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ABSTRACT

This Yearbook 1989-90 is made up of six articles. The articles depict the ongoing discussion taking place in Finland as it re-assesses the state of its mathematics education and planning measures for its development. The opening article presents an analysis of the president of the International Commission on Mathematical Instruction of some current trends in mathematics education. The following three articles represent then recent Finnish research on the area of mathematics education. The last two articles offer views about mathematics teaching and teacher education in Estonia and Czechoslovakia. The articles are: (1) Some Contemporary Tendencies in Mathematical Education (Miguel de Guzman); (2) Mathematics, Science and Technology Teachers' Conceptions about Their Professional Knowledge and Skills (Yrjo Yrjnsuuri); (3) Study Orientations of Mathematics by Upper Secondary School Students (Raija Yrjnsuuri); (4) "A Contextual Approach to the Teaching of Mathematics: Outlining a Teaching Strategy That Makes Use of Pupils' Real World Experiences and Strategies, and the Results of the First Teaching Experiment of the Project (Tapio Keranto); (5) Developments in the Teaching of Mathematics in Estonia (Olaf Prinits); and (6) The System of Teacher Education in Czechoslovakia with Special Reference to Mathematics Teachers' Education (Jaroslav Bartak). (MDH)

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Theory into practice

Pekka Kupari (Ed.)

MATHEMATICS EDUCATION RESEARCH IN FINLAND
Yearbook 1989 – 90

UNIVERSITY OF JYVÄSKYLÄ

Institute for Educational Research. Publication Series B. Theory into Practice 66

MATHEMATICS EDUCATION RESEARCH IN FINLAND

YEARBOOK 1989 – 90

Edited by Pekka Kupari

Institute for Educational Research
The Finnish Association of Mathematics and Science Education Research

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The Yearbook 1989-90 includes six articles. In the first article the President of ICMI describes some tendencies which are actual and remarkable in the learning and teaching of mathematics. The following three articles represent then recent Finnish research on the area of mathematics education. And the two last ones offer views about mathematics teaching and teacher education in Estonia and Chechoslovakia.

Descriptors: mathematics education, teachers, profession, study orientation, contextual approach

PREFACE

The sixth yearbook on mathematics education is published at a time when Finland is re-assessing the state of its mathematics education and planning measures for its development.

The work was initiated by the so-called Leikola Committee (Committee of Basic Education in Mathematics and Science). In its final report in September 1989 the Committee presented its views on the development of both general and vocational education, and also took a stand on adult education and mass communication. The Committee considered lack of understanding of grave global and regional environmental problems to be the most important current issue. A second broad problem area mentioned by the Committee was the discrepancy between the need to use information technology and its educational supply.

As a result of the Committee's work, the National Board of General Education set up a work group, whose task is to assess the development needs in comprehensive school and upper secondary school mathematics in 1990-92 and to make development proposals. At the same time as the Committee has started its work, it has made a deliberate effort to arouse discussion on the reform of mathematics education.

The articles in the present yearbook aptly depict the ongoing discussion. In the opening article the President of the ICMI analyzes some current trends in mathematics education. Other topics are concerned with teachers' conceptions of their professional skills, students' study orientations, and contextual approach to teaching. External views are represented by reviews of mathematics teaching and teacher education in two countries.

As its predecessor, also the present yearbook covers two years.

Pekka Kupari

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SOME CONTEMPORARY TENDENCIES IN MATHEMATICAL EDUCATION

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Spain

In this paper I try to briefly explain some tendencies that, in my opinion, are at the moment quite noticeable in the international panorama of the teaching and learning of mathematics at the primary and secondary level of education.

1. A situation of change

The last 30 years have been the scene of very profound changes in mathematical education. If one has to judge by the strong efforts that the experts in mathematical education are still making in order to find adequate patterns for teaching and learning it is quite clear that we are as yet living in a phase of exploration and change.

The reform movement of the decades of the sixties and seventies towards "modern mathematics" brought with itself powerful transformations in education, not only in style but also in the new contents that were introduced. Among the main traits of these changes one can underline the following ones. One started emphasizing the abstract structure in some areas, specially in algebra. One tried to strengthen logical rigour and comprehension that was considered in contrast with the operative and manipulative aspects. This lead in a natural way to an interest in the foundations of elementary mathematics by means of some rudiments of set theory and in the cultivation of the algebra, where rigour is easily attainable, although its problems, at an elementary level become little more than tautologies. Geometry instead, where rigour is difficult, was laid aside, in spite of the richness and formative value of its problems. The mistrust of intuition contributed even more to the almost absolute exile of elementary geometry.

In the seventies one started to perceive that many of the changes one had introduced were not very appropriate. With the substitution of geometry by algebra, elementary mathematics had been emptied of content and of interesting problems. The lack of spatial intuition was another of the disastrous consequences of the banning of geometry from our programs, a defect one can very obviously perceive in those persons that had their education in those years. One can say that the disadvantages that resulted with the introduction of the so-called "modern mathematics" were by far greater than the questionable advantages that one had thought to be able to attain such as rigour in the foundations, comprehension of the underlying mathematical structures, the modernity and the approximation to contemporary mathematics...

2. Some general contemporary tendencies

During the eighties there was a rather extended acknowledgement that one had considerably exaggerated in what respects the emphasis in the abstract structure of mathematics. It is necessary to foster and to cultivate intuition and the operative manipulation of space, of course without abandoning the comprehension and understanding of what one is doing, but also without letting that this effort towards understanding may relegate to the background the intuitive contents of our mind in its approach to the mathematical objects. To each phase of mental development, as to each phase of historical and scientific progress, corresponds its own level of rigour.

Today one generally agrees that for a healthy conception of mathematics one needs the persistent support in the concrete, in the reality from which the mathematical concepts and problems arise in a natural way. And that, in order to understand this fruitful interaction between the real world and mathematics it is necessary to resort, on the one hand to the history, that unveils this emergency process of our mathematics along time, and on the other to the applications of mathematics, that make manifest for us the fecundity and power of this science. According to today most accepted conception, our mathematics in its ordinary tasks is much more similar to other empirical sciences than one had thought in the past. Mathematics, as the other sciences, proceeds also by tentative approaches, by trial and error, by experiments, sometimes fruitful, sometimes sterile, till it reaches a more mature form, although always perfectible. Our mathematical education should try to reflect these profoundly human traits, gaining thereby in accessibility, dynamic power, interest and appeal.

One of the most clearly defined and most extended general tendencies today is the emphasis on the transmission of the mathematical thinking processes, i.e. the abilities for problem solving, rather than on the mere transference of contents. Mathematics is, above all, know how, it is a science where the method is clearly more important than the contents. For this reason one grants today a greater role to the study of the questions, in a great part neighbouring with cognitive psychology, regarding the mental processes involved in the activity of problem solving. In our intellectual and scientific world, so rapidly changing, it is much more worthwhile to store up useful thinking processes than knowledge contents that get very quickly absolete.

In this direction one has to classify the efforts to transmit heuristic strategies that are appropriate for the general activity of problem solving, rather than just offering receipts that might be useful for each subject matter.

The existence of such powerful tools as the pocket calculator or the computer is influencing the attempts to orientate our primary and secondary education in such a way that the potential of these instruments may be fully used. It is quite clear that, because of different circumstances such as cost, inertia, novelty, teachers impreparation, hostility on the side of many ..., there are as yet no generally accepted patterns of use of these tools with entire satisfaction. This is one of the important challenges for an immediate future. But is is already clear that our style of mathematical education and even its contents are going to change quite drastically. The accent has to be located, also because of this reason, on the understanding of the mathematical processes rather than on the execution of certain routines that, in today situation, take a great amount of the energy of our students.

Another general preoccupation one can observe in the atmosphere leads to the search for means to motivate the students from a more ample point of view, not just resorting to the values of mathematics in itself. One has to make manifest the strong mutual impacts that culture, history and the development of society on one hand, and mathematics on the other have caused upon each other. In our intellectual environment, with a strong tendency towards the dehumanization of science and the depersonalization brought about by the computerization of our culture, it is more and more necessry to reach a truly human knowledge so that man and machine may in it occupy the right place that to each corresponds. An adequate mathematical education can effectively contribute in this task.

3. Some advisable methodological changes

At the light of these general tendencies one can point out a few methodological-principles that might appropriately guide our education.

(1) Towards the acquisition of the typical processes of mathematical thinking.

How should one proceed in the mathematical teaching and learning at any level? In a way similar to the one man has been using in the invention or creation of mathematical ideas and methods. One first gets in contact with the mathematical portion of the real world that has given rise to the concepts that we want to explore with our students. We try to stimulate their autonomous search, their own gradual discovery of simple mathematical structures on problems that reality itself provides in a natural way. It is clear that we cannot expect that our students may discover in a week what mankind has elaborated in many centuries. The guided search for patterns, without annihilating the pleasure of discovery, must be however the noble goal of the teacher, so that the detection of useful techniques and strategies may be readily assimilated by his students. Their application to the problems that formerly appeared as unattainable will then be a true source of intellectual satisfaction and pleasure.

(2) The importance of motivation and presentation.

Our students are intensely bombarded with very powerful techniques of communication. It is a very strong competition that we have to face when we try to get a substantial part of their attention. This is something we should constantly have in mind in order to foster in ourselves the use of such tools as video, Tv, comics, ludic activities and direct participation.

Our teaching has to be directed to a very great number of students that in principle have an almost absolute lack of motivation. However the society in which they live and also their own benefit requires that they acquire at least a basic formation in mathematics. It is necessary to use all the motivating aspects which we can think of, real life, mathematical recreations, history of mathematics, its philosophy, biographies of famous mathematicians, etc.

(3) Fostering the taste for mathematics.

Physical activity is a pleasure for a healthy person. Intellectual activity too. Mathematical activity practiced as an autonomous know how, under an appropriate quide, may be an attractive exercise. The taste for mathematical discovery is possible and strongly motivating in order to go through other routine aspects of the mathematical learning. The appreciation of the possible applications of mathematical thinking in modern scientific and technological activity can fill with astonishment and pleasure many of our students more oriented towards the practical aspects. Others will perhaps be more touched by the contemplation of the impacts that mathematics has had on the history or philosophy along the centuries or by the biography of someone or other of the great mathematicians.

We need to put aside by every means at our disposal the very ingrained idea, probably coming from initial blocks in the childhood, that mathematics is and has to be arid, boring, abstruse, abstract, useless and very difficult.

4. Some tendencies regarding the contents

The above mentioned general tendencies suggest in a natural way some reforms related to the contents of the programs in use that, with more or less decision, and in some cases in an experimental and tentative form, are being introduced in our mathematical education.

(1) Towards a discrete mathematics?

The mathematics of the 19th and 20th centuries has been above all mathematics of the continuum where mathematical analysis, because of its powerful applications to other sciences and to technology, has played a predominant role.

The coming of age of computers, with their immense capacity of rapid computation, has opened to investigation a great number of different fields, with their origin not any more in the physical sciences, as in previous centuries, but in some other sciences such as economy, organization sciences, biology, etc. Their problems appeared before quite opague and impenetrable, partly due to the enormous amounts of numerical information one had to handle in order to acquire valuable mathematical intuitions that could effectively help on the way towards a solution. On the other hand, the importance of the discrete algorithms, so much needed in the computer sciences and in the modellization through the computer of different phenomena, has given rise to a shift of emphasis towards discrete mathematics. Certain portions of discrete mathematics are sufficiently elementary to successfully form part of a program of elementary mathematics. Classical combinatorial theory and some modern aspects of it like graph theory or combinatorial geometry could be some of the appropriate candidates for this purpose of introducing discrete mathematics in our initial studies. Elementary number theory, a field that in some countries never disappeared from the initial programs of education, could be another one.

Several attempts have been made to introduce these and other similar elements of discrete mathematics in the curriculum. It seems that this is only possible at the expense of setting aside some other more classical portions of mathematics that one is rather unwilling to omit. Although it is quite clear that the flavour of the mathematics of the future will be rather different because of the presence of the computer, we do not yet clearly see how this is going to influence the contents of primary and secondary mathematics.

(2) The impact of modern computation methods.

Until not many years ago it was quite frequent to devote great amounts of time and energy in our schools to routine tasks such as long division of a six figure number by another one with four figures or to the extraction of the square root of a six figure number with three exact decimals. One also consumed a good amount of time in the secondary education to the use of the table of logarithms with its intricate labyrinth of interpolations. Today, the presence of the pocket calculator has made almost all of us agree that this energy and this time are better spent in some other tasks. Such algorithms are very interesting as profound and intelligent algorithms, but as routine abilities for the human mind are completely superfluous.

Today, in our secondary education, we dedicate pretty much time and energy in order that our students acquire dexterity and agility in the computation of derivatives, antiderivatives, in the resolutions of linear systems, the multiplication of matrices, the graphic representation of functions, the computation of the standard deviation, etc. There are in the market already pocket calculators that are able, just by pressing some keys, in a few seconds, to find the derivative of

$(1+1/x)^x$, to compute its Taylor polynomial up to the term in x^3 around $x=2$, to graphically represent this function in the neighbourhood of a certain value or to compute its integral between 2 and 3 with very good approximation. The inversion of a matrix 8x8 makes the machine busy for a few seconds, a minimal portion of the time one uses to introduce the data into the calculator. The computation of the standard deviation of a great mass of data is immediate. The roots of a seventh degree algebraic equation, even with complex values, are furnished by the machine in a rather easy way.

If this is the situation it is quite clear that our classes of calculus, algebra, and statistics have to follow in the not too distant future quite different paths than the ones we now have. It will be necessary to stress the accent in the comprehension and interpretation of what one is doing, but it will be superfluous to spend much of the energy we now devote to acquire strong skills in routine tasks that the machine does with much greater reliability and speed. The emphasis will also be set on the experimentation that the calculator allows us now. To give a complicated example: Does the sequence $a_n=n((n)^{1/n}-(n+1)^{1/n+1})$ converge? With the pocket calculator I have written the formula that gives me a_n and then I have asked it to compute a few significant values. It answers:

a_{100}= .037421803; a_{1000}= .00594325; a_{10000}= .0008217;
a_{100000}= .000105; $a_{1000000}$= .00002

This experiment gives me some confidence to conjecture that, although rather slowly, a_n tends to 0, and it is well known how powerfully a good conjecture can help towards the solution of a problem. On the other hand the calculator gives me a graphic of the function $y=x((x)^{1/x}-(x+1)^{1/x+1})$ that reinforces my conjecture.

The capacities of this new pocket calculator for calculus, algebra, statistics and graphic representation enhance the possibilities of school mathematics for more realistic applications that until now were out of reach in our courses because of the amount of time that would be necessarily spent on tedious numerical and symbolical computations.

(3) Towards a recuperation of geometrical thinking and spatial intuition.

After an unjustified abandonment of intuitive geometry in our programs at the time of the "modern mathematics" movement one feels today that it is absolutely necessary, from a didactical, scientific and hisorical point of view, to try to recover the spatial and intuitive content in all of our mathematics, not just in geometry. However it is not yet quite clear how one should plan an adequate geometrical formation. It is necessary to avoid the exaggerations that were present, for example, in the development of the geometry of the triangle of the 19th century. One should also avoid to follow a persistently rigorous and axiomatic way in the introduction of geometry, too arduous and tedious for today students. Perhaps a healthy middle way, from the point of view of the rational construction of elementary geometry, could consist in the establishment of an opertive basis through a few obvious principles upon which one could build some interesting local developments of classical portions of geometry chosen because of their depth and beauty. Some of the works by Coxeter could serve as nice examples to follow.

(4) Expansion of statistics and probability.

Probability and statistics are very important components of our culture and in many of our sciences. They should constitute an important portion of the basic cultural baggage of our citizens. Many are the countries that include these subjects in their secondary education programs, but in too few of them the teaching is done with the desired effectiveness.

MATHEMATICS, SCIENCE AND TECHNOLOGY TEACHERS' CONCEPTIONS ABOUT THEIR PROFESSIONAL KNOWLEDGE AND SKILLS

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The purpose of this study was to inquire conceptions of mathematics, science and technology teachers about the adequacy of their knowledge and skills acquired in the basic teacher training. As a part of a wider survey 56 teachers of these subjects answered a questionnaire. Seven dimensions were structured by a factor analyses: subject knowledge, education knowledge, institution knowledge, didactical skills, evaluation and planning skills, skills in teaching methods and interactive coping. They were interpreted to be in relationship with the three components of Kerr's theory of intentional activity. Most mathematics, science and technology teachers think that their knowledge and skills are adequate in subject knowledge, in didactical skills, in education knowledge and in evaluation and planning skills. A majority of teachers think that their institution knowledge is inadequate. The ratings of the studied mathematics, science, and technology teachers were significantly higher than those of the other teachers in subject knowledge and in evaluation and planning skills. They were significantly lower than those of the other teachers in institution knowledge and in methodical skills.

1. Introduction

Student teachers have a long and demanding training. They must study the subjects they will teach, education and school legislation. They must also practice teaching before they become professional teachers. However, teachers can not always during their career answer for the challenges of the daily work only on the ground of their initial teacher training. They must almost continously re-evaluate the adequacy of their competency, and they must learn new knowledge, skills and coping strategies.

The teachers of the different subjects and levels have got very different teacher training. In school they have the shared task to educate children, and in this work the different subjects are their main tools. Their work is strictly connected with the subject matter they are teaching but they have also some institutional tasks outside teaching. How adequate they find the knowledge and skills acquired in the basic teacher training?

The purpose of this paper is to report some results about the conceptions of subject teachers in the Finnish comprehensive school. The problem of this study is, how experienced mathematics, science and technology teachers evaluate the adequacy of their knowledge and skills acquired in the basic teacher training.

2. Teachers' work

The paradigm of this study is determined by the conceptions of teachers and of the teachers' work. In this study, the teachers' work was assumed to be a series of intentional activities. There are possibly also some unintentional activities, but they are left outside interest, because a teacher can not be worried about the adequacy of his knowledge and skills in unintentional activities. It is also assumed that the teachers reflect their intentional work. They can evaluate the results of their actions, and they can analyse the details of their activities, most often the details of the process of teaching and learning.

Because the teachers' work is assumed to be intentional, and the teachers are assumed to reflect their work, it is reasonable to think that they have - as a part of their self-concept - conceptions about the adequacy of knowledge and skills needed in their work.

Two general tests of adequacy for any teaching action can be differentiated. The first, the test of subjective adequacy, queries whether A's action fits his relevant beliefs and values. The second test features objective adequacy. While A might believe that what he is doing is adequate for teaching, it may not be adequate for teaching on standards either of the knowledge community or of the moral and political context, or of both. (Kerr 1981, 77)

The teachers themselves can have a conception only of the subjective adequacy of their knowledge and skills. Because I think the most important phenomena are the teachers' own conceptions about their work, I will limit the problem of this study to the subjective adequacy of teachers' knowledge and skills.

Teachers have acquired their knowledge and skills on different ways. There can be separated basic teacher training, in-service training, work and other experiences. I assume that the teachers can self-evaluate how adequate the knowledge and the skills acquired by the basic teacher training are. I think they have conceptions about the adequacy of this knowledge and of these skills. I tried to measure these subjective conceptions directly by asking the teachers themselve~

Kerr (1981, 74) has proposed that any purposive, goal-directed activity might be 'factored' into three components: one that regards choice of goal, one that concerns a choice of means or plan, and a third that regards acting on the plan. When teachers' work is assumed to be intentional it should include these three components. In a case of teaching, an action would fall under the teaching description if it concerns choosing a learning to encourage, as designing a plan to encourage that learning, or as acting on that plan to encourage that learning.

