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AUTHOR	English, Lyn D.
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ABSTRACT

Seventy-two Australian children aged from 4 years 6 months to 9 years 10 months were individually administered a set of six combinatorial problems involving the dressing of toy bears in all possible combinations of clothing items. Six age groups were represented: eight children were in each of the 4, 5, and 6 year categories; and 16 children were in each of the 7, 8, and 9 year categories. Because the problem domain was novel, children had to use their existing general strategies to help them solve the problems. A series of increasingly sophisticated solution strategies (reflecting a knowledge of the combinatorial domain), plus several scanning actions serving primarily in a monitoring capacity (reflecting an application of general strategies) were found. Significant associations existed between children's solution strategies and their scanning actions on each problem; the children changing the nature of their scanning as they adopted more complex solution strategies. The nature of this association was a key factor in problem success, especially when there was an additional constraint on goal attainment. The results are examined concerning changes in children's principled knowledge base and in the nature of their general strategies. Cases involving problem failure in the face of sophisticated domain knowledge highlight the importance of children applying the appropriate domain-general strategies in both novel and routine problem solving. Four tables present study findings, and there is a 53-item list of references. (SLD)

CHILDREN'S USE OF DOMAIN-SPECIFIC KNOWLEDGE AND DOMAIN-GENERAL STRATEGIES IN NOVEL PROBLEM SOLVING

Lyn D. English
Centre for Mathematics and Science Education
QUT
PO Box 284
Zillmere
Australia, 4034

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ABSTRACT

Seventy-two children aged 4 to 9 years were individually administered a set of six combinatorial problems involving the dressing of toy bears in all possible combinations of clothing items. Because the problem domain was novel, the children had to use their existing general strategies to help them solve the problems. Analyses of the children's responses revealed a series of increasingly sophisticated solution strategies (reflecting a knowledge of the combinatorial domain), plus a number of scanning actions serving primarily in a monitoring capacity (reflecting an application of general strategies). Significant associations were found between children's solution strategies and their scanning actions on each problem, with the children changing the nature of their scanning as they adopted more complex solution strategies. The nature of this assocation was a key factor in problem success, especially when there was an additional constraint on goal attainment. The findings of the study are examined in terms of changes in children's principled knowledge base and in the nature of their general strategies. Cases involving problem failure in the face of sophisticated domain knowledge highlight the importance of children applying the appropriate domain-general strategies in both novel and routine problem solving.

The importance of domain-specific knowledge and domain-general strategies in the development of problem-solving competence has been a much debated issue (e.g. Alexander & Judy, 1988; Borkowski, Carr, & Pressley, 1987; Bransford, Vye, Kinzer, & Risko, 1990; Glaser, 1984; Keating & Cane, 1990; Lawson, 1990; Owen & Sweller, 1989; Sternberg, 1989). It is a well established fact that individuals with a comprehensive knowledge of a given domain will outperform those with a limited knowledge (e.g. Chi, Hutchinson, & Robin, 1989). Likewise, those who employ effective planning and self-monitoring will perform better than those who do not regulate their actions (Flavell, 1979; Lawson, 1990). What is not well established however, is the manner in which domain-specific knowledge and general strategies interact during problem solving and learning (Alexander & Judy, 1988).

Studies which emphasise the importance of domain-specific knowledge usually contrast the depth of the expert's schematic knowledge with the superficial structures of the novice (e.g. Chi, Glaser, & Farr, 1988; Chi, Hutchinson, & Robin, 1989; Larkin, 1985). Larkin (1985) maintains that effective problem solving is not possible without the understanding derived from a comprehensive knowledge base. From this perspective, many of the difficulties experienced by novices are due to their focus on the literal or surface features of a problem rather than on the underlying principles (Chi, Glaser, & Rees, 1982).

Proponents of the domain-specificity of problem-solving skills frequently downplay or dismiss the role of general strategies. Sweller (1989, 1990) contends that skilled problem solving is dependent on schema acquisition and rule automation and that general problem-solving strategies are of little value. He argues that there is insufficient evidence for emphasising skills such as planning and monitoring in developing students' problem-solving competence. Without automation, novel problem solving is prone to error, inflexible, and "likely to be a difficult or impossible task for the vast majority of people" (Sweller, 1990, p.458).

Such a stance has provided a bone of contention for researchers who stress the importance of general strategies, these being applicable across a range of tasks and domains (e.g. Belmont, Butterfield, & Ferretti, 1982; Brown, 1978; Kuhn, Amsel, & O'Loughlin, 1988; Peverly, 1991; Schoenfeld, 1985; Van Haneghan & Baker, 1989). The most commonly cited strategies are those that perform a self-regulatory or metacognitive function and include skills such as planning, predicting, monitoring, checking, and revising (e.g. Brown & Campione, 1981; Lawson, 1984; Pressley & Ghatala, 1990; Scardamalia & Bereiter, 1985; Sternberg, 1985). It has been claimed that these self-regulatory mechanisms contribute not only to immediate performance but to continued growth of the cognitive system (Scardamalia & Bereiter, 1985). The development of students' inherent self-regulatory mechanisms can assist them in acquiring new knowledge through self-generated activity.

