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AUTHOR	Hutter, Ruth
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ABSTRACT

This booklet is the third in a series of nine from the Teacher Training Institute at Hofstra University (New York) and includes 11 samples of computer program lessons designed to enhance the teaching of secondary school mathematics. An introductory section includes the purpose and rationale for this series of software programs, the role of the computer in school mathematics, and contact information for availability of programs. An outline of the specific topic, the description of the program sequence, the required mathematical background, the program uses, the suggested homework assignments, and a pedagogical commentary are included for each of the following program topics: (1) the sine relationship in the first quadrant; (2) the cosine relationship in the first quadrant; (3) the sine and cosine relationships in all quadrants; (4) the linear combination of sines and cosines; (5) the inverse of a parabola; (6) the inverse of the sine function; (7) the straight line; (8) the upright parabola; (9) more graphs of parabolas; (10) the absolute value function and its graph; and (11) general parameters for graphing functions. (JJK)

HOFSTRA UNIVERSITY

TEACHER TRAINING INSTITUTE

Department of Mathematics and School of Secondary Education
Hofstra University
Hempstead, NY 11550

DISSEMINATION PACKET - SUMMER 1989

Booklet #3

RUTH HUTTER

THE COMPUTER GOES TO THE FRONT OF THE CLASSROOM

NSF Grant # TEI8550088 8741127

This booklet is the third in a series of nine booklets which constitute the Hofstra University Teacher Training Institute (TTI) packet. The Institute was a National Science Foundation supported three-year program for exemplary secondary school mathematics teachers. Its purpose was to broaden and update the backgrounds its participants with courses and special events and to train and support them in preparing and delivering dissemination activities among their peers so that the Institute's effects could be multiplied.

This packet of booklets describes the goals, development, structure, content, successes and failures of the Institute. We expect it to be of interest and use to mathematics educators preparing their own teacher training programs and to teachers and students of mathematics exploring the many content areas described.

"Software in the Classroom" was a basic topic offered as part of the TTI's cycle of courses. This booklet des^ribes the work of Ruth & Rudolph Hutter on software which assists in the teaching of various topics of the high school mathematics curriculum, such as the behavior of the trigonometric and quadratic functions. This software emphasizes understanding above drill and makes use of the computer's unique capabilities in unison with the teacher, who acts as an indispensible guide.

TEACHER-TRAINING INSTITUTE REPORT #3:

THE COMPUTER GOES TO THE FRONT OF THE CLASSROOM

by

Ruth Hutter
Mathematics Department
Borough of Manhattan Community College
New York, NY 10007

The accompanying disk is for use on the Apple II computer. A review of the original software, of which the disk is a sampler, is to be found at the end of this booklet, and was provided through the courtesy of R. & R. Hutter

The Computer Goes To The Front Of The Classroom.

by
Ruth Hutter
Mathematics Department
Borough of Manhatten Community College.
199 Chambers Street
New York , N.Y. 10007.

The programs included in this report are samples of a whole group of software designed to enhance the teaching of the subject matter contained in the curriculum of the High School. Each program is accompanied by printed information giving a detailed description, and a variety of pedagogical suggestions.

Purpose and Philosophy.

It is my belief that in order to do well in mathematics at any age level it is necessary to understand the concepts on which definitions and operations are based. Successful applications of mathematics to explain and describe quantitative relations in our environment can only be achieved if learning is conceived as understanding.

Drill, which is offered in abundance on most educational software, should only be used when a mechanization of certain operations leads to an economy of thinking.

"Math Anxiety" is most often nothing but fear that our mechanical memory will fail us, and that the failure will cause us to make the most absurd mistakes.

I am convinced that the difficulties experienced in learning do not lie in the nature of mathematics itself, but that the reason for poor performance is the method by which it is taught.

The role of the computer.

The computer could be very helpful in the teaching of mathematics. Its capability to create dynamic pictures is often far more effective than drawings in textbooks and on blackboards in clarifying the meaning of concepts and operations.

Unfortunately, the majority of educational programs presently on the market are tutorial, drill, or testing programs; they do not make use of the unique capabilities of the computer just discussed, even though they are often technically competent. Tutorial programs are no replacements for a good teacher since the procedures programmed into the computer are by necessity inflexible.

Teachers often hesitate to use the computer in the classroom because they believe it to be time-consuming. It has been the experience, however, that understanding of fundamental ideas can shorten considerably the time spent on repetition and review. It also leads to faster generalization of ideas and lets the student himself see the variety of applications without discussing a large number of examples.

Other objections, voiced frequently, are that the teacher does not possess the necessary computer knowledge. Our programs were designed to require hardly any knowledge other than being able to read and to "push buttons".

It should be emphasized again that the computer is only an excellent assistant to the teacher but does in no way replace him or her. Since the computer is not a mind-reader it cannot sense the difficulties a student may experience ir a particular situation.

Additional notes.

If the receivers of the report-packages have tried the programs on the accompanying disk, the author is interested in their comments. The author is also offering to conduct workshops, where the use of these and other programs can be demonstrated in simulated classroom situations.

Time, place, and specific subject matter can be arranged by contacting Mrs. Ruth Hutter, 445 East 80 th. Street, New York, N.Y. 10021, Tel. (212)-249-4563.

The programs included with this report are:

1. SINE (1.QUADRANT)
2. COSINE (1.QUADRANT)
3. SINE AND COSINE (ALL QUAD'S)
4. COS(BX)+SIN(AX)
5. INVERSE-PARABOLA
6. INVERSE-SINE
7. STRAIGHT LINE
8. UPRIGHT PARABOLA
9. GRAPH PARABOLA
10. ABS(X)
11. FUNC-PLOT (PARAMETERS)

Information on other programs may be obtained from the author.

