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AUTHOR	Titus, Janet C.
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ABSTRACT

This study investigated hearing-impaired students' understanding of the mathematical concept of fractional numbers, as measured by their ability to determine the order and equivalence of fractional numbers. Twenty-one students (ages 10-16) with hearing impairments were compared with 26 students with normal hearing. The study concluded that hearing-impaired subjects lagged behind their hearing peers in arithmetic computation and the development of the concept of fraction. The hearing-impaired students were capable of ordering the same types of fractions as younger hearing students. In general, they made the same kinds of errors as the younger hearing students, employed strategies in a similar way, and were negatively influenced by the size of the counting numbers composing the fractions. Possible explanations for this developmental delay are explored, and implications are discussed. A copy of the assessment instrument is included in an appendix. (Contains 26 references.) (JDD)

The Concept of Fractional Number Among Hearing-Impaired Students

Janet C. Titus
University of Minnesota
Department of Educational Psychology

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The Concept of Fractional Number Among Hearing-Impaired Students

Relatively little is known about the mathematical knowledge and conceptual development of hearing-impaired students. Since the early 1970s, educators have openly discussed the dearth of information in the area and have called for a shift in attention from research in language development to research in mathematics education, one of a number of neglected areas in the education of many hearing-impaired children (Johnson, 1977; Lang, 1989; Sinatra, 1977; Suppes, 1974). The comparative lack of attention in the hearing-impaired research literature parallels the lack of attention mathematics has received in the classrooms of many hearing-impaired children. The secondary role of mathematics is somewhat surprising given the results of four national assessments. These studies consistently indicated that although hearing-impaired students lag far behind their hearing peers in both reading and mathematics, they score highest on mathematics computation subtests (DiFrancesca, 1972; Gentile & DiFrancesca, 1969; Karchmer & Allen, 1984; Trybus & Karchmer, 1977).

No research to date has been reported on the development of rational number concepts among hearing-impaired children. However, there is clearly a need for hearing-impaired students to be meaningfully educated in rational number topics. Educators from the Model Secondary School for the Deaf (MSSD) and the National Technical Institute for the Deaf (NTID) have noted specific deficiencies in post-secondary hearing-impaired students' knowledge of rational number topics such as fractions, decimals, percents, number lines, ratio, and proportion (Bone et al., 1984).

During the past ten years there has been much research in rational number concepts with hearing children. The importance of rational number concepts lies in their foundational role in the development of proportional reasoning, the "capstone of elementary math" and the "cornerstone of high school math" (Post, Behr, & Lesh, 1983). In particular, the realization that rational numbers have size, just as counting numbers have size, appears to be fundamental to children's development of rational number concepts, relations, and operations, including their ability to order rational numbers, internalize the concept of equivalent fractions, and have a meaningful understanding of the addition and multiplication of fractions (Behr, Lesh, Post, & Silver, 1983; Behr, Wachsmuth, Post, & Lesh, 1984).

Given the fundamental nature of the "quantitative notion" of rational numbers (the realization that rational numbers have size), a number of studies have focused on hearing children's abilities to demonstrate the size of rational numbers. One particular technique developed is an "order and equivalence" task (Behr, Wachsmuth, & Post, 1985), in which children indicate the relative magnitude of fractional numbers presented in a group of at least two fractions. That is, children are instructed to indicate which of the order relations - greater than, less than, or equal to - is true for particular fraction pairs (Behr, Wachsmuth, Post, & Lesh, 1984; Post, Wachsmuth, Lesh, & Behr, 1985). Students are then asked to explain the strategy they used to order the fractions. By focusing on the strategies students employ to determine the order and equivalence of fractional numbers, much has been learned about the conceptual development of their thinking about rational numbers.

Purpose of the Study and Research Questions

The purpose of this study is to investigate hearing-impaired students' understanding of the mathematical concept of fractional number. Specifically, this study focuses on hearing-impaired students' understanding as measured by their ability to determine the order and equivalence of fractional numbers. The specific questions investigated in this study are:

Question 1: What is the level of performance of hearing-impaired students on fractional number problems requiring the determination of order and equivalence, and how does this performance compare with that of their hearing peers?

Question 2: What strategies do hearing-impaired students employ to determine order and equivalence of fractional numbers, and how do these strategies compare with those of their hearing peers?

Method

Table 1 displays the design of the study as well as a listing of the variables and measures employed. The age groups included in the design are meaningful from a fractional number perspective: elementary school students between the ages of 10 and 12 are typically beginning to work substantially with fractions, and junior high school students between the ages of 13 and 15 are typically making the transition to algebra (for which fractional number concepts and proportional reasoning are important foundations.) More detailed descriptions of the variables and measures will follow.

Participants

A total of 47 students, 23 boys and 24 girls, participated in the study. Twenty-one of the students were hearing-impaired while 26 had normal hearing. All students were between 10 and 16 years of age and were enrolled in or had just completed a grade in the 3 through 10 range at the time of data collection.

Hearing-impaired students The sample of 21 hearing-impaired students was recruited from a summer science program located at the Minnesota State Academy for the Deaf (MSAD). The students' characteristics included: an educationally significant hearing impairment (unaided hearing loss greater than 40 decibels across the speech frequencies of 500, 1000, and 2000 Hz); onset of deafness occurring prior to two years of age; and current recipients of special education services. Thirteen students were in the 10-to-12 age group, and eight students were in the 13-to-16 age group. (Although the age range for the older group of students was proposed to be 13 to 15 years of age, data from one 16 year old hearing-impaired girl were included because of the small sample size. This particular student had recently completed the eighth grade.) The average age of the younger students was twelve years one month, while the average age of the older students was fourteen years eleven months.

The hearing-impaired students were from a variety of educational placements: six students attended school at a residential placement on a regular basis, five students attended school in a self-contained classroom, and the remaining 10 students were mainstreamed and receiving itinerant support services. Grade placement ranged from grades three through ten. Specific information on the hearing-impaired students' fractions background during the previous school year was not available to the researcher. However, according to a teacher and researcher associated with the Minnesota State Academy for the Deaf, the mainstreamed students would have been exposed to the same curriculum as their hearing peers, and the residential students (all were students at MSAD) used a curriculum that is used in regular education settings. No information was available on the background of the students from self-contained classrooms. However, it was determined that all hearing-impaired students whose data were used in the final analyses were familiar with fractions and had worked with them in their math classes.

Hearing Students The hearing sample consisted of 26 students who were recruited from two public, regular education classrooms in the Twin Cities metro area; 12 were enrolled in a fourth grade class and 14 were members of an eighth grade pre-algebra class. The average age of the fourth grade students was ten years three months, and the average age of the eighth grade students was fourteen years three months.

