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ABSTRACT

Reports about U.S. schools have indicated the need for improvement of science teaching and learning. One of the solutions advocated calls for classroom contexts that allow for authentic practice under the guidance of teachers who model pertinent skills as practitioners in the field of study. The metaphor used to describe such teaching-learning situation is that of cognitive apprenticeship: teachers model scientific skills and coach students in their attempts to handle the practical and conceptual tools in the sciences. This paper summarizes the results of a two-part study. During the first part, the learning outcomes when grade 8 students, in small groups of two and three individuals who are framing, developing, and comleting their own research agendas, are examined. The second part of the study reports on the interactions between a gifted 10th-grade student and the researcher of this study, who acted as a mentor and a coach. An interpretive research methodology was used in both parts of the study. Underlying the study was a constructivist view of knowledge acquisition. The findings of the study are based on: direct observations of teachers and students; interviews with teachers and students; audiotaped sessions of tutoring relationship; and interviews with the tutee in the relationship. The results of this study confirm the viability of the concept of cognitive apprenticeship for science teaching and learning. The discussion addresses the issue of helping teachers to change their strategies through a change of metaphors and the issue of research based in and conducted from the school level. Forty-five references are included. (KR)

ASPECTS OF COGNITIVE APPRENTICESHIP IN SCIENCE TEACHING

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The level mathematics and science achievement exhibited by our nation's children is alarmingly low, when compared to that of other nations (*Science Achievement in Seventeen Countries*, 1988). At the same time, student interest in taking science courses at the high school or university level is waning. In their search for explanations of this trend, science educators, scientists, and scientist-journalists have pointed to the discrepancies between science in the schools and that practiced by scientists as possible causes (Sagan, 1989; Suzuki, 1989; Tobin, 1990). More so, what students learn in school is not only different from the activities of scientists and everyday activities of ordinary people, but the whole school culture may be antithetical to any useful domain learning (Brown, Collins, & Duguid 1989). This divergence between school and other aspects of life has become of increasing interest to researchers in cognition and education: it is increasingly becoming clear that knowing, and thus learning, cannot be considered independently from the situation in which it occurs.

During the past decade, a number of researchers from various backgrounds reported on the kind of problem solving activities in which scientists engage in their natural setting (Latour & Wolgar, 1979; Qin & Simon, 1990; Schön, 1983; Suzuki, 1989) and on the problem solving of people in everyday activities and on the job (Lave, 1977; 1988; Lave, Murtaugh, & de la Rocha, 1984; Scribner, 1984). The problem solving activities in all these situations is a function of the social and physical context in which they take place. Posing (framing) and solving problems are affected by both the physical and the conceptual tools of a profession--a view captured by notion of "paradigm" (Kuhn, 1970). These tools, physical or conceptual can only be learned and understood through their use in realistic and authentic situations. At the same time, the meaning of these tools is not invariant but is a product of continous negotiations between the members of the community, and thus is always in the making (Greeno, 1988; Latour & Woolgar, 1979; Toulmin, 1972). In traditional school practice, on the other hand, the meaning of concepts are fixed, and learning is reduced to memorizing definitions and factual statements. This static view of knowing and learning has come under severe criticism and alternative approaches to teaching have been proposed (Burton, Brown, & Fischer, 1984; Lampert, 1986; Roth, 1990; Schoenfeld, 1985).

Wiggins (1989) recently called for an education that would help students "using content knowledge, as contextually appropriate, to recognize, pose, and solve authentic knowledge problems" (p. 47); that is, he called for an education which helps students in developing the habits and standards of those cultures from which the experts come whether they are mathematicians, physicists, or historians. Reports on teaching-learning experiments that implemented such an approach have appeared over the past few years (Lampert, 1986; Roth, 1990; Schoenfeld, 1985; Schön, 1987). All these teaching-learning contexts had in common that they provided contexts--classroom, laboratory, or field--where students learned to see the world through the eyes of practicing mathematicians, physicists, or biologist-agronomists. Collaboratively, these students were involved in explorations; they developed a sense for asking questions and learned to design of experiments to answer them; they discovered alternative solution paths; and they constructed and negotiated the meaning of events and concepts. In this way, the activities in these teaching-learning contexts were authentic by engaging students in the ordinary practices of an intellectual culture. Thus, questions arise regarding the nature of authentic practice and regarding the contexts that will support authentic practice in the classroom.