DYNA-GRAPHS

BY

RUTH RUDOLF

HUTTER

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Please note:

Program #1 SINE (1.QUADRANT).
General Topic: Trigonometry.

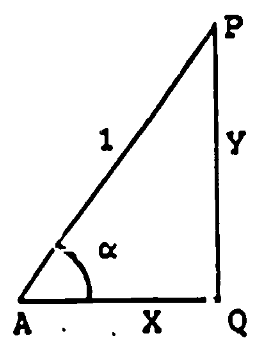
A. Specific Topic:

1. Values of the sine-function of an angle α, $0^\circ <= \alpha <= 90^\circ$.
2. Graphic representation of the sine curve for $0^\circ <= \alpha <= 90^\circ$.

B. Description of Program:

Draws a sequence of right triangles in a unit circle. In every triangle AQP (see figure below) sin(α) is then represented by the ratio PQ/PA = y/|1|=y.

The angle values (in degrees) are then transferred to the extension of the horizontal axis of the coordinate system, the corresponding y-values are drawn in, and their endpoints are connected by a curve, the sine curve.



C. Required Mathematical Background:

1. Knowledge of the definition of the sine as a ratio.
2. Graphing in rectangular system of coordinates.
3. Knowledge of relationship between degree and radian measure of an angle.

D. Use of Program:

To get a good understanding of the values of the sine-function in the first quadrant and the graph of the function.

Visualization of the increasing nature of the sine-function in the first quadrant.

(use of space bar permits quick interruption of program in progress!)

E. Class- and Homework:

Let students draw a unit circle on graph paper and have them draw a curve similar to the one on the screen, using y-values from the unit circle.

F. Pedagogical Commentary:

The definition of the trigonometric ratios starting with a right triangle often makes it difficult for the student to associate these ratios with the idea of a function.

The visualization on the screen makes it easy to understand the meaning of the sine function in terms of the sides of a right triangle.

Placing the quarter circle into a cartesian coordinate system and describing the trigonometric ratios in terms of the coordinates of points on the circumference of the circle and their distance from the origin leads to an easily intelligible introduction of the trigonometric functions for angles larger than 90 degrees.

The signs of trigonometric functions in the four quadrants are quickly learned; no great amount of memorization drill is necessary, provided the student is familiar with the cartesian system of coordinates.

Program #2 COSINE (1.QUADRANT).
General Topic: Trigonometry.

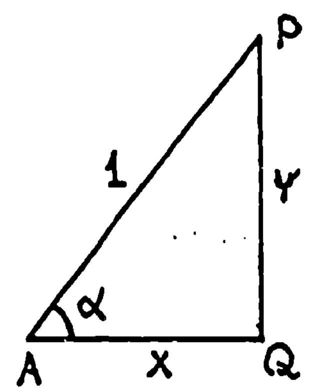
A. Specific Topic:

1. Values of the cosine-function of an angle $0° <= \alpha <= 90°$.
2. Graphic representation of the cosine curve for $0 <= \alpha <= 90°$.

B. Description of Program:

Draws a sequence of right triangles in a unit circle. In every triangle AQP (see figure below) $\cos(\alpha)$ is represented by the ratio $AQ/AP = x/|1| = x$.

The angle values (in degrees) are transferred to the extension of the vertical axis of the coordinate system, the corresponding x-values are drawn in, and their endpoints are connected by a curve, the cosine curve rotated by 90°.



This results in a plot of the cosine function for angles between 0° and 90° in an unconventional position. The next step in the program rotates the cosine ordinates (without changing their length) counterclockwise by 90 degrees and thereby bringing the cosine curve into the conventional position.

C. Required Mathematical Background:

1. Knowledge of the definition of the cosine as a ratio.
2. Graphing in rectangular system of coordinates.
3. Knowledge of relationship between degrees and radian measure of an angle.

D. Use of Program:

To get a good understanding of the values of the cosine function in the first quadrant and the graph of the function.

Visualization of the decreasing nature of the cosine function in the first quadrant.

(the use of space bar permits quick interruption of the program in progress!)

E. Class- and Homework:

Let students draw a unit circle on graph paper and have them draw a curve similar to the one on the screen, using x-values from the unit circle.

F. Pedagogical Commentary:

See program #1.

Program #3 SINE AND COSINE (ALL QUAD'S).
General Topic: Trigonometry.

A. Specific Topic:

1. Values of sine and cosine functions ($0^\circ <$ $<= 360^\circ$).
2. Graphical representation of sine and cosine curves for $0^\circ <= \alpha <= 360^\circ (2\pi)$.

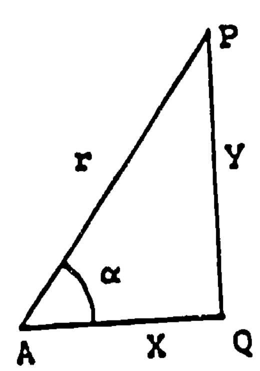
B. Description of Program:

Draws a sequence of right triangles in a unit circle. In every triangle AQP (see figure below) sin(α) is then represented by the ratio PQ/PA=y/|1|=y, and cos(α) is represented by the ratio AQ/AP= x/|1|=x.

The angle values of α in degrees are then transferred to an extension of the horizontal axis (for the sine values) and to an extension of the vertical axis (for the cosine values). The corresponding y-values (ordinates) are then drawn in for the sine curve above and below the x-axis. For the cosine curve the x-values (abscissas) are drawn to the right and the left of the (negative) y-axis.

The endpoints of the ordinates (abscissas) are then connected to give the sine (cosine) curves.