Information on the two groups of hearing students' fractions background during the school year was provided by their cooperating teachers. The younger group of students had spent approximately four weeks engaged in a variety of fractions topics, including shaded regions, equivalent fractions, reducing to lowest terms, comparing the sizes of fractions, addition and subtraction of fractions with like denominators, and addition and subtraction of simple fractions with unlike denominators. Students worked in workbooks, the text, and with a variety of manipulatives. The older group of students were introduced to signed fractions and fractions with variables for approximately two and one-half weeks during the first semester of the school year. This knowledge was applied throughout the school year when solving equations with rational number expressions.

All hearing students who participated were considered "average" math students by their teachers' judgements.

Measures

Dependent measure The dependent measure is a two-part fractional number instrument consisting of 18 fraction pairs. Seven distinct types of fractions were included on the instrument. The definitions of the seven fraction types as well as examples of each are displayed in Table 2. These specific categories of fractions have been employed in past research on the order and equivalence of fractions (Behr, Wachsmuth, Post, & Lesh, 1984; Roberts, 1985).

Part 1 of the fractional number instrument consists of 14 items which require students to indicate which of two fractions presented in a pair is the "bigger" value. Students indicate their choice by filling in one of the corresponding three circles below the fraction pair. The 14 items were obtained by including two items from each of the fraction categories listed in Table 2. They are arranged in an assumed easy-to-hard order.

After choosing their answer, students indicate how sure they are their answer is correct by circling one of three sentences: "I am very sure my answer is correct.", "I think my answer is correct.", "I guessed the answer." These options were included in an effort to help control for the relatively high chance of guessing the correct answer on any given item. After each of the three sentences is a small drawing of a face. The faces are graphic depictions of each sentence, and are intended to serve as a language cue for the hearing-impaired students.

Part 2 of the fractional number instrument contains four fraction items similar to those found on Part 1. However, after choosing the larger-valued fraction, students are asked to explain how they solved the problem (indicate their strategy) by drawing a picture or solving a math problem or writing a sentence. This task was included to try to understand the strategies the students employ when deciding which fraction has the larger value. The four items included on Part 2 of the instrument were chosen to represent the full range of difficulty of fraction items.

Both Parts 1 and 2 of the fractional number instrument begin with a set of illustrated directions and four sample problems. To guard against teaching to the task, the four sample problems in each Part contain whole as opposed to fractional numbers. Thus, for the sake of clearly explaining the task at hand, students are asked to order whole numbers as well as explain their strategies when ordering whole numbers. Although it is conceivable the students could be negatively influenced by initially ordering whole numbers, this approach was used for two reasons. First, as mentioned above, the presentation of fractional numbers could serve as an instance of "teaching to the task". And second, because students are essentially being assessed on the solidity of their fractional number concepts, asking them to order whole numbers should not negatively influence them given they truly understand fractional concepts.

Although the example problems as well as the items are identical for both the hearing and hearing-impaired groups, the illustrated directions on the hearing-impaired version are presented in a slightly different way. On the hearing-impaired version of the instrument, small pictures of hand signs are included for a number of key words (Lawrence, 1979; Riekehof, 1978). These are intended as language cues for the hearing-impaired students, and impart no additional information over that found on the hearing version.

Covariate measure To account for possible initial differences between the hearing and hearing-impaired groups, a covariate measure was used. The covariate measure consists of 12 math computation items involving operations on whole numbers and fractions. This seems appropriate to use as one's score on a math computation test will most likely be correlated with one's score on the fractional number instrument. Further, all items are between a first and seventh grade level. This assures that all students would have been exposed to at least some if not all of the items within the range. The items on the covariate measure are arranged in an assumed easy-to-hard order.

A copy of all instruments administered to the students is in Appendix A. The directions for Parts 1 and 2 of the fractional number instrument are those found on the hearing-impaired version of the instrument.

Procedures

Hearing-impaired group administration Data were collected from the hearing-impaired students on two separate occasions. The students were grouped by age, with the younger group (ages 10 to 12) participating one week prior to the older group (ages 12 to 16.) The same procedures were used during each testing.

Students were seated at a number of tables scattered throughout a small room. The chairs and tables were arranged with ample space between them in an effort to discourage cooperation among students while answering the fraction problems; also, all seating was in clear view of the experimenter and the interpreter.

An interpreter skilled in American Sign Language assisted with the explanation of the directions and the administration of the instruments to the hearing-impaired students. Those students needing additional help with the directions were given individual attention. The students proceeded as a group while completing the instruments. The time involved for data collection took approximately one hour with the younger group and 45 minutes with the older group.

Hearing group administration The hearing group data were collected during the late spring of 1990. The explanation of the experimental tasks to the hearing students was identical to those given to the hearing-impaired students with the obvious exception that an interpreter was not needed. The participating fourth grade students completed the tasks in a quiet room in the basement of their school, while the eighth grade students completed the tasks in a reserved section of the school library. In both instances, the experimenter initially arranged the seating with ample space between places in an effort to discourage cooperation among students while answering the fraction problems; also, all seating was in clear view of the experimenter. The time involved for data collection took approximately 45 minutes for each group.

Results

Question 1: What is the level of performance of hearing-impaired students on fractional number problems requiring the determination of order and equivalence, and how does this performance compare with that of their hearing peers?

In order to answer this question, results from Part 1 of the fractional number instrument were analyzed using total scale score and item analysis techniques.

Total scale score analysis

As mentioned previously, data on a covariate measure were collected to account for possible initial differences in mathematics performance between the hearing-impaired and hearing groups. Also, data on the students' level of confidence in their answers was collected to help control for guessing and improve the reliability of the fractional number instrument. However, the data from both of these sources were not used in the total scale score analysis: the assumptions for the analysis of covariance were not satisfied, and a variety of weighting schemes failed to appreciably improve the reliability of the fractional number instrument. Thus, the unweighted data were analyzed using a crossed two-way analysis of variance. (The results of the hearing-impaired and hearing students performance on the covariate measure are included in Appendix B.)

Tables 3 and 4 display the descriptive statistics and analysis of variance table, respectively, for Part 1 of the fractional number instrument. Figure 1 displays the results graphically. As can be seen, main effects were found for hearing status and age group: the hearing students outperformed the hearing-impaired students, and the older students outperformed the younger students. (These results are identical to those found on the mathematics computation measure.) Further, a significant interaction was found, suggesting that performance on the fraction measure was not similar within the hearing-impaired and hearing groups at the two age levels.

Follow-up pairwise t-tests using Scheffe's method ($p < 0.05$) revealed the older hearing students significantly outperformed all other groups of students (older hearing-impaired, younger hearing-impaired, and younger hearing students.) No differences were found between the two age groups of hearing-impaired students, the younger students (hearing-impaired and hearing), and the younger hearing and older hearing-impaired students.

