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ABSTRACT

Reform proposals for mathematics teaching and learning have clear implications for significant change in curriculum and in traditional teacher roles. One contribution to the reform effort was the SummerMath for Teachers program, an inservice program committed to helping teachers develop constructivist orientations to teaching and learning by working with them in ways congruent with those constructivist orientations. The program began with a two-week residential summer institute, followed by a year of intensive individual follow-up with teachers in their own classrooms. This study investigated the learning of two teachers who participated in the program from 1987 to 1989. The study focused on: (1) what each teacher brought to the program; (2) each teacher's experience of the program itself; and (3) changes in each teacher's visions and practices over two years. The teachers' experiences illustrate the ways in which individuals create their own "patchworks" of practice as they merge prior knowledge and experience with the new ideas presented to them as learners and teachers of mathematics. While the two teachers responded quite differently to the Institute experience and the intensity of the challenges it posed, both found the support and respect they needed to begin changing their practices. The contrast between these two cases points up a paradox inherent in teaching teachers: how to effect significant and specific changes in mathematics teaching while acknowledging that teachers themselves need to be active constructors of their knowledge and practice. (Author/LL)

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CHANGING VISIONS AND CHANGING PRACTICES: PATCHWORKS IN LEARNING TO TEACH MATHEMATICS FOR UNDERSTANDING

Suzanne M. Wilson and Deborah Loewenberg Ball

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Abstract

Talk of the reform of mathematics education abounds. But there is little known about the processes by which elementary school teachers learn to change their practices and adapt to these reforms. In this paper, the authors investigate the learning of two teachers—Ms. Leland and Ms. Rosen—who participated in SummerMath, a staff development program focused on mathematics teaching. The teachers' experiences illustrate the ways in which individual teachers create their own "patchworks" of practice as they merge their prior knowledge and experience with the new ideas presented to them as learners and teachers of mathematics. The contrast of these cases enables the authors to explore a paradox inherent in teaching teachers: how to effect significant and specific changes in mathematics teaching while acknowledging that teachers themselves need to be active constructors of their knowledge and practice.

CHANGING VISIONS AND CHANGING PRACTICES: PATCHWORKS IN LEARNING TO TEACH MATHEMATICS FOR UNDERSTANDING[1]

Suzanne M. Wilson and Deborah Loewenberg Ball[2]

Calls for changes in mathematics teaching and learning abound. The current reforms, such as those articulated in the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* (NCTM, 1989) and *Professional Teaching Standards* (NCTM, 1991), press teachers to alter content and to reframe mathematics instruction—to move from an emphasis on speed and accuracy to an emphasis on reasoning, from an emphasis on memorization and procedures to an emphasis on conceptual understanding. Teachers are encouraged to alter the structure of their classrooms to promote group interaction and discourse, cooperative learning, and group discussion. Moreover, threaded throughout is a clear commitment to the idea that learning is a constructive process. These reform proposals have clear implications for significant change in curriculum and in traditional teacher roles.

Important to note is that teachers, not policymakers or researchers, are the ones called upon to make these changes in content and pedagogy; little will change unless teachers change it (Cohen, 1990; NCTM, 1991). Yet this presents a paradox, for teachers are themselves products and producers of the traditional instruction that the reformers seek to change (Cohen & Ball, 1990). Teachers' understandings, attitudes, images, and assumptions have been shaped in traditional mathematics classrooms with traditional conceptions of content, pedagogies, forms of assessment, and ways of organizing students for instruction (Cohen, 1988). What teachers know and believe about mathematics and about the teaching and learning of mathematics is, in many cases, limited by what they have been exposed to both as students and teachers in American classrooms. What they do is what they know, what they have learned. Yet these reforms require that teachers teach content they have never learned in ways they have never seen (Cohen & Ball, 1990). This

[1]An earlier version of this paper was presented at the April 1991 annual meeting of American Educational Research Association, Chicago.

[2]Suzanne Wilson, an associate professor of teacher education at Michigan State University, and Deborah Loewenberg Ball, an associate professor of teacher education at MSU, are senior researchers with the National Center for Research on Teacher Learning. The authors would like to thank the other members of the research staff of the Center, especially James Mead for his help with data management. In addition, they would like to acknowledge colleagues David Cohen, Ruth Heaton, Penelope Peterson, Richard Prawat, Ralph Putnam, Janine Remillard, and Nancy Jennings for helping them to think about teacher change and teacher learning. This paper has also benefitted from Ball's earlier work with both James Mosenthal and Richard Prawat and comments by Deborah Schifter and Martin Simon.

paradox is at the heart of the work of current mathematics teacher education, both preservice and inservice.

An additional factor complicates the picture: If learning is a process of construction, then teachers themselves also construct understandings of teaching and learning, of subject matter and students. Consequently, they will often reach conclusions and develop practices different from those desired by teacher educators. This presents a tension for teacher educators committed to effecting particular changes in teachers' practices and is not unlike the dilemmas faced by K-12 classroom teachers who are committed to children exploring and developing their own ideas, but who also feel responsible to make sure that their students acquire particular knowledge. In order to investigate how this dilemma plays out, we examine challenges faced by the SummerMath for Teachers program, an inservice program committed to helping teachers develop constructivist orientations to teaching and learning, and committed to do so by working with teachers in ways congruent with those constructivist orientations.

We begin with a description of the program itself and an introduction to the thinking of some of the program staff. Then, in order to illustrate the challenges and realities of their work, we investigate the learning of two teachers—Ms. Leland and Ms. Rosen[3]—who participated in the SummerMath for Teachers program. Ms. Leland's and Ms. Rosen's experiences illustrate the ways in which individual teachers had their own unique "patchworks" of practice, created out of changes in their visions and woven with their own prior knowledge of mathematics and assumptions about teaching and learning. The portraits demonstrate how teachers critically evaluate, adopt, and adapt new ideas about the content and pedagogy of mathematics depending on their goals for instruction; their ideas about learning, teaching, and knowledge; their understandings of mathematics; and the contexts in which they work. We picked these teachers because of the contrast they offer along many of the dimensions. We conclude the paper by raising questions about the variation portrayed by these cases of teachers' change, questions that might highlight dilemmas inherent in trying to design an inservice teacher education program that would press for significant and specific changes in content and pedagogy.

Method

The SummerMath for Teachers program was one of the 11 teacher education programs studied by the National Center for Research on Teacher Education (NCRTE)

[3]The names of the teachers are pseudonyms.

between 1987 and 1989. The study examined what teachers were taught and what they learned, combining case studies of programs with longitudinal studies of participants' learning. We followed groups of teachers and prospective teachers over time, tracking whether and how their ideas and ways of thinking, skills, and dispositions changed while they were participating in these teacher education programs. We used questionnaires, interviews, and observations; NCRTE researchers investigated respondents' understandings of subject matter, ideas about teaching and learning, and notions about working with diverse students. The data were gathered over time: before the program, at several points during the program, and after the conclusion of the program itself.[4]

Three foci guided our analysis of the interview and observational data for Ms. Leland and Ms. Rosen over the two-year period, 1987-1989: (1) what each teacher brought with her to the program—her extant visions and practices; (2) each teacher's experience of the program itself; and (3) changes in each teacher's visions and practices over the two years that we followed her. In order to provide the reader with some perspective for the program context—the visions and practices of SummerMath for Teachers—we also drew on observations of program sessions and interviews with program staff.

The SummerMath for Teachers Program

Program Visions

The SummerMath for Teachers staff aimed "to help teachers develop their abilities to teach in a way that involves students in a problem-solving, active-learning approach to the learning of mathematical concepts." Most teachers, according to one of the program's directors, have no explicit theory of learning; they think about teaching and about what they do but not about what students do with it. Consequently, teachers are inclined to tell and show students how to do mathematics instead of creating activities that help students to construct understanding of the content. According to the program staff, teachers must learn to "give up responsibility for getting the students to the answer" and become facilitators rather than deliverers.

