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ABSTRACT

For change to occur in mathematics instruction, teachers need control of significantly different instructional sequences, evaluation schemes, and curriculum and to think beyond procedural views of mathematics. Two-week inservice and coursework separate from classroom experience are not sufficient to achieve these goals. Inservice and support must bring about conceptual understandings and parallel actual classroom implementation via extensive co-teaching and modeling. Sufficient time must be spent to integrate these new and desired understandings into teachers' routines. This paper describes implementations where such change took place and the model which led to joint ownership in project and outcomes. Valley Crest Project is a collaborative teacher/university researcher program intended to help change the teacher's instructional ideas and behaviors. The original instructional approach by the teacher was behavioristic and associationalist with an emphasis on correct answers. To change instruction, the project provided stages in which the teacher observed the university researcher, identified a problem area to work on, learned along with students as the university researcher taught, co-taught with the university researcher, and received long-term support after the project was over. The end result of this project was a transition of control to the teacher and a viable implementation of effective mathematical change at the school level. The teacher became indoctrinated into the culture of "real world" mathematics and was instrumental in disseminating this culture to the students and to other teachers. (Contains 16 references.) (Author/MDH)

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True collaboration:  
An analysis of an elementary school project in mathematics.

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## ABSTRACT

For change to occur in mathematics instruction, teachers need control of significantly different instructional sequences, evaluation schemes, and curriculum and to think beyond procedural views of mathematics. Two-week in-service and coursework separate from classroom experience are not sufficient to achieve these goals. In-service and support must bring about conceptual understandings and parallel actual classroom implementation via extensive co-teaching and modeling. Sufficient time must be spent to integrate these new and desired understandings into teachers' routines. This paper describes implementations where such change took place and the model which led to joint ownership in project and outcomes.

## INTRODUCTION

For a meaningful change to take place in the mathematics instruction of our young people, teachers must be in charge of a significantly different instructional sequence, evaluation scheme, and curriculum. Merely stating the need for these items, however, is not enough to ensure it will take place. A major barrier to implementation is that elementary teachers are simply not in a position to implement such changes (Peck & Connell, 1991b). Teachers first must be able to reach beyond procedural views of mathematics to grasp essential conceptual constructs themselves.

It is important to note, however, that neither are simple "two-week" in-service, or additional coursework separate from actual classroom experience, sufficient if these goals are to be reached (Fenstermacher and Berliner, 1983; Hart, forthcoming; Levine, 1987; Little, 1984). Teacher in-service and support must bring about conceptual understandings on the part of the teachers and parallel actual classroom implementation via extensive co-teaching and modeling by master teachers throughout the course of the intervention. Sufficient time must be spent that the new and desired understandings are thoroughly integrated into teachers' normal routines.

A case in point is that of elementary mathematics education. It is currently plagued by various conceptions regarding mathematics as held by practicing mathematicians and those within the school environment where mathematics is taught. These diverse belief systems have led to the creation of a dichotomy in which there is the world of "school mathematics" of the teacher and that of the "real mathematics" of the mathematician and scientist (Hess, 1991). This dichotomy causes severe problems for education as practicing teachers are only aware of the world of school mathematics.

To see how this dichotomy plays out in the daily life of students consider the following characteristics of "school" versus "real world" mathematics.

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INSERT TABLE ONE ABOUT HERE

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For our students to be adequately prepared for the demands of the evolving society, there must be a significant change in the view of "School" Mathematics to enable an induction into "real

mathematics" as envisioned by mathematicians and scientists. Yet, in order for change to occur practicing teachers must become aware of and members of the "real world" culture of mathematics as recognized by the practicing mathematician and scientist. An induction which cannot occur without the active and willing participation of the teachers themselves.

Although most teachers are unable to specify the exact nature of this dilemma, many are nonetheless aware it exists and would like to do something to remedy it. It has been a common occurrence in our work to be invited by classroom teachers to work with them to "do something to help my class". This paper will describe the results of one such invitation, the program which it helped to generate, and the subsequent impact this program had upon the classroom teacher and the students this teacher served.

#### Description of the Valley Crest Project

The original invitation was facilitated by an existing working relationship with a PDS (Professional Development School) school at which school faculty were already serving as adjuncts to university faculty. However, as in the case of each successful intervention in our experience, the invitation was initially at the teacher level. Once a working arrangement had been achieved with the teacher involved administrative support was obtained. This is in marked contrast to traditional top-down reform efforts.

The collaborative nature of this project affected the curriculum, evaluation, and implementation at many levels. Researchers provided materials, lessons, and much of the instruction. The classroom teacher selected topics and concepts in accordance with the district and state guidelines for fifth and sixth grade classes and contributed to the instructional effort. All decision making was a team effort with the researchers and the classroom teacher working in concert. To provide an overview of the resulting project, a brief summary will be provided of the curriculum, instructional focus, and implementation procedures used.

