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ABSTRACT

This booklet is the eighth in a series of nine from the Teacher Training Institute at Hofstra University (New York) and contains descriptive information about two courses included in the institute's program. The first course, by David Knee, William McKeough, and Robert Silverstone, is "Discrete Mathematical Models," which deals with topics from graph theory, set theory, logic, combinatorics, probability theory, statistics, and finite algebraic structures. The second course, by Joyce Bernstein and William McKeough, is "Spreadsheets in the Classroom" and focuses on spreadsheet-based conjecturing and problem solving activities involving geometric properties, number theoretic principles, conditional probability, trigonometric relations, and graphing capabilities. For each course this booklet includes: (1) the course description and requirements; (2) the pretest/posttest, the midterm assignment, and the final examination; (3) course handouts for various projects and assignments; (4) sample project results from some of the participants; and (5) a proposal by a participant for an inservice peer group workshop. (JJK)

HOFSTRA UNIVERSITY

TEACHER TRAINING INSTITUTE

Department of Mathematics and School of Secondary Education
Hofstra University
Hempstead, NY 11550

DISSEMINATION PACKET – SUMMER 1989

Booklet #8

DAVID KNEE, WILLIAM McKEOUGH & ROBERT SIVERSTONE
DISCRETE MATHEMATICAL MODELS

JOYCE BERNSTEIN & WILLIAM McKEOUGH
SPREADSHEETS IN THE CLASSROOM

NSF Grant # TEI8550088, 8741127

This booklet is the eighth in a series of nine booklets which constitute the Hofstra University Teacher Training Institute (TTI) packet. The Institute was a National Science Foundation supported three-year program for exemplary secondary school mathematics teachers. Its purpose was to broaden and update the backgrounds its participants with courses and special events and to train and support them in preparing and delivering dissemination activities among their peers so that the Institute's effects would be multiplied.

This packet of booklets describes the goals, development, structure, content, successes and failures of the Institute. We expect it to be of interest and use to mathematics educators preparing their own teacher training programs and to teachers and students of mathematics exploring the many content areas described.

"Teaching Mathematical Concepts via Spreadsheets" was a basic topic of the TTI's cycle of courses, while "Discrete Mathematical Models" was planned as a 'coda' course in response to TTI participants' requests. The coda was an added Institute component, run during five weeks of the summer of 1988, and created to round off the program after two year-long cycles had been offered (June 1986 through May 1988). The other two courses of the coda were: "Calculus in the Secondary Classroom" and "Problem Solving via Pascal Data Structures".

This booklet describes both the "Discrete Math" and the "Spreadsheet" courses. It gives their syllabi, tests, sample units and a sampling of participant projects: a spreadsheet on voting methods, and outlines for both an in-service discrete mathematics course and a spreadsheet-based course.

TEACHER TRAINING INSTITUTE

Discrete Mathematical Models

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Spreadsheets in the Classroom

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Booklet #8

Contents

1. Course Description and Requirements

Discrete Mathematics is the label that recently has been applied to a collection of topics from finite mathematics (the previous,roughly equivalent label) that, since the 1950's began to appear in college, high school and elementary school curricula. "Discrete" is meant as an opposite of "continuous" and refers to structures, models and topics such as graph theory, set theory, logic, probability and statistics, finite algebraic structures, combinatorics (the theory of counting), linear programming, mathematical linguistics and so on. The definition is not strict - some of these topics do have continuous (as in "continuum", i.e. the real numbers) aspects.

The applications of this bundle of topics is immense and often novel. Some educators go so far as to say that Discrete Math rivals the calculus in importance on the college level for serious students of mathematics and the sciences. Discrete topics may be more important than calculus for those concerned with computers, management science, or the social sciences. In New York State where "Sequential Math" has been introduced in the high schools, logic, probability and statistics, and abstract algebra are now part of the curriculum. Few educators doubt the relevance, power, application and beauty of these new additions, although debate still exists on how to incorporate them without watering down other basic competencies.

As with the other Coda courses (Summer 1988), this course was chosen with participant input. Happily, an important and delightful book, "For All Practical Purposes, an Intro to Contemporary Mathematics", a cooperative effort of COMAP (Consortium of Mathematics and Its Applications) directed by Solomon Garfunkel, became available that Spring. We chose this as our text (which we supplemented with classroom material) and also presented to the class a sampling from the set of 26 half hour videos that accompany the text. These videos, which have appeared on various educational television stations, give an entertaining, folksy and high level introduction to the mathematics, history, personal[illegible] and especially the applications of the book's five areas: Management Science, Probability and Statistics, Social Choice (Voting Schemes), Size and Shape, and Computers. The text can be used in colleges and might also make an excellent addition to 12th year math course choices in the high school.

Course Syllabus

1. Graph Theory, chapters 1 and 2

* management science, operations research, optimal solutions, algorithms.

* Basic definitions and examples of graph theory: vertices, edges, vertex degree, path connectedness trees, directed graphs, Euler circuits, Hamiltonian circuits.

* Postman problem, traveling salesman problem, 4-color problem, the Euler formula for polyhedra, NP completeness, spanning trees, greedy algorithms.

* Combinatorics: fundamental counting principle permutations and combinations.

* Applications: communications networks, routing problems for airlines, measuring the complexity of an algorithm, etc.

2. Linear Programming Chap. 4

* linear inequalities, feasible regions, convex sets, optimizing profit or cost, corner principle, graphical solution.

* Simplex method, Dantzig, Karmarkar, Khachian: greatest use of computers is for LP problems.

3. Probability & Statistics Chap. 5-8

* collecting data, random and biased samples, averages and variability, quartiles, histograms, baseball stats Latin squares.

* Basic probability, gambling, sample space, normal curve, central limit theorem, expected value.

* Linear regression, computer graphics.

* Statistical inference, confidence intervals.

* Fermat, Pascal, Bernoulli, R.A. Fisher.

* Applications: efficacy of a new drug or treatment procedure, design of experiments, opinion polls, quality control, sports statistics, social science research.

4. Voting Schemes Chap. 9

* Majority rule, plurality vote, sequential voting, Condorcet winners, Borda count, approval voting.

* Kenneth Arrow's impossibility result and its conceptual connection with Heisenberg's Uncertainty Principle and Goedel's Incompleteness Theorem. Balinski and Young: There is no satisfactory solution to the seat allocation problem.

5. Codes Chap. 19

* Logic, truth tables, binary representation, logical circuits, computer arithmetic.

* Error-detecting and error-correcting codes, data protection, information theory.

* Boole, Hamming, Shannon, Huffman.

Videos (from the 26 half-hour shows that accompany the text):

#1 Management Science, an overview

#2 Street Smarts/Street Networks

#5 Juicy Problems/Linear Programming

#8 Picture This/Organizing Data

#9 Place Your Bets/Probability

#10 Confident Conclusions/Statistical Inference

#12 The Impossible Dream/Election Theory

#24 Creating a Code/Encoding Information

HOFSTRA UNIVERSITY

NSF - Teacher Training Institute July 1988

Further Bibilography on

VOTING PARADOXES

(with thanks to Bob Silverstone)

1. UMAP Modules In Undergraduate Mathematics, 55 Chapel St., Newton, MA 02160

Decision Analysis For Multicandidate Voting Systems, Samuel Merrill, III, Department of Mathematics/ Computer Science, Wilkes College, Wilkes-Barre, PA, UNIT 384

Methods of Congressional Apportionment, Milton P. Eisner, Department of Mathematics, Mount Vernon College, Washington, DC, UNIT 620

An Application of Voting Theory, James M. Enelow, Department of Political Science, SUNY, Stony Brook, NY, UNIT 386

2. The Choice of Voting Systems, Richard G. Niemi and William H. Riker, SCIENTIFIC AMERICAN, June 1976 Vol. 234, No. 6

3. Paradoxes of Preferential Voting, Peter C. Fishburn, Steven J. Brams, MATHEMATICS MAGAZINE, Vol. 56, No. 4, September 1983

4. Parliamentary Coalitions: A Tour of Models, Philip D. Straffin, Jr. and Barnard Grofman MATHEMATICS MAGAZINE, Vol. 57, NO. 5, November 1984

Participants started the course with a pre-test took a semi-collaborative take-home midterm and ended the course with a final examination, the first part of which was the post-test (exactly the same as the pre-test). Students were also required to submit two projects, a high school classroom unit (or units) and a prospectus for an in-service course for mathematics high school teachers, both covering topics from the course.