SummerMath for Teachers was based on a view of mathematics learning, labeled by the program staff as "constructivist," which holds that individuals must construct their own understandings of mathematical principles and concepts. As one of the staff members

[4]All words in quotations are taken from data collected in interviews, classroom observations, and document analysis. See NCRTE, 1988, for further information about the Teachers Education and Learning to Teach Study. See also, Kennedy, Ball, McDiarmid, & Schmidt (in press) for data collection strategies and rationales.

explained, "What 'constructivism' means to us is that people don't take on meaning by hearing the meaning it has for someone else. They've got to have experience with the phenomenon to create meaning for themselves." Central to this view of learning is the belief that learners are "constantly accommodating and assimilating, constructing, changing one's own notions by reflecting and resolving contradictions." Teachers, then, must provide the physical and intellectual space necessary to do so in productive ways.

According to this view, students must be actively involved, and their engagement must move from the concrete to the abstract levels, if they are to develop conceptual understanding and the ability to solve mathematical problems. Applying mathematics to novel situations, inventing strategies, and assessing the reasonableness of one's solutions are among the hallmarks of understanding. Telling and explaining are less the teacher's trade in this approach. Instead, the teacher serves as a guide, facilitating students' learning by posing problems and asking questions aimed at helping students clarify their thinking (e.g., "What are you trying to do?" or "What does the 1/2 refer to here?").

One of the program directors emphasized that the teacher's role, while not centered on telling and showing, was nonetheless crucial:

> It's not a matter of putting kids in a room with a bunch of interesting things and they'll discover the theory of relativity. No, a teacher plays a very important role in directing and making decisions and focusing students on particular concepts.

Still, she worried about how this could be distorted, for expectations color experience: "Frequently when we're asking a student [a question] we've already decided what that student has to connect it to." Another staff member emphasized the importance of the kind—that they must be tasks that students are ready for and that will provoke some conflict that they have to resolve. All the staff members agreed on the need for teachers to learn how to ask good, probing (as opposed to *leading*) questions.

Describing a teacher who was doing well, one of the program directors outlined three key pedagogical strategies that could be identified with this kinds of teaching:

> One is the *increased use of manipulatives* . . . second is that *questioning* is really improved . . . to the point where she is asking the kids questions that really get them to thinking about the "why" behind what they are doing and she is giving *problems that challenge kids* that aren't just routine exercises and she is starting to think about *extensions* to problems.

In sum, the program's vision of good teaching aimed to put teachers in the unfamiliar role of guiding students to explore and make sense of mathematics. They envisioned classrooms in which students would use manipulatives regularly to represent and solve problems, where word problems would be common tasks, and where much more of the class period would be spent with pupils working in pairs and small groups. And they assumed that all of these activities would be aimed at promoting the development of conceptual understanding, not just procedural skill.

Program Practices

Program staff were committed to creating and conducting a program that was congruent with their constructivist commitments. They do so for many reasons, not the least of which is that in classrooms of constructivist orientation, the processes involved in the construction of understanding are inseparable from the products—the understandings—of that construction. Students leave such experiences understanding new concepts, and how these concepts are generated and validated. One of the program directors remarked that, in designing the program, they try to keep themselves "honest":

> The thing that is very interesting and sometimes hard for people to think about is the fact that . . . the content is also the process—because why teach it as content if it doesn't work, and if it works, why not *use* it?

Participating teachers commented on this aspect of the program frequently, for, in their experience, this was unusual. One teacher explained that she had been to many inservice programs where "people tell you lectures aren't any good—and then they sit there and lecture to you!" The participants in this program, in contrast, were treated as learners who needed time and space to be engaged in mathematical and pedagogical tasks.

The program had two components, beginning with a two-week residential summer institute, followed by a year of intensive individual classroom follow-up with teachers in their own classrooms. The summer experience was intended to challenge and alter the teachers' assumptions about the learning and understanding of mathematics and, consequently, about the teacher's role in fostering learning. When working with experienced and successful teachers, these are not small things to tamper with. One of the program directors explained that "there is a strong psychological component" involved in asking experienced and successful teachers to

> take a hard look at what they do in their classrooms and be willing to see that perhaps what they have been doing for 42 years, or 30 years, or 10 years, is not as effective as it might have been. There are major changes that could be made to make it much more effective. That's a tremendously anxiety- and dissonance-producing situation.

The program staff paid thoughtful attention to the program's setting, creating an environment for the summer institute that could support the kinds of anxieties and tensions that the teachers might experience as they were confronted with new challenges and risks. Held in midsummer, the setting of the program was a beautiful old college dormitory where teachers slept, ate, and worked. Each day began with breakfast at 7:00. Mornings were focused on mathematical problem solving; afternoons included an hour and a half of physical education (dance and tennis), followed by a two-hour class in Logo, computer language. Following dinner, teachers wrote in their journals, pursued unsolved problems from the morning's activities, or worked on computer programs. Throughout their stay, the teachers developed a strong sense of camaraderie and group spirit.

Program activities involved teachers as learners in doing mathematics and solving problems, alone and in small groups. For example, over the course of two mornings in small groups, the teachers constructed numeration systems. They wrestled with ways to represent different quantities and how operations with those quantities might be symbolized and performed. In the computer lab, in the afternoons, teachers worked in pairs on problems using the programming language of Logo.

Staff members tried to work with teachers using the same approaches they wanted the teachers to use with children. They asked questions that probed the teachers' ideas as they worked; never did they show how to solve a problem or confirm an answer. Neither did they praise teachers for their successes in solving problems.

At times, staff members would ask teachers to step out of their role as learners to examine the pedagogy of the mathematics lessons. For example, on the day after the numeration activity was completed, one of the program's directors opened the discussion by saying,

> You've had the experience over the past three days of being math students, and I daresay that what you experienced is at least somewhat different than what you experienced growing up. I would like for us as a group to pull together some ideas from that experience, what it has been like, some reflections on what it means to us, what sense we are making of it. I'd like to start out by having you pull out some of the characteristics you have

> noticed about the teaching. And when I say "the teaching," I'd like you to include the design of the lesson, the small-group activities, and the large-group activities. What's *not* part of the math lesson is when we ask you to step *out* of the role of being a student and talk about the teaching and learning.

In this particular session, the teachers had noticed a number of things. For example, they commented that they had been "guided, not led" to figure out solutions, that they had used manipulatives, that they had had opportunities to listen to others which helped them to expand their own ideas. Others disagreed with the "no praise" approach. They argued that everyone needs "a pat on the back" and that they wished that the staff would sometimes show approval for their ideas. One of the staff members pointed out that in most classrooms, students come to depend on the teacher or the answer key. Although it may be "more anxiety-producing for people who are used to looking to the outside for approval," one staff member observed, developing independent learners meant that the teacher needed to step back.

After the summer institute, a year of intensive in-class follow-up was provided. Each SummerMath staff member was matched with between 2-10 teachers. Staff members visited teachers' classrooms weekly, observing lessons, sometimes co-teaching, sometimes conducting demonstration lessons. After each lesson, the staff member and the teacher would discuss what had happened, how it went, and ideas for changes or modifications. Because this aspect of the program depended on the one-on-one interactions of SummerMath program staff member and a teacher, each experience was unique in some ways. In our observations of the program participants, we observed four members of the program staff negotiate meaningful follow-up with their respective teachers. Two such relationships are described briefly in the following sections.

