ED 355 112 SE 053 473

AUTHOR	Birenbaum, Menucha; And Others
TITLE	Diagnosing Knowledge States in Algebra Using the Rule Space Model.
INSTITUTION	Educational Testing Service, Princeton, N.J.
SPONS AGENCY	Office of Naval Research, Arlington, VA. Cognitive and Neural Science Div.
REPORT NO	ETS-RR-92-57-ONR
PUB DATE	Oct 92
CONTRACT	N00014-90-J-1307; R&' .21559
NOTE	34p.; Distribution li›.. on last five pages may be difficult to reprodu~e due to poor print quality.
PUB TYPE	Reports - Research/Technical (143)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	*Algebra; *Cognitive Measurement; Cognitive Style; Diagnostic Tests; *Educational Diagnosis; *Equations (Mathematics); Foreign Countries; Grade 8; Grade 9; Junior High Schools; *Knowledge Level; Mathematics Achievement; Mathematics Education; Problem Solving; *Student Behavior
IDENTIFIERS	Israel (Tel Aviv); *Rule Space

ABSTRACT

This paper illustrates the use of rule space as a tool to support cognitive analyses of students' mathematical behavior. The rule space approach is explained and is then used to classify students into one of two methods for solving linear algebraic equations in one unknown and to diagnose their knowledge states in this topic. A 32-item test with open-ended questions was administered to 231 eighth and ninth graders in a high school in Tel Aviv, Israel. The following outcomes of the rule space model were presented: (1) a classification of examinees into knowledge states resulting from the two solution approaches at the group level along with individual examples; and (2) tree-diagrams of the transitional relationships among the states for each strategy. Implications for using the feedback provided by the rule space model in the context of instruction and assessment are discussed. (Contains 20 references.) (Author/CW)

RR-92-57-ONR

DIAGNOSING KNOWLEDGE STATES IN ALGEBRA USING THE RULE SPACE MODEL

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This research was sponsored in part by the
Cognitive Science Program
Cognitive and Neural Sciences Division
Office of Naval Research, under
Contract No. N00014-90-J-1307
R&T 4421559

Kikumi K. Tatsuoka, Principal Investigator

Educational Testing Service
Princeton, New Jersey

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
10/1/92

3. REPORT TYPE AND DATES COVERED
Interim, April 1989 - August 1992

4. TITLE AND SUBTITLE
Diagnosing Knowledge States in Algebra Using the Rule Space Model

5. FUNDING NUMBERS
C-N00014-90-J-1307
61153 N
RR 04204-01
R&T 4421559

6. AUTHOR(S)
Menucha Birenbaum, Anthony E. Kelly and
Kikumi K. Tatsuoka

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

8. PERFORMING ORGANIZATION REPORT NUMBER
ETS RR-92-57-ONR

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Cognitive Science Program (1142CS)
Office of Naval Research
800 N. Quincy Street
Arlington, VA 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release
Distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

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14. SUBJECT TERMS
Cognitive diagnosis, Classification, IRT, Algebra

15. NUMBER OF PAGES
26

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

Diagnosing Knowledge States in Algebra

Using the Rule Space Model

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Running Head: Knowledge States in Algebra

Diagnosing Knowledge States in Algebra
Using the Rule Space Model

Abstract

This paper illustrates the use of rule space as a tool to support cognitive analyses of students' mathematical behavior. The rule space approach is explained and is then used to classify students into one of two methods for solving linear algebraic equations in one unknown and to diagnose their knowledge states in this topic. A 32-item test with open-ended questions was administered to 231 eighth and ninth graders. The following outcomes of the rule space model were presented: (a) a classification of examinees into knowledge states resulting from the two solution approaches at the group level along with individual examples; (b) tree-diagrams of the transitional relationships among the states for each strategy. Implications for using the feedback provided by the rule space model in the context of instruction and assessment are discussed.

Diagnosing Knowledge States in Algebra
Using the Rule Space Model

Indices that may be quickly and inexpensively generated from standard mathematics tests include total number correct for each student, measures of central tendency, dispersion, and standard errors of measurement. While these summary and descriptive statistics are of value in ranking students or comparing a student's performance to the performance of students on some larger normative sample, they do not provide much diagnostic information about the mathematical operations that the student has mastered or has not yet mastered.

