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ABSTRACT

The relationship between teachers' and students' personal constructs regarding intrinsic motivation in the mathematics class were examined in this study. Participants were six middle-school mathematics teachers and 30 students from 5 classes. Videotape, direct observation, individual interviews, and repertory grid tasks focused on the ways in which teachers attempted to build their students' motivations into their lessons, and the belief systems of teachers vs. students. Recent mathematics activities for each class served as elements for construct elicitation in the repertory grid task. Teachers and students were presented with random pairs of activities and were asked to determine what made one activity more fun than the other. Responses (constructs) were entered into a computer program that paired each activity with each construct and asked participants to rate how well each construct described each activity. Results revealed that the individuals studied whether teachers or students were similar in their construct systems. Despite the similarities, the differences that were apparent seem to be problematic to the extent to which teachers can anticipate the motivation of their students. Teachers did pay attention to motivating their students in developing their lesson plans, but the ways in which they attempted to build motivating exercises seemed to be more dependent upon the teachers' personal conceptions of intrinsic motivation than their beliefs about their students. Most of the studied teachers had little notion of the motivational beliefs of their students. Teachers' and students' cognitive organization of construct supported the model proposed by Middleton, Littlefield, and Lehrer (1992). Both students and teachers tended to stress the interrelationship between arousal and control levels in determining the intrinsic motivation of mathematics activities. Results are examined in relation to the need to inform teachers regarding the dynamics of student motivation, and to pay particular attention to the individual differences in students' motivational beliefs. In general, results indicate that when teachers are able to predict their students' beliefs, they are better able to fine tune their instruction to turn kids on to mathematics. (Contains 24 references.) (Author)

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Teachers' vs. Students' Beliefs Regarding Intrinsic Motivation
In the Mathematics Classroom: A Personal Constructs Approach

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Abstract

The relationship between teachers' and students' personal constructs regarding intrinsic motivation in the mathematics class were examined. Participants were six middle-school mathematics teachers and 30 students: Three highly motivated students and three lower motivated students from five classes.

Videotape, direct observation, individual interviews, and repertory grid tasks focused on the ways in which teachers attempted to build their students' motivations into their lessons, and the belief systems of teachers vs. students. Recent mathematics activities for each class served as elements for construct elicitation in the repertory grid task. Teachers and students were presented with random pairs of activities and were asked to determine what made one activity more fun than the other. Responses (constructs) were entered into a computer program that paired each activity with each construct and asked participants to rate how well each construct described each activity.

Results revealed that the individuals studied whether teachers or students, were similar in their constructs systems. Despite the similarities, the differences that were apparent seem to be problematic to the extent to which teachers can anticipate the motivation of their students. Teachers did pay attention to motivating their students in developing their lesson plans, but the ways in which they attempted to build motivating exercises seem to be more dependent upon teachers' personal conceptions of intrinsic motivation rather than their beliefs about their students. Most of the studied teachers had little notion of the motivational beliefs of their students.

Teachers' and students' cognitive organization of constructs supported the model proposed by Middleton, Littlefield, & Lehrer (1992). Both students and teachers tended to stress the interrelationship between arousal and control levels in determining the intrinsic motivation of mathematics activities. Results are examined in relation to the need to inform teachers regarding the dynamics of student motivation, and to pay particular attention to the individual differences in students' motivational beliefs. In general, results indicate that when teachers are able to predict their students' beliefs, they are better able to fine tune their instruction to turn kids on to mathematics.

Teachers' vs. Students' Beliefs Regarding Intrinsic Motivation In the Mathematics Classroom: A Personal Constructs Approach

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Research has shown that when children are motivated intrinsically to perform an academic activity, they spend more time engaged in the activity, learn better, and enjoy the activity more than when they are motivated extrinsically (Lepper, 1988). Clearly, getting children to engage in learning "for its own sake" is a primary goal for educators in that not only will children learn better at the immediate task, they will also tend to seek out similar activities in the future. Thus, designing intrinsically motivating activities is of paramount importance for reaching the goal of developing life-long learners.

It is unclear, however, just why children prefer intrinsically motivating activities. Is it because there is something "intrinsic" to the activity that induces them to engage, or might it be that the activity affords certain characteristics that fits the individual's notion of motivation. The distinction between these two cases may seem trivial at first. Does it really matter to the educator if the motivation stems from the activity or from the child? In the case of mathematics, it seems that yes, it really does matter. Descriptive studies have shown that some children enjoy mathematics, seek out mathematical problem situations, and excel, while others ("math anxious" students) have a real fear of mathematics, and thus avoid engaging in mathematical problem situations, and ultimately fail (Hoyles, 1981; Widmer, 1980). In addition, the utility and importance of mathematics is at least acknowledged by students even if not understood completely, yet this knowledge is not sufficient to motivate them to continue taking mathematics courses (Dossey, Mullis, Lindquist, and Chambers, 1988). If "mathematics" is intrinsically motivating to some students, but not for others, it seems unlikely that there is any factor inherent to mathematics that is motivating to all children. It is reasonable to assume then, that individual differences among students, and the ways in which mathematics education compliments these differences determines to a large extent the degree to which mathematics is motivating.

In an earlier study, myself and colleagues (Middleton, 1990; Middleton, Littlefield, & Lehrer, 1992) studied gifted 4th and 5th graders' personal constructs regarding fun in academics. This work was an attempt to test a theory of how academic activities come to be regarded as fun. We hypothesized that children's conceptions of academic intrinsic motivation would tend to be organized into three constructs, reflecting their interests, the degree of personal control afforded by an activity, and the degree to which the arousal afforded by the activity was optimal for engagement.

Cluster analysis revealed that children tended to organize their constructs into three general categories: Arousal, or the cognitive stimulation afforded by an activity; Personal control, the degree to which the activity was considered a free choice or of appropriate difficulty; and a loosely defined construct the we termed Interests, pertaining to the degree to which the students liked the activity, the importance of the activity, and their ability at performing the activity.

We also found that students, and girls in particular, seemed to identify with their teacher in evaluating the motivational value of academic tasks. In addition, we found that children tended to rate mathematics as less fun as they progressed from elementary to junior high school.

From the results of the study, a model of academic intrinsic motivation was developed. We suggested that children construct representations of the motivational value of academic activities, and use these representations to evaluate whether it is worthwhile to engage in the activity for its own sake (See Figure 1).

First, given the possibility of engaging in an activity, the person must ascertain whether or not they have classified the task as fun[1]--intrinsically motivating earlier, i.e., they will attempt to ascertain its "fit" with their constructed interests. If an "interest" match is found, the label of "Fun" is placed on the activity, and the individual can engage without further evaluation. If a "not an interest" match is found, the label of "Not Fun" is placed on the activity and the person will exit the system.

If the activity does not match any of the entries in the interests construct, it must be evaluated on the perceived degree of arousal and the perceived degree of control it affords the individual (i.e., how meaningful success in the activity will be). In general, if an individual has never entered into an academic activity, she will tend to evaluate the stimulation (challenge, curiosity, fantasy) it provides, and the personal control (free choice, not too difficult) it affords. If both domains are perceived to be adequate, then she will place a tentative label of "fun" on the activity and enter into it. While engaged, the individual constantly monitors arousal and control. If either condition becomes insufficient (or too much as in the case of so much control the task becomes boring), she will attempt to disengage from the task, unless some extrinsic motivator (grades, coercion) influences her to continue.

If, however, arousal and control conditions are met consistently, the individual may choose to place the activity into her interests. Then, if she gets the opportunity to enter into the activity in the future, she need not evaluate it. She merely has to check for a "match" in her interests construct, and engage, assuming the activity will be fun.

So far, I have discussed the processes by which a mathematics activity can become classified as an interest. Unfortunately, our current knowledge indicates that children seem to *remove* mathematics activities from their interests as they grow older. What processes might affect this declassification? Although untested as yet, with a few modifications, our model may also explain this tendency. McCombs and Marzano (1990) present a model of student task-evaluation similar to our own. They postulate that the individual evaluates activities based on their relevance to the self-system and the degree of threat the activity presents. If the activity is perceived to afford low

[1]Throughout this paper, I will use the terms "Intrinsic Motivation" and "Fun" interchangeably. The colloquial term "Fun" is better understood by students and teachers (and researchers), and carries connotations of positive affect that "Intrinsic Motivation" may not.

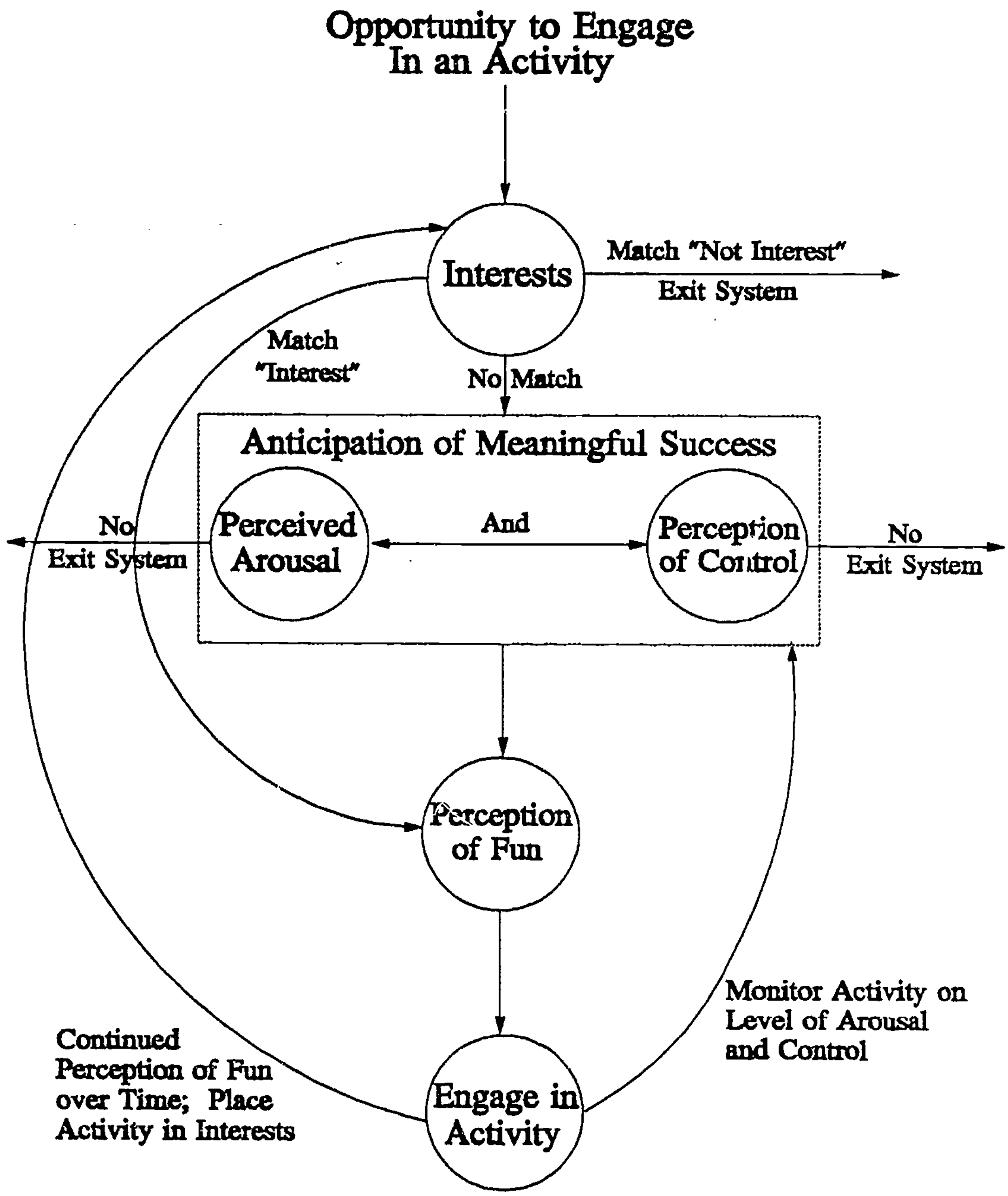

Figure 1. A Model of Academic Intrinsic Motivation

relevance (i.e., is unimportant), and presents a high threat to the self, then they will engage in compensatory activities to maintain the self-system (i.e, will have little motivation to engage in the task).

