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ABSTRACT

This paper discusses the central thesis that new research on estimation and mental computation will benefit from more focused attention on the situations in which they are used. In the first section of the paper, a brief discussion of cognitive theory, with special attention to the emerging notion of situated cognition is presented. Three sources of expertise as contextual are proposed: social theories about language and the development of thought; anthropology and the study of situated cognition; and the history and philosophy of scientific domains. In the second section, "Mathematics and Sense Making", a review of work on problem solving that dealt with the importance of context, especially with respect to the interpretation of problem solutions is presented. Situational factors that influence children's problem-solving performance are described though research findings on division with remainders in problem-solving situations. Finally, a situated perspective on further research regarding serious consideraiton of mental computation and estimation as situated mathematical processes is discussed.

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Treating Estimation and Mental Computation as Situated Mathematical Processes[1]

Edward A. Silver

My assigned task in this paper was to identify some potentially fruitful theoretical formulations for research on alternatives to paper-and-pencil computation, such as estimation and mental computation. Since I approach the topics of estimation and mental computation as an "outsider" who has conducted research primarily in the area of mathematical problem solving rather than computation, I am biased toward consideration of the use of these mathematical processes in problematic situations, and this bias is reflected herein. The central thesis is that new research on these topics will benefit from more focused attention on the situations in which mental computation and estimation are used.

The paper begins with a brief discussion of cognitive theory, with special attention to the emerging notion of situated cognition. I have chosen to discuss this particular theoretical position both because it has not yet become established in mathematics education research and because it may provide support for many in the field who are seeking to make progress investigating contextualized aspects of estimation and mental arithmetic. Next, I discuss some of my problem-solving research that deals with situational factors as influences on children's problem-solving performance. Finally, I discuss a possible direction for research that might emerge from our serious consideration of mental computation and estimation as situated mathematical processes.

Knowing in Context: A Challenge to Cognitive Theory

There are many theoretical perspectives which might form a suitable basis for instructional research on computational alternatives. Case (1989) has reviewed several of these and suggested productive directions starting from empiricist, logico-mathematical developmental, and cognitive structuralist perspectives. Most of the work done to date on the topics of estimation and mental computation has proceeded from these theoretical assumptions. For example, studies of expert estimators or mental calculators have attempted to identify and describe the cognitive processes and mental structures that appear to account for the expert performance (e.g., Nohda, Shimizu, Yoshikawa, Reys, & Reys, 1990; Reys, Rybolt, Bestgen, & Wyatt, 1982). Other studies have examined children's development of competence on tasks of increasing complexity as they mature (e.g., Case & Sowder, 1990; Sowder & Wheeler, 1986). These lines of work have been solidly grounded in or closely allied with cognitive theory, and they represent fruitful directions for further research.

Most of the research has been influenced by a general information-processing framework. According to this view, knowledge and cognitive processes are dissected into components and are hypothesized to reside in cognitive structures in the mind of the knower. As Greeno has described it:

> *An information-processing analysis assumes that a person constructs a representation of the situation and his or her goal and reasons by manipulating the symbols in the representation. The person's knowledge includes information structures corresponding to the concepts and propositions. ... Analysis of a specific cognitive capability , in this framework, involves hypothesizing a set of processes for representing situations, a set of knowledge structures -- most frequently in the form of schemata -- that provide organization for the information an the reasoning processes, and a set of processes that make inferences of various kinds.* (1989c, p. 49)

This general information-processing framework has been used productively to analyze many areas of mathematics learning and performance, including estimation and mental arithmetic. Nevertheless, there are important aspects of mathematics knowledge and its use that are not completely captured within this perspective. An alternate theoretical perspective may be helpful.

Some recent cognitive research has begun by defining learning and expertise in task domains not from the perspective of knowledge structures or abstract general mental processes but rather from the perspective of knowledge use in practice (Brown, Collins, & Duguid, 1989; Greeno, 1989a: Pea & Greeno, 1990). The word "practice" is used here in the sense of Scribner (1984), who used the term to describe a culturally organized set of significant activities with its own technology and symbol system. The notion that expertise is contextual has entered the theories of some cognitive scientists from at least three different directions: social theories about language and the development of thought, anthropology and the study of situated cognition, and the history and philosophy of scientific domains.