Authentic Practice

In his comparison between the "unrealistic model" for current science teaching and his own practice in genetics research, Suzuki (1989) described how one of their projects actually progressed. The experiments based on the team's starting question caused more "questions to flood in," which led to more and more investigations and to the development of new apparatus and tools. When Suzuki and his team finally were ready to report results, they "rifled through their records and selected the ones that said what [they] wanted" (Suzuki, 1989, p. 192). But Suzuki emphasized at the same time that the published report conveyed nothing of the hard experimental work, the disappointments and exhilaration of the search, or the original reason for doing the experiment. These aspects of the work of scientists have also been reported by other noted discoveries such as the double helix structure of DNA, (Watson, 1968), the development of the transistor (Nelson, 1962), the modeling techniques of chaos theory (Gleick, 1988), or other research in other fields (Knorr-Cetina, 1981; Latour & Woolgar, 1979). This pattern in the search for solutions to problems is repeated at all levels of real-world research programs. In contrast to school book problems, real-world problems are ill-defined and the first step towards a solution is to recast--frame--the situation in such a way that the conceptual tools available to the scientists can be applied (Brown, Collins, & Duguid, 1989; Bruner, 1961). The investigator then follows the implications of this frame, through a cycle of framing questions, testing, and reflection which becomes a "three-fold transactional experiment" (Schön, 1983). Recently, the term "cognitive apprenticeship" has been used to conceptualize the teaching-learning context of authentic practice in the classroom.

Cognitive Apprenticeship

Experimental teaching-learning contexts where the students were involved in authentic practices have been described by Schoenfeld (1985) and Lampert (1986) for the teaching of mathematics and by Roth (1990) for physics teaching. These teaching-learning contexts bear a lot of ressemblance with traditional craft apprenticeship and have been conceptualized as "cognitive apprenticeship" (Brown, Collins, & Duguid, 1989; Collins, Brown, & Newman, 1989). Another example was provided by Schön (1983). The teacher, a systems engineer, provided a context of authentic practice to teach systems engineering to peasant children in Columbia. The students learned techniques of sytems engineering by going through repeated cycles of framing and posing questions, experimenting, and reflecting critically upon the meaning of experimental results. In this context of authentic practice, the "students were led to experience the experimental method before any explicit mention of its principles was made" (Schön, 1983, p. 201).

In traditional apprenticeships, the novices became "enculturated" into a trade by learning to use its conceptual and practical tools. Similarly, apprentice scientists are initiated to the professional community of their field during their graduate and post-graduate work (Toulmin, 1982). In this context, the doctoral and post-doctoral students learn to master an intellectual practice, that is, they learn the use of conceptual and practical tools and the available modes of discourse (J. Hawkins & Pea, 1987). The teaching-learning situation, then, becomes one were a practicing member of the culture models the use of conceptual and practical tools. The advantage of such teaching through authentic practice is that it that it can help to overcome the problem of the master practitioner's implict knowledge. In traditional teaching the implicit assumptions of an intellectual culture were never made explicit. In the practice of cognitive apprenticeship, master and apprentice may feel the need to uncover implicit assumptions, the meaning of which is inseparable from the context of this practice (Greeno, 1988; Schön, 1987). But the context of cognitive apprenticeship may also provide other than linguistic modes of discourse; thus, many implicit assumptions do not have to be explicated because they are embedded in the context which the students experiences and learns as a whole (Bateson, 1980; Rogoff, 1984; Schön, 1987).

PURPOSE

The present study was driven by the following questions. Given that students are placed in an open-inquiry context where they are allowed to pursue their own questions, What are the knowledge and skills achievements? What effect does an increased familiarity of the physical and conceptual context have on the learning outcomes? What discourse patterns do teacher and student use in small group interactions? How does long term coaching affect the cognitive development of students? Is cognitive apprenticeship a sufficient metaphor to sustain effective teaching?

METHOD

An interpretive research methodology (Erickson, 1986) was adopted throughout the study that was based on a constructivist view of learning (von Glasersfeld; Bruner, 1986). The findings of the study were derived from direct observations of teachers and students; from interviews with teacher and students; from audio-taped sessions of a tutoring relationship; and from interviews with the tutee in the relationship.

Sample

Three classes of grade eight science students from an urban private school and their teacher participated in part one of the study. The teacher had both a bachelor's degree in biology and in education, and was in the process of completing a masters degree in biology; the study was underway toward the end of his first year of teaching. To elucidate small-group teacher-student interactions, a second study was conducted. A gifted student, who had just completed the grade 10 in the same school, and this author as mentor-coach (with experience in physics and educational research) were the participants in part two of this study.

