DOCUMENT RESUME

ED 347 069 SE 052 532

AUTHOR	Horak, Virginia M.
TITLE	Students' Cognitive Styles and Their Use of Problem-Solving Heuristics and Metacognitive Processes.
PUB DATE	Apr 90
NOTE	16p.; Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics (Salt Lake City, UT, April, 1990).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Processes; *Cognitive Style; Computer Uses in Education; Games; *Heuristics; Junior High Schools; Mathematics Education; *Metacognition; *Problem Solving
IDENTIFIERS	Microworlds

ABSTRACT

This study investigated junior high school students' use of problem-solving heuristics and metacognitive processes and the relationships that might exist between the students' use of these processes and their cognitive style. Using a computer microworld called "Nimbot" based on the ancient game of Nim, 10 seventh- and eighth-grade students' problem-solving heuristics and metacognitive activities were observed during game sessions via audiotape and a record of the student's key strokes. The various heuristics and metacognitive activities utilized were identified and listed. Cognitive style was measured as a score on the Group Embedded Figures Test to determine the students' location along the field-dependent-independent continuum. General conclusions related to heuristic utilization included: (1) junior high students use the heuristics of trial-and-error, look-for-a-pattern, draw-a-diagram, compare-and-contrast-data, account-for-all-possibilities, simplify-the-problem, break-set, and work-forward-in-solving-a-problem; (2) all but one student used the trial-and-error heuristic; and (3) all students utilized the work-forward heuristic. General conclusions related to metacognitive processes utilization include the following: (1) all students selected a strategy to help them understand the problem; (2) all students focused initially on not losing the game; (3) all students at times were inefficient in monitoring conclusions and generalizations they made while playing the game; and (4) none of the students utilized a highly nonproductive strategy. Interactions between the students' field-dependence-independence and their use of heuristics and metacognitive activities were noted. (16 references) (MDH)

Students' Cognitive Styles and Their Use of Problem-solving Heuristics and Metacognitive Processes

Virginia M. Horak

College of Education
The University of Arizona
Tucson, AZ 85721

Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Salt Lake City, Utah
April 1990

Students' Cognitive Styles and Their Use of Problem-solving Heuristics and Metacognitive Processes

Purpose and Significance

This study investigated junior high school students' use of problem-solving huerlstics and metacognitive processes and the relationships which might exist between the students' use of these and their cognitive style. Since the late 1970's, few other topics in mathematics education have received the attention that problem solving has received. That focus continues today with problem solving being the first standard for each grade-level category in the National Council of Teachers of Mathematics _Curriculum and Evaluation Standards for School Mathematics_ (1989).

In spite of the energy and wisdom that has been devoted to improving the problem-solving ability of our students, many educators feel the results are disappointing. Mathematics educators have come to realize that problem solving is more than knowing the heuristics that can be applied to a situation. As Lester (1983) has stated, "more is involved than possessing a repertoire of skills, facts, algorithms, and strategies . . . . This repertoire is essential, but not sufficient" (p.43).

Many educators and researchers believe that students' metacognitive abilities and subsequent activities may provide the missing links that account for the mental activity that goes into successfully applying and evaluating these problem-solving strategies. The _Standards_ frequently suggests and encourages a concern for metacognition. Included in the Problem Solving, Communication, and Reasoning Standards are goals which envision that students will be able to "reflect on the process and how it relates to prior problems" (p.76), "evaluate appropriate strategies" (p.77), "reflect on and clarify their own thinking about mathematical ideas and situations" (p.78), and "validate their own thinking" (p.81). All of these goals are metacognitive in nature, in that they focus on the learners' knowledge concerning their own cognitive processes, i.e., knowledge about their knowledge (Sternberg, 1985).

The cognitive and learning styles of learners have long been of interest to educational researchers. It would seem that if a learner's cognitive style describes how s/he is able to identify the parts of a structure and place his/her organizational scheme onto it, then this style should also affect how that learner perceives and structures (controls) his/her own cognitive processes. Stice (1987), in writing about problem solving, metacognition, and the use of cognitive style indicators, states that he is "convinced that application of these instruments can be of great help to anyone who truly wants to help students learn" (p.106).