In order to produce the conventional position of the cosine curve the horizontal abscissas are rotated into a vertical position. The sine and cosine curves are then shown again in their usual positions. The four quadrants are indicated by vertical lines drawn at π/2, π, 3/2π, 2π.



C. Required Mathematical Background:

1. Definition of sine and cosine functions: sin(α)=y/|r|, cos(α)=x/|r|.
2. Graphing in cartesian coordinates.
3. Knowledge of relationship between radian and degree measures of angles.

D. Use of Program:

To get a good understanding of the behavior of the sine and cosine functions in the four quadrants and to illustrate the relationship of the sine and cosine curves.

The program can also be used to demonstrate the periodic nature of the sine and cosine functions.

The program will facilitate the memorization of the function values (together with their signs) and it can be used to introduce the idea of 'reference' angle.

E. Class- and Homework:

The program can be used to illustrate reasonableness of an answer involving function values, to visualize points of intersections of the sine and cosine curves, to show maxima and minima values, and to give an approximate solution to simple inequalities, such as sin(α)<1/2.

F. Pedagogical Commentary:

Program #3 is an extension of programs #1 and #2 ,the user should look at those programs agai .

Simultaneous graphs of sine and cosine functions permit the student to learn many facts about these functions.

For instance, points of intersection, comparison of the behavior of these functions in certain intervals of the domain (increasing or decreasing, maxima or minima).

Under the assumption that the circle is a unit circle the ranges of the functions can easily be established.

The graph shows the behavior of the functions in the four quadrants. The radian measure in the graph might be converted to degree measure by the student as a homework assignment.

Program #4 COS(BX)+SIN(AX). General Topic: Functions.

A. Specific Topic:

Combination of functions by addition of ordinates.

B. Description of Program:

Sketches the graphs of y=sin(ax) and y=cos(bx) with user-selected frequencies a,b. Demonstrates visually the addition of the ordinates of the sine curve to those of the cosine curve, and then produces the curve of the sum of the two functions.

Two speeds are available to show the process.

C. Required Mathematical Background:

1. Knowledge of trigonometric functions.
2. Curve plotting.

D. Use of Program:

Contributes to the study of graphing of more complicated curves. The student might be asked to graph curves with varying frequencies and to compare his results with those produced by the computer.

E. Class- and Homework:

Since the method of adding two sinusoidal functions can also be applied to other types of functions, the student may be asked to graph functions such as

$y=x+\sin(ax)$

or

$y=3x^2+\cos(x)$,

and then use program #11 for comparison of results.

F. Pedagogical Commentary:

The program presents a graphical representation of the addition process applied to two functions.

It attempts to show visually the addition of two functions by adding the ordinates of one function to the other.

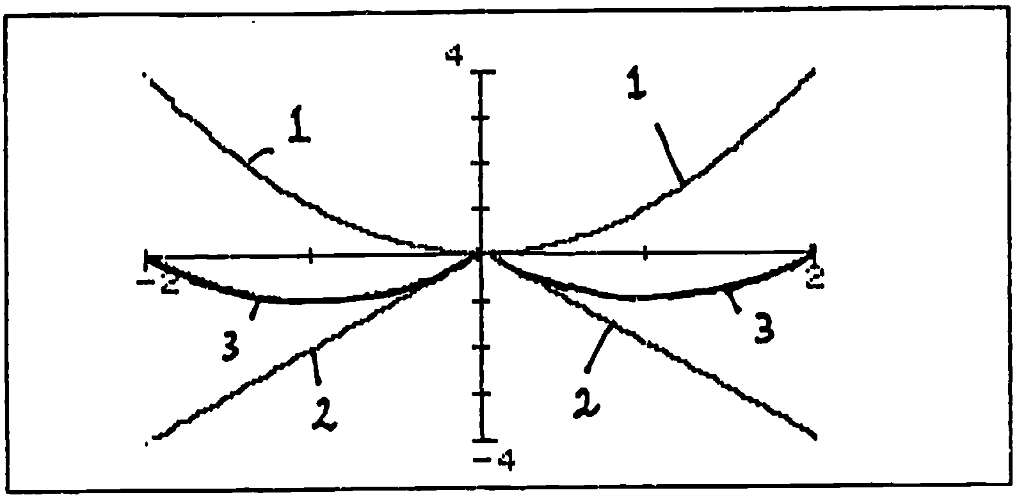
Let students use the same procedure for the following functions:

$f(x)=x^2-2\cdot|x|$

and

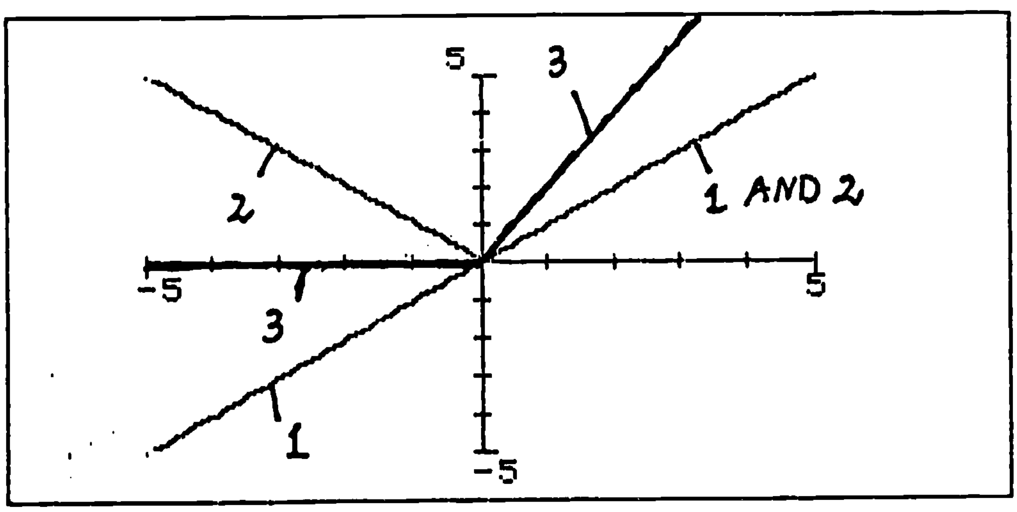
$g(x)=x+|x|$.