Curriculum focus. The curriculum used in this project was conceptually based and utilized a five phase approach which allowed students to construct mathematical intuition via physical materials

(Peck and Connell, 1991a). In this approach, the initial two phases made use of physical materials in a much different fashion from traditional approaches. Rather than using manipulatives to demonstrate procedures or rules, problems were posed which required active student involvement with physical materials to model mathematical situations, define symbols, and develop solution strategies via actions with the materials. As the children used these physical materials to solve problems, they actively constructed the operations and principles of arithmetic. The third phase required sketches of the physical materials and situations experienced by the students to encourage a move toward abstraction. The sketches then served as the basis for additional problems and as tools for thinking. In the fourth phase, the children constructed mental images through imagining actions on physical materials. The experiences with mental images provided a basis for the fifth phase where students constructed strong arithmetic generalizations and problem solving skills.

The computer in this project was just another "tool" available to the students in their ongoing efforts to construct meaningful methods of dealing with the problems they encountered. The nature of this "tool", which was provided for the students to "think-with", came to shape their performance and cognitive styles. When a computer was available for the students use the problem solving situation shifted toward the identification and selection of what data to include in the problem, identification of the problem goals, and choice of appropriate procedures and control statements to obtain and verify the desired results. As a consequence of the instructional sequence outlined above the children constructed a series of related mathematical concepts. When these concepts and applications were overlearned the students instructed a MacIntosh via Hypertalk to carry out the necessary instructions and operations which they had derived (Peck, 1989). The computer played a pivotal role in this project, albeit a much different role than that usually associated with CAI. Rather than using the computer for its speed, the computer's patience and need for exactness of logic and clarity of expression was utilized. The computer assumed the role of an active listener that would do exactly what it was told, as opposed to a pre-programmed instructor requiring a specific type of answer.

Throughout the project, a major goal of the curriculum was to enable the successive internalization and abstraction of the preliminary physical experiences the children shared. Each of the

outlined phases was viewed as a step along the path toward eventual mathematical abstraction. For example, the sketches drew much of their power from earlier experiences with objects. In a similar fashion, the mental images reflected the sketches and manipulations performed by the students. The interrelated nature of these experiences set the stage for abstractions and the intuitive foundation upon which the abstractions could safely rest. These abstractions, rather than being based upon a single demonstration of rules, rested upon a tightly woven network of understandings.

Instructional focus. An explicit instructional objective was to help each child find a way to answer the question, "How can you tell for yourself?" for all portions of the mathematics they were learning. The instructors shared the common belief that children must be allowed to figure things out and be responsible to themselves, not a teacher or answer key, for their results. It was felt that if children are to engage in thinking about and solving problems for themselves, then they must have a "place" to go in order to be able to determine if they are making sense. Physical objects in this instructional model served this purpose. These beliefs, coupled with the earlier described curriculum focus, led to the following principles:

1. The instructor did not explain. The instructor served as a problem poser, skeptic and question asker focussing upon student explanations.
2. Manipulations with physical materials defined meanings which were associated with arithmetic symbols and operations. Problems were developed requiring an appeal to those objects and meanings.
3. The instructor attempted to enable the children to internalize and abstract their experiences by requiring them to work problems in the absence of the physical materials.
4. The instructor used a meaning-centered evaluation scheme (Peck, Jencks, & Connell, 1989).

Evaluation focus. Evaluation as used in traditional instruction often appear designed to identify and reward "winners" over "losers" using information acquired from measures of success or failure on narrowly prescribed sets of cognitive tasks (Corrigan, 1990). When every child is to be given the chance

to construct the necessary understandings to make them a "winner", however, this approach is not overly helpful. The authors certainly did not want the designated "losers" opting out of further mathematics education.

The need for a shift was needed toward evaluation methods which could be used to guide instruction aimed at maximizing the number of "winners". To accomplish this a two-step evaluation scheme was used to guide classroom instruction involving the use of Sato's Student-Problem Chart (Sato, 1990, Switzer and Connell, 1990) and a follow up teacher interview (Peck, Jencks and Connell, 1989). Each of these techniques are quite effective alone, but when used together they have been found to provide a very efficient methodology in assessing student understandings.

The information provided in Sato's reporting format allows for quick identification of both problems and students with unusual patterns of responses - indicating potential sources of difficulty and identifying key questions to ask selected students. At a simple level, a Student-Problem chart (see Table 2) is a systematic ordering of student item responses to teacher selected sets of problems (Sato, 1990). First, the problems and their associated student responses are ordered from left to right beginning with the easiest (as determined by those problems having the highest number of students responding correctly) to the most difficult (those problems with the least number of students answering correctly). Once this is done, the students are ordered from top to bottom by highest total score to lowest total score.<sup>1</sup>

Each row of the Student-Problem Chart contains the responses of an individual student. The sum of "+"s in each row corresponds to the raw score (total score) for each student. In Table 2, "+" indicates a correct response. Incorrect responses are indicated with either " " if the student did not attempt the problem, or the value of the response if the problem was answered incorrectly. Each column

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<sup>1</sup> The computer program, SPPC, which creates Student-Problem Charts and additional statistical information is available for IBM compatible computers from:

The Office of Educational Testing, Service, and Research  
51 Gerty Drive  
Champaign, Illinois 61820.



corresponds to an individual item on the test. Reading down a column reveals how students responded to that item.