In the case of solution of linear equations in one unknown, for example, a teacher may wish to know more than that a given student is "poor at algebra" because his or her score was one standard deviation below the class mean. Ideally, the teacher would wish to know which of the many components of performance in algebra is causing difficulty for a given student and for the class as a whole. Adequate performance in the algebra of linear equations requires more than skill in applying an algorithm. It rests upon adequate performance in and understanding of a larger body of mathematics that ranges from mastery of simple operations such as addition to mastery of more difficult concepts such as the distributive law and quotients. Armed with this diagnostic information, the teacher may then examine the difficult area(s) for the students in terms of misconceptions or faulty skill performance using any of the interview and protocol analytic tools provided by researchers in cognitive science. In considering group-level performance, the teacher may wish to examine teaching methods to determine if these are responsible in any way for the students' mislearning.

The value of a diagnostic profile that points out deficiencies and strengths in the students' performance in mathematics has long been recognized (VanLehn, 1982). However, the computational problems involved in "teasing out" the dimensions underlying students' performance are formidable. VanLehn (1982) noted that thousands of hours of work by trained experimenters were required to determine students' "bugs" in subtraction. The problem is exacerbated when the instructor has available only the student's correct/incorrect score on each item, or has little time to deal with detailed levels of assessment.

To illustrate the combinatoric problem involved in producing profiles of mastery/non mastery on task subcomponents, imagine that one can describe the solution of an item in terms of the mastery of four underlying dimensions. Thus, it may be argued that a student who fails this item (who is not guessing) may have failed to master all four dimensions, or failed to master any three, or any two, or any one. For a problem with four dimensions, there are $2^4 - 1$ patterns that could account for an incorrect answer. In general, for an item with k dimensions, there are $2^k - 1$ patterns that could account for an incorrect answer. As the number of dimensions increases, the number of patterns to consider climbs exponentially.

Tatsuoka developed the rule space methodology to address the combinatoric problem associated with diagnosis of mastery of underlying dimensions of an item (called attributes), (e.g., Tatsuoka, 1983, 1985, 1990, 1991; Tatsuoka & Tatsuoka, 1987). An attribute of a task is a description of the processes, skills or knowledge a student would be required to possess in order to successfully complete the target task. Attributes are not generated by rule space; they are generated by a domain expert (usually in concert with a cognitive scientist). They may include, but are not limited to, a student's ability to perform some procedures. Attributes may also include a student's use of heuristics, or adoption of a strategy. In general, rule space can handle any expression of an underlying dimension of a task that can be specified to the extent that certain items tap that attribute and other that do not. By examining a student's differential performances on the items, rule space categorizes students into the attribute mastery pattern that would best account for the student's individual, item-response pattern.

For the mathematics educator, rule space can provide the following diagnostic information: (a) a description of each student's mastery (and nonmastery) of the attributes judged by a domain expert to be necessary for successfully completing the test; (b) a description of group level mastery patterns obtained by aggregating across the individual profiles; and (c) partial-mastery charts that can be used to aid in the design of remediation (Tatsuoka & Tatsuoka, 1992). A detailed description of rule space is beyond the scope of this paper. A simplified description follows.

Rule space is a statistical methodology for classifying students' responses to a set of items into one (or more) prespecified attribute-mastery patterns. In practice, a domain expert and cognitive scientist would identify the attributes of the target task that are of interest. They would then write items that sample from this set of attributes. The resulting items and attributes would then be arranged in an attribute-by-item matrix (referred to as a Q matrix in rule space). Unfortunately, a student's actual mastery or nonmastery of a set of attributes cannot be measured directly, but must be inferred from the student's pattern of responses to the items. In an ideal case, a student who had mastered some, but not other, attributes would answer correctly only those items that contain attributes that he or she had mastered, and answer incorrectly those items that contain at least one attribute that he or she had not mastered. Such a student would produce an ideal item-response pattern. Within rule space, specialized functions, called Boolean Description Functions (BDF), are used systematically to determine the knowledge states of interest (i.e., those that describe ideal behavior in terms of attributes) and to map them into ideal item-response patterns (Tatsuoka, 1991; Varadi & Tatsuoka, 1989). Rule space then plots the ideal item-response patterns in terms of two variables: θ (theta), and ζ (zeta).

θ and ζ. The ability continuum derived from an item-response (IRT) analysis (Lord & Novick, 1968), θ, is used as one dimension along which to describe the ideal item-response patterns. Thus, a high-ability student (scoring high on θ) would have an ideal item-response pattern with many 1s and few 0s (for correct and incorrect responses to items, respectively); conversely, a student at the lower end of the ability continuum (scoring low on θ) would display an ideal item-response pattern containing mostly 0s. A student of high ability who gets some easy items incorrect, or a student of low ability who gets some hard items correct would be measured high on an "unusualness of response" scale, which is what ζ is (Tatsuoka, 1984; Tatsuoka & Linn, 1983). ζ is the second dimension that rule space uses to describe students' responses.