The results may be verified by using program #11 with proper parameters.
(see attached graphs which were obtained by this method).



$F(x) = A \cdot x^2 + B \cdot (-2 \cdot |x|)$

#	A	B	C	D
1	1	0	0	0
2	0	1	0	0
3	1	1	0	0



$F(x) = Ax + B \cdot |x|$

#	A	B	C	D
1	1	0	0	0
2	0	1	0	0
3	1	1	0	0

Program #5 INVERSE PARABOLA.

General Topic: Functions and Relations.

A. Specific Topic:

1. Inverse functions and relations.
2. Conic sections-Parabola.

B. Description of Program:

Plots the inverse of $y=x^2$ by showing that the image graph $y=\pm\sqrt{x}$ is produced by reflection of the original plot on the line $y=x$.

Also illustrates that if P(object) has coordinates x,y, the coordinates of P(image) are y,x.

C. Required Mathematical Background:

Knowledge of the coordinate geometry of the conic sections.

D. Use of Program:

Demonstrates that if $y=f(x)$, $y=f^{-1}(x)$ is not always a function. A one-to-one relationship between x and y for the inverse $y=f^{-1}(x)$ exists only in an interval where the original function $y=f(x)$ is either increasing only or decreasing only.

Example: If $y=x^2$ and $x>=0$ then $+\sqrt{x}$ is the inverse function. Similarly, if $y=x^2$ and $x<=0$ then $-\sqrt{x}$ is the inverse function.

E. Class- and Homework:

Select problems from textbook. Let students investigate whether a given function has an inverse function and in which interval.

F. Pedagogical Commentary:

The program illustrates two methods of obtaining "inverses". One method is based on reflection of points of the original curve on the line $y=x$. The other method obtains the inverse by interchanging x and y and solving the resulting equation for y.

If the original curve is either monotonically increasing or decreasing in the defined domain then its inverse is a function. This is also illustrated in the graphs.

Program #:6 INVERSE SINE.
General Topic: Functions and Relations.

A. Specific Topic:

Inverse functions.

B. Description of Program:

Plots the sine curve between $-2\pi <= x <= 2\pi$ and reflects it on the line $y=x$. The graph of the reflected curve (arcsin(x), inverse sine) is obtained by drawing perpendiculars from points on the original curve to the line $y=x$ and extending the perpendiculars to image points symmetrical to the points of the original function. $y=x$ becomes the perpendicular bisector of the lines connecting the object and image points.

Since the sine curve is monotonically increasing for $-\pi/2 <= x <= +\pi/2$, $y=\arcsin(x)$ can be considered a representation of the arc-sine-function for $-1 <= x <= +1$. This is shown in the graph by reenforcement of the portion of the arcsin-curve between $x=-1$ and $x=+1$.

C. Required Mathematical Background:

1. Understanding of the function concept.
2. Knowledge of trigonometry.
3. Curve plotting.

D. Use of Program:

In addition to its specific purpose of discussing the arcsine-function, the program can be used to introduce the student to the general concept of inverse relations and functions.

E. Class- and Homework:

Let students graph the arcsine-function by the demonstrated procedure.

F. Pedagogical Commentary:

See the commentary for program #5.
In the present program the method of interchanging x and y is not illustrated.

It should be emphasized that the periodicity of trigonometric functions creates problems of interpretation related to the inverse trigonometric function. Clearly, the relation y=arc sin(x) is not a function. To each value of x from the domain of the arc sine ($-1<=x<=1$) there correspond infinitely many values of y, where $-\infty < y < +\infty$. Therefore, the range is restricted to its "prinipal value", i.e. $-\frac{\pi}{2}<=y<=\frac{\pi}{2}$. That range is considered the range of the arc sine function. The graph indicates that range.

Program #7 STRAIGHT LINE.
General Topic: Analytic Geometry.

A. Specific Topic:

Analytic Geometry of the straight line.

B. Description of Program:

Draws a straight line through two user - chosen points x_1, y_1 and x_2, y_2. Shows the slope of the line by drawing $y_2-y_1=\Delta y$ and $x_2-x_1=\Delta x$, and prints its numerical value. It then asks for another x-value $x=x_3$, computes the corresponding y-value $y=y_3$, and by drawing y_3-y_1 and x_3-x_1 shows by similar triangles that:

$$\frac{y_2-y_1}{x_2-x_1} = \frac{y_3-y_1}{x_3-x_1} = \left(\frac{y_3-y_2}{x_3-x_2}\right)$$

C. Required Mathematical Background:

1. Elementary Algebra.
2. Knowledge of similar triangles.
3. Graphing of straight lines.

D. Use of Program:

This program emphasizes the constancy of slope of a straight line and, therefore, can be used for the derivation of the equation of a straight line:

$$\frac{y-y_1}{x-x_1} = m$$

Different forms of the equation can then be obtained from the above formula as follows:

$y-y_1=m(x-x_1)$ (point-slope form)

$y=mx+(-mx_1+y_1)$

Letting $a=m$ and $b=-mx_1+y_1$ we have

$y=ax+b$ (slope-intercept form)

This can also be written as:

$$Ax+By+C=0 \qquad \text{(standard form)}$$

which can lead to:

$$\frac{A}{-C}x + \frac{B}{-C}y = 1 \qquad \text{or} \qquad \frac{x}{\left(-\frac{C}{A}\right)} + \frac{y}{\left(-\frac{C}{B}\right)} = 1$$

Letting:

$$p = -\frac{C}{A} \qquad \text{and} \qquad q = -\frac{C}{B}$$

we have

$$\frac{x}{p} + \frac{y}{q} = 1 , \qquad \text{(intercept form)}$$

where p and q are the x- and y-intercepts, respectively.
Zero slope and infinite slope can also be demonstrated.