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INSERT TABLE TWO ABOUT HERE

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Examination of a Student-Problem Chart made it a simple matter to determine which students to interview and what questions to ask to pin down conceptual understandings. To see how this is done, consider student number 5094 from Table 2.<sup>2</sup> This student has done very well on the test. A score of 87.5% identifies him as a "winner" and little further concern would typically be given regarding the developing understandings. Looking across the row of item responses (see Figure 1), however, a disturbing observation is made. The student has missed problems 12 and 10. These problems as indicated by classmate's performance are significantly easier than other problems to which the student had responded correctly. There may exist a potentially dangerous gap in understandings. A talk with this student about these problems is in order.

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INSERT FIGURE ONE ABOUT HERE

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This method of looking at student data can also provide information showing unsuspected strengths of students. For example consider student 2163 (see Figure 2), who despite a very poor overall performance on the test has correctly answered problems 23 and 8, two of the more difficult problems.

This approach enabled us to determine who to talk to and what questions to ask them to maximize our effectiveness in the interview itself. Student interviews were then conducted to evaluate student explanations and problems. These follow-up interviews with these key students and problems provided for meaningful feedback on the results of instruction and any non-productive conceptualizations that may have been constructed. In short, a closer examination of what the students are thinking and not just what they were doing.

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<sup>2</sup> This Student-Problem Chart is taken from an actual classroom set of response. The student names for this example have been replaced with ID numbers in order to protect student privacy.

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INSERT FIGURE 2 ABOUT HERE

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To see the impact of this combined approach consider student 5094 (Figure 1). As a result of examining the Student-Problem Chart the teacher decides to interview student 5094 concerning problems 10 and 12. The student did not yet know whether his answers were right or wrong. These problems both dealt with a common concept - multiplication of fractions. Taking a clue from this information, the teacher presented the student with a problem similar to the test problem and questioned the student about it. In the course of the conversation, the student described what he thought of while doing the problem.

...well, multiplication makes things bigger, see. Like 12 - I mean  $3 \times 4$  is bigger than 3 or 4. So  $1/3 \times 1/4$  has gotta be bigger than  $1/3$  or  $1/4$  and the way I did it first it wasn't.

The success he had experienced was due to familiarity with a procedure which only applied part of the time - not on a useful understanding of the meanings surrounding fraction multiplication. Further discussion with the child revealed that this view of multiplication as "making things bigger" was interfering with his development of adequate understandings.

#### Description of Teacher Impact

At the beginning of the project the participating instructor held many of the characteristics earlier identified as those typical of "school" mathematics (Hess, 1991). In particular, the instructional posture was behavioristic and associationistic with emphasis upon correct answers and consistent forms of problem solution. In keeping with this orientation, the instructional emphasis was placed upon memorizing algorithms and equations for later application. When problem solving was presented, it generally referred to the decoding of word problems provided from the text where there was only one correct answer and one "correct" method of solution.

For the first year of the project the university team took over much of the classroom instruction, with the classroom teacher serving as a monitor and support. During this period a shared "language" was

developed to facilitate communication. It became extremely common to hear them discussing a student's performance as being "kind of like a 4, (referring to the Structured Interview Groups) but I don't really think he understands that much" or "good scores, but she seems to miss some of the easiest questions - like a B (referring to the Sato's SP Groups), I guess". Soon this language was extended to groups as well as individual students as evidenced by comments such as, "This activity will be great for my 2's, but I don't know if the 4's will get it" and "these 4's are going to drive me nuts".

With time and experience the tentative nature of the communication became less tentative with statements such as "definitely a 4" or "A responses for sure" becoming more and more common. An important aspect of this developing language was to promote a continuing and conscious examination of both student processing and conceptual understanding. The vocabulary described in the previous paragraph allowed the expression of a richer understanding of students performance and convenient discussion of differential effects of the curriculum.

Model of inservice provided. In attempting to introduce the participating teacher into the "Real" mathematics culture the university faculty began with discussions of underlying mathematical framework and how it could be articulated in her class. Differences of perceptions and possibilities were then discussed and worked through. Once the ideas required were identified, an inservice plan covering these topics were identified.

In doing this, the teacher was required to become a learner - with all of the associated learner characteristics. For the case of inservice, the teacher already possesses knowledge and beliefs about the content to be learned which form a filter through which new information is processed and understood. The process of learning, therefore, involved more than a simple adding on or replication content. It involved the development of a new conceptual perspective through which content--facts, principles, instructional practices--can be personally mediated and understood.

Taking the view that a teacher cannot teach using a method in which there is no experience as a learner, the inservice followed the same five phase instructional plan outlined above that would later be used with the students. Treating the teacher as learner, then, involved the creation of a commonly understood set of definitions and terms with which problem-solving could take place. Just as with the

students, problems solving using concrete materials and use of them to develop new problems and problem representations followed. The inservice proceeded to develop abstract representations using graphics, leading to the creation of mental images serving as a bridge to the formalized mathematical symbols. A result of this process was that the teacher was able to generate physical, graphical, and symbolic representations of mathematic problems in the same fashion as that which would later be presented. This treatment of the teacher as learner is viewed as a crucial aspect of the intervention. Without personal understanding from experience of this manner of learning it is unlikely that later teaching could be effective.