Under our framework, the arousal/control interaction optimizes the task relevance by providing sufficient challenge and personal control. Using Kelly's (1955) Choice corollary[2] as the agent that provides some synthesis of ideas, activities that are perceived as motivating will be perceived as being relevant to the extension of the individual's construct system (i.e., they provide stimulation), and will present little threat because they are under the control of the individual. Conversely, activities that are not perceived as motivating will provide little relevant information (little stimulation), and/or will not be controllable by the individual. Thus, when mathematics is originally included as a student's interest (i.e., when the student anticipates that mathematics activities will be fun), and when the student is presented with activities that provide little stimulation or control, the strength of activation of "mathematics" as an interest will be decreased with each successive activity.

The preponderance of students' recollections of bad experiences regarding school mathematics (e.g., Hoyles, 1981) explains in part why students tend to decrease in their liking of mathematics as they get older, and why enrollment in higher level mathematics courses has declined. Students do not see mathematics as being integral to their academic self-concept, and they try to avoid the anxiety caused from involvement in mathematical tasks. In addition, since students tend to attribute causal relationships between their motivational attitudes and their teachers' attitudes, it seems likely that some degree of mismatch between students' motivational constructs and their teachers' constructs accounts for this perception.

The three domains contributing to students' perceived intrinsic motivation have turned up elsewhere in the literature on motivation. Hidi (1990) for example, discusses the positive effects of individual interests on motivation and learning within the interest domain. However, she rejects the incorporation of individual interests into the classroom milieu as being impractical, especially when the teacher-student ratio is high. Instead, she focuses on the "interestingness" of academic tasks: The degree to which a task elicits curiosity and attention. Hidi suggests that this task-specific interest is based on the degree of arousal generated by presentation of the task. Her description of task-specific interest is echoed by other researchers. Malone's (1980, 1981) notion of the arousal producing conditions of challenge, fantasy and curiosity contributing to intrinsic motivation is highly similar. Likewise, Anderson, Shirey, Wilson, and Fielding (1987) suggest that four attributes contribute to interest: Novelty, activity level, character identification, and life themes. Like the model proposed in Figure 1, these attributes focus on arousal (stimulation) and control level. In our framework, Hidi's task-specific interest is elicited by the student's evaluation of arousal and control levels, and individual interests is conceptualized as developing from consistent patterns of task-interest.

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[2]Kelly's Choice corollary maintains that, given a choice between two alternative constructs, an individual will choose that construct that serves to extend the utility of the system, and will reject the construct that does not extend the system, or that serves to undermine the system.

Since the perception of being intrinsically motivated is determined by the individual, it seems reasonable to assume that individuals with differing "Interests" define motivating activities in different ways. This seems especially true for students vs. teachers. The focus of this paper is to discover both teachers' and students' beliefs about what makes the study of mathematics motivating intrinsically such that similarities and differences can be distinguished. The degree to which a student's conception of intrinsic motivation is similar to the teachers' conception is hypothesized to contribute to the intrinsic motivation exhibited by the student in the mathematics classroom.

Method

Participants

Participating teachers were volunteers, solicited from five middle-school mathematics classes (one 6th grade, two 7th grade and two 8th grade) in a rural Wisconsin school district. Each teacher was paid a small honorarium of $30 for their participation to be used for purchasing classroom materials for the classes studied. Two of the teachers taught one of the studied classes in a team fashion.

Teachers were asked to rank order their students based on their perceptions of their students' motivation to learn mathematics. Six students, three ranking highest in perceived motivation, and three ranking lowest, were selected from each class. The two groups of students were matched on ability within each class to eliminate stable ability biases. Thus, the total number of participants was 36: One teacher and six students from each class (plus one extra team-teacher). Eight students and one teacher did not complete all of the data collection tasks, so their data will be presented in its incomplete form where appropriate.

Procedure

Classroom Observation and Videotaping. Prior to individual data collection, each teacher's class was observed and videotaped for one 43-minute period. Each visit was scheduled such that the lesson observed would be what the teacher considered "typical" of their mathematics class. During the lesson, the experimenter made field notes describing the classroom atmosphere, the content covered, the overall motivation of the class, and the teachers' attempts to motivate their students.

Initial Teacher Interviews. Each teacher was interviewed individually. As stated above, teachers were asked to rank order their students based on their perceptions of the students' motivation to learn mathematics. This procedure was chosen deliberately to build in the bias of the teachers to the study. Teachers were asked whether these students identified were different in mathematical ability, and students with widely disparate abilities were eliminated, and other students substituted such that the final pool of students in each class were matched fairly closely on ability. Gender, however has been shown to be an important differentiating factor in the motivation literature (see Fennema & Peterson, 1985), so no attempt was made to match students on gender across groups. The number of males and females did differ for the two groups, with males representing a significantly higher proportion in the lower motivated group (10 males versus 5 females), while females represented a higher proportion in the highly motivated group (9 females and 6 males).

Following the ranking procedure, teachers were asked to list the ten mathematics activities they covered for the previous month of instruction they felt were the most important, including the videotaped lesson. Some teachers who worked on several larger projects elected to list activities they performed earlier in the semester to provide a significant number of activities for analysis. After the lessons were articulated, each teacher was asked the following questions:

1. *Please describe your lesson plans you made for teaching the videotaped lesson.*

The teachers were presented with the videotaped lesson and were asked to describe the process they went through as they developed their plan for teaching the lesson. Teachers' plans were tape-recorded, and paper was provided for teachers to organize their thoughts.

2. *Please rank the following considerations in the order of the importance you placed on them when you developed your plan.*
    A. *Mathematical Content.*
    B. *Student Abilities.*
    C. *Student Motivation.*
    D. *Method of Instruction.*
    E. *Manipulatives/Visual Aids.*

This question served as a springboard for the following questions. The primary interest here was not on the rank order, per se, but to get teachers to think about motivation in relation to their other lesson-planning goals.

3. *I notice you have ranked student motivation number [number here]. Can you tell me why you placed this relative importance upon it?*

Question number 3 was developed in order to allow teachers to explicate their beliefs regarding the importance of motivation in the process of instruction. It was hoped that responses to question number 2 would illuminate the general process teachers go through in developing their lesson plans.

4. *How did you account for the motivational characteristics of your identified students [names here] in developing your plan?*

Question 4 was aimed at getting the teachers to focus on their highly motivated students and their less motivated students, and to describe how their plans might differ for the two groups. This was also used in conjunction with teachers' ratings on their students' repertory grids (to be described later) to determine how teachers' beliefs regarding their students' motivation are translated into action.

5. *How do you normally account for the motivation of your students in your classroom?*
    A. *How do you account for your students' interests?*
    B. *How do you stimulate your students in mathematics?*
    C. *What control do you give your students in your instruction?*
        i. *Choice of activities?*
        ii. *Tailoring activity requirements to ability?*

Question 5 was aimed at gathering a more broad description of the lesson planning strategies of the teachers. It may be that planning for the videotaped lesson is quite different from planning for lessons in general. This information should illuminate difficulties teachers have in motivating large numbers of their students. In addition, it was designed to direct teachers' attention to aspects of intrinsic motivation relevant to the model of academic intrinsic motivation presented in the introduction (Figure 1).

6. *How do you define motivation in mathematics?*

Question 6 was designed to uncover teachers' overall views towards what motivation is, and to determine what factors contribute to their ratings of their students' motivation. In addition, this question was designed to determine if aspects of intrinsic motivation are considered important to teachers' definitions of mathematics motivation.

7. *What do you know about student motivation?*

This question was asked to garner information regarding the level of knowledge teachers have gleaned from their coursework, inservice experience, or outside reading pertaining to motivating students in the classroom.

8. *Have you had any formal training in motivation?*

Going beyond informal knowledge, question 8 is intended to get an idea of the level of formal training teachers may have had in their teacher education courses.

9. *Additional comments regarding the videotaped lesson, and/or teachers' philosophy of education.*

Question 9 was designed to put teachers' beliefs and knowledge in perspective with their overall orientation towards education, and especially their role in educating students.

Student Construct Elicitation and Ratings. Procedures were adapted from those outlined by Kelly (1955). Students worked individually on both the construct elicitation task and the construct ratings task. Student were presented with each of the ten activities provided earlier by their teachers to anchor students' constructs in the instructional sequence, and to provide a basis of comparison to teachers' constructs. Dyads of activities were presented at random on a written form, and the students were asked, "What makes the first activity more fun than the second?" The number of dyads varied for each class due to the number of important activities the teachers generated. To eliminate fatigue and frustration, the number of dyads presented was limited to 20.

Students' responses to the dyad presentations (constructs) were entered into a computer program that randomly paired each of their constructs with the activities proffered by their teachers (see Lehrer & Guckenberg, 1988 for a more detailed explanation of the computer program). All possible combinations of constructs and activities were rated. The program presented the students with each construct and asked them to rate on a scale of 1 to 5 (1 = lowest rating, 5 = highest rating) how well each construct described the activity. For example, if problem-solving was one of the ten activities engaged in over the past month, and if that student also indicated that one of her constructs that defined fun was "I am good at it," she would be asked to rate how well the construct "I am good at it" describes "Problem-solving." To reduce boredom, the students were allowed to quit the ratings task if they liked, and return to it at a later time. Ratings were automatically entered into a data matrix file.

Teacher Construct Ratings. The procedure for eliciting teachers' personal constructs followed the same format as that of the students. Teachers were presented with the same written elicitation form as their students. Teachers' responses to the elicitation (constructs) were entered into the computer program described above, and teachers were asked to rate on the 1 to 5 scale how well each of their constructs described each activity.

Following their own personal constructs ratings, teachers were given copies of each of their students' unrated computer programs. Teachers were asked to rate each construct/activity pair presented as they perceived the student would.

Analyses

An additive clustering technique (e.g., Sattath & Tversky, 1977; Lehrer, 1988) was applied to the ratings matrices. The distance metric used compares the proximities between constructs by taking into account all other constructs in the analysis (Lehrer, 1988). Constructs that are simultaneously close to each other but distant from other constructs are considered most proximal. This comparative distance has distinct advantages over traditional distance metrics in that it performs fewer transformations on the original data. For example, most methods use an average distance metric for determining cluster membership. By using average distance, the clustering algorithm would be unable to distinguish between two extreme ratings and two middle ratings with the same average value. Further, many metrics force original data which may be asymmetrical into a symmetric distribution. Comparative distances are more consistent to the original data (and therefore to the cognitive representation that generated that data) in that they do not use averaging techniques to create the distance metrics.

