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ABSTRACT

Some research on the learning of mathematics and on learning in general suggests that students may acquire knowledge more efficiently when presented with worked examples rather than the traditional lecture-and-then-practice format. Good worked examples may facilitate the development of mathematical schemata, while a means-end approach may retard schema development. In the three studies reported here, this research was extended from a laboratory setting to the classroom in which the topics were presented as part of the algebra curriculum by the regular instructor. The length of the studies ranged from 2 days to 13 days. Students in three algebra classes in an urban public high school were assigned to either a worked example (WE) or a conventional practice (CP) learning condition. On posttest measures (posttests, in-class worksheets, and homework), students in the WE condition did as well or better than students in the CP condition. A number of these differences favoring the WE group were statistically significant ($p < .05$). ANOVAS also found a significant Achievement X Learning Condition interaction with low achievers benefiting more from the worked examples than high achievers and in some cases performing as well as high achievers in either instructional group. Additionally, students in the WE learning condition completed the lessons more quickly, completed more homework, and worked more independently. (Author)

THE USE OF WORKED EXAMPLES IN TEACHING ALGEBRA

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ABSTRACT

Some research on the learning of mathematics and on learning in general suggests that students may acquire knowledge more efficiently when presented with worked examples rather than the traditional lecture-and-then-practice format. Good worked examples may facilitate the development of mathematical schemata, while a means-end approach may retard schema development. In the three studies reported here, this research was extended from a laboratory setting to the classroom in which the topics were presented as part of the algebra curriculum by the regular instructor. The length of the studies ranged from two days to thirteen days. Students in three algebra classes in an urban public high school were assigned to either a worked example (WE) or a conventional practice (CP) learning condition. On posttest measures (posttests, in-class worksheets, and homework), students in the WE condition did as well or better than students in the CP condition. A number of these differences favoring the WE group were statistically significant ($p < .05$). ANOVAs also found a significant Achievement X Learning Condition interaction with low achievers benefiting more from the worked examples than high achievers and in some cases performing as well as high achievers in either instructional group. Additionally, students in the WE learning condition completed the lessons more quickly, completed more homework, and worked more independently.

AIMS

The major purpose of these studies was to extend the work of John Sweller and his colleagues from a laboratory setting to the classroom. Building upon research on domain-specific knowledge, schema theory, and expert-novice differences, Sweller hypothesizes that a greater use of worked examples in the teaching of mathematics and science should facilitate learning by reducing the cognitive load and freeing attention during practice (see J. Sweller, Journal of Educational Psychology, 81(4), 457-466). Furthermore, good worked examples present categories of problems in their initial state and illustrate the correct subsequent moves, the information that is necessary for schema development. In contrast, a conventional practice format and means-end problem solving may interfere with learning because attention will be split between the initial state, the goal state, the current state, and the sub-goals, thereby retarding schema development. In support of their hypotheses, Sweller and his colleagues have found that students who use worked examples spend less time on practice and make fewer errors on posttests than students in a conventional practice condition.

Apart from *cognitive load theory*, there are several reasons why worked examples be a useful instructional tool:

1. Worked examples may greater encourage mental participation on the part of the student. Less time may be spent on lecture and demonstration of procedures and more time on productive problem solving.

2. During conventional practice, a limited number of examples are presented, allowing students to make faulty inductions and construct incorrect procedures. Explicit examples may help to constrain errors during this practice time, when much of the student´s meaning for mathematics is constructed.

3. Many students have no one at home to assist them with high school mathematics. By the time they begin working at home,

decay in learning has taken place, leaving students who already have gaps in their mathematical knowledge unable to solve the homework problems. Worked examples should serve as an extension of the teacher, providing scaffolding during practice at home and in class.

STUDY I: WRITING EQUATIONS

METHOD

SUBJECTS

The subjects were first-year algebra students in a general high school located in a large Midwestern city. The population of students is racially and culturally diverse. Three algebra classes which were being taught by the author were used in this study. Within each of the three classes, students were divided into a worked examples (WE) group and a conventional practice (CP) group. In order to partition the groups, students were given a mathematics test constructed by the author. Students in each class were paired and randomly assigned to either the WE or the CP group. The scores on the placement test were used to define a second variable, Achievement.

PROCEDURE

One week prior to this study, all students were given a 12-item pretest on writing equations from English expressions (e.g., Five less than a number is twelve). Students who had one or no errors took part in the study, but were not included in the analysis. Prior to the instruction, the students had been working with solving algebraic equations.