Thus, rule space generates a two-dimensional coordinate space (with θ on the x-axis and ζ on the y-axis) in whose plane certain points represent the θ and ζ of the ideal-response patterns. However, students' performances on the test items are often subject to fluctuations. Producing an ideal response pattern is likely to be rare. Students' item-response patterns that deviate from an ideal response pattern are considered as "fuzzy" response patterns. Points corresponding to the fuzzy response patterns swarm around their respective ideal response patterns, and generate regions within probability ellipses with the ideal response patterns as their centers. A 90% probability ellipse encloses 90% of the fuzzy-response-pattern points; a 95% probability ellipse encloses 95% of them; and so forth. Rule space then uses information on a student's actual score, measured on θ, and ζ, to decide where in the two-dimensional space spanned by these measures the student's fuzzy item-response pattern lies (Tatsuoka & Tatsuoka, 1987). A student is classified to the ideal response pattern that embraces his or her point in the smallest associated ellipse. This determination is made by measuring how far from the centroid the student's point is, in terms of Mahalanobis' distance. Once the most likely ideal item-response pattern is identified, the most conservative attribute-mastery pattern for that ideal item-response pattern is assigned by rule space to that student. The most conservative pattern is chosen for instructional purposes. The most conservative pattern will err in the direction of suggesting that a student has not mastered the identified attributes, when he or she may have mastered them. Thus, the conservative diagnosis would spur a remedial strategy that would be most likely to target the student's weaknesses.

Rule space entails a statistical pattern classification approach. Its accuracy of classification depends on how well the items are written, how well they test (as unambiguously as possible) the attributes that were established by the domain expert, and the amount of error in the student's responses. Since rule space does not produce the attributes, the onus lies on domain experts and cognitive scientists to provide it with useful descriptions. For areas that are well-defined (e.g., subtraction of fractions, signed numbers operations), rule space has been shown to perform quite well (Tatsuoka, 1990; Tatsuoka & Tatsuoka, 1992).

From among the several methods possible to solve a given linear equation in one unknown, we have chosen to demonstrate the use of rule space using two different approaches (expressed as two different Q matrices). One method involves the use of a simple heuristic -- initially evaluating the equation to determine if a simpler solution path would result by not rewriting the equation in standard form until the final step (method I). The other method involves consistently rewriting the equation in standard form (i.e., with variables on the left-hand side of the equation and constants on the right (method II). More details regarding the two methods are given in the method section. The purpose of the present study was to illustrate the application of the rule space model for diagnosing students' knowledge states in linear equations based on the two specified solution methods. Thus, we will see how rule space can be used to identify students who may need further remediation, to identify subcomponents of linear algebra that may be causing difficulty for the entire group of students, to produce partial mastery charts that may form the basis of fruitful remediation, and to identify students for whom it may be of value to study further in terms of their particular strategy use.

Method

Subjects

The sample consisted of 231 8th and 9th graders (age 14-15) from an integrative high school in Tel Aviv. Fifty-seven percent of the subjects were girls. The students studied mathematics in high and low achievement groupings (106 in the former and 125 in the latter).

Instruments and procedures

A 32-item diagnostic test in linear algebraic equations in one unknown was developed by Gutvirtz (1989). (The test items appear in Appendix A).

The internal consistency of the 32-item test as measured by Cronbach's Alpha coefficient was 0.95. The item difficulty indices (percent correct) ranged from 0.41 to 0.93 with an average of 0.74. The item discrimination indices (item-total correlations) ranged from 0.40 to 0.75 with an average of 0.60.

Two sets of attributes were specified for the two solution methods (see Tables 1 and 2) and these sets used to produce two separate Q matrices (see Appendices A & B). The two sets of attributes result from a strategic decision made at the outset. In method I, a heuristic,"evaluation," is applied, wherein the student scans the equation in its initial form to determine if it is likely to be simpler to delay writing the equation in standard form until the final step. For example, the evaluation rule could be applied to item 5 in the test. When the evaluation heuristic is applied the solution unfolds, thus:

$4x + 21 = 10x + 17$	
$21 - 17 = 10x - 4x$	(evaluating, and subtracting an x-term and a constant from both sides)
$4 = 6x$	(adding or subtracting variable terms)
$6x = 4$	(applying the symmetry law)
$x = 4/6$	(dividing across by the coefficient of x, when $a>b$)

These operations are denoted in Table 1 as 12, 2, 6, 11, 10, and 14 (see also the corresponding row for item 5 in Appendix A).