Design, Data Sources and Data Collection

Part One of the study took place over a 10-week period from April through May of 1990. During this time, the students from three grade 8 classes conducted research on micro-environments (biomes). To conceptualize the teacher's role during the unit, we used the metaphor of a graduate advisor assisting his students in doing research. This author coached the teacher throughout the unit. Each of the teacher-selected biomes had an area of approximately 10 x 5 meters; the plots were distributed to the students by means of a lottery. The students, who worked in groups of two, spent their weekly double period in the field, mapping the campus and the research site and gathering descriptive physical and biological data, and gathering specific data needed for answering their research questions. Once a week, the teacher conducted a research methodology session during which the students learned (a) investigative techniques such as strip sampling, quadrat sampling, and random sampling; (b) means of representing data such as bar graphs, histograms, and lin graphs; and (c) the use of specific instruments such as a soil moisture meters or light meters. The primary data sources for this part of the study were student field notes, student formal reports, field notes of observations in the classroom and during the students' field study, and interviews with the teacher.

Part Two of this study took place during an eight-week period in the summer of 1990; this author coached one gifted grade 10 student to get a grade 11 equivalent in physics, chemistry, and mathematics. During these eight weeks, the author formally met with the student once or twice daily for a total of 56 hours, about 20 of which were audio-taped. The recordings were transcribed for subsequent analysis. Both also met informally for discussions for a total of about 100 hours. The content of these informal discussions was documented in daily field notes. Together with the artifacts generated during the formal interactions (computer print outs, student work, teacher notes), the transcripts and field notes made up the primary data sources for part two of the study.

FINDINGS

The findings are presented below in the form of four assertions. Assertions 1 through 3 indirectly relate to cognitive apprenticeship in that they assert learning outcomes from a teaching-learning context conceptualized as an advisor-graduate student relationship. Assertion 4 directly relates to the cognitive apprenticeship model in that it was based on the discourse between a "student-apprentice" and a "teacher-coach."

Assertion 1. The students engaged in and developed sustained research programs that gradually developed to high levels of sophistication.

As the student groups became increasingly familiar with their biome, we observed patterns of crystallizing research programs. Students' research topics varied widely. Although their first study was not successful ("What different types of animal and plant life live in different amounts of light?"), it had helped Jere and David to find a focus and productive questions "flooded in," just as described by Suzuki after his research team had done some preliminary, un-focused studies (Suzuki, 1989). The students' research now focused on issues surrounding the topic of soil, its composition, its moisture content and the factors affecting the later. This focus was reflected in the questions which the two recorded in their field note books or which they began to investigate. Some of these questions read, "Is there a relation on our slope between soil moisture and the air temperature and what kind of plants fit into this relationship?", "What different types of soil porosity, texture, compounds, and colour are there in our area?", and "What is the percent soil moisture and organic content in three different parts of our area?" In subsequent investigations, the two boys went on to research questions and to test new hypotheses which they had generated during earlier investigations. This progress paralleled that in the work of scientists as described by the Canadian geneticist Suzuki (1989) for his own work and followed a general pattern for research (Lawson, Reichert, Costenson, Fedock, & Litz, 1989). For example, Jere and David tested soil type and its rates of passing water. Their hypothesis which had linked ground cover and soil moisture had been refined and was joined with the proposal of "fertile ground from organic material" to arrive at a hypothesis which linked organic content and soil moisture. The boys subsequently investigated porosity and organic soil content to see if they are linked to the distribution of moisture in the soil. Further hypotheses related the amount of sunlight, without which "the water can't evaporate [because it] is turned into heat," and moisture. Conversely, other groups focused on patterns and factors related to growth by investigating growth as a function of a plant's location, the amount of growth over time, or the effect of soil and weather variables on growth. Others again focused on a single plant and researched its life cycles, living conditions, and growth patterns. During class discussions, students shared their findings with others. In this way, the descriptions and relationships of individual groups were disseminated throughout the class; and students learned about data collection strategies, data and data tranformation techniques that had worked or failed for others. Most importantly, the students in this study learned to frame their own questions, the most crucial part to scientific inquiry and reflective practice (Newman, Griffin, & Cole, 1984; Qin & Simon, 1990; Schön, 1987).

