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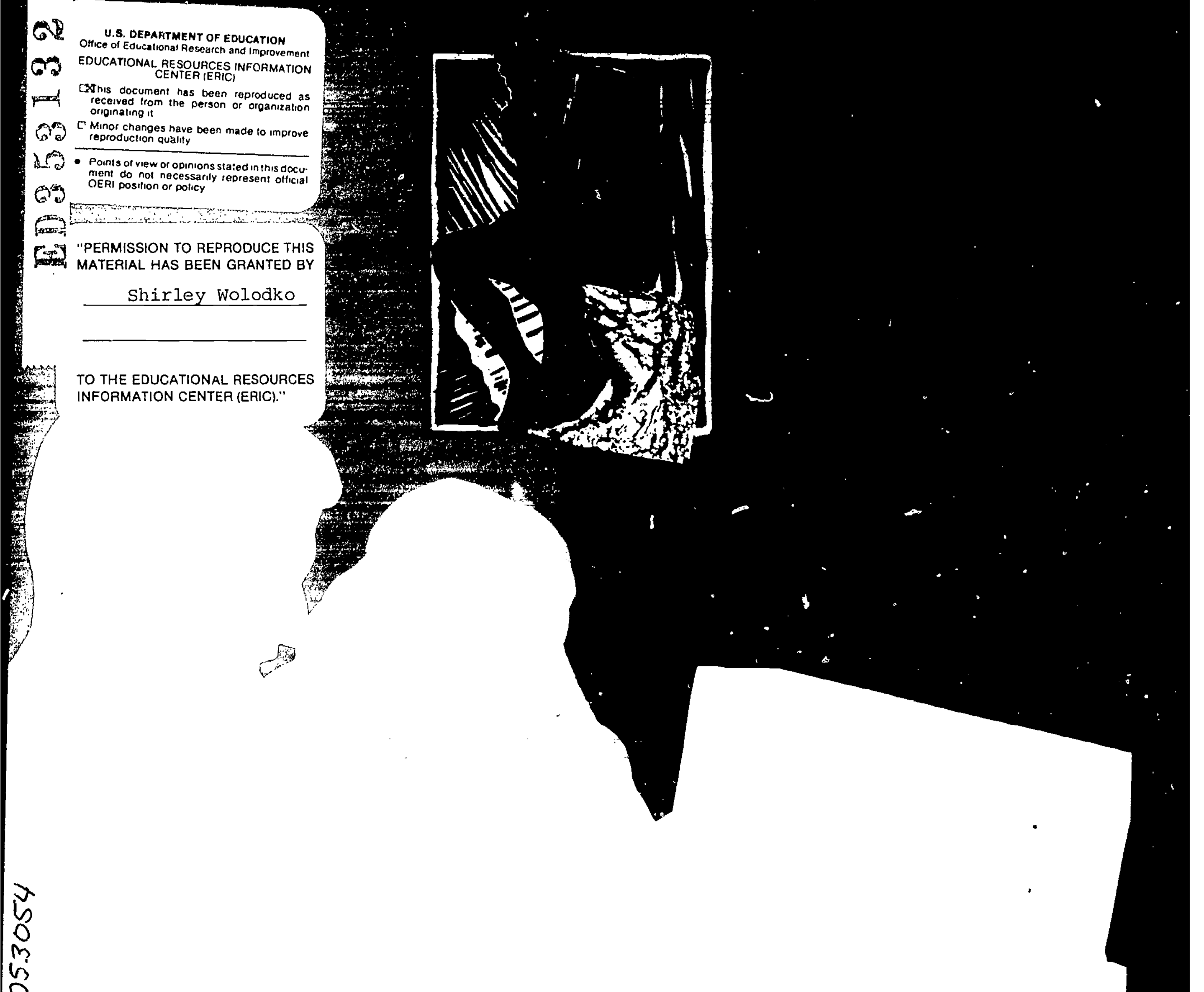
ABSTRACT

This document is designed to assist teachers by providing practical examples of real world applications of high school mathematics. Fifteen problems are presented that individuals in industry and business solve using mathematics. Each problem provides the contributor's name, suggested skills required to solve the problem, background information for solving the problem, ideas to consider in the problem-solving process, and the solution. The problem topics include: (1) straightening steel rods shipped to customers in coils; (2) laying out the horizontal alignment of a highway, (3) calculating the dimensions of unusually shaped windows; (4) calculating parking capacities on city street; (5) transmitting telephone calls; (6) adjusting a parabolic satellite antenna; (7) calculating the materials required to construct a highway; (8) calculating the forces on a bridge; (9) mounting a gas heater for optimal efficiency; (10) maximizing the efficiency of a telephone network; (11) forecasting energy needs of electrical utilities; (12) calculating the terminal velocity of a metal sphere; (13) ensuring adequate visibility at a railway crossing; (14) analyzing spot speed data to determine highway speed limits; and (15) calculating minimum area requirements that metal plates must cover in the construction of wooden trusses. (MDH)

MATHEMATICS

AT WORK IN ALBERTA



Students First

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MATHEMATICS

AT WORK IN ALBERTA

Students First!

Alberta
EDUCATION

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More than ever, how well students learn depends on how well they see the connections between what they learn in school and the real world. Students need to see the ways in which school subjects such as social studies, biology or mathematics serve them both in their daily living and in their future work.

We can help students make these connections, but to be successful at it we need the efforts of community members. I'm happy that more and more people are taking this to heart. Educators, business and professional groups, and members of various other public groups, are establishing partnerships in support of students' learning. This resource book is the outcome of just such a partnership – between members of the business and professional community and mathematics educators.

I'm delighted to share with TransAlta Utilities the launching of this engaging book. I believe that students and teachers will be inspired by these high school math problems that Albertans encounter in their everyday work. And I hope this book will encourage others in the community to work with us in the interest of educational excellence for all our students.

Jim Dinning
Minister of Education

How To Use Mathematics at Work in Alberta

Mathematics at Work in Alberta is designed to assist teachers by providing practical examples of "real world" applications of high school mathematics. Problems are presented that individuals in industry and business solve using mathematics.
The design of the resource is as follows:

Contributor
The contributor's name, affiliation and telephone number.

Skills and Suggested Courses
The suggested skills required to solve this problem and the courses where introducing this problem may be appropriate. This is by no means an exhaustive list.

Background Information and Problem
Background information for solving the problem, the context of the problem and the problem itself.

Problem-Solving Steps
Throughout the presentation of and solution to problems, ideas you might wish to consider in the problem-solving process are included. Again, this is not an exhaustive list but just ideas of questions that students might think about when solving the problem.

Solution
A solution to the problem is presented as suggested by the contributor. These solutions are not the only ones possible. Students may develop other solutions.

Some of these problems may require extended time to solve.
They could be used as projects in the mathematics program.

Contents

PROBLEM 1 *Straightening Steel Rod*

Continuous lengths of round steel rods are usually wound up in coils as they are manufactured. These coils are then shipped to customers. To cut or otherwise process the steel rod, these coils first have to be straightened using a set of rollers, as shown in diagram I. Each roller is wedge-shaped to hold the rod in place (diagram II). As shown in diagram III, the roller is wedge-shaped at an angle of 30°. A small diameter rod might contact at point (a) whereas a larger one would contact at a point higher up the roller wedge. Any particular roller is good for a range of rod sizes. However, the largest rod that can be straightened by the rollers without being gouged would contact at point (b). Given a roller of width (W), what is the radius (r) of the largest rod that can be straightened?

Diagram I:
Side View

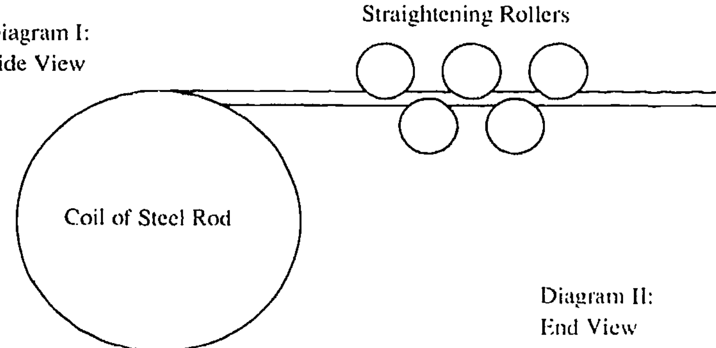
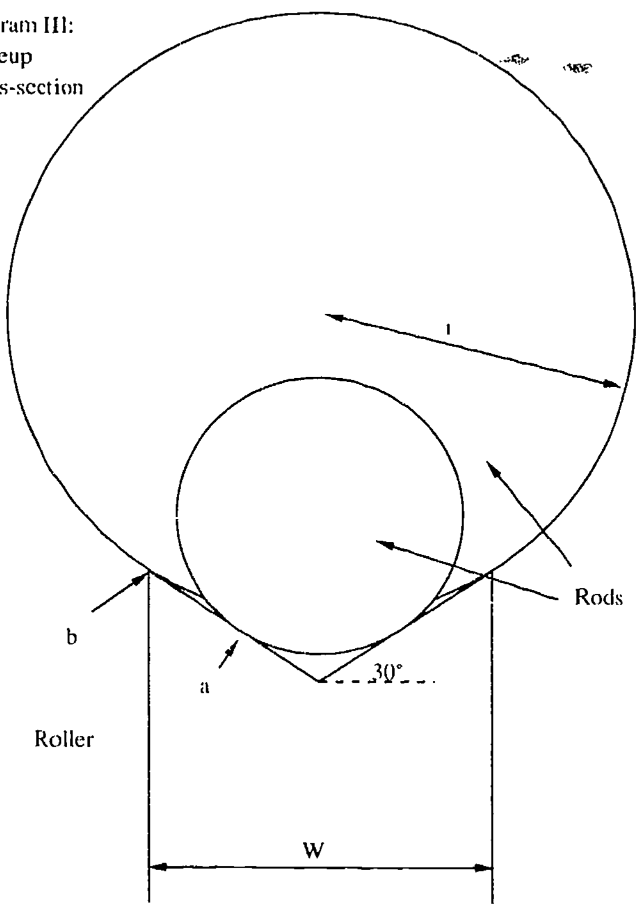


Diagram II:
End View
Cross-section

Diagram III:
Closeup
Cross-section



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Project Engineer

ORGANIZATION:

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SKILLS:

- Trigonometry
- Circle Geometry

SUGGESTED COURSES:

Math 10, 20, 23

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Investigate why steel rod is put on rolls.
- Investigate how steel rod is manufactured and whether this will make a difference in solving the problem.
- Identify the information required, needed and wanted.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

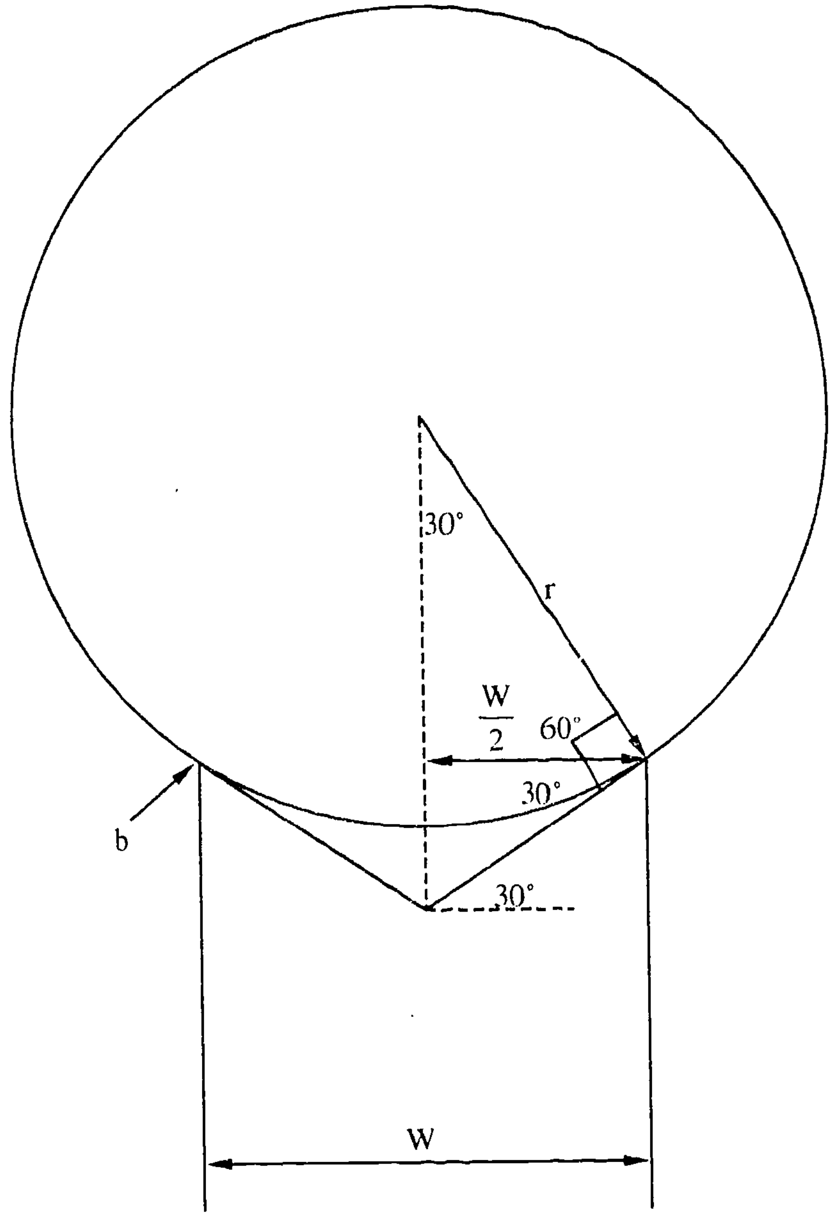
- Making a conjecture as to what the result will be.
- Solving a simpler problem by using numerical values for *W* or *r*.

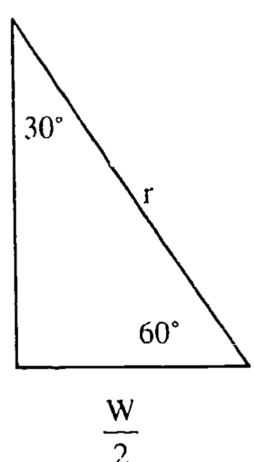
IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- The symmetry of the roller.
- The properties of tangents to a circle and the sum of angles in a triangle.
- Parallel lines and their alternate interior angles.
- Complementary angles.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- If it makes sense to say that the width of the roller will equal the radius of the largest rod.
- If there are wedges at angles other than 30°.
- What the radius is of the smallest rod that can be straightened.
- What industries in Alberta would use steel rods.
- What other products might be shipped in rolls and require straightening.





$$\sin 30^\circ = \frac{opp}{hyp} = \frac{\left(\frac{W}{2}\right)}{r}$$

$$\frac{1}{2} = \frac{(W)}{(2)(r)}$$

$$r = W$$

Therefore, the largest rod that can be straightened by a 30° roller has a radius equal to the width of the roller.

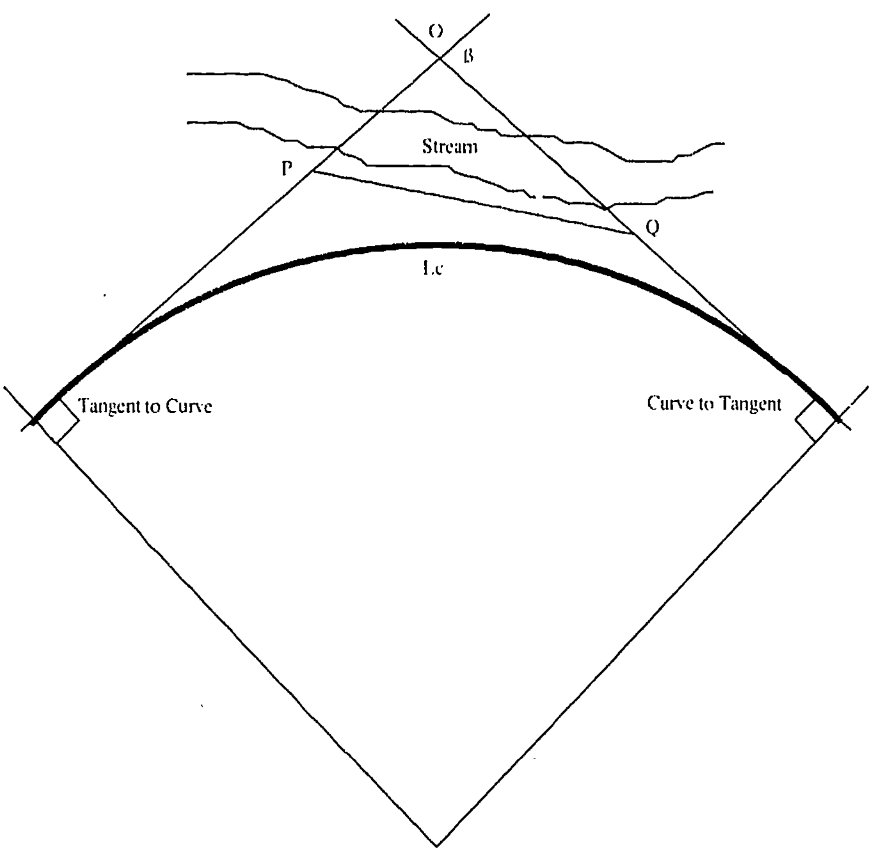
PROBLEM 2 *Highway Curve*

Before construction of a highway can begin, the surveyor is required to lay out the horizontal alignment of the highway (plan view layout or map view). The alignment may consist of straight segments and curves. Under normal circumstances, the surveyor can access the points of intersecting lines on a curve and is able to turn the required deflection angles.