A training and modelling period followed this inservice during which the university faculty co-taught with the school faculty in the classroom. In this manner, university faculty became part of classroom instruction and served as models to which the teacher could relate. The university faculty provided support, and served as a scaffold (Collins, Brown, and Newman, 19xx) from which the teacher developed independent strategies and methods.

The student growth and progress observed by the teacher led to a refinement and stronger adoption of the project ideals and goals. Her own teaching characteristics shifted by the end of the second year toward the "real world" mathematical culture. The following characteristics were observed: instruction became student centered and constructivist in nature; the instructor's role became that of question asker and problem poser; and problem solving, persistence, and resourcefulness on the part of the students became highly valued.

The long term support of the instructor continued long past the length of this study. This true collaborative nature has resulted in the creation of a support system within the school and district whose impact upon instruction has outlasted the daily presence of the researchers. The project is currently being disseminated by the initial instructor to other schools within the district.

#### Description of Student Impact

This study included a wide variety of measures including both qualitative and quantitative strands of evidence. Although other data were gathered, the description will focus upon two strands of

evidence, one quantitatively and one qualitatively based. The mixed methodology discussed in previous sections utilizing both S-P Charts and structured interviews were carried out throughout this study as a means of guiding instructional focus.

Quantitative findings. The Valley Crest Mathematics Inventory was used to gather student pre and post data.

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INSERT FIGURE THREE ABOUT HERE

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This assessment had been used in earlier studies by the authors and mapped nicely to the curriculum of the school. For this study, validity controls were constrained to face and content validity as determined by the teachers and investigators taking part in the study. It should be noted that an earlier extensive cooperative effort with a district level evaluation team (from another state) had been undertaken in test construction in which extensive item analysis was performed to select the best items and establish the item to objective mapping used in this study. Reliability estimates using Cronbach's Alpha were calculated for both pretest ( $\alpha=.74$ ) and posttest ( $\alpha=.84$ ).

An initial examination of the pre and post total scores as shown in Figure 3 illustrate that growth was indeed made during the course of the year. A T-test on these scores found a mean difference of 13.95 and a value for T of 7.93 which was significant beyond the .001 level.

Although heartening, this finding must be tempered with the realization that this intervention took place over the course of a year. Had a significant difference not been found it would have been cause for great alarm on the part of the investigators, not to mention the local school authorities. In looking at the content areas measured, see Figure 4, it is possible to make some additional observations.

There are several increases worthy of notice in light of the instructional focus spent during the year. Although Geometry and Statistics were not formally presented during the year they increased none the less. It is in the areas of Extended Mathematics (pre-algebra) Problems, Miscellaneous problems (which required a variety of problem solving strategies, and Estimation (which although not formally discussed was inherent in all student work) that the greatest increase in student performance may be

observed. The near doubling in student performance in each of these areas provides strong evidence that the instructional emphasis upon student problem solving was effective for this group.

An additional support for this may be found in examining the Modified Caution Signs computed for the students using the pre and post assessment. This index may be interpreted in the following manner: an A type response indicates high levels of performance and consistent patterns of item response, B indicates high performance and inconsistent response, C indicates low levels of

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INSERT FIGURE FOUR ABOUT HERE

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performance and consistent responses, while D represents low performance and inconsistent patterns of response. In looking at Figure 5 it should be noted that the number of students identified as having type A responses (High and Consistent) nearly trebles over the course of the intervention while the number of students in B increases. This is accompanied by a corresponding decrease in the number of students showing a C or D response pattern.

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INSERT FIGURE FIVE ABOUT HERE

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Qualitative findings. The reported findings derive from two sources collected during the first and second years. The first source of evidence is the interviews conducted throughout the year. The second source is the student notebooks in which the students wrote a five minute reflection of their work at the end of each session.

One of the major observations from the student interview lay in the student perceptions of the problem and the associated problem solving efforts which they attempted. In particular, there was a consistent "reversion to form" on the part of the students. As long as the problems made sole use of newly constructed information the students were able to utilize their developed understandings. They were able to demonstrate effective problem solving strategies which required both conceptual and procedural understandings. This situation shifted dramatically, however, whenever prior knowledge was required as part of the problem solving efforts.

One case in particular stands out as illustrative of this tendency. During the course of the interview the student had been asked to "share" 72 counters with 9 people as shown in Figure 6.

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INSERT FIGURE SIX ABOUT HERE

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In working this problem the student successfully completed the exchanges necessary and achieved the correct answer. Self-generated procedures, which were highly effective for this student, were used and in the subsequent explanation the student was able to describe numerous situations in which such "sharing out" would be desirable. The student was then presented with the problem shown in Figure 7.

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INSERT FIGURE SEVEN ABOUT HERE

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The student's response was both immediate and discouraging.

**Student:** This is a Dear Miss Sally Brown problem! We learned how to do these ages ago... see, you just divide, multiply, subtract, bring down...Dear Miss Sally Brown.

In applying the steps of Divide, Multiply, Subtract, and Bring Down, however, the student failed to perform a single step correctly - with the result that the answer was 720! Furthermore, the attention to sense-making and reality checks used in the first problem situation were nowhere to be found. The strength of Dear Miss Sally Brown and its associated "right" procedure" proved too much for the student.