The additive algorithm was chosen to extract clusters of constructs because it is less restrictive than hierarchical or complete clustering algorithms. Additive techniques allow for constructs to be included in more than one cluster, which seems to be more reflective of human thinking.

For example, in the proposed model of intrinsic motivation, perceived arousal and perceived control are highly interrelated. A construct such as "It is challenging" may be seen by a child as providing an optimal level of arousal (neither too high nor too low). It may, however, be seen concurrently as providing a facilitative degree of control over the activity by not being too difficult to prevent success. To detect this relationship between domains in a cluster analysis, the algorithm must be capable of placing the construct into more than one cluster.

Proximities between constructs are displayed using an additive tree graphical representation. Such a representation provides a visual indication of the hierarchical structure of students and teachers' representations of intrinsic motivation in mathematics. By examining both the type of constructs elicited, and their organization, similarities and differences between students' and teachers' beliefs can be explicated.

Teachers' ratings of their students' repertory grids were also subjected to the additive clustering. By examining the structure of the teachers' ratings vs. the students' ratings, one can determine the degree of similarity between students' motivations and teachers' beliefs about students' motivations both descriptively (i.e., how similar are clusters for teachers vs. students?), and empirically (through correlational analysis of ratings).

Pearson correlations were computed by pairing cells in the teachers' predictive grids with cells in each of the students' grids. It was predicted that the correlation between high motivated students' and teachers' ratings would be significantly higher than correlations between low motivated students' and teachers' ratings.

Teachers' responses to the interview questions were transcribed and analyzed with reference to their own construct organizations, students' construct organizations, and the teachers' ratings of their students' constructs. It was hypothesized that teachers' constructs regarding intrinsic motivation in their mathematics classes would be congruent with the types of activities teachers design. For example, if teachers place a heavy emphasis on stimulation, then they should design activities that provide novelty, curiosity and fantasy. However, research is divided upon whether teachers' plans will reflect any of their beliefs regarding their students' motivations. Clark and Peterson (1986) for example, reviewed ten research studies regarding teachers' plans. They reported that teachers focus primarily on content goals, and secondarily on learning objectives. Very few studies reported teachers focusing on student motivation.

It may be that teachers do not place a heavy degree of emphasis on student motivation in making their instructional plans. If this is true, then evidence of congruence between students' constructs regarding fun in mathematics and their teachers' constructs could be one possible determinant of student motivation--i.e., if a teacher does not take their students' motivation into account, then the more similar their *constructs* regarding the motivational value of mathematics are to their students', the easier it will be for the teacher to motivate their students.

If teachers' do place a heavy degree of emphasis on student motivation in their lesson planning, then congruence between their *planning strategies*, and their students' constructs could be considered evidence of the motivational facility of their plans.

Results

The results of the study are problematic with respect to reporting in a concise fashion. Each teacher's classroom represents a separate case, independent of the other teachers'. Thus, a detailed report of each case is not feasible for the scope of this paper. However, commonalities did become apparent across cases, and these commonalities form a principled set of conclusions that serve to inform theory, and that suggest ways in which teachers can begin to raise the motivational level of their mathematics classes. I will describe a single case--Mrs. Morris' classroom as an example of the richness of data uncovered in this study, and to provide a flavor of just how teachers' and students' beliefs play out in the mathematics classroom. Following this description, I provide a summary of the results, pointing out commonalities among the five cases, and highlighting some of the important differences. Now, let's take a look at Mrs. Morris' class.

Classroom Observation and Videotape

Mrs. Morris' Classroom.

The following excerpts from the experimenter's field notes describe the overall structure and conduct of Mrs. Morris' classroom.

Mrs. Morris is an experienced mathematics teacher. She originally taught high school mathematics for four years, then took ten years off. After her hiatus, she went back to teaching, this time in the middle grades. She has been teaching 7th grade mathematics and science for three years.

Her classroom can best be described as a blend of order and disorder. The desks are in neat rows; special mathematics and science problems[3] and student work are displayed prominently on the walls, ceiling, and any available counter space. Motivational posters were strategically placed by the students' work. Examples of these include: Math is Far Out! Do you have a direction, a plan, a goal? Or, are you just taking any path? Dream, Prepare, Apply. Weigh your options, learn math. In addition, Mrs. Morris has an IBM computer station at the back of the room with three networked personal computers. One-half hour *prior* to class, these stations were full of kids working on mathematics and science reports.

The class was organized according to four stations where children worked in small groups of approximately 6 students: The Computers, a Problem Solving Station, an Activity Station, and one for Bookwork. Mrs Morris rotates children from station to station each day so that each child has the opportunity to work at each station by the end of the fourth day of the unit. On the fifth day of each unit, children take a quiz on the material covered. At the start of the period (the first in the day), after the announcements and role-taking, Mrs. Morris very specifically outlined the objectives of the unit, and reminded the students where they were with respect to stations. Students rushed to their stations with some alacrity, and very quickly got on-task. Students stayed on-task for most of the period, with some intermittent rambunctious play.

During their work, students' conversations were mostly on the task at hand. Students were encouraged to help each other as far as checking answers and explaining problems. Some children did elect to work alone...for a while, but the structure of the problems (which will be addressed later on) influenced most to seek out their classmates.

At the end of the period, Mrs. Morris assigned a journal entry. Children were asked "What is still confusing that you need to figure out Monday?" Five minutes was set aside for writing answers. The last five minutes of the period were filled with more announcements of an upcoming field trip, free talk, busywork, and finishing up the journal entries.

Teacher Interview

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[3]An example of an extra credit mathematics problem: If you slice off each corner of a block of cheese, how many faces, edges, and vertices will the new figure have?

Question 1. *Please describe your lesson plans you made for teaching the videotaped lesson.*

Mrs. Morris plans start with the mathematical content. She first determines two to three concepts she can cover in four days, and then goes to the book and assigns pages that coincide with those concepts for the Bookwork station, including worksheets and book assignments. She then develops activities that involve physical manipulation of some object pertaining to the content, and assigns them to the Activity station. Since the observed lesson was on symmetry and space, examples of activities include designing and constructing their own 3-dimensional figures and determining their line symmetry. Students were free to choose any common geometrical form, or they could develop their own shapes. The Problem Solving station was structured such that students needed to go above and beyond the content objectives. For example, students were given paper figures, and were asked to determine the line symmetry of the figures, ranging from stars to circles, etc. Mrs. Morris reserves the Computer station as a review station. Students on the computers were presented exemplars and non-exemplars of geometric figures to define features of circles: Chords, radii, diameter, etc. The computer program allowed several exploratory tries before giving a rule.

When probed to explain why she chose to divide the class up into four stations, Mrs. Morris replied that she chose the stations as a method of incorporating the computers in the classroom as a motivating factor. She would rather have had three stations, but the small number of computers necessitated creating another station so that the maximum number of students at any computer would be two to avoid chaos at the back of the room.

Question 2. *Please rank the following considerations in the order of the importance you placed on them when you developed your plan.*

A. Mathematical Content.
B. Student Abilities.
C. Student Motivation.
D. Method of Instruction.
E. Manipulatives/Visual Aids.

Mrs. Morris ranked student motivation number four, behind mathematical content, manipulatives, and student abilities. Her reasons for its relative placement reveal that she pays specific attention to student's abilities and appropriate manipulatives/visual aids for those ability levels. She feels that appropriate challenge motivates her students of diverse ability levels. These responses are further illuminative with respect to the third question.

Question 3. *How did you account for the motivational characteristics of your identified students in developing your plan?*

Mrs. Morris feels that she doesn't have to worry about her highly motivated students. She stated, "I can tell them to do the book for the whole day, and they do it." So, she concentrates more on her less motivated students in developing her plans. She tries to think of activities that will keep them on task such as extensive work with manipulatives on more concrete tasks.

As stated before, one of the reasons for creating the stations was to help motivate kids by placing them on the computers. However, Mrs. Morris discovered other benefits of her class structure.

I find that when we do whole group activities, and incorporate a lot of book work, they (the high group) get bored too, because they are beyond what the average one is doing. So, doing the stations has a real tendency to be able to go beyond and do more enrichment activities...In the Problem-Solving station, I try to make that my enrichment station. So, it frustrates my middle-level kids But...each group has a higher-level person in that, and the higher-level person will usually take it and continue with it. The other ones, they'll do it while the person (high level) is there to help them with it and then usually they'll terminate.

As we delved further into the difficulty of taking the individual differences of 30 kids into account when designing instruction, Mrs. Morris alluded to an important theme in her conception of student motivation in mathematics: Expectations.

Question 4. *How do you normally account for the motivation of your students in your classroom?*

Before I went to this type of concept with the stations, I used to do a problem-of-the-day which seemed to be a motivator to get them into it and get them started...But it ended up getting real routine. I lost my good kids faster. They would get done, and then they would get absolutely bored. The ones that are not motivated are struggling a little bit now...they're kind of bottomed out now, but they are starting to come back because they know what the expectations are.

When asked to elaborate on this further, Mrs. Morris commented on students' initial confusion when she initiated the stations. Because students did not know what she expected at the end of a unit, they "were kind of like fish out of water." She then began laying out her expectations explicitly at the beginning of each class period.

...And that they knew what was expected so that when the quiz came up, those things would be on it, they should know them. And it seems to be helping out. They seem to know where to focus their attention, and those that want to go beyond can. But they still get the core activity.

So, she attempts to keep students on-task with respect to content and activities by making them aware of the expectations before-hand, and then she refocuses at the end of the day to discover what students still do not understand so that groups can concentrate directly on the difficult areas.

Question 5. *How do you account for your students' interests when you develop your plans?*

Mrs. Morris' attention to ability differentials becomes evident in her responses to this question. Because her students are ability grouped, she focuses on activities that are a bit harder for the more able students, activities that are concrete and more basic for the less able, and a combination of the two for the average students. As will become clear later, this attention to challenge is integral to Mrs. Morris' personal conception of what makes mathematics motivating.

Questions 6 and 7. *How do you stimulate your students in mathematics; and What control do you give your students in mathematics.*

When Mrs. Morris was asked how she stimulates her mathematics students, after an initial flippant remark about Fridays bring a primary stimulant, she immediately referred to the computers:

They (the students) can make something work, and it (the computer) doesn't really judge them. They can go back and redo it. It takes them step by step, and I think it makes them feel pretty successful.

In addition to Mrs. Morris' attention to the matching of ability with task requirements, she tries to provide some range of activities that students can choose from...when they finish the core requirements:

> I would say though, that the only time they focus on math (in the free choice activities) is when they are on the computers. If they are at the other stations, I don't see them going beyond and trying something different.
>
> I think the activities have to be varied which I think the computer does. I mean it can vary, you can turn it on and off, or do whatever to switch. And they have control. That's one nice thing about the kids. They have control over that. You can direct them, but they end up taking charge. And I think it *is* a motivator for children. When they feel they are taking charge of something, and they are in control

As stated before, Mrs. Morris initially perceived the computer station as a motivator for her class, and the students' reactions confirmed those initial perceptions. So, her beliefs in their value for her class has strengthened over time.

Question 8. *How do you define motivation in mathematics.*

Mrs. Morris has strong beliefs that students' home environment is the major determinant of their motivation in her mathematics classroom: Some adult that has high expectations for them to succeed. Without that home influence, Mrs. Morris feels that is nearly impossible to reach all of her students.