2. Examinations

NSF-TEACHER TRAINING INSTITUTE
Professors William J. McKeough - David Knee

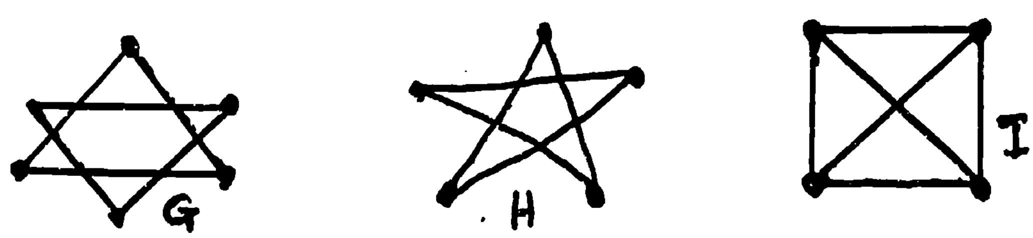
MATH 309 MID-TERM
July 12, 1988

1. Minimize $2x + 5y$ subject to

$$\begin{aligned} x,\ y &\geq 0 \\ x + y &\geq 3 \\ 2x + y &\geq 4 \\ 2x + 5y &\geq 10 \end{aligned}$$

2. Solve - Lethal Brothers manufactures 2 types of laundry detergent, Sludge and Slime, each of which contain sodium borax, mercury phosphate and an enzyme called Miracle Blight. A one pound box of Sludge contains 10, 4, and 2 ounces of sodium bortax, mercury phosphate and Mircle Blight respectively. A one pound box of Slime contains 7, 5, and 4 ounces of sodium borax, mercury phosphate and Miracle Blight respectively. Lethal Brothers has in stock 12,000 ounces of sodium borax, 6,000 ounces of mercury phosphate and 2,000 ounces of Miracle Blight. If the profit on Sludge and Slime is 40 and 45 cents respectively, how many boxes of each should Lethal Brothers manufacture in order to maximize profit?

3.



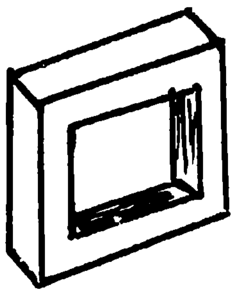
which of these graphs have which properties?

	planar	connected	has a bridge	has Euler circuit	has Euler path
G					
H					
I					

4. Find the Euler number (V-E+F) for (show V, E & F individually)

   a) the planar graphs above

   b) the regular dodecahedron

   c)  "The Picture Frame"

5. Using a Latin Square design, describe a procedure to assess relative effectiveness of three very different texts on math achievement, as measured by a standardized test.

   - what questions or precautions would you raise/take early on?

   - what would the design be?

   - how would you administer the experiment?

   - what would you do with the results?

6. Using only the data on p.135 which is marked with an asterisk:

   - calculate Pearson's r for price, weight

   - calculate a linear regression of price onto weight

   - how well does the L.R predict
     a) the weights of the 2 Volvos?
     b) the weights of the 2 Oldsmobiles?
     c) the weights of the new Blitzfire 6 with the Thunderclap Engine, from Serbo-Croatia, costing $4,500?

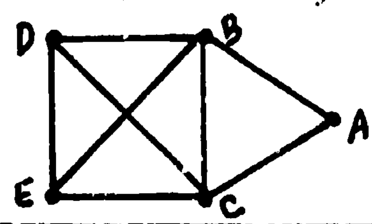
comment on/analyze your responses

NAME ____________________

HOFSTRA UNIVERSITY
NSF Teacher-Training Instititue
Summer Session II, 1988
PART I OF FINAL: POST TEST

1.a Define "Euler circuit." ____________________

1.b Find an Euler circuit for this graph -or- say why it is impossible to do so.



2. Consider this LP problem: Maximize profit = $2x + 3y$ under the constraints $x \geq 0$, $y \geq 0$, $x+y \leq 5$, and $3x+y \leq 9$. Show work.

2.a Sketch the feasibility region, below.

2.b Present the problem's solution.

3. Define briefly:

Condorcet winner: ____________________

Arrow's Impossiblity Theorem: ____________________

NAME ______________________

4. Create a 9-element Latin Square different from [not isomorphic to]:

A	B	C
B	C	A
C	A	B

5. State the "Law of Large Numbers," using the term or concept of expected value and not using the term, gastroenterologist.

____

____

____

6. Explain, with seemly brevity, the mathematical meaning of the weather forecaster's statement: "There is a 70% chance of rain tomorrow."

____

____

____

7. Consider the contingency table, below, of responses to a questionnaire in which one item could be answered "yes" or "no" and another could be answered "always", "sometimes", or "never".

	always	sometimes	never
yes	14	50	26
no	16	40	14

What is the probability of a randomly-selected answer-pair being:

7.a "always", given a "yes" response to the other question?
7.b "yes", given "always"?
7.c "yes" or "always"

8. Use truth tables to show that, for all values of A and B, the expression NOT (A OR B) is logicaly equivalent to (NOT A) AND (NOT B).

NSF-TEACHER TRAINING INSTITUTE
SUMMER CODA 1988

SED 309 FINAL Part II

1. Using the 7-bit with 4 data bit system of binary coding described in "NOTES ON BINARY CODES", demonstrate whether the following 'message' will a) prove correct; b) detect and correct an error; c) detect but not correct a multi-bit error.

1101111

2. a) How many different preference schedules are possible in ranking 3 candidates if ties are allowed? Show work.

   b) 90 voters have the following preference schedules for 3 candidates, A, B, & C.

	40	30	20
1st place	A	C	B
2nd place	C	B	C
3rd place	B	A	A

   Compute the winners (if any) by Borda, Condorcet & eliminate successive losers methods.

3. Given the character/frequency chart below:

Character	Frequency
A	8
B	12
C	6
D	1
E	2

a) design a Huffman code
b) find its weight
e) decode the message 01101101100

3. Participant Projects

SED 309 Patrice McDonald

Spreadsheet on Voting Methods

The spreadsheet I created using SC3 shows the results of 5 different voting methods for a three candidate preference schedule. The 5 methods are plurality, Condorcet, Borda Count, elimination by least first place votes, and elimination by most last place votes. Sincere voting is assumed for all of the methods.

I think the spreadsheet has different uses depending on the teacher and grade level of the class for which it is being used. On upper levels, after the different methods have been discussed and demonstrated, the spreadsheet could be used to show how different methods can produce different winners. This can be done quickly without tedious calculations by using the spreadsheet. A discussion of how close or different the numbers of voters must be to generate drastically different results could be done and then strategic voting could be looked at.

In lower grade levels, the concept of different voting methods could be discussed especially among gifted and talented students. To me, one of the harder methods is the elimination by first or last place votes, since this involves changing the preference schedule. The spreadsheet constructs new preference schedules for both situations based on the original input. If the teacher then wants the students to do the calculations to determine the winner using those schedules, the spreadsheet can be changed to hide those entries below each preference schedule. In fact at any point, the teacher can hide all of the entries where the calculations are done and have the students do the calculations for arithmetic practice.

Social Studies teachers might also want to use it. They would most likely use it intact and just enter different numbers to generate various results. Their approach and use of the spreadsheet would probably be somewhat different from the mathematical approach.

I have attached copies of the results of the spreadsheet with two different sets of input. I have also included the formula printout. In addition, I included the disk with the program on it, since I really don't think the printout shows what this spreadsheet really looks like in use.