Although designed by students without restrictions, the research agendas were strongly determined by the material objects and by the equipment of the classroom environment. For example, because soil humidity meters were available, about 30% of the student groups designed at least one investigation involving such a device. However, the questions and relationships studied by the students were quite different from group to group. While one group studied the relationships between soil moisture and organic content of the soil or between soil moisture and surface temperature, others looked at relationships such as those between the soil moisture and expiration of water from horse tail plants, or the relationship between topology of the site and soil moisture. Similar variations of questions an dhypotheses around an instrumentation could be observed for other instruments. From this finding we concluded that the logic of the students' research projects was very much opportunistic; and we concluded that student selections of concepts and instruments were contingent on the site of their investigation and on the instruments and other tools (such as reference books, sieves, and refrigerator) available in the laboratory. However, the very same opportunism and contextual selectivity has been reported from the work of successful--in terms of recognition, publications, and citations--research laboratories (Knorr-Cetina, 1981; Latour & Woolgar, 1979). These researchers noted local idiosyncracies in experimental preparation and instrumentation and in the interpretation of results. Projects took specific turns because certain pieces of equipment were available or because a certain technology was *en vogue* while an individual who knew the technology was a part of the research team. The difference between the students' work and that of scientists seems to lie in the sophisitcation, but not in kind. The major differences exist in the process of reporting. While scientists decontextualize and objectify their work through the process of reporting "findings" in scientific papers (Knorr-Cetina, 1981; Latour & Woolgar, 1979; Suzuki, 1989), students' findings and reports are still very much contextualized. However, the contextualization of students' work, when combined with collaborative activity as in this study, has been reported to lead to high cognitive outcomes (Brown & Palincsar, 1989). Brown and Palincsar reasoned that allowing students to

> "generate and own their particular knowledge--form a community of learners, responsible for each other, etcetera--[will] lead to a sea change in spontaneous activities that promote deeper understanding and a search for causal explanations" (p. 5).

Similarly, it has also been suggested that a great deal of scientific discovery comes from the interest in everyday phenomena, and if the questions grow out of existing interest (D. Hawkins, 1990; Wells, 1990).

Assertion 2. Supported by an increasingly familiar context, the students developed a wide variety of complex research and analysis skills.

Over the course of their investigation, the students came to use various research methods and skills. The students collected the data often using maps to locate the specific sites of measurement or by using large and unwieldy data tables. Because these forms of data presentations were difficult to understand by their peers, students saw the need to summarize data in some form and resorted to various graphical methods. These included line graphs to show relationships, bar graphs, and histograms. However, the choice was always driven by the students' needs and the data they had generated. With the help of the teacher-coach during the weekly methodology session, or through feedback from peers, appropriate methods of representing the data were selected. The students also felt the need to learn about sampling methods. Questions that were raised during the planning and collection of data were "Are these data representative of our plot of land? How should we collect the data to eliminate biases?" Thus, the students chose random sampling methods to determine the percent soil coverage in different parts of their lots; they chose a factorial design to compare the moisture levels in the soil as a function of area (low-, mid-, sloped-, high-field) and surface temperature; or they resorted to more descriptive methods. In this way, students learned both descriptive and factorial designs, that is, they learned descriptive-interpretive and hypothetico-deductive approaches to gathering and interpretation of data, epitomized in the work of anthropologists and natural scientists. This also permitted students to take a self-critical stance and evaluate their own procedures and claims: "While generalizing, which disconfirming evidence did we disregard?" For example, on the basis of their data, Tom and Mark had claimed that there was a relationship between the topology of their site (high-, mid-, and low-area) and soil moisture. But one of their moisture measurements taken in the high area showed a high moisture content. In the manner of researchers using an interpretivist approach, they looked for an alternate explanation:

> "This [discrepant, high moisture] might be because there was about 80% ground cover (refer to scale on map). What this might mean is that as the water evaporates, the leaves and other dead twigs might catch the moisture, and this might allow it to fall bag to the ground."

The students were also very careful not to reverse cause and effect. In their investigation concerning the relationship between soil moisture and organic content, Jere and David reasoned:

> "This relationship could work either way. The soil organic content might be higher when the soil moisture is higher, because the moisture in the soil makes an ideal environment for bacteria to break down dead twigs, leaves, etc. into organic material. On the other hand, the soil moisture might be higher when organic content is higher because decomposed bits of material might make soil absorb moisture easier."

All student groups showed a dramatic development of skills as the familiarity with their site increased; as they became more familliar with the different methodologies of researching, though the data, and their presentations were intimately linked to features of this context. Burton, Brown, and Fischer (1984) reported similar patterns of development in learning to ski; they credited the success to the wholistic approach of teaching skills through utilization of context variables. Thus, effective teaching situates skills, concepts, and ideas in the context of an application (Prawat, 1991). As a consequence, the goal of such instruction must then be to encourage the simultaneous development of a holistic understanding and a differentiated sensory awareness of the learners' environment.

Assertion 3. The students constructed new means of representation, their own, new vocabulary, and new concepts to talk about and report the results of their research.

