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ABSTRACT

Recent calls for mathematics instruction reform recommend that a problem-solving inquiry-oriented (PSIO) classroom environment be established. Students, however, may view mathematics as a static, ruled-based discipline that runs counter to the PSIO approach. A study examined the student/teacher interactions as seen from the viewpoint of five honors geometry students holding a rule-based viewpoint of mathematics as they participated in a PSIO approach to learning mathematics for an entire year. A case study methodology was employed to gain insight into students' beliefs, thinking, and behavior. Results are reported according to themes that emerged concerning the students' beliefs about understanding mathematics, learning priorities, learning strategies, and mathematical power. Findings indicated that rule-based learners in the PSIO environment: (1) chose to ignore opportunities to think about relationship and concepts in-depth; (2) used inappropriate cues based on emotion and external factors to decide when understanding is achieved; and (3) remain uninvolved in their learning and became more passive towards the end of the study. Appendices provide definitions, teacher observation checklists, and a student survey. (Contains over 100 references.) (MDH)

A PROBLEM-SOLVING INQUIRY-ORIENTED APPROACH TO LEARNING MATHEMATICS: STUDENT/TEACHER INTERACTIONS OF RULE-BASED LEARNERS

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This paper was presented at the annual meeting of the American Educational Research Association, San Francisco, California, April 22, 1992. The paper is based on the author's doctoral dissertation completed at the University of South Florida--Tampa in April, 1991 under the direction of A. Edward Uprichard and E. Ray Phillips.

Results from the Fourth National Assessment of Educational Progress (NAEP, 1986) clearly indicate that students are learning more than mathematics from their lessons. Eighty percent of students indicated that they conceived mathematics to be a rule-based subject rather than a process-oriented one (Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988). Another 50% felt that learning the subject involved mostly memorization. The results of recent studies have shown that the very way the teaching of mathematics is approached, the practices and rituals that occur within the context of school mathematics, influences student beliefs about the nature of mathematics to the point that it cripples student ability to function mathematically (Schoenfeld,1985, 1989; Oaks,1987, Lampert,1990). Researchers (Schoenfeld, 1989; Lampert, 1990, Hatfield, 1989) have recommended that further examination of classroom environments that foster effective interactions where students are encouraged to think mathematically rather than use rule-based approaches be pursued.

In response to calls for reform, the National Council of Teachers of Mathematics (NCTM), has recommended that a problem-solving inquiry-oriented (PSIO) classroom environment as embodied in the _Curriculum and Evaluation Standards for School Mathematics_ (1989) and in the _Professional Standards For Teaching Mathematics_ (NCTM, 1991) be established for all mathematics instruction. Yet, when teachers concentrate on creating an atmosphere in which problem solving is encouraged through investigation of phenomena, discussion of observations, and formulation of hypotheses, massive resistance is encountered from students. Teachers' attempts to teach using a problem solving approach to learning are not taken seriously when role definitions are changed to require students to become active learners and when traditional textbook approaches are left behind (Stephens & Romberg, 1985; Cooney, 1985). Student discomfort with these demands may encourage teachers to abandon new strategies after one or two trails (Joyce & Showers, 1988). Hence, if the NCTM standards are to be implemented by teachers, it is imperative to determine what happens to students who hold conceptions of mathematics that may run counter to a problem-solving inquiry-oriented (PSIO) approach to learning mathematics. What is the experience of students who hold a static, rule-based view of learning mathematics when they encounter teachers who enthusiastically persist in using an instructional orientation that is primarily concerned with the enhancement of problem-solving behaviors? What factors contribute to effective interactions and a change of mathematical viewpoint for rule-based learners? The purpose of this paper is to answer these questions and to provide some guidelines to educators who wish to modify their teaching of problem solving to assist students to arrive at an understanding of mathematics as a powerful process of inquiry. The discussion involved in this paper centers around an examination of an in-depth case study of a select group of students who held a static, rule-based viewpoint of mathematics as they engaged in a problem-solving inquiry-oriented approach to learning mathematics as embodied in the NCTM Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). At the heart of this discussion is an attempt to create a coherent image of how conceptions, beliefs, and behaviors are intertwined in effective student/teacher interactions in which students learn to engage in mathematically empowering behaviors as outlined in the NCTM standards.

Mathematical Understanding

Numerous mathematics educators have written about mathematical understanding. Some have attempted to distinguish between types and qualities of understanding (Brownell, 1935; Skemp, 1977, Heibert, 1984, Greeno and Riley, 1987) while others have postulated models for understanding and problem solving (Schoenfeld, 1985; Schroder and Lester, 1989) However, few researchers have investigated the beliefs students express about their own process of understanding mathematics within the context of the classroom and how beliefs may have influenced this process. Schoenfeld (1985,1989) has suggested that the way students view mathematics, what they feel is required in doing mathematics, dictates their behavior in problem-solving situations. While researchers have studied the conceptions that middle school and post-secondary students hold about mathematics (Franks, 1988; Oaks, 1987) and how those conceptions contribute to learning, few extensive investigations have focused on student/teacher interactions as seen from the students' vantage point at the secondary school level. How do rule-based students at the secondary school level engage in the learning of mathematics in the context of a PSIO classroom environment as recommended in the NCTM Curriculum and Evaluation Standards (1989)? (See Appendix 1 for definitions used in this study.)

Methods

A case study methodology was employed to gain an in-depth understanding of student beliefs, thinking, and behavior as they engaged in a PSIO learning environment. This type of methodology has the potential to move one toward a fuller understanding of the natural experience acquired in ordinary personal involvement, to extend the explanation of that which one knows (propositional knowledge), and to increase one's understanding of that which is the foundation of what one knows (tacit knowledge) (Stake, 1978).

The case study design was organized in to three phases: (a) identification phase, (b) interaction phase, and (c) verification phase. The observation and selection of the teacher along with observation and selection of the five student participants comprised the identification phase and took place during September 1989. The interaction phase consisted of four cycles that were repeated from October 1989 through April 1990. Each cycle consisted of two separate areas of observation: student classroom interactions and student small-group problem-solving behaviors. The cycles of observation were scheduled to commence at three--week intervals culminating in a two--week validation phase.

Data Sources

Data sources included classroom observation of participant behavior, small-group problem solving sessions, student interviews, sorting tasks, and student course papers. During the 1989-1990 school year, five students from the selected teacher's honors geometry class were followed through an entire year in a PSIO learning environment. A questionnaire concerning student beliefs about mathematics and problem-solving followed by interviews to verify participants viewpoints was used to identify the five participants who held beliefs in the direction of being rule-based. (See questionnaire, Appendix 2.) This comprised the identification phase which took place during September of 1989.

In the interaction phase, students were observed during class to determine how they reacted to instructional processes involved in a PSIO environment that contributed to the growth of math-empowering and relational behaviors. An audio recording along with detailed notes were kept on each student showing the type of math-empowering behavior solicited by the teacher, the nature of student responses and whether relational thinking was outwardly demonstrated by student participants. Following each class a stimulated recall interview was held with individual students in order to gain in-depth information about student math-empowering behaviors, relational thinking and reactions to a PSIO learning environment. This technique was adopted from the work of Alba Thompson (1982) and modified for the present investigation. In-depth information pertaining to student behavior and experiences was solicited by replaying an audiotape of the lesson and requesting each participant to comment on portions that were considered mathematically empowering. In addition small-group problem solving sessions were used to determine if selected students could perform math-empowering and relational behaviors. Identification of math empowering and relational behaviors was cross validated by a trained investigator. After being trained in the coding methods of this study the outside investigator was able to independently reach agreement with the researcher in 82% of the situations that were coded.

A content analysis of interviews and analysis of the percent of performance of math-empowering and relational behavior was made for classroom interactions and problem-solving sessions. Inferences gleaned during the study were verified using triangulation of data, in the validation phase in May, 1990 using sorting tasks, individual interviews, and questionnaires.

Triangulation of Data

From this information a descriptive interpretation of each participants' perceptions and experience was developed. A triangulation, as proposed by Burgess (1984), of data in terms of time, space, and people was used to verify and organize the descriptions. Emerging patterns were drawn and confirmed by multiple data sources and each description was rechecked by the participants to verify interpretation and amended accordingly. A between subject analysis in which common themes and patterns of common perceptions and experiences between all the subjects and clusters of subjects was completed. Thus, "thick description" (Geertz, 1983) was provided on the primary sources of data using a time, space, and persons data triangulation procedure (Burgess, 1984) to enhance validity.

Selection of the Teacher Participant

During the 1989-1990 academic year this case study focused on five ninth grade honors geometry students who held a static rule-based (instrumental) viewpoint about learning mathematics. More specifically, this study explored the themes, patterns, unique differences of reflections on (a) beliefs about mathematics, (b) beliefs about problem solving, (c) perception of teacher behavior, (d) perception of the learning process, (e) perceptions of themselves as learners, and (f) changes in beliefs perceived by these students as they engaged in a problem-solving inquiry-oriented learning environment.