DIFFERENT VOTING METHODS
Written by Patrice McDonald

A PREFERENCE SCHEDULE for 3 candidates is listed below.
You must enter the number of people in favor
of each different preference order.
Enter the numbers in D16,F16,H16,J16,L16, and N16.
After the numbers have been entered, page down or
use down arrow to see the results of different voting methods.
Each method of voting assumes sincere voting is done.

	1st Sched.	2nd Sched.	3rd Sched.	4th Sched.	5th Sched.	6th Sched.
Number in favor of each preference schedule	20	5	7	12	6	17
1st choice	A	A	B	B	C	C
2nd choice	B	C	A	C	A	B
3rd choice	C	B	C	A	B	A

METHOD 1
Plurality Method

	A	B	C
	20	7	6
	5	12	17
TOTALS	25	19	23

THE WINNER IS A WITH 37.31 PERCENT OF THE VOTE

METHOD 2
Condorcet Method

	A	vs. B	B	vs. C	C	vs. A
	25	19	19	23	6	25
	6	17	20	5	12	7
TOTALS	31 to	36	39 to	2J	18 to	32

THE CONDORCET WINNER IS B

METHOD 3
Borda Count Method

IMPORTANT You must assign point values for each choice:

Enter in B50 3 points for a 1st place choice
Enter in B51 2 points for a 2nd place choice
Enter in B52 1 points for a 3rd place choice

	A	B	C
	75	57	69
	26	74	34
	29	11	27
TOTALS	130	142	130

[illegible]ORDA COUNT WINNER IS B

METHOD 4
Elimination of candidate
with the least 1st place votes

When the candidate with the least 1st place votes is eliminated, the PREFERENCE SCHEDULE now looks like this:

	1st Sched.	2nd Sched.	3rd Sched.	4th Sched.	5th Sched.	6th Sched.
Number in favor of each preference schedule	20	5	7	12	6	17
1st choice	A	A			C	C
2nd choice		C	A	C	A	
3rd choice	C		C	A		A

	A	B	C
	20		6
	5		17
	7		12
TOTALS	32	0	35

THE WINNER IS C

METHOD 5
Elimination of candidate
with the most last place votes

When the candidate with the most last place votes is eliminated, the PREFERENCE SCHEDULE now looks like this:

	1st Sched.	2nd Sched.	3rd Sched.	4th Sched.	5th Sched.	6th Sched.
Number in favor of each preference schedule	20	5	7	12	6	17
1st choice			B	B	C	C
2nd choice	B	C		C		B
3rd choice	C	B	C		B	

	A	B	C
		5	20
		6	7
			12
		17	
TOTALS	0	28	39

THE WINNER IS B

Carolyn Walters
July 16, 1988
NSF - TTI
Math 309
Prof. William J. McKeough
Prof. David Knee
Course Description

As a math teacher, one of the questions that I am asked most often is: "When are we ever gonna use this?" I thought about this and will use this as the underlying theme of the course.

I. City sidewalks, busy sidewalks - sounds familiar, we could use this to introduce some elementary graph theory.
   a. definition of graphs - planar and connected
   b. Euler circuits and paths
   c. the Chinese postman problem - find the "best" path
   d. applications to city planning - directed graphs used for sanitation pickups and meter maid routes
   e. WHEN ARE WE EVER GONNA USE THIS???
      The seniors at Mount Vernon High run the entire homecoming weekend. This includes a motorcade through the city. If we start and finish at Memorial Field (Euler circuit) and since there are one-way streets (directed graphs), what is the best route to take. Students could work on the problem and submit their solutions. Perhaps a prize to the one who submits the actual motorcade route.

II. Linear Programming
   a. what does it mean - graphing regions
   b. how to find the optimal solution - examining the corner points of the graph
   c. discussion of integral solutions
   d. WHEN ARE WE EVER GOING TO USE THIS???
      As part of homecoming there is a dance. The admission is $3 for singles and $5 for couples. The fire laws allow no more than 900 persons. It has been practice to have one chaperone for every 60 singles and one chaperone for every 45 couples. There are 15 adults willing to chaperone the dance. How many singles and couples should be admitted to the dance so that the income from the dance is a maximum?

III. Statistics
   a. mean, median, mode, range
   b. normal distribution curve
   c. standard deviation
   d. confidence intervals
   e. scatter graphs
   f. linear regression
   g. Pearson's r
   h. WHEN ARE WE EVER GONNA USE THIS???
      The senior class has a raffle as part of the Homecoming. They would like to analyze past raffles to determine the average number of chances that a student will sell. This will help to determine how many raffle tickets to order. In addition, they would like to be able to predict the attendance at the Homecoming football game. They could then sell more tickets if if the attendance is high. There seems to be a correlation between temperature and attendance. Using the data from previous years and knowing the weather forcast, they could use linear regression to predict this year's attendance.

IV. Theory of Voting
   a. majority wins - if only two choices
   b. plurality vote - run off vote
   c. sequential voting
   d. Condorcet winners
   e. preference voting
   f. Borda count
   g. approval voting
   h. WHEN ARE WE EVER GONNA USE THIS???
      How will the Homecoming Queen and King be determined? Which method of voting should be used? Is one better than another. Students will have to decide which to use and this decision itself is a vote. We could go on forever deciding how to do it, sort of proving Arrow's impossibility theorem.

So now at least I can answer that dreaded question: "WHEN ARE WE EVER GONNA USE THIS????" I am sure that the teachers who would take this course would also be able to answer this question.

Spreadsheets in the Classroom

1. Course Description and Requirements

HOFSTRA UNIVERSITY

MICROCOMPUTERS IN SECONDARY MATHEMATICS EDUCATION

SECONDARY EDUCATION 308A, MAY 1988

NCTM's long-standing support for Problem-Solving, later made explicit in its 1989 Standards, complements a recent NY State mandate of an additional year of secondary mathematics for all students as a condition for graduation. The Teacher-Training Institute (TTI) sought to respond to these two initiatives by providing training in the application of electronic spreadsheets to the high-school curriculum. Participants knew generally of the potential of these multi-purpose programs, but wanted to learn more about their potential for enriching and extending mathematics instruction. Specifically, many wanted to design new courses to serve the new State requirement, courses which would promote problem-solving, would tap the power of the micro-computer, and would enable explorations in mathematics which were not feasible earlier. Participants also wanted to incorporate this microcomputer application into existing courses, again to broaden the scope of students' understanding. Spreadsheets are too often seen as business-only applications and not enough as general-purpose problem-solving tools; SED 308A was designed to remedy that misapprehension.

Used with skill and understanding of their power and limits, electronic spreadsheets can foster students' problem-solving skills. Their intrinsic "what if" capability allows teachers and students to raise and answer more probing, more interesting, more calculation-bound questions. The computer removes the time delay and drudgery which otherwise might impede such investigations. Spreadsheet programs which have intrinsic graphing capabilities or which link to external graphing programs provide additional educational power.

Examples of spreadsheet-based problem-solving activities developed for or by our TTI Participants include:

1] {Middle School Level} Explore perimeters, areas, and volumes of common figures and forms by asking "what if" one doubled all dimensions or changed selected dimensions.

2] {High School Level} Ulam's Conjecture or Hailstone Numbers can be explored for a wide range of "input" values, varying by such characteristics as prime/not-prime, parity, factors, etc.

3] {High School or College} Conditional Probability allows exploration of (a|b)-type propositions (the occurrence of event a, given the occurrence of event b) with varying underlying probabilities.

4] {High School} Exploring Ambiguous Triangles using built-in trigonometric functions or generating functional values through "look-up" tables.

5] {High School} The graphing capabilities of many spreadsheets increases, visually, students' comprehension of functional relationships, e.g., "How the graph changes when you change the coefficients of a conic section."

Each Participant demonstrated the ability to design an instructional unit on a topic in secondary mathematics of her or his own choosing. Each topic had to be susceptible of implementation on an electronic spreadsheet and had to emphasize an element of mathematical problem-solving. Copies of each Participant's unit have been distributed to all other Participants to enhance dissemination.