Therefore, the problem of this study can be detailed according to these components. It can be asked, whether there is empirical evidence of the dimensions, which are in relation to these components, and how - in these dimensions - the comprehensive school mathematics, science and technology teachers evaluate the subjective adequacy of their knowledge and skills acquired in the basic teacher training.

I have found earlier that teachers' conceptions about the adequacy of their knowledge and skills changes greatly in the transition from teacher training to school and during the first years of work (Yrjönsuuri 1987). Many others have also presented results showing teachers' changes and changing conceptions of the teachers in this transition (e.g. Perho 1988, Oser 1987, Miklos & Greene 1987). In this study, I am interested in the conceptions of experienced teachers who have been teaching several years.

3. Instrument, sample and method

A questionnaire with 30 items was constructed on the basis of the objects of teacher training (Opettajanvalmistuksen opetussuunnitelmatoimikunnan mietintö 1968). The teachers rated the degree of adequacy of their knowledge and skills acquired in the teacher training in a five point Likert-type scale from fully adequacy to fully inadequacy. I have earlier used almost the same instrument, which was found to be reliable (Yrjönsuuri 1987).

The questionnaire was sent to 600 comprehensive school teachers (300 women and 300 men, 300 class teachers of the lower level and 300 subject teachers of the upper level, 200 teachers born in 1938, 200 teachers born in 1948, 200 teachers born in 1958). It was returned by 428 teachers (71.3 %). I have described the sample in an other paper (Yrjönsuuri 1989).

The number of mathematics, science, and technology teachers among the returners was 56. Their aswers are reported in this paper.

The answers were analysed statistically. Frequencies show how many teachers considered the knowledge and skills to be adequate or inadequate. It was analysed by the t-test whether the mean significantly differs from the midpoint of the scale (3). The dimensions of the teachers' knowledge and skills were searched by factor analyses. A principal component analysis was made with the sample of all the teachers who answered (N = 423), and the means of the mathematics, science and technology teachers' oblique factor scores were calculated from the results of this analysis. Their means were compared with the means of the sample of all teachers (M = 0.000, s = 1.000). The significance of the difference was analysed with the t-test.

4. Results

I have reported the results of the factor analyses earlier (Yrjönsuuri 1989). The factors could be interpreted in relationship to the three components of an intentional activity. Three factors were interpreted to be related to the first component (choosing objectives). They were named: subject knowledge (SKn), education knowledge (EKn) and institution knowledge (IKn). Three other factors were related to the second component (designing a plan). They were named: didactical skills in subject matter (DSk), evaluation and planning skills (PSk) and skills in teaching methods (MSk). Only one factor was related to the third component (acting on the plan). It was named: coping with interactive situations (ICo).

Of the 30 variables 23 could be grouped according to the factors. Seven variables have been left outside the following examinations, because they had almost equal loadings on two or more factors. The means and one standard deviation error bars of variables of all answered teachers have been presented in the Figure 1 so that the variables, which have the biggest loadings on the same factor, are drawn in a same box. If the mean is over the midpoint, it means that most of the teachers aswered that their skills are adequate; if it is under the midpoint, so the majority answered their skills are inadequate.

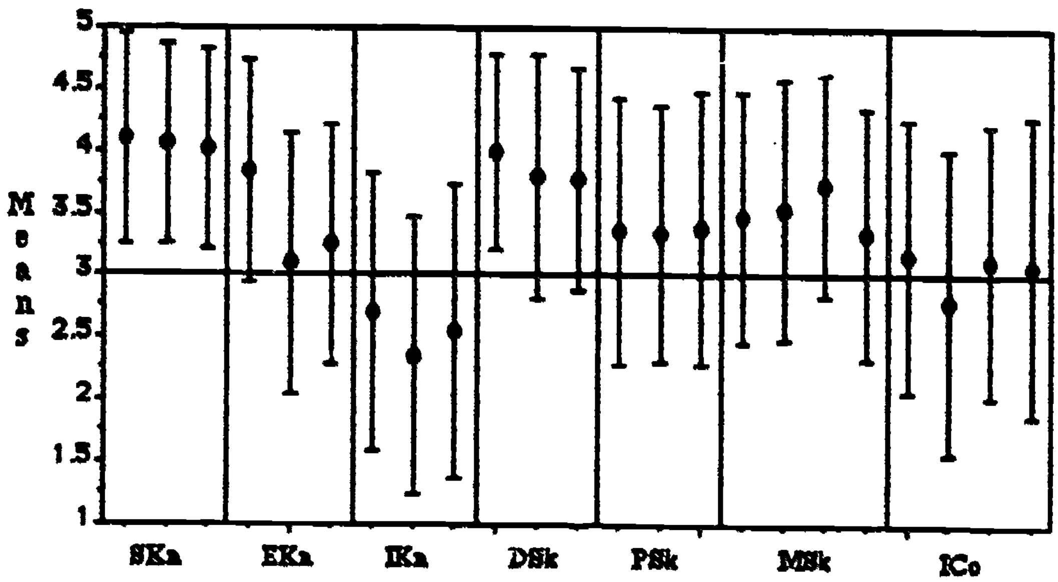

FIGURE 1. The means and one standard deviation error (plus/minus) bars of the teacher skill variables (N = 428)

As it can be seen in the Figure 1 the variables in the same box often have almost the same mean. It can be interpreted that the variables presenting the adequacy of knowledge and skills in a same dimension show that the adequacy is almost the same. The means of the variables in the first box (subject knowledge) and in the fourth box (didactical skills in the subject matter) are especially high, but also the means of the variables in the fifth (evaluation and planning skills) and sixth (skills in teaching methods) box are all over the midpoint. The means of the variables in the third box (institution knowledge) are especially low. The means of the variables in the last box (interactive coping) are close to the midpoint.

In the following, the frequencies, means and standard deviations of variables of mathematics, science and technology teachers are examined in the order of the factors. Sum scores and factor scores of subject knowledge (SKn), of education knowledge (EKn), of institution knowledge (IKn), of didactical skills in subject matter (DSk), of evaluation and planning skills (PSk), of skills in teaching methods (MSk) and of interactive coping (ICo) are used for these analyses

Adequacy of subject knowledge were measured with three items (Cronbach alpha = .88):

b01 = content knowledge of subject matter (oblique loading on the factor .90)

b21 = mastery of teaching matter (loading .90)

b24 = mastery of subject matter (loading .88)

TABLE 1. Frequencies (1 = fully inadequate, 2 = nearly inadequate, 3 = difficult to say, 4 = nearly adequate, 5 = fully adequate), means, standard deviations, and probabilities for M = 3

	1	2	3	4	5	M	s	p<
b01	1	1	3	23	28	4.36	.82	.0001
b21	1	1	6	23	25	4.25	.86	.0001
b24	0	1	2	28	25	4.38	.65	.0001
Σ	2	3	11	74	78			
%	1	2	7	44	46			
s1	max 2.00 min -1.00					1.33	.67	.0001
f1	max 1.79 min -2.60					.42	.92	.001

s1 = sum scores, s1 = (b01 + b21 + b24):3 - 3
f1 = factor scores

This dimension of teachers' professional knowledge is reliable, and the teachers in the sample are almost unanimously satisfied with the adequacy of the subject knowledge acquired in the teacher training. Only very few teachers think that their subject knowledge is not adequate. Most of them think that it is nearly or fully adequate. There are 90 % of ratings in all three variables showing nearly or fully adequate knowledge, but only 3 % of ratings showing nearly or fully inadequate knowledge.

The means of these three variables are the highest of all variables, and they are very significantly over the midpoint. The sum scores give in one scale the same result as the variables. The factor scores show that the mathematics, science, and technology teachers think even in a higher grade than other teachers that their subject knowledge is adequate.

Adequacy of education knowledge were measured with three items (Cronbach alpha = .69):

b17 = knowledge of the basic principles of education (oblique loading on the factor .83)

b18 = knowledge of problems in school education (loading .58)

b23 = knowledge of pupils' psycho-physical development (loading .80)

TABLE 2. Frequencies, means, standard deviations, and probabilities for M = 3

	1	2	3	4	5	M	s	p<
b17	0	7	12	27	10	3.71	.91	.0001
b18	5	21	11	13	6	2.89	1.19	.50
b23	6	18	13	18	1	2.82	1.06	.21
Σ	11	46	36	58	17			
%	7	27	21	35	10			
s2	max 1.67 min -1.33					.14	0.80	.19
f2	max 1.82 min -3.50					-.25	1.14	.11

s2 = sum scores, s2 = (b17 + b18 + b23): 3 - 3
f2 = factor scores

There are differencies between the items of this dimension. The answers in the first item are significantly different from the answers of the second and third item. Only seven teachers think that the knowledge of the basic principles of education is nearly inadequate, but 37 teachers think it is nearly or fully adequate. The mean of this item is over the midpoint on a very significant level. In the two other items, there are more teachers who think the knowledge is inadequate than others who think it is adequate. But the means are not significantly under three. In all these variables, there are relatively many ratings - about one fifth of all - for difficult to say.

The mean of the sum scores is not significantly over the midpoint of the scale. As seen by the factor scores, the teachers of mathematics, science and technology do not think on a different way as other teachers about the adequacy of the education knowledge.

Adequacy of institution knowledge was measured with three items (Cronbach alpha = .76):

b03 = mastery of the institutional activities in the school (oblique loading on the factor .90)

b13 = knowledge of the school administration (loading .87)

b16 = management of the co-operation between school and home (loading .66)

TABLE 3. Frequencies, means, standard deviations, and probabilities for M = 3

	1	2	3	4	5	M	s	p<
b03	9	17	22	5	3	2.57	1.04	.003
b13	17	19	8	9	3	2.32	1.22	.0001
b16	21	18	10	7	0	2.05	1.03	.0001
Σ	47	54	40	21	6			
%	28	32	24	12	4			
s3	max 1.67 min -2.00					-.69	.93	.0001
f3	max 1.58 min -2.86					-.32	1.01	.03

s3 = sum scores, s3 = (b03 + b13 + b16):3 - 3
f3 = factor scores

Most ratings (60 %) in these three variables, show nearly or fully inadequate knowledge. Especially, the knowledge of the school administration and the management of the co-operation between school and home have been inadequate, while it has been difficult to say, whether the skills in the mastery of the institutional activities in the school have been adequate.

The means of these variables are the lowest of all variables. But this is a very understandable result, because there are in the teacher training very limited possibilities to become acquainted with the circumstances of various schools. The factor scores show, that the teachers of mathematics, science and technology consider their knowledge in this dimension to be even more inadequate than the other teachers.

Adequacy of didactical skills in subject matter was measured with three items (Cronbach alpha = .71):

b04 = presenting the subject matter in teaching (oblique loading on the factor .90)

b05 = daily and weekly planning of lessons (loading .65)

b06 = structuring subject matter in teaching (loading .72)

TABLE 4. Frequencies, means, standard deviations, and probabilities for M = 3

	1	2	3	4	5	M	s	p<
b04	0	5	8	32	11	3.88	.83	.0001
b05	0	5	11	25	15	3.90	.91	.0001
b06	0	5	12	24	15	3.88	.92	.0001
Σ	0	15	31	81	41			
%	0	9	18	48	24			
s4	max 2.00 min -1.00					.88	.74	.0001
f4	max 3.73 min -2.55					.07	1.19	.67

s4 = sum scores, s4 = (b04 + b05 + b06):3 - 3
f4 = factor scores

Nobody thinks he has fully inadequate skills on this dimension, and only very few mathematics, science, and technology teachers think that their didactical skills in the subject matter are nearly inadequate. Most of them think that the skills in this dimension are nearly or fully adequate. There are 72 % of ratings in all three variables showing nearly or fully adequate knowledge, but only 9 % of ratings showing nearly inadequate knowledge.

The means of these three variables are the second highest of all variables, and they are very significantly over the midpoint. The sum scores give in one scale the same result as the variables. But the factor scores show that the mathematics, science, and technology teachers do not think in a higher grade than the other teachers that their didactical skills are adequate.

Adequacy of evaluation and planning skills was measured with 3 items (cronbach alpha = .76):

b27 = long term planning of teaching (oblique loading on the factor .52)

b29 = continuing evaluation of pupils (loading .81)

b30 = giving marks (loading .87)

TABLE 5. Frequencies, means, standard deviations, and probabilities for M = 3

	1	2	3	4	5	M	s	p<
b27	3	11	9	27	6	3.39	1.09	.01
b29	3	10	13	23	7	3.38	1.09	.01
b30	4	7	10	25	10	3.54	1.14	.001
Σ	10	28	32	75	23			
%	6	16	19	45	14			
s5		max 2.00	min -2.00			.44	.96	.002
f5		max 3.12	min -2.63			.35	1.11	.02

s5 = sum scores, s5 = (b27 + b29 + b30): 3 - 3
f5 = factor scores

There are no big differencies between the ratings of the three items in this dimension. About one fifth of the teachers in the sample think that it is difficult to say, whether these skills are adequate or inadequate, and about one other fifth think that the evaluation and planning skills given by teacher training are nearly or fully inadequate in their practical work. But most of them (59 %) think that the evaluation and planning skills are adequate.

The means of these variables are significantly over the midpoint of the scale, and also the means of the sum and factor scores are over zero. The mathematics, science, and technology teachers evaluate their skills in this dimension on average higher than other teachers, because the mean of factor scores is almost significantly (p < 2 %) over zero.

Adequacy of skills in teaching methods was measured with 4 items (Cronbach alpha = .83):

b07 = motivating pupil (oblique loading on the factor .58)

b08 = using various teaching methods (loading .92)

b09 = demonstration and visulization in teaching (loading .83)

b10 = using different models of teaching (loading .92)

TABLE 6. Frequencies, means, standard deviations, and probabilities for M = 3

	1	2	3	4	5	M	s	p<
b07	1	18	14	20	3	3.11	.99	.42
b08	2	15	13	22	4	3.20	1.03	.16
b09	1	6	18	26	5	3.50	.87	.0001
b10	3	18	17	16	2	2.93	.99	.59
Σ	7	57	62	84	14			
%	3	25	28	38	6			
s6		max 2.00	min -1.33			.19	.67	.04
f6		max 1.47	min -2.86			-.33	.91	.01

s6 = sum scores, s6 = (b07 + b08 + b09 + b10):4 - 3
f6 = factor scores

28 % of the teachers in the sample think that the methodological skills are nearly or fully inadequate, 28 % can not say whether they are adequate or inadequate, and 44 % think that they are nearly or fully adequate. Only the mean of demonstration and visualization in teaching is significantly over the midpoint of the scale.

The mean of the factor scores is significantly ($p < .01$) under zero. The negative mean of factor scores shows that the mathematics, science, and technology teachers do not evaluate the adequacy of their skills in teaching methods as high as other teachers.

Adequacy of coping with interactive situations was measured with four items (Cronbach alpha = .89):

b11 = flexibility in teaching situations (oblique loading on the factor .97)

b12 = coping with unexpected situations (loading .82)

b25 = flexibility in changing teaching arrangements (loading .79)

b26 = maintaining discipline in the classroom (loading .73)

TABLE 7. Frequencies, means, standard deviations, and probabilities for M = 3

	1	2	3	4	5	M	s	p<
b11	6	10	17	15	8	3.16	1.20	.33
b12	12	16	10	11	7	2.73	1.34	.14
b25	6	9	18	19	4	3.11	1.11	.48
b26	11	14	12	12	7	2.82	1.32	.32
Σ	35	49	57	57	26			
%	16	22	25	25	12			
s7	max 2.00 min -2.00					-.19	.81	.10
f7	max 1.91 min -2.52					-.05	.97	.68

s7 = sum scores, s7 = (b11+b12+b25+b26):4 - 3
f7 = factor scores

No one of the means of the variables of this dimension is significantly over or under the midpoint of the scale. But 38 % of the ratings in these items show nearly or fully inadequate skills in this area. That is a big number. When there are 25 % of ratings for difficult to say, so only 37 % of the mathematics, science, and technolohy teachers think they have nearly or fully adequate skills in this dimension.

But the factor scores show that the mathematics, science, and technology teachers do not think in a lower grade than the other teachers that their coping with interactive situations are adequate.

5. Discussion

Seven dimensions of teachers' professional knowledge and skills were interpreted in the study. They can be interpreted to be related to the three components of intentional activities so that the first three dimensions are related to the first component, choosing objectives, three other ones are related to the second component, designing a plan, and the seventh dimension is related to the third component, acting on the plan. Teachers' self concept can be analysed in these dimensions, and their conceptions about their knowledge and skills acquired in the basic teacher training vary in a logically interpretable way in these dimensions.

Concerning subject knowledge (SKn), most mathematics, science and technology teachers think that their knowledge is adequate. In education knowledge (EKn), there are more of those teachers who think their knowledge is adequate than of those who think it is not adequate. As to institution knowledge (IKn), a majority of mathematics, science, and technology teachers think that the knowledge is inadequate. About didactical skills of subject matter (DSk), most teachers think their skills are adequate. In evaluation and planning skills (PSk), a majority of teachers thinks their skills are adequate. In the skills of teaching methods (MSk), there are not significantly more of those who think their skills are adequate than of those who think they are not adequate. Concerning coping with interactive situations (ICo), the difference between those who think the skills are adequate and those who think they are not adequate is also not significant. These situations of the means of the sum scores are illustrated in the Figure 2.

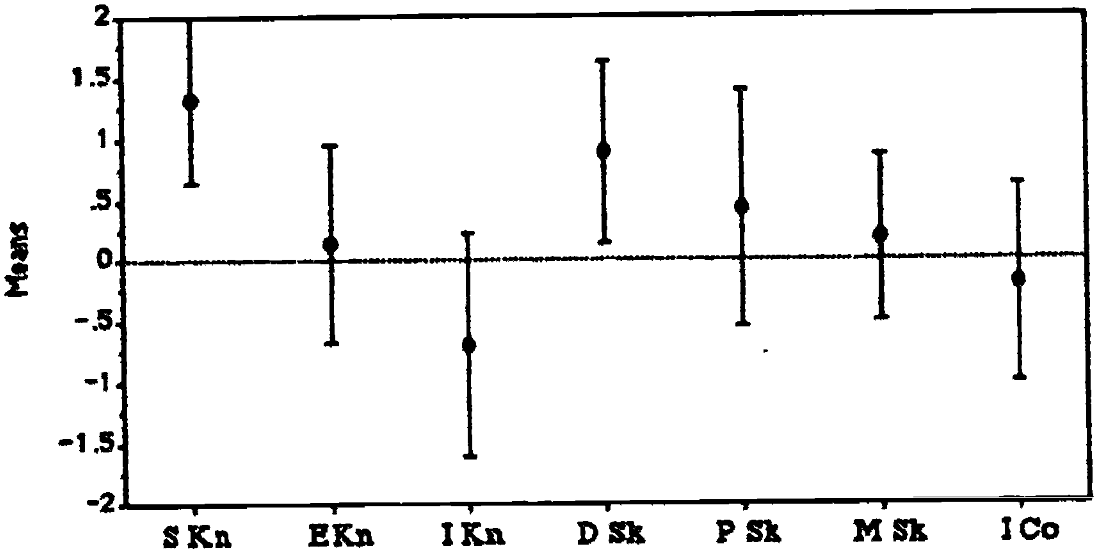

FIGURE 2. The means and one standard deviation error (plus/minus) bars for the sum scores of teachers' knowledge and skills

The ratings of the studied mathematics, science, and technology teachers were significantly higher than those of the other teachers in the following two dimensions: subject knowledge (SKn), and evaluation and planning skills (PSk). They were significantly lower than those of the other teachers in the following dimensions: institution knowledge (IKn), and skills in teaching methods (MSk). There was no significant difference between the ratings of mathematics, science and technology teachers, and the other teachers in the following three dimensions: education knowledge (EKn), didactical skills in subject matter (DSk), and coping with interactive situations (ICo). The mean of the education knowledge is under the midpoint of the scale but the significance is only 11 %. These situations of the factor scores are illustrated in the Figure 3.