In the sections that follow, we present two teachers who participated in the SummerMath program from 1987-89. Ms. Beatrice Leland, who taught third grade in a white middle-class suburban community had been teaching for over 20 years when she enrolled in the SummerMath program. Ms. Leland, who described herself as most comfortable with a "structured" classroom, was attracted to the SummerMath program to satisfy a personal goal of developing a sense of competence and confidence with mathematics. A lively woman in her mid-30s, Ms. Belinda Rosen had over 10 years of experience teaching in a neighboring college town. For the past several years, Ms. Rosen had been teaching fourth grade. Her school regrouped children by "ability" across classes for mathematics and reading instruction, and Ms. Rosen was responsible for the weakest students in the fourth grade. Ms. Rosen decided to participate in the SummerMath program because she thought she could learn some things to help her get better at discipline (her least favorite aspect of teaching) as well as "some more creative things to do in class." The stories we present provide a glimpse into the teachers' experiences of the program—how it interacted with other features of their personal and professional lives. They offer insight into how the visions and practices of the SummerMath program staff played out over time in these two teachers' thinking and practice.

Ms. Beatrice Leland

Ms. Leland Before She Entered the Program

Ms. Leland, self-possessed and friendly, claimed to be relatively weak in math ("There is a math learning disability in my family"). Prior to attending the SummerMath summer institute, Ms. Leland was looking forward to learning new things—albeit hesitantly:

> I really do not know what they are going to do in the program. I do not know if my math is going to be strong enough for it or not. Or how comfortable I will be. . . . I do not have a strong math [background], and I told them that. But I think I am at the point where I want to learn and I am ready to learn and that is why I am going.

Ms. Leland went on to explain that she hoped to acquire some new

> ideas that will become part of my repertoire. . . . I will do things which are successful. That is really what I'm looking for. I am not looking for anything extraordinary, [just] something that is going to become part of my repertoire to make things understandable.

Ms. Leland was in search of these new ideas largely because of her concern for helping students "understand" mathematics. She described repeatedly the gap between successfully mastering algorithms and understanding why something works, noting that as a student she had only learned the procedures and not the underlying reasons. She frequently disparaged her own mathematical knowledge ("All this stuff I just learned by rote") and was committed to teaching her own students differently:

> All these things I learned rotely and never had any visual picture or understanding for why. And this is why I am trying to put more

> understanding into the children, so that they can hang it on something. Not because the teacher said it.

Ms. Leland talked about mathematics rather matter of factly, even though she did suggest she had experienced some math anxiety as a child. Mathematics to Ms. Leland was a set of rules. "certain rules that have been developed. And we try to develop the rules and learn them." A critical piece of "understanding" the mathematics, however, involved being able to visualize it. She mentioned several times that understanding a math problem meant knowing what it looked like. By observing, handling, manipulating, and experimenting with visual representations of mathematical rules and ideas, students would be able to visualize and "understand" the mathematics they were learning. Students learning to divide might split up a pitcher of Kool-Aid; students learning about triangles might find some in the real world, "so they could feel them, hold them, really get the sense of what a triangle is." She was also concerned that students learn how to solve word problems, noting that she had never had much success with solving problems concerning passing trains.

Full of good intentions, Ms. Leland was also quick to note that there were limitations to what she would be able to do in her classroom. She felt that there were real differences in what certain students could learn about mathematics, for only the very able seemed well-equipped to deal with the subject at times. Time was also a big issue, as well as district curriculum guidelines:

> I have to meet my obligations in the school system. And I cannot throw the baby out with the bathwater by experimenting. I have to do things within my curriculum and if I can make it more interesting to my children, if they can enjoy math and understand math—Great! But I haven't fouled them up . . . so far things seem to be working and I'm not going to ruin that.

When we first visited Ms. Leland's classroom, she had just started teaching long division to the top group of third graders, 27 in all. It was a muggy spring morning and the children and the room were damp and warm. The classroom walls were covered with charts and pictures and, in one corner of the room, a display of flags stood proudly. The students' desks were arranged in clusters of five. The back wall displayed large charts that recorded the children's mastery of basic facts and their timed test performance for addition, subtraction, multiplication, and division. There was also a pegboard with small commercially packaged bags of bright new manipulatives: base ten materials, geoboards, decimal squares, tape measures, and spinners. These looked unused.

Ms. Leland's math lesson lasted one hour, 20 minutes of which was devoted to the long division work. The remaining time (40 minutes) was spent on basic facts drill in the form of games and timed written worksheets. The following excerpt shows the way in which Ms. Leland worked with the class during the whole class segment on long division.

She opened the division work by asking students to get scrap paper out to do the division review, and she wrote the following division problem on the board:

$$4\overline{)72}$$

Ms. Leland: 72 divided by 4. I want to know how many groups of 4 are there in 72. Where am I going to start, Christy?

Christy: 4 into 7.

Ms. Leland: 4 into 7. Start at the left. Division is the *only* time that we start at the left-hand side. How many 4s are in 7? What do you think, Joe?

Joe: 1.

Ms. Leland: What's next?

Another student suggested: Write it up.

Ms. Leland: (writing 1 above the bracket) Am I all done? (The students shake their heads.) Why not?

Student: You have to subtract.

Ms. Leland: Why do I have to subtract?

Student: You have to compare.

Ms. Leland: And when we compare, we're finding the difference. Chrissie?

Chrissie: 3.

Ms. Leland: All done?

Student: Write it down.

Ms. Leland: Okay, what am I going to do now?

Student: Bring down the 2.

Ms. Leland: Ahh. We have a new number to divide. How many 4s are in 32?

Student: 8.

Ms. Leland: How do you know?

Student: 'Cause 8 x 4 is 32

Ms. Leland: Am I all done, Dave?

Dave: You have to compare.

Ms. Leland:	You have to compare by *subtracting*. (She subtracts.) Is there a remainder? Jennifer?
Jennifer:	No.
Ms. Leland:	No. But you may write the remainder up here like this if you wish. (She writes R 0.) Okay, it looks like you remember quite a bit. How about if we try another one?

At this, Ms. Leland seemed satisfied that the students had retained most of what she had shown them on the previous days. She moved briskly on to another example ($98 \div 5$) and, as before, her students supplied the steps as she asked for them.

This mode of asking questions to focus the students on the steps pervaded the entire lesson. Despite the heat, the class was well-behaved and attentive. Ms. Leland would have received high ratings for her capacity to keep 27 perspiring children focused on the steps of the long division algorithm. Yet, little attention was given during this lesson to the mathematical meanings of what the students were doing. Ms. Leland referred to the problem she had given as "How many 4s there are in 72?" She reminded them that "when you compare, you are finding the difference." And later, when Ms. Leland asked the students to divide 98 by 5, she paused before beginning to lead them through the steps again. "Do you think there is going to be a remainder on this one?" she asked. When a student answered yes, she asked how he knew. "Because the 5s have either a 5 or a 0," he replied. "Right!" asserted Ms. Leland. She did not ask the student to explain what he meant. His answer was simply right.

Ms. Leland said she was pleased with this lesson, for the students were remembering more than she had expected. She felt long division was hard because "there are so many steps to remember, and [you have to know] where to put the answer." When asked how she had initially introduced it, she laughed, "I told them some genius figured out a system that *works*—if you do everything they tell you to do. If you leave something out, it doesn't work!" For Ms. Leland, the goal of mathematics teaching seemed clear: Develop each child's computational skill.

Ms. Leland After the Summer Institute

In the fall following her participation in SummerMath, Ms. Leland seemed almost angry—although she claimed that she had "enjoyed the experience." She accused the program of being "a cult," of using a lot of "brainwashing activities," of "zeroing in on one approach . . . being too narrow." Part of her anger was fueled by her own experiences as a learner in the workshops: She had found the staff's tendency to answer all of her genuine questions with additional questions frustrating. As she put it, "Now that's okay occasionally, but 8 days of that is ridiculous!" Concerned that she need not "reinvent the wheel," Ms. Leland found it frustrating and inefficient that the staff asked her to reinvent everything. Instead, she wanted to learn from the experiences of the staff and take their knowledge into her classroom, selecting things that worked, winnowing things that seemed less helpful.