I will say little about the social theory roots; the interested reader can find an excellent account in Cole and Griffin (1980). Anthropologists have found that a contextual view of expertise is useful as they have tried to describe the nonformal knowledge of unschooled but highly skilled practitioners in various complex performance domains, such as navigation (Hutchins, in press) and tailoring (Lave, 1977). Other support for the view that context plays an important role in understanding expertise comes from studies which have emphasized the study of mathematical knowledge acquisition and usage from the perspective of particular cultural practices (e.g., Carraher, Carraher, & Schliemann, 1985; Saxe, 1988) or related work which has questioned the value of knowledge and principles acquired in academic settings for the purposes of practical problem solving (e.g., Lave, 1988; Rogoff & Lave, 1984). In general, these studies have pointed not only to the situated nature of much quantitative reasoning in out-of-school settings but also to the importance of situations in evoking particular kinds of performances. In general, this work, along with several decades of work in psychology and education that has noted a general lack of knowledge transfer across tasks and situations, has caused many to question a theoretical perspective that posits abstract knowledge in the head of the knower as he moves freely among situations to which that knowledge might be applied. This has led to the development of an alternative perspective that places performance contexts more centrally at the theoretical core; namely, situated cognition (Brown, Collins, & Duguid, 1989). It is interesting to note that some researchers in this field hgave explored what they call "oral mathematics," a term which refers to non-paper-and -pencil mathematics. This oral mathematics, which is apparently used extensively outside the classroom in everyday quantitative situations, can be thought of as situated mental computation, often involving approximate numbers. It is interesting to contrast this notion of "oral mathematics" with Pestalozzi and Colburn's use of the same term to denote classroom exercises in rote, mental arithmetic (cf. Reys & Barger, this volume).

A third impetus for the notion of highly contextualized expertise has been developments in the philosophy of science which have suggested that understanding science requires understanding the practice in which scientists engage. In the area of mathematics, the work of Lakatos (1976) on the social processes of debate and argumentation in the development of mathematical ideas, of Toulmin (1958) on the development of disciplinary concepts through a process of selection and debate, and of Kitcher (1983) on the reference potential of concepts and the importance of metamathematical aspects of new developments in mathematics have been particularly influential in providing a foundation for this new epistemological view which emphasizes notions of disciplinary practice, social construction of knowledge, and situated cognition.

Situated cognition is a theoretical position designed to explain the relationship between knowing and doing. The theory of situated cognition suggests, as its name implies, that all knowledge is situated in contexts; i.e., knowledge resides in a jointly constructed space of mind and the situation in which mind finds itself confronted with a problem (Greeno, 1989a, 1989b). Although somewhat controversial and not yet widely accepted in the research community, this view may provide the needed theoretical basis for analyzing some important aspects of mathematical cognition.

As Resnick has noted, work done from the general information-processing perspective has tended to incorporate two fundamental assumptions, which she calls decomposition and decontextualization:

> *The decomposition assumption refers to the notion that competence can be completely defined by a collection of independent elements of knowledge or skill....The decontextualization assumption refers to the idea that competence exists independently of the performance that it enables; that there is some pure or abstract form of knowing that remains intact no matter what the conditions of its use; that knowledge is fully defined as something inside an individual's head, independent of the situation in which the individual acts.* (1989, p. 35)

These assumptions are fundamental impediments to our efforts to understand such widely observed phenomena as students who possess certain competences but fail to demonstrate them on specific tasks that call for the competences, or students who appear to lack a competence in classroom situations but perform well on apparently similar tasks in out-of-class settings. Although one can study mental arithmetic and estimation as decontextualized and decomposed processes, there may be much that is gained by setting aside these assumptions and treating them instead as situated processes. In fact, I believe that estimation and mental computation are especially well suited to a more contextualized perspective.

Consider, for example, the oft-noted tendency of students to resist using estimation strategies to answer estimation problems. The most commonly observed behavior is for children to calculate an exact answer and then round off the answer to produce a number that appears to be more like an estimate. Although somewhat difficult to understand and interpret from the perspective of mental structures, this behavior may be better understood from a more contextualized perspective. In particular, it can be argued that children have a general tendency to seek exact answers to computations, especially when presented in school or school-like settings. In the context of school mathematics, this general tendency would be honored and supported by typical classroom teaching practice in which exercises and problems that require exact answers are posed. Given the general tendency of students to seek exact answers, given the general similarity between the stimuli in tasks requiring exact answers and estimates (e.g., same symbol systems, both presented as stories), and given that the two kinds of tasks are both likely to be seen by students as school tasks, the behavior of the students is not surprising. They are only slightly modifying the performance of exact computation that is generally valued and rewarded in the school situation.