In this example, the point of intersection of the two alignment lines is not easily accessible because of the stream. Therefore, the surveyor is required to use geometry and trigonometry to assist in laying out the highway alignment.

Given:
- -line PQ = 695.21m
- -angle OPQ = 35° 18' 30"
- -angle OQP = 25° 20' 10"
- -desired curvature of 4° / 100 m arc

Find:
1. The deflection angle ß.
2. Angle POQ.
3. Length of PO and QO.
4. The total arc length of the highway curve (Lc).



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SKILLS:

- Trigonometry
- Circle Geometry

SUGGESTED COURSE:

Math 20

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Find examples of surveying around obstacles on the roads in your area.
- Investigate what methods and instruments surveyors use to sight and mark straight lines and curves.
- Review the given information.
- Examine the reasons that a horizontal alignment is important in constructing a highway.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

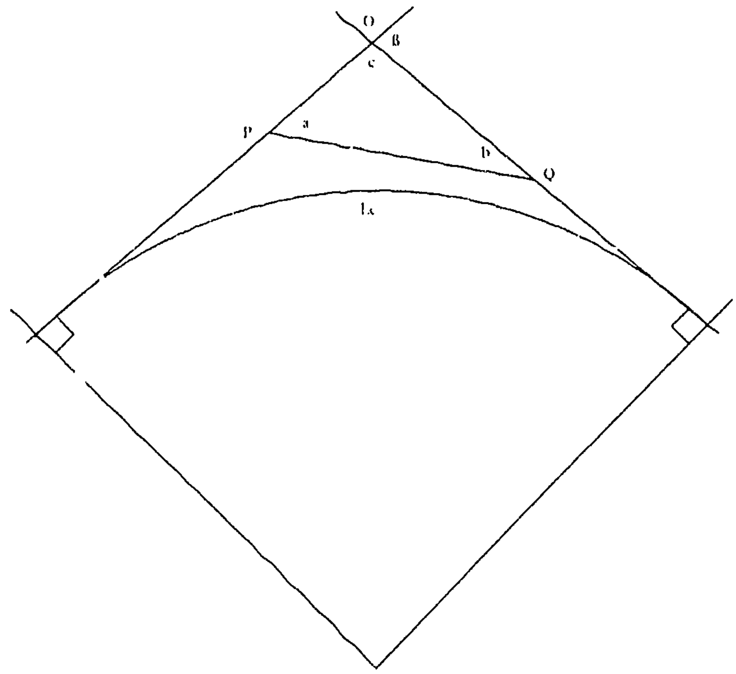
- The reason for finding the deflection angle.
- How angle POQ and the lengths of PO and QO affect the total arc length of the highway curve.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Supplementary angles.
- The sum of angles in a triangle.
- Circle tangent properties.
- When and how degree/minute/second angle measurements are converted to decimal degrees.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- How the result in #4 can be related to radian measure.
- How the results in this problem could be used when surveying around other obstacles.
- Another way of solving this problem; that is, was all the information that was calculated needed to solve the problem?
- If the size of the deflection angle affects the arc length of the highway curve.



1. $\angle ß = 180° - \angle c$

$\angle c = 180° - \angle a - \angle b$

$\Rightarrow \angle ß = 180° - (180° - \angle a - \angle b)$

$\Rightarrow \angle ß = \angle a + \angle b$

$\Rightarrow \angle ß = 35°\,18'\,30'' + 25°\,20'\,10''$

$\Rightarrow \angle ß = 60°\,38'\,40'' = 60.6\overline{4}°$

2. $\angle POQ = \angle c = 180° - \angle ß$

$= 180° - 60.6\overline{4}°$

$= 119.3\overline{6}°$

3. $$\frac{\sin 119.3\overline{6}°}{695.21 \text{ m}} = \frac{\sin 25.336°}{\overline{PO}}$$

$\overline{PO} = 341.36$ m

$$\frac{\sin 119.3\overline{6}°}{695.21 \text{ m}} = \frac{\sin 35.308°}{\overline{QO}}$$

$\overline{QO} = 461.05$ m

4. $$\frac{4°}{100 \text{ m}} = \frac{\angle ß}{Lc}$$

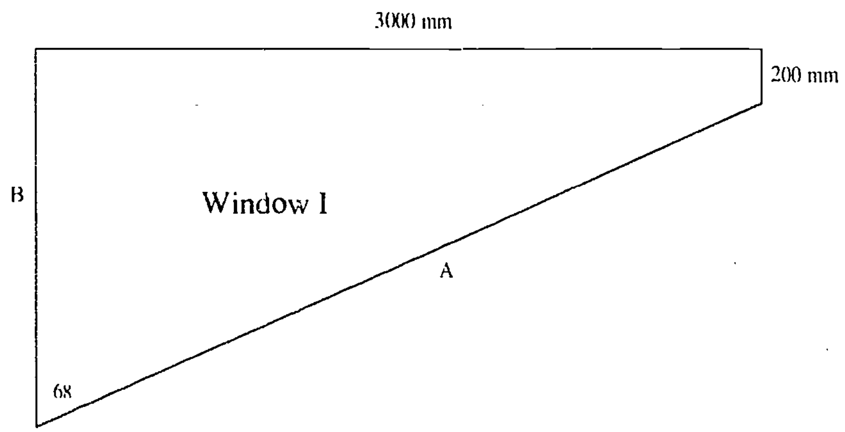
$$\frac{4°}{100 \text{ m}} = \frac{60.6\overline{4}°}{Lc}$$

$Lc = 1516$ m

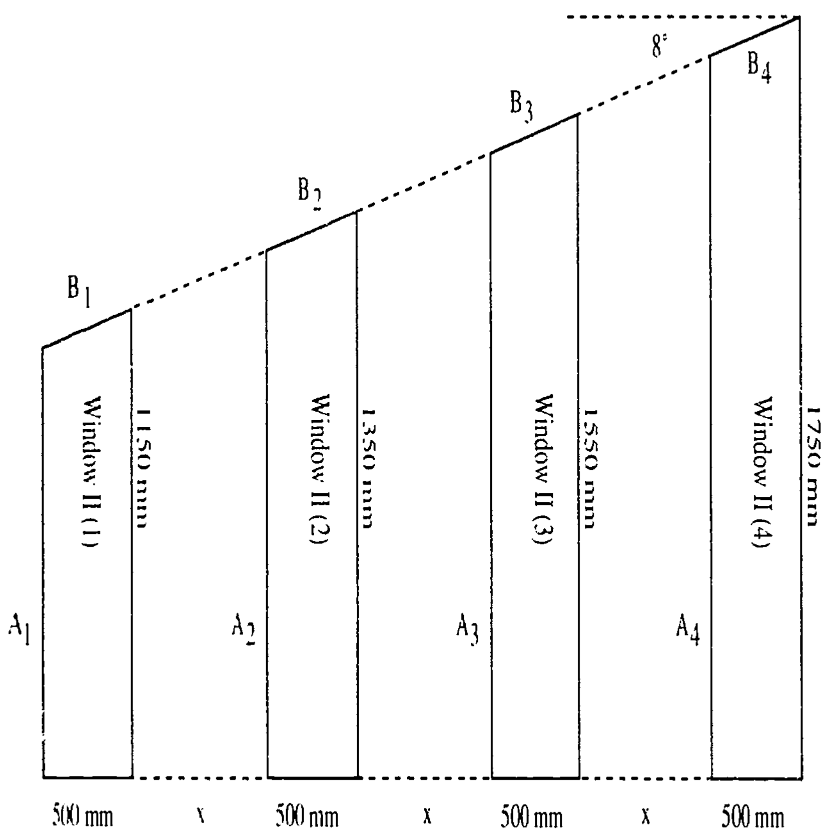
PROBLEM 3 *Church Windows*

A church architect in Red Deer requests some unusually shaped windows. The dimensions given do not completely specify the window size and framing requirements. The following calculations need to be made for costing and manufacturing.

1. Find the length of sides A and B for window type I.



2. Find the length of sides A_1 - A_4 and B_1 - B_4 for the window type II series. Also find the even spacing (x) between the four windows.



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SKILL:

• Trigonometry

SUGGESTED COURSES:

Math 20, 23

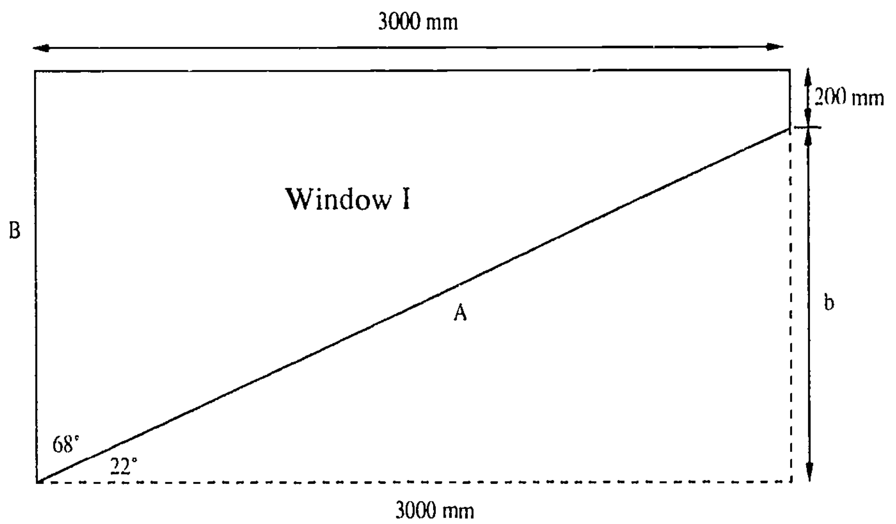
IN *UNDERSTANDING* THESE PROBLEMS, YOU MAY WISH TO:

• Review the diagrams to make sure you clearly understand what they are illustrating.

• Examine how glass is made and what its properties are (i.e., fluid properties). Will these properties affect these problems?

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Breaking the problem into smaller parts.
- Developing an algebraic formula to represent the spacing between the windows in the second problem.
- Using geometry to create rectangles in both problems.



1. $$\cos 22^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{3000 \text{ mm}}{A}$$

$$A = \frac{3000 \text{ mm}}{\cos 22^\circ} = 3236 \text{ mm}$$

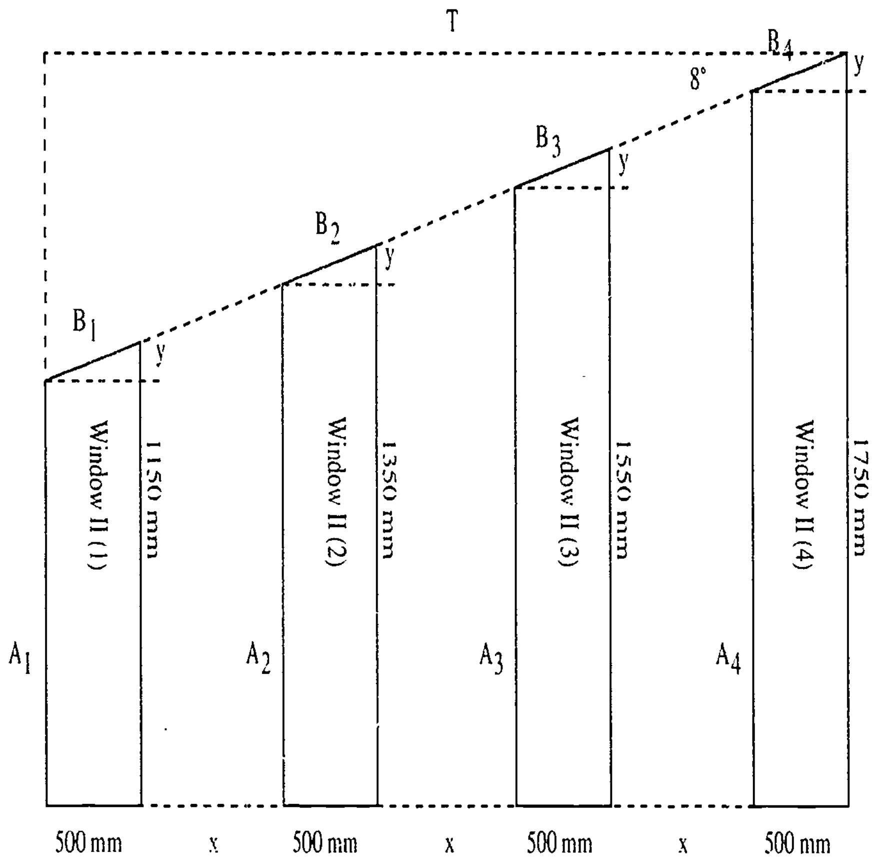
$$B = 200 \text{ mm} + b$$

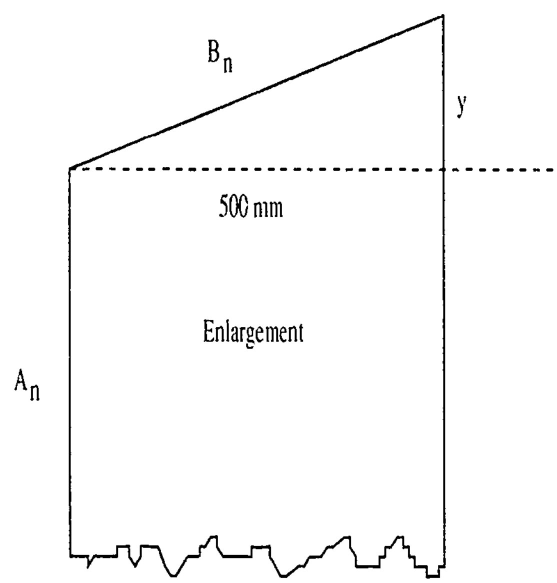
$$\Rightarrow \tan 22^\circ = \frac{\text{opp}}{\text{adj}} = \frac{b}{3000 \text{ mm}}$$

$$\Rightarrow b = (\tan 22^\circ)(3000 \text{ mm}) = 1212 \text{ mm}$$

$$B = 200 \text{ mm} + 1212 \text{ mm} = 1412 \text{ mm}$$

PROBLEM 3 *Solution*





IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- The use of symmetry.
- Complementary angles.
- The use of trigonometric ratios.

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IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- What factors, other than the measurements calculated in these problems, affect the cost of specially manufactured windows.
- What it would cost Indal Building Products if an error were made in calculating the size of windows.
- The reason that the measurements are given in millimetres.

2. $\tan 8^\circ = \frac{\text{opp}}{\text{adj}} = \frac{y}{500\text{ mm}}$

$y = (\tan 8^\circ)(500\text{ mm})$

$y = 70.3\text{ mm}$

$\therefore A_1 = 1150\text{mm} - 70.3\text{ mm} = 1080\text{ mm}$

$\therefore A_2 = 1350\text{mm} - 70.3\text{ mm} = 1280\text{ mm}$

$\therefore A_3 = 1550\text{mm} - 70.3\text{ mm} = 1480\text{ mm}$

$\therefore A_4 = 1750\text{mm} - 70.3\text{ mm} = 1680\text{ mm}$

$\cos 8^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{500\text{ mm}}{B_{1-4}}$

$B_{1-4} = \frac{500\text{ mm}}{\cos 8^\circ} = 505\text{ mm}$

$T = (4)(500\text{ mm}) + (3)(x)$

$(3)(x) = T - (4)(500\text{ mm})$

$\Rightarrow \tan 8^\circ = \frac{\text{opp}}{\text{adj}} = \frac{(1750\text{ mm} - 1079.7\text{ mm})}{T}$

$\Rightarrow T = \frac{(1750\text{ mm} - 1079.7\text{ mm})}{\tan 8^\circ}$

$\Rightarrow T = 4767\text{ mm}$

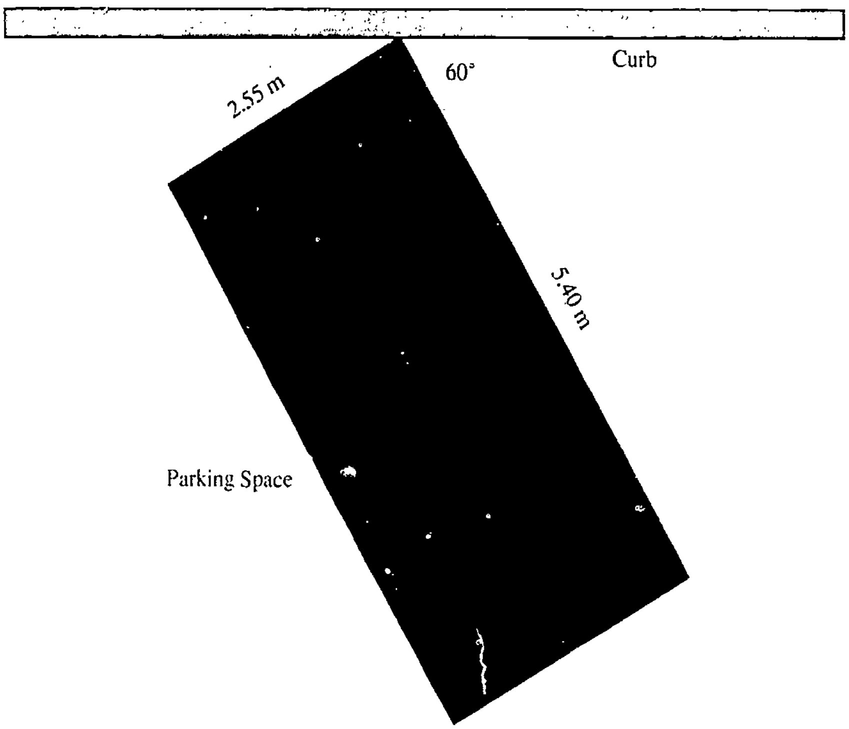
$(3)(x) = 4767\text{ mm} - 2000\text{ mm}$

$x = \frac{2767\text{ mm}}{3}$

$x = 922\text{ mm}$

PROBLEM 4 *Parking Spaces*

An average automobile requires an angle parking space with the dimensions of 2.55 m width and 5.40 m length. If the spaces are being calculated for parallel parking, each automobile requires an additional length of 1.20 m as room to manoeuvre. A small town's main street currently uses 60° angle parking:



The town council has contracted you to provide information for town planning decisions regarding parking capacity.