Her concerns for the wastefulness of "reinvention" transferred into her teaching concerns as well. When asked how she might describe the program to another teacher, she explained:

> It's basically reinventing arithmetic through experiences. And the teacher is a facilitator. And I don't have a problem with that. But to have them reinvent everything—I just don't have the time for that. I'm under time constraints and I'm also under constraints of the curriculum I'm supposed to teach.

But it may be that Ms. Leland *did* have some objections to SummerMath's assumptions about the role and place of teachers in mathematics teaching and learning. At one point, she had confided in us that the best inservice program she had ever experienced was sponsored by Disney World and went so far as to bring in the materials that she had acquired on that project. For a paper that she wrote as part of that project, Ms. Leland claimed that her motto was "Know your stuff, know whom you're stuffing, and stuff 'em," noting in her paper that she "intend[ed] to make the front of my classroom center stage and the teacher as the focal point." If Ms. Leland's assumptions about learning and teaching include the assumption that teachers need to be center stage and the belief that learning is related to "stuffing," it may be that part of her discomfort with SummerMath involved a clash between the program's very different assumptions about teachers' roles and the nature of learning.

Ms. Leland was also disappointed that the staff did not provide the teachers with more options, more variety: "I thought they'd give us a variety of manipulatives, a variety of approaches. I didn't realize that we were going to be zeroed in on one approach." Moreover, she believed that the program claimed that children do not need praise, and she disagreed vehemently with this stance: "Most children at this age—contrary to what the director and friends espouse—*do* like the assurance that they are right." Sarcastically, she noted, "I don't think Piaget ever said to ignore pats on the back!"

Despite these disappointments and disagreements, Ms. Leland did feel that she learned some things through the summer program. She thought she'd work on the idea of "wait time," use more manipulatives, experiment with having students work in pairs ("But those are the largest groups I'll try"). When she discovered that one of the program directors was to be her staff person for the year, she expressed some concern about working with him, although she was looking forward to seeing what types of things he might do with her students and what she might be able to observe and learn from him.

When we observed Ms. Leland's classroom the second time, her one-hour class began with a 15-minute round of a fast-paced competitive basic facts drill game called "Gotcha!" Then Ms. Leland told the class that they would be reviewing missing addends in their workbooks and that they would work in pairs to use cubes to solve the problems. Efficiently, she distributed zip-lock bags containing the Unifix cubes[5] to each pair of students. Ms. Leland asked someone to read number 13 on page 19 out loud, without giving the answer. She called on a boy, who read "5 plus blank equals 13." Ms. Leland repeated what he said. She wrote 5 + ___ = 13 on the board and drew a big square in the spot of the missing addend.

"Could you take your cubes and, working together, find out what the answer is? When you find the answer, put a square on your paper and write the answer on the paper. *Don't* attach them together—just put them in groups—we don't have time to attach them." Ms. Leland walked around, asking, "Did you put your answer in the square?" She continued to circulate, telling some more partners to be "kind enough" to put their answers in the square so that when she looked down at their paper, she would be able to see it fast. She asked students, "How are we doing? Did you put your answer in the square?"

Ms. Leland wrote ____ - 18 = 14 on the board, saying that this one is "a little tougher." She said she did not want anyone to raise their hands right away. "At 25 past we'll try to share, okay?"

After a couple of minutes, two girls approached her and announced that it was 32. Ms. Leland responded by asking them only if they had "proved" it. After a few moments, they came back to say they *had* proved it. Ms. Leland smiled, nodding noncommittally. As she walked around, she saw an answer that was wrong. "Why don't you try that one again?" she suggested. Walking around, Ms. Leland reminded the children that this was a "tough problem." When a child proposed 31, Ms. Leland replied, "You're very close—why don't you

[5] Unifix cubes are plastic interlocking blocks that can be used as concrete models for addition, subtraction, multiplication, division, fractions, measurement, and other ideas in elementary mathematics teaching.

check it?" Jennifer said she thought it was 4. "A lot of people think it is 4, but it would be hard to take 18 dollars away from 4, wouldn't it?" replied Ms. Leland.

She continued suggesting to people that they check their answers with the blocks. To one group she said: "You're in the ballpark, but you're still a long ways from a home run." To another pair, who proposed 42, Ms. Leland responded: "Well, you don't *have* 42 blocks, so I wouldn't give you a problem that was *that* high." One group announced proudly that they had an answer and they did it with blocks. They asked whether it was right. "Well, you know it's right if you did it with blocks," nodded Ms. Leland. After about five minutes, Ms. Leland called on the pair of girls who had "proved" 32 with their blocks and asked them to demonstrate how they did it. After this, the remaining 15 minutes of class was spent on a basic facts sheet—72 problems.

After the lesson, Ms. Leland said she thought she would be doing more group work, reporting that the SummerMath staff were "very much for group work." She thought working in groups was worth experimenting with, but she was not going to spend time arranging the groups—the kids could just work with whomever was close by. "Moving around takes too much time—two minutes here and two minutes there—it adds up to your whole penmanship lesson. She was also glad to be using the Unifix cubes because this year's group was very "into concrete-types of things."

Ms. Leland Later That Same Year

Visiting Ms. Leland's classroom several months later, we saw the same patterns. Again, she was working with Unifix cubes. Children had been working in pairs, using the cubes to model student-written word problems. For example:

Purr-fect kitty litter costs 69¢. I give the clerk 75¢. What is my change?
What is the cost of two bags of kitty litter?

Ms. Leland had the class review the answers to the problems. One girl, up at the front of the room, demonstrated how she used 6 white Unifix cubes and 9 blue ones to represent 69¢. She said she "regrouped" 75 to be 6 tens and 15 ones and then she subtracted till she got to 69. Ms. Leland repeated the girl's explanation. The class period continued in this mode for about 20 minutes, with some students seeming to listen and a few others appearing to be more involved in constructing things with the cubes. The class was orderly; problems were demonstrated and confirmed apace. Next Ms. Leland spent 20 minutes working on telling time and writing time using analog clocks. The third segment of the class consisted of independent seatwork: textbook word problems.

Afterwards, Ms. Leland said she was still having some "management" problems, for it is not easy for students to listen to one another. But she really liked using the manipulatives because third graders are so concrete. She said that she still overplans and that it is hard to fit in everything in the hour she allots to math.

Ms. Leland Two Years Later

Echoes of Ms. Leland's concerns about mathematics teaching could still be heard two years later when we interviewed her for the last time. She was still concerned with time and still concerned with staying within the boundaries of her district curriculum. Her attitude toward math remained matter of fact: Mathematics was a tool, something to help students to master so that they could balance a checkbook, measure a wall. She appreciated the utility of mathematics, she said, as a worthwhile tool:

> I find math as a tool to achieve something. To me, math is a tool. I am not fascinated by mathematical relationships like people in astronomy or something like that. But what you can learn through math and what you can achieve and where you can get to by using math is important. You've got to make the tool fit the purpose.

Although much seemed the same, the hostility she felt when reeling and fresh from the SummerMath summer program had evaporated. Having spent a year working with the program director in her classroom, she no longer spoke with cynicism and sarcasm, only with respect and admiration. She and the program director had talked about the summer institute, and he had admitted being anxious and tired near the end of the program. They had also talked of how he might avoid re-creating the emotional reactions that some of the teachers had experienced as learners. Noting that he was a "fine role model," she explained:

> When it actually came to practice, he could compromise. He was worldly enough but up there he came across as living in an ivory tower and he is not an ivory tower person in practice. He realizes that you can't reach the ideal, but for the [summer institute], it was the ideal only.