The situated perspective may not only help us understand certain of students' errors and failures but also assist us in designing research and instruction to develop a more solid grounding in the processes of estimation and mental computation. In order to do so, we will need to examine the settings in which these processes are used. Several authors (e.g., Hope, 1986; Trafton, 1978, 1986; Usiskin, 1986) have provided a <u>use-oriented</u> rationale for the importance of estimation and/or mental computation as topics in the school curriculum. With respect to estimation, for example, it can be argued that there are only a few situational characteristics which give rise naturally to the use of estimation. In particular, estimation is natural or useful in these three settings: (a) prior to engaging in an exact computation in order to gain advance information that might aid in error detection or selection of a computation method; (b) after an exact computation in order to check the reasonableness of the obtained answer; or (c) as an alternative to an exact computation in settings for which the numbers are infelicitous in some way, such as being very large or very small, or being unknown or impossible to know exactly. These three estimation situations could form the basis for estimation instruction, in which the use of estimation was modeled and practiced in these three settings and particular estimation strategies emerged in the situated use of estimation as a mathematical process. Such instruction would likely look quite different from current instruction which tends to teach in sequence a set of decontextualized estimation strategies that have been arranged into a hierarchy of hypothesized complexity. Such instruction would follow naturally from an attempt to implement many of the curricular and instructional changes suggested by the NCTM Curriculum and Evaluation Standards for School Mathematics (1989) such as integrating richer, more situational problems into mathematics instruction or placing greater emphasis on the interpretation, rather than simply the computation, of problem solutions.

Mathematics and Sense Making

In this section of the paper, I review some of my work on problem solving that has also dealt with the importance of context, especially with respect to the interpretation of problem solutions. Although providing a brief general account of the work, I will concentrate on those aspects of the work that may have particular relevance to the topics of mental arithmetic and estimation. Moreover, I present the work as an example of the ways in which the perspective of situated cognition may enrich our understanding and interpretation of data.

In a brief but persuasive article, Terezinha Carraher (1989) argues that the time may be right for a fruitful cross-fertilization of paradigms in research on children's thinking and use of mathematical knowledge. In commenting on several analyses (e.g., Baranes, Perry, & Stigler, 1989; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Carraher, Carraher, & Schliemann, 1985) of children's ability to solve computation tasks, school-like story problems and contextualized problem tasks, she demonstrates that different theoretical assumptions can influence both the nature of the data considered and the interpretation of results: "When researchers start from different conceptions, they ask different questions about their data and find different answers. It seems, however, that the time for rapproachment has come" (p. 322). I offer the following account of my research on interpretation of solutions to story problems as an example of how the information-processing perspective from which the work originally developed can be integrated with a situated cognition perspective to provide a more complete understanding of children's performance.

There are many anecdotal reports, and some empirical confirmation, that children do not generally develop in school a disposition toward making sense out of numbers or, more generally, of any mathematics they learn. Data from the Fourth National Assessment of Educational Progress (e.g., Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988; Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988) indicate that many students in the United States see mathematics as a subject that is not creative, that consists mainly of facts to be memorized, and that is about symbols rather than about ideas. The prevalence among students of such views of school mathematics provides the underlying support for a belief that mathematics is not necessarily supposed to make sense and that learning mathematics has little, if anything, to do with sense-making.

Division with Remainders

Confirmation of the assertion that students tend to see mathematics as disconnected from sense-making also comes from research I and my collaborators have been conducting over the past few years on students' difficulties in solving story problems involving division with remainders, such as "The science teacher at Marie Curie School has been given 730 frogs. The frogs will be kept in tanks. Each tank holds 50 frogs. How many tanks are needed to hold all the frogs?"

The widespread failure of American students to succeed in solving problems involving whole number division and remainders has been documented through the National Assessment of Educational Progress and several state assessments. Only 24% of a national sample of 13-year-olds was able to solve correctly the following problem which appeared on the Mathematics portion of the Third National Assessment of Educational Progress (NAEP, 1983): "An army bus holds 36 soldiers. If 1,128 soldiers are being bused to their training site, how many buses are needed?" A similar division problem appeared on the 1983 version of the California Assessment Program (CAP) Mathematics Test for Grade 6 and was answered correctly by only about 35% of the sixth-graders in California In both assessments, students commonly erred by choosing non-whole -number answers.