Thus, performance between age groups revealed no difference in order and equivalence concepts for the hearing-impaired students, while a significant difference was observed for the similarly aged groups of hearing students. Further, both age groups of hearing-impaired students performed similarly to the younger hearing students.

Item level analyses

Strategies for correctly ordering fractions are not the same across fraction types. Thus, students' patterns of correct and incorrect response choices were examined by fraction type. The results from these two item analysis procedures (performance by fraction type and an analysis of students' errors) prompted an additional post hoc analysis.

Performance by fraction type Table 5 displays the percent correct for fraction items of each type for the hearing-impaired students; Table 6 contains the analogous information for the hearing students.

Most of the hearing-impaired students as well as the younger hearing students were able to correctly order fractions with like denominators. Ordering positive fractions with like denominators follows the same counting rule as ordering whole numbers: the fraction with the larger-valued numerator is the larger-valued fraction. However, beyond ordering fractions with like denominators, the hearing-impaired and younger hearing students appear to have had difficulty with the task. Although about two-thirds of the older hearing-impaired students and nearly all of the younger hearing students were able to correctly order the "like numerator - unit" fraction pair 1/2 vs. 1/3, ordering the remaining fractions with like denominators (unit and non-unit) was troublesome. (The inconsistency in error rates between the fraction pair 1/2 vs. 1/3 and its counterpart, 1/15 vs. 1/17, may be due to the familiarity of the fractions 1/2 and 1/3.) Further, the majority of the hearing-impaired students and younger hearing students did not recognize equivalent fractions in the form of equivalent multiples; no hearing-impaired students and only one younger hearing child recognized equivalent non-multiples.

The results of the last two fraction types, "non-equivalent multiples" and "non-equivalent non-multiples", were assumed to be the most difficult fractions to order, and yet a sizeable portion of the hearing-impaired and younger hearing students correctly ordered both of the "non-equivalent multiples" as well as one of the two "non-equivalent non-multiple" fraction pairs. It is interesting to note that, in the case of the "non-equivalent multiples", the larger fraction pairs were indeed composed of the "bigger numbers". That it, while 9/10 is a larger fractional value than 3/5, 9/10 is also composed of larger valued counting numbers (9 and 10) than is 3/5 (3 and 5). Further, with respect to "non-equivalent non-multiples", 8/9 is the "bigger number", and it is indeed composed of "bigger" counting numbers. When it comes to 3/11 vs. 11/3, the independent counting numbers composing each fraction are the same. In fact, many hearing-impaired students indicated these particular fractions were equivalent in value.

Perhaps, when faced with a difficult ordering problem (ie: "non-equivalent multiples" and "non-equivalent non-multiples"), the hearing-impaired and younger hearing students were influenced by the values of the independent counting numbers that composed the fraction pairs. Behr, et. al. (1984) found that hearing children in the early stages of fractional conceptual development tended to order fractions based on the whole number values of the individual numbers that composed the fractions (termed "whole number dominance"). This hypothesis will be examined in the post hoc analysis.

The older hearing students had little difficulty with ordering "like denominators" (although the younger hearing students outperformed them on this easiest task) and "like numerators" (both unit and non-unit). They also recognized equivalence, although less so in the form of "equivalent non-multiples". Most older hearing students also correctly ordered the "non-equivalent multiples" and "non-equivalent non-multiples". Their performance on these last two fraction types is not noteworthy in the same way it was for the hearing-impaired and younger hearing students since the majority of the older hearing students were able to correctly order all fraction types.

In summary, when performance by fraction type is examined, the two groups of hearing-impaired and the younger hearing students performed similarly to each other. Although most were able to correctly order fractions with like denominators, their performance on more difficult fraction types was weak. The majority of the older hearing students correctly ordered all fraction types.

Error analysis of students' responses To further investigate students' level of performance, students' incorrect response choices (ie: their errors) were analyzed. What were the popular distractors? Was there anything noteworthy about the errors students made? It is important to remember that the information discussed below refers only to those students who chose an incorrect distractor to an item. A number of trends appeared as a result of the error analysis.

First, both within and between hearing-status', those students who chose an incorrect response choice were in high agreement on the incorrect answer. Although the older hearing students did not make many errors overall and there was no identifiable trend to the errors they did make, even they tended to agree with the hearing-impaired and younger hearing students-in-error on the wrong answer.

Second, both the entire group of hearing-impaired students and the younger hearing students rarely chose the equivalence option. That is, when presented with two fraction pairs and asked to choose the larger of two fractions or indicate if they are equivalent, those students who chose the wrong option almost exclusively chose the remaining fraction option, not the equivalence option. (This was true even when the fractions were equivalent.) The only exception to this trend occurred for the younger hearing-impaired group on the "non-equivalent non-multiple" item "3/11 vs. 11/3". Here, the majority of the students who were in error chose the equivalence option.

Given the hearing-impaired and younger hearing students-in-error did not correctly choose the equivalence option, which option did they choose? On the equivalent items, the hearing-impaired and younger hearing students-in-error chose the fraction with the "bigger numbers", the fractional value composed of "bigger" independent counting numbers.

Recognition of the hearing-impaired and younger hearing students' tendency to identify the equivalent fraction composed of the larger counting numbers as the larger valued fraction as well as their relatively low error rate on the most difficult-to-order fractions prompted a post hoc pattern analysis from the perspective of the "bigger number" hypothesis.

<u>Post hoc pattern analysis of students' responses: The "bigger number" hypothesis</u> The response choices of all fraction pairs were reanalyzed from the viewpoint of the "bigger number" hypothesis: students with a weak conceptual understanding of the order and equivalence of fractions will tend to chose the fraction composed of the larger counting numbers as the larger valued fraction. Behr, et al. (1984) observed this whole number dominance phenomenon in their research.

To carry out this analysis, the percent of students from each group who chose as larger the fraction with the bigger numbers was calculated for each fraction pair. (It is important to note that for five of the fourteen fraction pairs, the fractions composed of the larger valued independent counting numbers were indeed the larger valued fractions. Of these five pairs of fractions, three were presumably the most difficult to order (they fell into the categories "non-equivalent multiples" and "non-equivalent non-multiples")).

The analysis revealed that the majority of the entire group of hearing-impaired students chose the fraction composed of the larger valued counting numbers as the larger valued fraction for <u>all</u> fraction types (median percent endorsement was 77% for the younger hearing-impaired students and 62% for the older hearing-impaired students.) Two exceptions to this trend were: 46% of the younger hearing-impaired students rated the "non-equivalent non-multiple" item 3/11 vs. 11/3 as equivalent (even this outcome seems to indicate the students were influenced by the values of the independent counting numbers); 38% of the older hearing-impaired students rated "1/2" as larger than "1/3".