In method II, the student performs the mathematical operations necessary to bring the x-terms to the left-hand side of the equation, and the constants to the right in all cases. Thus when the evaluation heuristic is not applied, the solution path is more complex, since it now involves operations with signed numbers:

$4x + 21 = 10x + 17$	
$4x - 10x = 17 - 21$	(subtracting an x-term and a constant from both sides)
$-6x = -4$	(adding or subtracting variable terms and operations with signed numbers)
$6x = 4$	(multiplying both sides of the equation by -1)
$x = 4/6.$	(dividing across by the coefficient of x)

These operations are denoted in Table 2 as 2, 7, 8, 11, 13 (see also the corresponding row for item 5 in Appendix B).

The rule-space analysis:

1. The adequacy of the two attribute matrices was tested by regressing the vector of item difficulties on the set of attribute vectors. The entire set of attributes accounted for 95% of the variance (R^2=.95; R^2_{adj}=.91) for method I, and 77% of the variance (R^2=.77; R^2_{adj}=.63) for method II in the total sample.

2. The BILOG program (Mislevy & Bock, 1983) was used for estimating the item parameters (a's and b's) of the IRT two-parameter logistic model. The a values ranged from 0.55 to 2.20 with a mean value of 1.21; the b values ranged from -2.12 to 0.45 with a mean value of -0.84.

3. In order to determine the ideal item-response patterns corresponding to the attribute mastery patterns, the BUGLIB program (Varadi & Tatsuoka, 1989) was used. As a result, 461 ideal item-response groups (representing 461 different knowledge states) were generated for method I, and 453 for method II.

Results

A. Method I classification results

The classification of the actual students' response patterns into the 461 predetermined knowledge states resulted in 55 non-empty groups. A summary of the classification results is presented in Table 3. As can be seen in the table, 15 groups had frequencies of 2 or more, the maximum having 10 students in a group. The table also presents the states into which one or more students were classified, ordered by IRT θ. Figure 1 is a tree representation of those states. Each state is represented by a node indicating the non-mastered attributes in that knowledge state, and located on the IRT θ-value scale, which is given on the left side of the table. The arcs connecting the nodes indicate transitional relationships among the states. A transition from one state of knowledge to another is said to be possible whenever the set of non-mastered attributes associated with the second state is a proper subset of the first state. Thus, arcs connect lower knowledge states to higher ones, where a higher state is defined as a state having at least one less non-mastered attribute than the lower state connected to it.

Insert Table 3 and Figure 1 about here

B. Method II classification results

The classification of the students' response patterns into the 453 predetermined knowledge states resulted in 51 non-empty groups. A summary of the classification results is presented in Table 4. As can be seen in the table, 20 groups had frequencies of 2 or more, the maximum having 8 students in a group. The groups are ordered by IRT θ. Figure 2 is a tree representation of those states.

Insert Table 4 and Figure 2 about here

C. Classifying Students into the two Solution Methods.

A decision rule was set to determine which of the two methods a given student was more likely to have used. The shorter of the two distances (Mahalanobis' distances) between a student's response pattern and that of the nearest ideal item-response group in each method was chosen to indicate the student's group affiliation. Applying this decision rule resulted in 104 students being classified into method I and 89 into method II. Of the rest, 13 students had identical Mahalanobis' distances for both methods; 19 answered all items correctly, and 6 answered all items incorrectly -- thus the method used by these students could not be determined. The students' average ability/proficiency levels as measured by IRT θ were -0.08 and 0.05 for methods I and II, respectively (with SDs 0.98 and 0.84, respectively). Thus, the difference between the two groups in mathematics ability as inferred form their performance on the current test was insignificant. Among the students who were classified into methods I and II, 80% and 81%, respectively, were within the 95% probability ellipses of knowledge states--the ideal response patterns ($\chi^2_{cv\ 3df\ \alpha .05}$ = 0.35). (For a complete discussion of probability ellipses in this regard see Tatsuoka & Tatsuoka, 1987).

Examples of Classified Responses for Method I.

To illustrate the outcomes of the rule space model for method I, three students who were better classified to this method are now described.

Student 13 correctly answered 6 items (items 6, 9, 14, 17, 23, 29) and erred on 26 items. This student was classified into knowledge state No. 437 with a Mahalanobis' distance of 0.00, indicating a perfect match between the student's response pattern and the ideal response pattern represented by that knowledge state. As can be seen in Table 3 the IRT θ value for that state is a low -1.73, and it is characterized by non-mastery of the following attributes (see Table 1): 1 (adding a term to both sides of the equation), 2 (subtracting a term from both sides of the equation), 3 (applying order of operations), 4 (applying the distributive law), 5 (applying the commutative law), 7 (applying signed numbers operations), 8 (dividing across by the coefficient of x, resulting in x = b/a, when a =b), 10 (dividing across by the coefficient of x, resulting in x = b/a, when a >b), 12 (evaluating the equation), 13 (applying order of operations and the distributive law), and 14 (applying the symmetry law and evaluating the equation).