Initially, an outstanding teacher who used a problem-solving inquiry-oriented approach to learning mathematics as embodied in the NCTM Curriculum and Evaluation Standards for School Mathematics was identified. Mathematics supervisors were contacted and asked to recommend those teachers best able to carry out criteria of excellence for a mathematics program as envisioned in the NCTM standards. Supervisors completed a survey about each recommended participant in order to ascertain the extent to which classroom environment, learning activities, teacher role, and assessment reflected the standards. Further verification of the prospective teachers' instructional practice was made using a three-day classroom observation checklist. (See Appendix 3.) Teacher behaviors were recorded using the checklist every three minutes. Assessment practices were determined during a teacher interview that followed observation. Results of the observation checklist were totaled and rank-ordered for each teacher. Surveys similar to those administered to the mathematics supervisors were given to prospective teacher participants and a peer teacher chosen by the prospective teacher. Results of the surveys were totaled and rank-ordered. Finally the survey portion and observation portion of the selection process were averaged and rank ordered. The teacher with the highest rank was selected for this study.

Description of the Teacher Participant

Although the selected participant, Mr. Mark Meadows, a national awardee for teaching excellence, maintained he was still in the process of finding ways to implement the newly created standards effectively, observation showed substantial progress in all areas of concern. A brief discussion of instructional practice in the area of classroom environment, learning activities, teacher role and assessment practices follows.

Classroom Environment. Mr. Meadows created an active learning situation for students during whole group and small group instruction that included the continuous use of computers, calculators, and manipulatives. Questions such as "Suppose you do this?", "What if?" "Can you give me an example of this?" "Is this always true?" "Is this never true?" were asked frequently to probe for in-depth understanding of processes. He honored student thinking by considering every question carefully and methodically. Multiple solutions to problems were stressed and compared throughout instructional activities.

Although large-group learning situations predominated instruction, cooperative learning was implemented on a limited basis. Mr. Meadows assumed the role of facilitator during these occasions, circulating about the room observing and asking questions to help students think about the problems they were solving. Students were instructed, "Each of you get together and solve these problems. Then convince the other that your thinking is correct."

Learning Activities. The Standards call for a variety of problem-solving activities that allow students to explore, conjecture, test hypotheses, make generalizations, and draw conclusions. Although Mr. Meadows' primary method of instruction was didactic in nature, it was supplemented by numerous opportunities to explore standard and non-routine problems and to think critically. Class sessions were conducted on a question-and answer-basis. This allowed for a high degree of interaction between the students and their teacher and especially encouraged problem posing demanded by the NCTM in the Standards. Often questions posed by the students generated a lively dialogue that moved beyond the scope and nature of the curriculum. Mr. Meadows challenged students to test conjectures and defend their answers by using such phrases as: "Mary explain to Joe how to do number three and state what reason you would use to do this problem."; "Raise your hand if you think this statement is true or false."; and "Tell me why you think that." The Standards call for opportunities for students to ask and to formulate questions about real world situations. Mr. Meadows was especially adept at generating student curiosity by discussing such applications. Mr. Meadows challenged his students by drawing problems from a variety of texts and especially from math competitions. Mr. Meadows' concentrated on making connections which he stressed during instruction and in testing situations. Students often commented, "In Mr. Meadows' class, you don't ever know when you'll see this again. Things seem to pop up every where."

Teacher Role. The selected teacher was primarily interested in helping students to develop thinking skills. As Mr. Meadows stated, "I want students to be able to think and grow." As a result, Mr. Meadows led students through large group problem-solving situations in which he guided students to think carefully about principles involved in a problem and to draw their own conclusions. He very skillfully probed students about their thinking processes, presented counter examples to help them stay on track, and sometimes raised questions to guide them into correct thinking processes. During small-group sessions the same behaviors were observed when he dealt with individual students.

Assessment. Assessment techniques were traditional in Meadows' evaluation plan. While the Standards call for an on-going written assessment of student problem-solving behaviors, none were evident. However, Mr. Meadows used an all-response technique that allowed him to monitor the responses of his class continuously, diagnose problem areas and give instantaneous feedback.

In addition, Mr. Meadows' required students to complete a long-term project, which was highly consistent with the standards, related to geometry and to present it to the class at the end of the semester.

In summary, Mr. Meadows' teaching reflected the criteria set forth in the NCTM Curriculum and Evaluation Standards for School Mathematics (1989) in several areas: in the environment he created for learning, in his instructional approach to activities, in the role assumed as a facilitator, and in his assessment procedures. Although students could have been provided with more opportunities for problem exploration during small-group activities, journal writing and assessment, a majority of the criteria called for in the Standards were being implemented.

Results

Themes that emerged from the research questions concerning the nature of the participants' beliefs about **understanding mathematics**, **learning priorities**, **learning strategies**, and **mathematical power** are organized into four sections that follow.

Understanding mathematics

Since this investigation was concerned with the nature of the participants' intentional use of their belief about understanding mathematics, belief about understanding mathematics served as a global organizing theme. The five participants--Ashby, Robert, Homer, Sarah, and Arlene-- presented varying thoughts about how one could best understand mathematics. For the participants, the notion of understanding as it is related to mathematics and problem solving had more than a single referent. The five participants actually expressed three different perceptions. They encompassed the idea that it is only necessary to:

1. know a procedure to understand a problem (Level I understanding),

2. know the procedure needed to solve a problem and when to apply that procedure (Level II understanding),

3. know the procedure needed to solve a problem, when to apply it, and why (Level III understanding).

Level I Understanding or Pure Instrumental Understanding which is characterized by concern with process only was expressed by Ashby, Robert, and Homer in their initial interviews. Their views about math and problem solving were probed.

> Ashby: I have all kinds of rules to memorize: all the time, especially in geometry. I usually spend many hours on math. . . . it's like a process and it becomes a pattern, so when you see that kind of problem you automatically recognize it and learn the rules and do it. . . . Like rules, I just study them over and over to myself.

> Robert: I guess it is because I try to memorize everything. So you know that way I'll just know it. That's the way I feel secure with it. . . . I think you have to know the procedure in order to do it

Homer: To understand it, you should probably be able to follow a certain procedure and do it over and over. Because if you do a different one you get mixed up. . . . Because it's hard for me to relate how they use a certain procedure. And they have different procedures for different problems. 'Cause it is easier to follow a certain procedure. And I can't remember all of them sometimes.

Level II understanding or semi-instrumental understanding was expressed by Sarah who was concerned primarily with knowing how to execute a process and where to apply it. Sarah expressed this view early in the year.

Sarah: Like if you have a problem and you have to work it out by the rule, but you might not know how to put 'em together, and if he only knows the rules and theorems or whatever, then he probably can't work out the problem. I don't just memorize things. I see what kind of problem it is, and when you learn how to work that kind out, then when you come to that kind of problem you know how to work it out, but sometimes you don't know exactly why.

Students who fall into the first two levels will also be referred to as rule-based students.

The third level of understanding or relational understanding involves more than rote performance of a set of rules and knowledge of appropriate situations to apply them. Throughout the study, Arlene adhered to the belief that behavior reflective of Level III Understanding was necessary, the belief that one needed to know how, when, and why to use rules in problem situations. Arlene's view of understanding math was stated in the following comment.

Arlene: (When referring to Joe, a student who used memorization only to learn facts and rules.) He'll do good for a little while and then when he has to visualize pictures like in geometry, he'll blow it. . .

. That's one way I learn, but also I have to visualize it and see it in front of me. . . . But you also need to know why you got the answer to understand it.

In terms of Skemp's (1978) conception of relational and instrumental understanding, Ashby, Robert, Homer, and Sarah would have fallen into the instrumental learner category: learners who give their attention to rules learned without reason. Based on Arlene's responses one would categorize her as a relational learner, a learner that is concerned with knowing both what to do and why. Arlene's response to the PSIO learning environment were included to provide a basis for comparison with those of pure instrumental and semi-instrumental learners.

Skemp hypothesized that instrumental students would not want to know all the careful background work or explanations given in preparation for topics in mathematics and would ignore attempts to convince him/her that this thinking is not productive in the long run. This study not only provides support for that hypothesis, but also explores reasons for the hypothesized behavior as well as other characteristics that accompany this belief system. What follows is a description of how the three beliefs concerning the levels of understanding of mathematics--pure instrumental, semi-instrumental, and relational--influenced the participants' beliefs about mathematics and problem solving, how these played a role in learning priorities, learning strategies, and the development of mathematical power.

In addition, changes in belief as perceived by these students will be considered in the context of a PSIO environment.

Changes in belief were accompanied by shifts in the participants' view of the teacher, the learning process and themselves. During the course of this study, Homer moved to a relational view of understanding mathematics which he conveyed in the third month of the study.

