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ABSTRACT

This booklet is the sixth in a series of nine from the Teacher Training Institute at Hofstra University (New York) and describes the dual approach this course used for the presentation of mathematics history along both a chronological outline and across themes such as: non-Euclidean geometry, women in mathematics, mathematics and mysticism, and mathematics in the United States. Included in this booklet are: (1) an introduction with rationale and purpose for this course; (2) course outline of mathematical chronology with selected bibliography; (3) the course outline with selected textbook; (4) a list of guest presenters and topics; (5) a sample of instructor class handouts and participants' presentations; (6) the diagnostic pretest, the midterm assignment, and the posttest final exam; and (7) an example of a peer workshop designed by a participant. (JJK)

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HOFSTRA UNIVERSITY

TEACHER TRAINING INSTITUTE

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DISSEMINATION PACKET – SUMMER 1989

Booklet #6

DAVID KNEE AND JANET BARBERA
HISTORY AND DEVELOPMENT OF MATHEMATICS

NSF Grant # TEI8550088, 8741127

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This booklet is the sixth in a series of nine booklets which constitute the Hofstra University Teacher Training Institute (TTI) packet. The Institute was a National Science Foundation supported three-year program for exemplary secondary school mathematics teachers. Its purpose was to broaden and update the backgrounds its participants with courses and special events and to train and support them in preparing and delivering dissemination activities among their peers so that the Institute's effects would be multiplied.

This packet of booklets describes the goals, development, structure, content, successes and failures of the Institute. We expect it to be of interest and use to mathematics educators preparing their own teacher training programs and to teachers and students of mathematics exploring the many content areas described.

"The History and Development of Mathematics" was a basic course offered as part of TTI's cycle of courses. This booklet describes the course's dual approach to the presentation of historical material: chronological and by overarching theme. Some of the themes discussed were: Non-Euclidean Geometry, Women in Mathematics, Mathematics and Mysticism, and Mathematics in the USA. The course was meant to create a context for participants' mathematical knowledge and a warm-up in problem solving for the rest of the Institute's offerings. Included are a course syllabus, outlines of class units, tests, a sample of

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participant presentations, and a discussion of the History of Mathematics as a peer workshop topic.

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HISTORY & DEVELOPMENT OF MATHEMATICS
Booklet #6

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Contents

1. Introduction and General Description

Often we learn mathematics as a lifeless, completed discipline. No one created it, it was always here. It comes from no country in particular, nor from any definite time. It never developed, it is a finished, dead thing.

Of course, in reality mathematics is a human endeavor, an art and a science to be delighted in, used, appreciated, talked about. Real people discovered and still discover it, invent it, teach it, reshape it, apply it, learn it, avoid it. You do, I do. Men and women have been fascinated by number and form since before history came to be recorded, long before. In the Institute's course we wanted to present the stories, the indelible images that can so enliven the teaching of our subject: an early caveperson fascinated by the moon's changing shape above the forest trees, a shepherd tallying his flock with notches on a stick, Archimedes of Syracuse engrossed in his figures in the sand, Galois' feverish letter to posterity the night before his fatal duel, little Gauss baffling his father and his teachers with his arithmetical speed. These incidents give our science flesh, blood, historical and human context and bring it to life. A secondary school teacher who has studied the history of mathematics does not have to plan to include the material in his or her classes. I've been told this by returning participants from the Institute's course. Stories and personalities spring naturally to mind and to mouth, add dimension to the lesson and help make our classrooms warmer, friendlier places.

The primary approach to the material was chronological. But certain threads or themes span centuries and millenia and are best presented as coherent stories rather than in segmented time slices. Among these stories are:

1. The development of Euclidean and non-Euclidean geometry, deductive systems.
2. Number representation.
3. Women in Mathematics.
4. Mathematics and mysticism.
5. Non-western contributions to mathematics.
6. The history of those areas of mathematics recently added to the high school curriculum.
7. Math, Art & Aesthetics.
8. Math in the USA

9. Infinity

10. Problem Solving

11. The Nature of Numbers

12. Crises in Mathematics

Professor Al Kalfus was a guest lecturer during Cycle I and he expanded his contributions for the next Cycle. His presentations involved themes #1, 6, 9, and 10 above. In theme #1, for example, Al described Euclid's geometrical system as presented in "the elements" and the attempts over many centuries, to prove the unweildy and unintuitive parallel postulate as a theorem. Saccheri, around 1700, turned to the reductio ad absurdum technique, assuming the oppostie of what one hopes is true and arriving at a contradiction. This fruitful approach produced a bizarre non-Euclidean world, later further investigated by Lobachevsky and others, and finally realized to be as consistent as Euclid's. This important story spans over 2,000 years, involves many great mathematicians as well as theorems and ideas of geometry, logic, and philosphy and illuminates the related birth fo abstract algebra, also a 19th century phenomenon.

Besides telling the stories of mathematics and laying out its chronology as a framework for participants' knowledge, the course served many other functions. It was offered at the beginning of both cycles as an introduction to the year-long program. Many participants had been away from college level mathematics work for a while and needed a warm up, needed to reacquaint themselves with the broad canvas of mathematics and with the process of problem solving (both alone and in groups) which would be so central in the entire Institute program. We wanted them to develop friendships among fellow participants, faculty and staff and get to feel at home at Hofstra. The intensity of the Institute workload, the group lunches, team projects and the use of the better prepared participants as coaches helped in the speedy formation of network that served to support participants in this course, and continued to function for the rest of the program and beyond.