The younger hearing students also appeared to be influenced by the size of the counting numbers, as many of them chose as larger the fraction with the larger valued counting numbers, regardless of the validity of the choice (median percent endorsement was 58%). However, their lower percents of endorsement for the fractions with the bigger numbers indicate they were not influenced to the same extent as the hearing-impaired students.

The older hearing students appear not to have been swayed by the size of the counting numbers (median percent endorsement was 14%). On the "like numerators - unit", "like numerators - non-unit", "equivalent multiples", and "equivalent non-multiples" items, there were very few instances of choosing the incorrect answer composed of the "bigger numbers". On those items in which the correct answer was in fact composed of the "bigger numbers" ("like denominators", "non-equivalent multiples", and "non-equivalent non-multiples"), the students displayed a low error rate. It is always possible the older hearing students were indeed swayed by the "bigger numbers" in these items, although that would most likely be limited to the most difficult items.

Throughout the analyses pertaining to Part 1, it was found that the hearing-impaired students of both age levels performed similarly to the younger hearing students. In an effort to further understand the hearing-impaired students' conceptual understanding of the order and equivalence of fractions, an analysis of the students' strategies for solving the problems was undertaken.

<u>Question 2:</u> What strategies do hearing-impaired students employ to determine order and equivalence of fractional numbers, and how do these strategies compare with those of their hearing peers?

The self-reported student strategies obtained from the four items on Part 2 of the fractional number instrument served as the data for the analyses that follow. However, before presenting the results, a number of problems with the data collection and the quality of the data will be explained. These problems appear to be related to the difficulty of the task.

Problems

Previous research on strategies used by students when solving order and equivalence fraction problems depended primarily on interviews (Behr, Wachsmuth, Post, & Lesh, 1984; Post, Wachsmuth, Lesh, & Behr, 1985; Roberts, 1985). Due to the immediacy of interviews, students' answers can be further probed if their initial reasoning is incomplete or not understandable. However, in the present study, the experimenter's lack of fluency in American Sign Language and other appropriate methods of communication prevented the use of interviews with the hearing-impaired students. Thus, self-reported strategies were communicated in written form. This method was not successful for a number of reasons.

Students from both age groups and hearing-status' appeared to have difficulty explaining their strategies. Many did not explain their strategies at all, but chose only the fractions they thought were "bigger". Some self-reported strategies were unclear or ambiguous, thus preventing categorization or, at the very least, precise categorization. Many students, especially the younger students, did not offer explanations for their answers but merely repeated their choice in sentence form. All of the problems mentioned above occurred more often for the hearing-impaired than for the hearing students. Indeed, the task of explaining one's strategy is a difficult one to begin with, and it appears as if this difficulty was exacerbated by the request to explain in writing. It is possible that students could successfully explain their strategies in an oral or manual method of communication during an interview and fail to clearly explain their strategies in written form. For all of these reasons, the data presented below should be considered exploratory.

Categorization and definition of strategies

The explanations offered by students were categorized based on the dominant or emphasized aspects of their reasoning. The categorization yielded seven "instructive strategies" (strategies that logically describe thought processes to guide a student to order fractions in a particular way) and six "non-instructive strategies" (non-strategies, they are responses to the question "How do you know?" that either did not describe thought processes or did not describe thought processes clearly enough to permit categorization.) Although instructive strategies did not always represent correct reasoning, they were informative in that one was able to follow the students' reasoning that led him/her to an answer. Each instructive strategy is defined in Appendix C and followed by an example taken from the categorized data. An asterisk (*) indicates the correct answer for each ordered fraction pair, while the lack of an asterisk represents an equivalent fraction pair. Of the seven instructive strategies defined, five have been described (at least in part) in previous research (Roberts, 1985).

Results of categorization of strategies

The reported use of instructive strategies was less than optimal and varied greatly by group membership. Overall, the students' rate for employing instructive strategies was 0.45. Thus, all analyses of strategies that follow are based on less than half of the potential pool of student explanations. Of those instructive strategies provided, 31% were from the hearing-impaired students. Table 7 displays the incidence of instructive strategies by each group of students on the four items that compose Part 2 of the instrument.

The older group of hearing students provided the largest number of instructive strategies, whereas the older group of hearing-impaired students provided the smallest number of instructive strategies. The younger groups of students (hearing and hearing-impaired) provided rates of instructive strategies similar to each other. Tables 8 and 9 list the frequency of specific instructive strategies engaged in by the hearing-impaired and hearing students, respectively, in determining the order and equivalence of the fraction pairs on Part 2 of the instrument. The most popular instructive strategy reported by both groups of hearing-impaired students was the Counting Numbers strategy, in which the students based their choice of the larger-valued fraction on the larger-valued independent counting numbers that composed the fraction. The younger group of hearing students also reported the Counting Numbers strategy most often. Further, the Counting Numbers strategy was used by both groups of hearing-impaired students and the younger hearing students as an "all purpose" strategy to order fractions of four different types.

The older group of hearing students reported strategies indicating a more mature understanding of fractional order and equivalence, and only rarely regressed to the Counting Numbers strategy for the most difficult fraction pairs.

Although the information displayed in Tables 8 and 9 represents only 45% of the total pool of the students' self-reported explanations, it is consistent with the findings of Part 1, as the most popular instructive strategy reported by the hearing-impaired and younger hearing students is the Counting Numbers strategy. The only fraction type for which the Counting Numbers strategy would have been appropriate is for fractions with "like denominators". However, some hearing-impaired students and younger hearing students attempted to apply the strategy to the other fraction types as well ("like numerator - unit"; "equivalent multiples"; "non-equivalent non-multiples"). Even a few older hearing students were influenced by the size of the counting numbers on the item "10/11 vs. 15/16" ("non-equivalent non-multiples").

In sum, hearing-impaired students from both age groups employed strategies in a fashion similar to younger hearing students.

Discussion

The main conclusion of this study is that hearing-impaired students between the ages of 10 and 16 years lagged behind their hearing peers in arithmetic computation and the development of the concept of fraction. In light of previous research on the mathematics achievement of hearing-impaired students, this is not a surprising result. Of more interest is the quality of the performance by the hearing-impaired students.

Throughout the performance and strategies sections of this study, the hearing-impaired students of both age groups performed similarly to each other. This was not the case for the hearing students. Not only were the hearing-impaired students capable of ordering the same types of fractions as the younger hearing students, but they made the same kinds of errors, employed strategies in a similar way, and, like hearing children who are learning initial fraction concepts, they were negatively influenced by the size of the counting numbers composing the fractions. Developmentally speaking, these results lead one to think that hearing-impaired students may develop rational number concepts in a fashion similar to hearing children, but the development is delayed. This particular pattern - same sequence of development with delay - has been previously observed in research on the cognitive and conceptual development of hearing-impaired children (see Greenberg & Kusche, 1989; Zwiebel & Mertens, 1985) as well as in more specific areas such as language, reading, writing (see Paul & Quigley, 1990), measurement, and money (Austin, 1975).

