DOCUMENT RESUME

ED 351 080 JC 920 559

AUTHOR	Hilbert, Steve; And Others
TITLE	Calculus: An Active Approach with Projects.
INSTITUTION	Ithaca Coll., N.Y.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	Nov 92
NOTE	10p.
PUB TYPE	Reports - Descriptive (141) -- Guides - Classroom Use - Instructional Materials (For Learner) (051)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Calculus; *College Mathematics; Course Descriptions; Courses; Higher Education; Instructional Materials; *Mathematical Concepts; *Mathematics Curriculum; *Mathematics Instruction; Mathematics Materials; Mathematics Skills; Teaching Methods
IDENTIFIERS	Ithaca College NY

ABSTRACT

Ithaca College, in New York, has developed and tested a projects-based first-year calculus course over the last 3 years which uses the graphs of functions and physical phenomena to illustrate and motivate the major concepts of calculus and to introduce students to mathematical modeling. The course curriculum is designed to: (1) emphasize on the unity of calculus; (2) focus on the effective teaching of the central concepts of calculus; (3) increase geometric understanding; (4) teach students to be good problem solvers; and (5) improve attitudes toward mathematics. The course centers on large, often open-ended, problems upon which students work both in and outside of class in groups, and individually, spending 2 to 3 weeks on each problem. Most of these projects are presented in such a way as to require a top-down analysis, in which the top level forces attention to a main idea, while the computations are required at the lowest levels. This approach enables students to recognize calculations as the "nuts and bolts" of a larger problem-solving process. Students' active participation, and clear written presentations of results are required. The course design is best represented by a spiral, which emphasizes the unity of calculus, while allowing for the continual review of the discipline's key skills and concepts, such as graphing, distance and velocity, multiple representations of functions, modeling, and top-down methodology. Three sample course problems are provided. (MAB)

Calculus:
An Active Approach with Projects

Steve Hilbert
John Maceli
Eric Robinson
Diane Schwartz
Stan Seltzer

Ithaca College
Ithaca, New York

November 1992

ITHACA COLLEGE CALCULUS

Supported by grants from the National Science Foundation and Ithaca College, we are teaching a projects-based first-year calculus course developed and classroom tested over the last three years. Our course contains a strong graphical component: we use graphs of functions and physical phenomena to illustrate and motivate the major concepts of calculus and as a means of introducing the students to mathematical modeling.

Our broad goals in this curriculum are to:

- Emphasize the unity of calculus
- Focus clearly on the central concepts of calculus and teach them more effectively
- Increase geometric understanding
- Teach students to be good problem solvers
- Improve attitude toward mathematics

We use large problems, often open-ended, to drive the curriculum; students work on these problems outside of class in groups of about three, spending two to three weeks on each project. Most of the projects are presented in such a way as to require a top-down analysis where the top level forces attention to a main idea, and the computations occur at the lowest levels. The projects require computational skills, but they enable the students to see calculations as the "nuts and bolts" of a larger problem-solving process. The projects focus the students' attention on a problem, often open-ended, that does not have a "five-minute solution" or even a five-hour solution. We insist on clear written presentation of results. However, it is important to emphasize that the projects are not simply added on to a traditional course.

We use a spiral approach in teaching the course, referring to a number of recurring ideas to which we return throughout the course. These "threads" emphasize the unity of calculus and are a source of continual review of the important concepts. Some of these are:

- Graphing
- Distance and velocity
- Multiple representations of functions
- Modeling
- Top-down methodology

Involving students actively is central to our approach. In one of the first classes of the semester the students form groups and the groups work on a problem. The students continue this kind of in-class problem solving activity throughout the semester, sometimes working individually and other times working in groups.

Interested in having one or two of us make a presentation to your department? Contact Steve Hilbert at Department of Mathematics and Computer Science, Ithaca College, Ithaca, New York 14850 (email:calculus@ithaca.edu).