E. Class- and Homework:

Select textbook problems for analytical discussion of linear functions and use computer for verification.

F. Pedagogical Commentary:

By selecting x2 equal to x1 and y2 differing from y1 a line parallel to the y-axis (with infinite slope) is obtained. Letting y2 be equal to y1 and selecting x2 different from x1 a line parallel to the x-axis (with zero slope) is produced.

The program should be helpful in demonstrating that the student needs only to remember one form of the equation of a straight line, namely

$$\frac{y-y1}{x-x1} = m \qquad (1)$$

The equation through two points only requires the calculation of the slope:

$$\frac{y2-y1}{x2-x1} = m$$

and the substitution of the coordinates of either point.
If the slope and the y-intercept (0,B) are given the equation is given by:

$$\frac{y-B}{x-0} = m$$

In case the x- and y-intercepts are given (x1,0 and 0,y2) we obtain the slope as:

$$\frac{y2-0}{0-x1} = m$$

and the equation becomes:

$$\frac{y-y2}{x-0} = \frac{y2}{-x1}$$

which leads to:

$$\frac{x}{x1} + \frac{y}{y1} = 1$$

which is the "intercept" equation.

Program #8 UPRIGHT PARABOLA.
General Topic: Analytic Geometry.

A. **Specific Topic:**

Locus properties of a parabola.

B. **Description of Program:**

Draws a line (directrix), a fixed point (focus) and points such that their distance to the fixed point equals the distance to the fixed line. The curve connecting these points is a parabola.

C. **Required Mathematical Background:**

1. Facility in algebraic manipulations.
2. Formula for distance between two points.
3. Knowledge of graphing.

D. **Use of Program:**

Illustrations emphasizing the locus properties of points lying on the curve facilitate the derivation of the equation of the origin-centered parabola.

In this program the axis of symmetry is along the y-axis. For demonstration of the inverse parabola, $x=y^2$, (axis of symmetry along the x-axis) use program #5 (menu #2).

For demonstration of influence of parameters on the shape of a parabola, use programs #9 and #11.

E. **Class- and Homework:**

Read pertinent material in textbook.

F. Pedagogical Commentary:

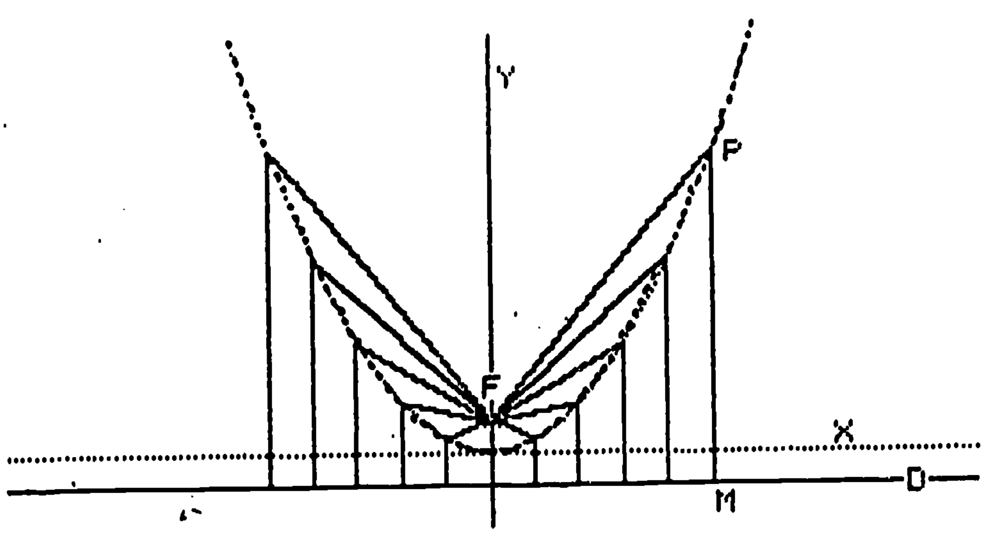
The student is often handed a formula describing a curve without being told how the formula was arrived at.

Using locus properties of a parabola the program shows the derivation of the formula for this curve.

The ratio of the distance of a point on the parabola to a fixed point (focus) to its distance from a fixed line (directrix) equals one:

$$\frac{FP}{PM} = 1$$

(using ratios greater or smaller than 1 will lead to equations of other conic sections).



PARABOLA FP/PM = 1
D = DIRECTRIX F = FOCUS

Program #9 GRAPH PARABOLA.
General Topic: Analytic Geometry.

A. Specific Topic:

1. Quadratic equations.
2. Graph of quadratic functions of the form: $y=A(x-H)^2+K$ (H,K coordinates of vertex).
3. Translation of function $y=Ax^2$.

B. Description of Program:

Plots the parabolic function: $y=A(x-H)^2+K$ for user chosen values of A,H, and K.

C. Required Mathematical Background:

1. Knowledge of quadratic equations.
2. Knowledge of graphing.

D. Use of Program:

Gives the students a visual display of the influence of the parameters A,H,K (shift of vertex, location of zeros).