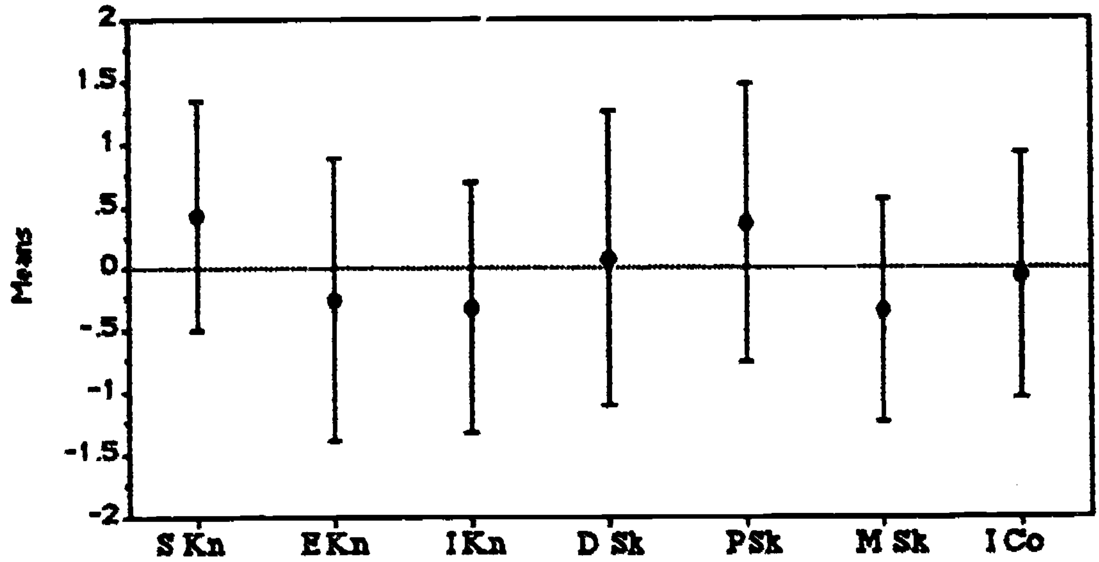

FIGURE 3. The means and one standard deviation error (plus/minus) bars for the factor scores of mathematics, science, and technology teachers' know ledge and skills

One of the weaknesses of this study was that the sample was not selected so that it would be representative for all mathematics, science and technology teachers in the Finnish comprehensive school. It would be a most interesting and important question to study with bigger and more representative samples the characteristics of various groups of teachers.

It must be remembered that an evaluation of adequacy does not indicate how much knowledge and skills are needed. If a skill is needed very little then a minor skill is adequate enough, if an other skill is needed very often and in variuos situations then a major skill could be inadequate. There is an other important limitation, too. All knowledge and skills that the teachers need can never be taught in the basic teacher training, e.g. institution knowledge concerning different schools in various circumstances can never be fully treated in the teacher training. So, from the point of view of the basic teacher training, the differencies between the various skills are normal and acceptable.

The used approach, perhaps, does not give a valid picture of teacher training as it really was years ago, when these now 30, 40 or 50 years old teachers were studying there. But it gives (or tries to give) a valid view of the inner reality in which they are working now. They have conceptions as a part of their self-concept about the knowledge and skills they have learned in the basic teacher training. These conceptions are a basically important part of their professional competency. The basic training of experienced teachers who work in schools can not be changed afterwards. It has been so adequate as possible, and it must be adopted as such. The results about teachers' conceptions could be used in planning of teachers' in service training and in improving teachers' working conditions in schools.

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STUDY ORIENTATIONS IN MATHEMATICS AMONG UPPER SECONDARY SCHOOL STUDENTS

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The article describes a socio-cognitive theory of orientation and its application to mathematics learning at the upper secondary schools. The social relationships linked with the study of mathematics were reviewed on the basis of the experiences of the students and the conclusions they had drawn from them. Four study orientations were defined on the basis of the socio-cognitive orientation theory: problem solving, social dependence, ego-defensive and giving-up study orientation. In the sample of the survey used in the empirical part of this study there were 452 upper secondary school students, 16-18 year olds. The investigated study orientations were typical of mathematics learning. Factor analyses gave 3 or 4 study orientations. In all factor analysis solutions, the last factors with the weakest explaining capacity were operational skill, the consideration of the similarity between solutions of problems, and self-confidence in problem solving. One common feature is that an upper secondary school student concentrates on the operational activeness of solving problems. A second common feature is social dependence and learned helplessness. Indifference for learning to understand mathematical language, texts, and problems as first and second factors were manifested somewhat differently in the students of long and short courses. The correlations between observed achievements and school marks in mathematics were statistically significant.

1. Introduction

The experience of meaningfulness at school is by its nature largely a product of social interaction. The constructive concept of learning presupposes consideration of differentiating instruction at the upper secondary school, which has become more problematic than earlier as a result of changing social conditions. Which situational factors does the student give greatest weight to in his orientation towards a learning task? In what ways is the study of mathematics by individual upper secondary school students similar or different?

The individual learner is among and exposed to many kinds of prestructured activities. The learning process cannot be described as a passive adoption of some structure of knowledge that is given from outside, but as an active process, where the individual is in continuous interaction with his environment. Study, as an element situated between teaching and learning, adds to the picture the traits of the students, self-concept as a learner. This article, which is based on an inquiry (Yrjönsuuri 1989), deals with the experiences, expectations and activities of upper secondary school students in the study of mathematics in Finland.

When curricula are drawn up, it is assumed that students have a positive attitude towards learning mathematics and that they experience learning it as meaningful. The learning process has, however, often instrumental meaning for an upper secondary school student. It does anticipate learning but the expectations or the social community. The student is directed by the social need of being accepted by the community and fulfilling its expectations. Learning difficulties arise, when the student lacks confidence in his own ability to learn.

Performance processes assume a positive meaning by being important to persons that the student esteems highly. As a result of failures the student starts to become estranged from his responsibility for learning and ends up in learned helplessness or gives up studying (Diener & Dweck 1978). Do the goals of instruction and their attainment go in different directions?

2. Situational orientations and study orientations

A socio-cognitive theory of orientation has been developed at the Department of Education in Turku under professor Erkki Olkinuora (1984, 1988; Salonen 1988). The purpose of the inquiry into the study orientations of upper secondary school students was to examine the orientations that students used in the study of mathematics and their relation to the success experienced as well as to the success expressed in school marks in mathematics. The subjects of the study were 453 upper secondary school students (Yrjönsuuri 1989).

In the inquiry, the social relationships linked with the study of mathematics were reviewed on the basis of the experiences of the students and the conclusions they had drawn from them. The conclusions were situationally linked with hope for success and fear of failure, owing to the student's own abilities, endeavours, difficulty of task or accident (Weiner 1985). A central concept of the socio-cognitive orientation theory is situational orientation. In the study of mathematics it was named study orientation. Study orientations include the conclusions that the student reaches in the interactional relationship between 'ego' and 'environment', when the purpose is to learn. The orientations are assumed to be directed towards the learning task, the ego of the learner, the social environment, or none of these. They are assumed to be results of learning, in certain ways permanent but also changeable. They depend on the subject, the task, the time and the place. The word 'situation' accentuates the change of orientations and their dependence also on each other. Four study orientations were defined on the basis of the sosio-cognitive orientation theory and they were named problem solving study orientation, social dependence study orientation, ego-defensive study orientation and giving-up study orientation.

The activity of a problem solving oriented student is directed by inner motivation and hope for success towards the solution of the task and the acquisition of knowledge that is required for it. The learning process is characterized by an orientation towards the future. The student interprets the elements of instruction and the teacher's views of the task as instruments for improving and reorganizing his own views of the task; thus it is the 'I' that rules and controls. The emotions arising from the task are directed towards the matter, not the ego. They are connected with schemes that are needed in the formation of the structures of knowledge to be learnt. For this reason cognitive conflicts are not experienced as menacing but as motivating for renewed endeavours. This type of problem solving study orientation is considered a normal and fruitful approach toward study and toward learning tasks in general.

In the inquiry, the problem solving study orientation was manifested in the underlining of operational activeness and in consciousness of goals. The activity of students concentrated on following the directions given by the teacher and on methodical solution of the mathematical problem. The concept of instruction that teaching technology has is limited to this kind of learning, but according to a new type of concept of learning this is not sufficient. This kind of learning is a direct result from the structure of the curriculum, becauce studying aims at an operational solution such as solving an equation or inequality, derivation, integration etc.

A student that is characterized by social dependence study orientation concentrates on the social environment of the moment. Performance processes get positive meaning by being important to persons that the student esteems. The student concentrates on the stage of instruction at hand, where the general linguistic frame of reference is more important than the structure of the learning task. The study process has instrumental meaning for the student.

This type of study orientation was manifested in many ways, both positive and negative. Students describe their study in following ways:

- *I do not see the point of the teacher's question, what answer he wishes to have;*
- *I do it like this, because we do so in the classroom;*
- *Teach me so that I will get maximum marks in the matriculation examination;*
- *Write the solution in full so that I can copy it corretly;*
- *Give us in the test a problem like 'this'.*

To a certain limit social dependence is necessary, so that instruction together with problem solving study orientation might be fruitful and bring forth profound results of learning.

The following two study orientations are directed away from problem solving and indicate learning difficulties. A student with ego-defensive study orientation attributes the causes or results of earlier failures to external reasons, for instance the difficulty of the task, rather than to his own endeavours. If he associates failure with his inability, it leads to a growing feelig of inferiority and to further failures. The student no more interprets the task independently. The past is predominating in the anticipation of the future. The student gives little thought to the fruitfulness of his study, and even then, it is mainly to protect his ego from failure (Peterson 1984). In positive situations he is able to express the real reason for his fear of failure, e.g. certain mathematical contents or a certain method.

The student aims at passing the course of mathematics. Owing to his fear of failure he believes he is more likely to attain this goal through accident and ego-defence. He begins to look for these means and forgets about learning mathematics. Typical features are, for instance, denial of the situation by telling 'I could do this if I only cared to try'. Or he may turn his distressing hostility to mathematics into exaggerated politeness towards the teacher, because it is important to him to retain good relations with the social community and so to manage the situation. In the student's classroom work features like the following can be observed:

- gives indistinct answers;
- guesses at results and claims to have solved the problem;
- tries to remember a formula, which he then uses whether applicable or not;
- begins many tasks in an accidental way and attributes his poor success to external reasons;

In the inquiry (Yrjönsuuri 1989), the ego-defensive study orientation was manifested in many kinds of hindrances to learning, such as the difficulty of mastering mathematical concepts and perceiving the problem, lacking self-confidence, and uncertainty of control, i.e. inability attributions.

A student characterized by giving-up study orientation does not experience the study of mathematics as meaningful to himself, but he may experience the study of some other subject as highly meaningful. The giving-up study orientation differs from the ego-defensive study orientation in that in a study situation the student does not experience motivational tension caused by the meaningfulness of the study. A student with ego-defensive study orientation may have even strong motivational tension, which he tries to overcome by resorting to defences available in given situations. Experiences of failure and the meaning given to them begin to change the student's structures of meaningfulness. The continuous dead-end experience influences the general basis of meaningfulness of the student's actions. In this case the change in the structures of meaningfulness means liberation from the motivational tension in learning mathematics. As the learner does not any more experience the study of mathematics as meaningful at all, he starts looking for compensation elsewhere, at best in other subjects or later outside school.

3. Study orientation and success

At the Finnish upper secondary school (16-18 year olds) all students study mathematics as a compulsory subject, choosing either an advanced course of 9 units or a short course of 6 units. In the final matriculation examination, students of the advanced course have to take a test in mathematics, whereas students of the short course may take it as an alternative part of their final examination.

When the students were grouped according to their choice of advanced or short course in mathematics, the difference between the averages of problem solving study orientation was statistically highly significant, as was also the difference between the averages of ego-defensive orientation, whereas the difference between the averages of social dependence orientation was only nearly significant.

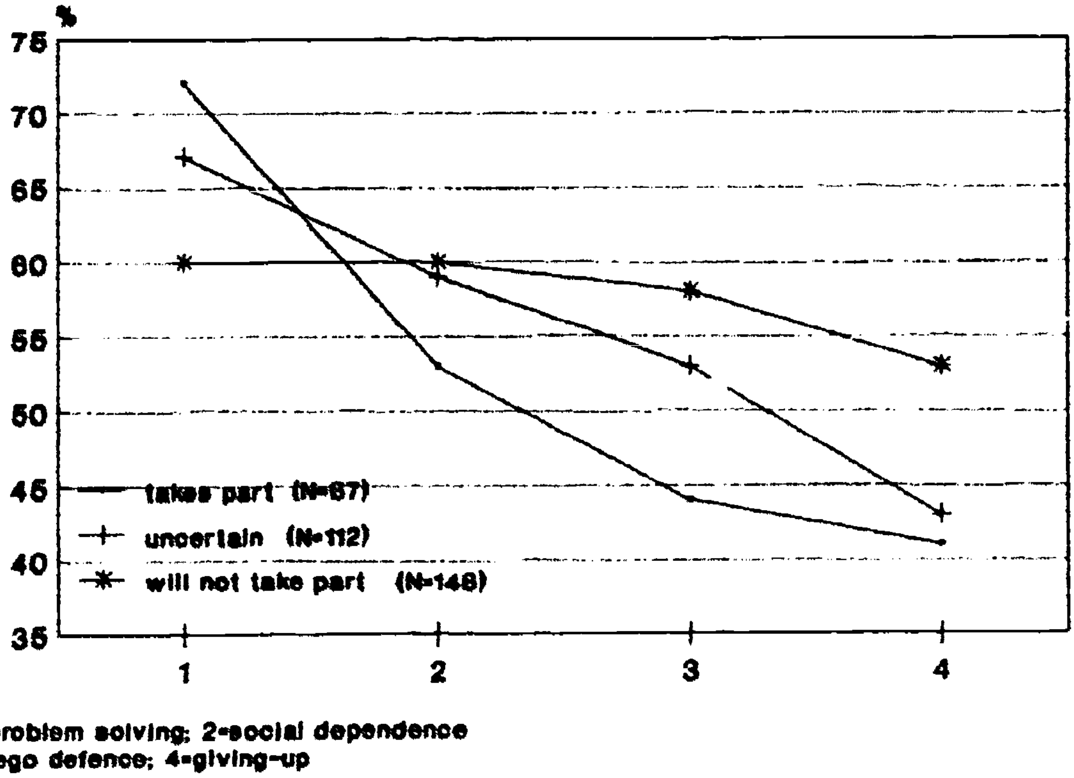

FIGURE 1. The profiles of the short course student's study orientations in the second year of the upper secondary school according to their intention of taking the mathematics test in the final matriculation examination. The vertical axis shows the persentages of the average orientations of the subgroups.

Figure 1 illustrates the proportion and differences of study orientations, when the students of short course were grouped according to whether they intended to take the test of mathematics in the final matriculation examination or not.

The average of problem solving study orientation among the students of the advanced course was 72 % of the maximum sum total, and that of the students of the short course 67 %. The average of ego-defensive study orientation among students in the advanced course was 46 % and in the short course 52 % of the maximum total.

The average percentage of problem solving study orientation averages is highest among the short course students who will take the mathematics test, among non-participants all other study orientation percentages are higher. Students of the advanced course show approximately the same profiles as the short course students who will take the mathematics test.

The success that the studens have experienced in mathematics explains the differences between the averages of study orientations. Students who have had excellent success have higher average of problem solving study orientation than students with below average success. The difference is highly significant. The average of study orientation that is directed away from the problem solving is lower among students who feel they have had excellent success than among those who feel they have had only passable success. Also this difference is highly significant. Those who in mother tongue and first foreign language have excellent or passable success do not show any difference in study orientations in mathematics.

4. Comparison between the study orientations of students of the advanced and the short course

The study orientations were investigated using factor analysis. The analysis of the advanced course produced four and that of the short course three factors that could be interpreted and had an acceptable explanatory capacity. These factors were called observed study orientations. They describe the situation among the students under investigation. How far this situation can be generalized depends e.g. on sampling, measuring instrument and the interpretation of factors. In any case it seems worth - while to review the result also by studying the investigation in greater detail and by reconsidering one's own teaching. A hierarchy of the formation of learning difficulties is given below as a comparison between students taking the advanced and the short course. Figure 2 gives a concise picture of what the investigation shows about the essential similarities and differences between the study orientations of advanced and short course students of mathematics.

In all factor solutions, the last factors with the weakest explanatory capacity were operational activeness, the consideration of the similarity between solutions to problems, and self-confidence in problem solving. As a common feature we can state that an upper secondary school student concentrates on the operational skill of solving problems.

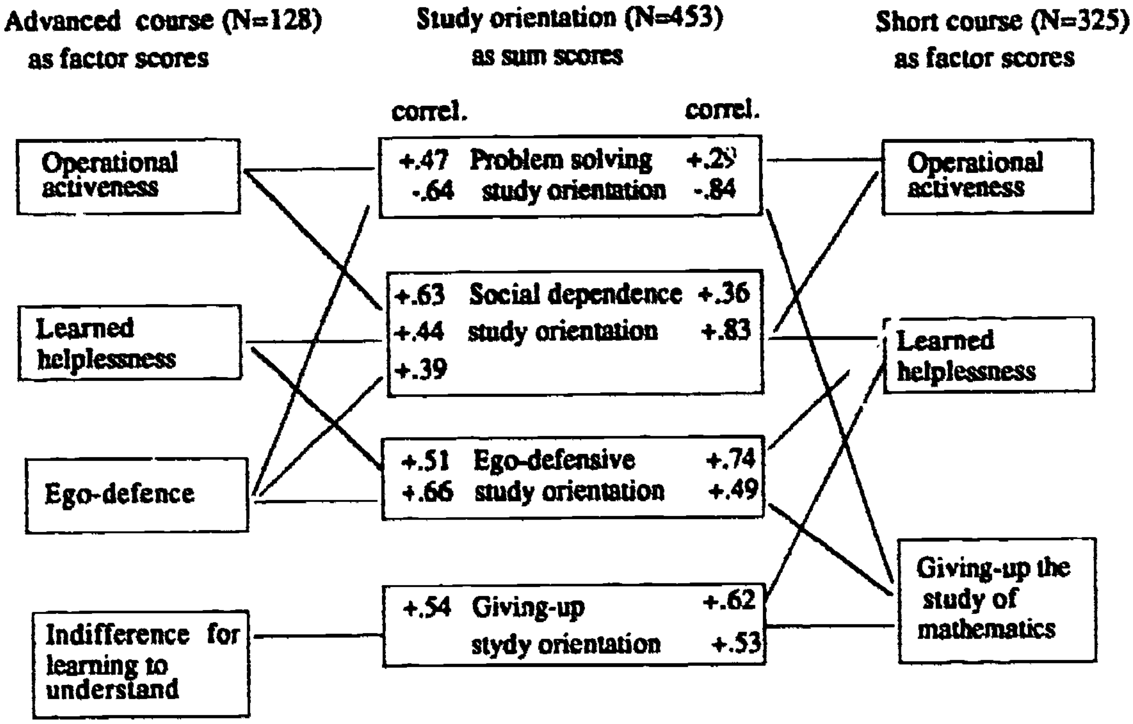

FIGURE 2. The correlations between the study orientations of upper secondary school students and the orientation according to the model of the socio-cognitive orientation theory

As the second common feature we can regard social dependence and learned helplessness, which came out in all factor solutions. This means the following: An upper secondary school student experiences his own activity as meaningless in his endeavours to learn mathematics, and mostly thinks about his social environment when seeking for accepted performance. Social dependence is central. It is not important for the learner to understand mathematics but to pass the course. He shifts the responsibility for his learning to the social community. In variance analysis, when the grouping variables were sex, course, and participation in the matriculation examination, there were no statistically significant differences in the averages of the study orientations mentioned in this and the preceding paragraph.