1. Develop a formula for the number of spaces (N) for a given curb length (L) for 60° angle parking.

2. Two years later, increased traffic along the main street makes angle parking unsafe. The town council wants to know how many spaces (N) they will have for a given curb length (L) if they switch to parallel parking.

3. The town's main street is 200 m long. If you want to retain the same parking capacity as before, how many additional spaces will have to be developed off main street in order to offset the spaces lost by the switch to parallel parking?

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SKILLS:

- Trigonometry
- Algebra

SUGGESTED COURSES:

Math 10, 20, 23

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

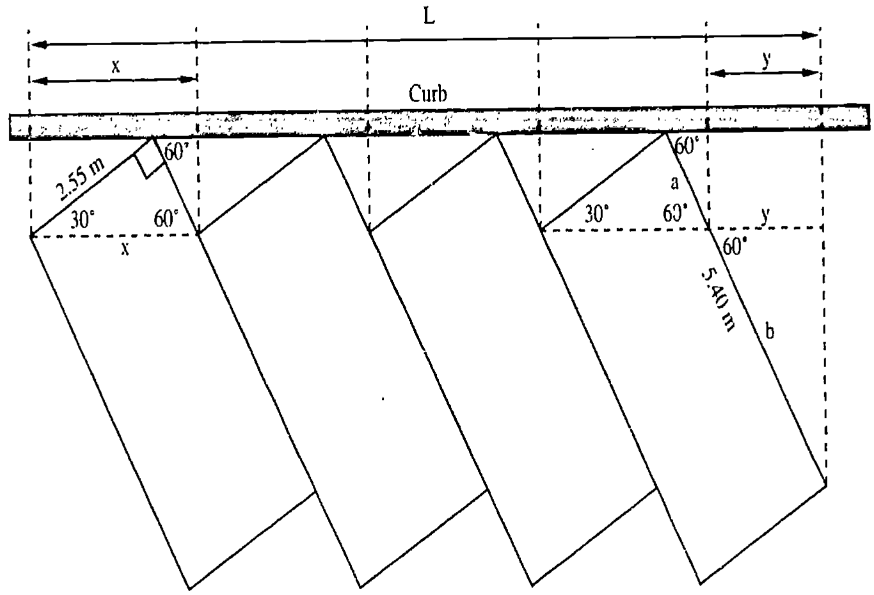
- Investigate the various types of parking around your town or neighborhood.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Establishing a hypothesis as to which type of parking will require more space.
- Sketching a diagram of angle-parked cars to examine how cars park at the end of the curbs on a street.
- Solving a simpler problem by looking at the differences in the types of parking for a given length of curb in your community.
- Developing an algebraic expression for the solution.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Complementary and opposite angles.
- Parallel lines and their angles.
- Alternate interior angles.
- Symmetric repetition.
- Examining whether a car can park in a fraction of a space.
- If parking is allowed on both sides of the street.



1. $$N = \frac{(L - y)}{x}$$

Where N represents the number of angle parking spaces for a given curb length L.

$$\Rightarrow \cos 30° = \frac{adj}{hyp} = \frac{2.55\text{ m}}{x}$$

$$\Rightarrow x = \frac{2.55\text{ m}}{\cos 30°} = 2.94\text{ m}$$

$$N = \frac{(L - y)}{2.94\text{ m}}$$

$$\Rightarrow \cos 60° = \frac{adj}{hyp} = \frac{y}{b}$$

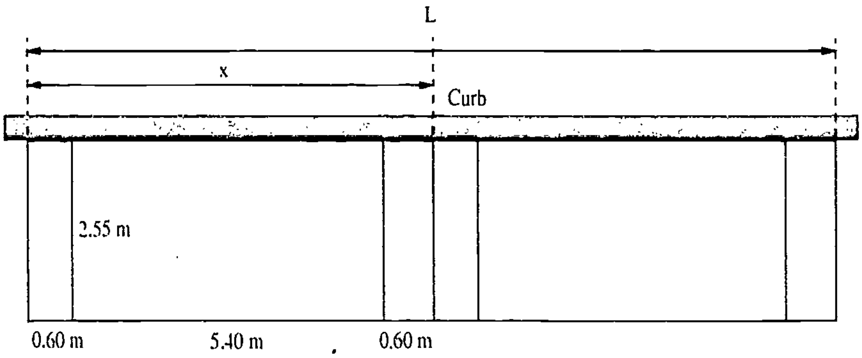
$$\Rightarrow \tan 60° = \frac{opp}{adj} = \frac{2.55\text{ m}}{a}$$

$$\Rightarrow a = \frac{2.55\text{ m}}{\tan 60°} = 1.47\text{ m}$$

$$\Rightarrow b = 5.40 - a = 5.40 - 1.47\text{ m} = 3.93\text{ m}$$

$$\Rightarrow y = (\cos 60°)(b) = (\cos 60°)(3.93\text{ m}) = 1.97\text{ m}$$

$$N = \frac{(L - 1.97\text{ m})}{2.94\text{ m}}$$



2. $N = \frac{L}{x} = \frac{L}{6.60\text{ m}}$

Where N represents the number of parallel parking spaces for a given curb length L.

3. $N = \frac{(200\text{ m} - 1.97\text{ m})}{2.94\text{ m}} = 67$ spaces / side

$(67)(2) = 134$ spaces total for 60° angle parking

$N = \frac{200\text{ m}}{6.60\text{ m}} = 30$ spaces / side

$(30)(2) = 60$ spaces for parallel parking

$134 - 60 = 74$

The town council will have to develop an additional 74 parking spaces off main street.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- If certain ways of parking (angle vs. parallel) are safer than others.
- How town councils arrive at the number of parking spaces required for a given area of town.
- What the difference would be in the number of parking spaces in this town if cars parked perpendicular to the curb.

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SUBMITTED BY:

Peter Hancock, P. Eng.
Assistant Program Head of Telecom Section

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Edmonton, Alberta
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SKILLS:

- Permutations and Combinations

SUGGESTED COURSE:

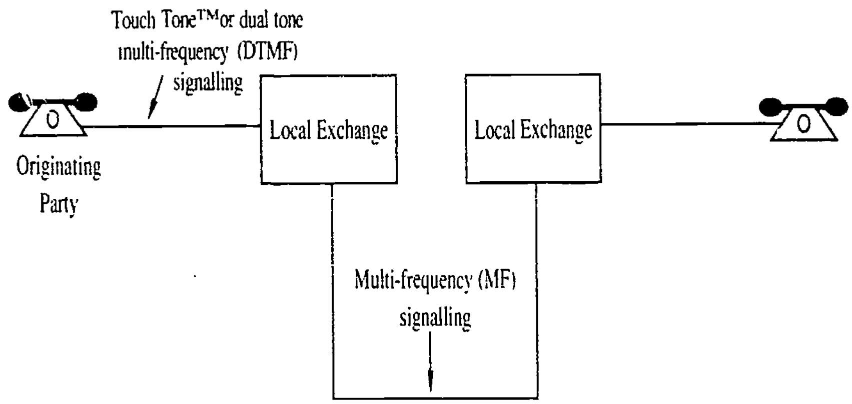
Math 30

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Examine how the rotary pulse (dial) system, which pre-dated the Touch Tone™ type, worked.
- Examine why both systems use two simultaneous tones.
- Examine what signal the local exchange needs to send to the receiving party's telephone to complete the call.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Drawing a sketch of the telephone key pad and labelling the buttons.



A typical telephone call follows the above route using this process. The originating party picks up the telephone and "dials" a number. The number is transmitted to the local exchange via Touch Tone™ or dual tone multi-frequency (DTMF) signalling. This method simultaneously transmits one tone from a group of four high frequencies and one tone from a group of four low frequencies. Once your local exchange knows the number that you want to call, it connects you to the local exchange of the party you are calling. This step is accomplished by an inter-exchange method of signalling known as multi-frequency (MF) signalling. This method of communication works by simultaneously transmitting two of a set of six frequencies. The local exchange of the receiving party then causes the receiving party's telephone to ring.

1. Inter-exchange telephone communications use the MF signalling system whereby two out of a set of six audio tones/frequencies are transmitted to send the digits 0 - 9 and some control codes (control codes signal events such as the beginning and end of a telephone number). How many of these control codes can be sent with this system?

2. The DTMF (Touch Tone™) system for communicating from a telephone to an exchange uses one tone from a group of four low frequencies and one tone from a group of four high frequencies to transmit a digit. How many buttons could you have on your telephone with this system (i.e., how many different combinations are available)? How many buttons are currently on your Touch Tone™ telephone keypad?

™ Bell Canada

PROBLEM 5 *Solution*

1. # of Combinations $\Rightarrow \binom{n}{i} = \dfrac{n!}{(n-i)!\,(i)!}$

$$\Rightarrow \binom{6}{2} = \frac{6!}{(6-2)!\,(2)!}$$

$$\Rightarrow \binom{6}{2} = 15$$

15 combinations - 10 digits = 5 control codes

2. $\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \left(\frac{1}{16}\right)$ or $4 \times 4 = 16$

$\therefore$ 16 combinations

1	2	3
4	5	6
7	8	9
*	0	#

The present telephone keypad uses only a 3 x 4 matrix. It therefore accesses 12 of the 16 possible combinations. A 4 x 4 matrix pad could be used to access all combinations.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

• If the order in which the audio tones are transmitted is important.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

• What keys could be included on a 4 x 4 matrix pad.

• What happens to the completion of the call if two Touch Tone™ buttons are pushed at once.

PROBLEM 6 *Satellite Antenna*

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SKILLS:

- Quadratic Functions
- Quadratic Relations

SUGGESTED COURSES:

Math 20, 30

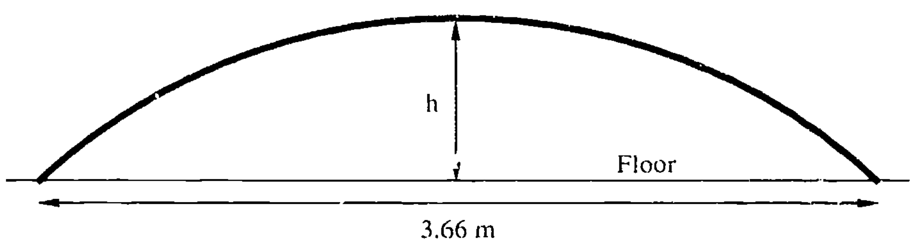
IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Restate the problem in your own words.
- Identify the information you are given and the information that you need.
- Identify what is found at the focus of the dish in a working system.

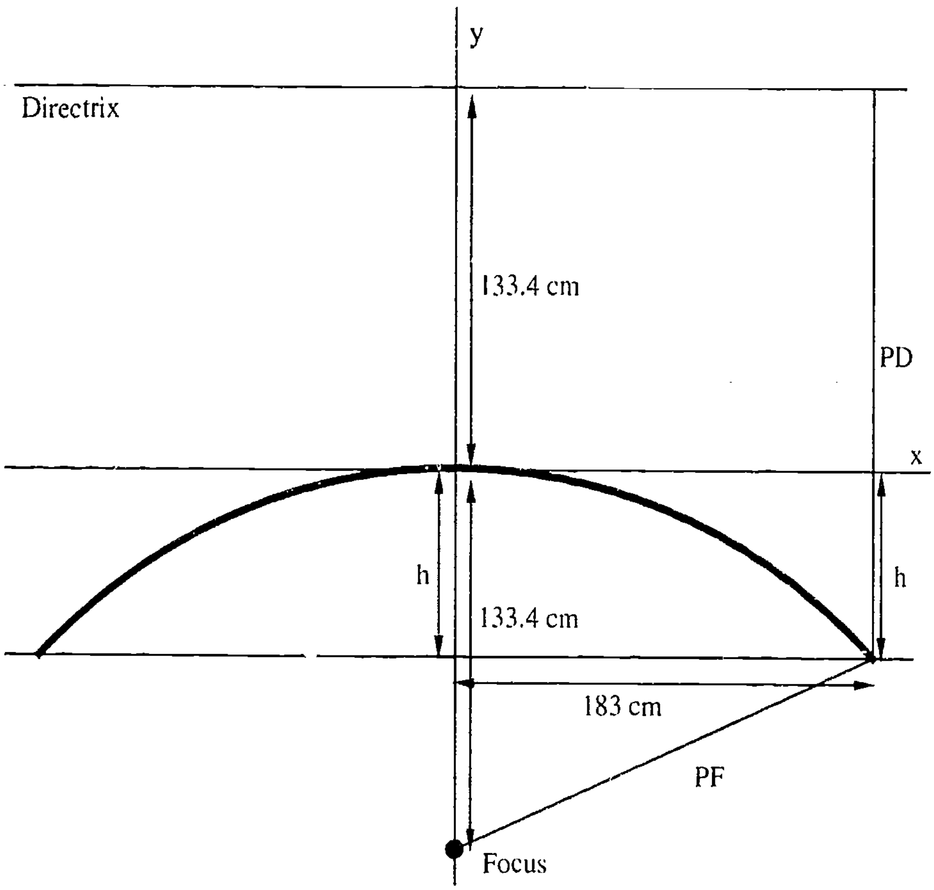
IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Solving a simpler problem by placing the vertex of the parabolic dish at the origin.
- Sketching a diagram and clearly labelling the location of all the given information.

A 3.66 m parabolic satellite antenna consists of a number of sections which are bolted together. There is some tolerance in the bolt holes and it is necessary that the antenna be "fine tuned" mechanically so that it is perfectly parabolic. The idea is to place the antenna on a flat surface, such as the floor, with the sections loosely bolted together. The centre of the dish will be jacked to the correct height. The perimeter of the dish is then clamped to the floor and the bolts tightened. If the focus of the dish is at 133.4 cm, determine the required height (h) to jack up the centre of the dish.



PROBLEM 6 *Solution*



IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Setting up a co-ordinate system for your parabola.
- The eccentricity for a parabola is 1.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- Why a satellite antenna is parabolic.
- Other applications that focus on this property of a parabola.
- Some of the effects if the antenna were an imperfect parabola.

$$e = \frac{|PF|}{|PD|}$$

$$\Rightarrow |PF| = \sqrt{(183)^2 + (133.4 - h)^2}$$

$$\Rightarrow |PD| = 133.4 + h$$

$$\Rightarrow e = 1$$

$$(133.4 + h) = \sqrt{(183)^2 + (133.4 - h)^2}$$

$$(133.4 + h)^2 = (183)^2 + (133.4 - h)^2$$

$$17795.56 + 266.8(h) + h^2 = 33489 + 17795.56 - 266.8(h) + h^2$$

$$533.6(h) = 33489$$

$$h = 62.8 \text{ cm}$$

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PROBLEM 7 *Road Construction*

SUBMITTED BY:

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SKILL:

- Area Calculations

SUGGESTED COURSES:

Math 10, 23

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Investigate why the road is crowned rather than flat on the straight section.
- Investigate why the road is sloped in the direction it is on the curve.
- Investigate why the road on the curve is not crowned in the same fashion as the road on the straight section.
- Examine what the shrinkage and expansion factors represent.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Restating the formula provided in your own words.
- Reviewing the diagrams provided to make sure they make sense to you.

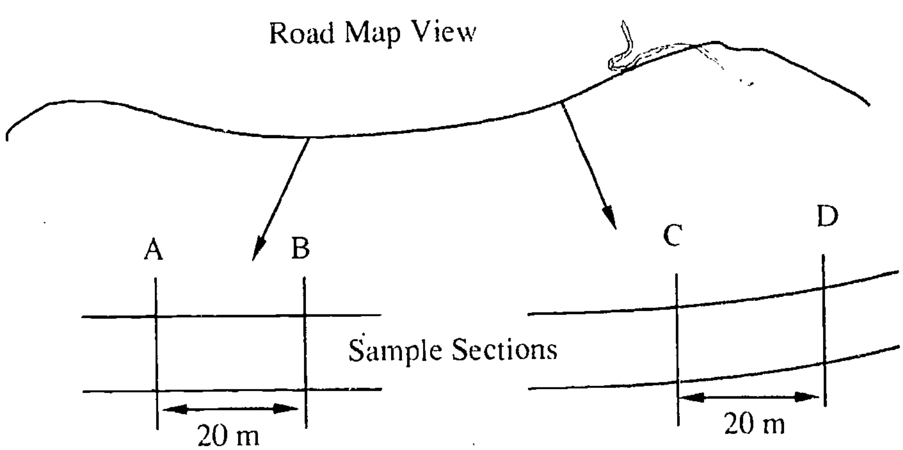
In the design of highways, the volume of material required to construct the highway is calculated from design cross sections. In this example we will calculate the volumes for two sections of the highway, one on a straight section and the other on a curve. The calculated volumes are then adjusted by multiplication factors to account for the shrinkage or expansion of the material. In this example we will use a factor of 1.25 for embankment/fill and 1.0 for cut. It is possible from these calculations to estimate the cost of the project based on quantity of material that must be moved. Answer the following questions using the planning information given on this and the next two pages:

1. Find the end areas and volume of fill (do not adjust with shrinkage or expansion factors) and cut between the two respective cross sections (see the bottom of this page and the following two pages) which are 20 m apart, given:

$$V = \left(\frac{A_1 + A_2}{2}\right) \times L$$

A_1, A_2 = area of cross section
L = distance between cross sections

2. Assuming that the total volume calculated for the 40 m sample is representative throughout a 5 km section of the project, how much would it cost to construct this portion if the cost to move every cubic metre of both fill and embankment (even if they happen to be the same material) is \$2.00/m³? (Note: now adjust volumes using 1.25 multiplication factor for fill.)