The graph below gives the velocity of a car that is driving along a straight highway through Wichita to Kansas City. When time was 0 the car was in Wichita. The graph gives the velocity for five hours.

A. Write a brief paragraph that describes what the car did during the five hours.

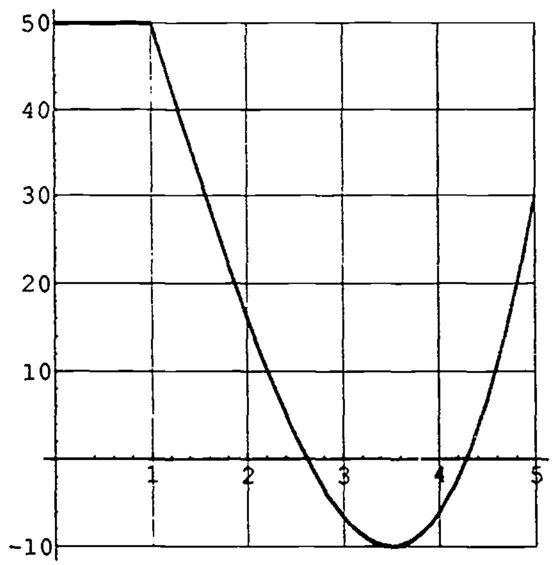
B. Sketch the graph of the distance from Wichita versus time for these five hours.

C. At what time during the five hours was the car closest to Kansas City? Explain.

D. At what time during the last four hours was the car closest to Wichita? Explain.

E. If the distance from Wichita to Kansas City is 150 miles, did the car reach Kansas City during the five hours? Explain.

F. Sketch the graph of the acceleration of the car versus time for these five hours.

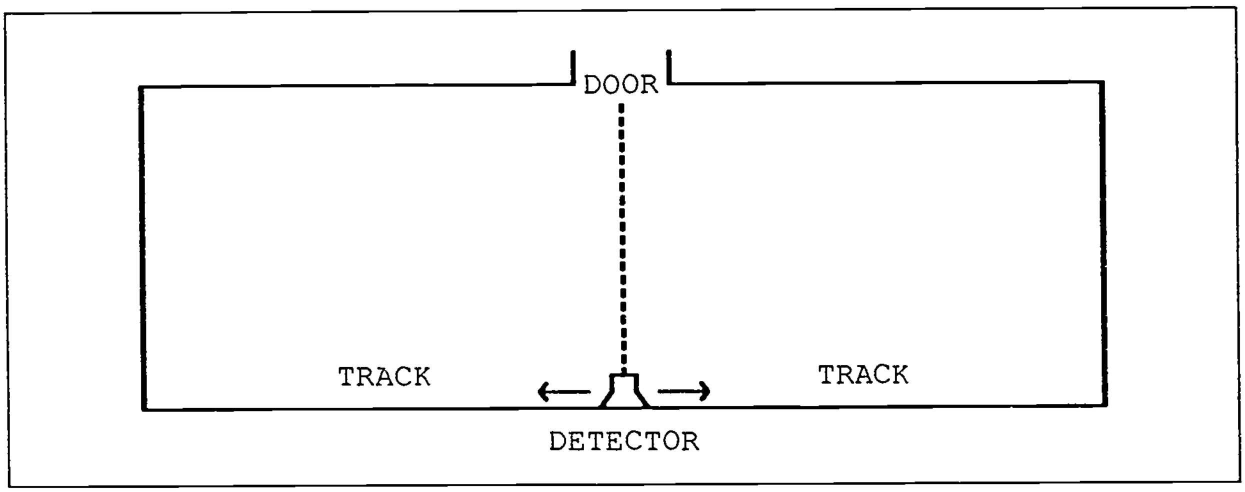


ITHACA COLLEGE CALCULUS

DESIGNING A DETECTOR

You are designing a security system for a hospital. The hospital keeps its supply of drugs in a storeroom whose entrance is located in the middle of a 40 foot long hallway. The entrance is a three-foot wide door. The hospital wishes to monitor the entire hallway as well as the storeroom door. You must decide how to program a detector to accomplish this. The detector runs on a track and points a beam of light straight ahead on the opposite wall. The beam reaches from floor to ceiling. Think of the hallway as a coordinate line with the middle of the door at the origin and the hallway to be watched as the interval [-20, 20]. You need to decide what x(t) is for t, where x(t) represents the position of the beam at time t.