To understand better the basis for the observed difficulty that students have in solving division problems involving remainders, several investigations have been conducted with students in grades 6, 7, and 8 (e.g., Silver 1986, 1988; Silver, Mukhopadhyay, & Gabriele, 1989; Silver & Shapiro, 1990; Silver & Smith, 1990). Overall, the findings of these investigations suggest that students appear to have little difficulty recognizing that division is an appropriate mathematical operation to be used in solving such problems and that they appear to have little difficulty carrying out the required computation. The failure to solve such problems successfully in school contexts appears to be directly related to students' failure to "make sense" of their computational result with respect to the problem situation.

Although students generally have little difficulty with different forms of expression for remainders in routine computation settings (e.g., 12 R2, 12 1/2, or 12.5), they often experience considerable difficulty when the computation is embedded in a problem situation. One source of the difficulty is that the same symbolic expression of a division problem can represent very different problem situations and have different answers that depend upon important aspects of the situational context and the quantities involved in the problem. For example, consider the following problems: "Mary has 100 brownies which she will put into containers that hold exactly 40 brownies each. (1) How many containers can she fill? (2) How many containers will she use for all the brownies? (3) After she fills as many containers as she can, how many brownies will be left over?" To solve each of these problems, one would perform the same calculation, 100 ÷ 40, but give a different answer to each problem. In the first problem, a quotient-only problem, the remainder is essentially ignored and only the quotient is given as the answer to the problem. For the second problem, an augmented-quotient problem, the existence of a remainder leads us to increment the quotient when answering the question. In the third problem, a remainder-only problem, the correct answer is the remainder itself.

Unlike most story problems that students solve in elementary school, sense-making is not optional activity for these problems. Since the same computational result is obtained for each problem, these problems do not to allow successful solution without semantic processing; i. e., without making sense of the situation, the quantities involved in the problem, and the context . A successful solution appears to depend to a great extent on mappings between and among at least three referential systems: the story text, the story situation, and a mathematical model. The distinction between the story text and the story situation, which has been articulated by Kintsch (1986), appears to be of critical importance in the solution of division story problems involving remainders, and it may help inform research on estimation story problems as well.

Consider a hypothesized version of a successful solution of the brownie problems from this perspective. A successful solver would map from the story (natural language) text representation of the problems into a mathematical model representation of $100 \div 40$, then perform the indicated computation within the referential system of mathematics, expressing the resulting answer with an appropriate mathematical representation. The solver would then map the computational result back either to the story text representation or to the implied story situation (in the "real world") representation in order to decide how to treat the quotient and remainder. Through such a process, the successful solver would finally obtain suitable mathematical and natural language representations of the solution that have accompanying interpretations and validity within the referential systems of real world situations and the knowledge domain of mathematics. Figure 1 provides a schematic representation of the mappings involved in this idealized problem solution.