E. Class- and Homework:

In class discuss the relationship between the two forms of the quadratic equation:

$$y=Ax^2+Bx+C \quad \text{and} \quad y=A(x-H)^2+K$$

Given

$$y=A(x-H)^2+K=Ax^2-2AHx+AH^2+K$$

Let $-2AH=B$ and $AH^2+K=C$

We have: $y=Ax^2+Bx+C$.

Similarly: Given (program #6)

$$y=Ax^2+Bx+C$$

$$=A\left(x^2+\frac{B}{A}x+\left(\frac{B}{2A}\right)^2\right)+C-A\left(\frac{B}{2A}\right)^2$$

$$=A\left(x^2+\frac{B}{2A}\right)^2+C-\frac{B^2}{4A}$$

Let $H=-\frac{B}{2A}$ and $K=C-\frac{B^2}{4A}$

$$=C-A\cdot H^2$$

we have

$$y=A(x-H)^2+K$$

Use program #11 to demonstrate the equivalence of the two forms:

$y=2x^2+2x+3$ (1)

$y=2(x+.5)^2+2.5$ (2)

since H=-2/4=-.5 and K=3-2/4=2.5

Use in program #11 the following expression:

```
F(x)=A*(2*x^2+2*x+3)+b*(2*(x+.5)^2+2.5)
```

and use successively :A=1,B=0 and A=0,B=1 to obtain a plot of (1) and (2). Note that both curves will fall on top of each other.

F. Pedagogical Commentary:

Start with using A=1,H=0, and K=0 to obtain the reference parabola:

$$y=x^2$$

Changes in A,H,K will produce a translation as well as a change in slope of the reference parabola.

Other pedagogical advice is contained in section E.

Program #10 ABS(X).
General Topic: Absolute Value.

A. Specific Topic:

1. Definition of absolute value.
2. Graph of absolute value function.

B. Description of Program:

Produces straight lines y=x and y=-x. Erases the part of the graphs for which y<0. The resulting v-shaped curve is the absolute value curve. It can be described as y=x for x>0, y=0 for x=0, and y=-x for x<0. This is the basic definition of the absolute value.

C. Required Mathematical Background:

Knowledge of graphing of linear functions.

D. Use of Program:

Facilitates greatly the understanding of the basic definition of the absolute value.

E. Class- and Homework:

Using the definition, let the student show that the absolute value of -2 is equal to 2 or that the absolute value of b-a is equal to the absolute value of a-b. Select similar problems from the textbook.

F. Pedagogical Commentary:

Every effort should be made to make definitions of mathematical expressions meaningful to students.
A difficult concept is that of the absolute value.
A graphic representation of y=|x| provides insight into this concept. The picture of the absolute value function on the sceen shows clearly that y=|x| is identical with y=x, but only for x>=0 and that y=|x| is identical with y=-x, but only for x<0.

These statements reflect the commonly found definition:
$|a|=a$ for $a>=0$, and

$|a|=-a$ for $a<0$.

(a being a real number)

<u>Example:</u> $|3|=3$ because $3>0$
$|-3|=-(-3)=3$ because $-3<0$
Therefore, any absolute value of a real number is always positive.

Program #11 FUNC-PLOT (PARAMETERS).
General Topic: Plotting of Functions.

A. Specific Topic:

The program plots user-chosen functions containing a maximum of four arbitrary parameters. Several plots for different sets of parameters may be obtained in succession.

The word 'parameter' means a 'constant' in an equation of one or more variables. Thus, in y=a+bx ,a and b are parameters which for a specific choice of their values give different straight lines.

B. Description of Program:

The user selects an arbitrary function of the form:

F(x)=F(x;A,B,C,D)

After selection specific values for the parameters A,B,C,D the computer will plot that function.

The user is then offered the choice to continue plotting the function for different values of the parameters.

C. Required Mathematical Background:

Familiarity with graphing of functions.

D. Use of Program:

To present pictures showing the influence of the values of the parameters on the shape of the curves given by the equation.

E. Class- and Homework:

In section F the utility of this program is discussed. The examples given there can be used by the student for class- and homework.

F. Pedagogical Commentary:

The program can be used in many different mathematical situations.

1. It can depict the addition of functions. In program #4 the process of addition of sinusoidal functions was illustrated. Program #11 can show the result of the addition or subtraction of up to four specific functions taking A,B,C,D as multiplier of these functions. For the example of program #4 we may write:

F(X)=CCOS(BX)+DSIN(AX)

and then take subsequently the following values for the parameters A,B,C,D:

#	A	B	C	D
1	1	2	1	0
2	1	2	0	1
3	1	2	1	1

which will yield the following curves:
cos(2x),sin(x) and their sum cos(2x)+sin(x).

2. It can be utilized to investigate the influence of parameters in periodic functions.

Example:

F(X)=ASIN(BX+C)

This function is used extensively in Physics to describe oscillations, presented often in the form:

$y=a\sin(2\pi ft+\varphi)$

where a is the "amplitude" f is the "frequency", and φ is the "phase shift". A,B,C,D must be interpreted appropriately. The following values could be serve as an example:

#	A	B	C	D
1	1	1	0	0
2	1	2	0	0
3	2	2	-1.57	0

3. Significant changes in the appearance of a function can be pictured by altering the parameters.
As an example you may consider the function:

$$F(X)=A\cdot e^{BX}\cdot(C+SIN(DX))$$

with the following sets of parameters:

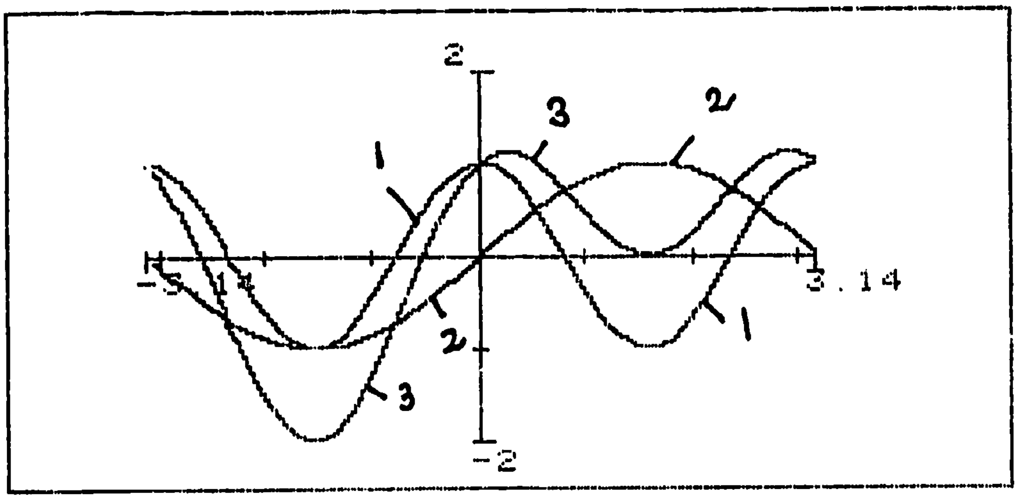
#	A	B	C	D
1	3	0	0	4
2	3	-1	1	0
3	-3	-1	1	0
4	3	-1	0	4

4. If a function has no parameters, it can also be plotted bt taking A=B=C=D=0. As an example consider a function that exhibits finite jumps"

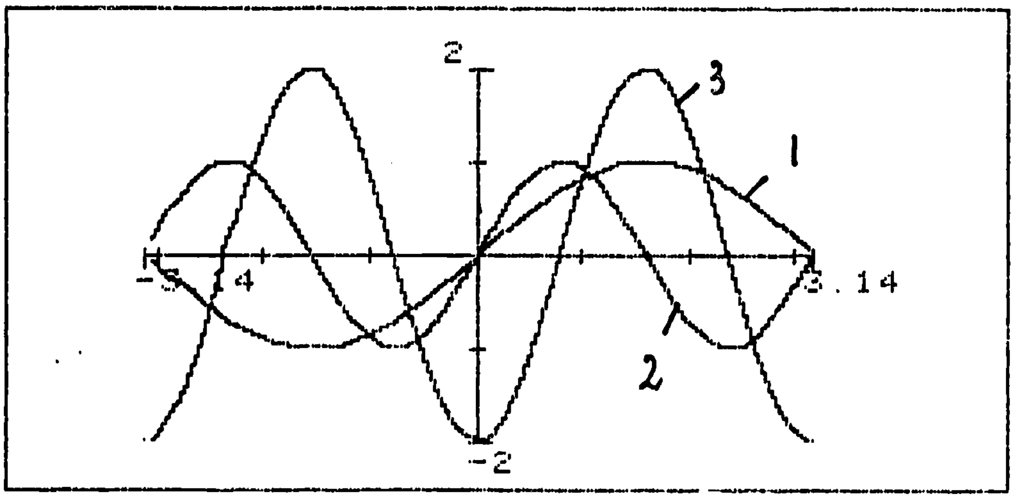
$$F(X)=\frac{X}{INT(X)}$$

A good domain is -3<=x<=+3 and a range 0<=y<=+2.
Note that the program requires the user to specify that the function being considered has "jumps".

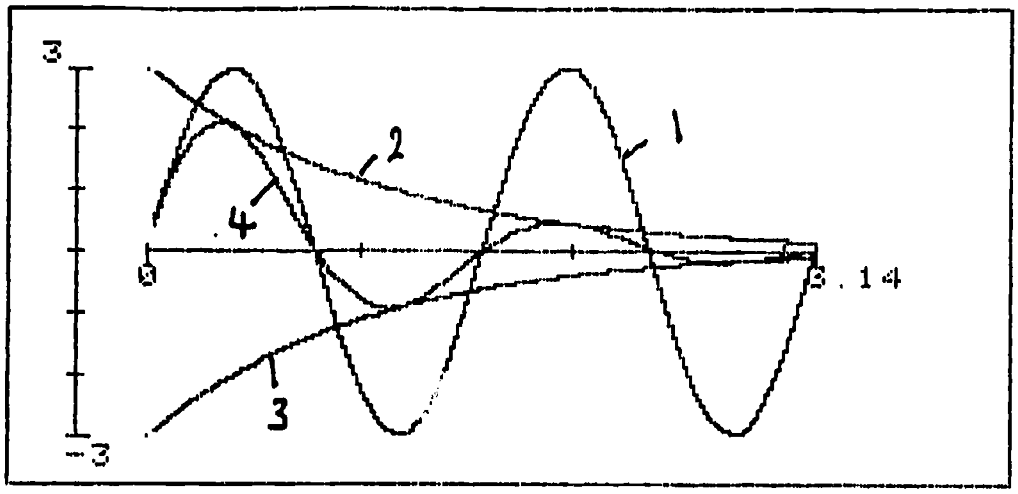
Print-outs of the graphs for these four functions are given on the following two pages.