What could explain a delay in rational number concepts among hearing-impaired students? One explanation, that which is often cited when a developmental delay is observed in research with the hearing-impaired, is the assumed effect of the delayed English language skills of hearing-impaired students. Hearing-impairment as an English language handicap is well documented throughout the research literature. In the words of Greenberg and Kusche (1989), "...[English] language deprivation influences the way information is processed, which in turn results in a type of experiential deprivation...; processing differences, in turn, appear to affect the development of further concept formation" (p. 101).

Another likely explanation for a delay is related to the language of mathematics and, in particular, the language of fractions. Even hearing children who are not "English language handicapped" have a difficult time with the language used routinely to explain rational number concepts (Post, Behr, & Lesh, 1986; Roberts, 1985). Specifically, mismatches between the students' ideas of particular words used during fractions instruction (such as "more" and "less") and researchers' intended meaning of the words have created confusion among students. It is reasonable that any language problem experienced by hearing students would be that much more of a problem for hearing-impaired students.

Another possible explanation for a delay is related to the quality of the hearing-impaired students' mathematics education. Given the highly publicized lack of emphasis on mathematics for hearing-impaired students and the often inadequate university level training in mathematics for many teachers of the hearing-impaired, a delay could be the result of an insufficient mathematics education. This would more likely be the case for students in residential or self-contained placements, as students who are mainstreamed for mathematics classes are exposed to the same curriculum as their hearing peers and have math teachers who are subject matter specialists.

Of course, it is possible that the findings could be influenced by limitations of the study. Because the researcher was not fluent in American Sign Language, it was impossible to individually interview the hearing-impaired students about their strategies. The request to explain strategies in writing may have prevented fraction-wise hearing-impaired or hearing students, who may have succeeded in an interview, from clearly communicating their strategy. The request to explain in writing could also have interacted with language difficulties. James (1981) believes that (with respect to hearing students) students need time to develop the appropriate oral language before explaining their thoughts on paper. Of course, it is always possible that students may understand concepts of order and equivalence and yet be unable to clearly explain their strategy in any communicative form: verbally, manually, or in writing. Williams (1975) noted that remedial hearing students experienced difficulty verbalizing fraction concepts they actually understood. Further, the results related to the instructive strategies describe only those students who provided instructive strategies. This group of hearing-impaired and hearing students may possess more advanced receptive and expressive communication skills - be more verbally advanced - than students who provided non-instructive strategies.

Although the results of this study are limited in their generalizability, they do suggest implications for the mathematics education of hearing-impaired students.

Implications

If, indeed, delayed development in fractional number concepts among hearing-impaired students can be partly explained by language difficulties, then capitalizing on non-verbal demonstrations of fractions concepts would be highly beneficial. Research from the Rational Number Project has shown that manipulatives or "embodiments" play an important role in the development of the order and equivalence of fractional numbers as well as of other rational number concepts (Behr & Post, 1988; Post, Wachsmuth, Lesh, & Behr, 1985). Given many hearing-impaired children communicate in a visual-spatial manner, concrete embodiments would be powerful aids.

Teaching the language of fractions within the context of rational number concepts may also facilitate students' learning. It is important, however, that "fraction words" not become the focus of the lesson. Although it is important to teach hearing-impaired students the language of mathematics, many educators and researchers believe that language per se should not be the focus of math class. Fraction words gain meaning as they are used in context.

It is also possible that a delayed development in fractional number concepts among hearing-impaired students can be partly explained by a lack of emphasis on mathematics for hearing-impaired children. If this is indeed the case, then we as educators and researchers need to seriously question the role of mathematics in the education of the hearing-impaired. If hearing-impaired people are to equally compete in the technological job market, if the statistics for unemployment and underemployment among the hearing-impaired are to improve, if the United States is to improve on the current deficit of scientists needed by industry and education, then mathematics needs a more valued place in the education of hearing-impaired children.

This "quality of mathematics education" issue speaks to the university level training of teachers for the hearing-impaired. The focus of training for teachers of the hearing-impaired is on the development of communication skills. Thus, many teachers are inadequately trained to teach subject areas such as mathematics (Paul & Quigley, 1990). Kluwin and Moores (1985) found that mainstreamed hearing-impaired adolescents achieved significantly better in mathematics than hearing-impaired students from self-contained classrooms, even after factors such as extent of hearing loss and prior achievement had been controlled. The differences were accounted for by the fact that regular education math teachers are subject area specialists and have more teaching experience. Thus, it seems that mathematics instruction needs a more valued place among the university level training programs in hearing-impaired education.

One final way in which the results of research on fractional number concepts may impact the mathematics education of hearing-impaired students can be summed up in an adage: "What's good for the goose is good for the gander". A plethora of research has been performed with hearing children on the development of rational number concepts. Much of what has been learned about teaching fractions to hearing children could also be applied to teaching hearing-impaired children. The importance of initial fraction concepts, teaching for meaning, the use of embodiments - all could be adapted for the hearing-impaired. To the extent that hearing-impaired children are similar to hearing children in terms of their pattern of development of rational number concepts, research on the developmental sequence of hearing children should be kept in mind and applied where possible.

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Table 1
Design, Variables, and Measures

Hearing Status

		Hearing	Hearing-Impaired
Age Level	10 - 12		
	13 - 15		

Independent variables:
* hearing status (hearing-impaired, hearing)
* age level in years (10 - 12, 13 - 15)

Dependent variables:
* total correct on a fractional number instrument
* quality and type of errors made by item type on a fractional number instrument
* strategies employed to determine order and/or equivalence on a fractional number instrument

Covariate:
* score on a math computation instrument

Table 2
Types of fractions

Category	example items
Fractions with like denominators	5/7 vs. 3/7
Fractions with like numerators	
a) unit fractions	1/2 vs. 1/3
b) non-unit fractions	2/5 vs. 2/3
Fractions with different numerators and denominators	
a) equivalent multiples ....fractions are equivalent, and numerators and denominators are related across fractions as integer multiples	1/2 vs. 2/4
b) equivalent non-multiples ....fractions are equivalent, but neither numerators nor denominators are integer multiples of each other	4/6 vs. 6/9
c) non-equivalent multiples ....fractions are not equivalent, but either numerators, denominators, or both are related as integer multiples	2/3 vs. 5/6
d) non-equivalent non-multiples ....fractions are not equivalent, and both their numerators and denominators are not related as integer multiples	5/6 vs. 8/9