While there are many studies that have focussed on either domain-specific knowledge or domain-general strategies, there are few that have examined the interaction of the two. This is particularly so with respect to studies in the preschool or early school years (Alexander & Judy, 1988). The present study was thus designed to investigate the interactive role of these two components in young children's problem solving. More specifically, the study sought answers to the following:

1. What domain-specific strategies do young children apply to the

solution of novel problems based on the mathematical domain of combinatorics?

2. In what ways do they monitor their progress towards goal attainment? (As used in this paper, monitoring encompasses the skills of checking, revising, planning, and predicting.)
3. How do their domain-specific and domain-general strategies interact to affect goal attainment?

To conduct such an investigation, it was necessary to design problems which would require children to use both types of strategies. It was hypothesised that if children had only an informal knowledge of the problem domain, that is, no knowledge of the "expert" means of solution, they would be more likely to apply their existing general strategies to help them solve the problems. It was assumed that the children would have acquired such strategies through their everyday experiences (cf. Kuhn & Phelps, 1982).

An important initial step in this investigation was to define the parameters of the study, given the inconsistency of the terminology in the literature (Keating & Crane, 1990). Within the present context, domain-specific knowledge encompasses the conceptual and procedural knowledge one possesses within a particular field. Procedural knowledge is the result of the compilation of conceptual knowledge into action units (cf. Anderson, 1987, 1989). Conceptual knowledge is akin to declarative knowledge ("knowing that", Anderson, 1985) and is conceived of as a hierarchical network of knowledge that is rich in relationships (Hiebert & Lefevre, 1986). As addressed in the discussion section, this conceptual knowledge is viewed in terms of a comprehensive set of domain-specific principles. Domain-general strategies, as the name implies, are applicable across domains and operate in a metacognitive capacity (e.g. Pressley & Ghatala, 1990; Yussen, 1985). Their functions include monitoring performance during problem solving, planning appropriate courses of action, predicting and evaluating the outcomes of these actions, and identifying when these actions need to be modified (English, 1988). As such, these strategies assist in the execution, regulation, and evaluation of a problem-solving task.

Given that the problem domain is a critical factor in the relationship between domain-specific knowledge and general strategies (Alexander & Judy, 1988), the development of a suitable set of problems was of major importance to this study.

CHOICE OF PROBLEM DOMAIN

For the problems to be challenging, yet still solvable, their parameters had to be unfamiliar to the children but their contexts familiar (Sternberg, 1985). The mathematical topic of combinatorics, involving the selection and arrangement of objects in a finite set, was chosen as the problem domain. The problems required children to dress toy bears in various combinations of clothing items, as indicated in the method section. Combinatorics was considered a worthy topic of investigation from both an educational and a developmental perspective. The domain comprises a rich structure of significant mathematical principles which underlie several areas of the mathematics curriculum, including counting, computation, and probability. In simple mathematical terms, combinatorics may be viewed as the operation of cross product. The cross product of two sets, A and B, is the set of combinations obtained by systematically pairing each member of A in turn with each member of B. Because there are several ways of forming these combinations, the domain is eminently suitable for designing problems that allow for different levels of solution. These range from simple, random matching of items through to the systematic pairing method. These different levels enable variations in the children's solution methods to be readily identified. In addition, the practical nature of the domain facilitates the development of meaningful problems involving hands-on materials which young children can manipulate to generate a solution (National Council of Teachers of Mathematics, 1989; Nelson, 1980; Worth, 1990).

From the developmental perspective, the establishment of a combinatorial system plays a central role in Piaget's theory of cognitive growth (e.g. Piaget, 1957; Flavell, 1963). This system is evident in a subject's ability to "link a set of base associations or correspondences with each other in all possible ways so as to draw from them the relationships of implication, disjunction, exclusion etc." (Inhelder & Piaget, 1958, p.107). The important cognitive strategies here are isolation or control of variables, and systematic combination. From their studies charting the development of the combinatoric operations, Piaget and his associates claimed that preoperational children form combinations only in an empirical manner by randomly associating two elements at a time; there is no systematic method in their actions. It is not until the concrete-operational period that children are seen to attempt some systematic procedure in forming combinations. The "different trials of systems" identified in this second stage are replaced by the one system, that of formal combinations, during the formal operations stage (Piaget & Inhelder, 1975, p.169). Subjects now have a systematic method for generating $n \times n$ combinations.

In sum, the problems chosen for this study drew upon a clearly defined body of domain-specific knowledge which catered for various levels of solution. When presented with the problems in a meaningful context, children could at least attempt a solution by applying their informal knowledge of the domain along with their existing repertoire of general strategies. The problems thus provided a suitable context for examining children's application of domain-specific knowledge and general strategies and the ways in which these interact to affect problem solution.