In their effort to record and to communicate information, the students constructed symbols, icons, and new concepts; they were thus engaged in the distinctively human activity of producing signs (Scribner, 1984). For example, Paul and Tom wanted to represent a survey of their area in a graphical from. They drew a cross section, but also created a map that represented not only type of vegetation, but also ground cover and steepness. To do this, Paul and Tom created their own key and constructed the map that contained all the information they considered relevant. At another occasion, the two constructed various maps containing a survey of the moisture levels, or percent ground cover. Other student groups created charts and keys for (a) the various transpiration rates of plants across their area of study, (b) soil temperature variations, or (c) growth densities of one type of plant. The students also felt the need to create new terms to increase the effectiveness of communication. By giving a definition, they made it clear how they wanted to have the term understood in the context of their study. Thus, in one study the following set of terms was used:

High-Field:	a flat area, maintained short grass, with occasional tree. The soil was above the water line, and moderate moist soil
Sloped-Field:	is a slanted area of shortly cut grass with few trees. The soil at the top is less moist than on the bottom. It links high field and low field.
Low-Field:	a lowland with maintained shortly cut grass dotted with trees. It is a flat plain except for a few lumps. Its soil is heavily saturated because it is near the water level.

As these examples show, the students were not merely involved in following "cookbook recipe" or "verify-the-law" labs. These students were actively involved in knowledge production; they constructed means of communication, means of recording, and means presentation; they experienced the need to create signs in their various forms; and they experimented with ways for communicating their ideas.

With regard to the concepts which the students learned, the teacher ascertained that each group of students learned all those concepts and skills that he would have "covered" in a teacher-centered class. This, however, is not a coincidence. We know since Kuhn's (1970) analysis of paradigms that the available conceptual and practical tools determine the questions asked by researchers and, in turn, determine the answers they will find.

Assertion 4. In small group and one-to-one interactions, the three components of cognitive apprenticeship, modeling, coaching, and scaffolding/fading promoted successful, high level cognitive learning outcomes.

The analyses of the data suggested that in small group and one-to-one interaction the teacher-coaches used the teaching methods of cognitive apprenticeship: modeling, coaching, and scaffolding/fading (Brown, Collins, & Duguid, 1989; Zmuidzinas, Stasz, & McArthur, 1990). A teacher in a learning environments based on such a method will teach thinking skills on a need-to-know basis in the context of authentic activity (Prawat, 1991). The students in all of our classes learned new skills and conceptual tools in the context of open-inquiry labs, that is, students determined in most instances their own research questions. New conceptual and practical tools were introduced when a particular situation necessitated it. Such skills included new computer programs, new measuring instruments, or new conceptual frameworks. In this manner, the context provided a meaningful backdrop against which students construct a framework for using the tool. Such contexts were meaningful, because the students had much input regarding the activities in which they were involved. For example, the use of a new computer program was modeled once, accompanied by a lot of "on-line" talk by the "teacher-coach," which, nevertheless left implicit much of the knowledge required to run the program. Then, with the support of the coach, the "student-apprentice" began using the tool on his own. The coach observed from the background and occasionally assisted with suggestions, reminders, and with help in other forms. While students held the tools (keyboard, mouse, or instrument), the talk of the teacher was accompanied by pointing, explicit verbal instruction, or, decreasingly, by doing part of the task. In a short amount of time, the support was faded out, and the students were using the new tool independently. The same pattern was found for such varied skills as dealing with problems in physical experiments, finding solutions to word problems in physics, discussing functions in algebra and calculus, and text book problems in chemistry. These descriptions of task and support were also given by Zmuidzinas, Stasz, & McArthur (1990) in support of the cognitive apprenticeship model for a coaching environment in algebra tutoring.

Cognitive Apprenticeship. When working on problems in a new topical area or with new problems that the student brought to the session, the teacher usually modeled the use of concepts, equations, and the solution of problems, thinking out loud as he went through the steps toward the solution. Once a solution was found, the problem was reviewed, the general principles high lighted, possible heuristics invoked that could help in the solution of similar problems. The student, thus, experienced expert behavior at two levels: First, they experienced first hand the expert's approach to the problem; then, they experienced metacognitive activities through the experts reflections on the process of searching a solution. The same two-tiered approach was subsequently used during the student's attempts in solving problems. First, the student worked, with the support of the teacher (scaffolding or coaching); then, the teacher assisted the student to reflect on their own problem solving process. Subsequently, alternate problem solving strategies, multiple forms of representation, or alternate conceptualization of the same problem were discussed between the teacher and the student.