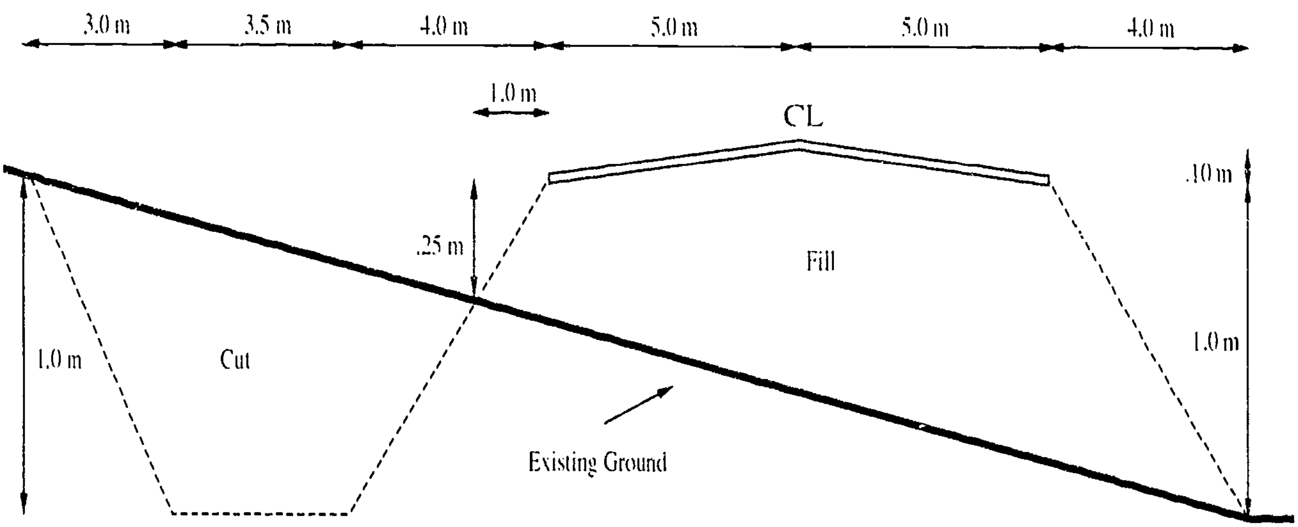
Note the centre line of the road marked by CL and that the cut forms the ditch.

In this problem, only four cross sections are being used. An actual project, however, would involve much more complicated shapes, layers, compaction characteristics and unusable cut such as topsoil, which is unsuitable for subgrade construction. The actual project may involve up to 600 cross sections. Computers are used for these calculations.

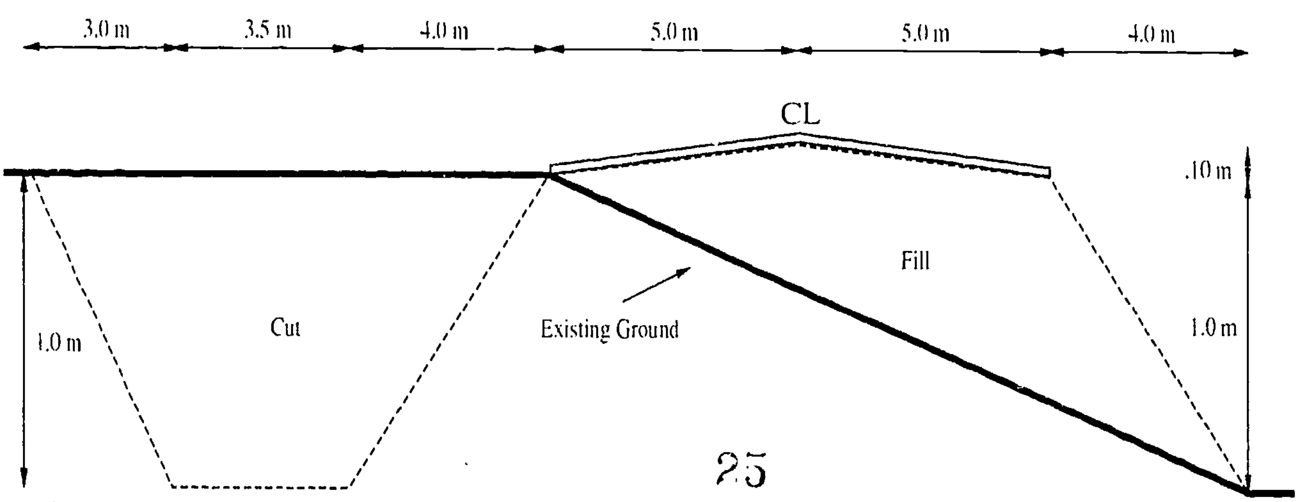
Straight Section Cross Sections 20 m Apart

(Note that horizontal and vertical scales are different.)

Cross Section A



Cross Section B



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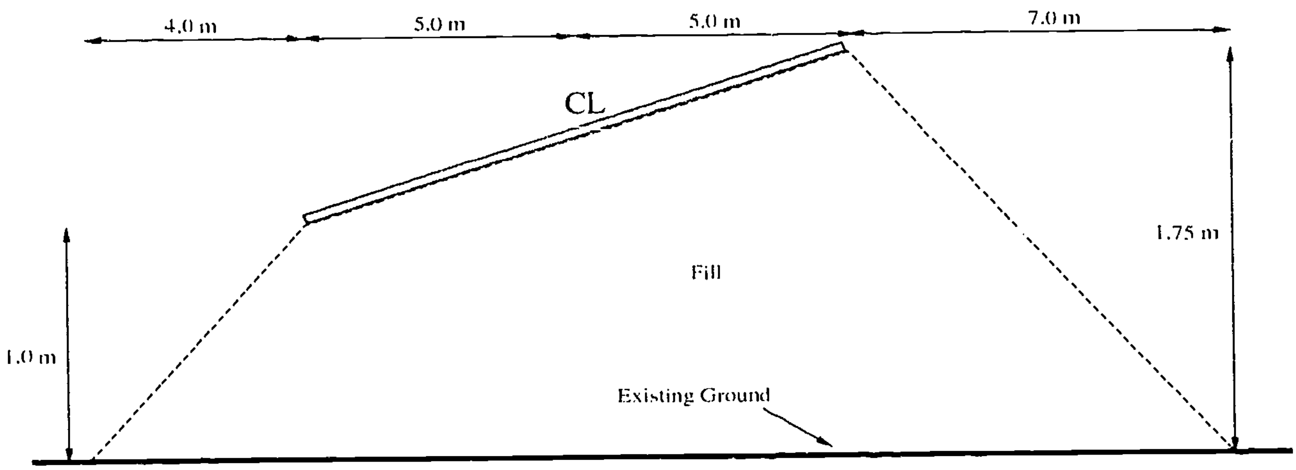
These thin cross sections allow for an accurate computerized integration along the length of the road to be constructed. Nevertheless, solving for these polygon areas instead gives an understanding of the basic process and why manual methods such as planimeters or strip-scaling are seldom used (investigate what planimeters and strip-scaling are).

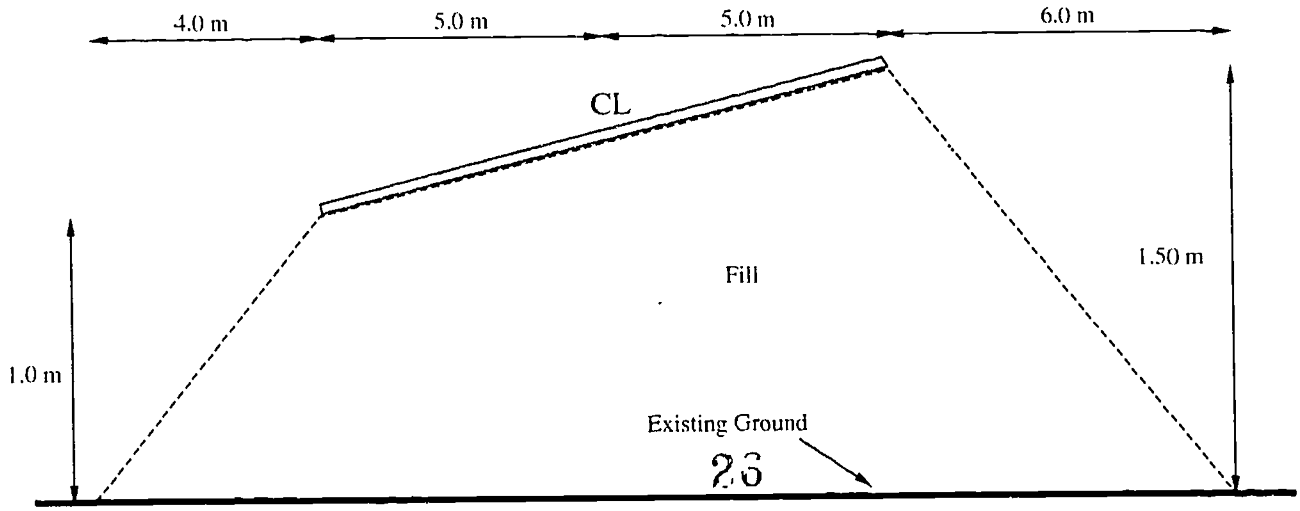
Curve Section Cross Sections 20 m Apart

(Note that horizontal and vertical scales are different.)

Cross Section C

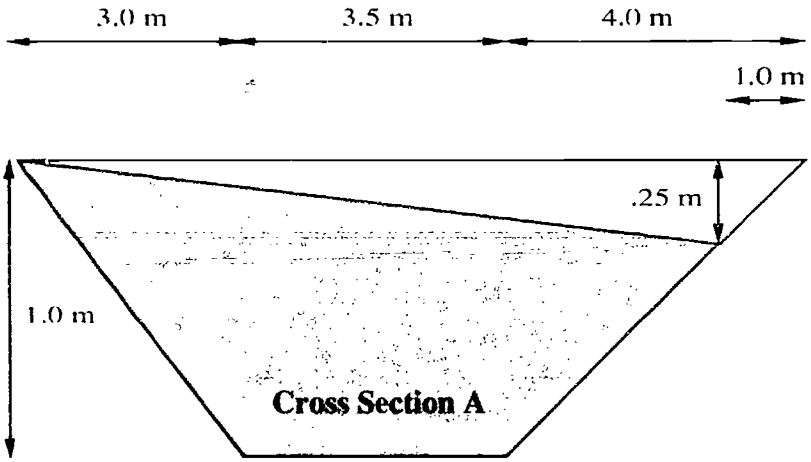


Cross Section D



1. a. Calculate the cut volume required:

Volume straight section cut (V_{sc}):

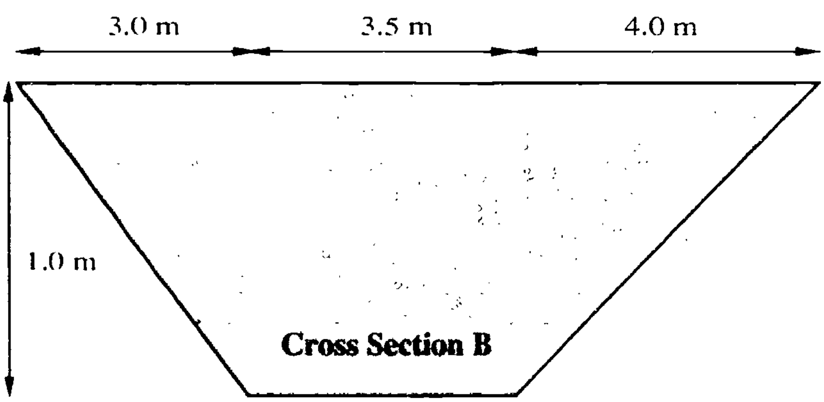


A_1 = area trapezoid - area triangle

$$\Rightarrow \text{area trapezoid} = (1.0\text{m})\left(\frac{10.5\text{ m} + 3.5\text{ m}}{2}\right) = 7.0\text{ m}^2$$

$$\Rightarrow \text{area triangle} = \left(\frac{1}{2}\right)(0.25\text{ m})(10.5\text{ m}) = 1.31\text{ m}^2$$

$$A_1 = 7.0\text{ m}^2 - 1.31\text{ m}^2 = 5.69\text{ m}^2$$



A_2 = area trapezoid

$$= (1.0\text{m})\left(\frac{10.5\text{ m} + 3.5\text{ m}}{2}\right) = 7.0\text{ m}^2$$

$$A_2 = 7.0\text{ m}^2$$

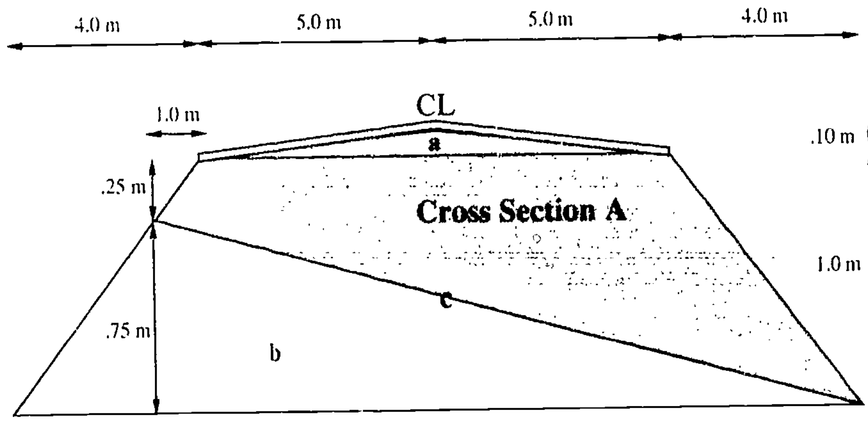
$$V_{sc} = \left(\frac{7.0\text{ m}^2 + 5.69\text{ m}^2}{2}\right)(20\text{ m}) = 126.9\text{ m}^3$$

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Cutting the irregular shapes to create regular shapes.
- Writing a simple computer program to perform these calculations.

1. b. Calculate the fill volume required:

Volume straight section fill (V_{sf}):



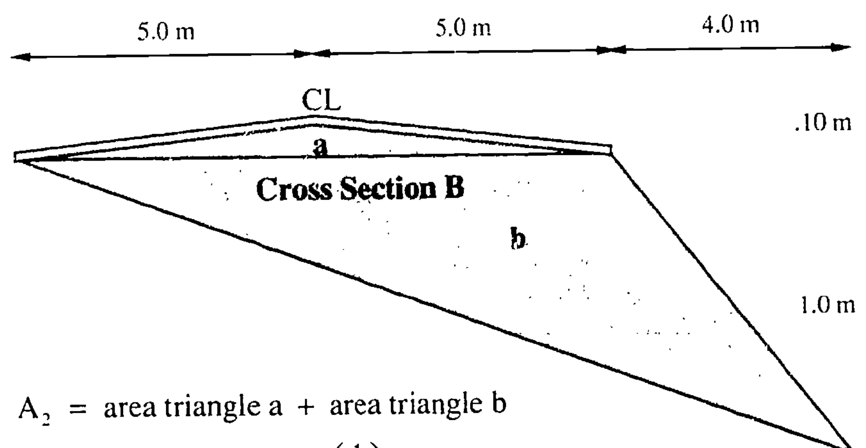
A_1 = area trapeziod c + area triangle a - area triangle b

$$\Rightarrow \text{area trapezoid c} = (1.0\text{ m})\left(\frac{0.75\text{ m} + 18.0\text{ m}}{2}\right) = 14.0\text{ m}^2$$

$$\Rightarrow \text{area triangle a} = \left(\frac{1}{2}\right)(0.1\text{ m})(10.0\text{ m}) = 0.5\text{ m}^2$$

$$\Rightarrow \text{area triangle b} = \left(\frac{1}{2}\right)(0.75\text{ m})(18.0\text{ m}) = 6.75\text{ m}^2$$

$$A_1 = 14.0\text{ m}^2 + 0.5\text{ m}^2 - 6.75\text{ m}^2 = 7.75\text{ m}^2$$



A_2 = area triangle a + area triangle b

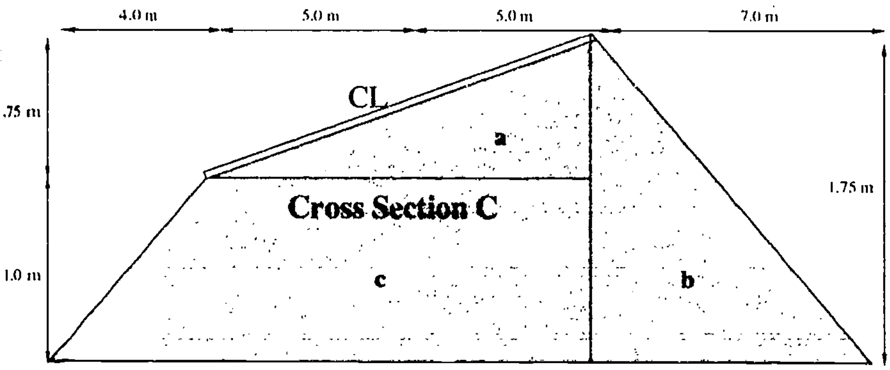
$$\Rightarrow \text{area triangle a} = \left(\frac{1}{2}\right)(0.1\text{ m})(10.0\text{ m}) = 0.5\text{ m}^2$$

$$\Rightarrow \text{area triangle b} = \left(\frac{1}{2}\right)(1.0\text{ m})(10.0\text{ m}) = 5.0\text{ m}^2$$

$$A_2 = 0.5\text{ m}^2 + 5.0\text{ m}^2 = 5.5\text{ m}^2$$

$$V_{sf} = \left(\frac{5.5\text{ m}^2 + 7.75\text{ m}^2}{2}\right)(20\text{ m}) = 132.5\text{ m}^3$$