Giving-up and ego-defence in the study of mathematics as first and second factors were manifested somewhat differently in the students of the advanced and the short course. Common features were the lack of interest in and indifference about learning the mathematical pattern of thinking, and the uncertainty of experiencing psychological and even cognitive meaningfulness. As differences we can state that the students of the short course give up the study of mathematics, the students of the advanced course show mostly indifference to having the mathematical pattern of thinking. On the basis of variance analysis the differences in these two above mentioned study orientations were statistically highly significant, when the grouping variables were sex, course, and participation in examination. The data on averages should not be generalized to apply to all students.

Defences emerging among students of the advanced course included mastery of mathematical concepts, use of language and discovery of the crucial problem in a given task, the presentation of the solution and the appraisal of the task as a whole. The abundant mathematical contents of the advanced course and the mastery of the situation enable the student to express readily even what is difficult for him, whereas the short course student is not able to do so.

The short course students's giving up of the study of mathematics is shown by the fact that in the second year of the upper secondary school, 30 % of students do not intend to take the mathematics test in the final matriculation examination, and 40 % are uncertain about their participation. The advanced course students's giving up is described e.g. by their transition to a short course in mathematics. About one-fifth make this transfer.

Finally the floor is given to some upper secondary school students who have successfully studied mathematics. They answered the question, what in their opinion was essential in learning mathematics, as follows:

> *"You learn mathematics when you are interested in the subject and you think only of mathematics."*
>
> *"The atmosphere in the classroom and the teacher's role contribute to making the study meaningful. ... but of course your own hard work has the greatest impact on success."*
>
> *"To have success in the study of mathematics you must work hard. You must learn to work independently and to understand the subject, merely learning formulae by heart is not enough. Instruction also has an effect on learning. The problems must be reviewed in detail*

together with the group and the teacher."

"I think the professional skill of the teacher has great impact on learning mathematics. To a great extent you cannot work out the problems on your own, as you can in subjects like history and biology and even languages. If you notice that the teacher really cares about whether you learn mathematics or not, it motivates you to try. Matter-of-fact dealing with questions and points that are not clear to the student also promotes learning; comments like 'it just is so' or 'but you should understand that' discourage in-depth study of the matter."

"What is the use of this inquiry again? If this is to be used in teacher training, I would rather draw attention to the teacher's attitude towards students and teaching. A good teacher is by far the best encouragement to learning mathematics, if you do not have enough enthusiasm otherwise. The attitudes of students change easily, if the teacher manages to make his lessons genial."

The central task of the teacher is to maintain social interaction and positive atmosphere in the classroom community by being interested in every student as an individual. The student underlines 'the teacher's professional skill', which he interprets as social, psychological and educational interaction. The student trusts the teacher's cognitive mastery of the subject, if he only 'was able and patient enough to give the students time to learn'. The central reason for the emergence of ego-defence is the disproportion of the time available and the contents to be learnt as well as the proportioning of the whole of the upper secondary school to extramural interests of young people, which also shall have an essential task in the development of a young person.

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A CONTEXTUAL APPROACH TO THE TEACHING OF MATHEMATICS: OUTLINING A TEACHING STRATEGY THAT MAKES USE OF PUPILS' REAL WORLD EXPERIENCES AND STRATEGIES, AND THE RESULTS OF THE FIRST TEACHING EXPERIMENT OF THE PROJECT

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This paper outlines an alternative teaching strategy to prevailing methods of teaching new mathematical concepts, one that takes into account from the very beginning of teaching the real world experiences that pupils have in connection with the content to be taught and their intuitive models of thought and action. At the same time as the "formalistic" teaching strategy is criticized, it is suggested what is needed and must be taken into consideration when teaching is planned and implemented in accordance with the contextual (modelling) approach. Finally, the main results of the first teaching experiment of the project are examined. This teaching experiment focused on developing the pupils' proportional reasoning in the continual and discrete mixing contexts. The experiences and results from this experiment suggest that it is possible and gainful to develop proportional reasoning in accordance with the contextual teaching strategy. However, to construct a really good contextual program for the teaching and learning of ratio and proportionality, a series of teaching experiments need to be performed which is also the purpose of this project.

1. The intention of the paper and a remark

This paper aims at outlining an alternative approach to the teaching of mathematics, one in which the pupil's related real world experiences and intuitive models of thought and action are taken into account and made use of from the very first stages of the teaching of mathematical contents. For this purpose, this article first examines and criticizes the prevailing "formalistic" model of teaching mathematics, outlining the various stages in the teaching of mathematics according to the alternative approach. Finally, I present some of the most important results of the first teaching experiment of the project entitled "Contextual

Approach to the Teaching and Learning of Mathematics" initiated in the spring of 1988. It is tentatively revealed in this context what is needed, what needs to be taken into consideration and how one should act when mathematics teaching is planned and put into effect in accordance with the contextual approach.

I want to stress it right away that the teaching strategy that is outlined in more detail below cannot and shall not be considered a general "mould" for the teaching of all mathematical subject matter. On the contrary, there is a lot in mathematics in which resting on real world experiences and models of observation may inhibit rather than support the acquisition of a valid idea of the mathematical system to be learned. It is clearly so in the case of negative integers. Their valid acquisition requires that one leans on the formal properties of the number system rather than on hard to find real world models of observation (for more details, see Freudenthal 1973, 280-282; Fischbein 1987, 97-102). On the other hand, it does not mean that the planning and implementation of teaching should not take into account schemes formed through previous learning experiences of mathematics, with which the pupil attemps to link the new mathematical knowledge and skills to be learned such as negative integers and mathematical operations with them. This can be summed up in the words of Fischbein (1987, 207):

> *"Our mathematical thinking remains profoundly rooted in our adaptive practical behavior, which implies spatial representability, concrete consistency, fluent continuity. The main problem is to learn to live with the intuitive loading of concepts necessary to the productive fluency of reasoning and at the same time, to control the impact of the very course of reasoning of these intuitive influences. For this, the student has to become aware of the exact formal meaning and the implications of the mathematical concepts, on one hand, and the underlying intuitions, on the other."*

2. On the bases and motives for the development of the teaching of mathematics in accordance with the contextual appi ach

The basic motive for the teaching of mathematics in schools can be seen in the fact the mathematics forms an essential part of the cultural historical heritage of mankind. Another fundamental motive is the fact that mathematics offers useful and partly irreplaceable tools for managing the practices of modern technological society, for utilizing them and for solving various scientific problems connected with the real wcrld. It is therefore desirable and presumable that the teaching strategies used in schools produce permanent and widely transferable mathematical knowledge and give a valid idea of both the constructive (mathematical modelling and development of axiomatic systems) and axiomatic deductive and applicative nature of mathematics (formal structures of mathematics and their application).

Close examination of existing curricula and text books adhering to them reveals that the teaching strategy used in the university teaching of mathematics first theory, then the applications has in many places and in various forms been absorbed to the teaching of mathematics in both the primary and secondary levels of education. Actually, this is not very surprising in view of the educational background of most of those who are currently responsible for the planning and implementation of the teaching of mathematics. In fact, these people have become accustomed to functioning within the confines of readily given deductive systems and to using the symbols and techniques developed by mathematicians to solve relatively ready formed and closed contextual exercises. They have less experience in the mathematical modelling of genuine real world situations and events and in why and how they have ended up in their formal systems. Some influence on the adoption of this "formalistic" approach in school teaching has obviously been had by the fact that teachers do not have available any teaching packages providing a proper example and opportunities to deviate from the customary models of thought and action.

From the viewpoint of adopting mathematical concepts, the "formalistic" strategy to teach is, to cite Hans Freudenthal, about the teaching of mathematical abstractions by trying to concretize them (see "the approach of concept attainment", Freudenthal 1983, 31-33, cf. "two compartment approach", Carss 1986, 202-204). This article, however, refers to this teaching strategy as a "formalistic" approach, because in it the teaching of mathematics focuses on the learning of formal systems and symbols and is only used a posteriori to solve real world problems (cf. de Villiers 1986, 13-15).

One sided use of the formalistic strategy involves a number of disadvantages. Firstly, it is indisputable that it largely overlooks mathematical modelling. Therefore the pupils may acquire a very biased and even erroneous notion of the nature of mathematics. Mathematics involves, after all, much more than just formal calculus according to agreed rules, or derivation of theorems, or proving based on given basic statements, although these certainly form an essential part of mathematical activity. As emphasized by de Villiers, acting within the confines of ready given deductive systems cannot give a valid idea of the meaning of axiomatization and of the functions in the process of developing and researching mathematical systems (1986, 15-21).

Another disadvantage and one that is closely associated with the one mentioned above, is the fact that the pupils are deprived of an opportunity to invent and develop by themselves the mathematical tools which are needed and were even originally developed for analyzing and utilizing the quantitative, spatial and temporal phenomena and events of the real world. There is the danger of a lack of meaningful experiences, which easily leads to the pupil gradually not even trying to understand the new mathematical knowledge and skills intended to be learned (see meaningful learning vs. rote learning, Ausubel & Robinson 1969; cf. also on the meaningfulness of studying, Olkinuora 1979).

A third disadvantage, closely linked to the two above, is the fact that the teaching of mathematics according to the formalistic approach very easily leads into the learning of mathematical knowledge and skills by rote, and thereby to their rapid oblivion and to feeble transfer. This is easily observed by examining in more detail for instance the teaching of ratio and proportion in conformity to the formalistic strategy. The strategy makes no effort to make explicit use of the pupils' proportional inferences for a complete and generally applicable solution (see Keranto 1989). It is thus actually not at all surprising that Kupari's nationwide survey of mathematics learning achievements among the basic school 9th graders recorded poor performance especially in the solution of contextual problems and specifically in ones involving proportionality (Kupari 1983, 124-137).

So, on the basis of what has been said so far, there are clear grounds for outlining an alternative strategy to the formalistic approach of teaching mathematics, one which takes into account the real world experiences and intuitive models of thought and action that the pupils have in connection to the topic to be learned.

3. Theoretical background and outlines for the teaching of mathematics in accordance with the contextual approach

The research conducted by the cultural historical school of Soviet psychology suggest clearly that the planning and implementation of a meaningful learning process requires that all mathematical knowledge and skills to be learned are linked to real world practices in such a way that the scientific concepts to be learned, such as ratio, proportion and proportionality, enhance this practice by modifying and enriching it. In other words, even the process of learning mathematical contents should be motivated by the need to understand and control one's own life practice (see the development of scientific concepts, Vygotsky 1962, Leontyev 1977; see also the subject problem of learning activity and the formation of theoretical concepts, Engeström 1983, 139-151). The empirical teaching experiments reported by Leontyev show in a clear and emphatic way that motivated learning processes such as these can actually be effected. In these experiments, the need to cope in the best possible way in practical problem situations caused an interest and an internally felt necessity to find out about the theoretical grounds of the problems and about models for explaining them (1977, 232-237).

Leontyev also thought that the studying of mathematical contents should be made part of a pupil's life practice. It is easier said than done! Nevertheless, it is very obvious that a meaningful learning process understanding the mathematical topic that is meant to be learned and is new to the pupils' learning process should endeavour to start from those real world contexts whose analysis really requires such mathematics and for whose control and utilization it was originally developed. In this way, the pupil is in a position to discover gradually the true meaning of the mathematical tools presented in a symbolic form, and the driving force crystallized in them through the work of the mathematicians of previous generations. An attempt is made to start the teaching of ratio and proportion, for instance, from such real world situations and problems involving direct real world proportionality, the analysis and solution of which really requires the scientific concepts in question. A gradual move is then made from mental solution to a written use of ratio and proportion. In the following, a teaching strategy like this that starts from the examination of real world situations and proceeds to their mathematical mastery is called the contextual approach (cf. "mixing approach", Carss 1986, 202; Hirst, A. & K., 1988, 243; "didactical solution", Freudenthal 1983; "modelling approach", Heller et al. 1989).

Figure 1 shows that the contextual approach to the teaching of mathematics is largely the inverse of the formalistic approach.

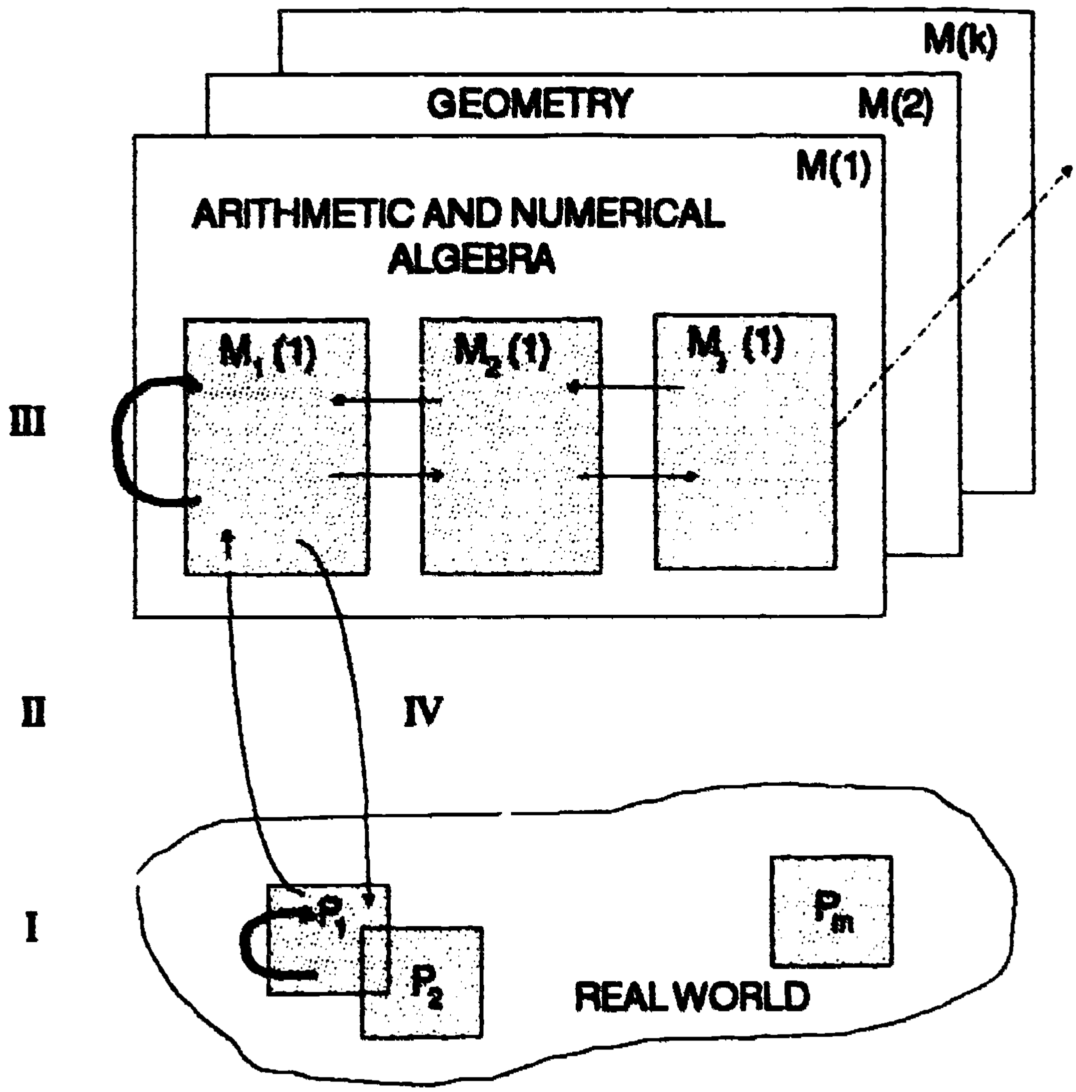

FIGURE 1. Teaching mathematics in accordance with the contextual approach: the teaching process.

The teaching of mathematics conforming to the contextual approach proceeds as follows (see Figure 1): First, those real world problem situations or that category of problems is examined whose solution really requires the mathematics that is to be learned. The aim is to find the basic principle or the basic relations for the category of problems in question. Secondly, an effort is made to model these basic relations. The third stage comprises of an internal elaboration of the mathematical model to enable the pupil to solve any problems belonging to the category under discussion. Fourthly, one goes back to solve the real world based problems that were referred to. The first cycle closes and another one can begin if it is found out that the mathematical model that was developed is not yet sufficiently general and complete for the solution of new problems. This process will be discussed in the following from the viewpoint of the pupil's learning acts.

4. Teaching in accordance with the contextual approach from the viewpoint of the pupil's learning acts

In the following, I will try to provide a brief outline of the contextual teaching strategy from the viewpoint of a pupil's learning acts. My purpose here is also to prepare the reader for an analysis of the first teaching experiment of the project initiated in the spring of 1988 ("Contextual Approach to the Teaching and Learning of Mathematics"). Some of the main results of will also be discussed.

An economical and illustrative way to start a discussion of the learning acts that are involved in teaching following the contextual approach is to examine the following model elaborated on the basis of ideas presented by Davydov (see Davydov 1982a, b; see also Engeström 1984, 13). Ratio, proportion and proportionality were chosen as the mathematical example in Figure 2 below, as their teaching/learning process is the most important object for research and elaboration in the project so far as content is concerned.