Through her collaboration with the program director, Ms. Leland noticed a gradual change in her thinking and teaching, an "evolution" that was not "revolutionary" but noteworthy nonetheless. Consider her recollection of some time spent in a kindergarten:

> They drafted me to come down to [the kindergarten] and they said, "We're studying 'A' and 'alligator is for A.'" And that was all. I found myself, instead of standing there and saying, "This is an alligator. It is . . ." I was suddenly off in the children. "What is an alligator? What does an alligator do? Where does one live?" Until they gave all the information of what an alligator was. And we ended up crawling on the floor like alligators, which is very different from what I probably would have done last year. [Then] I probably would have gotten a filmstrip and showed a picture. "This is an alligator." I would have told them, instead of the children telling me.

In addition to this self-reported change, there were other notable changes in Ms. Leland's talk about math teaching. She talked more often of using cubes and blocks, "tools to represent" and to help students "visualize." Although she still believed that for some subject matters, "like mathematics and advanced reading," there were differences in ability and aptitude that required remediation or acceleration, Ms. Leland now placed less emphasis on ability as a prerequisite for success in math: "You have a lot of people with ability who don't do bo-diddlely. And I don't think we give enough to the average youngster who might be working very hard." When asked how she decides whether or not students understand the material she is covering, she noted that she relied more heavily now on children's discussions and that she was paying less attention to what appears on their papers. This is a big change from the Ms. Leland we saw two years earlier who spent three-fourths of her math class on basic facts drill and who placed great emphasis on getting the right answer. Ms. Leland also noted one final change: She reported that one of her big changes was that she thought about math more often, "I notice math situations more often than I did before."

Next we move to Ms. Belinda Rosen, a teacher in a neighboring school district, whose students were much weaker academically than were Ms. Leland's. Like Ms. Leland, Ms. Rosen went to SummerMath as a fairly traditional math teacher. Like Ms. Leland, she was looking for a few good ideas or tips. Two years later, however, she too had made substantial—but, we think—somewhat different changes.

Ms. Belinda Rosen

Ms. Rosen Before She Entered the Program

Lively and full of energy, Ms. Rosen was constantly on the go—a self-proclaimed "people person." Before she entered the SummerMath program, Ms. Rosen reported that she had once been a "let's do manipulatives, let's do games" kind of teacher. But when she found that students couldn't add, she swung back to the basics, emphasizing the need to acquire skills that students would need in the classroom. But the emphasis she placed on computation made her uneasy, for she recognized that mathematics "is not just computation." She was torn between helping students master computational skills and providing them opportunities to problem solve, to do "things with time, money, equivalences, graphs, Cuisenaire rods." She felt pulled to do "a variety of things." At the same time, she acknowledged that her pupils would "have to be able to subtract if they are going to have a checkbook" or buy wallpaper. In addition, they would have to know how to add, subtract, multiply, and divide for fifth grade.

Like Ms. Leland, Ms. Rosen recognized that many students of math—including herself—learned to *do* things in mathematics but don't know what those procedures and processes mean. Reflecting on her own knowledge of fractions, she commented, "I know how to *do* it, but I really do not know . . . what it means to me." Asked why she thought this was, she noted, "Because I never got taught in a way that I was supposed to understand it, just that I was supposed to *do* it." Yet in her talk about teaching during our first interviews, Ms. Rosen focused almost entirely on teaching procedures. When asked, for example, how she would teach students about place value, she remarked that she would emphasize a "sequence" of "steps" to go through, starting with easy problems and then gradually introducing more complex ones.

Ms. Rosen thought that some students are perhaps "math disabled." They might have "great reasoning ability," but they were unable to remember what they had been taught from one day to the next: Every day is "a *brand* new day." While she cared about finding ways to help these students and believes she can "get to them *eventually*," she thought some approaches would not work with her class because they would not be able to handle them. She referred to "discipline" as her least favorite part of teaching, and she thought that these weaker students tended to be more distractable and have more behavior problems.

Ms. Rosen's goals for teaching math seemed to be shaped by her ideas about her pupils. Because they were weak, she believed she should emphasize following directions and understanding math vocabulary: "To really get clear when you say *product*, what does that mean, what does that word key? That it should be multiplication." She said she was trying to "inculcate" them with some of the essential material, so that "when and if something clicks," they'll have had exposure to it before. Ms. Rosen also wanted to help students develop more confidence in their abilities as learners, to take risks, to enjoy math class. She believed variety in instructional methods was important, for it kept students engaged and motivated.

Ms. Rosen decided to enroll in SummerMath because she wanted to "be more stimulated" in her mathematics teaching, and the program had received applause from several of her peers. She was looking forward to learning more about Logo, for she felt it had a lot to offer her students. When asked whether there were particular things she was interested in learning more about, she noted that she was weak on discipline and pacing, and would someday like to feel more in control of those features of her teaching.

Our first visit to her fourth-grade class (prior to her participation in the SummerMath summer institute) showed her teaching in much the same way as Ms. Leland. As a matter of fact, she too was teaching long division. Like Ms. Leland, she too led her students through structured questions:

Ms. Rosen: Do you remember what the first thing is that we're going to do?
Student: Divide.
Ms. Rosen: Divide.
Ms. Rosen: (writes "1. Divide" on the paper.) Do you remember what's next?
Student: Multiply.
Ms. Rosen: (writes "2. Multiply") After we multiply, what are we going to do?
Student: Check?
Ms. Rosen: Check's going to come at the fourth step. What are you going to do next?
Student: Bring down?
Ms. Rosen: Bring down will be our last step. Take a look over here (points at number under dividend). What do you need to do here? Is it subtract? Then we're going to . . .
Student: Subtract.
Ms. Rosen: And then?
Student: Bring down.

Ms. Rosen: And what do we do after we bring down? What happens whenever we divide another number, we're always going to do what? (pause) Put a number up here on top.

In the same lesson, Ms. Rosen reviewed estimating a product with a small group of students. The patterns of discourse were very similar:

Ms. Rosen: Tori, Josh, take a look at the next one.

$$\begin{array}{r} 348 \\ \underline{\times 5} \end{array}$$

Ms. Rosen: 48. Is 48 *less than* or *more than* 50?
All: Less.
Ms. Rosen: 48 is less than 50, so we're going to round *up* . . . or *down*?
Student: Down.
Student: Down.

This was, of course, dependent on the precision desired—something which had not been discussed on this day, although it may have been on an earlier day.

Ms. Rosen: (pursues it in her voice) *Down*? Josh, up or down?
Josh: 48 is close to 50, so we'd round to 50.
Ms. Rosen: Well if we were rounding to the nearest 10, that's true. If we're rounding to nearest 100, then we say 48 is less than 50, so we round down to 300. Eva, what's the next step? After we've rounded 348 to 300? (She is smiling and actively nodding at the students as she talks.)
Student: Multiply by 5.
Ms. Rosen: Multiply by 5, and what do you get?
Student: 1500.

Ms. Rosen wrote 1500 on the board. One of the students reminded her to put the comma in: 1,500.

Ms. Rosen, like Ms. Leland, managed her students effectively. Although she expressed a dislike for what she called "discipline," her class ran smoothly. She directed and led her students through the steps for division, for estimating. And, throughout the lesson, never did she genuinely solicit students' ideas or approaches. Instead, Ms. Rosen's questions were designed to get students to produce the right answers. Ms. Rosen was proactive and aggressive in helping students produce correct responses—and she did so enthusiastically, with élan. She smiled often, praised students for their efforts, cheered their work.

In talking with Ms. Rosen about this class, she said that she was pleased that students were remembering the steps. She said that the problem is that sometimes she ends up spending a lot of time on "the stuff they don't get" and she wonders how good that really is. For instance, she said, in trying to get them to know their multiplication facts, other things get left out. "These kids never get some things—like geometry—because they're always getting computational skills." She wonders if maybe she spends too long on some things, but she also notes that often they make so many errors, "it just would have been such a mistake to go on."