Table 3
Descriptive Statistics on Part 1 of the Fractional Number Instrument

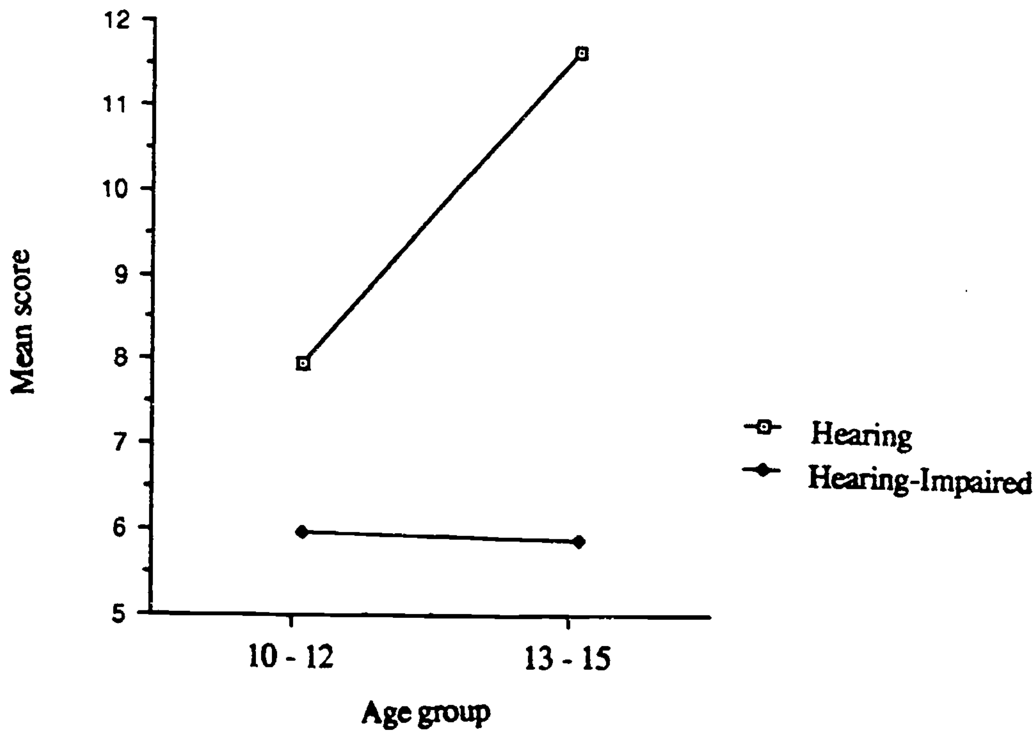
Hearing Status*	Age Group	N	Mean	Standard Deviation
HI	10 - 12	13	5.85	1.46
HI	13 - 16	8	5.75	0.83
H	10 - 12	12	7.83	2.27
H	13 - 15	14	11.50	2.75

* HI refers to "hearing-impaired" and H refers to "hearing".

Table 4
Crossed Two-Way Analysis of Variance Table

	SS	df	MS	F	p
Hearing Status	167.83	1	167.83	36.02	0.000
Age Group	35.74	1	35.74	7.67	0.008
Interaction	39.70	1	39.70	8.52	0.006
Within Cells	200.36	43	4.66		
Total	443.63	46			

Figure 1
Mean Performance on Part 1
of the Fractional Number Instrument



Note: Highest score possible is 14.

Table 5
Hearing-Impaired Students' Performance on Fraction Items by Item Type

item type	item	ages 10 - 12* % correct	ages 13 - 16* % correct
like denominators	5/7 vs. 3/7	85	75
	7/30 vs. 20/30	92	75
like numerators - unit	1/2 vs. 1/3	31	62
	1/15 vs. 1/17	23	38
like numerators - non-unit	2/5 vs. 2/3	23	25
	3/7 vs. 3/10	23	38
equivalent multiples	1/2 vs. 2/4	23	38
	5/7 vs. 15/21	15	12
equivalent non-multiples	4/6 vs. 6/9	0	0
	6/8 vs. 9/12	0	0
non-equivalent multiples	2/3 vs. 5/6	69	62
	3/5 vs. 9/10	77	75
non-equivalent non-multiples	3/11 vs. 11/3	38	12
	5/6 vs. 8/9	85	75

* For ages 10 - 12, sample size is 13. For ages 13 - 16, sample size is 8.

Table 6
Hearing Students' Performance on Fraction Items by Item Type

item type	item	ages 10 - 12* % correct	ages 13 - 15* % correct
like denominators	5/7 vs. 3/7	92	79
	7/30 vs. 20/30	92	86
like numerators - unit	1/2 vs. 1/3	92	100
	1/15 vs. 1/17	50	100
like numerators - non-unit	2/5 vs. 2/3	58	93
	3/7 vs. 3/10	58	93
equivalent multiples	1/2 vs. 2/4	50	100
	5/7 vs. 15/21	8	93
equivalent non-multiples	4/6 vs. 6/9	8	57
	6/8 vs. 9/12	0	57
non-equivalent multiples	2/3 vs. 5/6	75	71
	3/5 vs. 9/10	75	71
non-equivalent non-multiples	3/11 vs. 11/3	58	79
	5/6 vs. 8/9	58	71

* For ages 10 - 12, sample size is 12. For ages 13 - 15, sample size is 14.

Table 7
Incidence of Instructive Strategies on Part 2 of the Fractional Number Instrument, Hearing Status by Age Group

hearing status	age group	% instructive strategy offered
hearing-impaired	10 - 12	42
hearing-impaired	13 - 16	12
hearing	10 - 12	35
hearing	13 - 15	75

Table 8
Frequency of Instructive Strategies, Hearing-Impaired Students from Two Age Groups*

Item & Item Type	Ages 10-12		Ages 13-16	
1/5 vs. 1/12 (like numerator, unit)	Counting Numbers	8	(none)	
8/13 vs. 11/13 (like denominators)	Counting Numbers	2	Counting Numbers	1
	Area	2	Size of Piece	1
	Number of Pieces	1		
8/9 vs. 24/27 (equivalent multiples)	Counting Numbers	4	Counting Numbers	1
	Residual	1		
10/11 vs. 15/16 (non-equivalent non-multiples)	Counting Numbers	4	Counting Numbers	1

* Total N per group: ages 10 - 12, N = 13
ages 13 - 16, N = 8

Table 9
Frequency of Instructive Strategies, Hearing Students from Two Age Groups*

Item & Item Type	Ages 10-12		Ages 13-15	
1/5 vs. 1/12 (like numerator, unit)	Counting Numbers	2	Area	6
	Size of Piece	2	Number of Pieces	2
	Area	1	Size of Piece	1
			Residual	1
8/13 vs. 11/13 (like denominators)	Counting Numbers	4	Counting Numbers	3
	Number of Pieces	1	Area	3
	Number Patterns	1	Number of Pieces	2
			Size of Piece	1
			Residual	2
8/9 vs. 24/27 (equivalent multiples)	Multiplicative	1	Multiplicative	13
	Counting Numbers	1	Number Patterns	1
10/11 vs. 15/16 (non-equivalent non-multiples)	Counting Numbers	4	Counting Numbers	2
			Size of Piece	2
			Residual	3

* Total N per group: ages 10 - 12, N = 12
ages 13 - 15, N = 14

Appendix A

age ______ your birthdate ______

grade ______ Circle which: boy girl

school ______

Solve each of the following math problems. Please show all your work.