In order to reach state No. 0 (mastery of all attributes) from the state the student is currently in (state 437), a number of transitions need to take place, as can be seen in the tree diagram presented in Figure 1 one possible path is through states 429 (in which students have mastered attributes 1, 2, 4, 12, and 14), to state 244 (attributes 3 and 5), to state 3 (attributes 8 and 10), to state 1, by which time one attribute remains to be mastered (7), thus reaching a mastery of all required skills (state 0).

Student 50 correctly answered 31 items and erred on item 21. This student was classified into knowledge state No. 1 with a Mahalanobis' distance of 0.00. As can be seen in Table 3, five other students were classified into this knowledge state, which has a θ value of 1.23 and is characterized by non-mastery of only attribute 7 [Performing signed number operations].

Student 175 correctly answered 25 items and erred on items: 1, 13, 16, 19, 21, 25, 30. This student was classified into knowledge state No. 301 with a Mahalanobis' distance of 0.04 (the student's point is within the 99% probability ellipse for that state). As can be seen in Table 3, six other students were classified into this state which is characterized by non-mastery of the attributes 1 [adding a term], 3 [order of operations], 7 [signed number operations], and 13 [order of operations and distributive law]. In order to reach state No. 0 (mastery of all attributes) from the state the student is currently in, a number of transitions need to take place, as can be seen in the tree diagram presented in Figure 1. One possible route is through states 3, 1 to state 0.

Examples of Classified Responses for Method II

To illustrate the outcomes of the rule space model for method II, three students who were better classified to this method using the above decision rule are now described.

Student 148 correctly answered 27 items and erred on 5 items (items 1, 13, 16, 25, and 30). This student was classified into knowledge state No. 234 with a Mahalanobis' distance of 0.00, indicating a perfect match between the student's response pattern and the ideal response pattern represented by that knowledge state. As can be seen in Table 4 the IRT θ value for that state is .51 and it is characterized by non-mastery of attribute 3 (see Table 2): (applying order of operations). As can be seen in Appendix B, 61% of the subjects in method II group mastered that attribute.

Student 136 correctly answered 26 items and erred on the following 6 items [items 3, 5, 26, 27, 28, and 31]. This student was classified into knowledge state No. 59 with a Mahalanobis' distance of 0.05 (i.e., the student's point is within the 99% probability ellipse for that state). As can be seen in Table 4, four other students were classified into this knowledge state, which has an IRT θ value of -0.01 and is characterized by non-mastery of attribute 11 (dividing across by the coefficient of x, resulting in $x = b/a$, when $a > b$). As can be seen in Appendix B, 57% of the subjects in method II group mastered that attribute.

Student 142 correctly answered 11 items (items: 2, 6, 8, 14, 17, 18, 20, 22, 23, 24, 29) and erred on the other items. This student was classified into knowledge state No. 83 with a Mahalanobis' distance of 0.09 (a value within the 99% probability ellipse for that state). This state is characterized by non-mastery of the attributes 1 (adding a term to both sides of the equation), 4 (applying the distributive law], 6 (applying the distributive and commutative laws), 8 (applying signed number operations), and 11 (dividing across by the coefficient of x, resulting in x = b/a, when a >b). No other student in our sample was classified into that state.

D. Comparing the Results of the two Solution Methods.

The two methods, I and II, yielded overall significantly different results for item difficulties as was indicated by a discriminant analysis. Thirty five percent of the variance in item difficulty was explained by group affiliation to method I or II (Wilks Lamda 0.65, $\chi^2_{32df} = 76.01$, $p<.0001$). The discriminant function yielded substantive weights (>3) for the following items: 8 (.62), 16 (.57), 18 (.35), 21 (.55), 27 (-.33), 28 (-.34) , 29 (-.30). (The values in the parentheses are the standardized canonical discriminant function coefficients). As is evident from the signs of these weights, some items turned out to be easier for method I students and others for method II students. Item difficulties (percent correct) for each method appear in Appendices A and B.
The mastery level for the two groups also differ as can be seen by comparing the mastery level of similar attributes in the two groups given in Appendices A and B. These differences can not be tested statistically because even for the same attribute definition different items may apply in the two methods. However, a qualitative comparison of the interpretations based on mastery profiles for each method indicates that for students in method I the least mastered attributes (see table 1) are 7 (Performing signed numbers), 13 (Applying both arithmetic order and the distributive law in the same equation), 10 (Dividing across by the coefficient of x, [x=b/a when a>b]) and 5 (Applying the commutative law); whereas for method II students (see Table 2), the least mastered attributes are: 6 (Applying the distributive and commutative law), 4 (Applying the distributive law), 1 (Adding a term to both sides of the equation), and 11 (Dividing across by the coefficient of x, when a>b).