There were hints during this first visit that Ms. Rosen was open to experimentation. Although she described her "thrust" as trying to "keep [the children] on track" and trying to "inculcate" them with some of the essential material, Ms. Rosen also explored alternative curriculum and pedagogy. Every Friday for a month, she had been doing "problem solving" with her students, for example, and using Cuisenaire rods to help students work through the problems. She did this, she explained, because she had heard from someone who went to a "right-left brain workshop"—the importance of kids having their right brain stimulated, especially before 10 years of age, if they are going to do higher level thinking later on. She observed that some of her students do well with the Cuisenaire rods and with problem solving. For a while she *didn't* follow the textbook and made up more of her own stuff, but she wondered whether perhaps she was just "reinventing the wheel" and thought that maybe she wasn't thorough enough in her treatment of the topics and skills.

With the Cuisenaire rods, she said she has them making stairs, boxcars (equivalencies, Cuisenaire art). "I've never really been trained in that, though, so I'm not sure I'm doing it right."

Ms. Rosen After the Summer Institute

Ms. Rosen's experiences with the summer institute were strikingly different from Ms. Leland's. Calling it a "powerful experience," she claimed that "it would be hard for me to just go back and teach and *not* use any of the other skills I learned there." She loved the work she did on Logo, the problem-solving tasks she was given, the emphasis on the use of manipulatives. And even though she had used manipulatives in the past, she explained that her view of them had changed dramatically:

> I have to admit . . . I thought that manipulatives were for kids who don't get it. . . . [I changed my mind] when I saw that adults could really use them beneficially and come to a deeper understanding of problems. It made me stop and think. Most of the time I was able to stop and intuit--I had my own way--but I worked in groups where people were more concrete and they really used them.

Ms. Rosen liked especially being a learner of mathematics and found the work she did with her partner on Logo particularly satisfying. She did note that some other teachers seemed to have an emotional reaction to the content and the group work. Bemused, she thought that perhaps it was because "teachers are so unused to being learners." She too had felt a jolt from being put in the position of learner but found it exciting: "I had forgotten how much I liked that!"

Ms. Rosen drew heavily on her experience in SummerMath when she started teaching in the fall, using manipulatives and math journals, teaching her students Logo commands, reading the *Constructivist*, a publication for educators interested in constructivist thought in teaching and learning. Although she was leery of her students' abilities to construct algorithms, she was open to exploring the possibility.

Visiting Ms. Rosen's classroom after her participation in the Summer Institute, we saw some old practices, some new. The class period opened with the same timed tests, with one minute for students to do as many addition facts as they can, correctly, and Ms. Rosen was lively and energetic as we had seen the previous spring, praising students as she checked their papers, suggesting goals for tomorrow. When she saw an answer that was wrong, she pointed it out gently. In a low voice, she queried a small boy named Eric, "2 times 6 is 12, but 2 *plus* 6 is . . . ?" "8!" replied Eric. Ms. Rosen finished looking over his paper and told him, enthusiastically, "You did better than yesterday. *Great*, Eric!" Moments later, another boy cheered, pleased that he got only two wrong. Ms. Rosen looked at him. She said she was sure that he felt good about himself but she didn't want him to yell. Ms. Rosen continued on her rounds for a minute or two more. "Wow!! What an improvement! You got almost two times as many right as you did yesterday! Great!"

The timed tests were followed by half an hour of what she called "problem solving": representing and solving word problems using concrete materials--in this case, plastic chips. Before she launched the students on their work, she reviewed the "rules for problem solving": that they could "use counting," that they should work with and talk to other kids, and that they should record not only their answers but also *how* they solved each problem. The following excerpt provides a picture of Ms. Rosen's role in this work. In one small group, a boy named Jeff was reading the next problem aloud: "Mom baked some cookies. There were 16 cookies. She wanted to share them equally among 8 children. How many did the 8 children get?"

Immediately, someone in his group announced the answer: one cookie. Ms. Rosen tried to help. "How many cookies did Mom make? Should we read it again out loud?" Jeff reread the problem in a flat, monotonic voice. When he finished reading it, he agreed: "One cookie." Ms. Rosen suggested that the students get out chips and see. "What do you think we might use these chips for? What do you think they might stand for?" she asked. "Plus," said one boy. Ms. Rosen repeated the questions and someone else offered, "Cookies?" At this, Jeff looked at Matt, "If these are cookies, I'm going to eat 'em!" Ms. Rosen worked hard to guide them carefully in using the chips, asking "probing, not leading" questions, the kinds of questions advocated in SummerMath.

Ms. Rosen: How many cookies do you have to have?
Eric: 8 . . . I think.
Ms. Rosen: Are there 8 cookies or 8 children?
Eric: 8 children, so you have to have 8 more.
Ms. Rosen: 8 more *children*—or *cookies*? Are you adding cookies and children? I'm kind of confused about that.

Eric suggested using "take away" to solve the problem. Although Ms. Rosen had been thinking of this "sharing" problem as a division problem, she told him to "try it and see if it works." A few minutes later, he announced, excitedly, "There's 8 cookies left and 8 kids, so each kid can have *two*!"

Throughout the class period, Ms. Rosen worked at not showing the students what to do. She still hovered closely, guiding and prodding. But she did not answer their questions, nor did she validate their answers. Instead, she asked "probing questions" such as: "I'm not sure I understand what you did." "Jeff, what is it that *you're* doing?" "How many cookies did Mom start out with, Matt?" Helping Jeff, who was stuck, Ms. Rosen suggested that he pretend he had plates in front of him. Moving the counters around, she demonstrated: "Here's a plate, here's a plate, here's a plate. There's 8 plates. So how many cookies are your kids getting so far? (pause) How many are on each plate?"

Jeff: 1.

Ms. Rosen: They each get 1, but look at all the ones that are left over. Can we put any more on each of the plates?

Jeff: Yeah (and he proceeds to count out more chips).

Ms. Rosen was waging a tough battle with her students, for being asked to make sense in school was likely a novel experience for them. Continuing to struggle with representing the cookie-sharing problem, Matt had over 20 counters spread out on his desk. Ms. Rosen, noticing this, commented smoothly, "Oh that's interesting. How many of these things are out here?"

Matt: 15.

Ms. Rosen: 15?

Matt: Well, maybe more than 15 . . . 20.

Ms. Rosen: Hmm. Did you get that from the problem there? (Looking somewhat puzzled, she probes) Could Mom have had 20 cookies if she only baked 16? What do you think?

Matt: *I* don't know.

She persisted: If you baked 16 cookies, could you all of a sudden have 20?

There was silence. Ms. Rosen repeated her question.

Matt: Probably.

Another child offered to show Ms. Rosen his solution and Ms. Rosen invited Matt to watch. At the end, Matt announced, "Two for each child." Ms. Rosen seemed relieved to have him coming into line: "I *love* that answer, Matt." She walked away from the group, reminding them to record "2 cookies" and also to write down how they solved the problem. This approach to teaching was hard work.

Reflecting on the lesson afterwards, Ms. Rosen was pleased. "Originally I would have been reluctant to do [something like this], but this summer we saw that because the kids had counters they were able to do the problems." Focal for Ms. Rosen was trying to ask less leading questions—a skill she had honed and had been quite proud of prior to SummerMath. She said that she thought she was still leading the kids too much, like when she asked, "Now how many did you say the mother had left over?" or "If the mother bakes

16 cookies, could she end up with 24 cookies?" She said that these questions were too leading, but that this was *very* hard to avoid. She said she probably did it "tons of times." But she was also proud of herself: "At other times I was pleased that I really did try to say, 'Gee, I don't quite understand that—can you explain that to me?' . . . I am trying to get *them* to feel the discord if things aren't working."

Ms. Rosen said she was also trying to place "more emphasis on approaches—getting the kids to explain how they solved the problem—like one kid did it by subtraction and others had other strategies and they all got the answer." This, she said, was something she would *never* have done before:

> Definitely not! I would have said, "This is a subtraction problem." Or maybe just, "What do you think we're going to do here—add, subtract, multiply, or divide?" And they would have come up with "subtract" and I would have said, "Okay now go and solve it."