INSTRUCTION The instructional period began with students reading a short worksheet on writing equations from English expressions. This was followed by a brief instructional period which included solving three practice problems. Following instruction, practice worksheets containing 24 problems were distributed. In the WE group, the worksheet contained 12 problems where key phrases (five less a number) were linked to mathematical terms (x - 5) (see Figure 2). A similar practice problem followed each example. The CP group received a worksheet with the same 24 problems, but no problems were worked out.

During the 20-minute practice time, the instructor was available to assist students. Following practice, the worksheets were collected and a posttest was given. A twenty-item worksheet which differed by group was distributed for homework. Upon returning to class the following day, the homework was collected and a twelve-item test identical to the pretest was given.

RESULTS

Two way analysis of variances (Group X Achievement Level) were run on the test at the end of the first day (CLASSTEST), and at the end of the second day (POSTTEST) with number of errors as the dependent variable. ANOVAs were also run on the GAIN between pretest and posttest, and on HOMEWORK errors (only problems to be solved by both groups). The WE group outperformed the CP group on all four measures (see Table 1). The ANOVAs showed these differences to be significant on CLASSTEST, $F(1,43) = 4.56$, $p < .04$, and on HOMEWORK, $F(1,30) = 5.52$, $p < .03$. Achievement was also significant on CLASSTEST and HOMEWORK with high achievers making fewer errors.

In addition to these main effects, a Group X Achievement interaction was significant on CLASSTEST, $F(1,43) = 5.90$, $p < .02$, and the interaction approached significance on POSTTEST, $p < .10$ and HOMEWORK, $p < .07$. The nature of the interaction was similar in each of these cases and is illustrated in Figure 1. What is striking about the interaction is that the low-achievers in the WE group gained more than any other sub-group and in fact performed about as well as the high achievers in either of these groups.

An ANCOVA was also run on CLASSTEST and POSTTEST, with errors on the pretest as the covariate and errors on the posttest as the dependent variable. Group was significant for CLASSTEST, $F(1,43) = 5.20$, $p < .03$ and approached approached significance on POSTTEST $p < .10$. In both cases, the Group X Pretest interaction was significant, $p < .001$. An analysis of the Betas suggest that for the WE group, performance on the pretest is less of a factor on the posttests than it is for the

CP group.

Apart from quantitative measures, students in the WE group spent less time on practice, required less assistance from the instructor, and made both fewer errors and fewer types of errors on the practice worksheets.

STUDY 2: ADDITION AND SUBTRACTION OF POLYNOMIALS

METHOD

SUBJECTS

The subjects were the same as those in Study 1.

PROCEDURE

The procedures for instruction and practice were identical to Study 1. This study took place over three days, with a posttest given at the beginning of the second and the third day. For this study, each worked example was accompanied by three similar practice problems in response to students' comments which indicated that one worked example per practice problem was too high a ratio. The topic of instruction was adding and subtracting trinomials in both horizontal and vertical formats. Two-way ANOVAS (Group X Achievement) were run on the two posttests, the homework sheets, and in-class worksheets, with errors as the dependent variable. Because the answers involved trinomials, posttests and worksheets were scored both for Single Errors (either correct or incorrect) and for Multiple Errors (up to three errors per problem).

RESULTS

The WE group outperformed the CP group on all eight of the measures (see Table 2). However, these differences were not statistically significant, although the differences did approach significance on Worksheet 1 and 2, M.E., $p < .10$. A criterion of mastery (80% correct) had also been set for each of the posttest. On POSTTEST 1, 58% of the WE students and 29% of the CP students achieved mastery. The chi square statistic was significant, chi square = 4.15, df = 1, $p < .05$. There was no significant difference on POSTTEST 2.

STUDY 3: WHOLE CLASS INSTRUCTION

In the previous studies, relatively short instructional units were given to classes which split into WE and CP groups. Study 3 had the objective of investigating the use of worked examples in a typical classroom setting over an extended period.

METHOD

SUBJECTS

Three algebra classes taught by the author were used. The algebra with support (low achievers) and the honors class (high achievers) were placed in the WE condition and a regular class was placed in the CP condition. The expected performance pattern would be:

Honors Algebra > Regular Algebra > Algebra with Support.

Deviations from this pattern would be of interest.

PROCEDURE

All three classes covered the same topics involving multiplication and factoring or monomials and polynomials. All classes received identical worksheets except that those for the WE classes typically contained an example followed by three or four practice problems (see Figure 2). For the WE classes, lecture was minimized; more attention was given to self-learning while more time was given to initial lecture in the CP class. The instructor was available to assist students during the practice period. The instructional period lasted 12 to 13 days and three tests were given during this time.