> Well, I think that motivation comes from within. Activities that we do to stimulate them to do things, that's outside. But, wanting to achieve, wanting to do better, that comes from the inside, and I think that comes from whoever the home people are for that student. I think we are fooling ourselves in education if we think we can replace that. Some of us are able to key in on one or two kids, but other than that, unless the class sizes get down to where you can get one-to-one, and have these expectations for each one where they feel individual, and they feel special, I don't know if the motivation is there.

When asked if she could relate any instance where she saw a child change their motivational pattern, Mrs. Morris related examples of two children she had during that school year: "Ones who decided to check out." But as she initiated contact with the students' parents, and mutual expectations were set, consistent with her beliefs about the influence of the home, the students started to become more on-task.

> They have the desire to finish and get it done...basically when severe consequences are involved. You either have positive ones or negative ones. Usually, they go for the positive ones.

Question 9. *What do you know about student motivation? Have you had any courses or formal training in motivation?*

Mrs. Morris has had no formal training in motivation outside her preservice education classes.

Question 10. *Additional comments, teachers' philosophy of education.*

Mrs. Morris' philosophy of education reflects her belief that she should be an example for the students, not only academically, but in other areas of life as well.

> I consider myself...I need to be a role model for them in all areas. I try to show them that I enjoy math. When we try to solve some problems, or if I come up with a wrong answer, we don't have a coronary because it wasn't right on. Or we try to figure out where we went wrong. Some of them have a tendency to think that when you get a wrong answer then its too late. To be able to go back and redo is important. But also socially. I mean I want to set the example for them. I'm the type of person I think they should shoot for. I'm not perfect, that's for sure! But I certainly would hope that out in town...that if they saw me that they would be happy that I'm their teacher.

Mrs. Morris' Construct Organization

Figure 2 illustrates the organization of Mrs. Morris' personal constructs regarding intrinsic motivation in her mathematics classroom. As the three distinct branches of the tree diagram illustrate, three clusters of constructs emerged from the additive analysis. From the constructs' proximities to each other, it seems that Mrs. Morris, in concert with her beliefs regarding ability and task difficulty alluded to in the interview, organizes her beliefs with respect to these themes. For example, "It has a high skill level," and "It takes more thought" deal with high task difficulty, and constitute one of the three construct clusters in the diagram. "It is more tedious than other topics" deals with low task difficulty or monotony, and was organized closely to "It is too much eye work," another aspect of tedium. Both of these constructs were related to "It saves me mega time," referring to a tasks efficiency at reducing tedium.

Interestingly, "It is fun" was organized closely to "It is more hands on than other topics," "It uses higher-thinking skills," and "It forms links between concepts being developed." From the cluster analysis, and Mrs. Morris' interview, it seems likely that this upper cluster on the tree diagram represents a superordinate construct that is central to her beliefs system. Thus, it would appear that high task difficulty, coupled with the ability to manipulate objects and form associations between knowledge in a higher-order, conceptual fashion forms the basis for her intrinsic valuation of mathematics.

Mrs. Morris' Students' Construct Organizations

Highly Motivated Students. Upon examination, it appears that the highly motivated students in Mrs. Morris' class generally organize their motivational constructs similarly to their teacher (See Figures 3, 4, and 5). All three of the identified students, Andrew, Carlos and Bob, proffered constructs that refer to task difficulty and challenge, and learning and understanding mathematics better as central to their conception of what made mathematical activities fun. In addition, Carlos, the most highly motivated student in the class explicitly referred to the hands-on nature of mathematics activities as contributing to their motivational value.

Each of these constructs was central to the teachers' beliefs, and were central to how the teacher attempts to build motivation into her mathematics classroom. In addition, the highly motivated students' constructs reflect the model of intrinsic motivation presented earlier: The students' constructs revolved around themes of arousal, control, and interest.

Carlos' construct organization seems to be organized around two major clusters. The first cluster (the top cluster in figure 3) contains constructs most closely related to "It is fun." These constructs pertain to the interestingness of the topics, their aid to Carlos' learning of mathematics, and their hands-on nature. The second cluster contains constructs pertaining to aspects of arousal and control such as "It is easier to learn (than other topics)," "It is more hard (than other topics)," and "You can control what is going on."

Bob also seems to have two major clusters of constructs that define intrinsic motivation in his mathematics

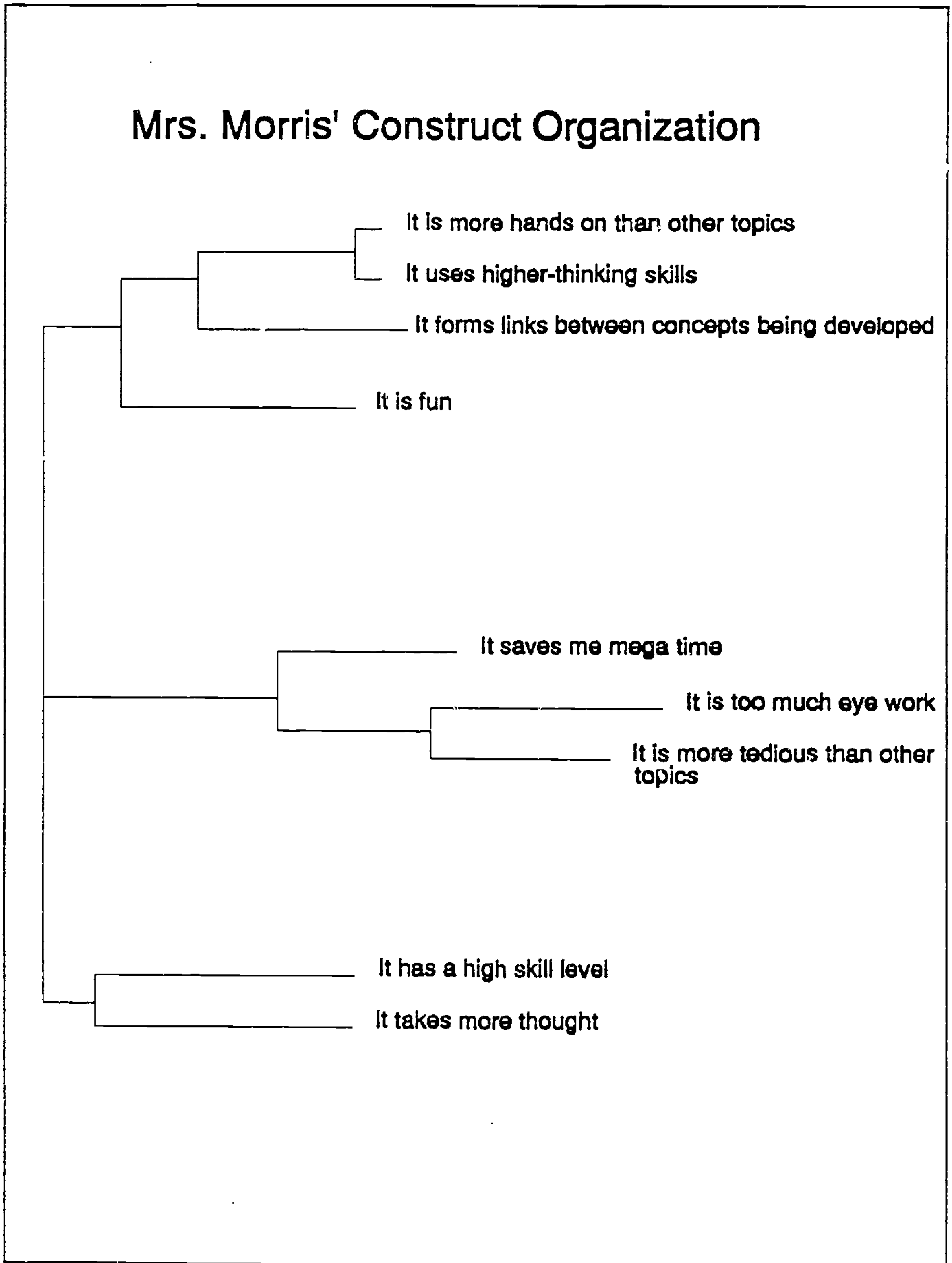

Mrs. Morris' Construct Organization
It is more hands on than other topics
It uses higher-thinking skills
It forms links between concepts being developed
It is fun
It saves me mega time
It is too much eye work
It is more tedious than other topics
It has a high skill level
It takes more thought

Figure 2

experiences. The constructs organized most closely to "It is fun" pertain to task ease and being able to learn faster. Interestingly, "It takes too long" was organized in this cluster. It is unclear exactly what this means at this time, however, one possibility is that it represents the negative pole of the construct "It helps you learn faster," and thus was rated similar due to being part of the same cognitive dialectic.

Andrew's construct organization is more difficult to describe. He has one clearly defined cluster of construct that seems to deal with challenge, understanding, and enjoyment. The three other constructs that were rated, "It is fun," "It is rather difficult," and "It is different (than what we usually do)," were also rated similarly to each other, but were not included in the same construct in the additive analysis. It is unclear whether these ideas are truly related significantly, or whether they represent other superordinate constructs not uncovered due to experimental error.

In contrast to the similarity of personal constructs between these three students, Mrs. Morris' attempts at rating her highly motivated students' constructs showed little relationship to her students' ratings. Correlations between ratings for the teacher and these students ranged from .168 for Carlos, to .020 for Andrew, to .012 for Bob (all non-significant, $p > .01$).

Upon examination of Mrs. Morris' organization of these students' constructs via Additive Tree diagram, it becomes clear that she seems to have organized these constructs with attention to their semantic similarities, and/or with respect to her *own* representation. For example, in reference to Bob's constructs (Figure 4), Mrs. Morris rated "It really has no right answer," "It is fun," "it helps you learn faster," and "You learn more (than other topics)" similarly. Bob's organization of these constructs in contrast, seems to be focus on his belief that having no right answer makes a topic less motivating. He organized "It really has no right answer" closely to "I hate it," and "It is stupid."

Lower Motivated Students. The first observation upon inspection of the construct organizations of the lower motivated students, one is struck with the similarity of themes these students came up with compared to the more highly motivated students (See Figures 6, 7, and 8). These students also seemed to focus on challenge and understanding mathematics better as making mathematics activities more fun. Also, two of these students, Eric and Duane, mentioned the hands-on, building aspect of Mrs. Morris' classroom. This is especially interesting with respect to Mrs. Morris' explicit attention to providing these types of activities for her less motivated kids.