Volume curve section fill (V_{cf}):



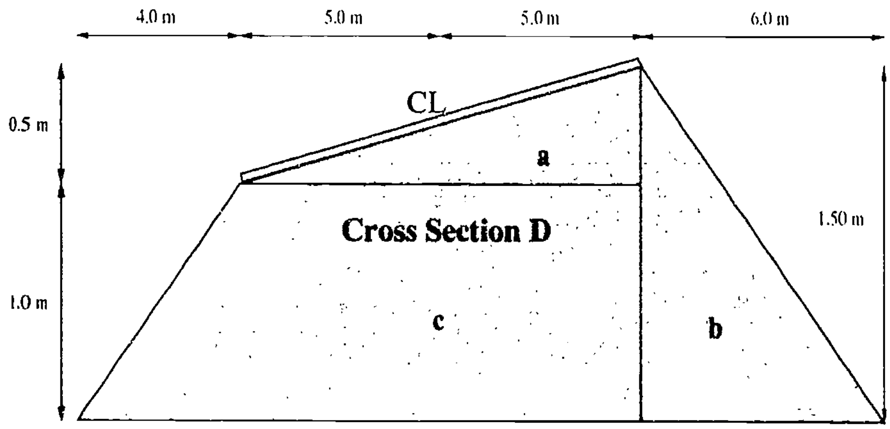
A_1 = area trapeziod c + area triangle a + area triangle b

$$\Rightarrow \text{area trapezoid c} = (1.0\text{ m})\left(\frac{10.0\text{ m} + 14.0\text{ m}}{2}\right) = 12.0\text{ m}^2$$

$$\Rightarrow \text{area triangle a} = \left(\frac{1}{2}\right)(0.75\text{ m})(10.0\text{ m}) = 3.75\text{ m}^2$$

$$\Rightarrow \text{area triangle b} = \left(\frac{1}{2}\right)(1.75\text{ m})(7.0\text{ m}) = 6.125\text{ m}^2$$

$$A_1 = 12.0\text{ m}^2 + 3.75\text{ m}^2 + 6.125\text{ m}^2 = 21.875\text{ m}^2$$



A_1 = area trapeziod c + area triangle a + area triangle b

$$\Rightarrow \text{area trapezoid c} = (1.0\text{ m})\left(\frac{10.0\text{ m} + 14.0\text{ m}}{2}\right) = 12.0\text{ m}^2$$

$$\Rightarrow \text{area triangle a} = \left(\frac{1}{2}\right)(0.5\text{ m})(10.0\text{ m}) = 2.5\text{ m}^2$$

$$\Rightarrow \text{area triangle b} = \left(\frac{1}{2}\right)(1.5\text{ m})(6.0\text{ m}) = 4.5\text{ m}^2$$

$$A_1 = 12.0\text{ m}^2 + 2.5\text{ m}^2 + 4.5\text{ m}^2 = 19.0\text{ m}^2$$

$$V_{cf} = \left(\frac{19.0\text{ m}^2 + 21.875\text{ m}^2}{2}\right)(20\text{ m}) = 408.75\text{ m}^3$$

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IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- Other factors that affect the cost of building a road.

$$V_{total} = V_{cs} + V_{cf} + V_{sf}$$
$$= 126.875\ m^3 + 408.75\ m^3 + 132.5\ m^3$$
$$= 668.125\ m^3$$
$$= 670\ m^3$$

Therefore, the total volume of dirt to be moved is 670 m^3.

2. Calculate the corrected volume for the 40 m, including the fill factors:

$$V_{total} = V_{cs} + (V_{cf} + V_{sf})(1.25)$$
$$= 126.875\ m^3 + (408.75\ m^3 + 132.5\ m^3)(1.25)$$
$$= 803.438\ m^3$$

Now multiply by the number of 40 m sections in 5 km and then the price per m^3:

$$\text{Price} = (803.438\ m^3)\left(\frac{5000\ m}{40\ m}\right)\left(\$\ 2.00/m^3\right)$$
$$= \$\ 200{,}859.50$$
$$= \$\ 200{,}000.00$$

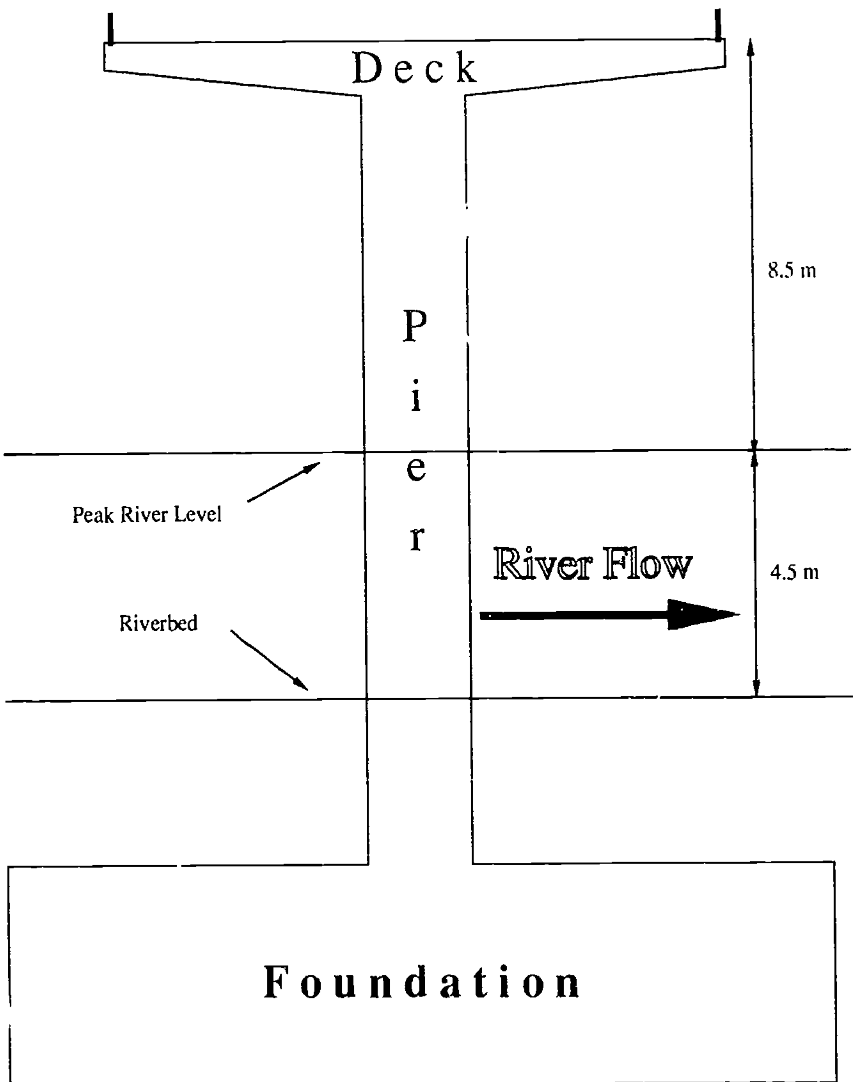
Hence, the cost to construct this portion of the road is $200,000.00.

PROBLEM 8 *Bridge*

In the design of a bridge, an important consideration is what size and shape of foundation is required to hold the weight (load) on the bridge and maintain its position. An engineer therefore has to calculate all the forces on a bridge that will have to be resisted by the foundation. These forces are usually measured in kilonewtons (kN). An engineering study yielded the following information on the structural loads for a proposed bridge design:

- Weight of the bridge deck with maximum traffic load, 500 kN
- Weight of the bridge pier, 400 kN
- Horizontal load on pier from average maximum river flow, 20 kN/m of pier that is under water
- Horizontal load of ice at breakup, 100 kN
- Horizontal load on pier due to maximum wind velocity, 10 kN

Calculate the maximum structural loads on the foundation.



SUBMITTED BY:

Wenona Urquhart-Cook
P. Eng.

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SKILLS:

- Vectors
- Trigonometry

SUGGESTED COURSE:

Math 31

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Examine what types of materials are used to build bridges.
- Examine methods that engineers might use to collect the data presented.
- Examine how and when you would measure the peak river level.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

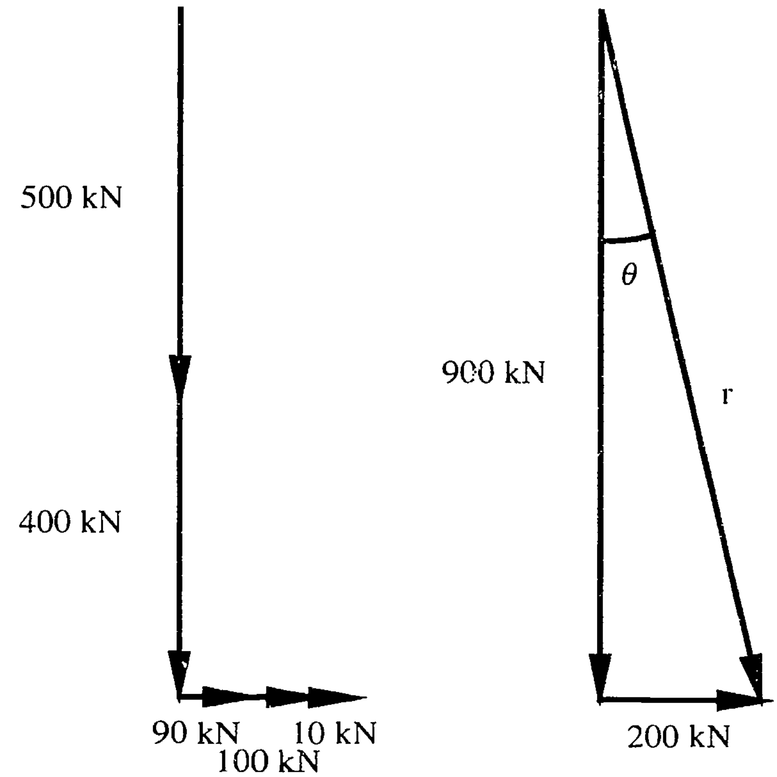
- Sketching a diagram to show the forces acting on the bridge.
- Reviewing trigonometric ratios that may be used in the solution.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- The use of radian measure when determining the angle measurement.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- The implication that this result has on the type of foundation for the bridge and on the area of the foundation.



$$r = \sqrt{(900)^2 + (200)^2}$$
$$= 922 \text{ kN}$$

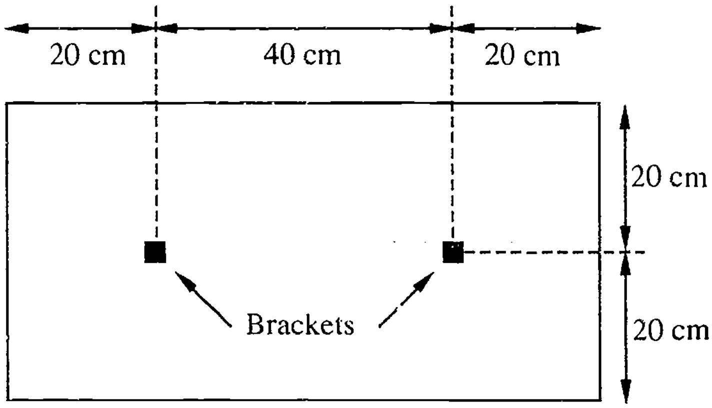
$$\tan\theta = \left(\frac{\text{opp}}{\text{adj}}\right)$$
$$\tan\theta = \left(\frac{200}{900}\right)$$
$$\theta = \arctan\left(\frac{200}{900}\right)$$
$$= 12.5^\circ$$

∴ Maximum structural load is 922 kN @ 12.5° downstream from the vertical.

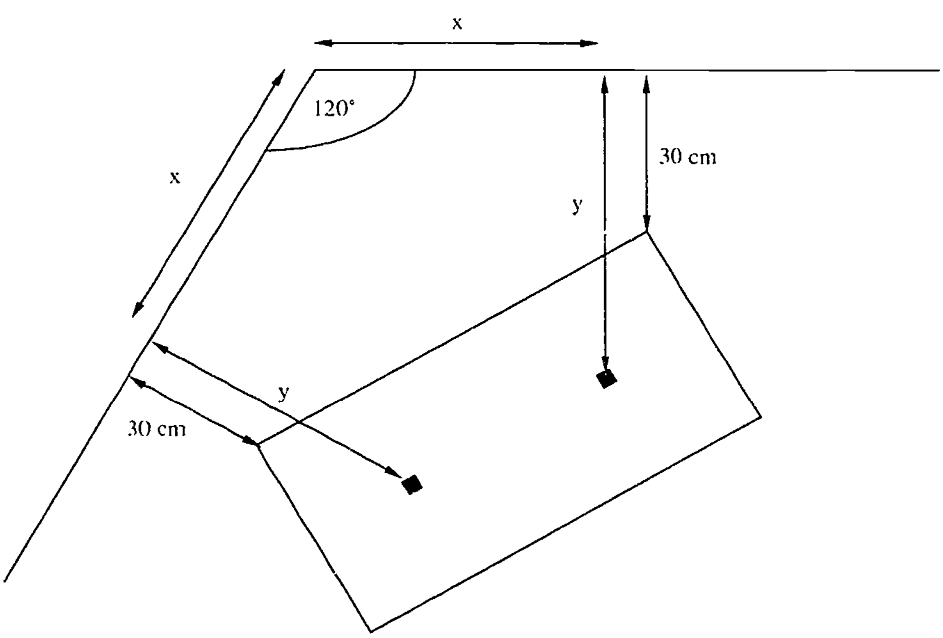
Armed with these data, a civil engineer can calculate the shape and size of foundation required to resist the vertical and horizontal loads. For instance, more vertical surface would be required to resist a greater horizontal load and more horizontal surface would be required to resist a greater vertical load. The magnitude of the force also dictates the total mass of foundation required.

PROBLEM 9 *Heater*

You have to mount a natural gas heater from the ceiling. The heater's mounting brackets are located on top of the heater, like this:



To conserve space, the heater must be mounted in a corner with an angle of 120°. To meet fire regulations, no part of the heater can be closer than 30 cm to the wall. In addition, the heater should be pointed towards the middle of the room for the best heat distribution. Thus, the optimum arrangement is the following:



What perpendicular distance (y) from the wall and distance from the corner (x) do the mounting bolts need to be placed in the ceiling?

SUBMITTED BY:

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SKILLS:

- Trigonometry
- Algebra

SUGGESTED COURSES:

Math 20, 33

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

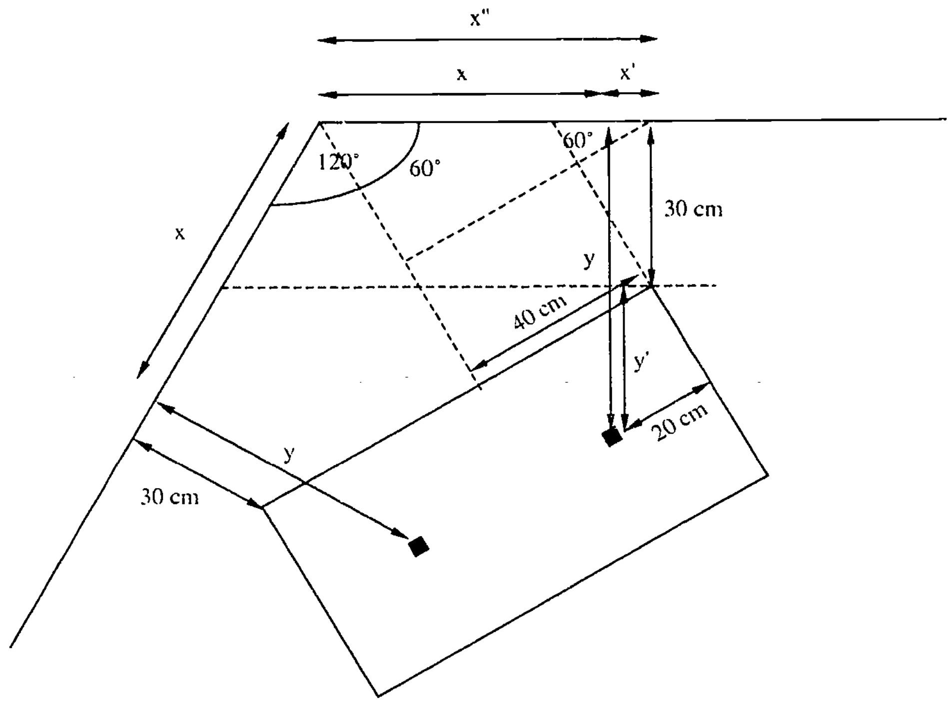
- Examine the fire regulations in your area.
- Examine alternatives to the "optimum" arrangement presented here.
- Determine why this arrangement can be called "optimum".