Ms. Rosen Later That Same Year

Visiting Ms. Rosen's classroom a few months later, we found her all the more convinced that the changes she was working on were worthwhile. One of Ms. Rosen's colleagues, another fourth-grade teacher who had not attended SummerMath for Teachers, was concerned that the students whom Ms. Rosen taught would not be ready for fifth grade. She had raised this concern with the building principal.[6] This incensed Ms. Rosen: "I have *total* confidence that my math program is of great benefit to *all* of Cindy's students." In a strong letter to the principal, Ms. Rosen was articulate about what she was trying to do:

> By checking the students for what they know and understand, I then present them with problems that help investigate those areas of weakness. *The children build the theories, not the teacher.* (italics added) I connect the concrete activity with the algorithm. I encourage students to think about their process, other kids' processes, and how those processes may be related. Kids have ownership of math as they construct concepts. I guide discovery.

The work in which Ms. Rosen was involved was challenging. On this visit to her classroom, we found several students still not responding easily to what she was doing. She was continuing to work on asking probing, not leading, questions. She was also trying to use

[6]Recall that students in this school were regrouped across classes. This teacher was concerned about students of hers who went to Ms. Rosen for math.

the strategy of asking students to "rephrase" what other students had said. Quite visible, too, was her effort to give her students experience with concrete material and to connect this "with the algorithm." In this lesson, which involved representing story problems with Unifix cubes, one boy spent most of his time very obviously constructing a long "snake" out of Unifix cubes and showing it, while giggling, to other children. Ms. Rosen was calm about this. As soon as she saw him using the cubes to solve the problem, she commented, "Matt, are you with us? I like the way you're building those towers of fours."

Most of this class period was spent working on the following problem, intended by Ms. Rosen to contribute to the students' understanding of multiplication: A man had a tire store and he knew that when he changed the tires on a car, he needed to change four tires, but he wanted a *fast* way of figuring this out. Ms. Rosen steered the children to determine how many tires the man would need to change the tires on three, four, or five cars. With questions, she pressed them to notice that they were counting by fours to figure it out.

Eric: Because it would be an easier way to go by 4s.
Ms. Rosen: Why? Matt?
Matt: Uhhh, I don't know.
Eric: (stumbling) Because you already know what . . . each tire has one, has two, each car has *four* wheels.
Ms. Rosen: (feigns surprise) *Oh*!! Is that why we chose to count by 4s?
Eric: Yes.
Ms. Rosen: Matt, why did we choose to count by 4s? What did Eric say? Can you rephrase that?
Matt: (pause)

Ms. Rosen waits, looking at him.

Matt: I forgot.
Ms. Rosen: Why did I put them in lines of 4s?
Matt: Because they're cars.
Ms. Rosen: They're cars. And how many tires does each car have?
Matt: 4!

Ms. Rosen began helping the students to construct a pictorial chart representing number of tires by number of cars. By the end of the period, they had drawn:

Number of cars	2	3	4	5	6
Number of tires	8	12	16	20	24

After the students finished figuring out how many tires are needed for three cars, Ms. Rosen instructed them to go on to four cars. Students built towers of 4 Unifix cubes. A small disagreement broke out: One boy thought it was 18, and another said 16. Eric announced that he knew that three cars used 12 tires, and "so I just added 4 more." "What a nice idea!" exclaimed Ms. Rosen. "Did you hear that, Matt? What did he do?" Matt was building a very long snake by attaching the cubes end to end.

Ms. Rosen: Matt. *Matt.* I'm asking you to focus in on what Eric said. Eric, say it again and I want you to listen, Matt, and see if you can rephrase it.

Eric: Okay. I put 'em in groups of 4s and I knew I had 12 over here, so I just counted and I added on and I got 16 from there.

Ms. Rosen: Okay, so you didn't have to count them again. You knew you had 12 with three cars and you just counted from there. Matt, can you explain Eric's way?

Matt explained what Eric had said and earned some praise.

Ms. Rosen: *Great.* Eric, let's see what Matt's way was now.

At this point, Matt complained because a piece of his Unifix cube snake broke off.

Ms. Rosen: (in a businesslike tone) Okay, but we're not doing that right now. We're doing problem solving. Eric, do you know what Matt did to get the 16? Matt, can you tell him?

After listening to Matt, Ms. Rosen announced briskly: "Okay. Let's do a couple more and see if we can see a pattern and start writing some *number sentences.*" By now Matt had a *long* Unifix cube snake. He showed it to Eric, but Ms. Rosen proceeded on calmly. "How many tires would you need with *five* cars?"

Ms. Rosen was fairly pleased with this class, although she thought that her SummerMath consultant would have "been a little upset with me" because she was still "telling them a little too much—like when I was trying to get them to see the math facts." Ms. Rosen thought that her consultant would advocate waiting until the kids "come up with it by themselves." She was enthusiastic about the kind of help and guidance her consultant was providing. "She is really good as a model. She does what she preaches and she is *really* supportive. She has helped keep me on track when I felt like giving up."

Ms. Rosen said that she was teaching like this every day. "I am totally sold," she said. Even though some of the kids were "negative," Ms. Rosen said, "it is good for them in the long run. It's real, *real* important to get this kind of understanding, I hope that they *do* get the algorithm later, but first they need to *think* that they can solve problems."

Ms. Rosen Two Years Later

Ms. Rosen's enthusiasm for SummerMath and mathematics as a field of study did not seem to wane over the next two years. At the time of our final interview, she was thinking about learning more about calculus because "It was always like this *word*. I want to know more about it." She felt that the program had taught her a lot about teaching: that people learn better when they have something concrete to connect to the abstract, that kids can learn from each other, that there are multiple ways to solve problems. She now felt a commitment to "dignifying thinking," to allowing children to "construct" their understandings so that they have some "ownership" of the work. When asked how she might teach students to think about place value and subtraction (the topic we first observed her teach), she explained that, instead of emphasizing a sequence of steps, she would have students "working in groups, talking, drawing, using manipulatives, drawing pictures and then sharing their ideas."

Reflecting on changes she observed in her own teaching and thinking over the past two years, Ms. Rosen commented on the fact that she was learning to give "a lot of the learning over to the kids." In addition to SummerMath, she had also adopted a whole-language approach and had participated in a workshop about constructivism. Together, these experiences seemed to converge, changing the way she thought about students' learning and about her role as a teacher:

> It's just all been fitting together. It's hard to tell which came first. Whether I was ready for all of this to happen or whether I had all these influences happen and I took advantage of them. I don't really know.

Whatever the source, Ms. Rosen's talk about teaching was significantly different in the final interview from when we first met and consistent with the change in her assumptions about how kids best learn and what teachers need to do to help students learn. She spoke of using manipulatives—cubes, blocks, drawings, maps—with frequency, of having students pair off or work in groups to learn from each other, of asking questions that tapped students' thinking and pushed them to gain insights. She continued to note that the students she taught presented particular problems—many related to discipline, management, and self-esteem—and that it was not always easy to implement the SummerMath vision in a room full of students well-practiced in misbehavior. But Ms. Rosen was committed to the idea that classrooms should be places where children explore and construct personal knowledge, and that in so doing they would develop both conceptual understandings and self-confidence.

Discussion:
Ms. Leland's and Ms. Rosen's Patchworks

The contrasting stories of Ms. Leland and Ms. Rosen highlight several issues about how teaching changes. Both teachers' practices appear patchworks of old and new: Ms. Leland still emphasizes computational skill and mastery of procedures, but Unifix cubes litter students' desks and she is prone toward asking more questions about what and how they are thinking. Ms. Rosen still directs and guides and praises students often, but she feels more comfortable slowing down, using manipulatives to deepen understanding, letting students teach one another. Ms. Leland worries about reinventing the wheel and wasting time; Ms. Rosen embraces the idea of knowledge construction and searches for ways to facilitate the development of students' thinking. Watching them teach, listening to them talk about teaching, we can hear old beliefs and concerns echo in their reflections. But there are also whispers of change, although they seem louder in Ms. Rosen's case than in that of Ms. Leland. What factors account for these patchworks? What shapes these eclectic mixes of old and new?