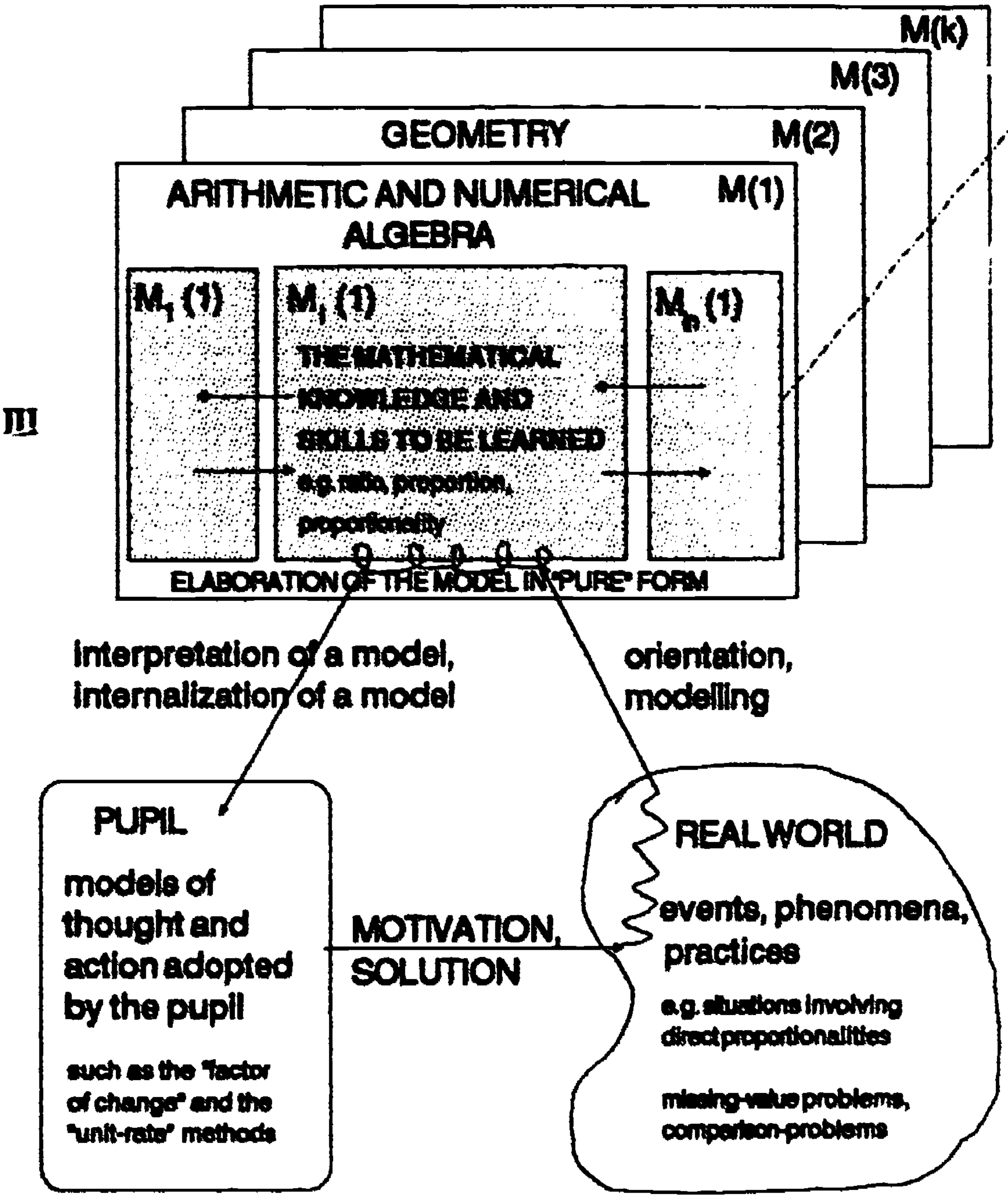

FIGURE 2. The whole of learning acts in the contextual approach to the teaching of mathematics.

In the first stage, the pupils' attention is directed to the examination of such real world situations, a valid analysis of which calls for mastery of the mathematical knowledge and skills that are meant to be learned. When the situations are analyzed, a cognitive conflict that may be established spontaneously and explicitly is generated between the pupils' output and the objective requirements how the problems should be solved. The main purpose of this stage is to generate an internal tension that motivates the pupils to find a valid model for the solution of the problems that were presented. In the teaching of ratio, proportion and proportionality, this conflict may be produced in a mixture context. The (erroneous) output by pupils obtained through mental reasoning is compared to the strengths of taste and colour in mixed liquids that were actually produced.

In the second stage, an attempt is made to direct the pupils' attention to features relevant for the solution of the entire category of problems. By modifying systematically the way in which the problems are put, the pupils are helped to find the basic principle. In the context of liquid mixtures, for instance, the basic principle for producing mixtures of the same strength is the invariance of the proportional shares of the liquids to be mixed. The within and between ratios remain constant.

In the third stage, an attempt is made to model the observed basic relations. The discovered mathematical model is developed in a "pure" form to such an extent that the entire category of problems that was originally under discussion can be solved in a valid way. For instance, mental identification and production of equally strong mixtures of liquid can be based on the observation of between and within ratios and on the formation of equivalent ratios (see within and between ratios, Karplus et al. 1983, Noelting 1980a, b; cf. the factor of change method, Post et al. 1988).

In the fourth stage, one reverts to solve the problems that proved too difficult at first with the aid of the mathematical tools that were learned. A new cycle starts when the pupils are taught to observe that the mathematical method that was developed is still defective or difficult to make good use of in certain problems. For instance, the pupils can be first taught to use the factor of change method to solve comparison problems involving integral ratios. At the end of the first cycle, one can present problems involving such difficult numbers and integral ratios that other methods are needed or worth using to sclve these problems, such as the unit-rate method or cross-multiplication method (for a summary of these methods, see Post et al. 1988).

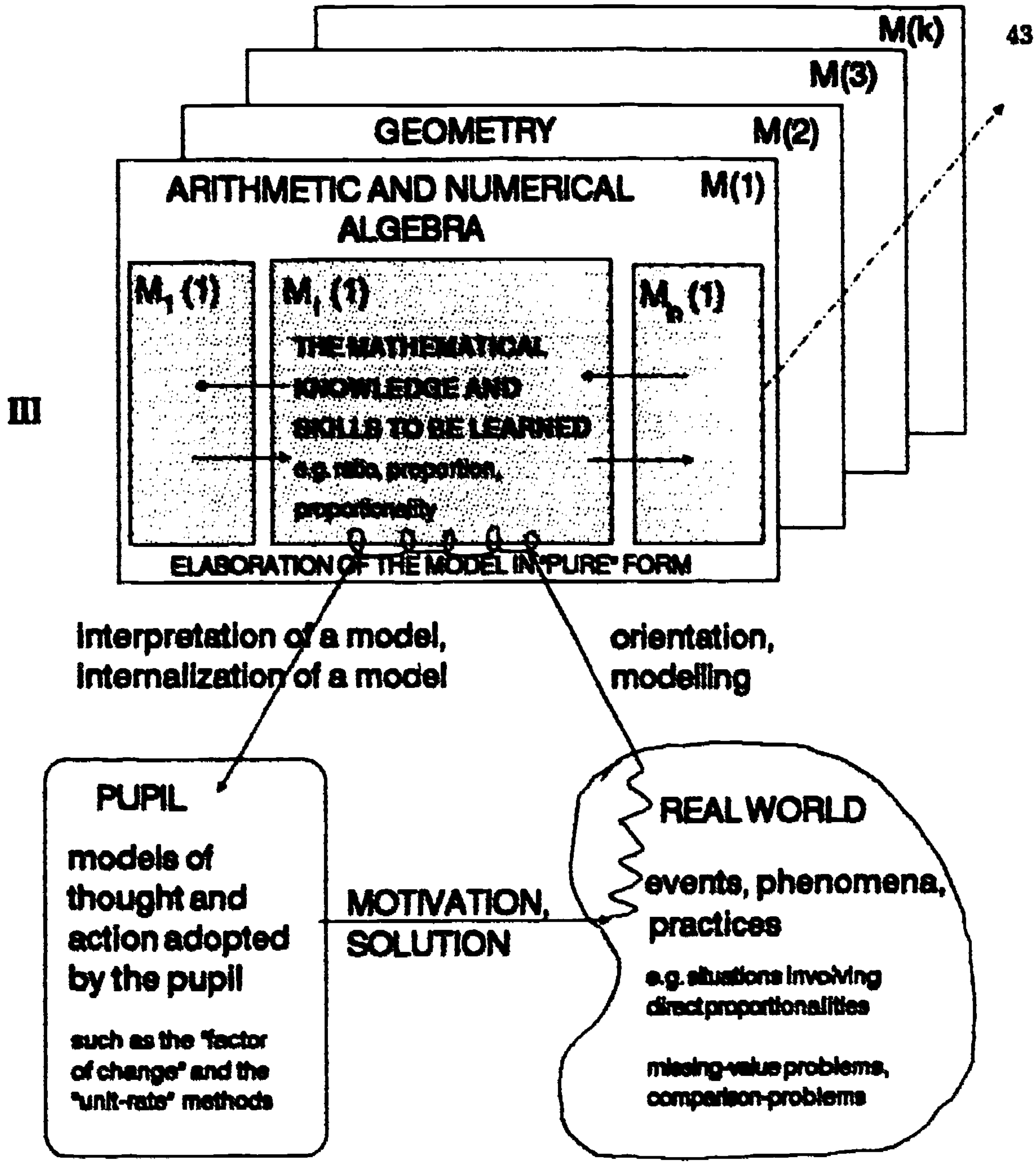

FIGURE 2. The whole of learning acts in the contextual approach to the teaching of mathematics.

In the first stage, the pupils' attention is directed to the examination of such real world situations, a valid analysis of which calls for mastery of the mathematical knowledge and skills that are meant to be learned. When the situations are analyzed, a cognitive conflict that may be established spontaneously and explicitly is generated between the pupils' output and the objective requirements how the problems should be solved. The main purpose of this stage is to generate an internal tension that motivates the pupils to find a valid model for the solution the contents used in the experiment. In other words, from the viewpoint of any learning experiences connected with ratio and proportion, both subgroups could be regarded equal when the teaching program started. Meanwhile, the sixth-graders had more learning experiences of other interpretations of the rational number and of calculations with these numbers. One of the fifth-graders missed so many lessons that none of his data was included in the analyses.

5.1.2. Measures

The author of this report constructed and implemented the teaching program to develop proportional reasoning in the pupils, with the teacher of the experimenting class providing assistance in the practical arrangements. The essential parts of all the 13 lessons (45 minutes each) were videotaped.

The teaching experiment had four phases:

PHASE I Assessment of the subjects' starting level in proportional reasoning by means of classroom tests developed for the purpose (comparison problems, continual mixing context, juice experiment I (1st lesson) discrete mixing context, lottery experiment I (2nd lesson); These pen-and-paper tests were preceded by an active demonstration at the beginning of both lessons, with the purpose of preparing the pupils for the experiment and to orientate them to the kind of problems that would be solved in the course of the teaching programme.

PHASE II The actual lessons (October 13 - December 15, 1988, nine lessons, 45 minutes each): The purpose was to learn how to solve mentally missing value and comparison problems involving integral pairs by observing the between and within multiplicity of these pairs and by forming equivalent ratios if necessary (cf, the factor of change method, Post et al. 1988). 1st lesson: Solving of comparison problems, creation of a cognitive conflict and examination of features relevant to the solution. 2nd lesson: Elaboration of a mathematical model, basic ratio 1:1 and equivalent ratios n:n. 3rd and 4th lessons: Ratios equivalent to the basic ratio n:m and vice versa. 5th and 6th lessons: Application of the elaborated model to the solution of real life missing-value problems. 7th and 8th lessons: Assessment of the applicability of the model to the solution of real world division in a given ratio and comparison problems. 9th lesson: Final assessment of the performance level attained in missing-value problems (discrete mixing context, lottery experiment II) and synthesis of what was learned.

PHASE III Final testing to assess the development of proportional reasoning in the subjects (discrete mixing context, lottery experiment I (12th lesson)).

PHASE IV Delayed final testing to assess permanency and transfer (constant mixing context, juice experiment I (13th lesson)).

As one may perceive, most of the actual lessons during the teaching experiment were used by mentally solving problems that require identification and formation of equivalent ratios. The pupils were guided to observe the within and between multiplicity of given pairs of numbers and to make use of equivalent ratios. The use of the unit-rate method or the cross-multiplication method was not taught to the subjects in the course of the first teaching experiment. Very little time was also available to solve more demanding problems (problems involving inequivalent ratios, see items {WBX, WX, BX and NX}). They were only dealt with in one lesson, which will inevitably show in the results discussed further below. It is also worth noting that the symbols used in the written solutions to the problems were adopted for the lottery problems. In other words, gradual symbolization of ticket combinations producing the same chance of victory formed the semantic basis for the understanding of the "pure" mathematics which was learned. It is obvious that verbal problems involving contexts other than the lottery one were also solved during the actual lessons. The liquid mixture context was only introduced in the initial test and in the seventh lesson in connection with the division in a given ratio problem. Beginning the fourth lesson, the program also contained independent solving and checking of series of proportionality problems, some of which were intended for differentiation of teaching. These series of four problems had been picked from a completed material of problem solution (such as the series "buying fruit", "number of steps", "playing" and "money exchange", Keranto et al. 1986b,c, 1987).

5.1.3. Classroom tests and analysis of their results

To assess the starting level of the subjects' proportional reasoning and cognitive development, three classroom tests were developed for the purpose: juice experiment I (22 comparison problems, continual mixing context), lottery experiment I (23 comparison problems, discrete mixing context) and lottery experiment II (14 missing-value problems, discrete mixing context). The number structures of the first two tests were identical with the exception of two tasks. The number pairs to be proportioned were positive integrals. The problems in lottery experiment II were similar in number structure to those problems in lottery experiment I in which the ratios examined were equivalent. Thus, for instance the comparison problem involving 4 winning tickets and 6 empty tickets vs. 2 winning tickets and 3 empty tickets had a corresponding missing-value problem with the mathematical structure 4:6=2:x. It may be also mentioned that the fourth member in the missing-value problems was always unknown one.

The starting point for the construction of these tests were the individual tests developed by Noelting and by the research group formed by Karplus, Pulos and Stage, as well as the classroom tests used by the author in a previous study (see the orange juice experiment, Noelting 1980a, b; the lemon juice experiment, Karplus et al. 1983; the juice experiments I & II, Keranto 1986a). For a summarized general idea of what kind of problems these tests include, which developmental level of proportional reasoning is represented by perfect performance in each problem and how the used invalid strategies are separated from each other and from valid multiplicative strategies, let us take a look at Table 1.

TABLE 1. The items of lottery test I, the developmental levels of proportional reasoning corresponding to valid performance in each item, the answer patterns derived from the consistent use of each strategy, and the order in which the problems were presented (question: "From which can, A or B, are you more likely to draw the winning ticket, or are the chances equal?"

	items: cans A (w e) vs. B (w e)	stage: Noelting	stage: problem structure Karplus et al	strategies & patterns: 1-dim (w)	1-dim (e)	An	Add	Pro	order of presentation
0.	3:2 vs. 2:3	IA		A	A	E	A	A	instruction
1.	4:2 vs. 5:2	IB		B	E	B	B	B	15
2.	2:3 vs. 2:5			E	A	B	A	A	1
3.	4:2 vs 3:3	IC	IE	A	A	E	A	A	6
4.	4:4 vs. 3:2			A	B	A	B	B	10
5.	2:2 vs. 3:3	IIA	E	B	A	B	E	E	14
6.	5:5 vs. 2:2			A	B	A	E	E	2
7.	1:2 vs. 2:4	IIB	WB	B	A	B	A	E	11
8.	4:2 vs. 2:1			A	B	A	A	E	18
9.	2:4 vs. 3:6		W	B	A	B	A	E	16
10.	6:3 vs. 2:1			A	B	A	A	E	3
11.	3:2 vs. 6:4		B	B	A	B	B	E	21
12.	5:2 vs.10:4			B	A	B	B	E	7
13.	4:6 vs.10:15		N	B	A	B	A	E	12
14.	4:10vs. 2:5			B	A	B	A	E	23
15.	2:1 vs. 4:3	IIIA	WBX	B	A	B	E	A	4
16.	1:2 vs. 2:5			B	A	B	A	A	20
17.	4:2 vs. 5:3		WX	B	A	B	E	A	13
18.	3:6 vs. 2:5			A	B	A	E	A	17
19.	3:2 vs. 6:5		BX	B	A	B	E	A	8
20.	4:5 vs. 2:3			A	B	A	E	A	19
21.	2:3 vs. 3:4	IIIB	NX	B	A	B	E	B	5
22.	4:3 vs. 3:2			A	B	A	E	B	9
23.	5:2 vs. 7:3			B	A	B	B	A	22

Note:
A, B, E: A, if the chance to draw winning ticket from can A is bigger; B, if the chance to draw winning ticket from can B is bigger; E, if the chances are equal.

w = the number of winning tickets
e = the number of empty tickets

Stages (Noelting 1980a, b):
IA (lower intuitive); IB (middle intuitive); IC (high intuitive); IIA (lower concrete operations); IIB (higher concrete operations); IIIA (lower formal operations); IIIB (higher formal operations).

Problems structure (cf. Karplus et al 1983, 5259):
IE (n:n vs. k:p), E (equivalency category 1:1), W (within combination ratio integral); B (between combination ratio integral); X (unequal ratios).

Strategies: 1dim(w), 1dim(e) (unidimensional comparison of winning/empty tickets); Am (additive comparison of the amounts of winning and empty tickets); Add (comparison of the differences of winning and empty tickets); Pro (proportional reasoning; multiplicative strategies, observation of within and/or between ratios, formation of equivalent ratios).

It is thus possible, in principle, to try to infer by this test the level of proportional reasoning in each subject in terms of strategies used and performance levels attained. In short, the strategies used are inferred as follows: the performance pattern of the pupil's choices is compared to an answer pattern resulting from consistent use of each strategy presented above (see ows 4 to 8 in Table 1). The procedure is the same both for items involving equivalent ratios {WB, W, B, N} and for ones involving inequivalent ratios {WBX, WX, BX, NX}.

If, for instance, a pupil's performance pattern in items involving equivalent ratios {WB, W, B, N} corresponds to an answer pattern resulting from consistent use of a strategy (with 0 to 2 deviations), this condition can be referred to as consistent use of the strategy in demanding equivalency items. Correspondingly, reference is made to consistent use of a strategy in demanding inequivalency items, when there are not more than two deviations from the corresponding patterns.

Similarly to a previous study by the author (Keranto 1986a), the empirically hierarchical nature of each test item was analysed by means of the Guttman scale. It was considered that the pupil had attained a certain developmental level in proportional reasoning if the problems in the item corresponding to it and in the items preceding it in empirical hierarchy had been correctly solved.

5.2. The results

In this article it is possible to summmarize just some of the most important results. To start with, the pupils' answers to the items in the classroom tests described above formed hierarchies which for the most part corresponded to their hypothesized order of difficulty. It was so both in the case of items {E, WB, W, B, N} and in the case of items {WBX, WX, BX, NX}. The one exception was item WB, which turned out to be systematically more difficult than item W in the lottery tests I and II. See Table 2 for a more detailed description.