A history course is a natural place to explore problems and ideas in number theory, geometry, algebra, probability and statistics, set theory, calculus, graph theory, logic and foundations, computers, etc. The rich problem sets of our text (we used David Burton's new book on The History of Mathematics) gave us ample exercise in seeing things from both modern and ancient points of view. Another (excellent) text with excellent problem sets is the classic, "An Intro to The History of Math" by Howard Eves, which could equally well have been chosen.

Guest speakers came to present, be questioned and stay for lunch. Team work was encouraged by recommending that teams of 2 or 3 do class presentations (although writing a paper or working individually were also acceptable). The midterm was a take home problem set; participants were expected to work together but do their final drafts alone.

Obviously, the course attempted too much. Its goals were too rich and varied to be squeezed into a five-week summer semester, especialy since a companion course was also required ("Software in the Classroom" during the first cycle, and "Problem Solving via Pascal" during the second). This problem became evident soon enough and the instructors of both summer courses engaged in periodic negotiation sessions with particiants. These took place in a healthy, coopertive atmosphere and usually resulted in a lessening of requirements or the postponing of a test. For example, during the second cycle when the history course shared the summer with the Pascal course, the "talk or paper" requirement was reduced to an "outline for a possible future talk or paper". The reader should be warned that the syllabus and course requirements as presented in this booklet are composites taken from the two cycles, do not include all the cutbacks that were arrived at, and so in places may go beyond what a class can reasonably be expected to complete in one semester.

The course began with a pre-test and concluded with post-test and a course and teacher evaluation. The latter evaluation form is included in booklet #1. The post-test was presented as part I of the final exam for the course which is included in this booklet. For a fuller discussion of the pre and post-test, see booklet #1.

2. Course Content, References

A general chronological outline of the course is given below along with a list of references.

Part I Brief Survey through 1600

A. Ancient Mathematics

1. Prehistory, number systems

2. Babylonian and Egyptian arithmetic, algebra, geometry

3. Early Greek Mathematics: Thales, Pythagoras, Eudoxus, Euclid - geometry, proof, number mysticism, irrationals

4. Later Greek Mathematics: Archimedes, Apollonius, Diophantus, Pappus, Hypatia - integral calculus, mechanics, conics, number theory, geometry

B. Chinese, Indian, Arabic Mathematics

1. 'I Ching', 'Arithmetic in Nine Sections', Chu's triangle

2. Bhaskara, Brahmagupta

3. Al-Khowarizmi, Omar Khayyam: geometry, cubic equations

C. Renaissance

1. Fibonacci, Cardano, Tartaglia, Ferrari, Vieta: arithmetic, series, notation, gambling, solution of cubic and quartic equations

2. Durer, Copernicus: geometry, magic squares, astronomy

Part II Modern Era

D. 17th Century

1. Napier and logarithms, Galileo and dynamics, Kepler and planetary motion

2. Geometry: Desargues, Pascal, Descartes

3. Number Theory: Fermat

4. Probability: Huygens

5. Calculus: Newton, Leibniz

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E. 18th and Early 19th Centuries

1. Extending and exploiting the calculus, analytic mechanics

2. Complex numbers, probability, series, geometry

3. Number theory, graph theory: Gauss, Euler

4. The Bernoullis, Euler, Lagrange, DeMoivre, Laplace, Legendre

5. Analysis: Fourier, Poisson, Cauchy, Dirichlet

6. Non-Euclidean Geometries: Saccheri, Lobachevsky, Boylai, Gauss

7. Abstract Algera: Abel, Galois, Hamilton, Cayley, Grassmann

8. Computers: Babbage, Lovelace

9. Women in Mathematics: Germain, Somerville, Lovelace

10. Mathematical societies and journals

F. Later 19th Century

1. Analysis rigorized: Weierstraas, Dedekind, Riemann

2. Geometry: Mascheroni, Klein

3. Foundations, set theory, logic: Cantor, Peano, Frege

4. Women in Mathematics: Kovalevskaya, Young

5. Mathematics education in American Universities reaches European level

G. 20th Century

1. Logic, foundations: Hilbert, Russell, Godel, Tarski, Cohen

2. Topology: Hilbert, Poincare

3. Computers: Turing, Post, von Neumann

4. U.S. Math becomes world class

5. Abstraction: algebra, category theory

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David, F.N., "Games, Gods and Gambling", New York, Hafner, 1962.

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Eves, Howard, "In Mathematical Circles", 2 vols., Boston, Prindle, Weber & Schmidt, 1969.

Eves, Howard, "Great Moments in Mathematics", 2 vols., MAA

Eves, Howard, "An Introduction to the History of Mathematics", Holt, Rinehart & Winston.

Burton, David, "The History of Mathematics: An Introduction", Allyn and Bacon, 1984.