One noteworthy difference between the two teachers relates to their experiences as learners. Ms. Rosen loved being a learner, rose to the challenge of the problem-solving tasks and was hungry to learn more mathematics. Ms. Leland seemed more interested in having the SummerMath staff tell her what they knew so that she could get on with the work of adapting their knowledge and experience to her practice; there was no need, after all, to reinvent the wheel. But Ms. Leland also enjoyed puzzling out problems. During our first interview with her, we presented her with a problem about division by fractions which she could not solve. She called her interviewer the next day and informed her that she had given the interviewer the wrong answer, and that she had spent the night before calling friends who could help her solve the problem. So if Ms. Leland also enjoys learning, why the stark difference in their experiences with the SummerMath summer institute?

Clearly, Ms. Leland did not respond well to the structure, content, and process of the summer institute. She was put off by the passionate commitment of the staff to their beliefs about teaching and learning, despite the fact that she claimed to believe that teachers should be facilitators. She disagreed with some central premises: that students don't need lots of positive reinforcement, that it is reasonable and possible to let children take the time to invent mathematical concepts and algorithms. When she left the Summer Institute, she was prepared to try out a few new strategies—some manipulatives, some wait time, some work in pairs—but she was not prepared to change fundamentally, "radically," the way that she taught. After all, things had been going pretty well, and she didn't want to disrupt her competent practice with too much experimentation.

Ms. Rosen, on the other hand, was inspired by the intensity of the workshop and found it profoundly moving. She enjoyed being asked questions, taking intellectual risks, puzzling out problematic situations. She recognized the parallels between how the staff treated the teachers and how the staff wanted the teachers to treat students. A constructivist approach to teaching and learning was appealing and comfortable to Ms. Rosen, for as a learner she had learned things in new and personally important ways. She left the Summer Institute believing in constructivism because she had experienced it. Ms. Leland had no such belief. Certainly it is nice to have students express their ideas, work with blocks for a while. But waiting for all the students to solve a problem on their own involved a great deal of time, time one couldn't afford to lose.

Ms. Rosen and Ms. Leland brought similar expectations to the SummerMath program. They thought the experience might offer them new ways to teach: Ms. Leland wanted to enrich her repertoire, Ms. Rosen wanted to increase the variety of methods so as to keep student interest high. It is not unusual for teachers to look toward inservices as sites for new ideas, new activities. Rather than hear some university professor spout eloquently about theories of teaching and learning, teachers feel the need for practical ideas—things that work in real classrooms, with real kids. SummerMath offers such practical ideas: Teachers learn about small group-work, about problem defining and solving, about Logo, about manipulatives. But the practices presented by SummerMath are embedded in a vision of mathematics teaching and learning. Ms. Rosen, who embraced the vision, experienced SummerMath as a rich source of practical ideas as well. Ms. Leland, who had reservations about the vision—and objected to what she perceived as a fervor in its presentation—saw fewer options available and was disappointed at the narrowness of the program's scope. One teacher left full of vision and new ideas for her teaching, the other left with a slightly smaller set of teaching ideas. Perhaps the patchworks, then, are influenced by the degree to which a teacher adopts the vision of teaching and learning that accompanies new teaching practices, for it would seem that Ms. Rosen's practices changed much more than Ms. Leland's.

But two years later, we did see change in Ms. Leland. Her teaching remained largely traditional, but she used more manipulatives and work in groups, and she was inclined to ask students what they know and believe. Her story of the alligators suggests that teachers—even when they don't adopt an entirely new vision for thinking about teaching and learning—can learn to make fundamental shifts in their beliefs about teaching. For Ms. Leland—and perhaps other teachers—the more powerful part of the SummerMath experience involved the sustained time with a colleague who could offer intellectual, practical, and technical support in the classroom. It was after a year of working with one of the program directors—watching him teach, listening to his ideas, observing how he translated the theoretical, "ideal" notions of SummerMath for Teachers into actual classroom practice—that Ms. Leland noticed changes in her pedagogical posture. Perhaps Ms. Leland and Ms. Rosen have different needs as learners, just as their students do, and the patchworks of practice that we see in these two cases are significantly colored by how these two women as learners and thinkers best learn new things about teaching and learning.

Finally, both cases suggest the powerful effect that context has on changing practice. Ms. Leland mentions over and again the press to cover curriculum, the need to "rob Paul to pay Peter." If she teaches students to solve some of these problems, it will take more time than her district curriculum allows, and she'll have to give up covering some material. As she said, "I haven't been able to do the robbing." She also noted how difficult it is for teachers to find time to reflect on what they are doing, and learn from their experiences. Ms. Rosen offers other examples of contextual constraints. Willing to take time and have students explore problems, she faces a group of students who'd rather see how much trouble they can cause with Unifix cubes and base ten blocks. She also reported some press within the school—from other teachers and from administrators—not to rock the boat, not to call for radical changes in curriculum or teaching methods. Without time—personal and curricular—and without support from students, teachers, and administrators, teachers like Ms.

Leland and Ms. Rosen find it difficult to take the risks necessary to learn to teach differently. The safest—and most sensible—thing to do in such a setting might be to mix the old with the new prudently.

Changing Visions and Changing Practices

What questions do these stories of Ms. Rosen and Ms. Leland raise about the challenge of changing practice? With a vision of mathematics teaching on the one hand, and a commitment to learning as active construction on the other, how can programs like SummerMath respect teachers as learners while helping them grow in particular directions? In working with experienced and competent teachers, the SummerMath staff designed experiences that were imbued with an intensity that they believed necessary to challenge the weight of experience. This intensity took several forms. The long days of the Summer Institute were filled with emotionally and intellectually challenging mathematical and pedagogical problems.

The year following, characterized by continuous conversation with a consultant about teaching mathematics, was equally full of challenge, for the teachers were constantly asked to rethink what they thought could and should happen in their classrooms. From the good food that Ms. Rosen described to the teasing familiarity that Ms. Leland exhibited toward the program director, the SummerMath staff actively and consistently tried to soften the experience of facing these challenges. They knew that they were asking teachers to take substantial risks, and they worked hard to create environments and relationships that would support and facilitate such risk taking.

Ms. Leland and Ms. Rosen responded differently to these forms of intensity. Ms. Rosen was profoundly affected by the summer institute while Ms. Leland responded to the close and careful attention from her consultant. Ms. Rosen embraced the vision first, and actively sought ways to change her practice; Ms. Leland, through gradual changes in practice, began to change her visions. As two different learners, Ms. Rosen and Ms. Leland both found the support and respect they needed to begin changing their practices. By challenging and supporting teachers in a variety of ways, the SummerMath program embraced the paradox of their task: effecting significant and specific changes in mathematics teaching while acknowledging that teachers themselves need to be active constructors of their practice.

References

Cohen, D. K. (1988). *Teaching practice: Plus ça change . . .* (Issue Paper 88-3). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Cohen, D. K. (1990). *Policy and practice: The classroom impact of state and federal policy.* Unpublished manuscript, Michigan State University, East Lansing.

Cohen, D. K., & Ball, D. L. (1990). Relations between policy and practice: An overview. *Educational Evaluation and Policy Analysis, 12.* 347-353.

Kennedy, M. M., Ball, D. L., McDiarmid, G. W., & Schmidt, W. (in press). *A study package for examining and tracking changes in teachers' knowledge* (Technical Series 91-1). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Center for Research on Teacher Education. (1988). *Teacher education and learning to teach. Journal of Teacher Education, 32*(6), 27-32.