Discussion

This paper illustrated the use of rule space to diagnose student's individual and group-level mastery of attributes related to linear algebra. Two different pre-specified solution models were identified and students were classified according to them. One model was chosen to be more mathematically sophisticated and involves mastery of the symmetry law and the application of a heuristic that allows for strategic decision making when solving the equation (i.e., to delay writing the equation in standard from until the final step). The other model represents a solution that progresses in a more standard fashion in which all the x-terms are brought to the left-hand side of the equation, and the constants to the right. Many other solution models could exist, of course. In order to test these models, unique Q matrices would have to be written.

Of this sample of students, 104 were more likely to be using the heuristic approach, and 89 students the standard approach. Supporting evidence for this distinction was found in that item difficulties differed for each Q matrix, indicating that the difficulty of an item is a function of the strategy used to solve it (since different attributes are called upon for each method). For example, attribute 7 (Performing signed numbers, negative subtraction and multiplication operations) posed the greatest difficulty for students classified as using the heuristic approach. This finding seems reasonable in that students who evaluate the equation to see whether it is easier (i.e., results in positive integer values) to bring x-terms to the right-hand side rather than to the left-hand side of the equation would generally not encounter operations involving negative numbers. Note that attribute 7 poses difficulty across all levels of ability (see Table 3). Attribute 13 also poses consistent difficulty (Applying both arithmetic order and the distributive law in the same equation). On the other hand, attribute 2 (Subtracting a term from both sides of the equation) causes difficulty for only the lowest ability students. For students using the standard approach, on the other hand, attribute 6 (Applying the distributive an commutative laws in the same equation) proved the most difficult.

When we consider the partial-mastery chart for students using the heuristic method (Figure 1), we see how the transitional states are interrelated when they are linked as proper subsets one of the other. One approach to remediation using this chart is to first identify the knowledge state that best describes the target student. Then, to consider the transition path that causes the least change as reflected on the ability measure, θ. Thus, a student classified to state 437 is more likely to respond to remediation that results in attaining state 429 (i.e., remediating attributes 1, 2, 4, 12, and 14), rather than to remediation that results in attaining state 303 (i.e., remediating attributes 2, 5, 8, 10, 12, and 14) -- since the latter state is associated with higher-ability students. For a more complete description of how to use transitional states for remediation purposes, see Tatsuoka and Tatsuoka (1992).

At the whole-class level, a teacher using the current analyses would know that a significant number of students were most likely not using the heuristic method. Therefore, the teacher could explicitly teach the evaluation heuristic, which would provide the students a choice of solution models, and would make algebra seem less mechanical and more mathematical. Concerning the class's performance on each attribute, the teacher could address each of the unmastered attributes using whole class instruction. Similar options would exist at the individual student level, in which the teacher could focus on the strategy-level decisions that the student is making or on remediation of the nonmastered attributes.

Comparing the rule space and buggy approaches. In recent years, cognitive scientists and psychometricians have contributed to the effort to better understand mathematics performance beyond simple indices (e.g., Birenbaum & Tatsuoka, 1987; Brown & Burton, 1978; Matz, 1982; Sleeman, 1984; Tatsuoka, 1990; Tatsuoka & Tatsuoka, 1992; VanLehn, 1990). An alternative modeling approach to rule space is the buggy approach, in which diagnoses are generated in response to the student's errors (Sleeman, Kelly, Martinak, Ward & Moore, 1989; Payne & Squibb, 1990; VanLehn, 1982). Many such errors may be "wild" or result from slips (e.g., Sleeman et al., 1989). As a consequence, remediation resulting from buggy analysis may lead the teacher and student far afield from the target task. To illustrate, consider an equation in the form $ax = b$. Bugs that have been noted for this case generate $x = b$ (Sleeman et al., 1989), $x = b - a$ (Sleeman et al., 1989; Payne & Squibb, 1990), $x = -(a + b)$ (Gutvirtz, 1989), $x = a - b$ (Gutvirtz, 1989), and $x = a + b$ (Gutvirtz, 1989; Payne & Squibb, 1990). To explain each of these cases, the teacher must make complex inferences about the underlying mathematical models of the student, and design remediation targeted to these inferences -- predicated on the questionable assumption that students are not generating many of these errors capriciously (Sleeman et al., 1989; Payne & Squibb, 1990).