> . . . for me, it's easier if I have each step, and I have it set out so I know what each step does. So I have to know what each step does. So I have to know a certain step to be able to work it out. And I need to know why or else I won't understand it, and I won't be able to do it on a test or something.

By the end of the study Sarah reflected this view in some aspects of her mathematical behavior. Sarah felt that knowing "why's" of a problem would lead one to be able to work independently and reflected this belief in problem-solving sessions. She stated, "If you know why somebody's doing it that way, then you can do it by yourself." Yet, she also conveyed that, "It's all right to work out a problem and not know why you do each step. . . . I figure if you understand it enough to get the correct answer, then it doesn't really matter." Since a change in mathematical dispositions is a goal of the NCTM standards, knowing what factors contributed to effective interactions that may have brought about a change in mathematical viewpoint for these rule-based learners will be considered.

<u>Beliefs About Mathematics-Relatedness of Concepts</u>. At the onset of the study, four of the participants believed that math was related because subjects followed a certain sequence in the curriculum or topics were presented in the text in a certain order. When asked if mathematics were related, all participants

responded in a manner similar to Ashby and Sarah.

> Ashby: Because to go from algebra to geometry you have to have geometry to do algebra II. They all are kind of similar. Each year you use something you had the year before.

> Sarah: Probably, true, because when you go further on, it has something to do with the other chapter and it just goes on from there.

Changes in perception-Validation Phase. In April, Robert, Sarah, and Ashby expressed views similar to those in the initial interview. By the validation phase of the study, Homer and Arlene held markedly different perceptions. Initially, Homer had viewed math topics as a disjointed set of topics that happened to fall under the heading of mathematics. By the conclusion of the study, he related topics in math because of their repeated use during instruction and the text.

> Homer: It's related in the sense that you use theorems and stuff that happened in the last chapter, and you have to. . . . you can't ever forget about one thing about math. It will probably show up somewhere else.

Arlene's belief about how topics in math are related grew to include the conception that new information could be developed using proof and logical reasoning.

> Arlene: I mean. We'll study the parallelograms, and then we'll study triangles and also the parallelograms and you can use them to prove the parallelograms, and you know, it's all related.

Arlene, the sole participant, who believed that knowing why was necessary to understanding, switched to the belief that math was related because one could construct new theorems and concepts by reasoning about those studied earlier. Three of the four participants who held pure instrumental and semi-instrumental viewpoints adhered to the notion that one could see that topics within mathematics were related to each other by examining the order of the curriculum or topics in a text. Thus, Arlene, a relational learner, and Homer, who switched his viewpoint to relational thinking during the study, were the only two participants who began to perceive of mathematics as a collection of related topics derived within the context of doing mathematics.

Beliefs About Mathematical Creativity. Initially all of the subjects believed that mathematics was in a constant state of change due to the knowledge explosion and rapid development of technology that has occurred recently. Ashby and Robert, who were pure-instrumental learners, felt that mathematical creativity was not within their capabilities. Robert best portrayed this thinking.

> Robert: I think that people are making up new discoveries every day because the world's changing every day, and I think that people are coming up with new discoveries. They may not all be right, but they are still coming up with them. . . . I don't think someone like myself could come up with . . . come up with something like a genius could.

Homer, Sarah, and Arlene expressed the belief that it was possible for them to create mathematics. Homer and Arlene felt that they would rather have someone guide them through their learning experience than create knowledge for themselves. Homer expressed his view about creativity.

> They probably do have some new theorems or the way they explain mathematics. And they uhm, I'm not sure. It sounds like they have new rules. I'm sure they got the basic outline a long time ago. . . . Probably (students

could discover mathematical concepts) but it is a little easier if you have someone whc knows more about it and can help you with it.

They desired to travel an easier path to acquiring mathematical understandings. Only Arlene, a relational learner, experienced mathematical creativity as a personal phenomenon she had encountered in the confines of the classroom.

> Arlene: You learn something new every day. We're growing in knowledge in science and math: I'm sure we've learned a lot in the past years. . . .
> (When asked about her ability to create math.) Yeah, an example is, like in solving a problem, my teacher will give us a method for solving it, and I can find another method that's easier for me to understand, and I'll do it my way and it comes out right, ... see I want to learn but I don't want to do my own stuff

Lastly, the other participants who held instrumental viewpoints did not indicate awareness of a personal experience with mathematical creativity.

Participant View of The Learning Process:

Beliefs About Learning Priorities

In the early months of the study the more rule-based learners, level one and two understanding, appeared to concentrate less during portions of the lesson that involved the development of definitions and relationships. These reactions were verified during interaction interviews. Ashby explained her choice.

> No, I can't remember what I was doing, but if you feel it's not really that hard, you don't even listen. . . . I wasn't really paying attention. So if you kind of know it, it's really not that difficult. Your attention span seems to get shorter and your mind strays because it's less important. I was thinking about what I was going to wear to the game tonight, too.

Furthermore, Ashby felt some of the information presented in class was unnecessary. She labeled it "trivia" or "mathematical trivia." She felt these questions were used by Mr. Meadows to trick the class. Usually, these questions were directed towards enhancing the student's understanding of the relational nature of a topic. For example, when Mr. Meadows wrote the formula A = lw and asked, "How many uses does this formula have? How many think it has only one use? How many think more? Why did you choose your answer?" Ashby's response was,

> I just thought this was one of his tricks here so I said it's just the area, it's just one way, but then he started saying that all the variables stand for somethin' and it's pretty obvious if you got the width and the area and you could find the length and the area and the length find the width, but I wasn't thinking about that, I wasn't really worried about trivia, and I was thinking about my outfit again. . . . I thought I knew it, but I didn't think about it, I just thought he was trying to trick us again. . . .

Whenever Ashby felt she was being tricked, she would shut down and allow her attention to wander. Unfortunately, these were the portions of the lesson in which relational understanding was the focus. It should be noted too that this behavior was observed in both exploration and didactic instruction as well. Even during cooperative sessions participants with level one and two understanding maintained a passive stance by waiting for others to supply relational information. As Ashby explained, "I live for the recaps."

To validate information gained in earlier interviews through the process of triangulation, the interviewer inquired as to participant priorities for learning during class. Ashby responded in the following manner.

> Interviewer: O.K. Let me ask you another question that is unrelated to this problem. When you are sitting in class, what's more important to you, to pay attention to proofs, theorems, definitions or naming the parts of a figure? Can you tell me what importance you place upon them? What priority would you give them?
>
> Ashby: Um, proofs, first 'cause I have a hard time with them. I want to know how to do them. Um, next I need to know my figures then definitions 'cause I can go home and memorize them.

Throughout the investigation, definitions needed to perform math-empowering behaviors and relational thinking were not valued by the pure instrumental and semi-instrumental learners--Ashby, Robert, and Sarah. When requested to prioritize the amount of attention they would devote to the learning of definitions, theorems, proofs, and the identification of figures, these three participants placed the learning of definitions third or below. As a result, Ashby, Robert, and Sarah had difficulty using definitions later to solve problems presented during instruction and in problem-solving sessions. Arlene, the only relational learner, maintained her view that the learning of definitions and theorems should be given top priority in order to understand the construction of proofs. Only Homer shifted his viewpoint by the end of the study by moving the learning of definitions and theorems to the top of his list of learning priorities.

This shift grew out of Homer's frustration with his performance (D+) in geometry. Homer realized he could not complete problems without understanding the definitions and theorems. At the same time, Homer became keenly aware of teacher behavior and began to anticipate and mimic Mr. Meadows who constantly required students to think about why a step was completed in a particular way and to find other ways to think about a proof. This behavior became automatic for Homer.

Another facet of student learning priorities was revealed by three of the four instrumental learners who disclosed that they concentrated on new or difficult material, but as soon as understanding was achieved, their focus was placed elsewhere. Sarah's description of her reactions was typical of the other participants.

> Sarah: And then when things aren't difficult, and I don't have to listen in class, I mean, 'cause that's usually the way I do it. And that's when I lose it, when I don't listen. Because, I think it's easy, that's usually when I forget how to do it.

However, problems arose for the instrumental learners since they used criteria for deciding when they understood a concept or theorem that omitted understanding mathematical connections and relationships. Criteria for understanding given by the instrumental participants included being able to: "raise my hand and give a correct answer," have "the teacher tell me its right," know "like I can really talk about it like I know what I'm saying," and have "it click in my mind and make me feel better." All of these cues excluded the activity of focusing on attributes involved in connecting concepts and forming relationships. Since knowing a process was given a higher priority than relational understanding, the participants screened out portions of the lesson that dealt with attributes of and in-depth explorations of concepts. Relational learning for some of the participants was considered trivia and ignored. Thus, the more rule-based students may have been unaware that they lacked in-depth understanding of mathematical concepts.