This reversion to an earlier, simpler, and for the most part inaccurate level of functioning occurred most often whenever time pressures came to play (such as those associated with a test) or an over learned piece of prior procedural knowledge was involved in dealing with the problem situation. The strength and persistence of this observation leads the authors to urge that great caution be taken regarding the nature of the initial mathematical experiences provided to children.

The student notebooks provided an interesting insight into student perceptions and difficulties presented in this approach. To illustrate this, selections from four students will be presented in sequence.

Student 1 (Female):

10-22. Today was easy and hard. First it was hard because I didn't know what to do. When he wrote the problems on the board... I looked at the board and thought "I can't do these" but once I got started it was easy but it took time because I had to use counters then I figured out how to do them without counters then I was going fast.

10-24. Today we worked with egg cartons it was very confusing I couldn't seem to do it. It looked easy but then I tried to do the next problem but I couldn't. It's just to hard.

10-25. We used the egg cartons again today. I learned alot today. I found a pattern to all of the problems when the answer (share with number) is 6 it's always half of the denominator I did 19 in 3 minutes thats super for me!!

11-27. Today we did these those and altogether (division and addition of fractions) I figured a short cut so then it was casier.

1-3. Today I learned to multiply fractions. they are very easy last year I learned to do them the hard way. I think I learned how to in third grade. This class isn't boring anymore.

1-5?. Today I went to computer I tried to multiply fractions it wouldn't work because all my fields were on background fields. I had to delete all of my fields and make new ones I tried it and it worked!! It was time to go but I had to try it one more time so I could see for sure it worked

Student 2 (Male):

10-2. I really like using the blocks because it helps me learn it better, and its fun... I think working with people smarter than me really helps me learn because they explain it very well

10-9. Today was complicating I didn't understand anything at first but after halfway K. explained it I still didn't understand a word she said. When she was finished explaining I understood. C was kind of wacky today

10-18. I really liked working today because I know I did good. One of the hardest problems was one that I made up.

10-23. Today all we did was play around wich (sp.) was exiting (sp.) because we discovered many different things...

11-?. Today I did 15 problems (wich (sp.) is good for me!) I started doing more problems after I had finished with out being asked.

11-8. Today I was at computer. There was three ladies here that watched us. We had fun. I feel like a genius (I like it). We got into buttons.

1-13. We did cakes on a piece of paper. What I mean is we drew a cake and shared it with a certain number... I feel smart. Almost as smart as C.

12-?. Today we did test review problems, it was fun! One of the problems I liked was  $\frac{2}{2/3}$  divided by  $\frac{1}{4}$ . I liked it because I had to draw 4 cakes Today was one of the funnest days I've had all year in math. I have many reasons why One I sit by C. Two I made tons + tons of progress! It was awesome. One of the problems was  $1 \frac{1}{2} + 2 \frac{3}{5}$ . I did it different than K but I got the same answer



1-?. Today we had a substitute. her name was Miss H. She confused us (C and I) because she did it the old fashioned way! but then I got better

Student 3 (Male):

10-2. I worked by myself. I finished my paper. I learned about share with. I cleaned up my work area. This math is easy.

10-3. We worked as a group. I don't really understand todays math the problems are too hard. I don't know if I got my answers right. I don't like this class its to hard

10-8. I used the counters which made it easy I don't like this very much I don't like this math

10-10. I worked with T. The math was a little hard. Then I understood it. Then it was easy. T. helped me I don't like this math much. I finished all the math I was supposed to do I don't like writing in this book

10-16. I understood some of the math today. I don't see what the big deal is. there's no big secret. I got finished. it was easy. K. confused me about some secret. Which I don't know. I hate those circles.

10-25. I worked on the computer. I learned to use the button and field I have my own stack I got lost a lot. I made a mess I always do but I clean it up. I made a formula today it was  $D*4-4=C$ . It is easy But thats what I think. We did seven problems they were pretty much easy. I thought it was fun using formulas... we might use them another day!

11-6. I finished the paper. I did most of it by myself. T and I worked together. I disliked problem 3/5 (?) I hated problem 2/13 (?) it was hard too. It was fun cutting the yard stick up.

11-8. I finished my work I cut 6 clocks it was fun. I made up two problem it was 3/6 and 2/4 they were easy. It was easy too! I learned what  $15*4$  was again. It is 60. I don't know why I forgot it. I bugged A. because of my counting but I was supposed to count the centimeters.

11-27. We learned WHAT? I understand the relationship between what and how many. It's easy now. I did extra work that I didn't have to do

Student 4 (Female):

10-8. entry journal today I figured out a really hard problem and I mean it was really hard I like to work in this class and I helped people out on there problems this math group is really fun the really hard problem was 4 counters and share with 37 and the answer was  $24 / 37$  it took me 5 days and I finally figured it out (Smiley face drawn in margin).

10-10. today I did some of my math problems and I got some of them wrong so I had to go through (self-enforced) every single problem and see I got them right and lucky I did because I would have missed almost every single one of them. I worked really hard today but thats okay

10-16. im trying to figure out this really hard problem because she (??) wants me to have six covered up and then make up problems and I'm trying to figure out a hard problem that no one can figure out and Dr. X. thinks he can fix me he thinks he can give me a hard problem and I can't figure it out but ill (I'll) show him!!!