What is quite different from the highly motivated students' constructs is the apparent lack of differentiation and organization of constructs made by these lower motivated students. Both the number and type of constructs elicited was smaller for the less motivated kids, and the additive trees appear to have less depth of organization. It is unclear at this time whether these represent stable findings. These children may have had less motivation to provide extra constructs, or may have tired of the ratings task on the computer. This seems unlikely, however, because the stability of their ratings was fairly high (Test-retest reliability ranged from .76 for Allison, to .96 for Duane, to 1.00 for Eric). In addition, the challenge aspect that was so apparent for the highly motivated students

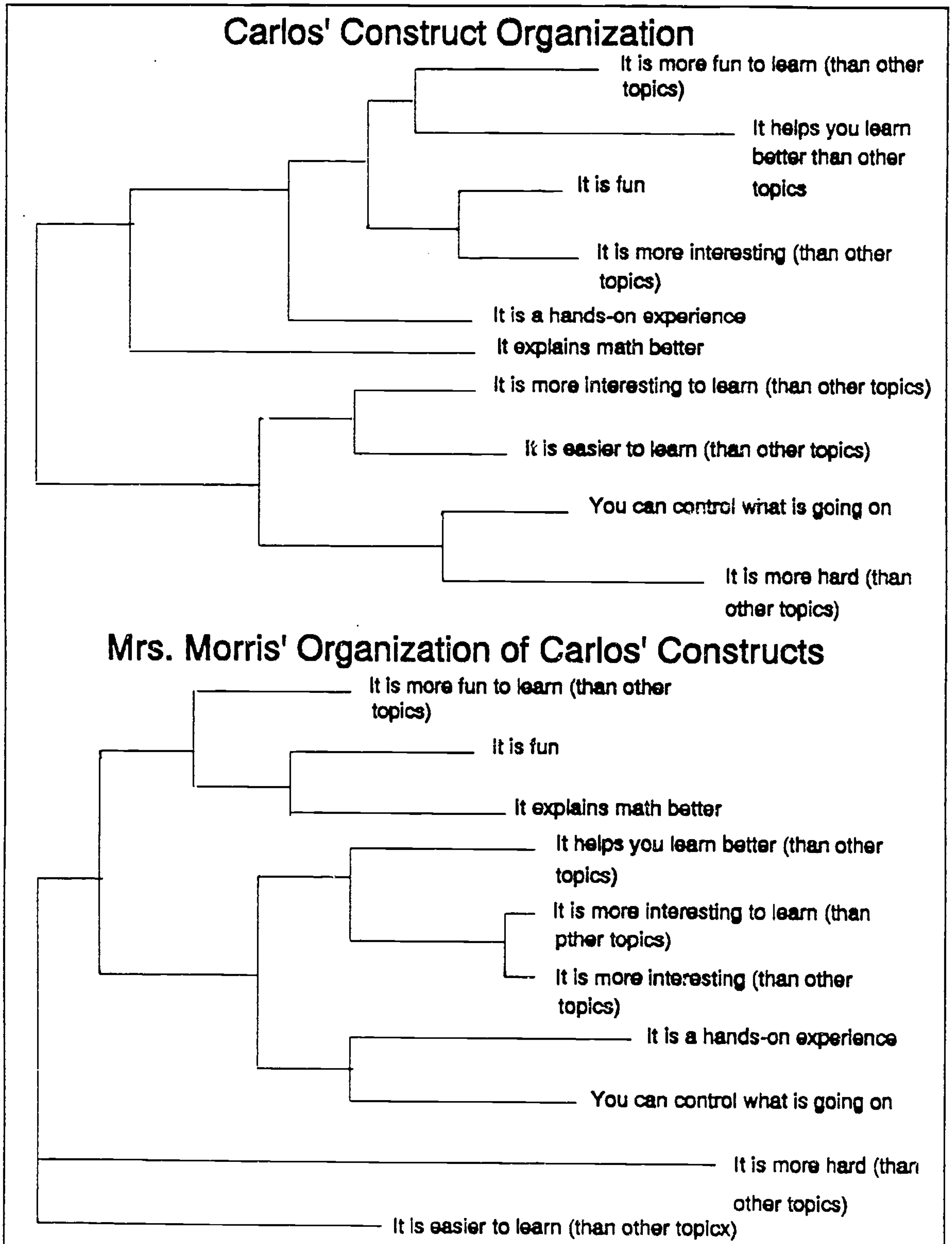

Carlos' Construct Organization
It is more fun to learn (than other topics)
It helps you learn better than other topics
It is fun
It is more interesting (than other topics)
It is a hands-on experience
It explains math better
It is more interesting to learn (than other topics)
It is easier to learn (than other topics)
You can control what is going on
It is more hard (than other topics)
Mrs. Morris' Organization of Carlos' Constructs
It is more fun to learn (than other topics)
It is fun
It explains math better
It helps you learn better (than other topics)
It is more interesting to learn (than pther topics)
It is more interesting (than other topics)
It is a hands-on experience
You can control what is going on
It is more hard (than other topics)
It is easier to learn (than other topicx)

Figure 3

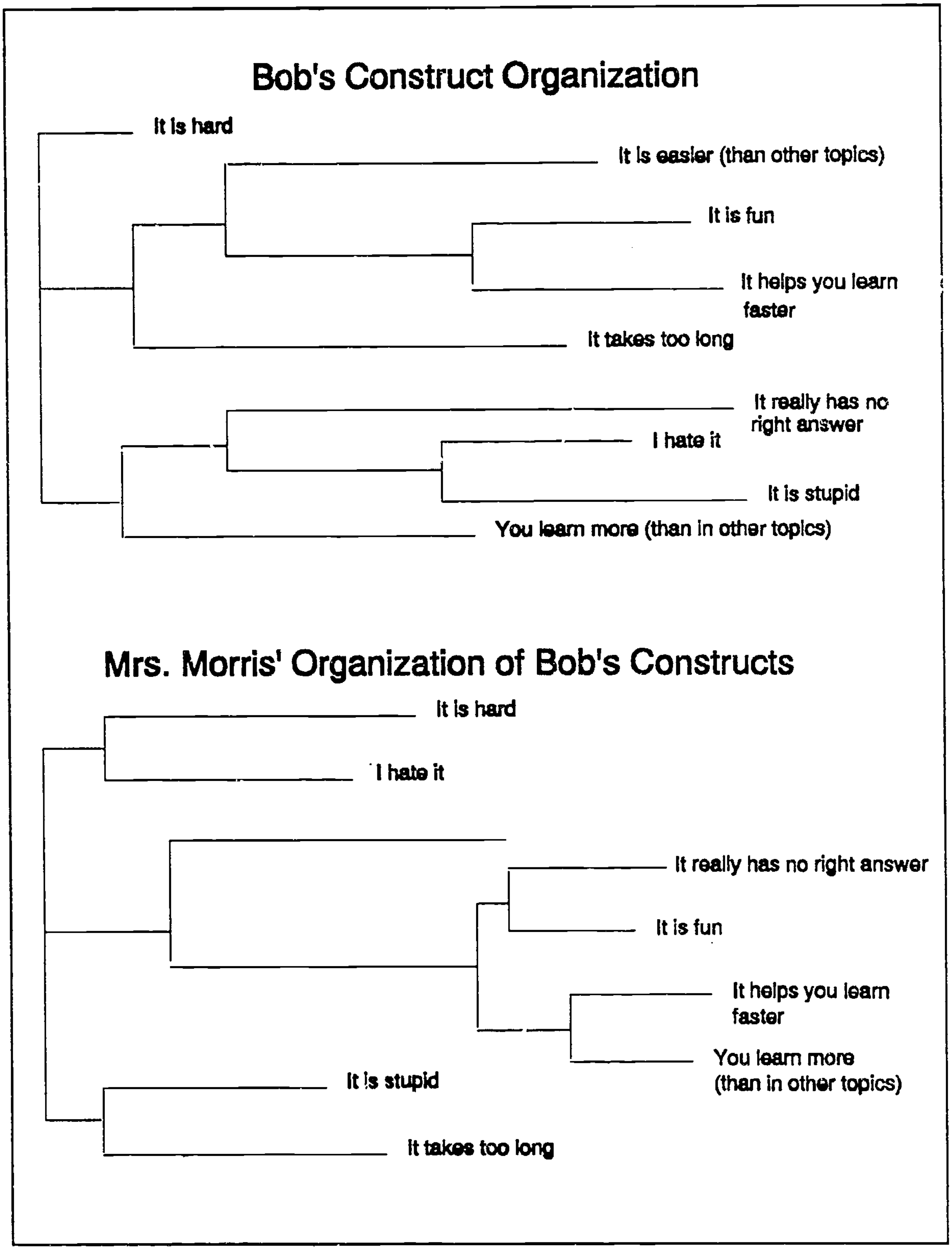

Bob's Construct Organization
It is hard
It is easier (than other topics)
It is fun
It helps you learn faster
It takes too long
It really has no right answer
I hate it
It is stupid
You learn more (than in other topics)
Mrs. Morris' Organization of Bob's Constructs
It is hard
I hate it
It really has no right answer
It is fun
It helps you learn faster
You learn more (than in other topics)
It is stupid
It takes too long

Figure 4

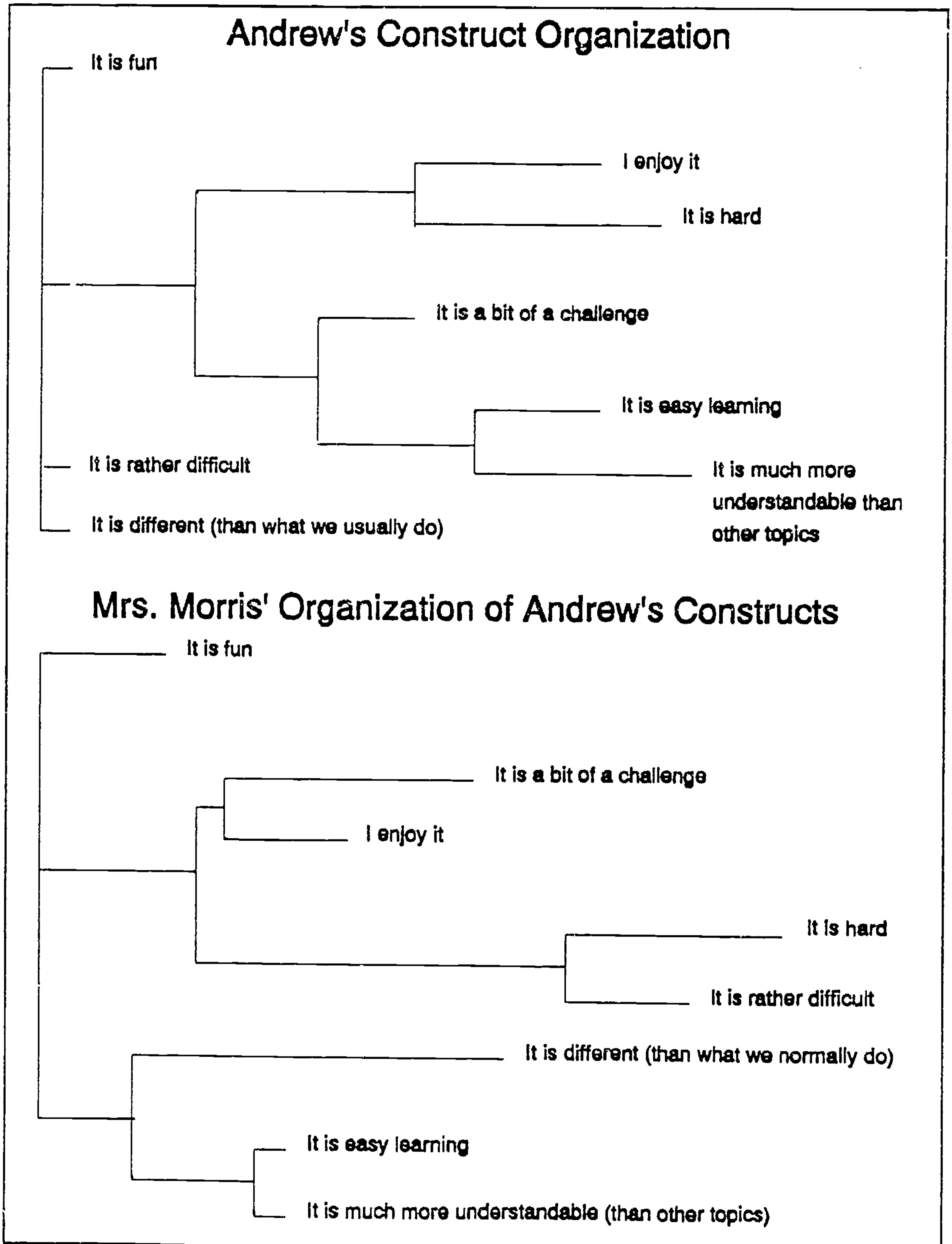

Andrew's Construct Organization
It is fun
I enjoy it
It is hard
It is a bit of a challenge
It is easy learning
It is rather difficult
It is much more understandable than other topics
It is different (than what we usually do)
Mrs. Morris' Organization of Andrew's Constructs
It is fun
It is a bit of a challenge
I enjoy it
It is hard
It is rather difficult
It is different (than what we normally do)
It is easy learning
It is much more understandable (than other topics)