Conceptual Framework

Considerable research has been done on problem solving in an instructional setting. Many researchers (Kraus, 1982; Kantowski, 1977; Goldberg, 1975, Schoenfeld, 1982) have found that teaching students strategies does indeed improve their problem-solving ability. However, it may not be sufficient to make students good problem solvers (Hatfield, 1978; Lester, 1983; Schoenfeld, 1983).

In many studies (Schoenfeld, 1982; Thomas and Grouws, 1984; Jensen, 1987), developing an awareness of and a concern for metacognitive processes in the students benefitted them in their problem-solving tasks.

Some students appear to be more inclined and able to use a variety of heuristics and to monitor the effectiveness of their chosen strategies. A student's cognitive style could have an effect on this ability. Although learners' cognitive styles have long been studied by educational researchers, results are frequently inconclusive. Witkins' well-known field-dependence-independence construct is one of the cognitive styles often investigated in studies of mathematics learning (Threadgill-Sowder and Sowder, 1982; Onyejiaku, 1982). Many studies show field-independent students more successful on the outcome measures. However, there have also been conditions under which field-dependent students out-performed field-independent students (Threadgill-Sowder and Sowder, 1982). It would seem likely that a learner's cognitive style will affect not only his/her use of problem-solving strategies, but also the use of metacognitive processes in choosing, monitoring, and evaluating these strategies.

Examining students' problem-solving and metacognitive activities can be carried out in a variety of settings. E. A. Silver (1985) pointed out that "the computer can be used as a tool to create environments in which people can be given the opportunity to think mathematically and solve challenging problems" (p.263). Games and game situations provide these microworlds in which students are able to use and display their problem-solving and metacognitive abilities.

Microworld Game Situations

A game which provides a rich environment for using many different problem-solving heuristics was used in this study. It is challenging to players, but not so difficult as to frustrate or discourage them. Nimbot is a computer microworld based on the ancient game of Nim. On the screen are three rows of robots with 5, 4, and 3 in the rows. A student plays against the computer, with each taking alternate turns removing any number of robots in a single row. The player's objective, of course, is to force the computer to take (shoot) the last remaining robot on the screen.

Research Questions and Procedure

This research made use of the above described microcomputer game situation presented to young people, age 12 and 13. By observing their interactions with these games, it was possible to address the following questions:

(1) What are the different problem-solving heuristics junior high school students use in a game situation?

(2) Do junior high school students monitor, evaluate, and change the heuristics they use?

(3) Is there a relationship between the cognitive style of a student (field-dependent-independent) and his/her use of problem-solving heuristics and metacognitive processes?

To investigate these questions, 10 seventh- and eighth-grade students were observed as they worked individually or in pairs to beat the computer in the game of Nimbot. During each observation, a record was kept of the student's key strokes as well as an audiotape of each session. Additionally, each student was administered the Group Embedded Figures Test, so as to determine his/her location along the field-dependent-independent continuum. The data collected were reviewed and analyzed to identify problem-solving heuristics and instances of metacognitive activities. Categories of heuristics were tallied and changes in strategies noted. Figure 1 provides a listing of common heuristics used in mathematical problem-solving situations. As will be seen, some of the heuristics are not appropriate when playing the game of Nimbot, but were included in the initial listing.