Finally, three of the four instrumental learners concentrated on ideas Mr. Meadows appeared to highlight during instruction and to delegate the most time. According to the participants, Mr. Meadows emphasized proofs and problem solving more than other subjects. Mr. Meadows stipulated that students should develop their ability to think through the use of problem-solving situations.

> We've talked so much about problem solving, but to me it's just about the most important thing that we are teaching. . . . I love to hear the kids prove things, argue back and forth . . . and I think we should encourage them to be more accurate in their mathematical statements and to prove their conjectures. It doesn't have to be a statement of reason proof. It can be justified verbally. Prove it to me. Convince me.

Within the confines of a classroom where problem solving was emphasized, these instrumental learners were even more prone to concentrate on memorizing processes presented in problem solving since they held the belief that they should also focus their attention on portions of the lesson the teacher stressed.

Participant View of The Learning Process: Learning Strategies

Initially, the instrumental participants, Ashby and Robert, expressed the belief that memorizing facts and rules enhanced their ability to understand.

> Ashby: Like rules, I just study them over and over to myself, and then after several hours, then I'd have my parents call them out to me in a different order, and

I'd recite the theorems and postulates to them.

> Robert: I like read over what the teacher had us . . . make things, you know that I think are important to know because like on different postulates. I'll read it, and if it stands out in my mind, I'll mark it with my pencil or something. Ah, I'll just keep on going back over that and drilling myself on it and make myself learn it. . . . . Until really I could just . . . if somebody asked me what it was, I could just name it off.

In the same interview the other participants of level 2 and 3 understanding used alternative strategies to enhance understanding. such as writing, visualizing, and creating problems similar to those posed in class and working them out independently.

> Sarah: I write everything down and look over it again and get help.

> Arlene: Usually, I'll sit down and read over my notes or read the book on the section a couple of times and then I'll go back and picture it in my mind and see what I can remember, the important parts of it.

> Homer: To study you can make up your own problems and work them out. . . . When I'm trying to study. They are hard to figure out sometimes to get them to have an even answer, so it works out equally.

Interviews conducted during the course of the investigation revealed that the participants who held a pure instrumental view, Ashby and Robert, chose strategies that either allowed for a low degree of involvement in learning activities or those strategies did not enhance connections between visual representations and concepts of geometry. Both Ashby and Robert chose to watch Mr. Meadows draw rather than draw figures themselves. Both students indicated that they found visualization difficult. During a problem-solving session, Ashby stated that she was able to set up trigonometric equations correctly in class because, ". . . he did a lot of them. He named each part." In these lessons students had been urged to complete drawings of each problem independently. When questioned further about her level of involvement, Ashby gave the following response:

> Interviewer: Were you drawing them along with him or just watching him?
>
> Ashby: Just watching him.

Examination of Robert's and Ashby's notes showed that they contained very few drawings. They remained uninvolved in situations that demanded drawing or visualizing.

When confused about situations, both used a "wait and see" strategy rather than question the teacher to clarify their misunderstanding. They feared revealing their inability to understand to their peers.

> Robert: I'm intimidated by the teachers really . . . . I'm just one of those people that get intimidated by people. . . . I try to figure it out for myself. It never works, though. I guess one day I'm going to have to learn to ask questions.

Occasionally, Robert and Ashby figured out how to solve a problem by referring to previous problems worked in class by Mr. Meadows. They used Mr. Meadows' examples as models to find patterns that led to the correct solutions. They later memorized the patterns; however, often they were unable to explain the reasoning involved. The following episode pertaining to the effects of Ashby's use of this strategy is typical behavior for learners with level one understanding. The class was asked to complete the problem shown below. Ashby had just looked at a set of examples she had copied from the board and responded with the correct answer.

AB = 18 BG = 15

EH = 10 AD =

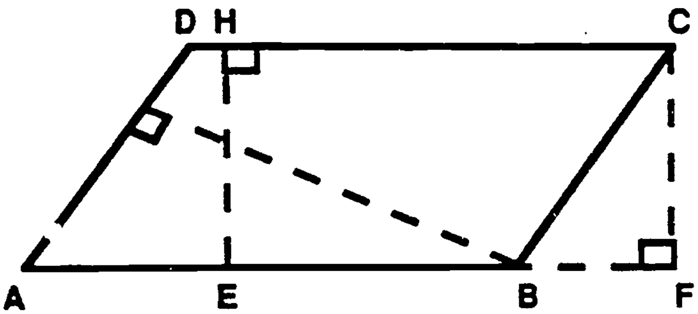

Figure 1. Ashby's Parallelogram Area Problem.

Ashby: Well, we had to find the areas; we multiplied the two . . . ah the base and the height which is 18 and 10 and then that gave you 180 and then BG is the height so you just divide that into the area and that gives you the base AD.

Interviewer: So what were you basically using? Tell me some of the principles or ideas involved in that problem?

Ashby: Uhm, I used the parallelogram formula a = bh and uhm. . . . not sure why we divide by 15 into that, but that's what we have always done, so. . . .

Interviewer: Can you tell me anything else?

Ashby: No not really. I never really knew what he let us know what to do, not really. I just know that's the way you do it. I never question why. . . .

Interviewer: So you looked at his model of what he was doing?

Ashby: Yeah.

By comparison, the semi-instrumental and relational learners, Homer, Sarah, and Arlene, involved themselves actively in learning by making mental visual inventories, diagrams, and visual comparisons of their work to others presented during instruction, by using mental repetition of reasons for definitions as problems were worked out in class to reinforce their learning. They expressed the following viewpoints about these strategies.

> Homer: I was going over the same thing that he said because I understood it. I was saying in my mind what he was saying out loud.
>
> Sarah: I was just going over it in my head while he was. . . . just listening to what he says and making sure I understand.

Arlene described similar behavior during a problem-solving session.

> Arlene: I set it up to equal 180° because they form a linear pair and they are supplementary. It gets in my mind when I say the definitions of linear pair; then I know why they are supplementary.
>
> Interviewer: You say it?
>
> Arlene: Yeah, kind of like doing a proof in my mind, a small proof.

As the study progressed, Homer and Sarah began to anticipate the teachers' moves, especially those that involved discovering the reasoning process used in a problem and finding multiple solution paths.

> Sarah: . . . in class, I try to think ahead of the teacher by figuring out why a problem is done that particular way and it's about the same thing. Because, then if you figure it out by yourself, then you can do it by yourself once you get home. . . . I figure it gets worked into your brain.
>
> Homer: . . . usually Mr. Meadows asks questions like, 'Why do you think we do this?', in class, and stuff. And so, first I didn't recognize he did that until I started noticing he does that a lot. So now I start looking ahead and see why we do that and look at the theorems and definitions.

Sarah reasoned that if she could discover "why" her ability to perform problems independently would be enhanced. On the other hand, for Homer finding out why was a result of his willingness to fulfill teacher expectations. Arlene looked on it as a way of reinforcing important conceptual understanding through repetition. She described her thinking about moving ahead of the teacher.

> Interviewer: Then what were you thinking about when you thought of the tests?
>
> Arlene: That, when we were doing these proofs like, the reasons why the alternate and interior angles, . . . to review, memory. Like to keep up with . . . .It's kind of like studying in a way, just to bring it up every now and then in your mind so you'll remember it. You won't have to cram for tests.

By the end of the study, Sarah and Homer had begun to concentrate on discovering why steps were taken to solve a problem. While Arlene had sustained a relational mind set since the onset of the study, during the course of the study Sarah and Homer began to integrate this way of thinking into their efforts at problem solving. Arlene, Homer, and Sarah believed that these actions would enhance their ability to understand and remember information gained during instruction--a perception that led them to be successful learners in problem-solving sessions and geometry.

Yet the pure instrumental learners, Ashby and Robert held a stance that allowed them to remain uninvolved in instructional activities. When given opportunities to focus on alternate methods of finding solutions to problems, these learners ignored such explanations. Robert felt that it was only necessary to learn one way. ". . . it's easier for me to learn. Not to learn so many things, I just learn one." Ashby, also blocked out second solution methods.

> I didn't know because if I don't understand one way I can't really think of another way. I know people that know a second way.

Thus, the instrumental learners, who tuned out many portions of instruction developed fewer learning strategies during the course of the study then the other participants. Robert employed only nine, the least number, followed by Ashby with ten. By the close of the study, the semi-instrumental and relational learners, Homer, Sarah, and Arlene, developed 12, 13, and 14 strategies respectively. As a result, the less rule-based participants (level 2 and 3 understanding) had developed more strategies for making mathematical connections.