Figure 5

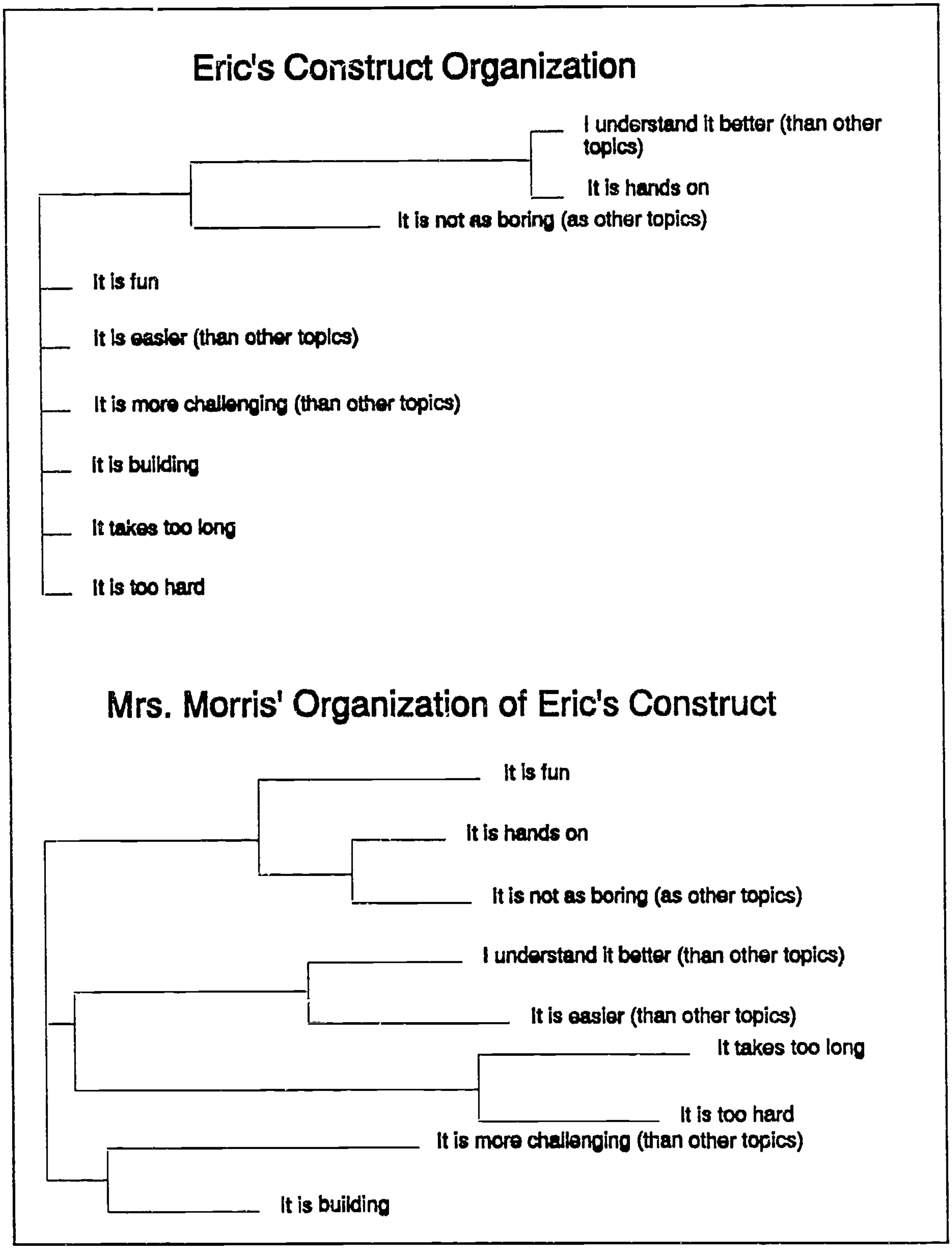
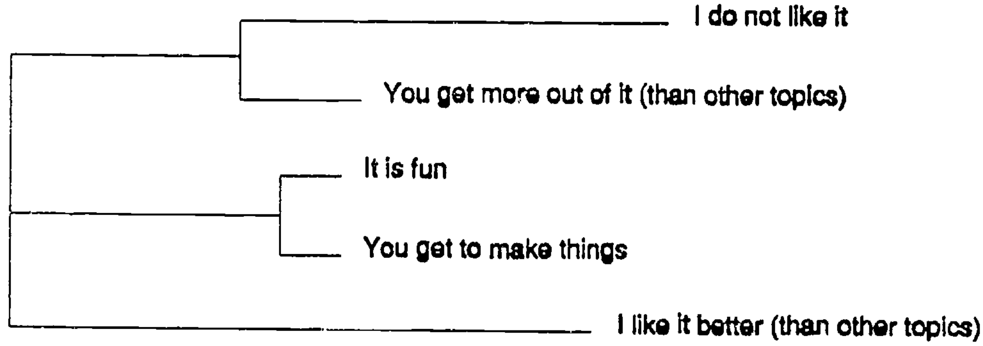

Eric's Construct Organization
I understand it better (than other topics)
It is hands on
It is not as boring (as other topics)
It is fun
It is easier (than other topics)
It is more challenging (than other topics)
It is building
It takes too long
It is too hard
Mrs. Morris' Organization of Eric's Construct
It is fun
It is hands on
It is not as boring (as other topics)
I understand it better (than other topics)
It is easier (than other topics)
It takes too long
It is too hard
It is more challenging (than other topics)
It is building

Figure 6

Duane's Construct Organization



Mrs. Morris's Organization of Duane's Constructs

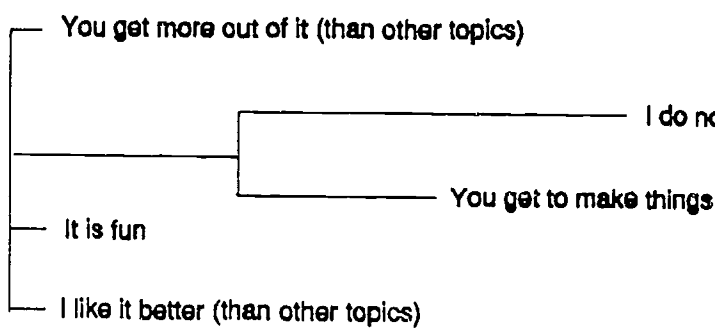


Figure 7

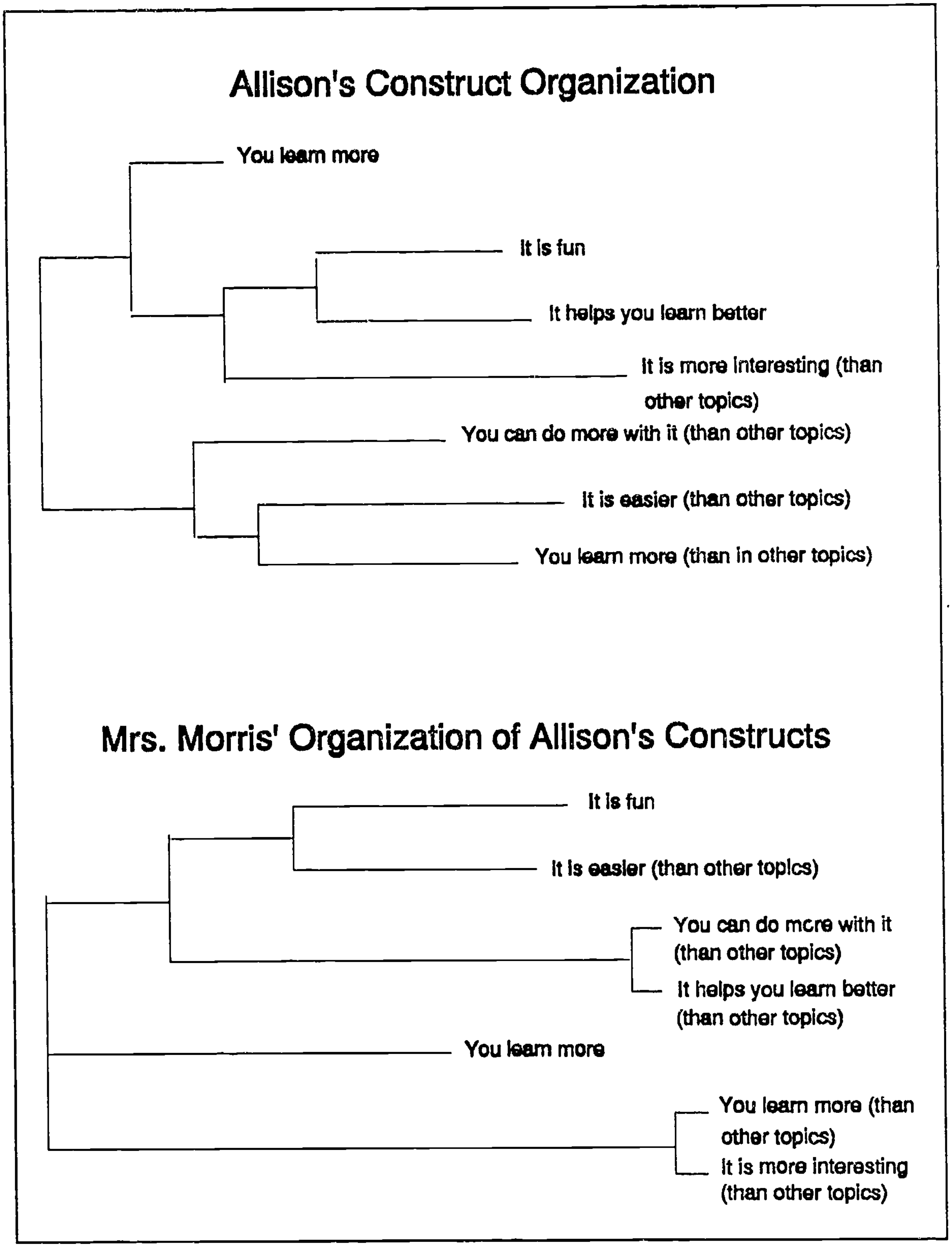

Allison's Construct Organization
You learn more
It is fun
It helps you learn better
It is more interesting (than other topics)
You can do more with it (than other topics)
It is easier (than other topics)
You learn more (than in other topics)
Mrs. Morris' Organization of Allison's Constructs
It is fun
It is easier (than other topics)
You can do more with it (than other topics)
It helps you learn better (than other topics)
You learn more
You learn more (than other topics)
It is more interesting (than other topics)

Figure 8

seems to be shifted to an aversion of too challenging tasks, or an attention to the ease of completing those tasks that are not too challenging. Thus, in Mrs. Morris' classroom, the beliefs about what makes mathematics motivating seem to be fairly stable across groups. However, the salience of the different poles of a construct in defining an intrinsically motivating activity (task ease vs. challenge, for example) seems to be related to a student's level of motivation.