VIDEOS

Nova Series: "The Mathematical Mystery Tour"

Ascent of Man Series: "Pythagoras"
"The Clockwork Universe - Newton"

3. Syllabus for "The History and Development of Mathematics"

Text: "History of Mathematics, An Introduction"
by David Burton, Allyn & Bacon, 1985

Text Assignments: Do any 6 (at least) problems listed, per assignment.

1.	July 6	Chap 1	p. 1- 20:Ex 1c, 2c, 3c, 4, 5c, 6c, 11c&e, 12c
		Chap 2	p. 56- 66:Ex 2a, 3-6, 9.
2.	July 9	Chap 3	p. 89-112:Ex 1-6, 8, 10, 11b&c, 12a, 14, 16 p.112-4, mid 117-mid 118, mid 123-4: Ex 2-5, 9, 15, 17, 21, 23 p.129-140:Ex 2, 4, 5, 7
3.	July 13	Chap 4	p.151-173:Ex 1-10, 12, p.186-201:Ex 1, 6-9, 14, 15, 16a, 18-20, 23, 26, 27 p.214-230 (but skip 1/3 p.219 through to 1st paragraph p.220, proof of Prop 2 on p.220, top 1/2 of p.222): Ex 2, 6, 9-11.
4.	July 16	Chap 5	p.232-238: 252-256:No Ex
		Chap 6	p.259-268, 276-283:Any Ex
		Chap 7	p.301-316:Ex 1a&d, 3b&C, 5, 15 p.317-318, p.320-323:No Ex
5.	July 20	Chap 8	p.325-347:No Ex p.360-mid 385, mid p.400-408: Ex 1, 2, 4, 7, 8a&c, 11
6.	July 23	Chap 9	p.411-436:Ex (on p.440) 1d, 9, 10b mid p.452-461:Ex (on p.464) 1, 4, 5, 6, 10
7.	July 27	Chap 10	p.468-484:Ex 1, 4, 6, 10 p.484-top 487, bot p.489-490, mid p.498-bot 503, mid p.505-516:Ex (on p.522) 1, 2, 4, 6
8.	July 30	Chap 11	p.525-2/3 p.530, p.533, mid p.537-538, p.548-552, bot p.553-top 555, 1/3 p.559-560, mid p.563-582 (but skip 2/3 p.526-1/3 p.527, bot p.527-top 528, 2/3 p.528-top 529): No Ex
9.	August 3	Chap 12	p.584-611:Ex 3, 7, 9, 10, 12 p.618-top 622, 625, 2/3 p.629-top 632, bot 632-5: No Ex.

Beware of text misprints and errors.

Grade based on:

(i) Take home problem set July 16, due July 23 (colaboration OK through Monday, July 20)

(ii) Team talk or team paper. (Cancelled for Cycle II - instead just an outline for a possible future talk or paper.)

(iii) In-class final on August 3 (1 hour 20 minutes).

(iv) Participation: blackboard problems, attendance, class discussion, etc.

Special Topics to be covered in class discussions, team talks or guest lectures: Women in Math, Math in the U.S., 20th Centur Math, using history in the secondary school classroom, problem solving, the new areas introduced in "Sequential Math" - Logic, Probability and Statistics, Transformational Geometry and Abstract Algebra, etc.

Team Talks

* 2 or more on a team
* 20 minutes per team member
  handouts containing bibliography, summary & possibly exercises (2 pages)
* reserve your date and topic
* make your talk useful & interesting, avoid just reading to your audience, consider using visual aids

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4. Participant and Guest Presentations

<u>Participant Talks & Papers Presented or Outlined</u>

Mathematics in the United States through the 19th Century

Islamic Art

Finger Manipulation Arithmetic: Chisanbop

How to Teach Topics of the New Sequential Curriculum:
Probability and Transformational Geometry

Classroom Applications of Pascal's Triangle

Stonehenge

Computer Arithmetic: 2's complement and Boolean Logic
Techniques for Teaching Transformational Geometry

Machine Limitations

Women in Mathematics

Math and Mysticism of the Great Pyramid

Paradox

Early Pythagorean Propositions

The Golden Section

Origins of Mathematical Probability

Polyhedra

Math & Music

Topics in Prime Number Theory

Gematria

Magic Squares

A History of Π

A History of Algebra

A History of Computers

Fibonacci & His Numbers

The Contributions of Black Mathematics

Guest Lectures

Profs. Mona Fabricant, Sylvia Svitak: "Women in Mathematics"

Prof. Robert Bumcrot: "My Mathematical Autobiography"

Mr. Alfred Kalfus: 1) The Development of Non-Euclidean Geometry & Deductive Systems

2) Teaching Graphing Using Transformations

3) Problem Solving

4) Using History of Mathematics in the Classroom

5) Infinity

Prof. Harold Hastings: "Fractals, Computers & Ecology"

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5. Sample Class Units.

In this section, we present two sample lesson outlines, originally planned by the instructor to be given during the course. The "Growth of Mathematics in the United States" was chosen as a team presentation by two participants and so the instructor was adequately able to advice the speakers during the course of their preparations. "Mathematics and Mysticism" was delivered by the instructor in bits and pieces throughout the summer and was part of a joint NCTM presentation (Long Island NE Regional Meeting, December 1987) with co-director William J. McKeough, who is also interested in this topic. We also present two participant class presentation outlines.