Mathematical Power

As in the Paulsen and Jackson study (1983), the participants based their view of themselves as learners on their performance during mathematics instruction. Since the participants; view of themselves was strongly linked to their performance, this section will focus on development of mathematical power. Mathematical power according to the NCTM Curriculum and Evaluation Standards for School Mathematics "requires the ability to use information to reason and think creatively and to formulate, solve, and reflect critically on problems"(1989, p. 205). Two facets of mathematical power will be discussed: the ability to think creatively about mathematics and the ability to assist others to reason about problems independently since they were emphasized in the PSIO approach to instruction that the participants encountered.

Mathematical Creativity. Four of the five participants became aware of creative activity, the ability to formulate alternate solution paths to a problem, as it had occurred among their classmates during instruction. Ashby, Robert, and Sarah described how they felt incapable of being mathematically creative. Three of the four instrumental learners except Homer, who switched his view about learning mathematics, believed they were not capable of creative mathematical activity--an activity that demanded relational thinking and reasoning. These behaviors had not fallen within their belief system. For those that believed that reasoning and relational thinking were not integral to their understanding and learning process, expression of these behaviors in the form of personal creative activity appeared impossible. Homer, who switched his beliefs about understanding to include knowing the reasoning involved in mathematical processes, began to perceive himself as being capable of independent creative activity. Arlene, who maintained a similar relational viewpoint from the beginning of the study, also experienced creativity as an individual phenomenon throughout the entire study.

Participant Perception of Giving Assistance. The manner in which the participants would give other individuals assistance reflected their beliefs about how one should understand mathematics. The pure instrumental and semi-instrumental learners described situations in which they offered assistance to others by showing them the steps needed to complete a process. A shift in mathematical power was also reflected in the way the participants finally chose to assist a fellow student, Joe, who was in trouble. By the close of the study, the less rule-based learners, Homer and Sarah, who initially held views similar to Robert and Ashby, described how they encouraged learners to think for themselves and to engage in searches for reasons needed for solutions to problems independently.

> Sarah: I would either show him what he's forgetting or just ask him, is there anything false you need to do or something like that. Because if I think he knew it, I would want him to come up with it himself. . . . But when you come up with it yourself, you remember it.

Sarah and Homer desired that the opportunity for autonomous thinking they had been shown be shared with others. They began to place value on becoming independent learners. By comparison, Ashby and Robert could not go beyond what they expected would enhance their learning. They desired to be shown processes without being required to reason, and this is mirrored in the assistance they offered. Ashby elaborated on this point when she described how she would assist Joe.

> Ashby: I'd tell him to find somebody else. Um, if I had to tell him I'd just tell him exactly what the teacher said. I'd never make it on my own.

They had not achieved the autonomy necessary to becoming mathematically empowered.

<u>Participant Perception of The Teacher</u>. Since the four participants who initially expressed an instrumental or semi-

instrumental viewpoint were concerned primarily with knowing the exact procedure involved in solving a problem, they held the expectation that the teacher should show them all the steps involved.

> Homer: Sometimes I don't like the way he teaches it. He should explain more instead of just giving us more homework.

> Ashby: I'm not told the answer but instead am asked about the process for solving a problem. Now he does that a lot. . . . You know it would be a lot easier if I could just watch him 'cause I've learned better watching him than doing it by myself. It makes me mad.

> Robert: I think a thorough teacher does present a step-by-step approach, but I also think the student has to study.

Despite the need to be shown, these participants still felt that Mr. Meadows was a good teacher. They simply were frustrated because they were accustomed to and desired to take a more passive approach to their learning that did not require them to think for themselves.

Another conflict arose when Mr. Meadows emphasized the need to learn the theorems in order to be successful. Because of their rule-based viewpoint (pure instrumental) about learning, Ashby and Robert interpreted learning theorems to mean memorization. They claimed that even though they had followed Mr. Meadows' directions they were unsuccessful.

> Ashby: I think the biggest misconception of Mr. Meadows', though, is he says to learn the theorems and you can't fail. And I think he's wrong. . . . He just says if you know 'em backwards and forwards, you can say them any time without hesitating, and you should have no problems. Or if you learn the formulas, then you'll have no problems, but, um, that's not true.

Robert: I might get lucky. And I'm not real sure. I never really feel like I understand the problem. And I understand the theorems and everything. But I don't ever understand the problem.

This differed with Mr. Meadows' conception of learning. When Mr. Meadows said "learn the theorems," he expected students to be able to do much more than memorize. He explained his expectation.

Now I don't really care whether they can write them out or not, but when they hear 'median' I want them to know how to draw a median and know what properties that median has or, otherwise, they're not going to be able to work the problem.

Ashby's and Robert's limited perception of the word "learn" placed them in a position in which they distrusted the teacher and were incapable of knowing how to apply their knowledge. Holding a pure instrumental viewpoint had become detrimental to Ashby and Robert and hindered their ability to function successfully in a PSIO learning environment.

<u>Changes in Perceptions of Teacher Behavior-Validation Phase</u>. Only those participants who developed or held a relational viewpoint, Homer and Arlene, exhibited a positive view of the teacher by the end of the study. Ashby, Sarah, and Robert, who maintained instrumental views, did not change their view of Mr. Meadows significantly during the study. Robert and Sarah held a view similar to the one expressed by Ashby who felt, "he's a good teacher for fast-moving people, honors. I don't think he's a good teacher for people who don't pick up quickly." Arlene, a relational learner, stated that Mr. Meadows was, "a good teacher. I really do think that. He explains things well." Homer disclosed a major change in perception of teacher behavior that

accompanied a shift in learning priorities.

> In the beginning I hated him 'cause I didn't understand. I thought it was his fault. But I'm starting to like him. . . . Uhm, the proofs, I never understood.

When Homer discovered that he was falling behind in class, he shifted learning priorities.

> Late October, early November, proofs really hit me hard cause I hadn't really memorized all the definitions and stuff that I had to 'cause I thought I knew them then. And what I did was, what I think I shouldn't do now, was I memorized them for that time, and I just thought I could forget about them. I didn't know they would come up again, so that got me in a bad situation. . . . Now I don't really memorize them, but I guess I do memorize them because to me memorizing them is understanding them too. To really memorize you have to be able to understand.

Homer additionally explained that he now checked his understanding by connecting what he was learning to problems and pictures he had drawn. Homer now focused his attention on learning definitions and theorems. He began to pair problems with procedures. "I think I started studying better and I started--instead of just looking at one procedure I put the two together, a procedure and a problem." And, he began to concentrate on reasoning. "And I need to know why or else I won't understand it, and I won't be able to do it on a test or something."

As a result, Homer's perception of Mr. Meadows had a corresponding change. He began to view Mr. Meadows as a competent professional who was very concerned about his students. "I think Mr. Meadows is extremely smart. He knows what he is doing. He tries to help us, I mean, you can tell how he really, like worries about whether we're going to need to do this or not. . .

Participant Perceptions of Themselves As Learners. At the onset of the study all the participants except Ashby were confident in their ability to perform mathematically. Ashby expressed the opinion that she was slow at mathematics and made A's through her own effort.

> . . . . I have to study real hard to make A's. I don't just make it like everybody else does. I've never been great at solving problems and that kind of stuff. . . .

Homer, Robert, and Sarah had a similar locus of control. They also believed that they could succeed through their own efforts; however, they believed they had some mathematical ability.

While the pure instrumental learners, Ashby and Robert, began the school year feeling confident about their ability to do mathematics, they ended the school year dissatisfied with their learning and lacking confidence in their ability to perform mathematically. Robert now believed he was not honors material.

> I don't know what the problem is. I mean, you mean, you know. I--maybe I'm just not the level of geometry that honors is. Maybe I shouldn't be there.

Frustrated over their struggle with problem solving, both Ashby and Robert, the pure instrumental learners switched their perceived locus of control. They now perceived that success was due to luck or some outside force beyond their control rather than effort. When successful, Robert and Ashby believed that either they were given an easy test as or that they made a lucky guess. Ashby described her probable reasons for success,

> I got lucky, because usually most of the time I guessed and lots of times I get things right when I guess. And

I feel good, but I know I don't really know what I was doing. I just got lucky.

Sarah, a semi-instrumental learner, indicated that her feelings of confidence in her learning fluctuated with her ability to perform mathematically. When asked if she lacked confidence, she stated,

> That's true because, um, well it depends on what time it is. Usually, I mean if I can't get the problems right the, um, I'd start going ahead and thinking even before I start doing the problems, that I'm not going to get it right. . . .

By the end of the study, Sarah felt she was happy with her learning, and as a result was confident. Occasionally, she incorporated relational thinking into her mathematical behavior, yet she still believed in the use of procedural thinking. She ended the school year satisfied with her learning in geometry although it had been a challenge for her.

The pure instrumental learners, Ashby and Robert, ended the school year dissatisfied with their learning, feeling that they had studied a mathematics vastly different from anything they had encountered.