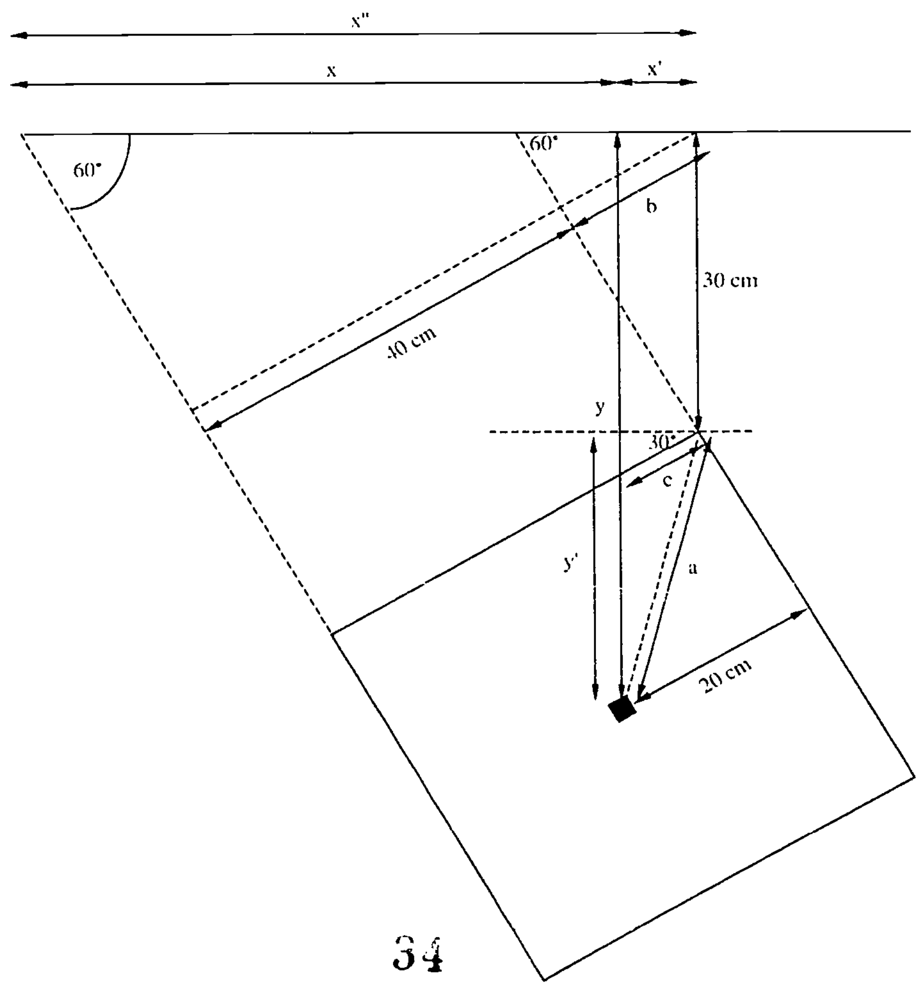
IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Labelling the diagrams in a manner that makes them clear to you.
- Building a model of the problem.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- The symmetry of the situation.
- The sum of angles in a triangle.
- Complementary and supplementary angles.
- Pythagorean theorem.
- Sine law.





$$x = x'' - x'$$

$$x'' = \frac{(40\text{ cm} + b)}{\sin 60°}$$

$$\Rightarrow b = (30\text{ cm})(\sin 30°)$$

$$x'' = \frac{(40\text{ cm} + (30\text{ cm})(\sin 30°))}{\sin 60°} = 63.51\text{ cm}$$

$$x' = (\cos 30°)(c)$$

$$\Rightarrow \frac{c}{\sin 15°} = \frac{a}{\sin 120°}$$

$$\Rightarrow c = \frac{(\sin 15°)(a)}{\sin 120°}$$

$$\Rightarrow\Rightarrow a = \sqrt{20^2 + 20^2} = 28.28\text{ cm}$$

$$\Rightarrow c = \frac{(\sin 15°)(28.28\text{ cm})}{\sin 120°} = 8.45\text{ cm}$$

$$x' = (\cos 30°)(8.45\text{ cm})$$

$$x = 63.51\text{ cm} - 8.45\text{ cm} = 56.19\text{ cm} = 56\text{ cm}$$

$$y = 30\text{ cm} + y'$$

$$y' = \sin(30° + 45°)(a)$$
$$= \sin(75°)(28.28\text{ cm})$$
$$= 27.32\text{ cm}$$

$$y = 30\text{ cm} + 27.32\text{ cm} = 57.32\text{ cm} = 57\text{ cm}$$

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- The differences in the perpendicular distance from the wall and distance from the corner if the distance between the heater's mounting brackets was farther apart or closer together.

PROBLEM 10 *Telephone Network*

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SKILLS:

- Probability
- Algebra

SUGGESTED COURSES:

Math 20, 23

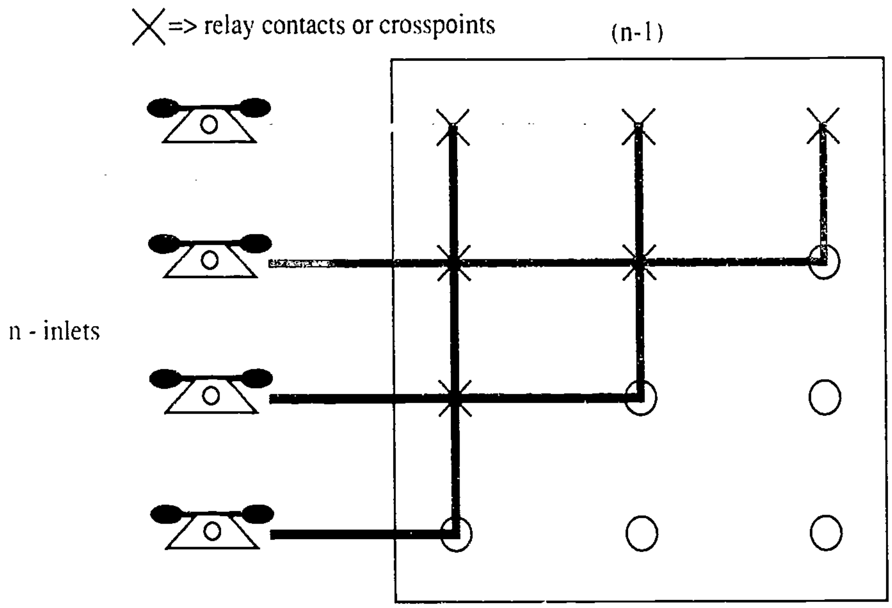
IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Examine similar networks.
- Examine how the first telephone can be connected with the third telephone.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Building a model using graph paper.
- Solving the problem for a fewer number of telephones or a fewer number of subscribers.

A telephone switching machine allows any telephone to be connected to any other telephone. Each telephone has its own line, which can be connected with the line of any other telephone on the network by closing one crosspoint:



1. Consider the switching network and develop a formula which will give the total number of crosspoints (x) for a switch with (n) telephones.

2. For a system with 20 telephones, calculate the number of crosspoints required for the switch.

3. Calculate the maximum number of crosspoints that can be in use with the 20-telephone system. Present this as a percentage of total crosspoints.

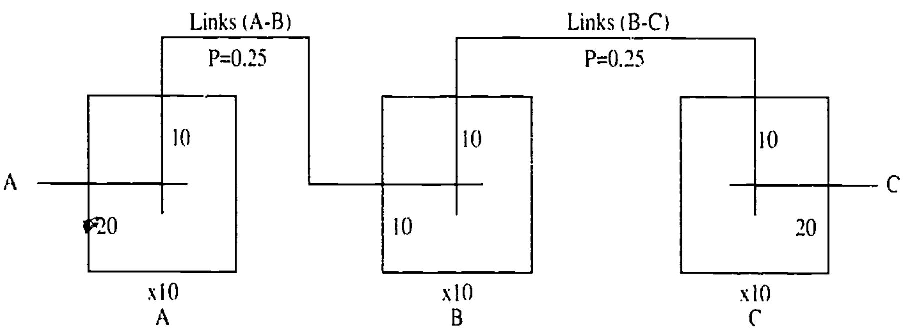
4. A typical central office (telephone exchange) might accommodate 10,000 subscribers. Calculate the size of a single switch that would allow any subscriber to talk to any other subscriber.

5. Calculate the percentage use ratio for the 10,000-line switch.

When n is large, the number of crosspoints in the single switch array becomes astronomical and the use ratio becomes miniscule. Hence, the single stage network is not, in general, practical for large-scale applications.

To minimize the number of crosspoints required, multi-stage networks are employed. In these networks, there are one or more intermediate stages that funnel the calls through fewer connections and switches, allowing any subscriber to talk to any other subscriber with fewer total crosspoints. This is at the expense of risking the possibility that some calls may be blocked because interconnections are shared. However, telephone traffic is random when dealing with a significantly large system. Probability theory can therefore be used to predict the quantity of equipment that will be needed to provide an acceptable level of service under most circumstances. For example, when we dial long distance most calls will get through on the first attempt. On Mother's Day, however, we may find all the circuits busy and will have to dial a number of times.

Consider a multistage design of which the following is one of 10 parallel units, all interconnected to form a three-stage network:



This network would service 200 telephones at each end. The primary (A) stage has 20 inlets and 10 outlets, the secondary (B) stage has 10 inlets and 10 outlets, and the tertiary (C) stage has 10 inlets and 20 outlets. This system is completely symmetrical and the A and C stages could be reversed. Each of the 10 lines out of the A stage connects to a different B stage unit and each of the 10 outlets from the B stage connects to a different C stage. Thus, anyone at A can talk to anyone at C.

6. If every link in the network is busy one quarter of the time (probability of being busy P = 0.25), determine the probability that all 10 of the link pairs are busy (that is, find the probability that a call from A cannot reach a particular outlet at C).

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- The probability of one link pair being busy and then extrapolating to 10 link pairs.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- Suggesting an explanation for the variable quality of long distance calls if telephone networks in general are multistage.
- Using the switch design in the previous parts and suggesting some ways in which two A stage telephones can be connected. Also try to visualize how the network can be "folded back" (enabling anyone from A to contact someone else at A) and draw simple diagrams to solve these problems.
- Assuming that folding back the network simply requires duplicating it backwards from B. Show that the crosspoint saving over one switch is still a factor of 14-16.

1. $$x = \frac{(n)(n-1)}{2}$$

2. $$x = \frac{(20)(20-1)}{2} = 190$$

3. If all n subscribers were connected, only $\left(\frac{n}{2}\right)$ crosspoints can be in use. Therefore, with x equal to the total number of crosspoints:

$$\frac{\left(\frac{n}{2}\right)}{x} = \frac{\left(\frac{20}{2}\right)}{190} = 5.3\% \text{ Usage.}$$

4. $$x = \frac{(10000)(10000-1)}{2} \approx 50 \text{ million}$$

5. $$\frac{\left(\frac{n}{2}\right)}{x} = \frac{\left(\frac{10000}{2}\right)}{50000000} \approx 0.01\% \text{ Usage.}$$

6. Any link busy: $F_B = 0.25$
   Any link idle: $P_I = (1 - 0.25)$

   Probability that both links AB and BC are idle:

   $$P_{2I} = (1 - 0.25)^2$$

   Probability that both links AB and BC are busy:

   $$P_{2B} = \left[1 - (1 - 0.25)^2\right]$$

   If there are 10 link pairs, the probability that all 10 pairs are busy is:

   $$P = \left[1 - (1 - 0.25)^2\right]^{10} = 0.000257$$

   Or fewer than 3 calls in 10,000 will not get through.

PROBLEM 11 *Electrical Utilities*

Electrical utilities must estimate, or forecast, the total amount of energy required for future years. Another measure of special interest which must be forecast is the "peak demand". The peak demand is the highest level of energy which is consumed in an area at one time of the year. The peak demand to be met in future years is a major factor in deciding if, and when, an electrical utility company will build another generating facility. To forecast both the energy needs and the peak demand of the utility's customers, you must know the kinds of customers the utility is serving. A load factor is a way of measuring the way in which customers use power. The load factor captures the relationship between the average demand and the peak demand.

$$\text{Load Factor} = \frac{\text{Average Demand}}{\text{Peak Demand}}$$

Where:

$$\text{Average Demand} = \frac{\text{Energy}_{\text{consumed in a year}}}{\text{hours}_{\text{in a year}}}$$

Therefore:

$$\text{Load Factor} = \frac{\text{Energy}}{(\text{hours})(\text{Peak Demand})}$$

Examples of Load Factors:

A residential customer's peak demand for a year will occur around suppertime on a cold winter day, when dinner is being made, the Christmas lights are on and the car is probably plugged in. A residential customer's average demand will be much lower because not as much power will be used during the summer or when the customer is at work or asleep. A typical residential customer's load factor will be around 50%.

On the other hand, an industrial customer may have an average demand which is very close to peak demand. The business may operate in such a way that everything is turned on almost all the time. An industrial customer's load factor could be very high, say around 85% to 90%.

Thus, represented through the load factor for all customers is the amount of reserve energy production required for an area over and above the average demand of that area. The load factor of the whole system will depend on how many customers with different types of load factors are being served by the electrical utility company in different areas.

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SKILLS:

- Statistics
- Algebra

SUGGESTED COURSES:

Math 10, 13, 23, 33

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Examine what causes the peak power demand to fluctuate.
- Restate the terms "load factor" and "average demand" in your own words.
- Investigate the objects you use that require electricity. Do you know what amount of energy you use in a given time period?

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Solving a simpler problem by predicting the load factors for people in your neighborhood.
- Investigating the need to maintain an awareness of the changes in types of customers.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Examining why an arithmetic average is not used in this problem.
- Comparing the two averages, total and harmonic, to examine their differences.

The following figures are known:

1989 -Peak demand = 5,000 MW
-Energy consumed = 35,000 GWh

1990 -Peak demand = 6,000 MW
-Energy consumed = 38,000 GWh

1991 10% increase forecast for total energy consumption

1. Estimate the peak demand for 1991 using the forecast increase in energy consumption and an average load factor of 1989 and 1990. There are two different ways to average the load factor, bearing in mind that it is a construct: a total average of its components and a harmonic average of the load factors. Calculate both averages and forecast, using the harmonic average, which is the one most commonly used by utilities.

Another important activity for utilities is billing customers. This can be confusing for the consumer as there are various rebates to be taken into account. For instance, in Alberta each utility customer is billed for the amount of kilowatt hours (energy) consumed in a given period. This amount is then reduced by a Provincial Discount, which is 8% of the **gross bill**, and a Temporary Rebate, which is 6% of the **gross bill less the Provincial Discount**. The resulting total is the net billing.

2. You want to check your utility bill. If the net amount (total) is $20.67, find the gross bill, the Provincial Discount and the Temporary Rebate.

PROBLEM 11 *Solution*

1.

1989

$$\text{Load Factor} = \frac{35000 \text{ GWh}}{(8760 \text{ h})(5000 \text{ MW})}$$

$$= 0.7991$$

1990

$$\text{Load Factor} = \frac{38000 \text{ GWh}}{(8760 \text{ h})(6000 \text{ MW})}$$

$$= 0.7230$$

⇒ Harmonic Average:

$$\Rightarrow \text{Load Factor Avg.} = \frac{1}{\left(\frac{(0.7991)^{-1} + (0.7230)^{-1}}{2}\right)}$$

$$\Rightarrow \quad = 0.7591$$

⇒ Total Average:

$$\Rightarrow \text{Load Factor Avg.} = \frac{35000 \text{ GWh} + 38000 \text{ GWh}}{(8760 \text{ h})(5000 \text{ MW} + 6000 \text{ MW})}$$

$$\Rightarrow \quad = 0.7576$$

1991

$$\text{Peak Demand} = \frac{\text{Energy}}{(\text{hours})(\text{Load Factor})}$$

$$= \frac{(1.10)(38000 \text{ GWh})}{(8760 \text{ h})(0.7591)} = 6286 \text{ MW}$$

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IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- "Reserve Margin." An electrical utility includes a reserve margin in the total amount of supply that it estimates is required to meet the needs of its customers.
- How the peak demand would change if all the customers in the system had a load factor of 85% to 90%.
- How an influx of industry into an area would change the system's load factor.
- How the total utility bill changes throughout the year, depending on the weather.

2. x = Gross Bill Amount

$$\$\,20.67 = x - (0.08)x - (x - (0.08)x)(0.06)$$
$$= x(1 - 0.08 - 0.06 + 0.0048)$$
$$= x(0.8648)$$
$$x = \frac{\$\,20.67}{0.8648}$$
$$= \$\,23.90$$

Gross Bill = $ 23.90

Provincial Discount = ($ 23.90)(0.08)
= $ 1.91

Temporary Rebate = ($ 23.90 - $ 1.91)(0.06)
= $ 1.32

The calculation of terminal velocity has many important applications in science and industry. It is used in the design of spacecraft, parachutes and undersea vehicles. In one application of terminal velocity, an important determination is the viscosity of a fluid. The viscosity of a fluid is the measure of how "thick" it is. One can see this difference when pouring water and pouring oil. The terminal velocity is highly dependent on the viscosity of the fluid.