Common Mathematics Problem-Solving Heuristics

Trial and Error

Make a Table

Look for a Pattern

Draw a Diagram

Restate the Problem

Compare and Contrast Data

Account for All Possibilities

Simplify the Problem

Break Set

Write a Mathematical Sentence

Make a Graph or Table

Make a Model

Work Backward

Work Forward

Figure 1

A brief explanation of a few of the heuristics might be useful at this point. Trial and Error was identified by random, initial moves with minimal subsequent analysis of the outcomes. Look for a Pattern was used when the player(s) made a generalization about the arrangements of robots that would result in losing the game. When using this heuristic, s/he was able to see a pattern in the arrangements, draw a conclusion from it, and make a prediction as to the outcome of the game. Compare and Contrast Data was identified by instances when a player recalled having seen an arrangement before and noting that it led to losing the game, whether or not a written record was made. Paper and pencil were always available for the students, although only one pair of girls made use of them. To Account for All Possibilities meant that a player examined an arrangement of robots and was able to systematically develop all possible moves from that arrangement in order to predict the outcome of the game. The players exhibited the Simplify the Problem heuristic in one of two ways. The more common way was to eliminate an entire row of robots to make the game more manageable. The other way was for a player to repeat the computer's moves, either within a game or in a following game. Work Forward meant that the player was able to predict one move ahead. After s/he made a move, s/he was able to predict what the computer was going to do for its next move. Break Set, in this study, was identified when a player changed how s/he approached playing the game. To break set meant that the player changed from playing the game as trying not to lose to playing the game as attempting to get the computer into a known losing arrangement.

The metacognitive processes of the players were also examined. To do this, the cognitive-metacognitive framework developed by Garofalo and Lester (1985) was used as a foundation. Figure 2 presents a list of metacognitive activities that were deemed appropriate for investigation in this study. Each category from the Garofale and Lester framework is included, with one or two identifying activities for each.

Metacognitive Activities

(taken from Garofalo and Lester, 1985)

Orientation - Select a strategy to aid in understanding a problem

Organization - Plan a course of action

Execution - Select an appropriate strategy to carry out the plan

- Monitor activities while executing the plan

Verification - Evaluate the outcomes of the strategy

- Revise or abandon nonproductive strategies

Figure 2

Findings and Conclusions

The analysis of the GEFT results, the audiotapes, and the records of the students' key strokes have revealed possible relationships and consistencies in students' use of metacognitive processes and problem-solving heuristics. Table 1 summarizes the data for this study. The left side of the table lists the heuristics used by any of the students while playing Nimbot. As can be seen, nearly half of the heuristics from Figure 1 are not included in this table. Many of the ones not used by the students would not, in fact, have been useful in this instance. Across the top of the table are the students in the study, identified by two-letter codes. The players identified with a comma between the letters indicate two students working as a pair. Also included in the table are the number of games each student (or pair of students) played before being able to beat the computer, the students' scores on the GEFT, and their gender.

The information revealed in Table 1 appears to indicate some general conclusions:

- Junior high students use the identifiable heuristics of trial and error, look for a pattern, draw a diagram, compare and contrast data, account for all possibilities, simplify the problem, break set, and work forward in solving a problem in a non-academic game environment.

- All but one pair of players used the Trial and Error heuristic, at least briefly. Three of them used it only minimally, to begin one or two games before they utilized another heuristic. For three players, it was their primary heuristic.

- All of the students utilized the Work Forward heuristic. They were usually successful predicting the computer's next move when the game was within one or two moves of ending.

Summary Table of Performance Data

	AN	CA	LE	BR	MO	RO	M,E	C,E
Trial and Error	x*	x	x	X	X	X**		x
Look for a Pattern	X	X	X				X	X
Compare and Contrast Data				X			x	X
Account for all Possibilities	X	X	X					
Simplify the Problem		X		X			X	X
Break Set	X	X		X			X	
Work Forward	X	X	X	X	X	X	X	X
Number of Games Played	12	16	18	23	19	32	19	10
GEFT Score(s)	14	12	6	14	13	5	11,10	8,11
Gender	M	F	F	M	F	M	F,F	F,F

* Lower Case x indicates minimal use of that heuristic.

** A X indicates the primary heuristic that the student used.

Table 1

- The three single students who relied primarily on the Trial and Error heuristic played more games than the other students before they were able to beat the computer.

- The two most field-dependent students played more games than the other players who used their same primary heuristic. This appears to indicate that it took them more games to be able to put a structure on the losing arrangements of robots.

- Four of the five most field-independent students/student pairs used the break set heuristic.

- Two of the girls working as a pair were able to beat the computer in the fewest number of games. They were also the only players to make a written record of losing arrangements and "bad" moves while playing the game. It appears to have been a successful and efficient heuristic for them.

An analysis of the audiotapes of the students as they played Nimbot provide further insights into their problem-solving abilities and their use of metacognitive processes. Some general conclusions related to the metacognitive processes listed in Figure 2 appear to be:

- All of the students easily selected a strategy to help them understand the problem. Most chose to use Trial and Error, but some did not utilize it very long before they changed to a more efficient heuristic.