After the teacher modeled one or two solutions, the student attempted problems on his own with concurrent support in the form of "scaffolding" (tight control of the problem and solution by the teacher) or in the form of "coaching" (more student control over the problem and ilts solution). One feature observed here was that diagnosis and remediation of errors was immediate. Such diagnosis is illustrated in the following transcript:

S: (mumbling). . . plus . . ahm .. . y . . . times 1.5 . . . OK . . . so 12 (inaudible)
T: Why do you write "minus"?
S: Oh . . . its "plus" . . . equals y
T: And where is 1.5, . . is in meters?
S: Oh . . . right (corrects error)

Although one might think that a more beneficial strategy would let students go into dead ends and find out their own mistakes. It has been assertained, however, that expert tutors most often use the instant repair technique (McArthur, Stasz, & Zmuidzinas, 1990). One possible reason for this behavior is that teachers prefer instant repair in the face of the time constraints of most teaching situations. Also, students might get more confused if they stray to far off successful solution patterns. We also observed instant remediation after misconceptions or incorrect usage of concepts in propositions were diagnosed. Then, the teacher stopped the students progress and model correct usage. In those cases as notational conventions, where the incorrect use did not interfer with the solution, corrective feedback was provided after the student completed the task. An example of such a situation was when the student wrote

$$\Sigma_{F_i} \quad \text{instead of} \quad \sum_i F_i$$

which is against the conventions in mathematics or physics; in t s case, the teacher waited with the repair until the student had finished. The tapes also rev_aled that during the coaching or scaffolding phase in a topic, the support and the corrective feedback was decreased and faded out as the student became more and more proficient.

After each problem, the teacher reviewed the general principles that governed the situation in the problem. For example, in high school problems involving torques, the systems are usually at equilibrium. Invoking Newton's second law, both the net force and the net torque, that is the sum of all forces/torques have to equal to zero. Thus, the teacher-coach offered for discussion different situations in which torques or forces did not add to zero. Review discussions also addressed the issue of multiple representations, or multiple routes to solution of problem. Figure 1 shows an example of mutiple ways of solving a system of two equations with two unknowns, and Figure 2 illustrates multiple ways of describing the acceleration of a system. Finally, the transcripts also revealed that the teacher modeled general heuristics such as classifying problems by general principles, qualitative assessment of answer, order of magnitude assessment of answers.

In order to come to a better understanding of the face-to-face or small group work, the notion of joint or shared problem space may provide us with a new conceptualization. Teachers--which are the designated representatives of the scientific community--and students must agree upon or negotiate the work to be done (Roschelle, 1990). The process of learning together is conceptualized as the joint or shared problem space. Its significance in the present study lay in the fact that the diagrams, computer displays, and jointly solved problems were tools for constructing, demonstrating, and repairing shared meaning.

Shared Problem Space. During the work on experiments, word problems, or "teaching" of new concepts, mentor and student developed shared problem spaces which they used to jointly construct meaning. This shared space was constituted in the work space (blackboard, paper) and the collaborative construction of diagrams, the collaborative mathematical work on the paper, and in collaborative calculations. This joint work can also be recognized from the even contribution to much of the discourse during problem solving sessions, particularly when the student had become more proficient in a topical area. Such joint work and equal contributions are shown in the following transcript:

S: That is not right!
T: Ahm . . . 3 . . . we must have calculated something wrong . . .
S: If there is a force here (pointing toward the pivot point on the bar)
T: But that doesn't count because it is the pivot point
S: No . . but this point definitely changes [to make a new pivot point] which is that this force doesn't count and so there is no reacting force . . . so it is not 10:50 [1050 newtons] but 10:50 [1050 newtons] minus 4:50 [450 newtons].

The objects around which the discourse evolved, that is the drawings, the writing, the calculator, and the computer became tools in the participants' effort to develop mutual intelligibility. In this effort, the student's talk comes more and more to ressemble that of the teacher, the representative of the community of experts.

The transcripts also indicated the heavy use of indexical expressions, that is, expressions which can be understood only in the context of the discourse. By using a diagram as a facilitating tool, and refering to parts of the diagram through pointing, the cognitive load of the conversation may have actually been lowered. In spite of the abundant use of indexical expressions in the following conversation, there was no indication that teacher and student did not understand each other:

T: I was just wondering if that could be written down in . . . as just one force diagram . . . so thateverything adds to zero . . . if all the forces
S: Well this can be broken down into this
T: Right . . .
S: . . . and this . . so the sum of those equals this so . . . if it equals zero that means that its not enough to overcome static friction
T: Let me see.
S: I guess you don't want it to equals zero in order to move.