Instructors: {Apple Computer Section} Joyce Bernstein, Adjunct Instructor in Secondary Education and TTI Cycle I graduate.
{IBM Computer Section} William J. McKeough, Professor of Secondary Mathematics Education Co-Director of TTI.

HOFSTRA UNIVERSITY
NSF Teacher-Training Institute
Hempstead, NY 11550

6 May 1988

SED 308A, Spring 1988
Bernstein & McKeough

There are three (3) (III) (drei) tasks required of you in this course: 1] a satisfactory needs-wants assessment for your dissemination project (in-service course); 2] a well thought-out, albeit tentative, course outline for it; and 3] a non-trivial spreadsheet application, together with detailed notes (unit or lesson plan) on how it should be used in a secondary mathematics class. The application should be submitted both on disk (You may put it on the SuperCalc3 disk you return at the end of the course.) and as two (2) (II) (zwei) printouts: in Display form and in Formula form.

For those of you who may have difficulty generating ideas for the spreadsheet application, there follows a list of possible areas, ideas, & topics which may stimulate your thinking. You are not limited to this list of potential applications! The list, however, should give you a rough fix on the levels of complexity and sophistication which is expected. All applications should be "polished," "bullet-proofed" to a reasonable degree, and suitable for classroom use by another teacher.

Please notice that some of these applications entail graphing, some use the MOD function, and some use a random-number generator. SC3 has all three. IBM-oids should load GRAPHICS.COM before running SC3 if they wish to produce printed graphs; triple or quadruple density printing is "encouraged."

The entries are in the order in which they occurred to the writer[s].

Generating primitive Pythagorean triples from various seeds
Conversion of periodic decimals to common fractions
- with non-zero integer part
- with negative values
- must handle cases of 1/17 & 0.033333...

Adding and subtracting common fractions
- reducing fractions to lowest terms

Demonstration whether spreadsheet numbers form a field
Truth tables for IMPLICATION, OR, AND, XOR
- handle converse & contrapositive

Pik's theorem: lattice-point areas
Solving triangles [entails trig functions]
- find area, given SAS, ASA, SSS
- find acute angles in right triangles, given sides
  - test for right-triangle-ness
  - find area, perimeter

Word problems

- motion, distance, mixture, age, &c.

Solving simultaneous linear equations

- via determinants
- via other methods

Approximating Pi

- Buffon's needle
- Archimedes', other methods

Find day of week, given date in YYYYMMDD format
Conversion from decimal to and from other number bases
Perfect and amicable numbers
Checking addition (or multiplication) of integers by "casting out nines"
Pascal's triangle
Conversion of units of measurement, currency

- both directions

Interest on a loan or bank balance

- simple
- compound
- variable compounding periods: monthly, daily

Sieve of Eratosthenes
Factoring quadratics with integer coefficients
Generating polynomials, given y-values for x = 1,2,3,...,n
Coordinate Geometry

- generate data sets from given polynomial
- explore effects on the graph of changing "details"
- polar coordinates, graphing therein
- colinearity, parallelism, perpendicularity

Phi, the golden ratio and fibonacci (& Lucas) sequences
Determining coordinates of incenter, circumcenter, orthocenter, etc. of a triangle, given only coordinates of vertices
Determine length of cevians (e.g., altitudes, medians, angle bisectors) in a triangle, the coordinates of whose vertices are given
Divisibility tests on integers
Diophantine equations
Graphing within modular systems
Farey sequences
Use MOD function to test for divisibility, primality
Decimals and fractions other than in base[ten]
Polygonal numbers: triangular, square, pentagonal, hexagonal, ...
Convert grade-book entries to standard or scaled scores
"Curve" grades to pre-set specifications
Permutations & combinations
Binomial theorem
Sums of finite & infinite series
Matrices and operations thereon
Complex-number arithmetic
Continued fractions
Magic squares
Multiplying polynomials
Synthetic division

2. Sample Class Units

SED 308A
Microcomputers in Secondary Mathematics Education

Sequences

A number sequence is a set of numbers that are arranged by some rule relating the respective terms.

Spreadsheet applications: design, relative copy, cell replication, use of functions and formulas.

Class Demonstration:

I. Arithmetic Sequence

Devise a spreadsheet which will generate any arithmetic sequence. Allow room near the top of the spreadsheet to enter the first term, a, the difference, d, and the number of terms. n. Let your series have 10 terms. Let the first term equal 50. Let the difference between terms equal 200. Find the last number by inspection and then by the standard formula $l = a + (n - 1)d$. Find the sum using the @SUM function and then by the standard formula $S = \frac{n}{2}(a + l)$.

II. Fibonacci numbers

The sequence of Fibonacci numbers 1,1,2,3,5,8,13,21,34,55,.. is defined recursively by the relation

$a_1 = 1,\ a_2 = 1,$

$a_n = a_{n-2} + a_{n-1} \qquad n > 2.$

The ratios of successive terms of the sequence have the following property:

$$\lim_{n \to} \frac{a_{n+1}}{a_n} \to 1.618034$$

This number, called the golden ratio, arises in the study of proortion in geometry, art, architecture, and biology. Verify this ratio.

Exercises:

1. If one of the initial terms is changed, the sequence generated is, of course, different. (Changing a_2 to 3 yields the Lucas numbers.) Investigate the effect a change in starting numbers has in the limit of the ratio.

2. The Fibonacci numbers have an interesting property concerning the sum of the squares of the first n terms. See if you can find this property.

3. A special case of Newton's method is a well known elementary algorithm for finding the square root of N: Let a_1 = guess. The $a_{n+1} = 0.5(a_n + \frac{N}{a_n})$.

Compare results obtained using this algorithm with the standard @SQRT function.

Factorials

For a positive interger n, n! (read "n factorial") is defined recursively by :

1! = 1

n! = n(n-1)!

4. Devise a spreadsheet which will have a descriptive heading and then generate two columns, n and n!.

5. The probability that in a group of n people, no two will have the same birthday is:

$$P_n = \frac{365}{365} \cdot \frac{365}{365} \cdot \ldots\ldots \cdot \frac{366-n}{365}$$

or recursively, $P_1 = 1$, $P_n = \frac{P_{n-1}(366 - n)}{365}$

Create a spreadsheet which will display the probability that for n = 1,2,3,..... people, no two have the same birthday and also the probability that at least two have the same birthday.

```
Arithmetic Sequence Demo
================================================================================

Enter 1st # in D4          50
Enter Difference in D5    200
Enter # of terms in D6     10

     50    250    450    650    850   1050   1250   1450   1650   1850

Last term:  1 = a + (n - 1)d=    1850
Sum:     @SUM(A8.J8)=       9500
Sum:     n/2(a+1)= .        9500
```

```
File: w1 arithmetic              REVIEW/ADD/CHANGE              Escape: Main Menu
=======A=======B=======C=======D=======E=======F=======G=======H=======I=======J===
  1|Arithmetic Sequence Demo
  2|=============================================================================
  3|
  4|Enter 1st # in D4       50
  5|Enter Difference in D5200
  6|Enter # of terms in D610
  7|
  8|+D4      +A8+200+B8+200+C8+200+D8+200+E8+200+F8+200+G8+200+H8+200+I8+200
  9|
 10|Last term:  1 = a + (n - 1)d=+D4+((D
 11|Sum:     @SUM(A8.J8)=  @SUM(A8
 12|Sum:     n/2(a+1)=      +D6/2*(
 13|
 14|
 15|
 16|
 17|
 18|
--------------------------------------------------------------------------------
D12: (Value) +D6/2*(D4+E10)

Type entry or use @ commands                                          @-? for Help
```