In looking at these student's notebooks several observations seem in order. First, in almost every case there is a marked increase on the part of the student toward self-posed problems as opposed to teacher directed problems. This shift took place at different times for different students, but was nearly uniform throughout the class. These self-posed problems came to be a driving force in the instruction and a source of student pride as evidenced by the student notebooks.

The appearance of a substitute, as evidenced in many notebooks, was a trying event for the children. Many of them dealt with this by doing it "the old way" on paper and then "talking about it" with their friends. Others merely "did it (the problems) the old way" and then complained about having to do worksheets. Time became a problem, not because of the time necessary for conceptual development, but because time would run out. The children's enthusiasm is evidenced by the comment "It was time to go but I had to try it one more time so I could see for sure it worked" which was echoed in many places in the notebooks.

In short, the students could be observed to be actively engaged in solving their own problems based upon group constructed meanings and procedures. The motivation came from the problem situation itself and the computer was viewed as a tool that was used to verify independently achieved results, not to dictate instruction. The result was a marked shift toward successful independent problem solving as indicated by the quantitative analysis and borne out by interviews and observations.

### Summary

In summary, the framework within which this cooperative effort took place began with the teacher as the initiator of the collaborative effort (we have found it seldom works if the principal calls). The researchers considered themselves as guests and as hands on workers with the children. The commitment was, and is, daily and long term (1-3 years for this project) for all parties involved. The end result of this endeavors was a transition of control to the teacher and a viable implementation of effective mathematical change at the school level. In this case the teacher not only became indoctrinated into the culture of "real world" mathematics, but was pivotal in dissemination of this culture and the project to the students and to other teachers within the school and district.

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TABLES

Table 1

"School" versus "Real World" Mathematics

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<b>SCHOOL</b>	<b>REAL WORLD</b>
Based upon associationism and behaviorism	Constructivist in nature
Values computational accuracy and efficiency	Values problem solving persistence and resourcefulness
Emphasis on memorizing algorithms and equations	Emphasis upon using mathematics to reason from external situations and objects
External determination of "right" or "wrong" is made by teachers or textbooks	Internal determination of "right" or "wrong" made by the individual based on actual situations and mathematical conceptions
"Problem Solving" means decoding word problems which apply a single well defined skill with	"Problem Solving" requires an active synthesis of knowledge and skills along with creativity and experience
Only one correct method leading to one correct solution is possible for each problem	Many methods exist for solving problems which may have one solution, many solutions, or no solutions
Technology and other resources are not to be utilized in school mathematics; that would be "cheating"	Technology and other resources should be fully utilized in problem solving
Supported by the structure of the curriculum, textbooks, and standardized tests	True competency in mathematics is achieved by immersion in meaningful problem solving

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Table 2

Example of a Simple Student Problem Chart.

			Problem Number																							
Student Number	Test (Raw)	Score (%)	0	0	0	1	1	0	1	1	0	0	2	1	1	2	0	1	2	2	1	0	2	1	1	
			-----																							
Answer Key =====>			4	4	2	1	2	1	2	3	1	3	4	4	2	1	1	3	2	3	1	3	1	1	3	5
			-----																							
5011	23	95.8	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
4064	23	95.8	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
4105	22	91.7	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2111	22	91.7	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
5094	21	87.5	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2170	20	83.3	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
5055	20	83.3	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2034	19	79.2	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
5115	19	79.2	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
5131	19	79.2	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
1016	17	70.8	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2105	15	62.5	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2182	14	58.3	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
3225	13	54.2	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2226	11	45.8	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2246	11	45.8	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
1046	10	41.7	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
1232	10	41.7	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
1102	09	37.5	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
2163	06	25.0	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	

FIGURES





Student Number	Test Score (Raw)	Score (%)	Problem Number	
			1	2
			1	1
			2	0
-----				
	Answer Key =====>		2	2
-----				
5011	23	95.8	+	+
4064	23	95.8	+	+
4105	22	91.7	+	+
2111	22	91.7	+	+
5094	21	87.5	+++ 3	+++ 3

Figure 1. Example of Student-Problem Chart use.

Problem Number

Student Number	Test Score (Raw)	Score (%)	Problem Number	Score
2163	06	25.0	2 3 + + + 3 1 + 2 4 2 1 3 4 4 2 3 4 4 2 ++	43
				++
				++
				11
				83
				02

Figure 2. Additional example of Student-Problem Chart use.