METHOD

Subjects

The study involved 72 children whose ages ranged from 4 years 6 months to 9 years 10 months. Six age groups were represented: 4 years 6 months to 4 years 10 months, 5 years 6 months to 5 years 10 months......9 years 6 months to 9 years 10 months. There were 8 children in each of the 4, 5, and 6 year categories and 16 children in each of the 7, 8, and 9 year categories. More children were included in the latter age groups because a pilot study had found little variation in the younger children's responses across the study problems.

The 4 and 5 year-olds attended a preschool in a middle-class suburb of Brisbane, Australia. The remaining children were from the first four grades of two state and two non-state primary schools in middle-class suburbs of Brisbane. The children within each age group were selected on a random basis and drawn equally from the four schools. The children had not been exposed to combinatorial problems in their school curriculum.

Materials and procedures

The children were individually administered a series of six problem-solving tasks in either a single 25 minute-session (for the 6 to 9 year-olds) or two shorter sessions on consecutive days (for the 4 and 5 year-olds). Each child's performance was videotaped, with the camera positioned to capture eye, head, and hand movements.

The problems were set within the context of dressing toy bears in all possible different outfits, each comprising a top and a pair of pants. The clothing items were backed with adhesive material which facilitated their attachment to the bears. The bears were made of thin wood and were placed on a stand. Once the bears had been dressed they were arranged in a line so that the child could clearly see the completed outfits. This arrangement also made it easier to keep track of the children's eye movements as they scanned along the line of bears they had dressed.

Problem no. 3 involved 9 combinations (3 sets of tops X 3 sets of pants) while each of the other problems was based on 6 combinations (2 sets of tops X 3 sets of pants; 3 sets of tops X 2 sets of pants). In the final two problems, the coloured clothing items were replaced with items of the same colour but with varying numbers of buttons. The goal in this instance was to dress the bears so that each bear had a different total number of buttons (the total did not exceed seven). These two problems assessed the children's "flexibility" in generating procedures for achieving the goal in a different context (Greeno, Riley, & Gelman,1984, p.122.).

Two of the problems had an additional constraint on goal attainment. The fourth problem required children to give the third bear a blue top. In this instance, the bears were arranged in a line from the outset. The purpose of this added constraint was to assess the "robustness" of the children's skills, that is, their ability to accommodate constraints not normally imposed in the problem (Greeno et al., 1984, p.122). The sixth problem contained a hidden constraint, namely, two combinations derived from different items had the same total number of buttons (one-button top/three-button pants and two-button top/two-button pants, both giving a total of 4). This meant that one of these two combinations had to be rejected.

A familiarisation task was administered to each child prior to the six problems. The goal for this task was simply to dress the bears. The task was designed to test children's colour recognition, as well as to establish an understanding of the terms, "outfit", and "same/different outfits". The latter term was crucial in the interpretation of the problem goal, especially when a common item was present. For example, the outfits, red top/blue pants, and red top/yellow pants are different from each other even though they have a common item. During this familiarisation period the children were not given any information that could bias their performance on the problems. For each of the remaining problems, the children were provided with more items (both bears and clothes) than were needed. This was to ensure that children did not think that item depletion meant problem completion. The children were expected to complete each problem without assistance.

IDENTIFICATION OF PERFORMANCE VARIABLES

The conceptual framework for the study determined not only the choice of problems but also which aspects of the children's performance were to be analysed and the form of analysis to be adopted (Ericsson & Simon, 1984; Uprichard & Engelhardt, 1986). An analysis of children's performance in a pilot study (25 children) revealed a number of key features in their behaviour of which empirically independent measures could be taken. These features were considered to reflect the cognitive components being addressed and included children's methods of item selection and combination, referred to here as their solution strategies, and the ways in which they checked their progress, namely, their scanning actions. It was also necessary to rate the children's overall performance according to the extent to which they attained the problem goal, as discussed later in this section.

Three main types of solution strategy had been identified from the children's performance in the pilot study. Each of the strategy types comprised two strategies, giving rise to a hierarchy of six, increasingly sophisticated procedures. These ranged from a random selection of items through to an efficient system of patterning, as described shortly. Because these strategies involved various means of selecting and combining pairs of items, they were assumed to reflect the children's knowledge of the combinatorial domain.

The children's scanning actions played a vital role in monitoring and regulating their performance as they progressed on the problems. These actions were thus considered to reflect the children's application of general strategies (cf. Gavelek & Raphael's, 1985, argument on children's use of metacognitive knowledge). Six different forms of scanning were identified, varying from inefficient "checking only after acting" approaches to more thorough checking procedures, as described later in this section.

Solution strategies

The three major types of solution strategies identified in the pilot study are referred to here as non-planning, transitional, and odometer strategies.

Non-planning strategies

The first of these strategies, labelled "strategy A" for ease of reference, involves a random selection of items in which there is no rejection of unsuitable items. In other words, children who use this strategy make no attempt to achieve the goal of all different outfits; they simply "dress the bears".