In lecture-based teaching situation, such construction of a joint problem space may not be established for a number of reasons. First, students follow a lecture and copy a teachers notes, but fail to record all indexical expressions which a teacher uses. The references made thus get lost and become irrecoverable. Second, because of the physical distance between teacher and students, additional modes of (non-verbal) communication do not operate and are not used to construct shared meaning (Bateson, 1980).

The physics, mathematics, and chemistry problems which teacher-coach and student-apprentice worked together, were not done for their own sake, but for the purpose of sharing in a common activity through which the student could learn, practice, and acquire the language of the subject. This form of bringing a newcomer into full participation in a community of shared knowledge through face-to-face talk in the context of on-going work has been termed "reflective practice" (Roschelle, 1990; Schön, 1987). Such forms of participatory learning have been developed in the form of residencies medicine, psychology, architecture, law clerkships, and in post-doctoral university research. Participatory learning relies on the discourse between "expert" and "novice" in a physical and social context similar to that of real world practice. As such, this context provides for a backdrop against which the interlocutors develop their shared interpretation (Roschelle, 1990). The construction of shared meaning in this context is facilitated by means of objects (such as diagrams, tools, instruments, and computer displays) in this shared space.

DISCUSSION

The foregoing assertions showed that the teaching in classrooms modeled on the metaphors of cognitive apprenticeship and teachers as coaches provided an environment conducive to high level cognitive outcomes. Classroom environments that are meaningful and purposeful function as inductive environments where students are involved and practice the logic of discovery. The students in these classes showed increasing facilities with formulating questions and hypotheses as well as with the testing of hypotheses, all crucial components in critical thinking. The study of one-to-one interaction to investigate aspects of cognitive apprenticeship supports the conjecture that such environments are conducive to the negotiation of meaning; conducive to reflective criticism and hypothesis generation; conducive to learning the use of conceptual and practical tools of science; and conducive to talking, to doing, and to seeing science in a very general sense. Finally, we could show that our students made sense of discrepant evidence and negotiated the meaning of divergent experimental results. The evidence from these observations forced us to take a critical look at crucial parameters of learning such as the effect of context, the effect of focusing on long-term rather than quick coverage of topics, the issue of transfer if skills are learned by long-term engagements in one topical area, and the effect of collaboration on scaffolding through peer interaction. Ultimately, this research also helped us to reflect on the effects of teachers as researchers in their classrooms. Each of the above issues, context, coverage, transfer, collaboration, and teacher-as-researcher will be addressed in the following paragraphs.

Context

When students operated in a familiar social (their research groups) and physical context (area under study), they become increasingly apt to attend to features that weren't salient when they first arrived at the scene. This increasing familiarity helps them to frame problems, to isolate variables and to investigate relationships between those variables whether these relationships were correlative or causal; and this familiarity also makes them more efficient in their collection and interpretation of the data which gave rise to new knowledge claims. The importance of context to cognition has so far been overlooked in the research on science teaching. However, in the area of naturalistic study of cognition by cognitive psychologist and anthropologists, an increasing number of researchers focus on the facilitating effects of the context to teaching. Thus, based on studies of mathematical behaviour of dairy workers, dieters, shoppers, and tailor and weaving apprentices in Liberia and Mexico, one can conclude that effective practice makes use of the environment to decrease mental effort expended (Greenfield, 1984; Lave, Murtaugh, & de la Rocha, 1984; Scribner, 1984; Lave, 1988). Also, the Columbian children who were taught systems theory in an increasingly familiar context began to develop an intense research agenda surrounding problems that led to the malnutrition in their home village (Schön, 1983). This interrelation of cognition with context can not always made explicit but, is available as tacit knowledge through its embeddedness in the context. As Rogoff (1984) remarked,

> "Skilled activities, such as arithmetic, communication, or skiing, may proceed through the use of contextual cues that interface with tacit knowledge rather than through the systematic application of explicit steps in problem solving" (p. 8)

This context then provides student teacher teams with support in their search for convergence of meaning in spite of the vagueness, ambiguity, or obscurity in the things about which they try to communicate (Schön, 1987; Brown, Collins, & Duguid, 1989). The supports that reside in the context may simply be in the form of memory aids, in the perception of the context as a whole, or in its facilitating role to mediate discourse between the actors in the situation.

Present schooling systems operate under the assumption that students will develop proficient and transferable skills through work in ever varying contexts or that we experience the logic of discovery if we never become familiar with with the context of the activity. However, "discovery favours the well prepared mind" (Bruner, 1961), and a "well-prepared mind" only comes with familiarity in a particular area. We all have heard of those accounts of scientists who had tinkered, investigated, and experimented in a topical area for years before they made any significant discovery (Watson, 1968; Suzuki, 1989). However, the discovery and construction of knowledge and skills is not limited to scientists but extends to everyday life. Lave (1988) reported the construction of highly efficient measurement and/or arithmetic skills--not taught at schools--by dieters after months in a weight-watching program, expert shoppers, or expert dairy workers.