File: w2 fibonacci

Fibonacci Numbers

Number	Fibonacci #	Ratio
1	1	
2	1	1.0000000
3	2	2.0000000
4	3	1.5000000
5	5	1.6666667
6	8	1.6000000
7	13	1.6250000
8	21	1.6153846
9	34	1.6190476
10	55	1.6176471
11	89	1.6181818
12	144	1.6179775
13	233	1.6180556
14	377	1.6180258
15	610	1.6180371
16	987	1.6180328
17	1597	1.6180344
18	2584	1.6180338
19	4181	1.6180341
20	6765	1.6180340

File: w2 fibonacci REVIEW/ADD/CHANGE Escape: Main Menu

	A	B	C
1	Fibonacci	Numbers	
2	========	========	========
3			
4	Number	Fibonacci #	Ratio
5	1	1	
6	+A5+1	1	+B6/B5
7	+A6+1	+B5+B6	+B7/B6
8	+A7+1	+B6+B7	+B8/B7
9	+A8+1	+B7+B8	+B9/B8
10	+A9+1	+B8+B9	+B10/B9
11	+A10+1	+B9+B10	+B11/B10
12	+A11+1	+B10+B11	+B12/B11
13	+A12+1	+B11+B12	+B13/B12
14	+A13+1	+B12+B13	+B14/B13
15	+A14+1	+B13+B14	+B15/B14
16	+A15+1	+B14+B15	+B16/B15
17	+A16+1	+B15+B16	+B17/B16
18	+A17+1	+B16+B17	+B18/B17

C1

Type entry or use @ commands @-? for Help

Hofstra - NSF Teacher Training Institute
Microcomputers in Secondary Education

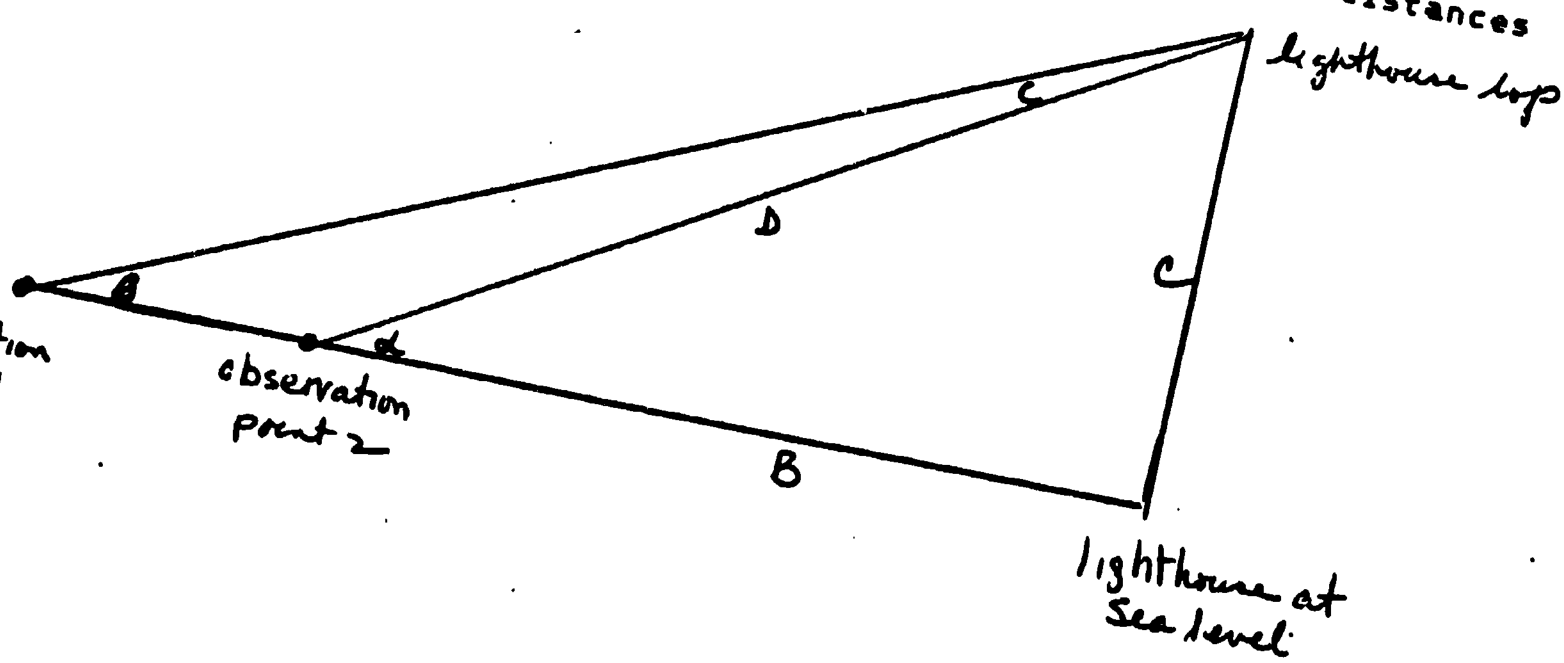
Test Generators Using Spreadsheets

Often, the extra time needed to teach spreadsheet manipulati makes the use of the microcomputer inappropriate for student interaction. Integrated Course III, for example, leaves little ro for creative exploration. The spreadsheet is still a valuable too a test generator. The following examples use the spreadsheet to handle the tedious calculations which accompany problems involving transformation geometry problems and problems involving trigonometr functions. Changing the input starting values changes the problem, making it possible to generate make-up tests quickly.

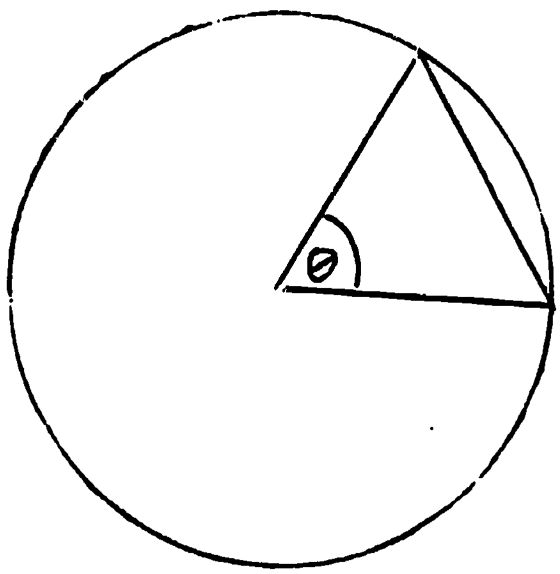
The following examples are done for illustraion only. Once you have mastered the use of @LOOKUP, you can generate countless other problems.

1. A good bonus question on a transformation geometry test asks for the image of a point reflected over a line which is not parallel to an axix. The file you are receiving, Transformation, contains one question.
   Try to generate a question which supplies a point and its image under a line reflection and ask for the line of reflection.

2. Trig functions are not supplied in Appleworks. If you look at the amount of memory consumed by the trig lookup tables, you will understand why. Look at the file, new Trig. The first worksheet describes the predictable question where a person is on land, looking at a lighthouse. Two viewpoints in line with the lighthouse are supplied, using the following diagram. Several angles and distances are requested.



3. The second trig problem is a worksheet for the area of a sector of a circle and the area of the triangle created by a chord connecting the endpoints of the sector.



Transformation Geometry

==

Transformation over lines not parallel to an axis:

In a transformation over a line, the line is the perpendicular bisector of the segment connecting a pointand its image.