The second strategy, referred to as "strategy B", is a trial-and-error procedure in which items are selected randomly and subsequently rejected if they prove unsuitable.

<u>Transitional strategies</u>

Strategies C and D are termed <u>transitional</u> because they are more efficient than the previous non-planning procedures but are not as efficient as the sophisticated algorithmic strategies. The distinguishing feature of the transitional strategies is the appearance of a pattern in item selection. This pattern is designed to generate a solution and is usually of a cyclic or alternating nature (e.g. red top, blue top, red top, blue top.....). However for strategy C, the pattern is only emerging and is not applied consistently throughout problem execution. That is, at some point during problem solution, the pattern is lost or is occasionally changed. When their pattern can no longer generate the required combinations, children revert to strategy B (trial-and-error).

Strategy D however, is characterised by a consistent and complete pattern in item selection. A cyclic pattern is used to generate all the required combinations, with the pattern usually applied to one item type only (e.g. all the tops, such as, *red top* / yellow pants, *green top* / blue pants, *blue top* / blue pants, *red top* / blue pants, *green top* / yellow pants, *blue top* / yellow pants).

<u>Odometer strategies</u>

The final two strategies, E and F, are the most efficient of all the strategies, this being due to the presence of an "odometer" pattern in item selection. This pattern, so named because of its similarity to the odometer in a vehicle (Scardamalia, 1977), retains the cyclic property of strategy D but incorporates a new feature, namely a "constant" or "pivotal" item. This item is repeatedly selected until all possible combinations comprising that item have been formed. Upon exhaustion of this item, a new constant item is chosen and the process repeated (e.g. *red top* / blue pants, *red top* / yellow pant, *red top* / green pants, *blue top* / blue pants, *blue top* / yellow pants, *blue top* / green pants, *yellow top* / blue pants, *yellow top* / yellow pants, *yellow top* / green pants*)*.

While strategy F is characterised by a consistent and complete application of the odometer pattern, strategy E displays one of several weaknesses. These include a failure to exhaust a constant item (frequently the omitted combination is formed at the end of task execution), an "over-exhaustion" of a constant item (the child normally detects the duplicated combination and corrects this without requesting assistance), or a failure to recognise problem completion upon exhaustion of all constant items (in this instance, the child attempts to create further combinations but soon realises this cannot be done).

Scanning actions

Prior to detailing the six scanning actions, consideration is given to the key features which serve to distinguish them, namely, the direction and focus of the children's scanning.

<u>Direction of scanning</u>

Children's scanning of their completed combinations was identified as either forwards along the line of dressed bears (from the first outfit formed through to the last) or backwards (from the last outfit formed through to the first).

<u>Focus of scanning</u>

To further define the nature of the children's scanning, the focus of their attention was identified as either one-dimensional or two-dimensional. One-dimensional scanning refers to eye movements along an array of items of the one type only, for example, along all the tops of a line of completed outfits. With two-dimensional scanning, the focus is on items of both types (tops & pants) within a completed outfit. Two-dimensional scanning involves attending to each outfit in turn, and comparing both of its items against the item pairs of other outfits.

The six scanning actions have been labelled 1 through 6 for ease of reference.

Scanning action 1

This designates an absence of any identifiable scanning.

Scanning action 2

Scanning action 2 involves two-dimensional, backward scanning along the line of dressed bears. This type of scanning is observed to occur only after item selection and combination; that is, children do not look along their line of completed outfits prior to dressing another bear.

Scanning action 3

This is characterised by scanning which alternates between the dressed bears and the unused items placed before the child. Scanning of this nature usually occurs during the formation of a combination.

Scanning action 4

Scanning action 4 involves scanning of the unused items only; there is no scanning of completed outfits. Scanning of this nature usually takes place prior to, rather than after, item selection and combination.

Scanning action 5

Scanning action 5 is defined by the direction and focus of the child's attention. It features one-dimensional, forward scanning along the line of dressed bears and may occur prior to, or after, item selection and combination. It does not include two-dimensional scanning, the absence of which distinguishes it from scanning action 6.

Scanning action 6

This encompasses the features of scanning action 5 but incorporates an additional feature, namely, two-dimensional scanning. Scanning action 6 usually occurs after item selection and combination where it serves in a verifying capacity.

Goal attainment

Children's attempts at goal attainment ranged from a complete dismissal of the goal through to efficient, error-free solution. Four levels of goal attainment were identified and the children's performance on each problem given a score of 1 to 4, as follows:

Score of 1: These children ignored the problem goal and simply dressed the bears.

Score of 2: Children given this score did not attain the problem goal or only did so with assistance from the interviewer. This score was also assigned to children who only achieved one of the two goals in the fourth problem.

Score of 3: These children made an error but detected and corrected it themselves.

Score of 4: The highest score was awarded to children who attained the problem goal without error and without assistance.

To assess the reliability of the three performance variables in the present study, the responses of 20 children were subjected to two independent ratings. A 90% level of inter-rater agreement was obtained. Most of the discrepencies occurred in the coding of the scanning actions, which was not unexpected given the detailed observations required. Once these discrepencies had been rectified, the responses of each of the remaining children on each of the six problems were coded in terms of the solution strategy and scanning action employed, and the extent of goal attainment.