TABLE 2. The succeeding frequencies for items E, WB, W, B, and N in lottery tests I and II, their order of difficulty and hierarchy values CR, MMR and PPR (N=19)

problem structure	problem combinations lottery I	lottery II	frequencies f(lottery I) t(a)	t(b))	f(lottery)II t(b)	the order of the difficulty of the items t(a)	t(b)	t(b)
E	2:2 vs. 3:3 5:5 vs. 2:2	2:2 vs. 4:? 5:5 vs. 2:?	18	19	19	I	I	I
WB	1:2 vs. 2:4 4:2 vs. 2:1	1:2 vs. 2:? 4:2 vs. 2:?	4	10	13	III	III	III
W	2:4 vs. 3:6 6:3 vs. 4:2	2:4 vs. 3:? 6:3 vs. 4:?	8	14	16	II	II	II
B	3:2 vs. 6:4 5:2 vs.10:4	3:2 vs. 6:? 5:2 vs.10:?	0	6	12	IV	IV	IV
N	4:6 vs.10:15 4:10vs. 6:15	4:6 vs.10:? 4:10vs. 6:?	0	2	5	V	V	V
					CR=	1.00	0.98	0.94
					MMR=	*	0.76	0.78
					PPR=	*	0.92	0.73

Note:
t(a) = initial test
t(b) = final test

Problem structure: E (category of equivalence 1:1); W (within composition ratio integral); B (between composition ratio integral); N (no ratio integral)

Besides the hierarchical nature of the pupils' answers, Table 2 also indicates that 12 pupils out of 19 have attained the level B in missing-value problems. These pupils learned to use both between and within multiplicity in such problems. Besides, five pupils learned to form "auxiliary ratios" to solve problems with no directly observable between or within integral ratios (see item N, missing-value problems). Equally good results were not achieved in the corresponding items of the comparison type, the solution of which as stated before was not adequate due to lack of time. Table 3 shows that the effect of the teaching program on the pupils was not homogeneous.

TABLE 3. Connection between previous performance (grades) in mathematics studies and the level of proportional reasoning (performance levels) at the beginning and end of the program (n=19, lottery I, initial and final tests). The arrows indicate shifts of individual pupils from one level to another.

mathematics grades		E(1)	IE(2)	W(3)	WB(4)	B(5)	WX(6)	BX(7)	N(8)	NX(9)
5-6	t(a)		1	1						
	t(b)		1	1						
7	t(a)	1	6		1					
				(3)						
	t(b)		2	4	2					
8	t(a)		2	1	1					
	t(b)		1		2			1		
9-10	t(a)		2	1	2					
	t(b)					2	1		1	1

Note:
t(a) = initial test
t(b) = final test

Level of performance: E (category of equivalence 1:1); IE (k:k vs. m:n, m=n=k); W (within composition ratio integral); B (between composition ratio integral); X (unequal ratios).

As seen in Table 3, a teaching program focusing on the use of equivalent ratios was most beneficial to pupils who had good (8) or excellent (9, 10) grades in mathematics. This connection was also to be seen in the correlations between these grades and the levels of performance (0.34 with the starting level test, 0.78* with the final test). In other words, when the teaching program started there was no clear connection between the pupils' earlier school achievement and performance in lottery I. At the end of the program, this connection was easily observable. It was also the case in missing-value problems (correlation with the final test 0.70*). It may further be noted that similar correlations were obtained in the liquid mixture problems (0.44 with the starting level test and 0.82* with the delayed final test). In fact, this was predictable. After all, the number structures in the problems of these tests were largely similar to each other. This fact is also borne out by the following correlations: in the final tests r(lottery I & lottery II)=0.65*, r(lottery I & juice I)=0.84* and r(lottery II & juice I)=0.64* (* $p<0.01$).

A pupil's age had no essential connection to achievement in the study program. It can also be mentioned that sex had no connection to test performance. This is clearly seen in the fact that there were three pupils from both grades among the six that did best, with four girls and two boys among them. The best performance was scored by two fifth-grade girls, one of whom may be considered a highly talented pupil.

It is clear that similar results can be seen in the data on pupils' solution strategies. After all, the use of a strategy largely determines, as seen in Table 1, which level of performance is reached by the pupil in the various tests.

TABLE 4. Connection between previous education attainment (grades) and strategies used by the pupils at the beginning and end of the programme (n=19, lottery I, initial and final tests). The arrows indicate changes in the strategies used by each pupil.

The strategies inferred from the pupils' answer patterns

		in items {WB, W, B, N}				in items {WBX, WX, BX, NX}			
		1-dim(w), 1-dim(e), Amount	Add	Mixed	Pro	1-dim(w), 1-dim(e), Amount	Add	Mixed	Pro
5-6	t(a)	1		1		1	1		
	t(b)		1	1			2		
7	t(a)	1	5	2		2	6		
			(5)				(4)	(2)	
	t(b)		6	1	1		5	3	
8	t(a)	1	1	2			3	1	
				(2)			(2)		
	t(b)			2	2		3	1	
9-10	t(a)		3	2			5		
	t(b)				5		2	1	2

Note:
t(a) = initial test
t(b) = final test

Problem structure:
W (within combination ratio integral); B (between combination ratio integral); N (no ratio integral); X (unequal ratios).

Strategies:
1dim(w), 1dim(e), Amount (unidimensional comparison of winning/empty tickets or of the amounts of winning and empty tickets);
Add (comparison of the differences in the amounts of winning and empty tickets);
Pro (proportional reasoning);

Mixed (more than two deviations from a pattern in the involved combinations of items).

As expected, in the case of pupils who have done well or excellently in their mathematics studies, the teaching program had a remarkable effect on their solution strategies of comparison problems in the lottery context. All of those pupils who had excellent marks in mathematics moved over to use multiplicative strategies in items containing equivalent ratios {WB, W, B, N}. However, some of them resorted to additive ones in items requiring formal operations {WBX, WX, BX, NX} even in the final test. As lack of time prevented their adequate solution in the course of the programme, such results were to be expected (cf. the results above for performance levels). On the other hand, research by Hart shows that it is difficult even for older pupils to abandon additive strategies (Hart 1987).

Let us finally examine the permanence of what was learned and the transfer to liquid mixture problems. The cross-tabulated diagram below shows that the use of strategies is relatively permanent and generalized also to the solution of liquid mixture problems (cf. the performance level correlations in the initial and final tests juice I and lottery II, r(starting level tests)= 0.57, r(final tests)= 0.84* (* $P < 0.01$).

TABLE 5. Connection between strategies used in the final test Lottery I and strategies used in the delayed final test juice I in the group of items {WB, W, B, N} (N=18).

strategies inferred from final test lottery I	strategies inferred from the delayed final test juice I →	1-dim/Am	Add	Mixed	Pro	Σ
	1-dim/Am	0	0	0	0	0
	Add	1	4	2	0	7
	Mixed	0	1	0	3	4
	Pro	0	1	0	6	7
	Σ	1	6	2	9	18

Note:
Problem structure: W (within composition ratio integral); B (between composition ratio integral); N (no ratio integral).

Strategies: 1dim(w), 1dim(e), Am (unidimensional comparison or comparison of amounts); Add (comparison of the differences in the amounts of winning tickets (juice concentrates) and empty tickets (waters); Pro (proportional reasoning); Mixed (more than two deviations from a pattern in the involved combination of items, for patterns see Table 1).

In addition to what was said above, Table 5 also indicates that three of the four pupils who were still using a mixed strategy at the end of the program had "matured" in two months to use multiplicative strategies. It is interesting to note that the time taken to "mature" what was learned improved rather than impaired the results obtained during the program. One can also note that as many as five pupils among the nine who used multiplicative strategies in the delayed final test were fifth-graders. Only one fifth-grader (marked 5) did not reach this level (with one fifth-grader missing so many lessons and tests that he was not considered in these examinations at all). The results obtained in the case of sixth-graders were not so good. It is very difficult to give an unambiguous answer to what might have caused this, in a sense, negative result. I will next try to present some possible causes of this poor performance of the sixth-graders as opposed to the fifth-graders. Finally, I will outline some ways to enhance the program for the second teaching experiment of the project.

6. Summary and conclusions

The results from the tests performed in the course of the first teaching experiment of this project confirm the previously established idea that the development of proportional reasoning can be assessed quite easily and reliably by means of multiple-choice classroom tests designed for the purpose (cf. Keranto 1986a, Noelting 1980a, b). The experiences and results yielded by the first teaching experiment also confirm the view that it is possible to affect the developmental level of the pupils' proportional reasoning by pointing out clearly and tangibly the inadequacy of the intuitive models of solution used by them and also by paying attention to features relevant to the solving of the problems.

On the other hand, the results obtained show consistently that the teaching program was mostly beneficial to pupils who had even otherwise done well or excellently in their mathematics studies. In the case of pupils who had performed poorly or fairly in their studies, the logical status of a solution method based on the observation of between and within ratios of pairs of figures and on the use and formation of equivalent ratios turned out to be too high. It is obvious that these pupils would have needed a method of comparing proportional values. Such a procedure has a lower logical status and it is more easily automated for the solution of the more demanding comparison problems. I assume that in this way the mediocre pupils in particular could have utilized their arithmetic skills better. In other words, one could assume that valid linking of the unit-rate method to the use of the factor of change method would have led to better success in the solution of more demanding comparison problems even in the case of mediocre pupils. This probably also explains the relatively poor performance of the sixth-graders, most of whom were mediocre pupils at best (8/13), in comparison to the fifth-graders. Most of the fifth-graders (4/6) examined had done well or excellently in their previous mathematics studies.

The hypothesis presented above will be tested in the second teaching experiment of the project. If the experiences and results derived from the experiment validate the hypothesis, there should be no obstacle of principle to adopting the teaching package for wider use in instruction. This would create a solid basis for the symbolic use of ratio and proportion also for the solution of more demanding comparison problems in the higher grades of the comprehensive school and in the upper secondary school. These problems will be discussed in more detail when the second and third teaching experiments are reported. At the same time, the pupils' attitudes to and experiences of the meaningfulness of the contextual approach to teaching will be examined. It is pleasing to cite here a sixth-graders' spontaneous question at the end of the second teaching experiment: "Professor, when do we continue studying mathematics in this way, next week perhaps?"

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DEVELOPMENTS IN THE TEACHING OF MATHEMATICS IN ESTONIA

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In this paper my intention is to give an overall view of the development of mathematics education in Estonian schools from the beginning of the 19th century until the 1970s.

1. Introduction

Relations between mathematicians in Estoria and Finland have not been particularly close. We have, to some extent, been able to become acquainted with Finnish mathematics textbooks, the journal "Dimensio" and some research in the field. We have had relatively little to offer you. Perestroika in the Soviet Union has created favourable conditions for rapprochement in our relations. Last year we established the Estonian Society of Mathematicians. In autumn 1988 a group of Finnish mathematics teachers was invited to a mathematics teachers seminar held in Valga, on the southern border of Estonia. To date, there have been fifteen such seminars. A sub-section of the Society of Mathematicians, for school mathematics, was established in Valga. I sincerely believe that through this Society we can form closer contacts in the future, possibly to the extent of organising Estonian-Finnish mathematics seminars.

2. The beginnings of mathematics teaching in Estonia

At the beginning of the 19th century, the name "Estonian" was almost unknown. Our forefathers and mothers, serfs, were known by the name "maarahvas" (country people). Endeavours to arrange education for children at that time were connected primarily with the activities of clergy and lords of manors influenced by the ideals of the Age of Enlightenment. The first impression of ABC-primers in Estonian date back to the 17th century. The first arithmetic course in Estonian appeared in 1805 in a reader by G.G. Marpurg, comprising some 40 pages. Next year (1806) the first arithmetic textbook in Estonian, "Arropiddamisse-Kunst" ("The Art of Calculation") was published. That book of P.H. Frey was designed primarily for home instruction. In the introduction, frey reminds the reader as follows: "It is a thousand times better if the father or mother can teach the children, rather than that they are taught by other, possibly unruly, children." It must be admitted that the standards of mathematics teaching in the first half of the 19th century in the village schools of Estonia were quite low.

Halfway through the century, upon the initiative of a Polva pastor, Johann Georg Schwarts, a series of eight school textbooks was published. One of these books was "Arvamise raamat" ("Book of sums") by F.F. Meyer, the pastor of the Johvi congregation. Another book, the book of Physics, in this series was written by Schwartz. Schwartz also wrote a supplement to his arithmetic textbook, entitled "Surveying", a book employing the first geometrical terms in Estonian.

The founding of the Society of Estonian Literati in 1872 was followed by a rapid succession of new school textbooks in Estonian, the first of these being "Moistlik rehkendaja" ("The Rational Calculator") by Rudolf Gottfried Kallas. This book was the avenue through which the basic requirements in teaching method were attained by the Estonian schoolmaster. Kallas stressed the need for concrete examples, explaining when and how to give them. The book states: "To calculate is to think." This requirement is reflected in the name of the book. Kallas was active in his opposition to so-called mechanical arithmetics. Kallas wrote a further methodological textbook "The most essential instructions for the rational calculator". In conjunction with these methodological books there appeared three sets of exercises and a booklet of exercises which was more demanding "12½ toopi pähkleid" ("Twelve and a half measures of nuts"). Whilst a student in the faculty of theology, Kallas was awarded a gold medal for his thesis in which he states his grounds for the teaching of arithmetics. He based his arguments on the metaphysical thinking of professor Teichmüller, the professor of theology at the University of Tartu.

3. The First national renaissance

The 1870s are known as the first renaissance of the Estonian people. In fact this is when the word "eestlane" (Estonian) first came to be more widely known and used. One of the first priorities was the furnishing of children with instruction using their own mother tongue. Upon the initiative of the Society of Estonian Literati quite many schoolbooks were published. Despite the fact that the curricula of the rural schools did not include courses in algebra or geometry, the first textbooks in Estonian in the field of geometry and algebra (one book in the latter area) were published during this decade. The writing of new textbooks brought up questions of terminology. The first person to do this publicly was A. Bilow in an 1884 issue of the newspaper "Olevik". He encouraged exchange of thoughts with the aim of creating Estonian equivalents to many concepts: e.g. horizontal - horisontaalne, solution - lahendus, equal - vordne.

The first renaissance produced new relationships with Finland in many different spheres. This, however, was not the case in the writing of textbooks. Rather, one can see the widespread influence of the German pedagogic school. One interesting fact is revealed in an article in an 1878 edition of the newspaper "Eesti Postimees". According to this article, U. Cygnaeus, the First Inspector of the Finnish primary school system and founder of the Jyväskylä Teachers'College had in his speech given vent to a matter which was at the time quite new, both in Finland as well as elsewhere. He had stressed the importance of women becoming teachers alongside men, since "the gentle nature and diligent work of woman bear abundant fruit wherever there are women teaching"

4. The development of teaching at the senior high school level

One of the first mathematics and physics teachers in the first Estonian high school was Jaan Sarv, later to be the first Estonian born professor of mathematics at the University of Tartu, when Estonian became the language of instruction. He was the primus motor in the creation of mathematical terminology in Estonian. In 1909 he published his "Mathemaatika sonastik" ("Mathematical Vocabulary"), with 600 terms in Russian, German and Estonian.

Jaan Sarv persevered in his efforts to give instruction in mathematics using Estonian language, despite new restrictions, even obstacles in the way of instruction in the mother tongue. In August 1917, he was the convenor of the first Estonian Mathematics Congress in Tartu. In addition to questions of terminology, the congress considered problems connected with the contents of the mathematics syllabus and with questions of teaching method. Jaan Sarv, for instance, demanded that the teaching of mathematics must be started from the concept of the set. This concept was also the basis for the latest widespread international effort to reform the content of school mathematics. A further demand set down was that mathematics should be taught as one subject. A scientific society was planned, the proposed members being the mathematicians, physicists and chemists who had participated in the first congress.

The same year, 1917, the second edition of the mathematical dictionary, with some 800 entries, was published.

5. The teaching of Estonian as an established subject

By the time the Estonian school system as a whole began using Estonian as the language of instruction in 1918, the majority of the problems in the area of terminology had been solved. Of course adjustments have been made out up to the very present.

Jaan Sarv became the first professor of mathematics in the Estonian-language University of Tartu. He made various study tours to universities abroad, acquired a deeper interest in geometry, and in 1928 he defended his doctoral thesis on the fundamentals of geometry. At this juncture, matters concerning school mathematics faded into the background. In 1920 Gerhard Rägo, whose family had been living in Russia since the latter half of the nineteenth century, returned home to the professor of applied mathematics. Furthermore he was to be the developing force behind school mathematics in Estonia for several decades.

During the years 1921-39 there were six congresses for teachers of mathematics and physics. While the stress in the initial years was on terminology and phraseology, the latter years saw a shift in emphasis to questions of pedagogy. One of the speakers at the 1937 Congress was School Councelor N. Kallio from Finland. In his talk he gave an account of the teaching of mathematics and physics in Finnish schools. Amongst other things, he mentioned that girls and boys in Finland follow the same syllabus in senior high school mathematics, but that girls have one more hour per week in the subject.

6. K. Väisälä and the teaching of mathematics in Estonia

During the first years of the Estonian-language University of Tartu, one of the creative forces behind the establishment and development of the standards was the young Finnish mathematics professor Kalle Väisälä. He was quick to learn Estonian and was soon lecturing in the language. We were fortunate enough to receive a letter twenty years ago from professor Väisälä, in which he said that he was writing a letter in Estonian for the first time in 45 years. He had not allowed the Finnish press to write about his 75th birthday. To us he wrote: "But Tartu may be an exception. Should they still want to remember officially one of the first academic staff of the university of the Republic of Estonia". An article on the occasion of his birthday was published in the journal "Matemaatika ja kaasaeg" ("Mathematics and the present".)

7. The reforms leave their mark

The reforms in the teaching of mathematics in the Estonian school system, effected by professor Rägo, based on the ideals of F. Klein and of other mathematicians, who had aimed at reforming school mathematics as early as the beginning of the century, especially as far as the teaching of functional dependence in the schools was concerned. G. Rägo was instrumental in putting these ideas into practice, and the thoughts he expressed at that time are still today influential in school mathematics in Estonia.

He insisted that the treatment of both the concept of the function as well as differentiation and integration are based on concrete questions and that they stay as such. In the preface to his 1921 mathematics textbook he writes: "I regard it as impermissible to teach the reader the differentiation of enormous terms and to find similar integrals. Neither do I ever solve a single one of the complex exercises for finding limits, the likes of which one finds in the older textbooks. One should not waste time in stressing the mind."

In 1930, the new mathematics syllabuses for the senior high schools were approved. An addendum to the syllabuses was a particularly thorough expose' written by G. Rägo. There he insisted that together with the teaching of calculations, mathematics should develop a love of truth, self-criticism, a sense of responsibility, an ability to see the problem, to understand it and to solve it. He regarded a logical way of thinking, the clarity of the expression of ideas and systematic work as important; he stressed the significance of accuracy the ability to concentrate, and discipline both in thought and deed.

Let it be stated that professor Rägo was a very enthusiastic and active man, with a good sense of organisation. He founded and developed the Mathematics Institute, affiliated with the university, he was a founder of the didactic-methodological seminar of the university, and he also acted as the head of the seminar during the 1930s. He set courses for mathematicians, applying also professor von Sanden's ideas on the more extensive use of numeric and graphic methods in university lectures. These ideas date to professor Rägo's two visits to the University of Göttingen for the purpose of furthering his studies. The syllabus devised by profesor Rägo was evaluated by professor Lietzmann from the University of Göttingen, professor Oseen from the University of Uppsala, and the Danish-born American professor Westergaard. They all considered the syllabus to be good; in fact, Oseen regarded it as the best in Europe at the time.

It might be mentioned here that in 1930 professor Rägo planned to make a study tour to Finland and Sweden. The tour, however, was not realized, despite the fact that funds were available.