The rule space analysis, by contrast, focuses diagnosis and remediation decisions on attributes that are integral to the task at hand. Then rule space analysis considers the extent to which the attributes for a given item are mastered over the entire test. For item 7 in the test [$8 + 4(x - 3) = 24$] method II, for example, the attributes to consider for this item would be 1 (Adding a term to both sides of the equation), 4 (Applying the distributive law), 5 (Applying the commutative law), 6 (Applying the distributive and commutative law), 8 (Performing signed numbers operations), and 10 (Dividing across by the coefficient of x, when $a<b$ [$x=b/a$]). The decision as to which attributes would be remediated would be based not on the given student's bug(s) for that item, rather on an analysis of how the attributes were mastered across the entire set of items by that student. In addition, the information gathered on the entire sample of students allows the teacher to consider a pathway to mastery for this student by considering the number of students assigned to each knowledge state (see Table 4 and Figure 2). The usefulness of remediation based on these knowledge states remains to be tested empirically. If they are found to be of value instructionally, remedial strategies can be proposed and scripted beforehand to address nonmastery of each of the attributes. Further, the rule space analysis permits the investigation of the application of these attributes at a strategic level (heuristic vs. standard methods in this case), which lends itself to remediation at this level. Finally, a careful examination of the Q matrix and the resulting group attribute mastery profiles can aid in designing future tests in that topic, thus increasing the validity of those tests. Regarding questions of validity, it should be noted that the two Q matrices (describing two different approaches to solving the linear equations) resulted in different item difficulties.

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Table Attributes Used to Describe Method I.

No.	Description
1	Adding a term to both sides of the equation
2	Subtracting a term from both sides of the equation
3	Applying arithmetic order of operations
4	Applying the distributive law
5	Applying the commutative law
6	Adding or subtracting variable terms
7	Performing signed numbers, negative subtraction and multiplication operations
8	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a=b$]
9	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a<b$]
10	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a>b$]
11	Applying symmetry law
12	Evaluating the equation to determine the simplest solution path
13	Applying both arithmetic order and the distributive law in the same equation
14	Applying symmetry law and evaluating the equation to determine the simplest solution path

Table 2. Attributes Used to Describe Method II.

No.	Description
1	Adding a term to both sides of the equation
2	Subtracting a term from both sides of the equation
3	Applying arithmetic order of operations
4	Applying the distributive law
5	Applying the commutative law
6	Applying the distributive and commutative law
7	Adding or subtracting variable terms
8	Performing signed numbers, negative subtraction and multiplication operations
9	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a=b$]
10	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a<b$]
11	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a>b$]
12	Number of distinct mathematical operations > 3
13	Multiplying both sides of the equation by (-1)

Table 3. For Method I the States with two or More Students Classified into Them Ordered by Theta (θ), and a Listing of Attributes Not Mastered.

State No.	IRT θ	Frequency	Attributes not Mastered
0 *	5.00	19	(all mastered)
1	1.23	6	7
3	.41	6	7, 13
6	.12	3	4, 7, 13
11	.02	3	4, 5, 7, 13
86	-.21	3	7, 10
107	-.52	2	1, 5, 7, 10
180	-.13	2	7, 8, 14
244	-.57	10	7, 8, 10, 13
301	.13	7	1, 3, 7, 13
303	-.08	3	1, 3, 4, 7, 13
304	.12	2	3, 5, 7, 13
348	-.59	7	3, 5, 7, 10, 13
376	-.24	2	3, 4, 5, 7, 8, 13
429	-.73	5	3, 5, 7, 8, 10, 13
437	-1.73	3	1, 2, 3, 4, 5, 7, 8, 10, 12, 13, 14
372*	-5.00	6	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 (none mastered)

* Students in these states were not included in the analysis since their method could not be determined.

Table 4. For Method II: The States with two or more Students Classified into them Ordered by Theta (θ), and a Listing of Attributes Not Mastered.

State No.	θ	Frequency	Attributes not Mastered
0*	5.00	19	(all mastered)
1	.34	2	12
3	.33	8	1, 6
10	.40	5	4
11	.27	2	4, 6
12	.00	2	1, 4, 6
14	-.12	2	1, 4, 6, 12
59	-.01	5	11
61	-.15	2	6, 11
73	-.66	2	1, 4, 6, 11, 12
213	-1.79	2	1, 4, 5, 6, 7, 8, 9, 11, 12
234	.51	2	3
237	.10	2	1, 3, 6
244	.09	5	3, 4
246	-.11	4	1, 3, 4, 6
285	-.48	2	1, 3, 6, 11
293	-.56	2	3, 4, 6, 11
294	-.65	2	1, 3, 4, 6, 11
304	-1.01	2	1, 3, 4, 6, 8, 11
336	-.15	2	3, 5, 6, 9
394	-.57	3	3, 9, 11, 12
453*	-5.00	6	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 (none mastered)

* Students in these states were not included in the analysis since their method could not be determined.