RESULTS

T-tests found no significant differences between the WE classes and the CP class on any measures. Group differences varied by test, although the WE group had slightly less errors on the summed score of the three tests as well as on homework.

Performance by class yielded more interesting results (see Table 3). The low achieving classroom, which is populated with students with learning disabilities, low mathematics achievement, and chronic failure did about as well as the other two classes on tests and on homework. While there is no clear pattern between learning conditions, it appears that as in

Study 1, the worked examples may have been especially beneficial to the low achievers who were able to use the examples as scaffolding during practice. In fact, some of the low achievers using worked examples outperformed students in the honors class on test scores.

This gain by low achievers is supported by observations of student behaviors. Students in the low achieving class who typically needed teacher support were often able to use the examples to solve problems correctly and work independently or in small groups. Consequently, the instructor was freed up to work more intensively with individual students who required assistance or to probe misunderstandings. Because less time was spent on initial lecture in the WE classes, more time could be spent on additional practice, review, and enrichment.

DISCUSSION

The three studies reported here are part of a larger body of research on the increased use of worked examples in the algebra classroom. In general, the studies lend support to the suggestion that an extended use of worked examples can be useful in teaching mathematics. Students in the worked example group did as well, and often better, on in-class worksheets, and on homework than did those in a conventional lecture and practice format on posttest. Furthermore, the WE students required less direct instruction and teacher assistance, spent less time on practice, and were often motivated by the examples. Most importantly, it was the at-risk students who seemed to profit the most from the worked examples, as indicated by the Group X Achievement interaction in Study 1 and by the class scores in Study 3.

It is not the author's contention that worked examples should be the primary means of instruction in the mathematics classroom. Traditional *lecture and practice* and *constructivist* ideas of learning are two models of teaching and learning mathematics; providing an increased load of worked examples for self-instruction and support is a third. Students have different learning styles and mathematics topics may require different or multiple means of presentation. In debates on instruction and learning, it is sometimes forgotten that procedural skills often precede conceptual knowledge and deep understanding. Furthermore, arguments between constructivist ideas and explicit teaching often ignore the fact that both are necessary components of a sound instructional program. An increased use of worked examples as a teaching and learning tool in the classroom provides a medium for presenting and reinforcing concepts and procedures, and for providing support as students attempt to construct meaning for mathematical tasks.

Table 1: Study 1: Scores By Group on Posttest Measures

Measure	WE group	n	[illegible]P group	n
Classtest	.9 (.82)	23	2.3 (2.58)	24
Posttest	1.3 (1.16)	20	2.6 (2.66)	23
Gain S.E. (Pretest minus posttest errors)	3.5 (2.16)	20	2.9 (1.77)	23
Homework Errors	1.2 (1.25)	17	2.9 (2.47)	17

NOTE: Standard deviation given in (   )

Only problems solved by both groups are counted for errors on worksheets.

Table 2: Study 2: Posttest Measures By Group

Measure	WE	n	CP	n
Posttest 1 Single Error	2.1 (1.80)	25	2.8 (1.83)	23
Posttest 1 Multiple Error	3.7 (3.77)	25	5.1 (3.98)	23
Posttest 2 Single Error	3.7 (2.43)	20	4.2 (2.78)	23
Posttest 2 Multiple Error	6.6 (5.23)	20	7.3 (5.92)	22
Homework Undone	.9 (2.17)	20	1.9 (3.41)	22
Homework errors	3.6 (2.72)	20	4.3 (2.59)	22
Worksheet 1 Single Error	3.3 (2.13)	25	4.4 (2.61)	22
Worksheet 1 Multiple Error	5.9 (4.82)	25	9.4 (7.08)	22
Worksheet 2 Single Error	3.6 (3.36)	20	4.9 (3.39)	22
Worksheet 2 Multiple Error	5.5 (5.40)	20	9.4 (7.98)	22

NOTE: Standard deviation is given in ( ).