> Robert: I don't like this year. But in the past year I have because I've done good and, you know, I've learned a lot. Which, I think I've learned a lot this year, but, ah, this year had just been a really different experience. . . Um, well. It's unlike any other math I've ever had. . . . A whole lot different.

Homer, who had shifted to a relational mind set, and Arlene, who maintained a relational way of thinking throughout the study, were satisfied with their performance and expressed complete confidence in their ability to perform mathematically. While the relational and semi-instrumental learners felt satisfaction with their learning, they ended the year feeling challenged.

> . . . . this year, the work is so different . . . I mean last year, it was pretty fun, but it was mostly because I understood everything. . . . This year it's, it's gotten pretty hard.

<u>Summary of Changes-Validation Phase</u>

In regard to the three levels of understanding mathematics as expressed by the participants--pure instrumental, semi-instrumental, and relational--only Homer expressed a change in viewpoint. Initially he held a pure instrumental stance. By the culmination of this study, Homer's viewpoint about understanding could be characterized as relational. The shift was evident during the second cycle of the study when Homer began to look for second solution paths, anticipate and emulate teacher behavior, and search for reasons needed to solve problems presented in class. Homer attributed this change to a desire to improve his grades and to an awareness that relational learning and understanding was necessary to function successfully in class. When questioned about the shift in thinking during the second cycle, Homer elaborated,

> Yeah, because I guess I was trying, I was trying to get better grades, trying to figure out different ways I could do it. . . . It's just something I figured out. I didn't even plan for it to happen. It was sort of like an accident, but I didn't realize it until just now, actually. What I was doing, I was just trying to figure out another way. Probably because I was a little bit bored, too . . . .

Homer possessed a strong desire to succeed. He used his ability to discern what the teacher expected and conformed to it which served as a catalyst for a change of strategies.

Homer: . . . usually Mr. Meadows asks questions like, 'Why do you think we do this?', in class, and stuff. And so, first I didn't recognize he did that until I started noticing he does that a lot. So now I start looking ahead and see why we do that and look at the theorems and definitions.

Although Sarah adopted relational thinking in some aspects of her behavior, she remained semi-instrumental in her belief about understanding mathematics. Sarah was motivated to think relationally in some instances because she believed that when she was able to reason in problems, then she was able to solve problems independently. However, she stated that it was not necessary to know why to understand mathematics, a rule-based stance.

The other pure instrumental learners, Ashby and Robert, maintained their belief system throughout the study. Neither of these participants were strongly motivated to succeed in mathematics. Ashby and Robert did not have as strongly formulated career goals that were math related as Homer who desired to be an architect. Both appeared to lack visual thinking skills necessary for success in geometry. Finally, both were strongly influenced by peer pressure and therefore unwilling to risk questioning the teacher and their peers to correct misconceptions that occurred during instruction. Even during small-group instruction and in problem-solving sessions, they maintained passive behaviors.

Throughout the study Arlene remained a relational learner. Her view of learning mathematics was consistent with the goals of a problem-solving inquiry-oriented approach to learning mathematics, and as a result, she was successful in geometry and in performing math-empowering and relational behaviors.

Corresponding changes in views about the relatedness of topics in mathematics, problem solving, perception of learning priorities, use of strategies, perception of the teacher, and personal performance in class accompanied Homer's change in belief system. Homer began to view math as a set of interrelated topics that he could create by reasoning. He began to focus on learning in-depth meanings of definitions and theorems instead of memorizing statements. He became highly involved in learning by anticipating teacher moves that called for reasoning and mathematical creativity, visualizing, and comparing his work with others. As his performance improved in geometry, Homer began to enjoy the challenge of problem solving and to appreciate Mr. Meadows' expertise as a teacher. By the end of the study, Homer felt capable of offering assistance to others in a way that focused on helping a person to perform mathematics autonomously. Although, challenged Homer felt satisfied.

By comparison, Ashby and Robert, who remained pure instrumental did not change their views about the relatedness of mathematics topics, perception of learning priorities, and use of strategies. Their belief that understanding mathematics meant memorizing process only led them to avoid concentrating on situations where concepts and relationships were developed in-depth. As a result, as the semester progressed, these students failed to perform math-empowering and relational behaviors because they lacked in-depth understanding of basic concepts. They began to mistrust a teacher who held a different view of the meaning of the word "learn." Learning strategies used by the pure instrumental learners such as copying problems and looking for patterns to follow allowed them to avoid reasoning and remain uninvolved. Although they became aware of creativity occurring amongst their classmates; they felt unable to perform these behaviors because they demanded the use of reasoning, a relational behavior. By the end of the study, they felt incapable of helping others with their mathematics and offered a rule-based form of assistance. Both had stopped believing in their ability to perform mathematically. If they succeeded, it was the result of luck or some outside force. Although these students felt challenged, they were dissatisfied with their learning.

Percent of Math-Empowering and Relational Behavior Observed During Classroom Interactions and Problem-Solving Sessions

The five student participants, who served as subjects in the current study, were interviewed during the 1989-1990 school year in 10 sessions that lasted approximately 1 to 1.5 hours in length. The sessions were audio-taped in the guidance office of Summit High School. The conduct of the study was divided into three phases: (a) identification phase, (b) interaction phase, and (c) verification phase. The observation and selection of the teacher along with observation and selection of the five student participants comprised the identification phase and took place during September 1989. The interaction phase consisted of four cycles that were repeated from October 1989 through April 1990. Each cycle consisted of two separate areas of observation: student classroom interactions and student small-group problem-solving behaviors. The cycles of observation were scheduled to commence at three-week intervals culminating in a two-week validation phase.

Analysis of Classroom Interactions. During the classroom interaction portion of each cycle students were observed to determine how they reacted to instructional processes involved in a PSIO environment that contributed to the growth of mathematically empowering behaviors. Detailed notes were kept showing the type of math-empowering behavior solicited by the teacher, the nature of student responses and whether relational thinking was outwardly demonstrated by student participants. One student was observed per class period. Each observation was followed-up with a stimulated recall interview until all five students had been interviewed. This technique was adopted from the work of Alba Thompson (1982) and modified for the present investigation. In-depth information pertaining to student behavior and experiences was solicited by replaying an audiotape of the lesson and requesting each participant to comment on portions that were considered mathematically empowering. Responses were categorized as math empowering or relational, both or neither. The categorization of responses was cross-validated by comparing the researcher's observations with that of a trained investigator who held a master's degree in the teaching of mathematics and currently teaches a college-level course in geometry. The investigator was given four hours of training in recognition of math-empowering and relational behaviors in order to establish interrater reliability. After receiving training in the coding methods of this study, the outside investigator was able to reach agreement with the researcher's categorizations 82% of the time. Figure 2 shows the total percent of behavior performed that was math-empowering or relational or both for each participant during the four cycles of observation. (See Figure 2 below.) Figure 2 shows that as the study progressed the percent of behavior classified as math-empowering or relational behavior or both performed by the two pure instrumental learners, Ashby and Robert, decreased. A corresponding increase in the percentage of math empowering or relational behavior or both was observed in Arlene who maintained a relational viewpoint from the inception of the study and in Homer and Sarah who were willing to adopt a relational viewpoint in some aspect of their learning.

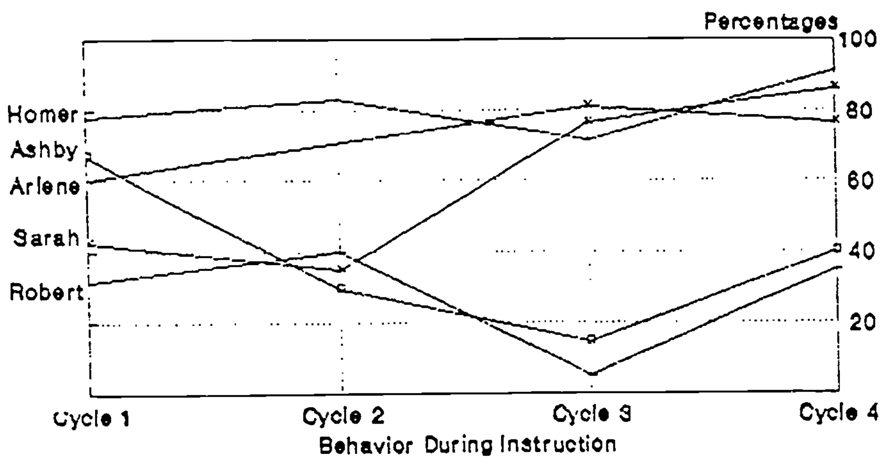

Figure 2. Total Percent of Observed Behavior in Cycles 1-4 Classroom Interactions that was Math-Empowering,

Analysis of Small-Group Problem-Solving Sessions. During the small-group problem-solving portion of each cycle the five participants grouped in dyads were asked to work non-routine problems containing concepts covered during class. The researcher audiotaped each interview and took extensive notes. Relational behaviors such as discussing why a rule was used and explaining how a definition was used to get a solution were noted. Along with these, math-empowering behaviors such as making a conjecture, testing a hypothesis, and comparing several solution paths were recorded. After each problem-solving session, students were questioned about their problem-solving behaviors to check whether verbal data gathered truly represented the analysis of these behaviors. Figure 3, displayed below, shows the total percent of behavior that was math-empowering or relational or both for each participant during the four sessions.