GROWTH OF MATHEMATICS IN THE UNITED STATES

A. 1500-1600

1. Practical mathematics only: arithmetic, surveying, navigation, astronomy, astrology, calendar

B. 1600-1700

1. First elementary schools established
2. Harvard (1636) and William & Mary (1692) founded
3. Colleges mainly for training clerics; Math curriculum roughly at level of our present day elementary schools
4. Almanacs

C. 1700-1800

1. Emergence of the American genius
    a. Benjamin Bannecker: descendant of slave; astronomy, almanac publisher, surveyor; praised by Jefferson, admired in Europe
    b. Benjamin Franklin: self-taught, encouraged development of American education, experimented with electricity, admired in Europe; founded University of Pennsylvania; 16x16 magic square, helped found American Philosophical Society
    c. Thomas Jefferson: studied math, astronomy, sciences, published articles, admired French educational system, metric system, encouraged growth of American education, University of Virginia, architect (favored octagonal shapes)

2. Math in colleges

   a. typical curriculum

   b. libraries

   c. John Winthrop, David Rittenhouse, Isaac Greenwood

3. Publishing

   a. Transactions of American Philos. Society

   b. Books on elementary math

   c. Americans publish articles in European journals

D. <u>1800-1875</u>

1. Colleges

   a. Many small colleges founded

   b. Math currciulum expands roughly to level of todays high schools

   c. Libraries

   d. Textbooks more in use, mainly from England and translations of French works.

   e. William B. Rogers MIT (1865)

2. Periodicals and problem-puzzle magazines

3. Figures in American Education

E. <u>1875-1900</u>

1. Math education revolutionized, studied for own sake

2. John Hopkins founded by Gilman; Eliot first president

   a. Based on advanced study and original research

   b. J.J. Sylvester develops Math Department - produces many fine Ph.D.'s

   c. Cayley on faculty

3. American Mathematical Socieity (1894) developed from New York Math Society (at Columbia) by Fiske, Bulletin and Transactions begun

4. Chicago Mathematical Congress of 1893 attended by many European and American mathematicians

5. Felix Klein holds Math colloquium

6. Many Americans study abroad, receive Ph.D. in Germany, England, France

7. Benjamin Peirce, C.S. Peirce, Emory McClintock, George Halsted, Josiah Willard Gibbs, etc.

8. University of Chicago, Math Department develops under E.H. Morore, Bolsa, and H. Maschek

9. History of Math develops as separate discipline and gains scholars

10. American circle squarers and other quacks

11. President Garfield proves Pythagorean theorem

E. 1900-Present

1. United States becomes world class: Growth of contributions in all fields: Topology, Algebra, Logic, Analysis, Computers

2. European emigres to United States, enrich our mathematical life: K. Godel, A. Einstein, E. Noether, J. von Neumann, S. Eilenberg, etc.

3. E. Post, N. Weiner, P. Halmos, S. Lefshets, A. Robinson, W. Quine, M. Gardner

4. World War II developments: computers, linear programming, numerical methods

5. 1966 International Congress in Moscow awards prizes to Stephen Smale and Paul Cohen

Further Reading

1. D.E. Smith & J. Ginsburg, "History of Math in American before 1900"

2. P. Beckmann, "A History of Pi"

3. Morris Kline, "Why the Professor Can't Teach" (Chapters 2 & 3)

4. Tarwater, ed. "Bicentennial Tribute to American Mathematics"

* * * *

NUMBER AND FIGURE MYSTICISM

A. Attitudes Towards Mathematics

1. Consumer, person-in-the-street, tradesperson: Math as the helpful gnome.

2. Student of a non-science: Math as difficult, probably useless, frustrating obstacle toward obtaining ones degree.

3. Scientist, engineer: Math, the respected, trusted assistant, can always be relied upon to come up with the right numbers.

4. Mathematician, philosopher: Math is an art, a world of beautiful, irrefutable truths and elegant proofs. Frank Morley's triangle as example of an aesthetic delight and surprise.

5. Mystic: Math is eternal truth, a way of elevating the spirit, a form of meditation, an entry to the source of wondrous and deep wisdom about the universe and ourselves. Ahmes: "Complete and thorough study of all things...and the knowledge of all secrets." Morley's triangle as example of esoteric wisdom.

B. Pythagoras and the Pythagorean Attitude

1. Pythagorean Brother (and Sister)-hood and their practices - music, mathematics, astronomy.

2. All is number. The content of mathematics is wisdom: the practice of math purifies and thus frees one from the necessity of rebirth.

3. Gematria: letters also serve as numbers in Greek and Hebrew alphabets. Amicable numbers. The gematric battle between Lutheran Michael Stifell and Catholic Peter Bungus.