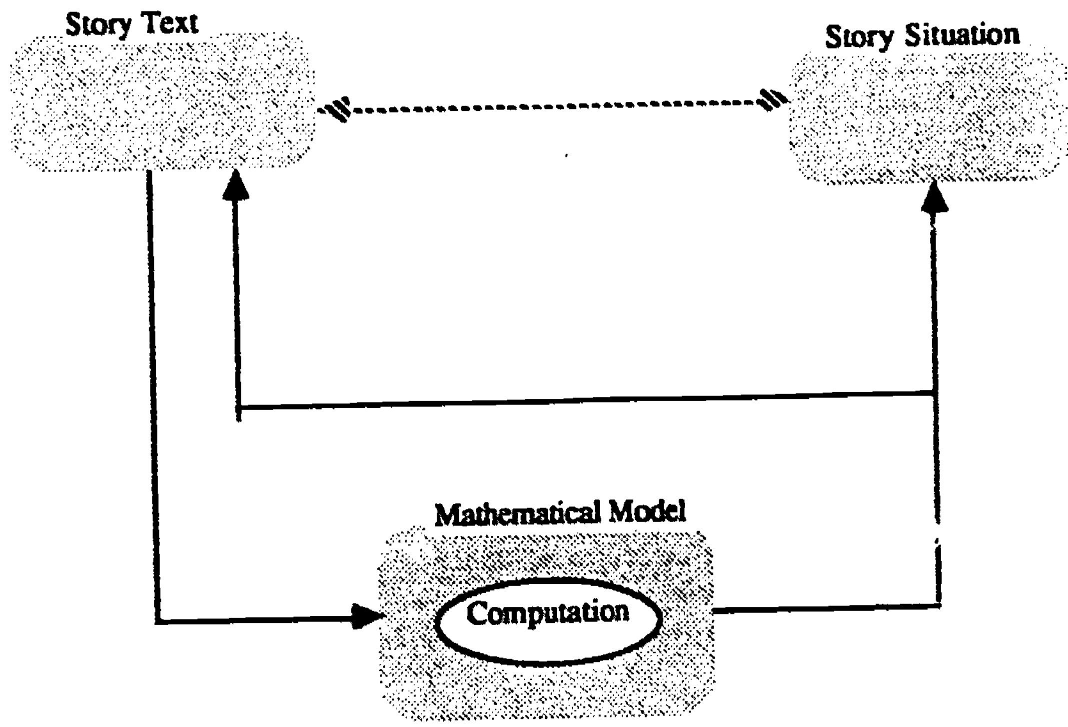

Figure 1 Schematic representation of an idealized successful solution

In an early study (Silver,1986) it was hypothesized that students were failing to attend to relevant information implicitly represented in the problem situation but not explicitly stated in the story text (e. g., no one is to be left behind; on some bus there may be some empty seats). Several problem variants were created that made relevant structural information more salient and students' performance on these variants was examined. The results of that study suggested that variations in the presentation of the problem, designed to make explicit certain implicit information in the problem or the required solution, significantly enhanced students' performance. Unlike most considerations of "relevant information" for problem solving in elementary mathematics, the focus of attention in the study was not so much on information that would enhance the mapping between the story text and the mathematical model but rather enhance the mappings between and among these two reference spaces and the story situation.

In follow-up research (Silver 1988a; Silver, Mukhopadhyay, & Gabriele, 1989) students' performance on augmented-quotient division problems and other division problem types (e.g., remainder-only problems and quotient-only problems) was examined. These studies examined the effects on students' performance of their solving other division problems that required the same computation and similar referential mappings. The results indicated that students' performance on each type of problem was enhanced by having students also solve related division problems. In general, the results were consistent with the explanation that enhanced performance was due to students' increased sensitivity and attention to the relevant semantic and referential mappings involved in the target problem solution. In particular, experience with the related problems may have drawn attention to the need for mapping into either the story text representation or the story situation representation after obtaining a solution to the target problem through use of a mathematical model.

Taken together, these results and the assessment findings suggest that students' failure to solve the division story problems may be due, at least in part, to their incomplete mapping among the relevant referential systems. In particular, students appear to map successfully from the problem text to a mathematical model (in this case, a division computation to be performed), compute an answer within the domain of the mathematics model, but fail to return to the problem story text or to the story situation referent in order to determine the best answer to the question. Figure 2 presents a schematic representation of a hypothesized version of a student's unsuccessful solution attempt.