## Assessment Performance

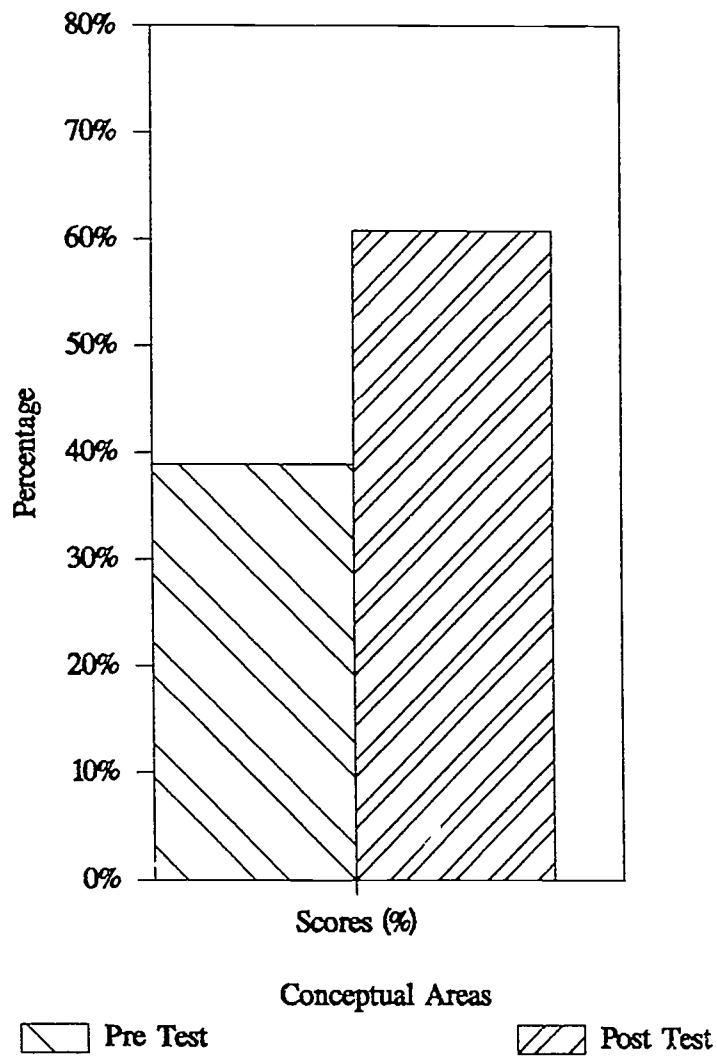


Figure 3. Pretest vs. Posttest total scores.

# Assessment Performance

by Content Area

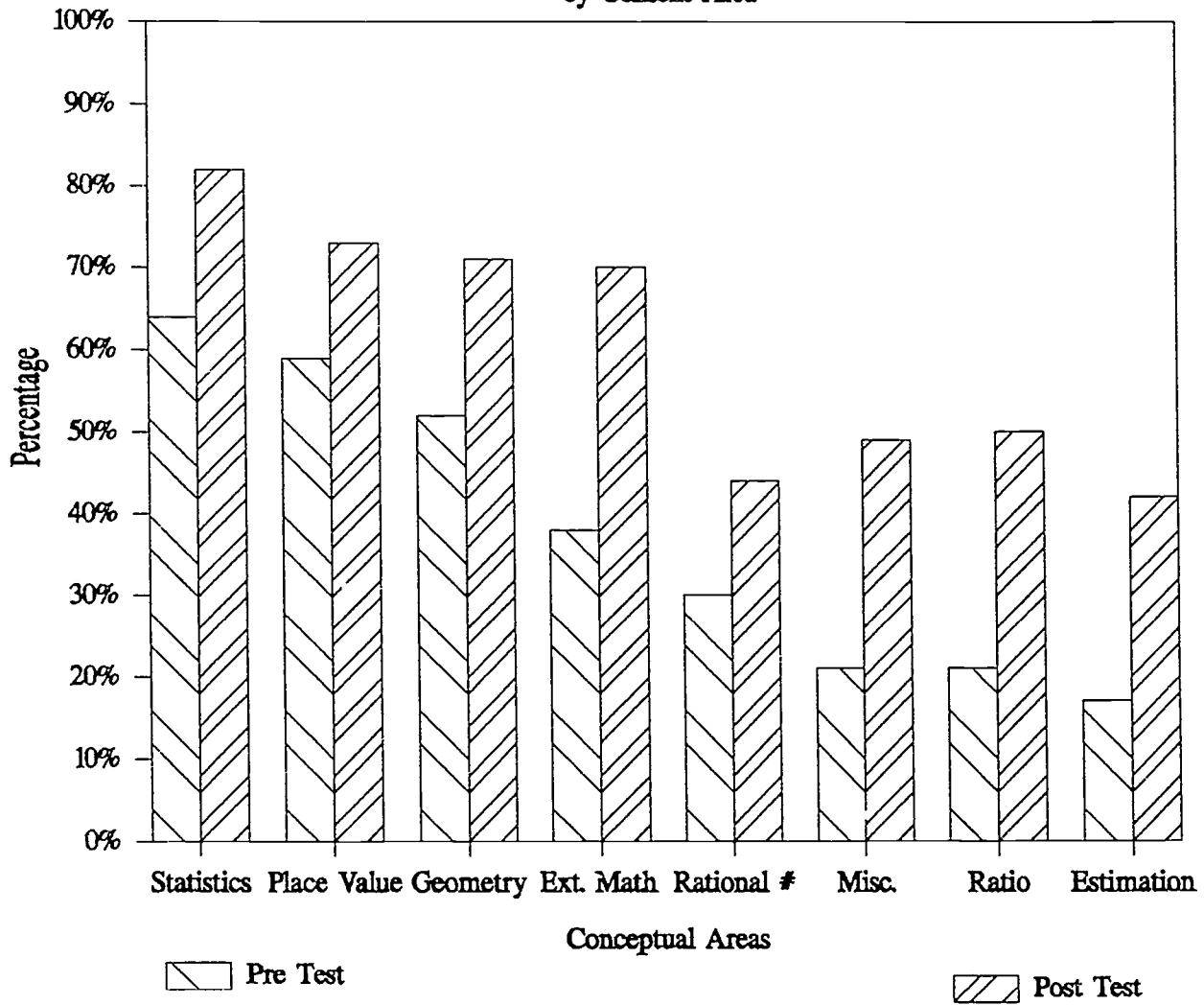


Figure 4. Pretest vs. Posttest total scores by content area.