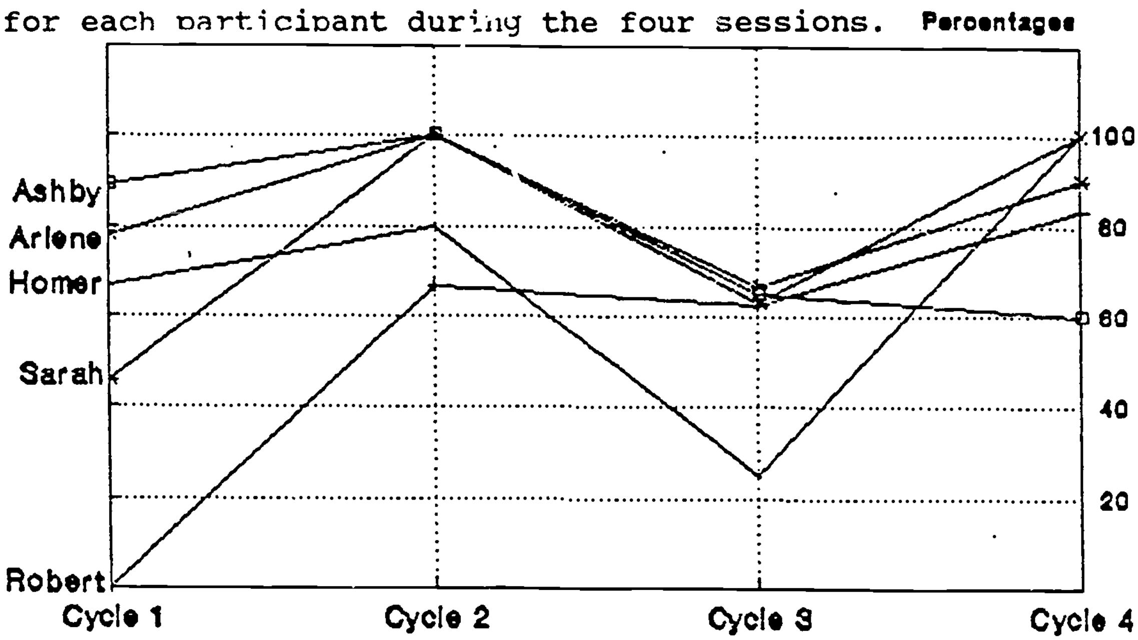

Figure 3. Total Percent of Behavior in Cycles 1-4 Small-Group Problem-Solving Sessions that was Math-Empowering, Relational or Both

The pure instrumental learners, Ashby and Robert, appeared to increase the total percent of these behaviors during the second cycle when they used empirical methods in place of logic and reasoning to verify their solutions. The total percent of math-empowering or relational behaviors dropped in cycles three and four as Asbhy and Robert became more passive and allowed others to supply definitions and relational information required for solutions. The total percent of math-empowering or relational behavior or those that encompassed both increased to 100% in the second cycle for Sarah, Homer, and Arlene. Again, these learners also did not rely on logic and reason. As in the Schoenfeld studies (1985), these participants turned to empirical methods to verify solutions. All of the participants had a corresponding decrease in the total percent of math-empowering and relational behavior during the third cycle since they had difficulty remembering definitions of trigonometric functions required for the problems they encountered. By the fourth cycle the three less rule-based learners recovered to perform at least 90% or more of the math-empowering and relational behaviors observed in this session. Sarah's, Homer's, and Arlene's increased ability to perform math-empowering behavior during classroom interactions became evident in problem-solving sessions also.

Discussion

This study provides an in-depth picture of rule-based learners as they engaged in a PSIO learning environment which is highly consistent with the recommended reforms encompassed in the

NCTM standards. It shows that rule-based learners who encounter this learning environment will choose to ignore opportunities to think about relationships and concepts in-depth. The participants, choice to ignore in-depth conceptual development actively during instruction may have contributed to an absence of domain-specific knowledge required to perform problem-solving behaviors. According to Newell and Simmon (1973) and Glaser (1984), domain-specific knowledge forms the foundation from which successful problem-solving behaviors emanate. In 20 out of the 50 opportunities for problem solving in small-group sessions, the rule-based participants demonstrated that they lacked knowledge of definitions which were integral to the successful performance of problem-solving behaviors. The results suggest that teachers and students involved in this approach to learning need to become aware of their belief systems about understanding mathematics and the manner in which beliefs may interfere with the learning process. Instruction should be designed to heighten awareness of mathematical beliefs about understanding.

Furthermore, this study shows that rule-based learners use inappropriate cues based on emotion and external factors to decide when understanding is achieved rather than use reasoning about relationships. The results suggest that instruction should be structured toward helping students develop a consciousness of factors that lead to the use of effective learning strategies and relational understanding. Instruction could be designed to increase student reflection through journal writing about learning strategies and successful understanding.

The rule-based participants of this study appeared to remain uninvolved in their learning and became more passive towards the culmination of this study. The results suggest that students must be encouraged to engage in active learning as the NCTM standards have recommended. These students may need opportunities that are less threatening in which they could engage in mathematical communication. Individual writing assignments about mathematical relationships, journal writing, and computer exploration may afford these students this opportunity. An environment that encourages adolescents to take risks without fear of harassment from peers should be developed and at the same time rule-based students should be assisted in developing skills for dealing effectively with peer pressure.

According to the NCTM teaching standards (1991),

> The experiences that mathematics teachers have while learning mathematics have a powerful impact on the education they provide their students. . . . Through these experiences, they develop ideas about what it means to teach mathematics, beliefs about strategies and techniques for teaching particular topics. Those from whom they are learning are role models who contribute to an evolving vision of what mathematics is and how mathematics is learned.

In the final analysis, the results support the argument that mathematics educators and instructors must address a vision of teacher education and instruction that will engage teachers in experience, tasks, and, discourse that build mathematical power.

This study provides a different picture of the manner in which rule-based learners may embrace thinking and behavior of a relational nature. If students unconsciously emulate teacher behavior modeled during class, then it is imperative that preservice and inservice teachers become adept at modeling appropriate problem-solving behavior and teaching using a problem-solving approach to learning. Teaching that fosters mathematical thinking as called for in the NCTM standards must permeate every aspect of instruction if students are to reach the point where they are able to construct their own mathematical understanding.

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APPENDIX 1 DEFINITIONS

Definitions

For the purpose of this study the following explanation of terms is provided as a guide to enhance understanding of the interactions involved in this research.

Beliefs or belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. (p.45 Schoenfeld, 1985)

Conceptions or conceptual systems are complex organizations of beliefs, disbeliefs, and concepts in a given domain (Scheffler, 1965).

Empowering Mathematical Behaviors, according to the NCTM Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), are:

> . . . an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems. (p. 5)

From this definition the following behaviors are defined as mathematically empowering:

1. exploring problem solutions,
2. making conjectures about a situation or problem,
3. posing a problem,
4. making a generalization,
5. drawing a conclusion,

6. testing a hypothesis or conjecture,

7. arguing logically to prove a hypothesis,

8. comparing solution paths,

9. using a strategy or heuristic to solve a problem, in terms of a Polya approach to problem solving, such as looking for a pattern and working backwards.

10. checking for the reasonableness of a solution, and

11. looking back at a solution to find further generalizations.

Instrumental viewpoint, according to Skemp (1978), is a viewpoint in which students believe that doing mathematics is knowing what to do without knowing why. These students look for a rule to use in a situation and consider mathematics as a disjointed bunch of facts and rules to be memorized.

Relational viewpoint is defined by Skemp (1978), to be a viewpoint in which students believe that doing mathematics is knowing how and why things work in mathematical activities. These students are concerned with discovering relationships and structures that are inherent in mathematics.

Math-empowering and relational behavior is a phenomena that occurs during instruction when students who have been asked to perform a math-empowering behavior by the teacher, carry it out, and are then able to give an explanation of why a formula or concept involved is related to the problem-under consideration.