4. Platonic solids and figurate numbers. Kepler and the harmony of the spheres.

5. Buckminster Fuller, modern pythagorean.

C. Chance Devices Used as Oracles

1. I Ching: binary arithmetic, magic squares, clock arithmetic.

2. Astragali dice, cards: probability, relation between gambling and fortune telling.

3. Kabbalah and the permutation of letters.

Further Reading

1. Augustus DeMorgan, "A Budget fo Paradoxes"

2. Martin Gardner, "Amicable Numbers", (article)

3. Wilhelm/Baynes with C.G. Jung Intro, "I Ching"

4. F.N. David, "Games, Gods & Gambling" (Chap. 2 & 3)

5. B. Russell, "A Hisotry of Western Philosophy" (Chap. 3)

6. H. Eves, "An Intro to History of Math" (Chap. 3)

7. Joshua Tractenberg, "Jewish Magic & Superstition"

8. B. Fuller, "Synergetics, Explorations in the Geometry of Thinking"

9. John McHale, "R.B. Fuller"

10. A. Koestler, "The Sleepwalkers"

11. Martin Gardner, "Fads & Fallacies in Name of Science"

* * * *

The Contributions Of Black Mathematicians

I. The impact of the African contribution to the history of mathematics.

A. Aristotle, in his book Metaphysics wrote: " The mathematical sciences originated in the neighborhood of Egypt...".

1. The Egyptian contribution to the development of geometry and arithmetic was also substantiated in the History of Herodotus, The Commentary On The First Book Of Euclid's Elements by Proclus, and the Déscription de l'Egypte that was written by scholars who were commis - sioned by Napoleon to research the culture of the Egyptians.

2. The Rhind Papyrus and the Moscow Papyrus were discovered and gave evidence that the Egyptians wrote on practical problems on calculations required for commercial, business, and government transactions. Simple geometric rules were were applied to determine boundaries of fields, contents of granaries, and building construction.

a. The Moscow Papyrus contained the masterpiece of ancient geometry - a calculation of the volume of a truncated square pyramid.

b. The Great Pyramid at Gizeh is a testement to the high development of engineering construction and geometric form. This pyramid was constructed around 2600 B.C.

B. Muhammad ibn Muhammad, an early eighteenth century astronomer, mathematician, and mystic, was born in Katsina in northern Nigeria. He wrote a manuscript on the magical use of letter symbols to represent numbers, topics on numerology, and some formulas and examples of the construction of magic squares with an odd number of rows and columns.

1. Katsina, one of the Hausa states in northern Nigeria, was a center of learning of many African scholars by 1700.

2. Timbuktu, in ancient Mali, was the center of culture and learning during the fifteen and sixteenth centuries.

C. Although history does not give much credit to Africans in the development of pure mathematics, there is reason to believe that some of the Arab mathematicians that contributed to the development of algebra and trigonometry were of African descent.

II. Benjamin Banneker, born in 1731 in Maryland, was the first American-born Black person to be recognized as a mathematician and astronomer.

A. With little formal education and available arithmetic books, Banneker was able to perform calculations unaided by astronomical tables to the point of publishing an almanac in 1792.

B. From a borrowed watch, Banneker built a clock that struck the hours by the time he was twenty-two. People marvelled at the accomplishments of the son of slave.

C. Banneker was a part of the team of men that helped to

III. The first four Black mathematicians to receive doctorates in pure mathematics in the United States were men.

A. Elbert Frank Cox, who received his Ph.D. in pure mathematics from Cornell University in 1925, was the first Black American to receive a doctorate in mathematics.

B. Dudley Weldon Woodard received a Ph.D. from the University of Pennsylvania in 1928, and William Claytor received his doctorate from the same university in 1933.

C. Walter Richard Talbot, the former chairman of the Mathematics Department at Morgan State College in Baltimore, Maryland, received his Ph.D in mathematics from the University of Pittsburgh in 1934.

IV. The first two Black women who received doctorates both earned their degrees in 1949.

A. Evelyn Boyd Glanville was granted at Ph.D degree from Yale University.

B. Margorie Lee Browne was granted a doctorate in mathematics from the University of Michigan.
   1. Dr. Browne in an authority on the theory of groups.
   2. She served as a Professor in the Dept, of Mathematics of North Carolina Central University.

V. One of the most honored contemporary Black mathematicians is Dr. David Blackwell, distinguished Professor of Statistics at the University of California, Berkeley.

A. David Blackwell received his doctorate from the University of Illinois in 1941.

B. He is the co-author of <u>Theory of Games and Statistical Decisions</u>. He is given credit for being one of the pioneers in the theory of duals and discovered of proof of the Kuratowski Reduction Theorem by constructing a game.

C. Dr. Blackwell was the first Black person to become a member of the Institute for Advanced Study at Princeton University and is the only Black mathematician in National Academy of Science. He is a member of the American Mathematical Society and the American Statistical Association.

VI. Three Black mathematicians who have published over forty research articles in mathematics are Dr. David Blackwell, Dr. J. Earnest Wilkins, Jr. and Dr. A.T. Bharucha-Reid.

A. Dr. Wilkins, Professor of Applied Mathematical Physics at Howard University, received his doctorate at the University of Chicago.

B. Dr. Bharucha-Reid is a professor at Georgia Institute of

Questions

1. What obstacles impeded the growth of mathematical activity in Africa?

2. Outside of ancient Egypt, were there African countries that contributed to the development of mathematics that were not recorded in historical documents?

3. Why is the number of Black people who have earned advanced degrees in mathematics relatively small?

4. What teaching techniques can be used to encourage Black children to appreciate the beauty of mathematics and to continue their training in mathematics?