Interestingly, Mrs. Morris was much better at anticipating her lower motivated student's construct ratings than her higher motivated students' ratings. Correlations between her ratings of Allison's and Duane's constructs and these students' own ratings were .482 and .469, respectively ($p < .01$). This is informative in that Mrs. Morris indicated in the interview that she pays less attention to her highly motivated students than her lower motivated students when she plans her instruction. It seems that her attention to these students' motivations is related to her ability to accurately predict their structure. However, for the lowest motivated student, Eric, Mrs. Morris' ratings were, again, very low ($r = .024, p > .01$). It is unclear at this time what this discrepancy might indicate, save that Eric's constructs were so undifferentiated that any attempt at organizing her thoughts may have led Mrs. Morris astray.

Summary across classes

The nature of the constructs generated by students was similar across the classes studied. Students invariably produced constructs that pertained to the difficulty of the tasks they were asked to perform, the challenge the tasks afforded, aspects of tasks they liked or were interested in, and the aspects of control the tasks afforded such as choice and ability fit. In addition, these constructs were similar in general to the constructs proffered by their teachers. Teachers also alluded to the task difficulty, the challenge of the tasks, the control the tasks afforded, and the novelty of tasks. For both students and teachers, the elicited constructs generally fit into the domains of interests, arousal, and control, although there was a great deal of overlap for individual representations.

Some of the variation across classes seems to be related to the influence of teachers' beliefs on their students' constructs. For example, students in Mr. Smith's class reported that the applications of mathematics to real life problems made some activities more fun than others. Mr. Smith placed a heavy degree of emphasis in his own constructs organization on the role of mathematical applications. Miss Burton' students were more attuned to the ease of performing their algebra activities than many of the students in other classes. Miss Burton's construct organization indicated that she also placed a heavy degree of emphasis on task ease when evaluating the motivational value of mathematics activities. Mrs. Morris (above) affirmed a belief that task difficulty and challenge, learning and understanding mathematics, and hands-on activities were central to her conception of what made mathematical activities fun. Her students, both highly and lower motivated groups, proffered similar constructs.

Although subjects in general were similar with respect to the nature and organization of their personal constructs, many differences existed between the highly motivated students and the lower motivated students. The highly motivated students tended to generate more constructs that pertained to the challenge and difficulty afforded by fun activities, while the lower motivated students tended to generate more constructs that pertained to task ease, and several indicated that just understanding what to do, or having seen similar activities before made the tasks more fun than others. Although these can all be thought of as relating to the degree of meaningful success students' anticipated having in the activities, it seems that the more highly motivated students tended to focus on higher arousal and less control in evaluating activities, while the lower motivated students tended to focus on lower arousal and more control. In addition, although the teachers' did not perceive group differences to be much of a function of ability, many of the teachers indicated that the lower motivated students had a lack of confidence in tackling mathematics problems. Thus, it seems likely that "Ease/challenge" constructs may pertain to ability attributions (or lack thereof) by the students--i.e., students may make a comparison of their perceived ability to the task requirements to determine the intrinsic motivation they feel the activities will afford.

Teachers in general, were poor at predicting their students' construct ratings. The mean correlation between teachers' ratings and their students' ratings was .2969, and the median correlation was similar (.2974). While this association is significantly greater than zero ($p < .01$), on average less than ten percent of the variance between teachers' ratings and students' ratings was common. Some interesting patterns did develop, however, when the differences between teachers' accuracy for their highly motivated students were compared with the lower motivated students. Table 1 lists teachers' rank order of their students with respect to motivation in the mathematics classroom, along with the correlation between teachers' ratings and students' ratings.

Similar patterns of accuracy were found between the two team teachers' predictions of their students' ratings. Both Mr. Martin and Mrs. Longman were more accurate than the other teachers in predicting their students' motivations. In addition, they were more accurate for their highest and their lowest ranked students than those students in the middle. In their interviews, they indicated that they paid more attention to their lowest motivated student, Archie, when developing their lesson plans, and in their instruction. Both teachers were most accurate for this student.

Mrs. Morris, who indicated in her interview that she focused more on the lower motivated students in her class when developing lesson plans, was most accurate for these students (Allison and Duane). Miss Burton, who focused more on the class average due to her perception of a generally high degree of motivation in her class, was much more accurate for her highly motivated students than her lower motivated students. It seems likely that, since she focuses on the class average, these highly motivated students who have similar construct organizations to their teachers', "fit" the classroom structure and the philosophy of the teacher better than the lower motivated kids, and thus make it easier for her to predict their belief structures. Mr. Smith, who did not place a great deal of emphasis on his students' motivations, at least on individual students' motivational characteristics, had very little idea of how his students would complete the ratings task. His predictions showed almost no relation to his students' beliefs.

From this analysis, it is apparent that the emphasis that teachers place on motivation in their planning,

Table 1.

Pearson Correlations between Teachers' Predictions of Students' Construct Ratings, and Students' Own Construct Ratings. Students are Ranked on the Degree of Motivation they Exhibit in the Mathematics Classroom (1 = highest).

Teacher	Student	Rank	*r*
Mr. Martin			
	Mark	1	**.5224**
	Toni	2	**.4872**
	Brenda	3	**.4385**
	Ada	4	**.4141**
	Archie	5	**.5727**
Mrs. Longman			
	Mark	1	**.5528**
	Toni	2	**.4835**
	Brenda	3	**.2831**
	Ada	4	**.4733**
	Archie	5	**.6245**
Mrs Morris			
	Carlos	1	.1676
	Bob	2	.0120
	Andrew	3	.0197
	Allison	4	**.4817**
	Duane	5	**.4690**
	Eric	6	.0248
Miss Burton			
	Asa	1	.2590
	Amy	2	**.3116**
	Barb	3	**.7320**
	Brian	4	.0267
	Chris	5	-.1555
	Don	6	.0996
Mr. Smith			
	Betty	1	.1637
	Chuck	2	.1259
	Billy	3	.0510
	Arthur	4	.0774

Note: All bold correlations are significant ($p < .01$).

especially how it pertains to the students they choose to focus on when developing and carrying out their plans relates heavily on the degree to which they can anticipate the motivational charac ristics of their students. Teachers who focus more on subgroups of children may be better able to predict these students' constructs due to the extra attention they give these students in planning the class.

Teachers who focus on the class as a whole, or the class average may be able to predict certain students' constructs better, especially in the case of highly motivated students, because the students' constructs are more congruent with the teachers' personal beliefs. Teachers who do not focus on students' motivation when developing their plans, not surprisingly may be unable to accurately predict any of their students' beliefs. In addition, the extra time available to the team teachers (Mr. Martin and Mrs. Longman) to discuss their plans, and to monitor the behaviors of their students seems to be an invaluable aid to their being able to predict both high and low motivated students' motivations in the mathematics class.

It is also apparent that the teachers studied are uninformed about the nature of motivation and its role in the mathematics classroom. This is especially the case with respect to intrinsic motivation, the focus of this study. Only one teacher, Mr. Smith, had courses specifically in the area of motivation, and few of the teachers studied had any instruction on motivation in their teacher education courses. In addition, even Mr. Smith's predictions of his students' motivations were very inaccurate, perhaps due to his lack of emphasis on motivation in developing his plans. This lack of knowledge about student motivation is evident in the reliance of the teachers on extrinsic motivators to stimulate their students. Most used reduced workloads, praise, and/or treats as rewards for good work. In addition, the use of humor in the classroom, though important to keep the atmosphere of the class light, often did not center around the content being taught, and may have diverted students' attention from the task at hand. Competition was important for at least two of the teachers studied (the two males), and not surprisingly, it seems to be more associated with challenge than control.

The lack of knowledge regarding motivation specifically within the mathematics education field is also manifest in teachers' attention to the utility and importance of mathematics for students' future lives. As stated earlier in this paper, students tend to believe that mathematics is both useful and important, but that these two features are not sufficient to motivate them to achieve. With the similarities found for these features in particular between teachers' and students' construct organizations, it seems that these beliefs are cycled from teacher to student, or from society in general to teachers and students, but that beliefs about the personal satisfaction individuals can achieve through mathematical problem solving is not. Surprisingly, teachers were quite candid about their lack of knowledge in this area, and most expressed some regret at not having the opportunity to learn more about student motivation.

None of the teachers were observed making the types of overt attempts to motivate their students by stressing that mathematics can be personally fun and exciting that were studied by Brophy, Rohrkemper, Rashid, and Goldberger, (1983). However, when teachers' personal conceptions of motivation in mathematics are examined it becomes clear that they did pay explicit attention to incorporating what they thought was motivating by using examples of real life applications, grouping practices, hands-on activities, etc., in their classroom. For example, Mr. Martin and Mrs. Longman used praise extensively throughout their lesson because both teachers believed strongly that praise made students feel good about their work. They also verbally reinforced their beliefs that ease made mathematics activities more motivating by telling the students, "This is not rocket science!" and other phrases that made it clear that the students had the ability to perform the assigned tasks. Mrs. Morris created her Problem-Solving station expressly for students to have the opportunity to extend their knowledge. Her construct organization revealed that she personally believes that extension of knowledge makes mathematics intrinsically motivating for her.

Mrs. Morris, Miss Burton, and Miss Saunders, the three teachers who had computers available for use all indicated in their interviews and on their construct organizations that they see the computers in their rooms as a prime motivator because they allow the students to be in control of their own learning. All three structured their lessons such that the computers could be incorporated.

From the classroom observations and examination of the videotaped lessons, it became evident that the teachers who were better able to predict their students' motivations in the construct ratings task also had a generally higher degree of motivation for their classes than teachers who were less able to predict their students' motivations. The team teachers for example who had the highest overall accuracy, had the highest overall class motivation. Their students were on task, they were more animated, and they participated in the class discussion more than students in the other classes. Mr. Smith, who had the lowest overall accuracy had students with an overall lower class level of motivation. Students were generally on task, but showed less animation, and several appeared to be somewhat listless and uninterested. Although these differences may be due to the differences in teaching style, the ability of the team teachers to monitor their students' behavior better than the other teachers, or just random differences in class personality, it suggests that teachers who understand their students belief systems may be better at motivating their classes in general than teachers who have less of an understanding of their students' motivations.

Discussion

Implications for Research

The results of this research indicate that although intrinsic motivation in the mathematics classroom is individual, differing from teacher to teacher, and from student to student, many commonalities are present in the overall types of constructs individuals use to evaluate mathematical activities, and these constructs fit nicely into the model of intrinsic motivation developed by Middleton, Littlefield, and Lehrer (1992). However, different individuals place differing levels of emphasis on the constructs they use in their evaluations. Some subjects placed a high degree of importance on stimulation--challenge, novelty, lack of boredom, etc. Interestingly, though not surprisingly, these individuals also tended to place less of an emphasis on issues of control such as free choice, and ability attributions. Other subjects placed a high degree of importance on levels of control, and less of an emphasis on stimulation. It is apparent that this trade-off between arousal and control is extremely important in developing a healthy appreciation for doing mathematics, or conversely, learning to dislike mathematics.