APPENDIX 2 TEACHER OBSERVATION CHECKLIST

ACTIVITIES:
During these activities the teacher:
(To be observed every 3 minutes on day 1)

________ 1. Uses problem explorations to introduce concepts and skills.

________ 2. Provides activities which encourages conjecturing and hypothesis formation.

________ 3. Provides activities which encourages students to draw conclusions.

________ 4. Provides activities which encourages students to test hypothesis during explorations.

________ 5. Challenges students with problem situations during concept and skill development.

6. Uses a variety of problems from
________ real world,
________ abstract, and
________ open-ended situations.
________ long term problems

________ 7. Provides activities in which students discuss their conclusions.

________ 8. Provides activities in which students and verify results of investigations with their peers.

________ 9. Provides opportunities to explain, conjecture and defend one's ideas orally and in writing using
________ journal writing
________ written reports
________ other written activity

________ 10. Provides activities in which students pose problems.

________ 11. Provides activities in which students formulate questions about real world situations.

________ 12. In large group work results of small group investigations are pooled and evaluated.

________ 13. Stresses the interrelatedness of mathematical topics and applications.

________ 14. Provides opportunities for students to model mathematical situations using concrete material.

________ 15. Provides opportunities for students to check the reasonableness of solutions to problems using estimation.

________ 16. Provides opportunities for students to look back at

solutions, make generalizations to extend these to new situations.

________ 17. Provides opportunities for students to review problem-solving control behaviors of planning, assessing, and evaluation.

Other Activities:

APPENDIX 2 CONTINUED

TEACHERS' ROLE (To be observed every 3 minuntes on day-2)

___ 1. Asks questions which actively engage students in the process of doing mathematics.
- ___process questions
- ___explanation of strategies
- ___examples of pattern
- ___extension of patterns
- ___comparison questions
- ___relational questions
- ___other questions

____2. Asks questions which encourage students to analyze their problem solving processes and give explanations Such as:
- ___ What if?
- ___ Why does that happen?
- ___ Can you give an example of this?
- ___ Can you find a counter example?
- ___ Do you see a pattern?
- ___ Is this always true?
- ___ Is this sometimes true?
- ___ Is this never true?
- ___ How do you know that?
- ___ other questions

___ 3. Acts as a facilitator and coach to students involved in the problem-solving process.

___ 4. Models the thinking processes involved in problem solving, especially planning, assessing, and evaluation.

____5. Models mathematical situations using concrete material.

Other behavior:

APPENDIX 2 CONTINUED

LEARNING ENVIRONMENT (To be observered every 3 minutes on day 3)

____ 1. Uses small groups to provide an environment where active participation in mathematics is encouraged through discussion.

____ 2. Provides concrete materials for students to model problem situations.

____ 3. Allows students to use calculators to free themselves of the drudgery of computation so they can concentrate on problem solving.

____ 4. Uses computer software to provide challenging problem solving situations.

ASSESSMENT (The teacher will be interviewed about assessment procedures if these are not in evidence during day 3.)

____ 1. Uses an assessment plan which provides for continuous feedback to students about all aspects of problem solving.
   ___ questioning techniques
   ___ use of information
   ___ conjecturing
   ___ use of problem solving strategies and techniques
   ___ verification of results
   ___ interpretation of results
   ___ correct solutions

____ 2. Uses an assessment based on evidence from multiple sources.
   ___ individual interviews
   ___ observation of group work
   ___ homework
   ___ tests
   ___ journals
   ___ class projects

____ 3. Reports of assessments to parents include results of problem solving activities and may be in the form of a student problem solving profile.

____ 4. Assessment of student problem solving behaviors are used to fine tune instruction on a continuing basis.

APPENDIX 2 CONTINUED

QUESTIONNAIRE FOR SUPERVISOR, TEACHER, AND DESIGNATED PEER TEACHER

Teacher____________________

Please rate the identified teacher using the following questionnaire. Circle a number from 5 to 1 for each item. 5=almost always demonstrated by the teacher 4=moderately demonstrated by the teacher 3=occasionally demonstrated by the teacher 2=rarely demonstrated by the teacher 1=never demonstrated by the teacher.

ACTIVITIES:

During these activities the teacher:

always				never	
5	4	3	2	1	1. Uses problem explorations to introduce concepts and skills.
5	4	3	2	1	2. Provides activities which encourages conjecturing and hypothesis formation.
5	4	3	2	1	3. Provides activities which encourages students to draw conclusions.
5	4	3	2	1	4. Provides activities which encourages students to test hypothesis during explorations.
5	4	3	2	1	5. Challenges students with problem situations during concept and skill development.
5	4	3	2	1	6. Uses a variety of problems from real world, abstract, open-ended situations and long term problems.
5	4	3	2	1	7. Provides activities in which students discuss their conclusions.
5	4	3	2	1	8. Provides activities in which students and verify results of investigations with their peers.
5	4	3	2	1	9. Provides opportunities to explain, conjecture and defend one's ideas orally and in writing using journal writing and written reports.
5	4	3	2	1	10. Provides activities in which students pose problems.

always				never	
5	4	3	2	1	11. Provides activities in which students formulate questions about real world situations.
5	4	3	2	1	12. In large group work results of small group investigations are pooled and evaluated.
5	4	3	2	1	13. Stresses the interrelatedness of mathematical topics and applications.
5	4	3	2	1	14. Provides opportunities for students to model mathematical situations using concrete material.
5	4	3	2	1	15. Provides opportunities for students to check the reasonableness of solutions to problems using estimation.
5	4	3	2	1	16. Provides opportunities for students to look back at solutions, make generalizations to extend these to new situations.
5	4	3	2	1	17. Provides opportunities for students to review problem-solving control behaviors of planning, assessing, and evaluation.
5	4	3	2	1	18. Asks questions which actively engage students in the process of doing mathematics. (Examples of question types are; process questions, explanation of strategies, examples of pattern, extension of patterns, comparison questions, and relational questions.
5	4	3	2	1	19. Asks questions which encourage students to analyze their problem solving processes and give explanations Such as: What if?, Why does that happen?, Can you give an example of this?, Can you find a counter example?, Do you see a pattern?, Is this always true?, Is this sometimes true?, Is this never true?, and How do you know that?.
5	4	3	2	1	20. Acts as a facilitator and coach to students involved in the problem-solving process.

always				never	
5	4	3	2	1	21. Models the thinking processes involved in problem solving, especially planning, assessing, and evaluation.
5	4	3	2	1	22. Models mathematical situations using concrete material.
5	4	3	2	1	23. Uses small groups to provide an environment where active participation in mathematics is encouraged through discussion.
5	4	3	2	1	24. Provides concrete materials for students to model problem situations.
5	4	3	2	1	25. Allows students to use calculators to free themselves of computation so they can concentrate on problem solving.
5	4	3	2	1	26. Uses computer software to provide challenging problem solving situations.
5	4	3	2	1	27. Uses an assessment plan which provides for continuous feedback to students about all aspects of problem solving. Such as; questioning techniques, use of information, conjecturing, use of problem solving strategies, verification of results, interpretation of results, and correct solutions.
5	4	3	2	1	28. Uses an assessment based on evidence from multiple sources. Such as; individual interviews, observation of group work, homework, tests, journals and class projects.
5	4	3	2	1	28. Reports of assessments to parents include results of problem solving activities and may be in the form of a student problem solving profile.
5	4	3	2	1	29. Assessment of student problem solving behaviors are used to fine tune instruction on a continuing basis.

APPENDIX 3 STUDENT SURVEY

Name______________________________

Part I. This survey asks you how you feel about mathematics or mathematics activities. There are no correct answers. The answer choices are "Strongly Disagree (SD)," "Disagree (D)," "Undecided (UD)," "Agree (AG)," or Strongly Agree (SA)." For each part, choose the one response that best described how you feel about the statement. Be sure to check the box for each question.

		SA	A	UD	D	SD
1.	Learning mathematics is mostly memorizing a set of facts and rules.	☐	☐	☐	☐	☐
2.	Learning mathematics is a creative activity that I have participated in my mathematics classes.	☐	☐	☐	☐	☐
3.	There is always a rule to follow in solving mathematics problems.	☐	☐	☐	☐	☐
4.	Learning mathematics is learning to do routine computations by following a set of rules.	☐	☐	☐	☐	☐
5.	Learning mathematics means exploring problems to discover patterns and make generalizations.	☐	☐	☐	☐	☐
6.	Learning mathematics is learning to follow a set of rules given by the teacher that are unrelated in any way	☐	☐	☐	☐	☐

APPENDIX 3 (Continued)

	SA	A	UD	D	SD
7. Learning mathematics is learning to follow a set of rules given by the teacher that are highly related to one another.	☐	☐	☐	☐	☐
8. Learning mathematics is learning a fixed set of rules that have not changed in years.	☐	☐	☐	☐	☐
9. No new ideas have been created in the field of mathematics in at least 100 years.	☐	☐	☐	☐	☐
10. Doing mathematics is a creative process like painting a picture.	☐	☐	☐	☐	☐
11. Doing mathematics is discovering for oneself how and why things are related to one another.	☐	☐	☐	☐	☐
12. When doing mathematics, it is only important to know how to do a process.	☐	☐	☐	☐	☐