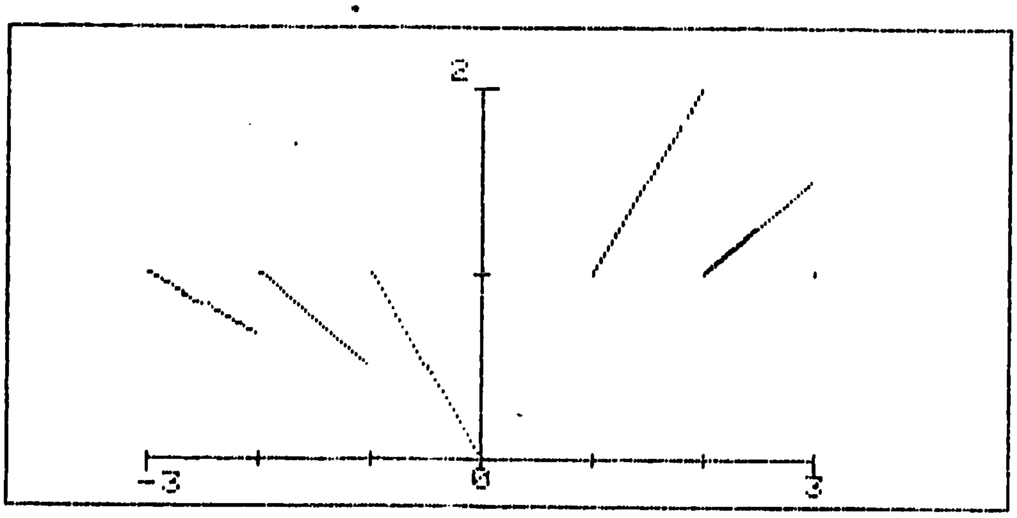
1. $F(x) = C \cdot \cos(Bx) + D \cdot \sin(Ax)$



2. $F(x) = A \sin(Bx + C)$



3. $F(x) = A e^{Bx} (C + \sin(Dx))$



4. $F(x) = \frac{x}{INT(x)}$

September 10, 1987

TO WHOM IT MAY CONCERN REGARDING PRE-CALCULUS AND CALCULUS SOFTWARE
BY RUTH AND RUDOLF HUTTER
MC GRAW HILL BOOK COMPANY

I would like to commend to you the above software without reservation. I have taught the calculus for more than twenty years, have been involved in working for the College Entrance Examination Board both as a workshop director and as an Advanced Placement Examination reader, am the co-author of a Pre-Calculus textbook, and have been involved in computer education as well, especially regarding its use in the teaching of mathematics.

I especially like the software under discussion since its primary function is as an aid to the teacher of calculus. Although it can be used by the student to provide understanding and reinforcement, its major strength is to be found in the classroom at the discretion of the teacher when developing new concepts and techniques. Indeed, it is the ultimate electronic blackboard for the study of Pre-Calculus and Calculus.

Much of the software I have found available for the study of the calculus emphasizes drill and practice in the techniques of finding the derivative and the integral. A necessary facet of such software involves a great deal of prose and explanation. I have always found this to be distracting and tedious for use in the classroom although perhaps appropriate for student use outside of the classroom.

On the contrary, the Hutters have left the explanations and the motivation to the teacher and have provided the graphic material which is essential to each major concept. Not only have they provided a way to improve upon the necessary blackboard work required of the teacher, they have added a realm of dynamics not available to the teacher except perhaps through the all too few films which have been produced in this field. Most of the programs provide changing situations which illustrate either limiting situations or the effect upon the graphs under discussion by changes in the parameters of their equations.

I have found many of the programs useful as a preliminary to the subsequent lecture given on a topic or for running while the lecture is underway. As an example, the excellent program showing the Riemann Sum through increasing numbers of randomly selected intervals is the lesson itself. The intervals are clearly shown with their including ordinates, a selected ordinate within each interval is designated, the corresponding rectangle is shaded, and the area under the curve is approximated. Then the number of intervals is increased, a new randomly selected partition is chosen, and the process is repeated. It becomes clear that the approximating area is independent of the specific selection of partition, and the error involved is a function of the largest interval selected (the norm). As the program is allowed to run, a discussion of the limiting process is motivated.

Most of the programs use pre-selected functions, chosen to bring out the concept illustrated. However, some of the programs are completely interactive in that they allow the teacher or other user to input a personally selected function, thus providing a means of exploring the properties under study. Some also allow for a variation of the speed of running so that the dynamics may be followed more readily. Some provide choices involving axes, specific parameters, computation of final results and the like. Occasionally during particulary lengthy computations a flashing note is made to the effect that one should "HAVE PATIENCE PLEASE. COMPUTER AT WORK".

The specific programs available are as follows:

PRE-CALCULUS: (One Disk)

- Trigonometric Function
- Inverse Function
- Analytic Geometry
- Absolute Value
- Functions (including function plot, intersections of curves, and linear inequalities).

CALCULUS: (Two Disks)

Integral Calculus:
- Upper and Lower Sums
- Midpoint Sums
- Random-Interval Sums
- Trapezoidal Sums
- Areas Between Two Curves (both x and y orientations)
- Polar Integrals
- Volumes by Disks
- Volumes by Shells (both x and y orientations)
- Surface of Revolution
- Calculations-Upper and Lower Sums
- Calculations-Midpoint Sums
- Evaluation of Definite Integral

Differential Calculus:
- Chords-Tangent (Increasing function)
- Chords-Tangent (Decreasing function)
- Function and Its Derivatives
- Slope-Derivative-Sine
- Derivative of ABS(x)
- Concavity
- Derivatives
- Related Rates
- The Lengthening Shadow
- Path-Velocity-Acceleration

Epsilon-Delta
Discontinuity
Taylor Approximation

The accompanying Teachers Manual identifies each of the following for each program and is very useful:

Applicable to Topic(s)
Description of Program
Required Mathematical Background
Use of Program
Class- and Home-Work

If you are interested in obtaining a true aid for the teacher in teaching PreCalculus or the Calculus, this material is for you!!

Lawrence D. Hawkinson

Lawrence D. Hawkinson
Instructional Supervisor, Mathematics (Ret)
Henry M. Gunn High School
Palo Alto, California