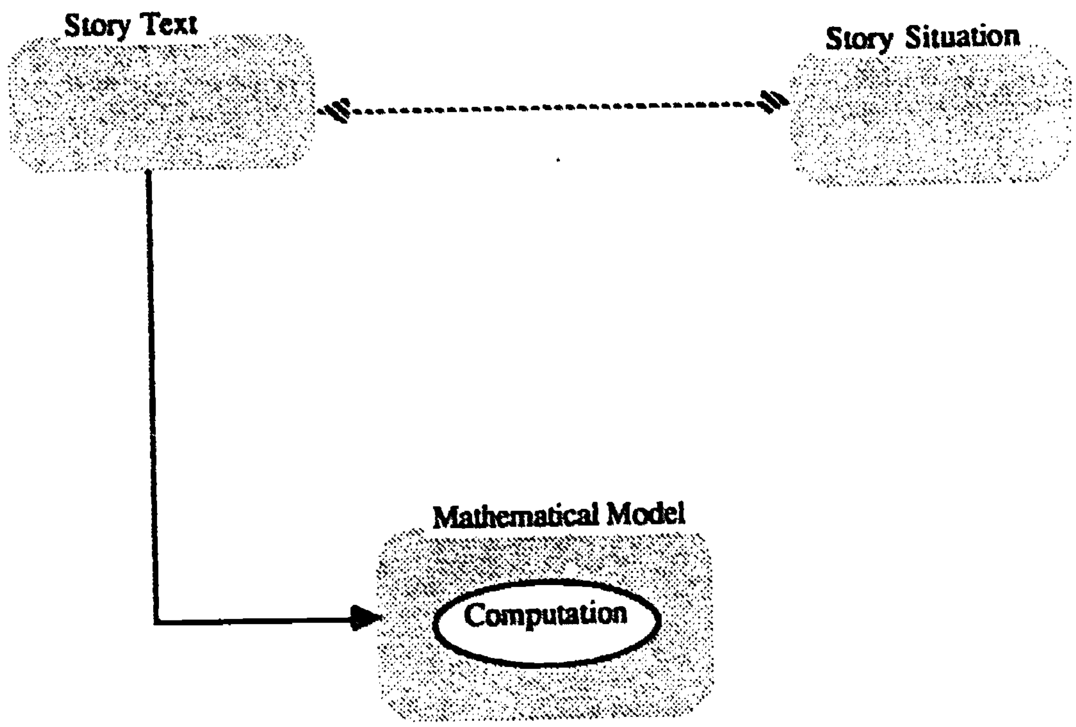

Figure 2 Schematic representation of hypothesized unsuccessful solution

Although the research has focused on interpretation of solutions with respect to situations, it should be clear that all of the work described thus far has been strongly influenced by the information-processing perspective discussed earlier. In particular, the description of mental representations and mappings is deeply embedded in that framework. As the work has progressed recently, a perspective more akin to situated cognition has been found to be useful. In one recent study, we have examined the problem-solving and interpretation performance of students in interview settings (Smith & Silver, 1990). This study with 12 middle school students revealed that some students who would have answered incorrectly if the tasks were presented in a multiple-choice format were able to offer interesting interpretations of their numerical answers. For example, students spoke of "squishing in" the extra students or of ordering mini-vans rather than a full bus for the extra students. This situated thinking and reasoning remained invisible in the multiple-choice format used in the prior research.

In order to explore the general prevalence of this kind of situated reasoning another study was recently conducted with about 200 middle school students (Silver & Shapiro, 1990). In order to move away from the multiple-choice format yet accommodate the large number of students, a paper and pencil, free response task was developed and administered. Much to our surprise, little evidence of situated reasoning was found in the responses of the students. In particular, nearly 60% of the students provided no written interpretation at all. Of the 30% of the students who gave the mathematically correct answer to the augmented quotient problem, about 2/3 gave an appropriate interpretation, but there was little evidence of any appropriate interpretation of alternative responses (e.g., mini-vans to explain fractional answers or "squishing" to explain other answers) by the remaining 70% of the students. After struggling with our interpretation of these findings, we discussed the results with the teachers who had administered the tasks to their students. The teachers revealed that many students discussed the problem after handing in their work and proposed alternate solutions and interpretations. This lively discussion, apparently rich in examples of situated reasoning, is evidence for the fact that students had the capability to provide interesting interpretations of their calculations. Yet, even in the free response mode, these interpretations remained invisible to us because the students did not believe their interpretations were appropriate to be included in the work that they handed in; i. e., their interpretations were not seen by them to be part of what would be accepted as mathematics work. This latter finding points to a potentially serious barrier to students' becoming adept at mathematical performances involving interpretation and sense making.

If estimation and mental computation are seen as components of a more general construct called number sense, then there are at least two important points to be made about the findings from the research on division story problems and the general topic of number sense. One point is fairly obvious: making sensible judgments about the reasonableness of answers (e.g., recognizing that one cannot have a fraction of a bus or a fish tank) or being able to offer reasonable interpretations of non-standard answers (e.g., mini-vans for fractions of buses) is surely one component of what we refer to as number sense. The second aspect is somewhat more subtle and relates more to the issue of students' disposition toward sense-making. The findings regarding students' difficulties with division story problems and their tendency to divorce their situated reasoning from their school mathematics solutions provide support for the notion that school mathematics education is not currently fostering the development in students of a general inclination toward sense-making in quantitative matters.

A Situated Perspective on Further Research

I have attempted to illustrate the value and potential power of viewing mathematical knowledge and performance from the perspective of the situations in which it is used. We have discussed situated cognition as an alternative theoretical perspective that may enhance our understanding of some important aspects of estimation and mental computation. And we have considered some research on students' interpretations of the results of their computations in a problem-solving setting from both information-processing and situated-cognition perspectives. I hope that both the general theoretical perspective of situated cognition and the particular model of semantic processing discussed earlier can be useful to researchers studying estimation and mental computation as situated mathematical processes. Although I do not advocate the abandonment of earlier theoretical perspectives, and one can find them well-represented in my own work, I believe there is much to be gained from considering the situated cognition perspective in our examination of new research issues in this area.