RESULTS

Prior to examining the associations between children's solution strategies and scanning actions and their impact on goal attainment, consideration is given to the trends observed within each of the variables.

Children's strategy preferences

Children in the 4 to 6 year age group favoured non-planning strategies in solving the six problems (refer Table 1). Strategy A (merely dressing the bears) was observed only in the preschool children (4 and 5 year-olds). Rarely did the 4 to 6 year-olds employ the transitional or odometer strategies. While the 7 to 9 year-olds displayed some non-planning behavior, particularly in the initial problems, they nevertheless favoured the more sophisticated strategies. This was particularly so for the 8 and 9 year-olds who showed a strong preference for the odometer strategies in solving the final problem. Although none of the children had been introduced to the most sophisticated strategy (F) in their formal schooling, 4 of the eight year-olds and 2 of the nine year-olds used it to solve the initial problem.

INSERT TABLE 1 ABOUT HERE

The results of chi-square analyses support the observed dichotomy between the types of strategies employed by the younger and older children. A significant relationship was found between age and strategy type on each problem, df = 25, $p < 0.001$ (except for problem no.2 where $p < 0.01$). A positive correlation was also found between age and children's change in strategy use between the first and last problems, $r = 0.24$, $p < 0.05$. A

McNemar test indicated no significant improvement in the types of strategies used by the 4 to 6 year-olds between the first and final problem, but a significant improvement in the older children (i.e. from non-planning or transitional strategies to odometer strategies), $X^2 = 13.5$, df = 1, $p < 0.001$.

A significant proportion of the variance in children's strategy change between the first and last problems was accounted for by their scanning actions. Children's scanning actions were dummy coded and a hierarchical regression completed with children's strategy change scores as the dependent variable. When children's scanning actions for problem no. 2 were partialled out, it was found that their scanning actions for problem no.7 explained a significant proportion of the variance in their strategy change scores, R^2 change = 0.45, F change = 10.58, $p < 0.001$. Further discussion on the association between children's solution strategies and scanning actions is presented in a later section.

Children's scanning preferences

Scanning action 3 (alternate scanning of completed outfits and unused items) was the most favored of all the scanning actions, as indicated in Table 2. Scanning actions 1 (absence of scanning) and 2 (scanning only after formation of combinations) were used mainly by the 4 and 5 year-olds, with a few 6 year-olds also displaying the latter scanning type. While the 7 to 9 year-olds made frequent use of scanning action 3, they also employed scanning actions 4 (scanning of unused items only), 5 (forward, one-dimensional scanning), and 6 (scanning action 5 plus two-dimensional scanning). The last scanning action however, was little used. The observed trends in the children's scanning preferences are supported by the results of chi-square tests conducted on the children's responses to each problem.

These results showed a significant relationship between age and scanning action (df = 25, $p < 0.01$ for each problem).

INSERT TABLE 2 ABOUT HERE

Goal attainment

To test for a relationship between age and goal attainment, an ANOVA was performed with age as the independent variable and the children's mean attainment scores across the six problems as the dependent variable. This produced a significant relationship, $F(5, 66) = 11.99$, $p < 0.001$. Post hoc analysis showed that the 7, 8, and 9 year-olds performed significantly better than the 4, 5, and 6 year-olds, with the 6 year-olds outperforming the 4 year-olds, $p < 0.05$. These findings are reflected in the data of Table 3. The majority of the younger children were either unable to attain the goal on each of the problems or could do so only with assistance. It is interesting to compare the attainment scores of the 7, 8, and 9 year-olds on the first three problems. Contrary to expectations, the 8 year-olds performed better than the 9 year-olds on the second and third problems (although not significantly so), while the 7 year-olds achieved the highest score of all the children on problem no.2. The decline in the older children's performance on the final problem, incorporating a hidden constraint, can be accounted for by examining the association between solution strategy and scanning action.

INSERT TABLE 3 ABOUT HERE

Association between solution strategy and scanning action

As indicated in Table 4, a significant association between children's solution strategies and their scanning actions was evident on each problem for each of the age groups, 4 to 6 years, and 7 to 9 years.

INSERT TABLE 4 ABOUT HERE

To illustrate the nature of this association, consideration is given to the children's responses on problem no.3, involving nine combinations. Here, 88% of the younger children (N=24) employed strategies A (simply dressing the bears) or B (trial-and-error) along with scanning actions 1 (absence of scanning), 2 (scanning only after completing combinations), or 3 (alternate scanning). Within the older age group (N=48), 54% of the children employed either a transitional or odometer strategy together with scanning actions 4 (scanning unused items only), 5 (one-dimensional, forward scanning) or 6 (one-and two-dimensional scanning). Twenty-nine percent favored scanning action 3 (alternate scanning) with these strategies. It is interesting to note that scanning action 3 was associated with all strategy types, while scanning actions 4 through 6 were only employed when there was a pattern in item selection (strategies C through F).