Table 3

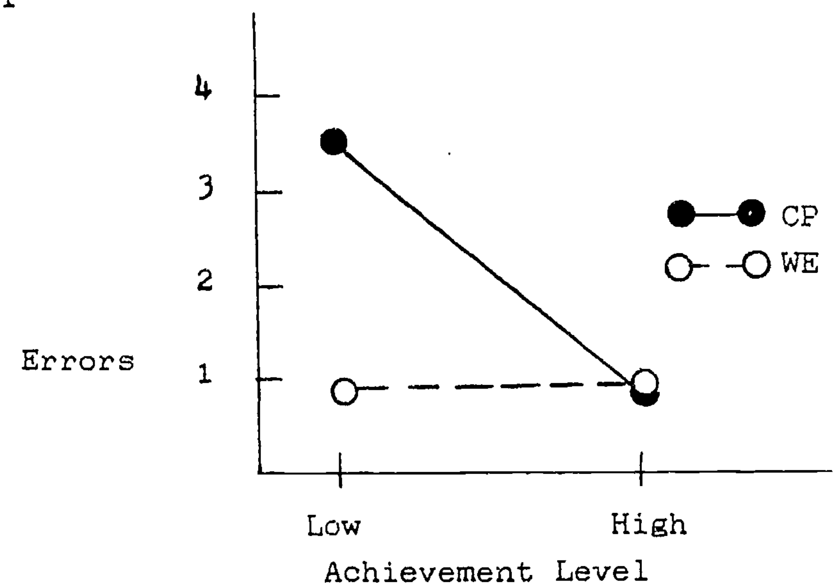
Study 3: Posttest Measures By Class

Measure	Algebra with support (WE)	n	Regular (CP)	n	Honors (WE)	n
Test 1 errors	3.3 (2.28)	17	2.9 (3.34)	13	2.8 (2.11)	17
Test 2 errors	2.7 (2.18)	16	2.4 (3.25)	14	2.7 (2.26)	19
Test 3 errors	1.4 (1.19)	16	2.1 (2.69)	14	1.4 (1.56)	19
Total error on Tests	7.3 (4.95)	16	7.5 (8.04)	14	6.7 (3.99)	17
Homework Average errors	2.7 (1.35)	15	2.7 (1.82)	13	1.7 (1.77)	19
Homework Undone	1.5 (1.93)	16	1.7 (2.09)	14	1.0 (1.33)	19

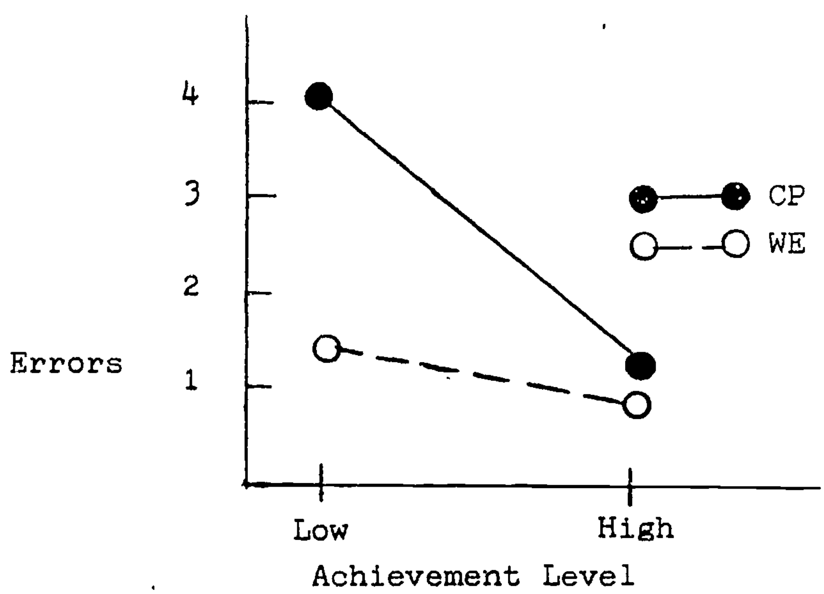
NOTE: Only problems solved by both groups is done on homework.

Homework average error is (errors)/(assignments done)

Figure 1



A. CLASSTEST
Interaction significant at $p < .02$



B. HOMEWORK ERROR
Interaction approaches significance at $p < .07$

FIGURE 2 EXAMPLES FROM WORKSHEETS

STUDY 1

1) Twelve less than a number is ten.

12 less than a number is 10

$$x - 12 = 10$$

2) Nine less than a number is eight.

STUDY 3

Differences of squares:

When Multiplying 2 binomials exactly the same except for one pair of signs, the product is a binimial like $x^2 - y^2$: a difference of perfect squares.

Example: $(x + 4)(x - 4) =$

$$x^2 + 4x - 4x - 16 = x^2 - 16$$

1) $(x + 3)(x - 3) =$

$$(x)^2 - (3)^2 = x^2 - 9$$

2) $(x + 7)(x - 7) =$

3) $(x - 9)(x + 9) =$