Writing the equation of the line of reflection in slope-intercept form,
Enter the slope(m) in cell E9 1
Enter the intercept (b) in E10 0

Enter the point you are reflecting entering the
abscissa in cell E13 1
ordinate in cell E14 2

The line perpendicular to

y =	1x +	0	through	1	2	is
y =	-1x +	3				
0 =	2x +	-3				

The point of intersection is 1.5 1.5
1.5 1.5is the midpoint of 1 2 and the image
The coordinates of the image are therefore 2 1

Lookup Table for Sin in columns AA and AB 2-362
Lookup Table for Cos in Columns AC and AD 2-362

Worksheet for problem 1:

put estimate for angle beta in E9: 30
Put estimate for angle alpha in E10: 60
Enter distance between measurements in F 11: 100
Supplement angle alpha = 120
Angle C = 30

side d = 100
side c = 86.6
side b = 50.00439

++

Worksheet for problem 2

Given a circle of radius 10 units
A sector is cut off with central angle 100 degrees
1. Find the area of the sector.
2. Find the area of the triangle formed by the sides of the sector and the chord connecting the endpoints of the arc.
3. Find the area of the segment between the arc formed and the chord connecting the endpoints of the arc.
Area of the sector = 87.26666
Area of the triangle = 49.24081
Area of segment = 38.02585

++

SED 308A
Microcomputers in Secondary Mathematics Education

Euclid's GCD Algorithm

Euclid's greatest common divisor algorithm for two positive integers is the following iteration:

If a & b are positive integers, a > b, divide a by b, obtaining a nonnegative quotient, q_1, and an integer remainder r_1, $0 \leq r_1 < b$:

$a = q_1 b + r_1 \quad 0 \leq r_1 < b$

If $r_1 <> 0$, divide b by r_1 and get

$b = q_2 r_1 + r_2 \quad 0 \leq r_2 < r_1$

If $r_2 <> 0$, divide r_1 by r_2 and get

$r_1 = q_3 r_2 + r_3 \quad 0 \leq r_3 < r_2$

Repeat the process until a zero remainder is reached. The last non-zero remainder is the GCD.

1. Design a spreadsheet to find the GCD of two positive integers.

2. Add a calculation for the least common multiple of the integers, LCM(a,b) = a*b/GCD(a,b)

Modular Arithmetic

Let n be a positive integer. The value of a modulo n (written a mod n) is the nonnegative remainder which results when a is divided by n. We can define sum and product mod n similarly. The demonstration exhibits an addition table mod 5.

3. Construct a multiplication table mod 5.

```
File:   w2 GCD                                                          Page  1

Euclid's Greatest Common Divisor Algorithm
==============================================================
Enter larger #                  Enter smaller #
   in B4                  510     in D4                     81

        Dividend          Divisor         Quotient       Remainder
             510               81                6              24
              81               24                3               9
              24                9                2               6
               9                6                1               3
               6                3                2               0
               3                0            ERROR           ERROR
               0            ERROR            ERROR           ERROR
           ERROR            ERROR            ERROR           ERROR
           ERROR            ERROR            ERROR           ERROR
```

```
File: w2 GCD                   REVIEW/ADD/CHANGE              Escape: Main Menu
===========A===============B===============C===============D========
  1|Euclid's Greatest Common Divisor Algorithm
  2|================================================================
  3|Enter larger #                  Enter smaller #
  4|   in B4          510              in D4         81
  5|
  6|        Dividend          Divisor         Quotient       Remainder
  7|+B4             +D4              @INT(A7/B7)    +A7-(B7*C7)
  8|+B7             +D7              @INT(A8/B8)    +A8-(B8*C8)
  9|+B8             +D8              @INT(A9/B9)    +A9-(B9*C9)
 10|+B9             +D9              @INT(A10/B10)  +A10-(B10*C10)
 11|+B10            +D10             @INT(A11/B11)  +A11-(B11*C11)
 12|+B11            +D11             @INT(A12/B12)  +A12-(B12*C12)
 13|+B12            +D12             @INT(A13/B13)  +A13-(B13*C13)
 14|+B13            +D13             @INT(A14/B14)  +A14-(B14*C14)
 15|+B14            +D14             @INT(A15/B15)  +A15-(B15*C15)
 16|
 17|
 18|
------------------------------------------------------------------------------
C1: (Label, Layout-L)  Algorithm

Type entry or use @ commands                                     @-? for Help
```

Modular Arithmetic

==

N = 5

+	0	1	2	3	4	NA	N
0	0	1	2	3	4		
1	1	2	3	4	0		
2	2	3	4	0	1		
3	3	4	0	1	2		
4	4	0	1	2	3		
NA							

*	0	1	2	3	4	NA
0	0	0	0	0	0	NA
1	0	1	2	3	4	
2	0	2	4	1	3	
3	0	3	1	4	2	
4	0	4	3	2	1	
NA	NA	NA	NA	NA	NA	

```
File: w2 mod arith                REVIEW/ADD/CHANGE            Escape: Main Menu
========A========B========C========D========E========F========G========H====
  1|Modular Arithmetic
  2|========================================================================
  3|N =       5
  4|
  5|
  6|           +         0         @IF(C6+1<@IF(D6+1<@IF(E6+1<@IF(F6+1<@IF(G6+1<
  7|           |         ---------------------------------------------------
  8|0          |         @IF(A8+C6@IF(A8+D6@IF(A8+E6@IF(A8+F6@IF(A8+G6
  9|@IF(A8+1< |         @IF(A9+C6@IF(A9+D6@IF(A9+E6@IF(A9+F6@IF(A9+G6
 10|@IF(A9+1< |         @IF(A10+C@IF(A10+D@IF(A10+E@IF(A10+F@IF(A10+G
 11|@IF(A10+1|         @IF(A11+C@IF(A11+D@IF(A11+E@IF(A11+F@IF(A11+G
 12|@IF(A11+1|         @IF(A12+C@IF(A12+D@IF(A12+E@IF(A12+F@IF(A12+G
 13|@IF(A12+1|
 14|
 15|
 16|
 17|           *         0         @IF(C17+1@IF(D17+1@IF(E17+1@IF(F17+1@IF(G17+1
 18|           |         ---------------------------------------------------
----------------------------------------------------------------------------
A3: (Label) N =

Type entry or use @ commands                                    @-? for Help
```

```
File: w2 mod arith                REVIEW/ADD/CHANGE              Escape: Main Menu
========A========B========C========D========E========F========G========H====
   1|Modular Arithmetic
   2|=====================================================================
   3|N =      5
   4|
   5|
   6|            +         0        @IF(C6+1<@IF(D6+1<@IF(E6+1<@IF(F6+1<@IF(G6+1<
   7|            |         -------------------------------------------------
   8|0           |         @IF(A8+C6@IF(A8+D6@IF(A8+E6@IF(A8+F6@IF(A8+G6
   9|@IF(A8+1<   |         @IF(A9+C6@IF(A9+D6@IF(A9+E6@IF(A9+F6@IF(A9+G6
  10|@IF(A9+1<   |         @IF(A10+C@IF(A10+D@IF(A10+E@IF(A10+F@IF(A10+G
  11|@IF(A10+1   |         @IF(A11+C@IF(A11+D@IF(A11+E@IF(A11+F@IF(A11+G
  12|@IF(A11+1   |         @IF(A12+C@IF(A12+D@IF(A12+E@IF(A12+F@IF(A12+G
  13|@IF(A12+1   |
  14|
  15|
  16|
  17|            *         0        @IF(C17+1@IF(D17+1@IF(E17+1@IF(F17+1@IF(G17+1
  18|            |         -------------------------------------------------
-----------------------------------------------------------------------------
D6: (Value) @IF(C6+1<B3,C6+1,@NA)

Type entry or use @ commands                                     @-? for Help
```

```
File: w2 mod arith                REVIEW/ADD/CHANGE              Escape: Main Menu
========A========B========C========D========E========F========G========H====
   1|Modular Arithmetic
   2|=====================================================================
   3|N =      5
   4|
   5|
   6|            +         0        @IF(C6+1<@IF(D6+1<@IF(E6+1<@IF(F6+1<@IF(G6+1<
   7|            |         -------------------------------------------------
   8|0           |         @IF(A8+C6@IF(A8+D6@IF(A8+E6@IF(A8+F6@IF(A8+G6
   9|@IF(A8+1<   |         @IF(A9+C6@IF(A9+D6@IF(A9+E6@IF(A9+F6@IF(A9+G6
  10|@IF(A9+1<   |         @IF(A10+C@IF(A10+D@IF(A10+E@IF(A10+F@IF(A10+G
  11|@IF(A10+1   |         @IF(A11+C@IF(A11+D@IF(A11+E@IF(A11+F@IF(A11+G
  12|@IF(A11+1   |         @IF(A12+C@IF(A12+D@IF(A12+E@IF(A12+F@IF(A12+G
  13|@IF(A12+1   |
  14|
  15|
  16|
  17|            *         0        @IF(C17+1@IF(D17+1@IF(E17+1@IF(F17+1@IF(G17+1
  18|            |         -------------------------------------------------
-----------------------------------------------------------------------------
C8: (Value) @IF(A8+C6<B3,A8+C6,A8+C6-(@INT((A8+C6)/B3)*B3))

Type entry or use @ commands                                     @-? for Help
```