The latter finding is particularly interesting in that a distinct change in the children's scanning was observed as they switched from strategies 2 (trial-and-error) or 3 (partial use of a pattern) to strategies 4, 5, or 6 where a uniform pattern was followed in item selection. Instead of using scanning actions 2 or 3 which involved two-dimensional backward scanning, they now employed scanning actions 5 or 6, both of which featured forward one-dimensional scanning. This involved a focus on items of the one type (e.g.

all the tops). Forty-two percent of all the subjects changed their solution strategies and scanning actions in this manner. It was apparent that the children were now checking the progress of their pattern of item selection. While scanning action 5 provided an adequate means of monitoring the implementation of an odometer strategy, it was inadequate for detecting the hidden constraint in the final problem. Scanning action 6, however, incorporated both one- and two-dimensional scanning and was thus most appropriate for the final problem. This point is revisited shortly.

Children's response to the additional constraint in problem no.4 ("Give the third bear a blue top") was to make greater use of scanning action 3. Of all the children, 63% used this more thorough scanning form, presumedly to ensure that the additional constraint was met. There was also a decline in the efficiency of the solution strategies employed by the older children as they moved from problem no. 3 to problem no.4. In contrast to the younger children who remained with their usual non-planning procedures, 35% of the older children reverted to transitional strategies or to non-planning behaviour. This was accompanied by a decrease in goal attainment, with 29% of the older children being less successful on problem 4 than on problem 3.

The association between solution strategy and scanning action had the greatest impact on the final problem which called for more thorough monitoring. When children failed this problem (i.e. obtained a score of 2), a significant association was found between the two variables, for both the younger children $X^2 = 19$, df = 6, $p < 0.01$, and the older children, $X^2 = 24.1$, df = 12, $p < 0.05$. This final problem required children to form combinations of numbers such that all totals were different. Because two combinations made from different items yielded identical totals, one of the combinations had to be discarded. The effective solving of this problem required the application of an odometer strategy, together with thorough checking procedures, these being either scanning action 3 (alternate scanning) or 6 (one- and two-dimensional scanning).

Fifty-eight percent of the older children (N = 48) and 29% of the younger group decreased their goal attainment score on this final problem. Failure to solve the problem was due primarily to two factors. Firstly, a non-planning strategy proved to be inadequate for meeting the needs of this problem. With the exception of one subject, children who used a non-planning procedure did not attain the goal, irrespective of the scanning action employed.

The second factor appeared to be a mismatch between children's solution strategy and scanning action. In other words, the children's scanning actions were inappropriate for the strategy they applied to this problem. This was particularly the case with the older group. Here, 16 of the 32 children who failed to solve the problem employed a transitional or an odometer strategy together with scanning actions 4 (scanning of unused items only) or 5 (one-dimensional forward scanning). Because these children either didn't bother to scan their completed combinations or only scanned along items of the one type, they failed to detect the identical totals. When asked why they had missed these two combinations, the children responded that their method had worked on the previous problems and they assumed that it would do likewise on the final problem. The children were clearly relying on the generative nature of their procedures to solve the problem. If they did monitor their progress, they focussed on the implementation of their odometer strategy rather than on their attainment of the problem goal. In contrast to these children, the 16 successful subjects used the most thorough scanning actions, namely 3 and 6, in conjunction with a transitional or an odometer procedure.

DISCUSSION

The children's responses to the study problems comprised two main components, namely, their solution strategies and their scanning actions. Because the solution strategies involved various means of selecting and combining items, they are assumed to reflect the children's knowledge of the combinatorial domain. Children's conceptual understanding of this domain is seen in terms of principled knowledge which places certain constraints on the nature of performance (English, 1988, 1990; Gelman & Greeno, 1989; Gelman & Meck, 1986; Ohlsson & Rees, 1991). Within the present framework, the general strategies are responsible for monitoring the proceduralization of this principled knowledge; this is in contrast to Ohlsson & Rees' model (1991, p.114) in which the principled knowledge is seen as the "device for the internal self-monitoring of performance".

Children's initial attempts at problem solution involved mainly non-planning or trial-and-error behavior, indicating a lack of sophisticated domain knowledge. However the children did require a certain minimum knowledge of the combinatorial domain in order to tackle the problems. The minimum set of domain principles which children needed for the problems has been examined elsewhere (English, 1988; 1990) and is not elaborated upon here. However it is worth citing a couple of the more significant of these principles, namely, the Principle of Difference and the Principle of Repair. The Principle of Difference addresses the uniqueness of combinations and asserts that two or more combinations of items will be different from each other if they differ in at least one item. The Principle of Repair states that any combination of items can be modified and that an inability to repair any combination indicates problem completion. Children's knowledge of these principles, albeit implicit, was fundamental to successful performance. It is argued that children had a basic understanding of this principled knowledge on problem commencement and were able to translate this into meaningful action. The resultant procedures were constrained by this principled knowledge as can be seen from the children's rejection of inappropriate items (i.e. items that will not produce a unique combination) and from the repairs children made to any identical combinations which occurred.