- The initial course of action followed by all of the students appeared to focus on not losing the game to the computer. Four of the students (all of whom were field-independent) were able to change their plan of action to focus on creating a losing arrangement for the computer in order to win the game. Doing this utilized the break set heuristic.

- All of the students at times were inefficient in monitoring conclusions and generalizations they made while playing the game. They clung to incorrect assumptions regarding arrangements that led to their losing the game. Two of the players who held on the longest to an erroneous rule for winning were both field-independent.

- To verify that they really had determined how to beat the computer at Nimbot, only the two most field-dependent players played additional games to confirm their winning sequence of moves.

- None of the students utilized a highly nonproductive strategy. Even though Trial and Error appears to have been less efficient than the other heuristics, the students who used it were quite successful.

In general, it appears that these junior high school students were quite capable of using a variety of problem-solving heuristics to successfully play the game of Nimbot. Furthermore, they demonstrated thoughtful planning and evaluating of the heuristics and the process of searching for the winning strategy. It also appears that there might be some possible interactions between the students' field-dependence-independence and their use of these heuristics and metacognitive activities.

This research was an investigation designed to build a foundation for the analysis of qualitative and quantitative data on the heuristics and metacognitive processes used by junior high school students. It also looked at the possible relationship between one dimension of a student's cognitive style and his/her use of metacognitve processes to monitor, evlauate, and make decisions about problem-solving heuristics. The current information could help to focus future research on appropriate teaching and curricular variables. It may also lead to the development of computer software that encourages and assists students in monitoring their own cognitive processes.

Bibliography

Commission on Standards for School Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston: National Council of Teachers of Mathematics, Inc.

Garofalo,J. and Lester, F.K. (1985). Metacognition, Cognitive Monitoring, and Mathematical Performance. Journal for Research in Mathematics Education, 16, 163-176.

Goldburg, D. J. (1975). The effects of training in heuristic methods on the ability to write proofs in number theory. Doctoral dissertation, Columbia University. Dissertation Abstracts International, 4989-b. (University Microfilms No. 75-7836)

Hatfield, L. L. (1978). Heuristical emphases in the instructin of mathematical problem solving. In L. L. Hatfield and D. A. Bradbard (Eds.), Mathematical problem solving: Papers from a research workshop (pp.21-42). Columbus, OH: ERIC Information Analysis Center for Science, Mathematics and Environmental Education.

Jensen, R. J. (1987, April). Microcomputer-based conjecturing environments. Paper presented at the meeting of the National lCouncil of Teachers of Mathematics, Anaheim.

Kantowski, M. G. (1977). Processes involved in mathematical problem solving. Journal for Research in Mathematics Education, 8, 16-180.

Kraus, W. H. (1982). The use of problem-solving heuristics in the playing of games involving mathematics. Journal for Research in Mathematics Education 13, 172-182.

Lester, F. K. (1983) Trends and issues in mathematical problem-solving research. In R. Lesh and M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 229-261. New York: Academic Press.

Onyejiaku, F. O. (1982). Cognitive styles, instructional strategies, and academic performance. Journal of Experimental Education, 51, 31-37.

Schoenfeld, A. H. (1982). Measures of problem-solving performance and of problem-solving instruction. Journal for Research in Mathematics Education, 13, 31-49.

Schoenfeld, A. H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh and M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 345-395). New York: Academic Press.

Silver, E. A. (1985). Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, N.J.: Lawrence Erlbaum Associates, Publishers.

Sternberg, R. J. (1985). The growth of reflection in children. New York: Academic Press.

Stice, J. E. (1987). Developing critical thinking and problem-solving abilities. San Francisco: Jossey-Bass Inc., Publishers.

Threadgill-Sowder, J. and Sowder, L. (1982). Drawn versus verbal formats for mathematical story problems. Journal for Research in Mathematics Education, 13, 324-331.

Thomas, W. E. and Grouws, D. A. (1984). Inducing cognitive growth in concrete-operational college students. School Science and Mathematics, 84, 233-243.