Appendix A

The Incidence Matrix for Method I for the 32 Items Using 14 Attributes with Percent Correct for each item and Percent Mastered for each Attribute.

Item		Attributes	% Correct	
		11111		
		12345678901234	Method I	Total sample
1	3+x=6+3*2	01100000000000	72	74
2	7x+7=14	01000001000000	83	81
3	16x=4	00000000010000	57	63
4	6x=2x+3	01000100010100	58	63
5	4x+21=10x+17	01000100011101	52	60
6	35=7x	00000000101000	95	93
7	8+4(x-3)=24	11010000100000	67	73
8	3+6x=18	01000000100000	69	77
9	60+12=6x+2x	00000100101000	88	81
10	4(2x+3)=10x	01010100101101	84	83
11	6+4x+x=22	01000100100000	75	77
12	98=7+7x	01000000101000	85	83
13	x-4=4+2*4	10100000000000	71	73
14	11x-3x+4x=44-12+4	00000100100000	87	87
15	4x+2=5+3x	01000100000100	84	84
16	2+2*3(2x+3)=22x	01110100101111	27	48
17	6x+8x=48+48	00000100100000	79	81
18	8+4x=26	01000000100000	83	85
19	6(x+3)=12x	01010100101101	80	81
20	5+3x+x=16	01000100100000	75	76
21	3+2*2(2x-3)=23x	11111110011111	20	42
22	75=5+5x	01000000101000	85	84
23	24=6x	00000000101000	95	92
24	12x+12=24	01000001000000	83	81
25	4+x=6+2*3	01100000000000	74	73
26	8x=4X+2	01000100010100	66	68
27	28x=7	00000000010000	53	54
28	14x+30=78-2x	11000100100000	80	78
29	5x+2x-3x=25+12-9	00000100100000	91	88
30	x-6=3+5*3	10100000000000	65	67
31	7+4x=28x	01000100011101	47	53
32	6+4(x-2)=18	11010000100000	66	70
% Mastered		69665905959827		
		44498519615937		

Appendix B

The Incidence Matrix for Method II for the 32 Items Using 13 Attributes with Percent Correct for each item and Percent Mastered for each Attribute.

Item		Attributes	% Correct	
		1111		
		1234567890123	Method II	Total sample
1	3+x=6+3*2	0110000000000	73	74
2	7x+7=14	0100000010000	80	81
3	16x=4	0000000000100	63	63
4	6x=2x+3	0000001000100	62	63
5	4x+21=10x+17	0100001100101	61	60
6	35=7x	0100000001001	97	93
7	8+4(x-3)=24	1001110101000	76	73
8	3+6x=18	0100000001000	85	77
9	60+12=6x+2x	0100001101001	81	81
10	4(2x+3)=10x	0101001101001	84	83
11	6+4x+x=22	0100001001000	79	77
12	98=7+7x	0100000101001	82	83
13	x-4=4+2*4	1010000000010	73	73
14	11x-3x+4x=44-12+4	0000001001010	90	87
15	4x+2=5+3x	0100001000000	85	84
16	2+2*3(2x+3)=22x	0111001101011	62	48
17	6x+8x=48+48	0000001001000	82	81
18	8+4x=26	0100000001000	89	85
19	6(x+3)=12x	0101001101001	83	81
20	5+3x+x=16	0100001001000	76	76
21	3+2*2(2x-3)=23x	1101111101011	52	42
22	75=5+5x	0100000101001	85	84
23	24=6x	0100000001001	92	92
24	12x+12=24	0100000010000	79	81
25	4+x=6+2*3	0110000000000	70	73
26	8x=4X+2	0100001000100	65	68
27	28x=7	0000000000100	44	54
28	14x+30=78-2x	1100001001000	73	78
29	5x+2x-3x=25+12-9	0000001001000	87	88
30	x-6=3+5*3	1010000000010	66	67
31	7+4x=28x	0100001100101	52	53
32	6+4(x-2)=18	0001110001000	71	70
% Mastered		1 1		
		5064729870579		
		1012681590728		

Figure Captions

Figure 1

A Tree Representation of the States in Method I to Which More Than One Student Was Classified

Note: The small numerals correspond to the State labels.

Figure 2

A Tree Representation of the States in Method II to Which More Than One Student Was Classified

Note: The small numerals correspond to the State labels.

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