We will calculate the terminal velocity of a metal sphere in water. The calculation depends on three major forces. The first is the buoyant force, which has been known since the time of the ancient Greeks. The buoyant force on an object is equal to the weight of the volume of fluid (in this case water) that it displaces. For a sphere:

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Mass} = \text{Volume x Density of water}$$
$$= \frac{4}{3}\pi r^3(\rho_w)$$

$$\text{Weight} = \text{Mass x Acceleration}$$
$$= \frac{4}{3}\pi r^3(\rho_w)(g)$$

$$\therefore\ F_B = \frac{4}{3}\pi r^3(\rho_w)(g)$$

The second force to consider is gravity, which pulls the object down. For our sphere of density ρ:

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Mass} = \text{Volume x Density of sphere}$$
$$= \frac{4}{3}\pi r^3(\rho)$$

$$\text{Weight} = \text{Mass x Acceleration}$$
$$= \frac{4}{3}\pi r^3(\rho)(g)$$

$$\therefore\ F_g = \frac{4}{3}\pi r^3(\rho)(g)$$

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SUGGESTED COURSES:

Math 30, 31

IN *UNDERSTANDING* THIS PROBLEM YOU MAY WISH TO:

- Restate the problem in your own words.
- Determine a definition for buoyant force.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Developing an experiment to determine whether the viscosity of oil is greater than that of water.
- Examining an application of this effect: in separating liquid droplets from a gas stream, the speed at which a gas stream crosses the top of a vessel determines if a droplet of a given size will be carried over or collected in the vessel.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Checking the final equation with dimensional analysis.
- Reviewing the algebraic expressions for accuracy in terms of describing the situation.

The final force acting on the sphere is the drag of the water it must displace to move. This force is roughly proportional to the cross-sectional area of the sphere, the square of the velocity of the sphere, and the density and viscosity of the water. We shall include the viscosity in a factor called the friction factor, which is approximately equal to 0.4 for a sphere of this diameter in water:

$$F_D \propto \text{Area x } \frac{\text{Velocity}^2}{2} \text{ x Density of Water}$$

$$F_D \approx (0.4)(\pi\ r^2)\left(\frac{V^2}{2}\right)(\rho_w)$$

Given that terminal velocity occurs when the downward force of gravity balances the upward forces of drag and buoyancy (in other words, the total force on the ball is zero so it no longer accelerates) and:

$$g = 9.81\ \text{m/s}^2$$

$$\rho_w = 1000\ \text{kg/m}^3$$

$$\rho_{steel} = 1000\ \text{kg/m}^3$$

$$r = 0.005\ \text{m}$$

find:

1. The terminal velocity of a steel ball in water.

2. The viscosity factor for a fluid whose terminal velocity for the same steel ball is 0.65 m/s and whose density is 918 kg/m^3.

1.

$$F_g = F_B + F_D$$

$$F_B = \frac{4}{3}\pi r^3(\rho_w)(g)$$

$$F_g = \frac{4}{3}\pi r^3(\rho)(g)$$

$$F_D \approx (0.4)(\pi\ r^2)\left(\frac{V^2}{2}\right)(\rho_w)$$

$$F_D = F_g - F_B$$

$$(0.4)(\pi\ r^2)\left(\frac{V^2}{2}\right)(\rho_w) = \frac{4}{3}\pi r^3(\rho)(g) - \frac{4}{3}\pi r^3(\rho_w)(g)$$

$$(0.4)(\pi\ r^2)\left(\frac{V^2}{2}\right)(\rho_w) = \frac{4}{3}\pi r^3(g)((\rho)-(\rho_w))$$

$$V^2 = \frac{20}{3}r(g)\left(\frac{(\rho)-(\rho_w)}{(\rho_w)}\right)$$

$$V = \sqrt{\frac{20}{3}r(g)\left(\frac{(\rho)-(\rho_w)}{(\rho_w)}\right)}$$

$$V = 1.50\ m\ s$$

2.

$$(0.4)(\pi\ r^2)\left(\frac{V^2}{2}\right)(\rho_w) = \frac{4}{3}\pi r^3(g)((\rho)-(\rho_w))$$

$$(0.4) = C$$

$$C = \frac{8}{3}r(g)\left(\frac{(\rho)-(\rho_w)}{(\rho_w)(V^2)}\right)$$

$$C = 2.34 \Rightarrow \text{Greater viscosity.}$$

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- The effect of a larger or smaller radius on the terminal velocity of the steel ball in water.

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- Algebra
- Triginometry

SUGGESTED COURSES:

Math 10, 20, 23, 33

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Discuss why design engineers use a truck and trailer unit to develop the worst case scenario.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

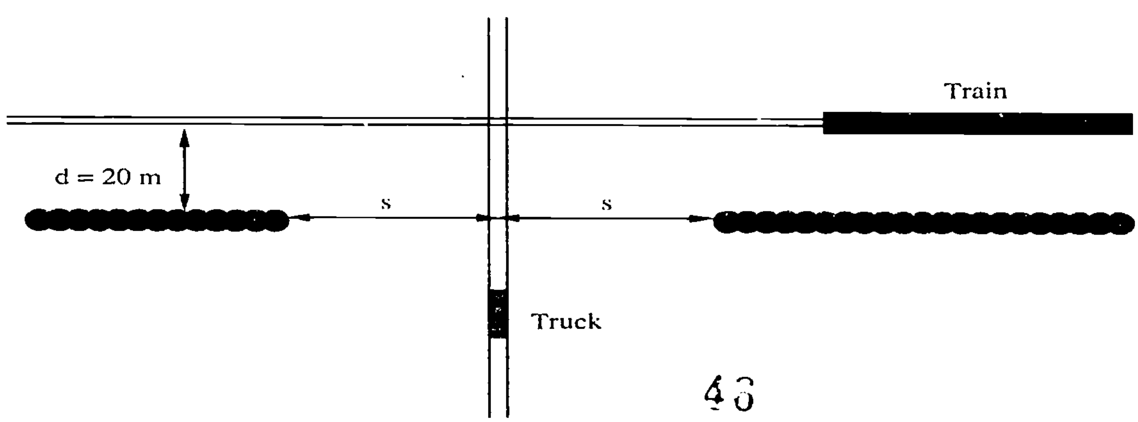
- Finding a railway crossing in your community and examining its visibility.

In the design of an uncontrolled railway crossing, the prime factor is ensuring adequate visibility so that an approaching vehicle can cross safely or stop. The crossing has to be safe for all vehicles, so design engineers often use a truck and trailer unit to investigate a worst case scenario. Use the following data for a truck and trailer unit:

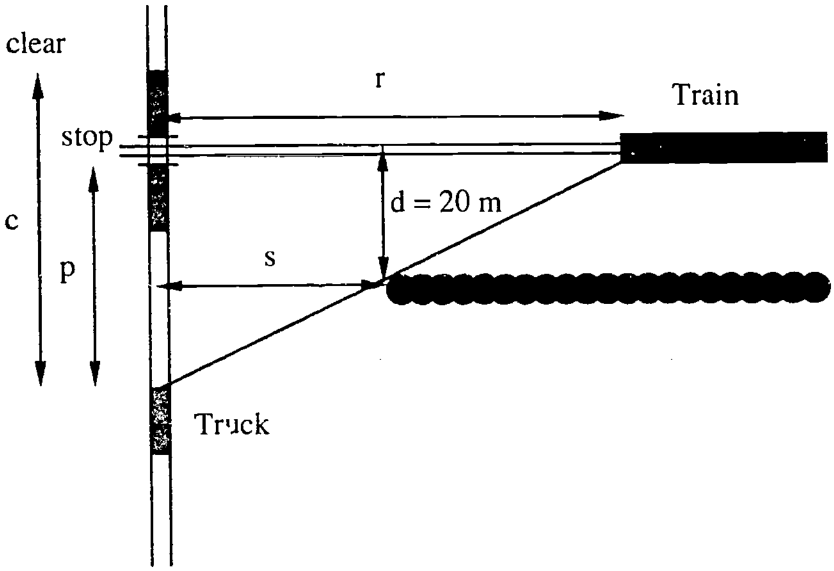
-Length of vehicle = 17 m
-Speed of train = 100 km/h
-Minimum distance from centre of track to safely stop or clear crossing = 5 m
-Speed limit approaching the crossing = 80 km/h
-Following table:

Design Speed	Reaction Time	Reaction Distance	Braking Time	Braking Distance	Minimum Stopping Distance
40 km/h	2.5 s	27.8 m	3.0 s	16.5 m	44.3 m
50 km/h	2.5 s	34.7 m	3.9 s	27.2 m	61.9 m
60 km/h	2.5 s	41.7 m	5.0 s	41.5 m	83.2 m
70 km/h	2.5 s	48.6 m	6.2 s	60.0 m	108.6 m
80 km/h	2.5 s	55.6 m	7.3 s	81.0 m	136.6 m
90 km/h	2.5 s	62.5 m	8.5 s	105.9 m	168.4 m

What minimum distance on either side of the centre of this country road does the row of trees need to be cut back so that a truck driver has the visibility to safely stop or cross. Assume that the driver sees the train immediately upon its coming into view, that the driver's view is from the centre of the road, and that the train's speed is constant.



PROBLEM 13 *Solution*



Total distance to stop = Minimum distance to stop @ 80 km / h
+ Minimum safe distance from centre of track

p = 136.6 m + 5.0 m = 141.6 m

Total distance to clear = p + Length of truck
+ Minimum safe distance from centre of track

c = 141.6m + 17.0 m + 5.0 m = 163.6 m

$$\text{Total time to clear} = \frac{\text{Total distance to clear}}{\text{Speed of truck}}$$

$$= \frac{163.6 \text{ m}}{80 \text{ km / h}}$$

$$= 7.36 \text{ s}$$

Thus, for a safe crossing the train must be at least 7.36 s away, so:

$$r = (7.36 \text{ s})((100 \text{ km / h})$$

$$= 204.5 \text{ m}$$

And by similar triangles:

$$\frac{204.5 \text{ m}}{141.6 \text{ m}} = \frac{(204.5 \text{ m } - s)}{20 \text{ m}}$$

$$28.884 \text{ m} = 204.5 \text{ m } - s$$

$$s = 204.5 \text{ m } - 28.884 \text{ m}$$

$$s = 175.6 \text{ m}$$

Therefore, the row of trees needs to be cut back 175.6 m from the centre of the road.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Using right angled triangles.
- Using similar triangles.

IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- How these results are affected in a town or city where there are buildings alongside a railway crossing.

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SUGGESTED COURSE:

Math 30

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

• Examine what "85th percentile" means.

• Examine how the data were collected.

• Examine the motivation for using the smoothed frequency in the speed cumulative distribution graph.

The behavior of traffic often gives a good indication of the appropriate speed limit which should be applied on a particular highway section. It is universally accepted that at least 85% of drivers operate at speeds which are reasonable and prudent for the prevailing conditions in any particular highway situation. Hence, the 85th percentile speed of a cumulative speed distribution is used as a first approximation of the speed which might be imposed on a section of highway, subject to the consideration of other factors such as accident rate, traffic condition, highway geometry and pedestrian traffic.

In analyzing spot speed data, a number of significant values are obtained. Some of these values are computed directly from the data and others are determined from a graphic representation of the cumulative speed distribution curve.

1. Mean Speed:

This is the arithmetic average of the observed data. It is a measure of the central tendency of the data and is computed from the formula:

$$\bar{x} = \sum_{i=1}^{n} \frac{f_i x_i}{n}$$

2. Standard Deviation:

All vehicles do not travel at the average speed, so there is a spread or dispersion of speeds about the mean. The standard deviation is a statistical measure of this spread. If we are dealing with an approximately normal distribution, the mean ± one standard deviation contains approximately 68% of the vehicles, ± two standard deviations contains 95%, and ± three standard deviations contains 99.8%. The standard deviation is calculated from the formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2}$$

3. Cumulative Speed Distribution Curve:

The calculated cumulative percentages of each group can be plotted as a smooth S-shaped curve where the 85th percentile or any other percentile can be read from the curve.

PROBLEM 14 *Highway Traffic Speeds*

The following is daytime speed data collected in 1988 on all Alberta four-lane, divided highways having a speed limit of 110 km/h for automobiles and 100 km/h for trucks. The total number of automobiles and trucks was 15,703 and 1,973 respectively:

Speed Class km/h	# of Vehicles in Class			Smoothed Frequency			Cumulative % Frequency		
	Autos	Trucks	Both	Autos	Trucks	Both	Autos	Trucks	Both
16	0	2	2	2.0	0.7	2.7	0.01	0.03	0.02
52	6	0	6	3.3	1.7	5.0	0.03	0.12	0.04
62	4	3	7	5.7	1.3	7.0	0.07	0.19	0.08
64	7	1	8	4.7	2.0	6.7	0.10	0.29	0.12
66	3	2	5	5.0	1.7	6.7	0.13	0.37	0.16
68	5	2	7	4.0	3.0	7.0	0.16	0.52	0.20
70	4	5	9	4.7	4.7	9.3	0.19	0.76	0.25
72	5	7	12	6.7	7.0	13.7	0.23	1.12	0.33
74	11	9	20	11.3	11.3	22.7	0.30	1.69	0.46
76	18	18	36	18.3	13.3	31.7	0.42	2.37	0.64
78	26	13	39	35.0	16.7	51.7	0.64	3.21	0.93
80	61	19	80	61.3	21.0	82.3	1.03	4.28	1.39
82	97	31	128	90.7	35.0	125.7	1.61	6.05	2.11
84	114	55	169	121.0	47.3	168.3	2.38	8.45	3.06
86	152	56	208	163.0	68.3	231.3	3.42	11.91	4.37
88	223	94	317	228.3	83.3	311.7	4.88	16.14	6.13
90	310	100	410	312.3	106.0	418.3	6.87	21.51	8.50
92	404	124	528	429.0	132.0	561.0	9.60	28.21	11.68
94	573	172	745	535.3	156.0	691.3	13.01	36.12	15.59
96	629	172	801	699.7	187.0	886.7	17.47	45.60	20.61
98	897	217	1114	850.7	209.0	1059.7	22.89	56.19	26.61
100	1026	238	1264	1044.3	214.0	1258.3	29.55	67.04	33.74
102	1210	187	1397	1175.3	187.3	1362.7	37.04	76.54	41.45
104	1290	137	1427	1244.0	134.0	1378.0	44.97	83.34	49.25
106	1232	78	1310	1263.7	94.7	1358.3	53.02	88.14	56.94
108	1269	69	1338	1283.0	69.3	1352.3	61.20	91.65	64.60
110	1348	61	1409	1244.0	55.3	1299.3	69.13	94.46	71.96
112	1115	36	1151	1149.0	39.3	1188.3	76.45	96.45	78.68
114	984	21	1005	957.0	22.3	979.3	82.55	97.58	84.23
116	772	10	782	758.3	15.3	773.7	87.38	98.36	88.61
118	519	15	534	563.7	10.7	574.3	90.98	98.90	91.86
120	400	7	407	396.0	10.0	406.0	93.50	99.41	94.16
122	269	8	277	289.7	5.7	295.3	95.35	99.70	95.83
124	200	2	202	194.7	3.7	198.3	96.59	99.88	96.95
126	115	1	116	143.0	1.3	144.3	97.50	99.95	97.77
128	114	1	115	94.7	0.7	95.3	98.10	99.98	98.31
130	55	0	55	78.0	0.3	78.3	98.60	100.00	98.75
132	65	0	65	45.0	0.0	45.0	98.88	100.00	99.01
134	15	0	15	37.3	0.0	37.3	99.12	100.00	99.22
136	32	0	32	20.0	0.0	20.0	99.25	100.00	99.33
138	13	0	13	20.7	0.0	20.7	99.38	100.00	99.45
140	17	0	17	14.7	0.0	14.7	99.48	100.00	99.53
142	14	0	14	17.0	0.0	17.0	99.58	100.00	99.63
144	20	0	20	11.7	0.0	11.7	99.66	100.00	99.70
146	1	0	1	14.0	0.0	14.0	99.75	100.00	99.78
148	21	0	21	20.0	0.0	20.0	99.87	100.00	99.89
150	38	0	38	19.7	0.0	19.7	100.00	100.00	100.00

IN DEVELOPING A PLAN, YOU MAY WISH TO CONSIDER:

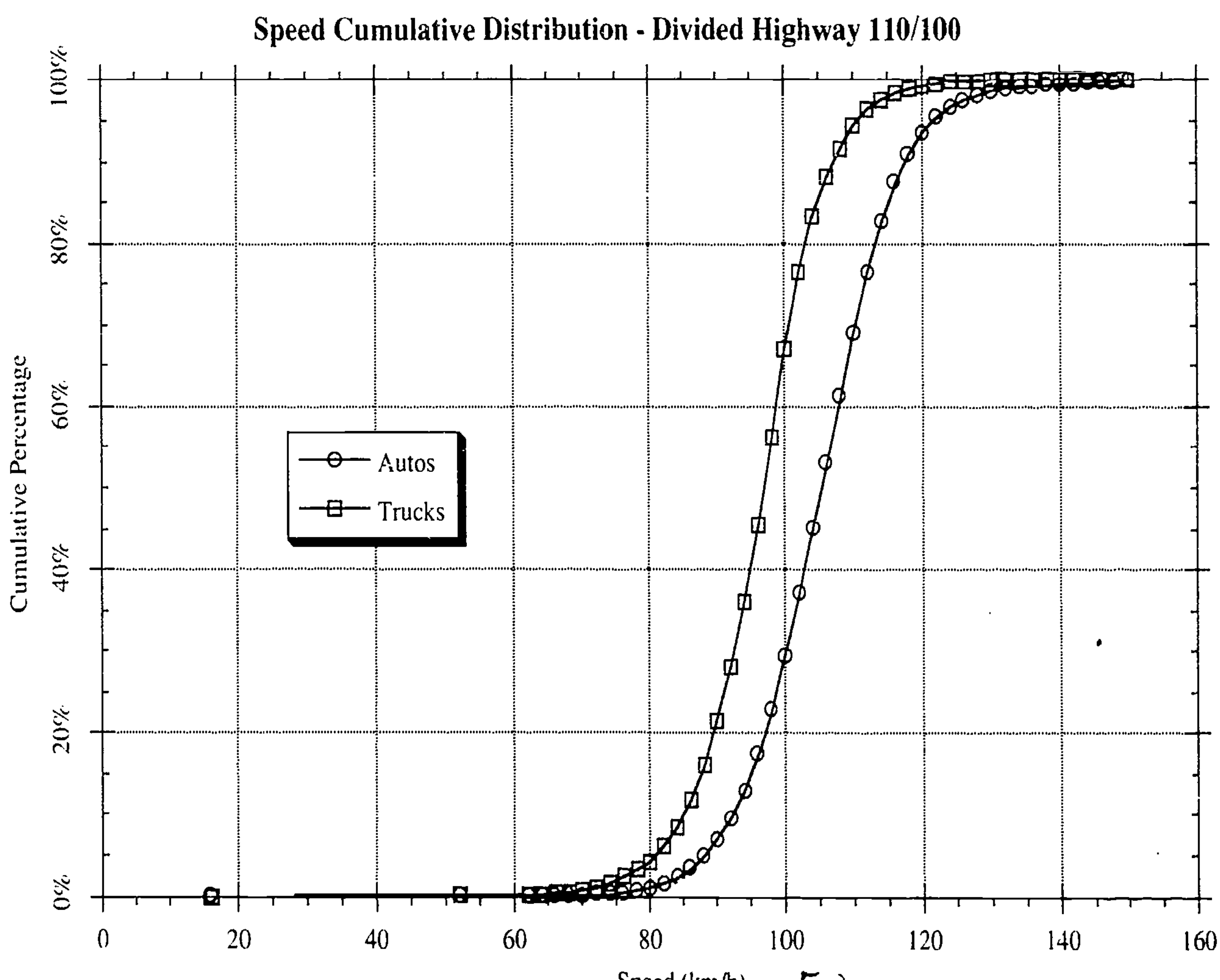
- Using technology to assist with the construction of the graphs.
- Writing a computer program that would analyze these data.
- Reviewing these data to suggest reasons for vehicles travelling 60 km/h.