5. What curriculum materials can be developed to emphasize the contribution that Black men and women have made in the development of mathematics?

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Carolyn Walters
History of Math
Summer 1987
Outline Report: Fibonacci and his numbers.

A brief history of Leonardo of Pisa, the son of Bonacci, otherwise known as Filius Bonacci, could start with how he became known as Fibonacci. Another historical fact to include is that Fibonacci was the first known winner in those famous math tournaments. He was presented problems by King Frederick II and this was the first recorded such event. Fibonacci introduced his sequence in his book Liber Abaci which was published in Pisa in 1202 AD. This sequence can be used to solve the hypothetical problem of the multiplication of rabbits.

Fibonacci numbers were ignored for centuries until the early 1960's when a Brother Alfred Brousseau became fascinated with them and established a group to investigate these numbers. As a result, The Fibonacci Quarterly has been published. There is a Fibonacci Biographical and Research Center at San Jose State College.

What is so intriguing about the Fibonacci numbers? They occur in nature: pine cone, sunflowers, and the reproduction of bees; in the stock market; in classical architecture; in music, the piano octave has eight white keys and five black keys for a total of thirteen - all Fibonacci numbers; and of course, in mathematics - this sequence is always showing up from Pascal's triangle to the golden ratio.

The sequence itself has many interesting properties, some of which we have seen in class. Some of the identities of the Fibonacci numbers include:

$F_{2n+1} = F_{n+1}^2 + F_n^2$; $F_1^2 + F_2^2 + F_3^2 + \ldots + F_n^2 = F_n * F_{n+1}$;

or the fact that the product of any two even-termed consecutive numbers plus 1 is a perfect square, namely:

$F_{2n} * F_{2n+2} + 1 = F_{2n+1}^2$ and $F_{2n} * F_{2n+4} + 1 = F_{2n+2}^2$ and

$F_{2n+2} * F_{2n+4} + 1 = F_{2n+3}^2$.

If we examine Pascal's triangle we can see the Fibonacci numbers along the diagonals but we must look at the triangle as such:

Sum								
1	1							
1	1	1						
2	1	2	1					
3	1	3	3	1				
5	1	4	6	4	1			
8	1	5	10	10	5	1		
	1	6	15	20	15	6	1	
	1	7	21	35	35	21	7	1 etc.

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We have seen in class the connection of this triangle with the coefficients of the binomial theorem and the number of combinations of n things taken r at a time.

The problem of the golden ratio is to divide a line segment into two segments such that one is the geometric mean between the whole line and the other segment. Algebraically this means: a/b = b/a+b. If we let x = b/a then the solution to the equation $x^2 = x + 1$ is the golden ratio. Let us examine the powers of the golden ratio.

$$x^3 = x * x^2 = x * (x + 1) = x^2 + x = (x + 1) + x = 2x + 1$$

$$x^4 = x * x^3 = x * (2x + 1) = 2x^2 + x = 2(x + 1) + x = 3x + 2$$

$$x^5 = x * x^4 = x * (3x + 2) = 3x^2 + 2x = 3(x + 1) + 2x = 5x + 3$$

Continuing in this same manner we see the coefficients of the linear expression which is equivalent to a power of the golden ratio are Fibonacci numbers.

In doing this research, I have learned some interesting facts about Fibonacci and his numbers. More research could only lead to my amazement with these numbers. I even learned a trick to use in the classroom. Take any two positive integers and create a sequence using the rule for generating the next Fibonacci number. Take the sum of the first ten numbers and it will be the product of the seventh term and eleven. It is easily seen if we start with a and b. The sequence is: a, b, a+b, a+2b, 2a+3b, 3a+5b, 5a+8b, 8a+13b, 13a+ 21b, 21a+34b. The sum of this sequence is 55a + 88b which is 11 * (5a + 8b), the seventh term of the sequence.

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6. Pre/Post Tests, Midterm and Final

Below are the exams used for Cycle I (Summer 1986). The midterm is a take-home problem set on which collaboration is invited but write-ups are to be done individually. The post-test is Part I of the final and is very similar to the pre-test (not included here) which was given on the first day of the course. For Cycle II, we made the pre-tests and post-tests exactly alike (see booklet #1).

History of Mathematics **Midterm Problem Set** July 1986
Due on Tuesday, July 22 Confer on solutions/individual write-ups

1. a) Find all prime factors of 50!
   b) Give prime decomposition of 10587.
   c) Find gcd (1729, 387)

2. P.65, ex 5: Show that the Babylonian formula for the volume of a truncated square pyramid reduces to that of the Moscow Papyrus.

3. P.315, ex 1d (corrected): Find all 3 roots of

$$x^3 + 64 = 6x^2 + 24x.$$

4. Prove that any positive integer can be written as sum of Fibonocci numbers, none used more than once.

5. Find (anywhere or anyhow) & prove correct a folmula for

$$\sum_{i=1}^{n} i^4$$

6. Find all integral triangles whose perimeter equals its area. (Hint in class.)

7. Verify Archimedes' assertion that area of parabola segment = 4/3 Δ for special case where the chord bounding the segment is perpendicular to the axis of symmetry of the parabola. Use calculus. You may assume that any parabola is representable as

$y = ax^2$.