Modular Arithmetic

==

N = 5

+	0	1	2	3	4	NA	N
0	0	1	2	3	4		
1	1	2	3	4	0		
2	2	3	4	0	1		
3	3	4	0	1	2		
4	4	0	1	2	3		
NA							

*	0	1	2	3	4	NA
0	0	0	0	0	0	NA
1	0	1	2	3	4	
2	0	2	4	1	3	
3	0	3	1	4	2	
4	0	4	3	2	1	
NA	NA	NA	NA	NA	NA	

3. Pre/Post Test

1. Identify at least two (2) ways in which random-number generators are used in software designed for the secondary level mathematics education market.

2. Describe (briefly, yet tersely) how a secondary mathematics teacher might use a spreadsheet to

   a) stimulate students' critical thinking; and

   b) enhance or expand the teaching of functions.

3. State at least three (3) ways in which microcomputers' floating point representation of real numbers fail to satisfy the definition of a <u>field</u>.

4. Discuss (succinctly, yet cogently) the cases for <u>and</u> against teaching the GOTO construct in microcomputer BASIC.

4. A Participant's Dissemination Project

Using the
APPLEWORKS© SPREADSHEET
in the
MATHEMATICS CLASSROOM

O.H.A.T Conference 23 April, 1988

©Robert Silverstone

The SPREADSHEET can be a **POWERFUL** tool for mathematical teaching, exploration and research. The APPLEWORKS[c] spreadsheet will be used in this demonstration, however the techniques can be applied to any other spreadsheet. Programs such as LOTUS 1-2-3[c] and VISI-CALC[c] include transcendental functions such as trigonometric, logarithmetic and exponential.

Four appplications of the use of the spreadsheet will be explored.

a) Examination of Polynomial Functions
   1. Making a table of values that can be used for plotting the graph of the function;
   2. Locating the ROOTS and MINIMUMS and MAXIMUMS;
   3. Using NEWTON's METHOD for approximating the roots;

b) Number Theory
   1. Constructing a table of FIBONACCI numbers
   2. Evaluating a 3 x 3 determinant

c) Trigonometric Functions
   1. Making a table of values
   2. Applications
      a) Calculate area of polygon
      b) Limit SIN(X)/X
   3. Linear Interpolation

d) A Modeling problem from Economics

```
File:                      REVIEW/ADD/CHANGE               Escape: Main Menu
========A========B========C========D========E========F========G========H====
  1|
  2|
  3|
  4|
  5|
  6|
  7|
  8|
  9|
 10|
 11|
 12|
 13|
 14|
 15|
 16|
 17|
 18|
--------------------------------------------------------------------------------
A1

Type entry or use @ commands                                      Ó-? for Help
```

Introduction to using a spreadsheet

The APPLEWORKS© screen shows a grid, whose horizontal scale is from A to H and whose vertical scale goes from 1 to 18, giving a grid of 9 x 18 = 162 entries. Each of the 162 entries is called a **CELL**, and each cell holds information. A cell is named by **LETTER INTEGER**, so that E3 or H1 are cell names. A1 is called the **HOME** cell.
Information is of two types:

VALUE This is **numerical** data which can be represented by:

a) a number, such as 0 or -4.56 or 3.141592

b) a formula, such as 22/7 or E3/(H1-4*A2)

c) A spreadsheet function. These functions are part of a spreadsheet, and must begin with the character @. Some of the functions we will look at are:

@Lookup
@Sum
@MIN
@MAX

All VALUEs **must** begin with any of the following:

A digit
A decimal point (.)
A negative sign (-)
A plus sign (+)
A @

LABEL This is any other type of information. It is used to describe or identify input and output.

All calculations are performed by operating with the CELL NAMES. For example, suppose that there is a number in cell E3 and another number in cell H2. The sum of these can be placed in cell A3 by FIRST moving the highlighted cursor to cell A3, then typing +E3 + H2. The initial + sign is needed to insure that we are using values, for the expression E3 + H2 is a LABEL (why?).

One word about computation. Spreadsheets perform the operations in the order that they appear. The expression 3 + 2 * 5 will have the value 25, for it will do (3 + 2) first then multiply that by 5. To insure getting the correct answer, use parenthesis liberally. The above expression should be written as 3 + (2*5).

Introductory examples on the use uf a spreadsheet

1. This example will demonstrate the use of **VALUE** and **LABEL** entries for a cell.

The problem is to enter two numbers, say A and B, and then display A + B , A − B , A * B , A /B and A^B.

In cell A1 enter the LABEL " A =

In Cell A2 enter the LABEL " B

The quotation marks " indicate that the entry is a LABEL

The actual values of A and B will be placed in cells B1 and B2

The above instructions are symbolozed by:

A1 : " A = B1 : value of A

A2 : " B = B2 : value of B

Now continue with the following **LABELS**:

A4 : " A+B

B4 : " A-B

C4 : " A*B

D4 : " A/B

E4 : " A↑B (↑ means RAISED TO POWER)

Now enter the following computations **(VALUES)**

A5 : + A+B (+ shows VALUE, not LABEL)

B5 : + A-B

C5 : + A*B

D5 : + A/B

E5 : + A↑B

Now, move the cursor to B1 and enter a value for A

Then move the cursor to B2 and enter a value for B

2. Plotting the linear function y = m*x + b

Problem: Given 2 points (x_1,y_1) and (x_2,y_2), calculate m and b

A1 : " X1 = B1 : value of x_1

A2 : " X2 = B2 : value of x_2

C1 : " Y1 = D1 : value of y_1

C2 : " Y2 = D2 : value of y_2

Now complete the problem with:

A3 : "Delta x = B3 : +B2 - B1

C3 : "Delta y = D3 : +D2 - D1

A4 : "Slope = B4 : +D3 / B3

A5 : "Y-inter B5 : +D1 - (B4*B1)

a) <u>POLYNOMIAL FUNCTIONS</u>

Problem 1: Make a table of values for the function

$y = 3x^2 - 2x - 2$ starting at $x = -3$ and going for 10 values.

In cell A2 enter the label **X** and in cell B2 enter **f(x)**

This will be shortened to

A2 : "X (the " means information is a label)

B2 : "f(x)

C2 : "Increment (The value will be placed in D2)

The values of x will be in column A, going from A3 to A12 and the corresponding values of y will be in column B, going from B3 to B12.

In A3 will go the starting value of x

A4 : +A3 + D2 (remember, D2 is the increment)

A5 : +A4 + D2 and so on to A12.

This process can be shortened by using the **COPY** command. **Move cursor to A4.** We wish to copy this formula in the successive cells, only changing the A4 to A5, then to A6, etc. Press **OPEN-APPLE C** for copy. Since we wish to copy only this cell, press **RETURN.** The copy process is to begin at cell A5. **Move cursor to A5 and press period. Then move cursor down to cell A12 and press RETURN.** The prompt at the bottom of the screen will ask **NO CHANGE** or **RELATIVE.** We want the A4 to change to A5 and so forth, so we want **RELATIVE.** The D2 remains constant, so press **NO CHANGE.** The values for X will appear in column A.

Now put the equation in cell B3.

B3 : **3*(x↑2) - (2*x) - 2** (notice use of parenthesis)

and now copy this formula into cells B4 to B12.

To use the process,

move the cursor to cell A3 and enter a starting value.

Move the cursor to cell D2 and enter an increment.

We can now find a table of values for this.