1) 3 + 5 =

2) 16 + 37 =

3) 18 − 5 =

4) 52 − 17 =

5) 3 x 2 =

6) 27 x 5 =

7) 24 ÷ 4 =

8) 51 ÷ 6 =

9) $\frac{1}{6} + \frac{3}{6}$

10) $\frac{1}{2} + \frac{2}{3}$

11) $\frac{9}{10} - \frac{7}{10}$

12) $\frac{8}{9} - \frac{1}{5}$

Please stop.

Do not turn the page until told to do so.

age ______________ your birthdate ______________
grade ______________ Circle which: boy girl
school ______________

Directions:

For each problem, you will see two numbers. For example,

1 3

Under the two numbers, you will see some circles that look like this:

1 3
○ ○ ○

Under the circles, you will see three sentences with three faces:

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

1) Look at the two numbers.

2) If the number on the left is bigger than the number on the right, write a mark in the left circle.

If the number on the right is bigger than the number on the left, write a mark in the right circle.

If the numbers are the same size, write a mark in the middle circle.

3) After you answer a problem, look at the three sentences with the three faces. Draw a circle around the sentence that says how sure you are that your answer is correct.

Let's do some examples.

example 1: Which number is bigger?

1 3
○ ○ ●

a) I am very sure my answer is correct.
b) I think my answer is correct.
c) I guessed the answer.

example 2: Which number is bigger?

5 4
● ○ ○

a) I am very sure my answer is correct.
b) I think my answer is correct.
c) I guessed the answer.

example 3: Which number is bigger?

7 7
○ ● ○

a) I am very sure my answer is correct.
b) I think my answer is correct.
c) I guessed the answer.

You try this one.

example 4: Which number is bigger?

7 3
○ ○ ○

a) I am very sure my answer is correct.
b) I think my answer is correct.
c) I guessed the answer.

Do you understand the directions? If you understand the directions, you may turn the page and try the rest of the problems. If you do not understand the directions, please ask your teacher for help.

1) Which number is bigger?

$\frac{5}{7}$ ○ ○ ○ $\frac{3}{7}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

2) Which number is bigger?

$\frac{7}{30}$ ○ ○ ○ $\frac{20}{30}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

3) Which number is bigger?

$\frac{1}{2}$ ○ ○ ○ $\frac{1}{3}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

4) Which number is bigger?

$\frac{1}{15}$ ○ ○ ○ $\frac{1}{17}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

5) Which number is bigger?

$\frac{2}{5}$ ○ ○ ○ $\frac{2}{3}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

6) Which number is bigger?

$\frac{3}{7}$ ○ ○ ○ $\frac{3}{10}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

7) Which number is bigger?

$\frac{1}{2}$ ○ ○ ○ $\frac{2}{4}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

8) Which number is bigger?

$\frac{5}{7}$ ○ ○ ○ $\frac{15}{21}$

a) I am very sure my answer is correct.

b) I think my answer is correct.

c) I guessed the answer.

9) Which number is <u>bigger</u>?

$\frac{4}{6}$ ○ ○ ○ $\frac{6}{9}$

a) I am <u>very sure</u> my answer is correct.

b) I <u>think</u> my answer is correct.

c) I <u>guessed</u> the answer.

10) Which number is <u>bigger</u>?

$\frac{6}{8}$ ○ ○ ○ $\frac{9}{12}$

a) I am <u>very sure</u> my answer is correct.

b) I <u>think</u> my answer is correct.

c) I <u>guessed</u> the answer.

11) Which number is <u>bigger</u>?

$\frac{2}{3}$ ○ ○ ○ $\frac{5}{6}$

a) I am <u>very sure</u> my answer is correct.

b) I <u>think</u> my answer is correct.

c) I <u>guessed</u> the answer.

12) Which number is <u>bigger</u>?

$\frac{3}{5}$ ○ ○ ○ $\frac{9}{10}$

a) I am <u>very sure</u> my answer is correct.

b) I <u>think</u> my answer is correct.

c) I <u>guessed</u> the answer.

13) Which number is <u>bigger</u>?

$\frac{3}{11}$ ○ ○ ○ $\frac{11}{3}$

a) I am <u>very sure</u> my answer is correct.

b) I <u>think</u> my answer is correct.

c) I <u>guessed</u> the answer.

14) Which number is <u>bigger</u>?

$\frac{5}{6}$ ○ ○ ○ $\frac{8}{9}$

a) I am <u>very sure</u> my answer is correct.

b) I <u>think</u> my answer is correct.

c) I <u>guessed</u> the answer.

Please stop.

Do not turn the page until told to do so.

age ____________ your birthdate ____________

grade ____________ Circle which: boy girl

school ____________

<u>Directions</u>:

For each problem, you will see two numbers. For example,

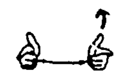
1 3

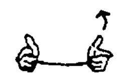
Under the two numbers, you will see some circles that look like this:

1 3

O O O

1) Look at the two numbers.

2) If the number on the <u>left</u>  is <u>bigger</u>  than the number on the right, write a mark in the <u>left</u>  circle.

If the number on the <u>right</u>  is <u>bigger</u>  than the number on the left, write a mark in the <u>right</u>  circle.

If the numbers are the <u>same</u>  size, write a mark in the <u>middle</u>  circle.

3) After you answer a problem, <u>explain</u>  how you knew the answer. You can write a sentence <u>or</u> draw a picture <u>or</u> solve a math problem to explain your answer.

Let's do some examples.

__example 1:__ Which number is __bigger__?

1 ○ ○ ● 3

How do you know?

3 is a bigger number than 1.

3 circles → ●●● ← more circles

1 circle → ●

So 3 is a bigger number than 1.

__example 2:__ Which number is __bigger__?

5 ● ○ ○ 4

How do you know?

5 squares → □□□□□ ← more squares

4 squares → □□□□

So 5 is a bigger number than 4.

__example 3:__ Which number is __bigger__?

7 ○ ● ○ 7

How do you know?

The numbers are the __same__ __size.__

7 circles ••••••• } same number of circles

7 circles •••••••

__You try this one.__

__example 4:__ Which number is __bigger__?

7 ○ ○ ○ 3

How do you know?