The 1920s and 30s were particularly outstanding decades in the development of Estonian culture. One of the areas to undergo rapid development at the time was indeed school mathematics. There is in existence an abundant tradition from this period.

During this time, textbooks, exercise booklets, sets of verbal problems and methodological instructions were published. The idea of the work-school, propagated by the renowned pedagogue Johannes Käis, found application in the field of mathematics teaching as well. At the beginning of the 1920s, the 3rd edition of the mathematics dictionary was published, comprising some 1400 terms. The authors were mathematicians from the University of Tartu, among them also professor K. Väisälä.

In 1937, new mathematics syllabi were adopted in Estonia, together with new textbooks. The reforms advocated and practised by professor Rägo had found their opponents. "These functions of G. Rägo are a divine curse and retribution", wrote one educationalist in his report. The new syllabi and the new textbooks based on them were written again by the Committee for Mathematics Teaching, chaired by professor Rägo. Some changes were made in the composition of the committee.

8. The teaching of mathematics in the Estonian SSR

With the year 1948 there began a transition to new syllabi covering the whole of the Soviet Union. The change was relatively rapid. As a result of this transition, the total number of lessons per week in mathematics increased by some eight lessons. The textbooks adopted were Estonian translations of books written in the 19th century by Kisseljov, Rybkin and others, these being the books used in the rest of the Soviet Union. Teachers were less than pleased with this change. The early study of geometry, the large number of mathematics textbooks (for instance six books in the sixth class), the teaching of mathematics-related disciplines as separate subjects and several other changes were all things the teachers were not used to. Attempts were started at creating a locally-biased curriculum, which would take into account requirements for the Soviet Union as a whole, but which would teach the mathematically related disciplines as one subject. It was also the prevalent wish that the textbooks were to be written by experienced Estonian teachers. In 1957 a mathematics committee under Elmar Etverk and in co-operation with the Ministry of Education began the work.

The teachers' journal "Noukogude Opetaja" published a draft mathematics syllabus and in 1958 some 5th and 6th classes in Estonian schools began trying out new textbooks. The testing was continued for the next few years. "Training camps" were arranged for teachers at the Bärska training centre and from autumn 1965 Estonian schools have adpted their own syllabi and textbooks.

It should be added here that it was not until 1959 that the first Conference of Mathematics and Physics Teachers was organised after the war. After that they have been held every three years. In order to activate pupils' interest in mathematics, students of mathematics at Tartu began to organise competitions for senior high school students in solving problems of the exact sciences. Similar competitions at the all-Soviet Union level were not begun until 1959. On three occasions a representative of Estonian schoolboys has, as a member of the Soviet team, got to compete in international mathematics competitions.

9. "The New Mathematics" in Estonia

The Department of Mathematics Teaching Method, founded in the university of Tartu in 1965, has been a guiding force in the development of Estonian school mathematics from the beginning of the 1960s. It has had a twofold task: to guarantee the methodological standards of future mathematicians, and, secondly, to be in the vanguard of the development of school mathematics throughout Estonia.

The latter half of the 1960s saw the initiation of the 'new mathematics', which has spread throughout the whole world. We too became enthusiastic about the concept of set in school mathematics. Alfred Lints was the instigator of the concept of set into school mathematics and Reet Ruga from the Tallinn Teachers' College did the appropriate introductory and background studies. The most radical of the reformers was Karl Ariva. His treatment of geometry, based on Weyl's axiomatics, turned out to be a difficult nut to crack, for both pupils and teachers. This concept of geometry, in fact, was the cause of the first doubts expressed as to the possibility of seeing the reforms to their conclusion.

Together with the high sighting of the concept of set, the teaching of mathematical logic, probability, mathematical statistics and linear vector optimisation became important. Evi Mitt, Kalle Velsker and Jaan Reimand, teachers of the staff of mathematics didactics, did the appropriate groundwork, tested them in the schools, and created thus the prerequisites for the adoption of these mathematical disciplines into the school curriculum and for the writing of the appropriate chapters into the textbooks.

At the 1966 International Congress of Mathematicians held in Moscow, we had the opportunity of presenting the results obtained in the teaching of mathematics in the Estonian schools. Here we also made new contacts which helped us along in our work. Professor Lilly Görke and her colleagues from Berline and Halle were of particularly great assistance to us, as well Rudolf Toelstra, the Dutch mathematics textbook writer. It should be also mentioned that our relationship with professor Paavo Malinen from Jyväskylä spans a period of 20 years.

In the 1970s some members at the Moscow Adademy of Sciences - Vinog... dow, Tihonow and Pontrjagin demanded adjustments in the school syllabus, which meant above all deletion of the set from the syllabus. Estonian textbooks adopted similar changes, although we have not abandoned the concept of the set completely. In the last year of high school we have replaced examinations based on Weyl's axiomatics with analytic treatment, where the vector has an important role.

As from the beginning of the 1970s, research in the Department of Mathematics Teaching Method has been concentrating on new questions. At the centre of these lie teacher training and in-depth subject studies. Scholars who have been involved in these fields sucessfully are Lea Lepman and Jüri Afanasjev. A problem of current interest is that of the psychology of mathematics teaching represented by Tiit Lepman.

The Chancellor of the Tallinn Polytechnical Institute, Boris Tamm, has informed us that Estonia has been granted the right to manufacture computers itself. Should this plan be realised, we can expect rapid developments in computer teaching in our schools.

A great triumph for our school mathematicians has been the adoption of the textbooks by Aksel Telgmaa and Enn Nurk as the compulsory textbooks throughout the whole Soviet Union in the 5th class and afterwards also in the 6th class. These textbooks won a competition of the area.

Recently the rate of development has been slowing down. We think that one of the best achievements in the new textbooks have been the chapters on the use of the pocket calculator. The recently founded Estonian Mathematicians' Society under the chairmanship of professor Ülo Lumiste has participated actively in solving the problems of school mathematics. Both Lumiste and professor Ülo Kaasik have been involved in the modernisation of the textbooks used in the schools. One of the tasks of the school mathematics section will be to involve school teachers in the development of school mathematics.

THE SYSTEM OF TEACHER EDUCATION IN CZECHOSLOVAKIA WITH SPECIAL REFERENCE TO MATHEMATICS TEACHER'S EDUCATION

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Due to very fast political changes starting at the end of 1989 in Czechoslovakia, every sphere of the life of Czechoslovak society including Czechoslovak school system is under the discussion and changes are expected. Main changes were caused by the abolishment of the leading role of communist party in Czechoslovakia. For example, the teaching of Russian language will not be compulsory, the teaching of Marxism-Leninism, ideological and political components of education, socio-political activities and socio-political practical trainings were completely abolished.

1. Teacher education system in general

The education of teachers and other educational personnel is divided into the following specializations:

a) Kindergarten teachers
b) Primary school teachers at elementary level (from the 1st to the 4th classes the teacher teaches all subjects)
c) Subject teachers for the upper level of primary school and for secondary schools (they are specialized in two subjects; this qualification can be extended to other subjects in post-graduate study)
d) Specialized subjects teachers
e) Teachers of practical subjects in secondary schools
f) Teachers in Popular Art Schools
g) Teachers in Language Schools
h) Teachers for special schools
i) Eduating personnel at boarding-type schools.

Education of kindergarten teachers is provided in secondary pedagogical schools. The training of selected teachers of pre-school education facilities is also being introduced at university level at pedagogical faculties. The teachers of primary and secondary schools must have university education.

Teacher education is concentrated predominantly in pedagogical faculties, where full or part-time training takes place in the specializations cited under b), c), f), g), h) and i). Qualification for teaching in specializations under c), f) and g) can also be obtained in art academies or at various university faculties. Qualification for specializations d) and e) can also be attained in economic, teachnical or agricultural schools of university level, either in the form of full-time study, or by parallel pedagogical study or by a follow-up course in pedagogical education. Any completed university pedagogical education provides the qualification for work as educational personnel in all types of educational facilities.

Czechoslovakia is divided into ten regions and two independent capital cities, Prague and Bratislava for administrative purposes. Each region has its own pedagogical faculty. They include pedagogical faculties of the following universities (see Figure 1):

* Charles University, Prague - its pedagogical faculty is located in Prague and Brandýs nad Labem
* Commenius University in Bratislava
* T.G. Masaryk University in Brno
* Palacký University in Olomouc
* P.J. Šafařík University in Košice, in Prešov and independent pedagogical faculties in cities of České Budějovice, Plzeň, Ústí nad Labem, Hradec Králové, Ostrava, Nitra and Banská Bystrica.

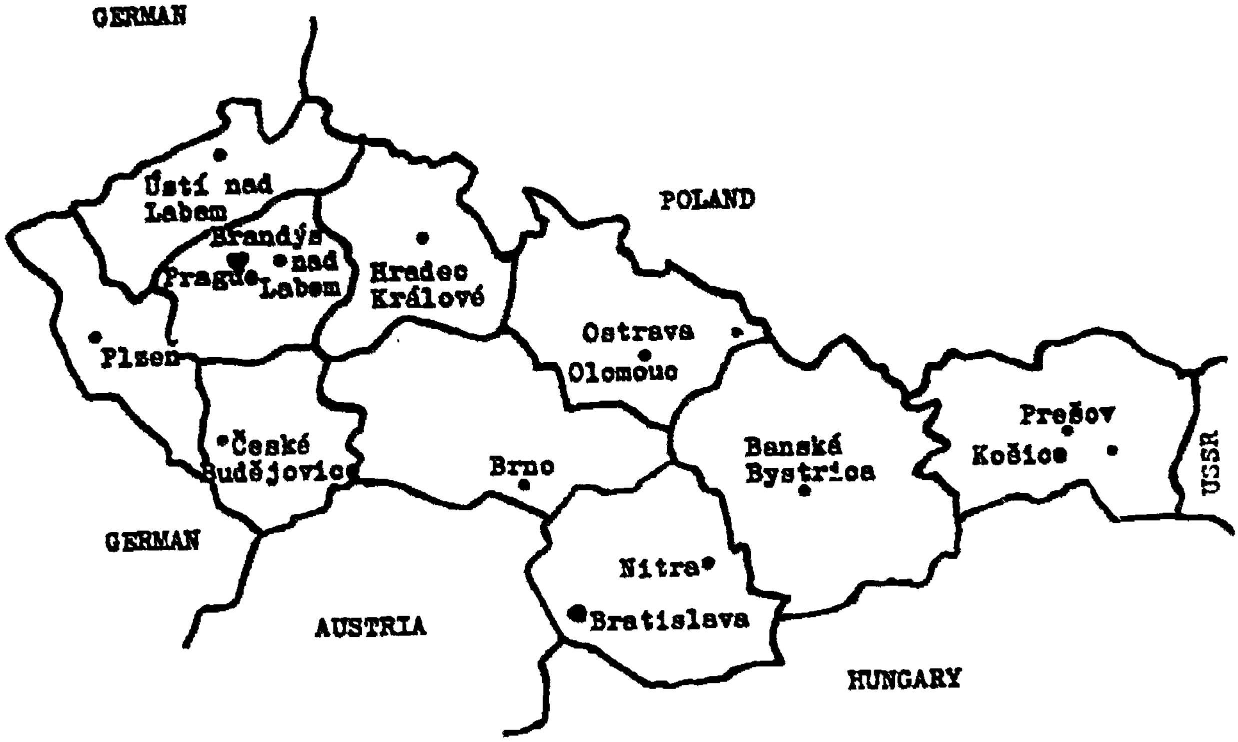

FIGURE 1. Pedagogical faculties in Czechoslovakia

2. Education of teachers for the elementary level of primary schools

Full- and part time education for teachers at the elementary level of primary school is available only at pedagogical faculties. The full-time course takes four years, the part-time course five years. The full-time course curriculum is shown in Table 1. The curriculum is compulsory throughout the country and includes an ideological and political component, pedagogical and psychological components and specialized subject-matter.

TABLE 1. Teacher's education curriculum of the elementary level of the primary school (numbers indicate weeks, excursions and other activities)

Year	term	instruction	specialized practical training	physical education	examination period	vacations
1	1	13	1	2	6	1
	2	15	-	2	6	5
2	3	15	-	-	6	1
	4	14	-	1	6	4
3	5	14	1	-	6	1
	6	14	-	-	6	4
4	7	15	-	-	6	1
	8	7	6	-	-	-
Total		107	8	5	42	17

The study of Marxism-Leninism lasts four years with a total of 450 teaching hours. The courses which provide a theoretical background, include educational psychology, pedagogics, special pedagogics, the biology of the child and school health care, teaching methodology and computer science and they represent a total of 550 teaching hours. The specialized subject-matter component of the course includes the mother tongue and literature, mathematics, physical education, art education, musical education and playing a musical instrument, polytechnical education, didactics of elementary science and elementary civics and the didactics of all the above mentioned subjects - with a total of 1800 hours throughout the whole course of study. All students are to follow this curriculum. From the 4th term they study one of the four specializations: physical education, art education, musical education or polytechnical education, to which a total of 270 hours are allocated, in addition to the basic course. Another 110 hours are devoted to learning a language.

The curriculum includes, beside the instruction described above, also various types of practical training which is divided into three types: specialized practical training, on-going practical training and other practical training.

2.1. Specialized practical training

During the first term an introductory week-long practical training period is organized where students visit classes in various types of schools. In the first week of the 5th term students participate in lessons in the first class of primary school for the whole week. They are present during the class and help teachers in teaching of some classes and in other activities. In the 8th term students teach at least 2 lessons a day continuously for 4 weeks and then at least two weeks are spent in a rural school with a reduced number of classes, where again students actively teach at least two lessons a day. The purpose of this continuous 6-week period of practical training is to provide the student with the opportunity to become comprehensively acquainted with the educational, administrative, social and political activities which the teacher is responsible for, as well as with the organisation of the school. It also provides an opportunity for a comprehensive application of the knowledge and skills, which the student has acquired, and of his or her ability to deal with real educational situations in instruction and outside it.

2.2. On-going practical training

In the 2nd, 3rd and 4th terms students visit classes at basic schools and at other educational facilities; in the 2nd term 2 hours a week and in the 3rd and 4th term one hour a week. After these weekly periods there is a one hour seminar at the faculty, analyzing the acquired experience. The purpose of this type of practical training is to acquire social communication skills. From the 5th term students also participate in teaching at primary school with two lessons a week. In the 5th and 6th terms these lessons are divided into mathematics instruction, instruction of the mother tongue and elementary science and civics, at a ratio of 0,5 lesson, 1 lesson and 0,5 lesson in the 5th term and 1 lesson, 0,5 lesson and 0,5 lesson in the 6th term. (The half lessons are usually taught as a whole lesson once every two weeks.) During the 7th term these two lessons are divided into practical training in musical education, art education, polytechnical education and physical education. During these practical training periods, students themselves teach the lesson in question. The aim of this practical training is to analyse the educational and teaching processes which are carried out in combination with the teaching of methodology of individual subjects at the elementary level of primary school and thus to train students for independent teaching and for their educational profession in general.

2.3. Other practical training

From the 1st to the 3rd year of study students participate in socio-political activities. Within the framework of these activities students work with Young Pioneer groups, they sit for exams called The Young Pioneer Leader Minimum, they participate in selected seminars, or they work as group leaders in Young Pioneers summer camps.

Beside this, students who have completed their second year of study also participate in three-week summer vacation practical training in Young Pioneer summer camps. An outline of various practical training is provided in Figure 2. (In Table 1, on-going practical training is included under the heading of "instruction".)

In the course of their teacher training students also participate in several courses, such as for instance summer and winter courses of physical training and ice-skating courses. The duration of these courses does not exceed 10 days.

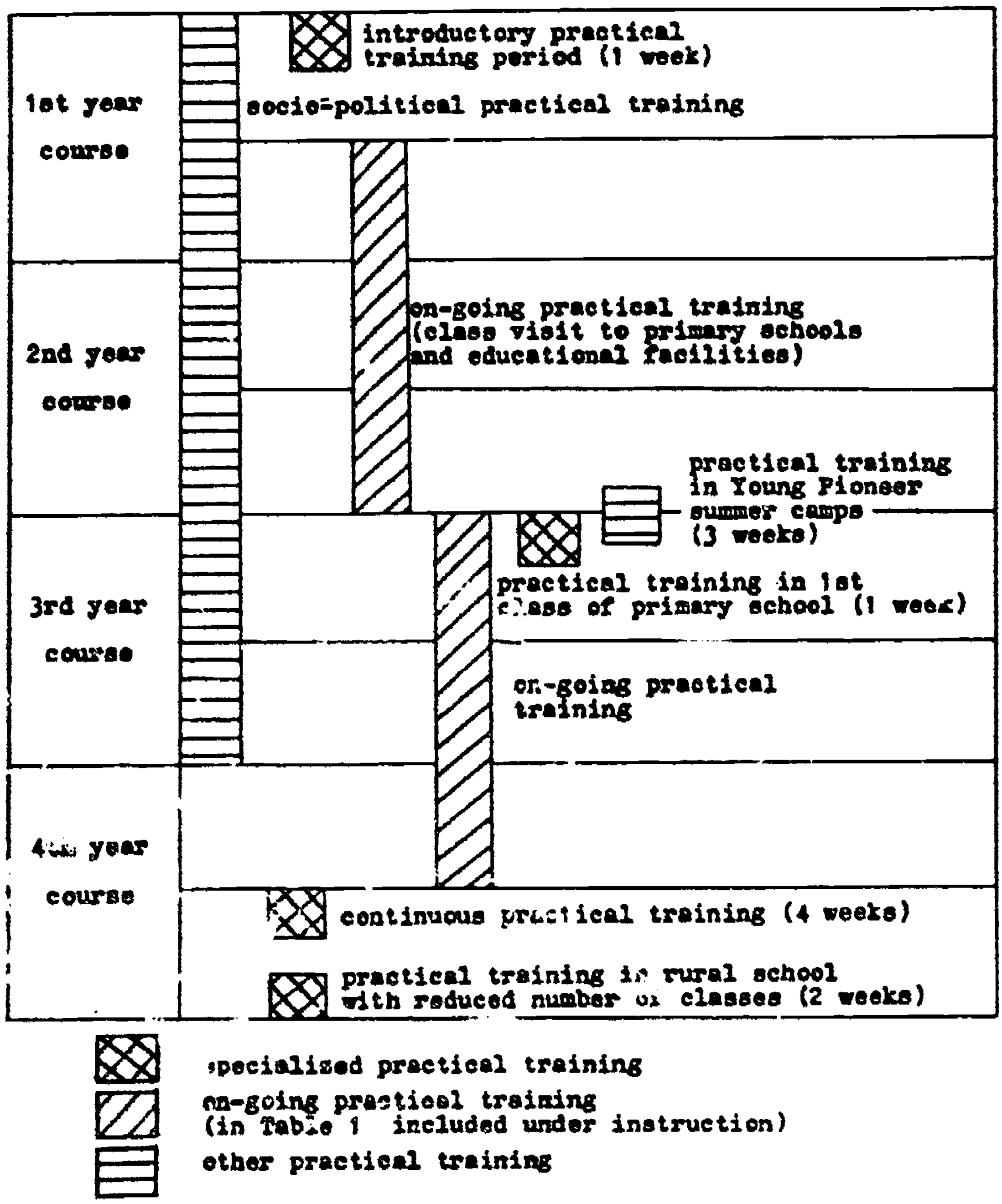

FIGURE 2. Outline of practical training in teacher education of the elementary level of the primary school

2.4. Mathematics instruction for teachers specializing in elementary level teaching

The mathematics course covers the whole four-year study and the instruction takes place in 7 of the 8 terms available.