# Student Performance Groups

(Based upon MCI)

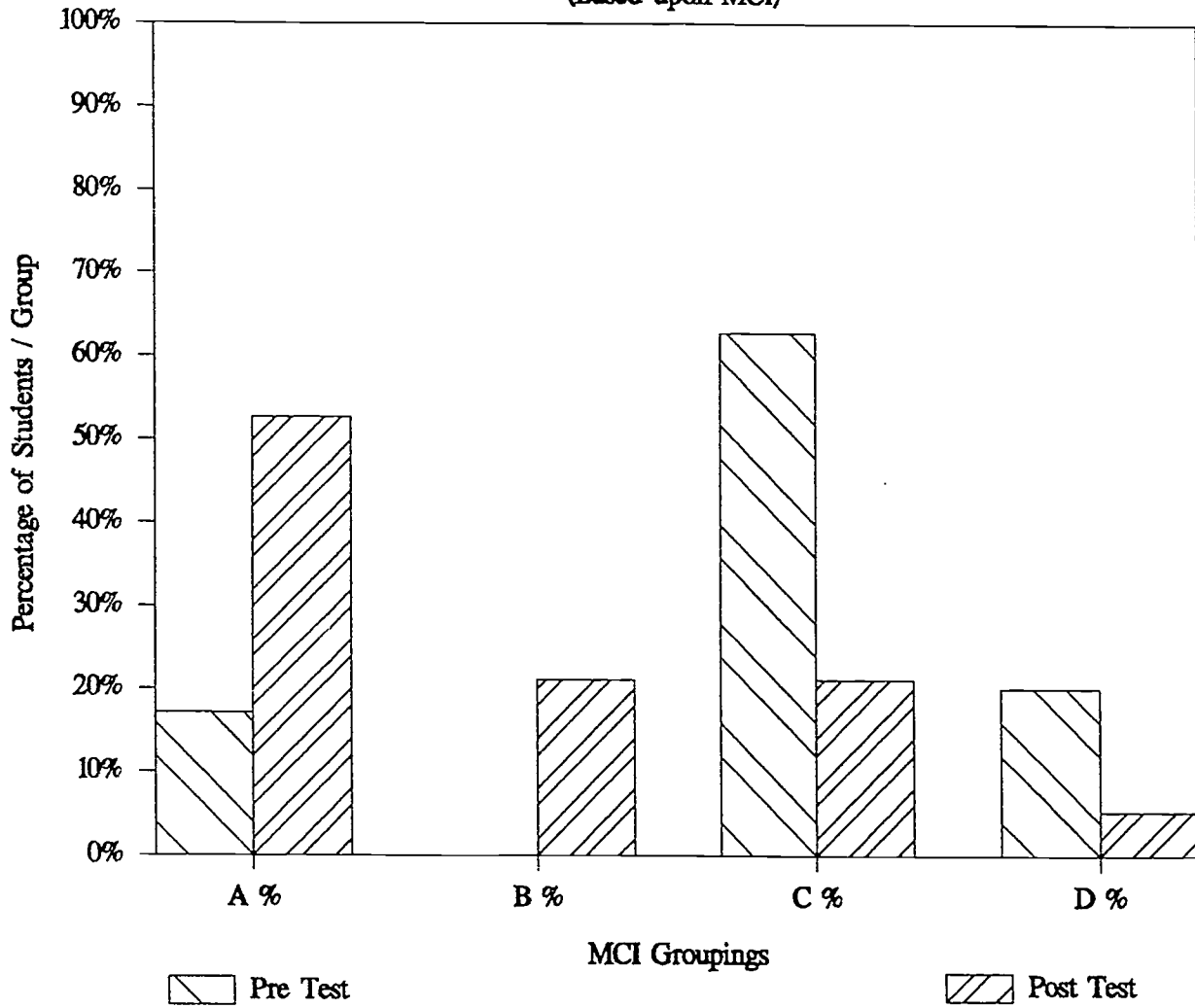


Figure 5. Pretest vs. Posttest Modified Caution Signs.

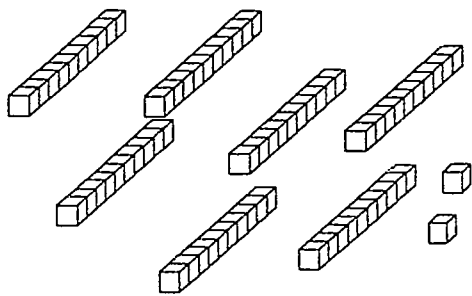


Figure 6. Sharing problem posed to student.

$$9 \overline{) 72}$$

Figure 7. Follow up problem posed to student.