The diagram below shows the beam pointing at the origin (i.e., the middle of the door), so if the detector was at this position at some time t, we would have x(t) = 0. As another example, x(5) = -15 means that the beam is pointing at the part of the wall 15 feet to the left of the middle of the door 5 time units after the detector starts.



Part 1.

A. Draw a graph of x versus time for 10 minutes for what your group thinks is a good choice for x(t). An important part of this is the reasons why you think this is a good choice.

B. The beam must stay on an object for at least one tenth of a second in order to detect that object. If the width of a person is one foot, decide whether your answer to A will detect a person standing anywhere in the hallway. Explain.

C. Investigate whether an intruder could get to the door by walking down the hallway without being detected by your system. Explain how s/he could do it and how likely you think it is. This may inspire you to to revise your answer to A.

D. For your answer to A, compute the longest time that the door will **not** be under surveillance. Remember the door is three feet wide and assume that as long as the beam is hitting any part of the door it is under surveillance.

BONUS. How fast must an intruder be able to unlock, open, and close the door and still avoid being detected?

Part 2.

A. Find a rule (function) for x(t) for the first 10 minutes. This part of your report should include any restrictions on possible rules for x(t) and reasons for these restrictions. For example, x(t) should never be less than -20 because the hall only goes from -20 to 20.

B. The beam must stay on an object for at least one tenth of a second in order to detect that object. If the width of a person is one foot, decide whether your answer to A will detect a person standing anywhere in the hallway. Explain.

C. Investigate whether an intruder could get to the door by walking down the hallway without being detected by your system. Explain how s/he could do it and how likely you think it is. This may inspire you to to revise your answer to A.

D. For your answer to A, compute the longest time interval that the door will **not** be under surveillance.

BONUS. How fast must an intruder be able to unlock, open, and close the door and still avoid being detected?

Part 3.

Compare parts 1 and 2.

Part 4.

Answer 1C and 2C if the person is running.

TAX ASSESSMENT

Due dates

Group meeting with me including detailed solution strategy: by end of first week
All parts due three weeks from today. (Last day for free readthrough: three days before due date.)

A class action suit has been filed against the county board of assessment by a group of landowners unhappy with the assessed value of their properties. Your group will model some assessment schemes and try them out on some lots. Each individual will analyze his/her own lot. Finally your group will analyze one of the disputed lots.

Real estate taxes depend on the assessed value of a property. The value of a piece of property is decided by either a single person (the assessor) or a group (the board of assessment). In most cases, the area of a piece of property is the most important factor in an assessment. However, other factors can give lots with the same area different assessments.

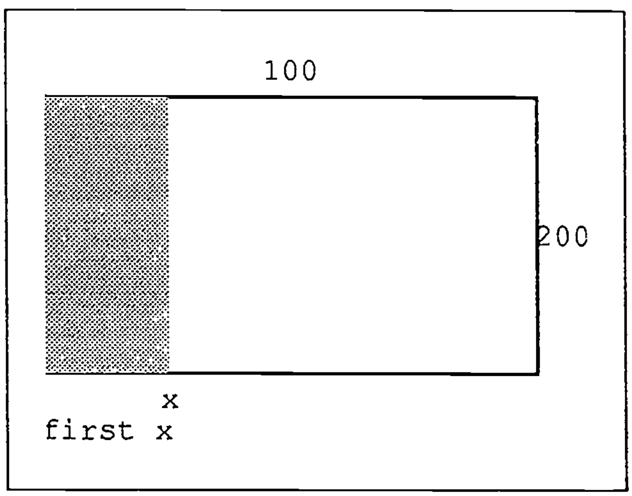
A. For our first example we will look at the simplest case: a rectangular lot that is 200 feet wide and 100 feet deep. This lot is situated in a tax district where land is assessed at $30,000 per acre. (Another way of saying this is to say the tax value of land is $30,000 per acre.)