During the first three terms the foundations of arithmetic are taught, with 1 lecture per week and 2 hours of seminar, a total of 129 hours. In the 4th term this is followed by the foundations of elementary geometry with 1 lecture and 2 hours of seminar per week, the total of 42 hours. In the 5th term there is 1 lecture and 1 hour of seminar a week and in the 6th term 1 lecture and 2 hours of seminar of mathematics teaching methodology a week, the total of 70 hours. In the 7th term there are also 30 hours of a selective seminar in mathematics for which there are course-unit credits. Here students have the option of choosing from among various subjects, for instance computer science and mathematics, selected methodology problems, the relationship between mathematical methodologies and general didactics, etc.

Figure 3. provides an outline of the mathematics instruction structure in teacher training for the elementary level of the primary school.

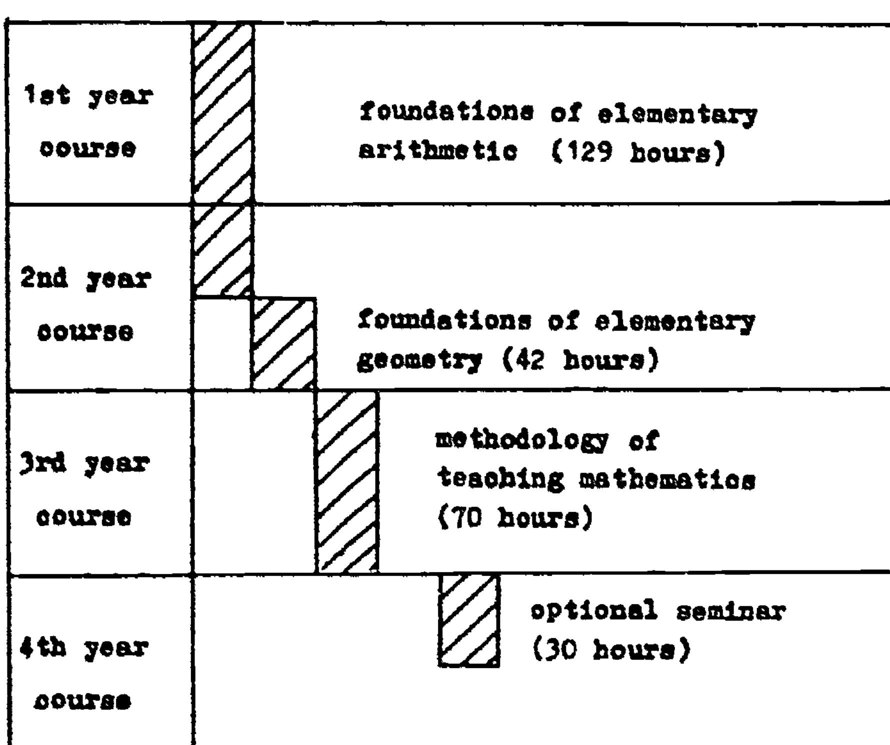

FIGURE 3. The curriculum for mathematics teachers of the elementary level of the primary school

3. Education of subject teachers

The full-time and part-time courses in education for teachers who will teach general educational school subjects are usually provided by independent pedagogical faculties, university faculties or art academies. The full time course takes five years, the part-time course six year. The full-time course curriculum is summed up in Table 2. This curriculum is compulsory throughout the country and includes an ideological and a political component, training in pedagogical and psychological subjects and training in individual school subjects.

TABLE 2. Teacher's education curriculum of general education subjects (numbers indicate weeks, excursions and other activities)

Year	term	instruction	specialized practical training	preparation of diploma paper	physical education course	examination period	vacations
1	1	15	-	-	-	6	1
	2	15	-	-	2	6	7
2	3	15	-	-	2	6	1
	4	15	-	-	-	6	4
3	5	12	3	-	-	6	1
	6	15	-	-	2	6	5
4	7	14	-	-	-	6	1
	8	11	4	-	-	6	4
5	9	9	6	-	-	6	1
	10	7	-	7	-	-	-
Total		128	13	7	6	54	25

From the 1st term, stress is placed on instruction in subjects which are common for all specializations (Marxism-Leninism, pedagogics, psychology, physical education, etc.), as well as on subjects for whose teaching the teacher will be qualified after graduation (the Czech language, mathematics, the Russian language, civics, physics, chemistry, geography, biology, the foundations of technology, musical education, art education, etc.). Usually from the 6th term the methodology of subjects in which the student specializes is taught.

Practical training is an inseparable part of the course. In connection with the change in the curriculum introduced in the academic year 1986-1987, changes have also occurred in the scope and organization of practical training. According to the new curriculum, in the 5th term students have practical training as educators in boarding-type facilities, in the 8th term practical training in teaching the subjects (they specialize in) and in the 9th term final practical training in their specialization.

Practical training as educators in boarding-type facilities in the 5th term (3rd year course) takes up 90 hours, i.e. 3 weeks. The purpose of this type of training is to provide the student with basic experience and skills in the highly varied conditions of educational activities which may arise in various educational facilities and/or Young Pioneer camps. Students learn to organize group activities of children, extracurricula activities in live-in-nature courses organized for several weeks each year by many schools, in after-school day-care centres, in boarding facilities or summer camps. They learn to exercise positive authority to be able to distinguish the optimum dividing line between an excessive authoritarian approach to children and unduly liberal one. They develop and exercise their own pedagogical perception and learn to create an atmosphere of cooperation between teacher and his pupils. At the end of this term the student receives course-unit credits on the basis of a report about his work which is made by the staff of the educational facility where his training took place.

Practical training of teaching subjects which takes place in the 8th term (4th year course) takes up 120 hours, i.e. 4 weeks and covers both specializations to the same e .tent (i.e. 2 weeks each). This training is organized either in one time or in an on-going manner; half of it takes place in the primary school and the other half in the secondary school.

During this practical training students learn to master successfully specific educational and instructional aims which were determined for each teaching lesson. On the basis of carefully prepared outlines they learn to teach independently in various types of lessons and other forms of instrution in compulsory, optional, non-obligatory and club-types of instruction. Practical training is organized in groups which visit lessons and then analyse what they have seen and heard. Then, 8 times, the student directly teaches a class of children in each of his subject, half of them at primary school and half at secondary school. Practical training is organized in groups of 3 to 5 students, and is supervised by a specialist in the methodology of the given school subject. Individual groups are supervised either by lecturers or professors from the faculty or by teachers at the school who have been entrusted with this work. Course-unit credits are awarded by the latter, according to an assessment based on a uniform outline.

The final practical training period in the teaching subjects takes place in the 9th term (5th year course) for 180 hours, i.e. 6 weeks. This time is divided between both specializations equally, 3 weeks for each subject. Again, half of it takes place at a primary school and half at a secondary school.

During this final period of practical training the student learns to do all the tasks of a teacher and his work approaches the demands made upon real teaching and work in a real school, including all extra-curriculum activities. He or she should acquire such a level of teaching skills that after starting his or her first teaching job he will be capable of carrying out all their obligations, at least at a basic level and with the aid of superiors. During these 6 weeks the student carries out 40 teaching lessons in each of his two subjects. If it is possible, practical training in both subjects takes place at the same time, on a parallel basis. During this whole period the student teaches 80 lessons. The focus of attention and the most important part of this training period are practical teaching lessons. The student is also expected to participate fully in the social and political life of the school and, beside instruction, also to participate in extra-curriculum activities, as set by the school's principal. Course-unit credits are again awarded according to an assessment based on a uniform outline by the school teacher entrusted with this work and by the school.

Beside the above mentioned periods of practical training, from the 1st to the 4th term the so-called social and political practical training takes place, where students usually work in Young Pioneer summer camps, in the course of the school year as group leaders in Young Pioneer organizations, etc. Figure 4 provides an outline of practical training in the training of teachers for individual school subjects.

The teacher training curriculum for general education, in subjects that are suitable (such as the foundations of technology, chemistry, art education, etc.) also includes various excursions or courses which take between 3 to 7 days.

During their study course, all students participate in a summer and a winter sports training courses each of which lasts 10 days.

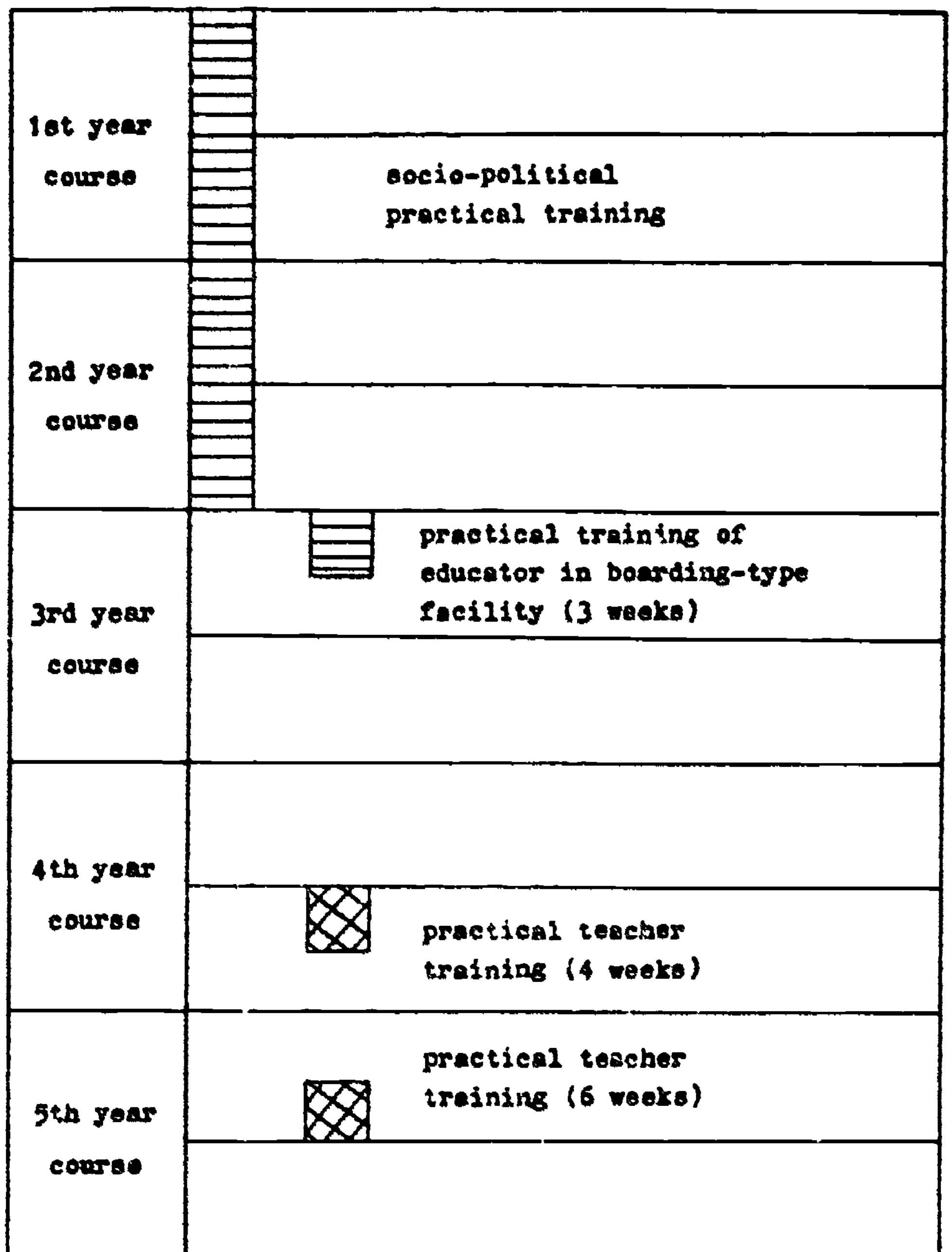

FIGURE 4. Outline of practical training in teacher education of general education school subjects

3.1. Mathematics instruction for subject teachers

The study of mathematics is evenly divided into all 10 terms (see Figure 5). The basic courses in the study of mathematics for teachers are mathematical analysis (a total of 5 terms - 333 hours), algebra and theoretical arithmetic (a total of 4 terms - 250 hours), geometry and descriptive geometry (a total of 4 terms - 258 hours). Much attention is also devoted to computer science (a total of 2 terms - 99 hours), as well as probability and statistics (a total of 2 terms - 91 hours).

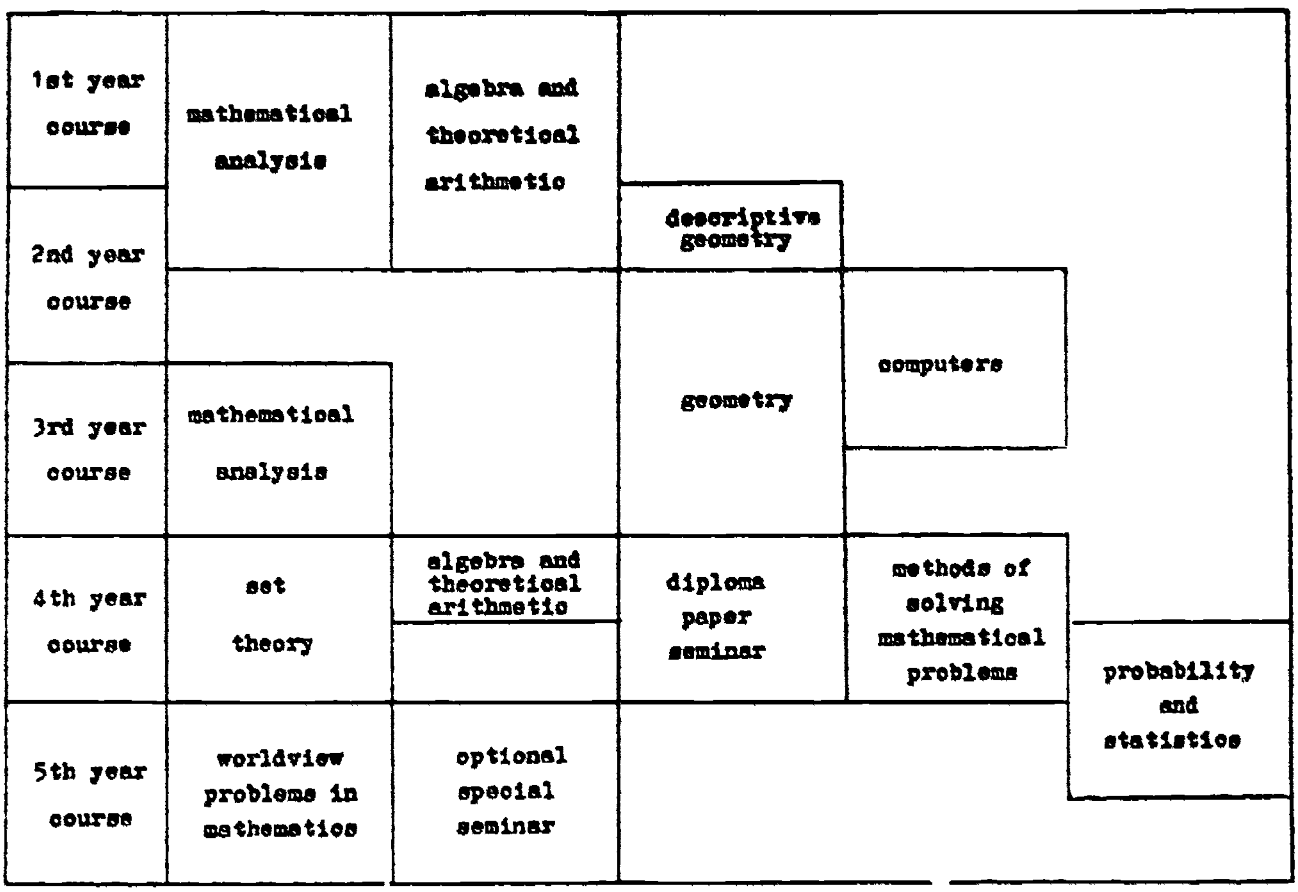

FIGURE 5. The mathematics curriculum for teachers of general education subjects

Table 3 provides a more detailed survey of all mathematical disciplines, taught in mathematics for teachers.

	1st year 1 / 2 course		2nd year 3 / 4 course		3rd year 5 / 6 course		4th year 7 / 8 course		5th year 9 / 10 course		Total
Mathematical analysis	$3/2^{c}_{ex}$	$3/3^{c}$	$2/2_{ex}$		2/2	$2/2_{ex}$					174/159
Algebra and theoretical arithmetic	$3/2_{ex}$	$2/2_{ex}$	1/2				$3/2_{ex}$				132/118
Descriptive geometry			$1/2^{c}_{ex}$								15/30
Geometry				$3/2^{c}_{ex}$	$2/2^{c}$	$3/3_{ex}$					114/99
Computers				3/2	$0/2^{c}_{ex}$						45/54
Diploma paper seminar							$0/1^{c}$	$0/1^{c}$			0/25
Methods of solving math. problems							$0/2^{c}$	$0/2^{cm}$			0/50
Set theory							$2/0^{c}$	$2/0_{ex}$			50/0
Probability and statistics								$3/2^{c}$	$2/2^{c}_{ex}$		51/40
Worldview problems of mathematics									$2/1^{c}_{ex}$	$0/3^{c}$	18/30
Optional special seminar									$0/3^{c}$	$0/3^{c}$	0/48
Weekly number of hou. for math. instruction	10	10	10	10	10	10	1C	10	10	6	599/653 1252
Number of mathematics examinations	2	1	2	1	1	2	1	1	2	0	13

TABLE 3. The mathematics syllabys for teachers of general education subjects

lectures/seminars
c ... course-unit credit
cm ... course-unit credit with marks
ex ... examination

4. Education of teachers for special schools

Teacher training for schools attended by youth requiring special care provides the qualification for teaching in schools and educational facilities for these adolescents in specialized classes for children with reading and writing disorders and with school adaptability disorders. Qualification is achieved in a full-time course or part-time training at independent pedagogical or university faculties. It also can be obtained by a supplementary study of special pedagogical disciplines within the framework of teacher training.

5. System of post-graduate study for pedagogical workers

The new concept of teacher training also includes plans for a further improvement of their qualification. The system of post-graduate study for pedagogical workers includes post-secondary school and post-graduate study courses and cyclical schooling of supervisory staff and special courses aimed at work with new curriculum, syllabi, teaching aids, didactical technology, etc.

Post-graduate study is regarded as a system of life-long education in close linkage to preceding education. It is implemented in three stages. The first stage immediately continues after teacher training graduation and its purpose is predominantly to assure job adaptation. Its task is to introduce the teacher to problems of practical teaching.

The second stage is predominantly concerned with inovations where teachers are acquainted with advances made in their specializations (for which they are qualified) and their methodologies as well as in pedagogics and psychology. On the basis of practical requirements this stage of post-graduate study can also have a form of increasing qualification and providing teachers with the opportunity to extend their qualification by another school subject or by other teaching or educational qualification.

The third stage is aimed at specialization.

Participation in some forms of study for attaining an academic degree is considered to be a desirable form of increasing the qualification of pedagogical workers.

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