History of Math
MATH 285, 286

Teacher Training Institute
FINAL

July 29, 1986
David Knee

PART I (Post Test)

NAME______________________________

1. Name 3 women mathematicians and give some facts about each (accomplishments, dates, places, biographical tidbits, etc.)

2. Identify these mathematicians:

   Lobachevsky

   Laplace

   Fermat

   Thales

   Diophantus

   Archimedes

3. a) Define amicable pair

   b) Find the amicable mate of 1210

   c) Show that if a and b form an amicable pair then one is deficient and the other abundant.

4. Identify each of these items briefly:

   Triangular number

   <u>Discours sur la mèthode</u>

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Saccheri quadrilateral

Egypt's greatest pyramid

fractal

$E = mc^2$

5. An urn contains 2 red balls and 5 yellow ones. One ball is drawn and replaced and then a second ball is drawn. What is the probability that: (don't work out the arithemetic)

   a) Both are yellow?

   b) They are of different colors?

   c) They are the same color?

   d) Both are red, given they are of the same color?

PART II

6. Match up by inserting correct letter in blank (not necessarily one-to-one)

Felix Klein	____	a.	Weierstrass
$x^n + y^n = z^n$, $n>2$	____	b.	Descartes
ε-δ definitions	____	c.	J. vonNeumann
Je pense, donc je suis	____	d.	Kronecker
100 = 17 + 83, for example	____	e.	Erlanger Program
Teacher of Kovalevsky	____	f.	Goldbach conjecture
Cantor's nemesis	____	g.	Paul Cohen
astragalus	____	h.	heel bone
$a \equiv b \pmod{n}$	____	i.	Gauss
a continuous, nowhere-differentable function	____	j.	Fermat's last theorem
π, for example	____	k.	transcendental number

7. Using Newton's method for approximating zeros of functions,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad \text{find } x_2$$

for $f(x) = 3x^2 - 4x - 3$ if the initial guess, x_1, is 5.

8. Prove that $\binom{n}{r+1} = \binom{n}{r} \cdot \frac{n-r}{r+1}$ (for $0 \leq r \leq n-1$)

9. Sketch the history of any <u>one</u> of these mathematical stories:

a) perfect #'s b) non-Euclidean Geometry
c) development of the Calculus

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7. Peer Workshops

One technique for multiplying the Institute's effectiveness was to have participants create in-service workshops for their colleagues back at their home districts. Ms. Janet Barbera of Walt Whitman High School in Huntington, NY presented an extensive workshop at her school on the History of Mathematics, Computer Graphics and Pascal. These topics were arrived at as the results of a Needs Assessment Survey.

We include here brief descriptions of the entire workshop and the History segment, three excerpts from the Time Line Ms. Barbera used as the underlying historical skeleton, and her historical bibliography.

Ms. Barbera writes:

Participation in the Hofstra-NSF Teacher Training Institute and fulfilling its Inservice Course teaching requirement were both positive learning experiences for me. I have a better understanding of the meaning of education from observing myself and others as we tested ourselves in unfamiliar, demanding situations. The outstanding support of the Hofstra faculty made it possible to risk failure and achieve success.

In planning the workshop offerings for my Inservice obligation I attempted to reproduce as much as possible of the TTI program. I had had prior experience with both history of mathematics and Pascal so assimilating that material and organizing it for presentation was challenging but manageable. However, the mathematical modeling course and its emphasis on computer graphics were totally new to me. The visual impact of computer graphics was so stunnung I was determined to share my excitement with the Inservice participants. Their reaction was enthusiastic and they are now more willing to use computers as an aid to instruction in their own classrooms.

My methods of presentation varied geatly from the usual chalk and blackboard mathematics lecture. For the history of mathematics I included slides on Islamic art, copies of paintings and folk art, recorded music appropriate to each particular time period, and examples of computer art such as fractals. For the computer graphics lectures I used a Macintosh computer with a large screen projector as well as copies of the programs and of their printouts. For the Pascal lectures I used the Mac and projector for the introducton to linked lists (Prof. R. Greenwell lent me a wonderful software demonstration), a short film on sorting techniques, and copies of sample programs with their printouts. I also simulated problems at the blackboard for stacks, queues, and trees before reviewing the actual program code.

I am very grateful to the Hofstra faculty for including me in the TTI program. Their excitement about mathematics and their dedication to teaching were truly inspirational. It was a privilege to work with scholars who encouraged exploration of new ideas and who welcomed insights from the class members. They have influenced my teaching in both mathematical content and classroom style. I am hopeful that I have communicated my enthusiasm to the participants in my workshops and to my students.

Outline and Relation to Teacher-Training Institute

I. History of Mathematics

The TTI course work formed the major part of this workshop. I presented the trends in mathematical thinking in chronological periods, systems of numeration and computation, applications of mathematics to societal needs, and the interrelation between mathematics and culture (philosophy, religion, art, and music). Biographical information on both men and women in mathematics was included from my own reading. A short discussion of computer science concluded the lecture.

II. Computer Graphics

Two sessions on graphics were presented. These talks were developed from the TTI course on mathematical modeling. I used the population models, random number evaluation, and fractal examples. For greater relevance to the interests of my class members, I also included sound, animation, and the graphing of algebraic and trigonometric functions. The discussions were directed to enriching comprehension through visual images as well as to understanding specific programming techniques.