My final comments relate to an issue -- number sense -- that becomes more salient if we take seriously a situated view of estimation and mental computation.

Number Sense and the Culture of the Mathematics Classroom

As noted in many of the papers in this volume, much of the current interest in estimation and mental computation in the United States is due to a general interest in the topic of number sense. Although it is difficult to define number sense precisely, behaviors like estimating before or after computing, judging the reasonableness of one's calculation, and using the relative size of numbers or numerical benchmarks (such as basic facts) to guide quantitative activity are all examples of se' se-making actions associated with numbers and numerical activity. Thus far, instructional attempts to increase students' competences in the area of number sense have tended to focus on discrete, teachable components of complex skills. Attempts to improve students' number sense often focus on overt cognitive behaviors whose manifestation will ensure that number sense is present and in use. For example, instructional units have been developed to help students learn many different techniques for estimating the result of a numerical computation, learn information about the precision of measurement or calculation and techniques for approximation, or learn strategies for mental mathematics in order to increase students' flexibility with respect to calculation strategies. Although this instructional approach is likely to increase the specific cognitive competence of the students who learn from it, it may not be completely successful if it fails to embed these competences in performances situated in authentic mathematical activity. The perspective of situated cognition may be helpful to us as we examine the design and effects of such instruction.

The situated cognition perspective also suggests that such instruction is unlikely to be completely successful in increasing students' number sense if it fails to address students' dispositions toward numerical activity, or more generally toward mathematics. It is apparent from much of the research discussed above that students do not believe that school mathematics makes sense or even that it should. Unless situations are provided in which students can participate in the authentic sense-making aspects of mathematical culture, it is unlikely that students will alter that belief. Decomposed and decontextualized instructional sequences to promote mental computation and estimation are likely to leave untouched this important aspect of the culture of the mathematics classroom. To promote the development in students of a finely tuned number sense, what is needed is nothing short of cultural revolution in school mathematics education.

Regardless of how well the instruction is designed, isolated instruction on individual components of number sense is unlikely to lead to the development in students of an adequate sense-making disposition with respect to quantitative processes and products. Considering the division example discussed above, it is clear that teaching students simply to answer such questions correctly is not the entire story. Our curriculum and our instructional methods must be restructured to emphasize sense-making in all areas of mathematics instruction and at all grade levels from kindergarten to grade 12. Moreover, our research needs to be directed at understanding at much deeper levels children's sense-making capabilities so that they can be built upon rather than ignored in classroom instruction.

As reform efforts proceed, care must be taken not to delete completely curricular topics, such as work with very large or very small numbers, that can provide important situations in which students can develop number sense. Likewise, we would be wise to resist the temptation to believe that extensive use of calculators will automatically develop good number sense. Although calculator activities can be used in situations that might promote number sense, they can also be used in ways that promote an over-reliance on the result found in the calculator display and increase students' failure to make sense of the situation, quantities, and numbers involved. Recall that performance on the NAEP army bus problem was poorer in the Third Mathematics Assessment (NAEP, 1983) when students used calculators than when they used only paper and pencil!

Given the need for fundamental reform, researchers may need to be more proactive in exploring these issues in a "transformative" research agenda (Silver, 1988b). Studies need to be undertaken that examine the long-term consequences of early mathematics instruction which emphasizes sense-making on the part of students and teachers. Curricular and instructional experiments related to number sense also need to be conducted with older students as they learn more complex procedures and solve more difficult problems; sense-making needs to be embedded into the fabric of mathematics courses for these older students as well. Although research must certainly also be done on specific components of number sense, it would be unwise for us to believe that the sum of the cognitive components would necessarily equal the complex whole of number sense. Studies directed at students' general dispositions toward quantitative sense making will deepen our understanding of these components. Moreover, we need research related to student exploration of rich situations that evoke mathematical sense-making behavior and its various components, such as estimation and mental computation. This research on estimation and mental computation should , when amplified by similar investigations in related topic areas, assist us in meeting a fundamental challenge for the next decade -- explicating the nature of the relationship between richly contextualized, situated quantitative understandings and abstract, powerful generalized conceptions of mathematical principles and structures.

Notes

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