The application of personal constructs psychology to the problem of understanding teachers' and students' motivation has proven to be powerful in so far as uncovering important individual differences between students, or groups of students, and their teachers. The multiple sources of information utilized (observation, interview, construct ratings) has created a method of triangulation that makes sense of the great diversity of teachers' and students' belief systems. Results from the teacher interviews augmented their construct elicitation and ratings, and provided examples of how teachers' constructs are useful in creating their instructional plans.

As a descriptive model, the model of academic intrinsic motivation developed by myself and colleagues

(Middleton, et al., 1992) held up well in the general sense. However, it seems limited at this time in describing any particular individual's system for determining the intrinsic motivation afforded by different mathematics activities. This is perhaps due to the "snapshot" quality of the present study--I looked at teachers' and students' constructs for activities they had already engaged in. It will be interesting to develop a study that looks at change in teachers' and students' construct organizations over time as they engage in specific activities that vary in the degree of arousal and control they afford. Thus, a more fine tuned, longitudinal experiment should reveal ways in which persons actively organize activities into their self systems, and should provide a more rigorous test of the theory.

The importance of studying motivational constructs within the domain of mathematics specifically was also reinforced by the results of this study. Most of the participating teachers revealed that they assess the mathematical abilities of their students, and this assessment is related to the degree of motivation students exhibit. Moreover, most of the students' mathematical deficiencies were reported by the teachers to be idiosyncratic with respect to mathematics, not manifesting themselves in other content areas. In addition, many of the constructs uncovered by teachers and students were directly tied to the types of activities they were asked to contrast in the elicitation task. For example, "You can make awesome geometric designs" was proffered by Lee, one of Miss Saunders' students. This construct seems to stem from the domain of mathematics, and not from other content areas.

With respect to the questions of interest for this paper, results are highly informative, and extend our knowledge with respect to the motivational characteristics of teachers and students, and how these might play off each other in the instructional milieu. In this respect, results are interpreted in terms what von Glaserfeld (1985) and Steffe (1988) called "radical constructivism," as opposed to "trivial constructivism."

> In trivial constructivism mathematics *for* children exists in ontological reality. In radical constructivism, the mathematical knowledge of the *other* is taken as being relative to one's own frame of reference and can be known only through interpreting the language and actions of the other--only by forming a possible conceptual model...It is taken as a "fit" rather than as a "match."

Results indicate that overall, students and teachers have developed similar belief structures with respect to their intrinsic valuation of mathematics. Both the highly motivated students and lower motivated students were concerned with aspects of arousal and control, as were their teachers. While these general categories were similar across groups, individuals tended to construct different representations depending upon the perceived trade-off between arousal and control, and their differing interests. In addition, several of the students made self-statements in their constructs organizations referring to the amount of pride they feel when they accomplish a difficult task, their beliefs about their abilities, and their prior knowledge of mathematics. These self-statements have been uncovered before (Middleton, Littlefield, and Lehrer, 1992). These results go beyond the attribution literature in that they help define the content of students' and teachers' attributions vis a vis individual differences in arousal and control tradeoffs, rather than merely defining the attributional structure common to large groups of individuals. Thus, the locus and stability of teachers' and students' attributions of success and failure do not seem to be important except as they relate to the ways in which individuals define success and failure in the first place.

Overall, teachers were poor at predicting their students' motivational constructs. However, results also indicate that teachers *can* assess the motivations of some of their students with a fair degree of accuracy but the degree of accuracy seems to be a function of which students teachers tend to focus upon when developing and carrying out their lesson plans, and to some extent, the similarity of the students' constructs with their own.

Indeed these findings make much more sense than the original hypothesis especially when teachers reasons for focusing on different students are taken into account. Teachers indicated that they focus on a small number of specific children (usually lower motivated students), subgroups of the class, or the class "average" to reduce the complexity of planning, and to reduce their own feelings of failure when they do not reach all of their students. In addition, at least one teacher did not focus on any particular student because she felt the class in general had a fairly high degree of motivation, and this freed her up to concentrate more heavily on the content being covered. Thus the realities of dealing with 30 students in a class inhibit a single teachers' ability to pay specific attention to most of their students, and as a consequence, many teachers have little idea of how their students are motivated intrinsically.

Teachers personal constructs of what makes mathematics intrinsically motivating do seem to play a major role in the types of activities and examples they choose and design. Teachers who believed that the utility of mathematics was a motivating factor tended to use more real-life examples. Teachers who believed that ease made activities more motivating emphasized the ease in which problems could be solved, and provided examples that illustrated lack of difficulty. Teachers who emphasized challenge, tended to create instances where students could go beyond the regular material. However, this knowledge must be tempered with the fact that the textbook was still viewed as the source of the curriculum for most of the teachers, and any modifications these teachers made in creating their plans were hampered by the traditional presentation of the texts. Most modifications centered around the development of examples that illustrated the content presented in the text, rather than development of a context through which the content could be discovered.

Teachers did attempt to provide examples that tied with what they believed their students' interests were. However, since the teachers did not focus on most of the participating students in developing their plans, they had to make a best guess at what their students might find interesting. Many times their guesses were successful, but many times they were not.

Teachers' attention to arousal issues such as novelty, fantasy, and challenge seemed to stem from their own belief systems, hampered by the particularly non-novel textbook presentation. When teachers perceived a way to make the material more stimulating for students, they were quick to incorporate stimulation into the lesson. However, it was interesting to note that several attempts to "stimulate" the students involved extrinsic rewards. It is unclear to what extent the teachers perceived these rewards to be stimulating for the students with regards to the task, or whether the rewards were seen as *stimulants* to induce the students to perform an unrewarding activity.

All of the teachers studied wanted to challenge their students in mathematics, and most were successful.

Each tried to allow students to go beyond the task requirements if they desired, and in different ways, they encouraged this in the classroom. Mrs. Morris developed the Problem Solving station for this purpose. Miss Burton allowed a student to create test items that would stump her. Mrs. Longman and Mr. Martin utilized mathematical puzzles and brain teasers. Mr. Smith used competition and had students gather data from home. Miss Saunders used difficult stock market data to allow students to draw out mathematical patterns. The most important finding with respect to stimulation in mathematics that comes from the present study is that what teachers feel is novel, challenging, or that requires imagination will determine to a large degree what they incorporate into the lesson.

With respect to control, all of the teachers indicated that they did not give the students much in the way of free choice of activities. They attempted to provide choices with respect to students' presentation of results, the order in which problems could be solved, and in the strategies students could choose in solving problems, but did not provide students with a wider variety of problem solving situations from which they could freely choose the activities that interested them.

In tailoring the requirements for participating in mathematics activities, the teachers did try to provide control for their students in various ways. Most teachers created a set of activities for the "average" student and allowed the students who would choose to do so go further. The three teachers who had computers available perceived that the technology allowed students to assert a higher degree of control over the learning process, saw this as beneficial, and incorporated computer programs into the curriculum. Several teachers used small groups so that students could teach each other, or used students in the class to teach the rest of the class.

Again, the ways in which teachers incorporated aspects of control into their classes seemed to stem from their own beliefs about control. As mentioned before, ease was one of the primary control factor several teachers attempted to make real for their students.

Implications for Instruction

In the process of interviewing the teachers I did notice that all of the teachers were in states of transition with respect to how they approached mathematics instruction. They were searching for new ways of presenting material, were developing units of their own they felt would be more motivating, and were aware that mathematical content and pedagogy are changing. Several of the teachers were relatively new to the field of mathematics teaching, and as such, were just beginning to develop a repertoire of pedagogical strategies and pedagogical content knowledge unique to their mathematics classes. Moreover, all of the teachers indicated that they were committed to the betterment of their students, and were committed to helping their students become more successful in their future lives.

These attitudes, though not unique, when coupled with the desire for change, seem to be influencing the participating teachers to begin the difficult struggle out of the inert system in which they were educated. It will be interesting to do as Mr. Smith requested, "...to be able to come back in five years and do the same sort of thing and see where I am as far as mathematics," for my intuition is that although the general philosophies of the teachers will be similar, the ways in which they view their students in relation to mathematics instruction will be quite different.

One prescriptive statement that should perhaps be made at this time is that explicit attention to the motivational characteristics of students in the mathematics classroom should be paid in the pre- and inservice education of middle school mathematics teachers. Teachers in the present study had little if any training with respect to student motivation, and this perhaps hampered their ability to predict the organization of the motivational constructs generated by their students. Future research should also attempt to inform teachers regarding their students' motivational systems, much in the same way as the Cognitively Guided Instruction (CGI) project (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) informs teachers about their students' knowledge and strategies with regards to addition and subtraction. Then the ways in which teachers' knowledge about their students are incorporated into their planning, instruction, and assessment can begin to be articulated. Admittedly, the explicitness of the information teachers receive in the CGI project cannot begin to be approached by the research on motivation, the general model of providing teachers with the latest knowledge regarding students' motivation in mathematics may at least give teachers a basis with which to make informed pedagogical decisions.

However, given that teachers are highly influenced by their own mathematics teachers in high school and college or their peers who are teachers, more than their teacher education courses (Middleton, Webb, Romberg, & Pittelman, 1990; Romberg & Middleton, in preparation), this kind of pre- and inservice education must be both continual and collaborative. Moreover, given that the mathematics content itself has historically been rigid, inflexible, rule-oriented and tedious, education on motivation itself will not suffice.

Conclusions

Intrinsic motivation in mathematics is a highly complex, individual process affected by a number of factors including classroom environment, teachers' beliefs about their students and mathematics, and students' beliefs about themselves in relation to mathematics. As its very name implies, intrinsic motivation is dependent upon the individual--the person's construction of reality, and their place in the reality they construct. Students can place themselves in the center of their mathematical reality, becoming agents impacting upon the content to augment their learning, or they can place themselves on the periphery of their reality, becoming "pawns" to be impacted upon (see de Charms, 1968).

The congruence between teachers' and students' conceptions of what makes mathematics instruction intrinsically motivating becomes less important in this respect than how teachers' and students' view each other and their roles in the structure of the classroom. The teacher who views their role as facilitator of knowledge, rather than the ultimate authority, or worse, the translator of the text, has the most opportunity to view the student as agent, and design activities that allow the students to take charge of their learning. However, it is still unclear whether students who have not ordinarily found mathematics to be motivating intrinsically, would actually take control of their own learning given the chance. If such students' self systems are highly preemptive, then Kelly's Choice Corollary (1955) would suggest that they would attempt to choose a view of their role as pawn over the new role offered since it serves to extend their self image. Thus, nothing less than a radical reorganization of roles where the old systems do not function seems to be the logical course of action. Only in a system where all actors are viewed by each other as agents, equal in authority, different in expertise, and willing to cooperate on important tasks can such entrenched self systems begin to be eroded.

It can only be hoped that the results of research on motivation in the classroom of which this study is but a small part, will provide practitioners with the knowledge they can use to effect such a change in their classrooms. However, one of the important issues not addressed in this study is the appropriateness of the actual content being explored. From the radical constructivist perspective, students and teachers will fabricate their own reality--their own vision of what mathematics is--regardless of the coherence and consistency of the curriculum. Thus, a student who is motivated intrinsically by inconsistent content, full of loopholes and gaps in logic, will develop a system that will cease to function adaptively given the need for consistency in representation for negotiation of meaning. Ultimately, such a student will be ill served by his or her education, the fun they experienced notwithstanding.

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