Children's scanning actions, which served in a monitoring capacity, reflect their application of general strategies and add support to the argument that children's initial procedures were executed with understanding (cf. Ohlsson & Rees' 1991 theory). When children initially used non-planning solution strategies they had to carefully monitor their actions if they were to succeed in solving the problems. As indicated in a previous section, children's methods of checking differed in their degree of thoroughness and efficiency and, when coupled with a non-planning strategy, determined the extent of problem success. For example, children who used a trial-and-error procedure and checked *only after* they had acted were naturally more error prone and less successful in goal attainment. In contrast, children who checked prior to acting or who checked continuously throughout problem execution, were less error prone and more successful in solving the problems.

As the children progressed on the problems, a significant improvement was evident in the solution strategies of the older subjects. The children moved away from non-planning behavior to a systematic method of solution. These observed changes in goal-directed behavior are considered to be mediated by changes in children's domain-specific knowledge base, in their processing efficiency, and in their domain-general strategies (Bjorklund & Harnishfeger, 1990). Children's adoption of the most sophisticated procedures (the odometer strategies), is indicative of a desire for a more efficient and more reliable means of problem solution. Children's change to a cyclic pattern of item selection, followed by their inclusion of a "constant" item, reflects a more comprehensive knowledge of the problem domain, in particular, a knowledge of the Odometer Principle (English, 1988; 1990). Children who displayed the most efficient procedure (strategy F) demonstrated an explicit or stateable understanding (Gelman & Meck, 1986) of this principle. That is, they were clearly able to describe their methods and could also explain why these were the most efficient for problem solution. The actions of children who used the less sophisticated odometer strategy (strategy E) indicate they possessed only a partial understanding of the Odometer Principle. That is, they did not appear to have acquired all of the four Odometer subprinciples of systematic variation, constancy, exhaustion, and completion (English, 1988, 1990). More specifically, these children failed to demonstrate a knowledge of either the subprinciple of exhaustion (reflected in a failure to generate all possible combinations from a given constant item) or the subprinciple of completion (failure to identify problem completion upon exhaustion of all constant items).

A particularly interesting feature of children's growth in domain knowledge was the associated change in nature of their general strategies. Children's initial monitoring actions may be considered to be compensatory, that is, they compensated for children's lack of efficient domain knowledge (Garner, 1990). From here, children adjusted their general strategies in one of three main ways. For some children, self-monitoring became unnecessary; they relied on the generative nature of their solution strategy to secure goal attainment. They were confident that their strategy would generate the required solution and hence did not bother to monitor the results of their actions. Other children changed their monitoring from a compensatory form to one which regulated procedural implementation rather than goal attainment. That is, their focus of attention was on their pattern of item selection rather than on the uniqueness of the combinations formed. Like the previous group, these children knew that their strategy would generate problem solution, but they nevertheless monitored its implementation. Another group of children displayed highly effective general strategies. They monitored both the implementation of their procedure and their attainment of the problem goal. These particular children were the most effective problem solvers and were the only ones who could solve the final problem comprising the hidden constraint on goal attainment. For this problem, no amount of sophisticated domain knowledge would help those children who did not engage in thorough self-monitoring.

It is also interesting to review children's shift from the most advanced solution strategy (F) to less advanced strategies on the fourth problem where an additional constraint was placed on goal attainment ("give the third bear a blue top"). Two explanations are possible here. One possibility is that children's conceptual knowledge of the Odometer Principle was not sufficiently robust to accommodate the additional constraint (Greeno et al., 1984), indicating a need for further experience in solving combinatorial problems. An alternative explanation pertains to the competence-performance issue. It draws upon Gelman & Greeno's (1989, p.132) notion of separating "competence within a domain from competence for assessing a setting within which to display the domain-specific competence as well as competence for generating a plan of action that honors the constraints of both the domain's and the setting's requirements." It is argued that children who had demonstrated an explicit knowledge of the Odometer Principle on a prior problem did possess the conceptual competence to solve this fourth problem, but lacked the required procedural competence (English, 1988; Gelman & Greeno, 1989). These children apparently failed to see how they could use the features of the problem setting to achieve the additional goal. They did not realize that, to give the third bear a blue top, all that had to be done was to commence the odometer pattern with a blue top. Because these children had difficulty in implementing their odometer procedure to solve this problem, they reverted to a less efficient method and, in so doing, changed the nature of their monitoring. Rather than focus their attention on their pattern of item selection, the children now concentrated on achieving the added goal of giving the third bear a blue top, while at the same time making sure that their actions produced unique combinations.