Do you understand the directions? If you understand the directions, you may turn the page and try the rest of the problems. If you do not understand the directions, please ask your teacher for help.

1) Which number is __bigger__?

$\frac{1}{5}$ ○ ○ ○ $\frac{1}{12}$

How do you know?

2) Which number is __bigger__?

$\frac{8}{13}$ ○ ○ ○ $\frac{11}{13}$

How do you know?

3) Which number is __bigger__?

$\frac{8}{9}$ ○ ○ ○ $\frac{24}{27}$

How do you know?

4) Which number is __bigger__?

$\frac{10}{11}$ ○ ○ ○ $\frac{15}{16}$

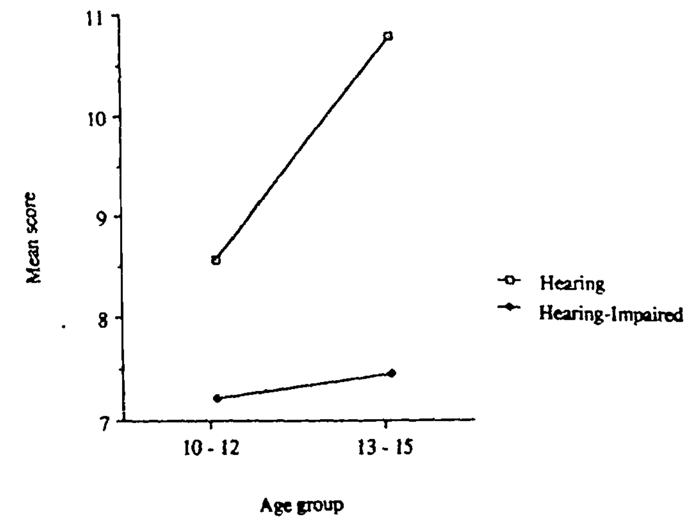
How do you know?

Appendix B

Descriptive Statistics on the Mathematics Computation Instrument

Hearing Status*	Age Group	N	Mean	Standard Deviation
HI	10 - 12	13	7.15	1.83
HI	13 - 16	8	7.38	2.83
H	10 - 12	12	8.50	0.87
H	13 - 15	14	10.71	1.33

Mean Performance on Mathematics Computation Items



Note: Highest score possible is 12.

Appendix C

Definitions and Examples of Instructive Strategies

1) Size of Piece: Students' reasoning focused on either the size of the piece represented by the fractional value (a 1/5 th piece), or on the size of an individual partition of the unit. When employing this strategy in the present study, students typically (but not always) drew a rectangular or round shape, partitioned it into equal-sized sections, and shaded a number of sections. The words "bigger" and "smaller" were observed in students' verbal descriptions. [Example: **1/5* vs. 1/12** The student drew two correctly partitioned circles and shaded the appropriate areas. Next to the circle indicating 1/5, he wrote "That piece is bigger", and chose the correct response. (age 10 hearing student); Example: **1/5* vs. 1/12** "1/5 is bigger than 1/12. Fifths are bigger than twelfths." (age 10 hearing student)]

2) Number of Pieces: Students' reasoning focused on the number of equal-sized parts into which the unit wholes were divided (represented by the denominator), or on the number of those parts being considered in the fraction (represented by the numerator). As in the strategy above, students typically drew a rectangular or round shape, partitioned it into equal-sized sections, and shaded a number of sections to explain their answer. The words "more" or "less" were observed in students' verbal descriptions when using this strategy. [Example: **8/13 vs. 11/13*** The student drew two wholes, each with 13 pieces. She shaded 8 sections in the first whole, and 11 sections in the second whole. Over the representation of 8/13 she wrote "few taken", and over the representation of 11/13 she wrote "more taken". She indicated 11/13 as the larger. (age 12 hearing-impaired student); Example: **1/5* vs. 1/12** Student drew two correctly partioned circles, each with one shaded piece. She chose 1/5 as the larger value. Next to the circles she wrote "cuz there are less pieces". (age 14 hearing student)]

3) General Area: In some instances, students correctly drew an area representation of the fractions, complete with shaded parts, but gave no further instructive explanation for their choice. Thus, it was not clear whether they employed a "Size of Piece" or "Number of Pieces" strategy, or any other one for that matter. As the "Size of Piece" and "Number of Pieces" strategies are both based on area representations, the present strategy was labeled "General Area" to emphasis the fact that students may have employed one of the area strategies defined above. [Example: **8/13 vs.**

11/13* Student drew 13 squares and shaded 8 of them. She drew another set of 13 squares and shaded 11 of them. She correctly ordered the fractions. (age 12 hearing-impaired student)]

4) Counting Numbers: Students' reasoning focused on the value of the numerator and/or denominator, and ordering of the fractions was based on the value(s) of the counting numbers composing each fraction. Note that this strategy is valid only when ordering fractions with like denominators. The "bigger numbers" hypothesis corresponds to this strategy. [Example: **8/9 vs. 24/27** The student wrote "I know that 24/27 is bigger because it has bigger numbers." (age 15 hearing-impaired student)]

5) Residual: Students compared each fraction to a value (or area) which was greater than both the fractions being compared. Further, the focus was on the difference between the fractions and the value 1 (or the unit whole). Students indicated this strategy in words or in both pictures and words. When employing a pictorial representation, they focused on the "leftover" pieces of the unit whole. Although this strategy could potentially be a powerful one, students employing this technique often employed it erroneously. [Example: **8/9 vs. 24/27** Student drew 9 circles, shading 8 of them. Over this picture she wrote "one left". She then drew 27 circles, shading 24 of them. Over this second picture she wrote "more left". She chose 24/27 as the larger. (age 12 hearing-impaired student); Example: **8/13 vs. 11/13*** Student drew two circles, each partitioned into 13 sections. She shaded 8 sections in one circle, and 11 in the other. She chose the correct answer, writing "There are less pieces left over". (age 14 hearing student)]

6) Multiplicative: Students focused on the relationships between the numerator and denominator in a pair of fractions, and ordering was based on the multiplicative relationships among individual terms in the fraction pair. Students used this strategy exclusively on the "equivalent multiples" items. [Example: **8/9 vs. 24/27** Student wrote "You can reduce 24/27 to 8/9", and indicated the fractions are equal in value. (age 14 hearing student)]

7) Number Patterns: This strategy included the identification of numerical relationships among the numerators and denominators in the pair of fractions to be ordered. Specifically, it was by way of an arbitrary "rule". [Example: **8/13 vs. 11/13*** Student drew two sets of thirteen lines. He

circled eight of them in one set, and eleven in the other set. He then wrote "Remember lowest one wins", and chose 8/13 as the larger. (age 10 hearing student)]