By changing the starting value and the increment, we can explore roots and extrema of this function.

The **@MIN** function. This will find the smallest entry in a list. We want to find the Minimum value of the B column.

B14 : @MIN(B3 . . B12) and this will display the smallest entry in the list.

B15 : @CHOOSE(@min(b3 . . b12) , b3 . . b12) will print out the **POSITION** of the smallest value in the list.

The **@MAX** function works in the same way, but chooses the LARGEST value in the list.

NEWTON'S METHOD

A standard algorithm for finding the SQUARE-ROOT of a number is called the DIVIDE AND AVERAGE method. Suppose that we want to find the square-root of 10. The process is

1. Make up a guess (first approximation) G_{old}
2. A new approximation is obtained by the formula

   $$G_{new} = (G_{old} + 10/G_{old})/2$$

3. If we are not finished, let $G_{old} = G_{new}$ and return to step 2.

We can program this in the following way:

```
A1  : " N =        ( Value goes into B1 )
A2  : " Guess =    ( Value goes into B2 )
B3  : ( B2 + (B1 / B2) ) / 2
```

and copy this formula into cells B4 to B10, leaving B1 as NO CHANGE.

This is a specialized case of what is called NEWTON'S METHOD which can be used to find any valid root of a number. Newton's formula is:

$$G_{new} = G_{old} - (G^{R}_{old} - N) / (R*G^{R-1}_{old})$$

(Verify, that if R = 2, you get the DIVIDE and AVERAGE method.)

Let the value of R be in D2 ;

In B3 : B2 - ((B2↑D2 - B1) / (D2*(B2↑(D2-1))))

b) **NUMBER THEORY**

Fibonacci numbers

Fibonacci numbers are generally defined recursevly in the following manner:

F(1) = 1
F(2) = 1
F(n) = F(n-1) + F(n-2)

A1 : "F(1) = (enter value in cell B1)
A2 : "F(2) = (enter value in cell B2)
B3 : +B2 + B1 (Copy this into cells B4 - B 18)

3 x 3 Determinant

Given the determinant

a b c
d e f
g h i .

The value is obtained by:

a*(e*i - h*f) - b*(d*i - g*f) + c*(d*h - g*e)

Suppose that the determinant is entered into cells

B3 - B5
| |
D3 - D5

B10 : +B3 * (C4*D5 - (C5*D4))
B11 : +C3 * (B4*D5 - (B5*D4))
B12 : +D3 * (B4*C5 - (B5*C4))
B13 : +B10 - B11 + B12

C. TRIGONOMETRIC FUNCTIONS

APPLEWORKS[c] does not include transcendental functions such as SINE, COSINE, LOG, EXP, etc. In order to perform calculations using thse functions, it is necessary to set of a **table of values**. Using a standard table of values, set up a SINE table in the following manner:

A30 : to A39 : Enter values 0 , 10 , 20 , ··· , 90
to represent the degree measure of the angle;

B30 : to B39 : Enter the SINES of the corresponding angles.

To use this table, the spreadsheet function **@LOOKUP** will be used. Enter an angle in cell C3.

C4 : @LOOKUP(C3 , A30 . . A39)

The value in cell C3 will be "looked up' in the list A30 to A39. C3 will be located in the interval $A_K \le C3 < A_{K+1}$ and the value in B_K will be returned.

To find the COSINE of the angle entered in C3 :

$Cos(x) = sqrt(1 - sin^2(x))$

Since the sin(x) is in cell C4, we can write:

C5 : SQRT(1 - (C4↑2))

To find TAN(X)

C6 : +C4 / C5 (Tan(x) = Sin(x) / Cos(x))

Application

The area of a triangle is (1/2)*a*b*sin(C)

If B1 : a
B2 : b
B3 : C
A4 : " AREA =
B4 : (1/2) * B1 * B2 * @LOOKUP(B3 , A30 . . A39)

Application

Find Lim (SIN(h)/h) (Denominator must be in RADIANS)
h->0

To convert Deg. to Radian: r = 3.141592*deg/180

A1 : 3.141592 (PI)
B1 : 10 (Starting value, or SEED)
B2 : +B1 / 2
C2 : @LOOKUP(B2 , A30 . . A39)/(A1 * B2 / 180)
Copy B2 and C2 into cells B3 - B18 and C3 - C18

INTERPOLATION

The table gives an approximation of the actual values of the SIN(x). By the process of LINEAR INTERPOLATION, we can get a better approximation. As an example, suppose that we wish to find SIN(46). The algorithm is:

ANGLE	SINE
40	.6248
46	X
50	.7660

(X - .6248)/(.7660-.6248) = (46-40)/(50-40)
or X = (46 - 40) * (.7660 - .6248) / 10 + .6248
Given C3 = 46 , all we need is the lower value of the interval (40). We can get it by

Divide 46 by 10	(4.6)
Truncate	(4)
Multiply by 10	(40)

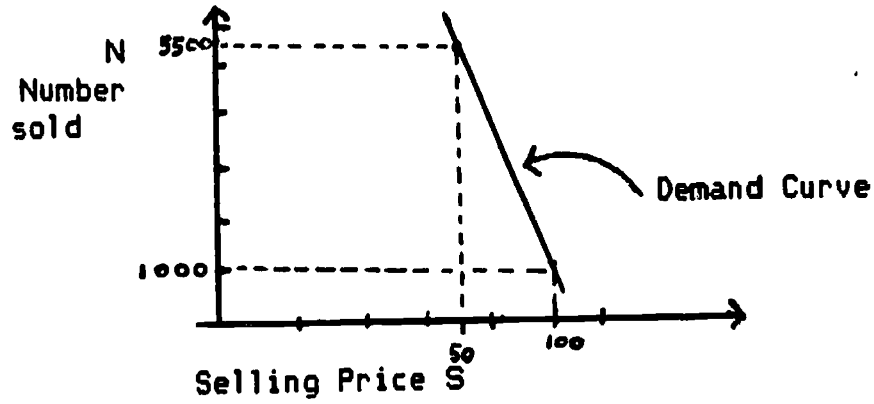
This is accomplished by the function @INT.
Suppose that we have:

```
C3 :   angle
C4 :   @INT( C3/10)*10
D4 :   @LOOKUP( C4 , A30 . . A39 )
C5 :   +C4 + 10
D5 :   @LOOKUP( C5 , A30 . . A39 )
D6 :   ( C3 - C4 ) * ( D5 - D4 ) / 10 + D4
```

and the interpolated value will be in D6.

d) A modeling problem from Economics

The Royce bicycle company wants to put a new cycle into production. It has been determined that the selling price of the cycle will strongly influence the number sold, and therefore the production rate. The Marketing Department has determined that if each bike sold for $50.00 that 5,500 bikes would be sold, and that if the price were $100.00, only 1,000 bikes would be sold. The Department believes that the relationship between the selling price (S) and the number sold (N) is given by the graph below:



What should the selling price be inorder to maximize income, and how many bicycles should be produced to achieve this?

How can the PROFITS be maximized rather than the income?

Solution

The INCOME is COST PER BIKE * NUMBER OF BIKES SOLD $I = S * N$

The TOTAL COST involves

a) A FIXED COST for research, etc F

b) A VARIABLE COST which we assume is directly proportional to the number produced $V = k N$

c) The COST of production is $C = F + V$

For this example, suppose we choose F = $4000.00 and k = $25.00 As an assignment, a similar problem, chosen from real-life can be presented and the students asked to research reasonable values for F abd k.

Problem: We wish to arrive at a relationship between INCOME and sales. We have, from (a), I = S * N. We need to eliminate N. To do this, the DEMAND CURVE (above) is used. Have students calculate its equation: N=10000-90S and substitite this into the I equation, resulting in

$$I = 10000*S - 90*S$$

Now set up a spreadsheet, where by entering S, values of I will be calculated.

We assumed that that V was linearly proportional to N What would happen if the relationship were quadratic, or square-r[illegible] or possibly even inverse?

For part 2, PROFIT = INCOME - COST