III. Pascal

The three Pascal workshops on data structures extended the information taught in the TTI Pascal course. Approximately one half of the workshop participants had already taken an inservice course I taught in Spring, 1987 in Pascal (two credits, 30 hours). I therefore presented new work for their benefit but I tried to keep key ideas uncluttered so that the novices could follow the main purposes of the data management techniques. I presented linked lists, stacks, queues, and binary trees (including preorder-inorder-postorder notation and the heap sort).

DESCRIPTION OF HISTORY SEGMENT

The Time Line served as a focus for all the material presented in the workshop. Mathematicians, inventors, musicians, artists, political events, authors, explorers, and other significant people/events were organized chronologically in order to give a sense of historical and cultural perspective for each mathematical discussion.

Short digressions enriched the chronological listing at appropriate points. Egyptian and Babylonian number systems as well as Egyptian volume formulas comprised the first short discussion. The Greek emphasis on proof, the use of figurate numbers, and some early Diophantine equations were the next topics under consideration.

The role of Islam in preserving Greek culture was discussed. A slide presentation of Islamic art emphasized the geometry of tessellations and the role of mathematics in the everyday life of the Islamic culture.

Presenting the Renaissance development of perspective in art and the advance of music theory offered an opportunity for examining paintings and listening to some representative recordings. The improvements in perspective were shown to relate to projective geometry and its application to map making.

The eighteenth century's emphasis on materialism and determinism was shown to be a direct result of mathematical and scientific advances during the Renaissance. Discussing the effects on philosophy, art, music, and literature again offered an opportunity for visual and aural entertainment. The discovery of non-Euclidean geometries was presented as a significant breakthrough in mathematical thinking.

The romanticism of the nineteenth century was a reaction to the rigidity of the previous century. Again, the workshop lecture used music and art to convey the new mood of the society. Riemann's spherical geometry was also discussed as a startling new direction for mathematics.

The workshop analysis of recent times indicated that rapid changes have taken place. The revolutionary mathematical thinking of the twentieth century was characterized by the theory of relativity, transfinite numbers, and logical paradoxes. The advent of computers brought a whole new world of mathematical techniques into being. Societal changes have been profound. Atonal music, cubism, new philosophies, religious movements, and many other significant new developments all indicate that old rules no longer apply.

Biographical anecdotes were used extensively throughout the entire lecture. Personalizing discussions of mathematics is a very successful technique in addressing any audience but is especially effective in speaking to high school students.

TIME LINE

4000 BC

Farmers along the Danube River
Towns on the Tigris-Euphrates Plain
Farming villages in northern China
Indians hunting and farming in North and South America
Organized society of hunters and herders in the Sahara

3000-1500 BC

Indus valley civilization in northern India: cities, houses with courtyards, modern drainage systems

2000-1500 BC

Mycenaean and Minoan Greeks
American Indians had permanent villages, domesticated dogs

1900-1400 BC

Stonehenge built

1523-1027 BC

Shang dynasty ruled China: class system, writing, money

1200 BC

Trojan Wars

850-600 BC

Iliad and Odyssey by Homer

605-562 BC

Nebuchadnezzar II King of Babylon: Hanging Gardens, Tower of Babel

569-501 BC

Pythagoras

551-479 BC

Confucius teaching in China

500 BC

Abacus invented (China)

1776

Declaration of Independence, Philadelphia

1776-1831

Sophie Germain: Higher arithmetic, elasticity

1777-1855

Carl Friedrich Gauss: Fundamental Theorem of Algebra, modulo arithmetic (biquadratic residue classes)

1780

Bifocals (Benjamin Franklin, USA)

1780-1872

Mary Fairfax Somerville: Mechanisms of the Heavens, Molecular and Microscopic Science, Tides of the Ocean and Atmosphere

1783

Balloon (J.M. and J.E. Montgolfier, France)

1788

U.S. Constitution ratified

1788-1867

Jean-Victor Poncelet: projective geometry (duality and continuity)

1789

French Revolution

1789-1857

Augustin-Louis Cauchy: limit theory, groups, indirect proof

1793-1856

Nikolas Ivanovich Lobatchevsky: non-Euclidean geometry

1797-1828

Franz Schubert

1935

Parking meter (Carlton C. Magee, USA)
Radar (Robert A. Watson-Watt, England)

1936

Spanish Civil War

1937

Jet engine (Fran Whittle, England)

1939-1945

World War II

1944

Mark I: First automatic, general-purpose, digital calculator (Howard Aiken, USA)

1946

Univac: electronic computer (J. Presper Eckert, Jchn W. Mauchly, and Harold F. Silver, USA)

1948

Xerography (Chester Carlson, USA)

1957

FORTRAN: Formula translation (John Backus and Irving Ziller, USA)

1959

COBOL: Common Business Oriented Language (Grace Hopper, USA)

1964

BASIC: Beginner's All-Purpose Symbolic Instruction Code (John G. Kemeny and Thomas E. Kurtz, USA)

1969

Pascal (Niklaus Wirth, Switzerland)

1975

Fractals (computer art and applications), Benoit Mandelbrot

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