1. Compute the area of the lot.

2. What is the tax value of the lot?

3. What is the tax value per square foot of the lot?

We will call the <u>front</u> of the lot one of the sides that is 200 feet long. If we draw a line parallel to the front through the lot at a distance x feet from the front, then we will call the part of the lot from the front to the line drawn the <u>first x feet</u> of the lot. See the picture.



4. Find the value of the first x feet in terms of x. (We will refer to this as the first x function, denoted f(x).)

5. Find the percentage of the value of the entire lot that is due to the first x feet in terms of x. (We will refer to this as the percentage due to the first x feet, denoted pf(x).)

6. Graph pf(x) for x in [0,100].

7. What value of x should you chose in order to divide the lot into two lots of **equal tax value**?

B. For many properties, frontage is an important attribute of property. For example, if the lot pictured above had one of its 200-foot boundaries on a busy highway, the lot would have 200 feet of frontage. When such property is assessed, one foot on the highway (along with the property behind it) is called one foot of frontage.

For commercial property, visibility and accessibility are important, so frontage makes a property more valuable. In such a situation, the front 50 feet of the lot should be worth more than the rear 50 feet of the lot. In the same way the first 25 feet should be worth more than the second 25 feet and so on. A method for finding the value of such a lot is to use what we will call a worth function w(x). The worth function, w(x), is defined as the value of a square foot of land whose center is x feet from the highway.

Answer **all** of the following questions (for a 200 foot by 100 foot lot with 200 feet of frontage) for both of the worth functions:

a. w(x) = \$(50 - .30x)/square foot

b. w(x) = \$($50e^{-.03x}$)/square foot

1. Compute the area of the lot.

2. What is the tax value of the lot?

3. What is the average tax value per square foot of the lot?

4. Find the value of the first x feet in terms of x.

5. Find the percentage of the value of the entire lot that is due to the first x in terms of x.

6. Graph pf(x) for x in [0,100].

7. What value of x should you choose in order to divide the lot into two lots of **equal tax value**?

C. Individual. Each member of the group should answer questions 1. through 7. of Part B for the worth function $w(x)= 50 - .30x$ for one of the **trapezoidal** lots described below.

Member 1: Frontage 200 feet, one side is perpendicular to the front with length 100 feet, the rear side is parallel to the front with length 320 feet, the fourth side is a straight line.

Member 2: Frontage 200 feet, one side is perpendicular to the front with length 100 feet, the rear side is parallel to the front with length 75 feet, the fourth side is a straight line.

Member 3: Frontage 90 feet, one side is perpendicular to the front with length 100 feet, the rear side is parallel to the front with length 120 feet, the fourth side is a straight line.

BONUS Also answer for $w(x) = \$50e^{-.03x}$/square foot

D. Group. A lot with 200 feet of frontage and 100 feet deep is bounded on one side by a creek that flows through a culvert that crosses under the highway. The distance of this side of the lot from the side perpendicular to the highway is given by $150 + 50\cos(.01\pi x)$ feet when you are x feet from the highway.

1. Sketch the lot.

2. Find the area of the lot.

3. Find the tax value of the lot for the worth function $w(x) = \$(50/(1+.01x))$ per square foot.

BONUS. What value of x should you choose to divide the lot into two parts of equal value?

E. Group.

1. The lot below was assessed at $1,000,000. Write a letter to the assessor that would support the claim that the land should be assessed for less. The letter should mention the worth function you used, what characteristics a "good" worth function has, and so on.

2. Write a letter from the assessor that justifies the assesment. The letter should mention the worth function you used, what characteristics a "good" worth function has, and so on.