Children's responses to these problems have significant implications for the elementary school mathematics curriculum. Firstly, it is apparent that children as young as 4 years are able to apply both domain-specific and domain-general strategies to the solution of novel mathematical problems set within a meaningful context. Given that the nature of the interactions between these strategies is fundamental to problem success, children need to be given the opportunity to utilize both strategy types in problem-solving situations. A related issue here is the danger of children's reliance on generative or algorithmic procedures and their consequent failure to monitor or evaluate their actions. Children need to realize the importance of engaging in self-monitoring, even in routine mathematical tasks. The use of cognitive apprenticeship teaching methods (Collins, Seely Brown, &

Newman, 1988) has proven successful in fostering the development of these metacognitive skills.

Secondly, it would seem that, provided the context is meaningful, children as young as 7 years are able to acquire a conceptual knowledge of the combinatorial domain at an earlier age than predicted by the Piagetian studies. This finding supports recommendations that elementary school children be given a range of informal experiences in applied mathematical topics such as combinatorics and probability (e.g. National Council of Mathematics, 1989; Travers, 1988).

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Correspondence and requests for reprints should be sent to Dr Lyn English, Centre for Mathematics and Science Education, Queensland University of Technology, PO Box 284, Zillmere, Brisbane, Queensland, Australia, 4034.

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TABLE 1

FREQUENCY OF SOLUTION STRATEGY USAGE BY AGE AND PROBLEM

AGE (YRS)	PROBLEM No.1						PROBLEM No.2						PROBLEM No.3						PROBLEM No.4						PROBLEM No.5						PROBLEM No.6					
	A*	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F	A	B	C	D	E	F
4	5	3					2	6					2	6					2	6					2	6					2	6				
5	5	1	1	1			4	2	1	1			4	3			1		4	2		1	1		4	3		1			3	3		1	1	
6		8						6	2					6		1	1			7	1					8						7	1			
7		9	2	3	2			5	2	4	3	2		5	4	2	2	3		4	6	2	3	1		6	5	1	1	3		4	6	2	1	3
8		5	3	3	1	4		6	3	2	3	2		2	4	2	2	6		3	4	2	5	2		3	3	2	3	5		2	2		2	10
9		5	3	4	2	2		3	2	3	4	4		1		7	2	6		2	3	1	8	2		1	3	3	3	6		1	2	3	2	8

NOTE: * Solution strategies

N(4 yrs) = 8; N(5 YRS) = 8; N(6 YRS) = 8; N(7 YRS) = 16; N(8 YRS) = 16; N(9 YRS) = 16.

TABLE 2

FREQUENCY OF SCANNING ACTION USAGE BY AGE AND PROBLEM

AGE (YRS)	PROBLEM No.1						PROBLEM No.2						PROBLEM No.3						PROBLEM No.4						PROBLEM No.5						PROBLEM No.6					
	1*	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
4	1	7					1	5	2				1	4	3				1	4	3				1	4	3				1	4	3			
5	3		3				2	2	3				2	2	3				2	2	3		1		2	1	5				2	2	3			
6		1	7					2	6					1	6		1			1	7					2	6					2	6			
7			15		1				13	1	1				9	1	6				12	2	2			1	13	1		1			10	2	2	2
8			14	1	1				13		3				6	2	7	1			10	2	3	1			10	3	1	2			8	4	1	3
9		1	12	1	2			2	10		4				7	2	7				10	1	5				7	2	7				6	4	4	2

NOTE: * Scanning action

N(4 yrs) = 8; N(5 YRS) = 8; N(6 YRS) = 8; N(7 YRS) = 16; N(8 YRS) = 16; N(9 YRS) = 16.

TABLE 3

MEAN GOAL ATTAINMENT SCORES (OUT OF 4) BY AGE AND PROBLEM

AGE (YRS)	PROBLEM No. 1	PROBLEM No. 2	PROBLEM No. 3	PROBLEM No. 4	PROBLEM No. 5	PROBLEM No. 6
4	1.6	2.1	2.1	1.6	2.1	1.6
5	2.4	2.1	2.3	1.3	2.1	2.0
6	2.8	3.1	2.5	2.4	3.0	2.0
7	2.9	3.8	3.2	3.3	3.5	2.6
8	3.5	3.7	3.7	3.3	3.5	2.6
9	3.5	3.6	3.4	3.3	3.6	2.7

TABLE 4

SIGNIFICANT ASSOCIATION BETWEEN SOLUTION STRATEGY AND SCANNING ACTION BY AGE GROUP AND PROBLEM

AGE GROUP (YRS)	PROBLEM No.1			PROBLEM No.2			PROBLEM No.3			PROBLEM No.4			PROBLEM No.5			PROBLEM No.6		
	X^2	*df*	*p*	X^2	*df*	*p*	X^2	*df*	*p*	X^2	*df*	*p*	X^2	*df*	*p*	X^2	*df*	*p*
4 - 6	32.2	9	<0.001	34.8	9	<0.001	35.7	12	<0.001	36.3	12	<0.001	10.9	4	<0.05	40.1	12	<0.001
7 - 9	22.8	12	<0.05	31.1	12	<0.01	28.4	12	<0.01	37.5	12	<0.001	41.1	16	<0.001	30.7	12	<0.01