	1.3	1.25	1.21	1.205	1.201	1.2001
x y	1.1	1.15	1.19	1.195	1.199	1.1999

   g. What is happening to y as x approaches 1.2 but is bigger than 1.2?
   h. What is happening to y as x approaches 1.2 but is smaller than 1.2?

3. $y = \dfrac{5-x}{x-2}$

   a. Where is there a break in this graph?
   b. What is the domain of the function?
   c. What is happening to y as x approaches 2 but is bigger than 2?
   d. What is happening to y as x approaches 2 but is smaller than 2?

4. $y = \dfrac{7}{x^2 + x - 6}$

   a. Where is there a break in this graph?
   b. What is the domain of the function?
   c. What is happening to y as x approaches 2 but is bigger than 2? What is happening to y as x approaches 2 but is smaller than 2?
   d. What is happening to y as x approaches -3 but is bigger than -3? What is happening to y as x approaches -3 but is smaller than -3?

5. Guess the values where the graphs of these functions break. Check by graphing.

   a. $y = \dfrac{5}{2x-6}$ b. $y = \dfrac{4x+15}{(x-5)(x+1)}$ c. $y = \dfrac{3}{x^2+1}$

   d. $y = \dfrac{x^2-25}{x-5}$ e. $y = \dfrac{4x+4}{x^2-4x-5}$

6. a. How did your predictions work in #'s 5c, 5d and 5e?
   b. Explain what happened in 5c.
   c. Use your trace and a friendly window to explain what happened in 5d and 5e.

7. a. A vertical line that a graph approaches and never touches is called a vertical asymptote. Using what you learned above, summarize how you would find the vertical asymptote of a rational function without graphing.

   b. Describe what a graph does on either side of a vertical asymptote.

Excursions with the TI-82 Graphics Calculators

Selected Answers

Page 1

3. [0, 3.65] approximately
5. 112 ft
6. After 1.5 seconds
7. 1 second, 116 feet
8. x-intercept approximately 3.69 seconds

Page 2

2. [0, 24]
4. At $x = 9$, temperature 70.2°
5. Approximately $x = 2.7$ or 8:40 a.m.; $x = 14$ or 8:00 p.m.
6. Never
7. 0, 22, 24 or 0° at 6 a.m., 4 a.m., and 6 a.m. next morning
8. 71.775° at 7.5 or 1:30 p.m.
9. Approximately - [illegible] at 23 hours or 5 a.m.

Page 6

2. x is approximately .34

Page 7

2. x is approximately -.02 and 6.58

Page 8

2. $[-110, 310]_x$ by $[-3,8]_y$ for example
3. $x = 179$ units and $y = \$3.18$

Page 9

1. $C(x) = 1.5x + 202{,}000$
3. $R(x) = 4x$
5. $x = 80{,}800$ $y = 323{,}200$
6. $x < 80{,}800$ $x > 80{,}800$
7. $P(x) = 2.5x - 202{,}000$
8. Zero

Page 10

3. 1 person
4. After 5 days
5. About 9.6 days
6. $y = 100$ because only 100 people total
7. Flat at first because few know; steep in the middle as rumor spreads faster; flat at the end because most people already know

Page 11

c. ii. (0,1) iii. $y = a$ iv. approaches zero

Page 13

1. c. The actual zeros are 1/3, 4, and approximately 1.19, -4.19
   f. $x = \frac{-3 \pm \sqrt{29}}{2}$

Page 14

2. c. None are rational d. -2.13

Page 15

1. c. 2, 2/3, -1/2 f. Because 2 is a double root
2. c. No d. .453
3. No real roots

Page 16 1. $y \to 3$ 2. $y \to 1$ 3. $y \to 0$ 4. 5. $y \to \infty$