The graph below is the cumulative distribution curve for the speed of the automobiles and the trucks. Using these data:

1. Calculate the mean and the standard deviation of the speed for automobiles, for trucks and for both combined. Graph the histogram of the frequency of both types of vehicles combined by their speeds. Comment on the distribution and the accuracy of the approximations for standard deviation percentages given in part 2 of the introduction.

2. Find the 85th percentile speeds for automobiles and trucks separately, and comment on possible reasons for their different speeds and the correlation between the limits and actual speeds.

3. If police stop any vehicle exceeding the speed limit by 10 km/h, how many vehicles will be pulled over in one hour if the traffic flow is 720 vehicles/hour and the ratio of trucks to automobiles is the same as that observed in the sample?



1. The mean and standard deviation of the speed are as follows:

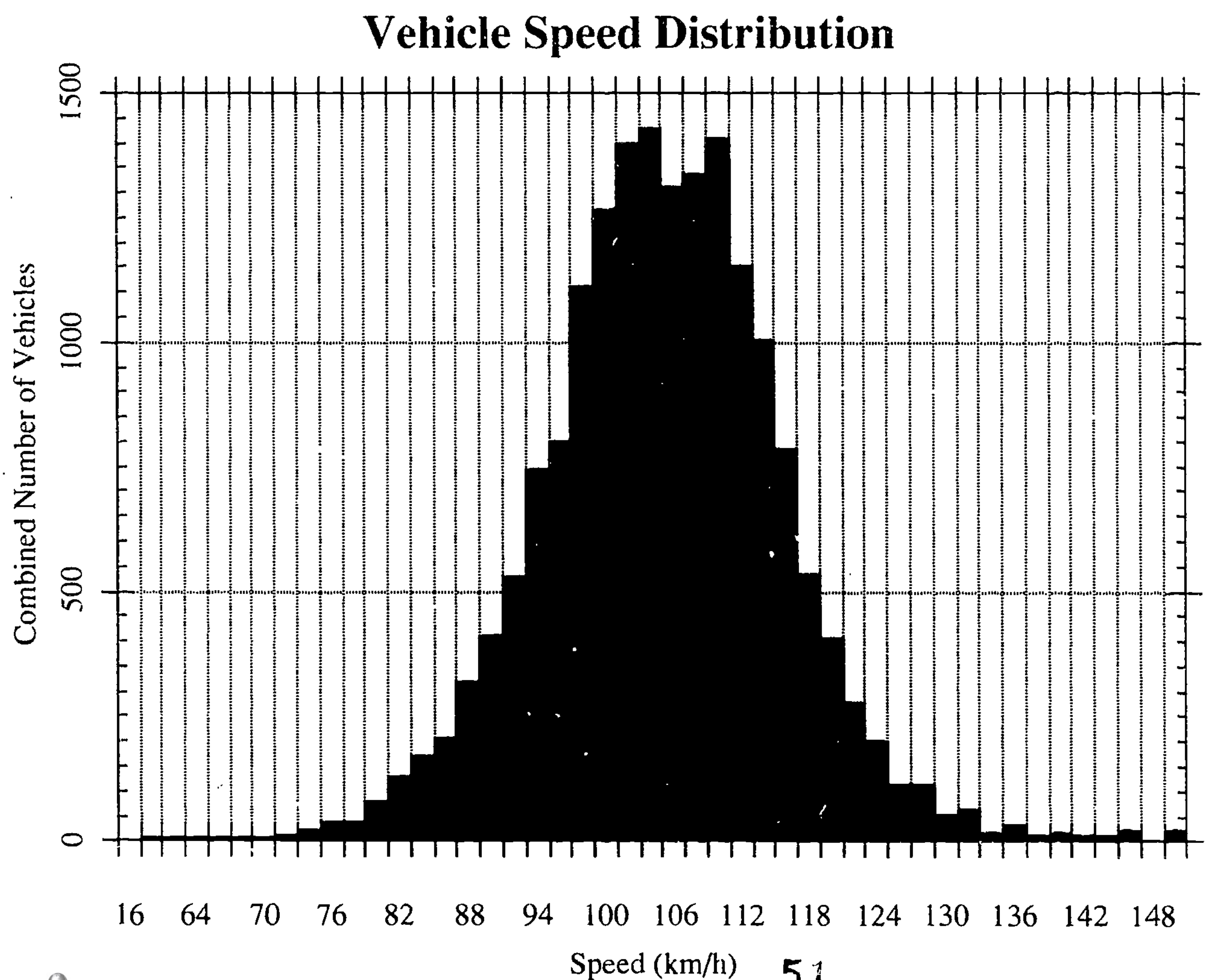
Vehicle	Mean Speed	Standard Deviation
Auto	106.1 km/h	10.5 km/h
Truck	97.2 km/h	9.2 km/h
Both	105.2 km/h	10.8 km/h

The histogram of the frequency data is shown below. It is very close to a normal distribution and thus the approximations of the percentages contained in various standard deviations are justified.

2. Reading from the cumulative distribution graph on the following page, you can see from the 85% line (1) that the 85th percentile speed is (A) 105 km/h for trucks and (C) 115 km/h for automobiles. The actual speed limits enforced on the highways are 100 km/h for trucks and 110 km/h for automobiles.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- The normal distribution and the percentages of data within one, two or three standard deviations of the mean.



IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- The differences in the data and results if you were examining the highway traffic speeds on a two-lane highway (that is, an undivided highway).
- The effect that these results have on establishing speed limits on highways.

3. On the graph below, reading up from 110 km/h (B) to the truck curve and across to (3), you can see that approximately 95% of trucks travel slower than 110 km/h; therefore, 5% of trucks exceed their speed limit by more than 10 km/h. For automobiles, reading up from 120 km/h (D) and across to (2), approximately 6% of automobiles exceed their speed limit by more than 10 km/h. Using the ratio of 15,703 automobiles to 1,973 trucks:

$$15703 + 1973 = 17676 \text{ Total vehicles for ratio}$$

$$\frac{1973}{17676} = \frac{\text{trucks}}{720} \Rightarrow 80 \text{ Trucks per hour}$$

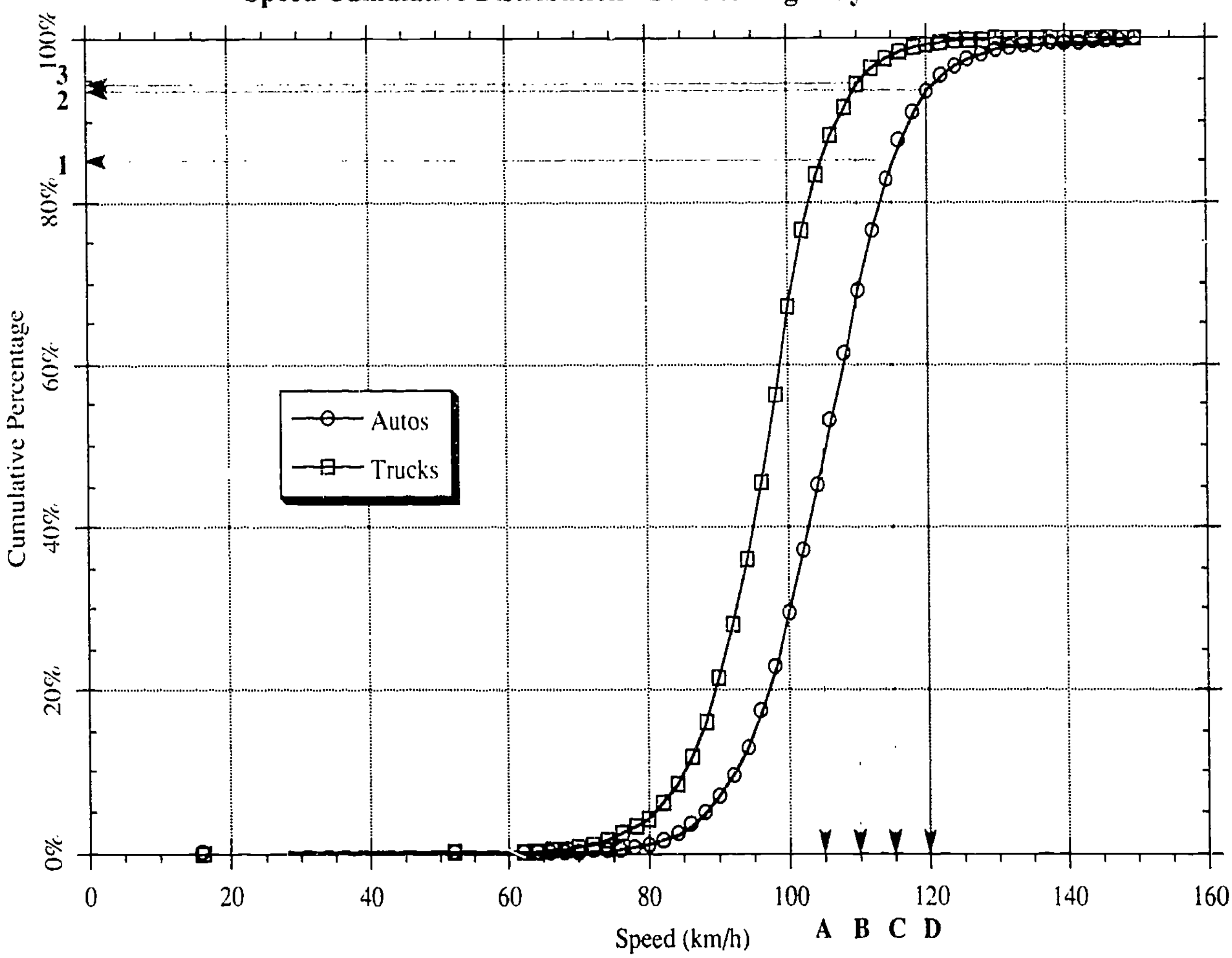
$$720 - 80 = 640 \text{ Automobiles per hour}$$

$$5\% \times 80 = 4 \text{ Trucks speeding per hour}$$

$$6\% \times 640 = 38 \text{ Automobiles speeding per hour}$$

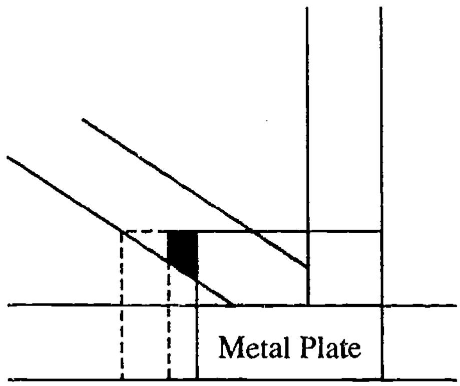
$$\therefore 42 \text{ Vehicles pulled over per hour}$$

Speed Cumulative Distribution - Divided Highway 110/100

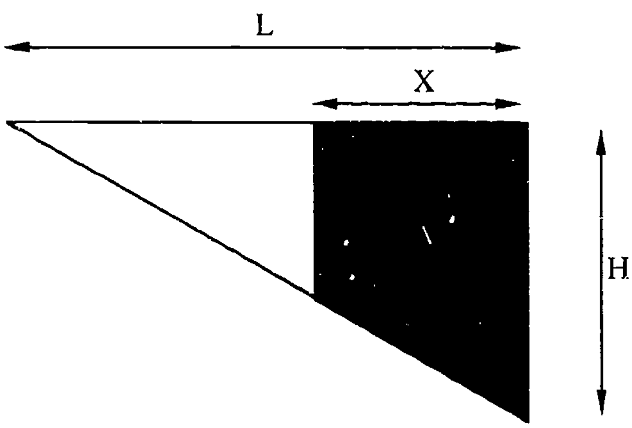


PROBLEM 15 *Metal Plate*

In the construction of wooden trusses and rafters, metal plates are used at the intersection of wooden members to hold them together. There are certain minimum area requirements that the plate must cover on each member in order to form a reliable joint. The joint looks like this:



The lightly shaded section is the existing plate. It has been determined that the wooden member at an angle needs more surface area covered to provide the necessary strength. The only way to do this is by lengthening the plate, as represented above by the dashed lines. For any additional area (A), represented by the black area, develop the formula for the additional plate length required using the following dimensions:



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SKILLS:

- Geometry
- Quadratic Functions

SUGGESTED COURSES:

Math 20, 33

IN *UNDERSTANDING* THIS PROBLEM, YOU MAY WISH TO:

- Investigate a new building to see exactly where the metal plates are used.
- Investigate who establishes the "minimum area requirements" mentioned in the first paragraph.

IN *DEVELOPING A PLAN*, YOU MAY WISH TO CONSIDER:

- Constructing a scale model on grid paper.
- Investigating the use of trigonometry.

IN *CARRYING OUT THE PLAN*, YOU MAY WISH TO CONSIDER:

- Establishing equivalent ratios based on similar triangles.
- The expansion of the product of two binomials.
- The equation for the area of a triangle.

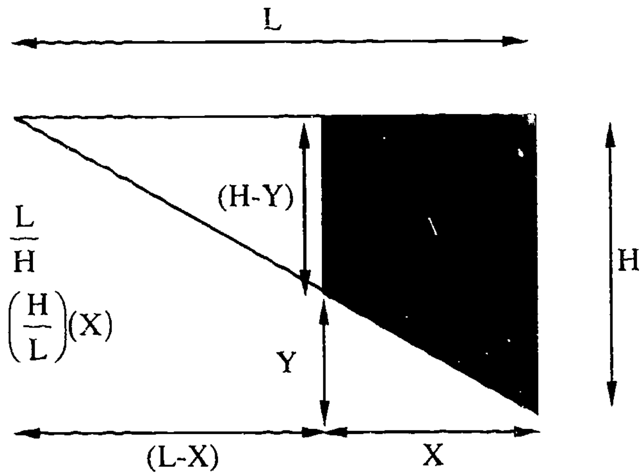
IN *LOOKING BACK*, YOU MAY WISH TO EXAMINE:

- The effect on the formula of changing the minimum area requirements.
- If this formula holds true for all cases.

The maximum area available is:

$$M = \frac{(H)(L)}{2}$$

Looking at the diagram and by similar triangles:



$$\frac{X}{Y} = \frac{L}{H}$$

$$Y = \left(\frac{H}{L}\right)(X)$$

So:

$$\frac{M - (\text{Area not covered})}{M} = \frac{A}{M}$$

$$\frac{HL}{2} - \frac{(L-X)(H-Y)}{2} = A$$

$$HL - (L-X)\left(H - \left(\frac{H}{L}\right)(X)\right) = 2A$$

$$HL - \left(LH - HX - HX + \frac{H}{L}X^2\right) = 2A$$

$$\frac{H}{L}X^2 - 2HX + HL + 2A = HL$$

$$\frac{H}{L}X^2 - 2HX + 2A = 0$$

$$HX^2 - 2HLX + 2AL = 0$$

And by the quadratic formula:

$$X = \frac{2HL - \sqrt{4(HL)^2 - 8HAL}}{2H}$$

$$X = L - \frac{\sqrt{(HL)^2 - 2HAL}}{H}$$

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