Learning to frame problems gives students an experience of science similar to that of scientists, because the real problem of scientific discovery is not to find the laws in the data but to define the problem (Qin & Simon, 1990). In imposing frame, the practitioner, whether she is a scientist, engineer, music teacher, or psycho-therapist, transforms the ill-defined problem into one that can be subjected to the knowledge constructed through past experience (Bruner, 1963; Schön, 1983). And in this framing, "the problem solver generates problem and solution shape at the same time; each entails the other" (Lave, Murtaugh, & de la Rocha, 1984). The effect of the context is not only in shaping the solution but also lies in its ability to give rise to powerful monitoring strategies, made possible because of the juxtaposition of problem, solution, and checking activities (Lave, Murtaugh, & de la Rocha, 1984). Framing ones own problems also has an enormous motivational effect. No longer are problems imposed by authorities--textbook or teacher--but all of a sudden, students "own" their problems and solutions.

Transfer

A traditional argument for the teaching of skills independent of context is that it will facilitate the transfer of knowledge and skills to new problem solving situations. However, as Lave (1988) pointed out in a thorough critique of transfer experiments, convincing evidence for such transfer is still lacking. More so, researchers in situated cognition recommend the abandoning the notion of context-independent concepts and skills (Brown, Collins, & Duguid, 1989). They hold that when a concept or idea is used in a particular situation, it is recast in terms of the situation, acquiring new meaning it did not possess before. The situation thus becomes part of the meaning of the concept (Prawat, 1991). On the other hand, Brown (1990) had reported successful transfer independent of surface structure if the deep structure of a domain is well understood and when children have differentiated the causal mechanisms in their emergent theories of the world. Inspite of these findings, most transfer studies show the special problematic of misleading surface features that counteract transfer.

Research in situated cognition, both from an anthropological and a cognitive perspective seems to indicate that the whole issue of transfer has been ill-framed (Lave, 1988). Traditional transfer research was looking for the transfer of normed and pre-determined skills to a new domain. Teaching for such transfer has been shown to be of limited scope. On the other hand, the research in situated cognition indicates that people, as they become increasingly familiar with new contexts become highly efficient problem solvers in the new context (Lave, 1988; Suzuki, 1989). Rather then looking at the transfer of specific skills between contexts with problems structured normatively by cognitive researchers, the study of problem solving revealed that in everyday contexts the solutions are very often already entailed in the way the problem was framed by the problem solver. Lave (1988: 169) noted that "very often a process of resolution occurs in the setting with the enactment of the problem, and it may transform the problem for the solver." Bruner (1961) and Schön (1983) cite examples of architects, engineers, and philosophers who by "imposing" metaphorical relationships onto the new context, frame and solve problems. The data emerging from our observations also indicate that as the students became more familiar with their setting, they began to frame problems and their solutions, attended to more than just surface features, labelled new phenomena, investigated relationships, and constructed new knowledge.

Rather than viewing an increased contextualization of the teaching-learning situation as a limitation, its benefits for instruction should be realized. By creating and working in a shared context, the teacher-expert and the student-novice can make use of of this context to transmit information and skills tacitly through pragmatic communication in the problem solving situation (Rogoff & Gardner, 1984; Schön, 1987; Brown, Collins, & Duguid, 1989). Keeping part of the task tacit allows the student to focus on the fundamentally important factors rather then on those not immediately relevant. Later, the initially tacit knowledge can be made explicit as needed, after the novice has become more familiar with the expert's knowledge and skills. The student can then devote more cognitive resources to details of the problem solving process in practical contexts. Rather then being context dependent, "skilled practical thinking is goal-directed and varies adaptively with the changing properties of problems and changing conditions in the task environment" (Scribner, 1984, p. 39).

Coverage

Some teachers have raised the criticism that letting students follow their own research agendas will not permit a specific content to be covered. However, one can easily counter that the traditional "coverage" has neither lead to a superior achievement compared to students in other nations (*Science achievement in seventeen countries*, 1988) nor to the transfer of this knowledge to higher grade levels. Many teachers can be heard to complain like that chemistry teacher who "had to re-teach everything because the students don't remember anything from previous years." Moreover,

> "there is no evidence that professors of college science expect entering students to have mastered specific knowledge or special skills peculiar to specific science. Most professors want intelligent, curious students; students with good study habits, students who want to study in a particular science, students with mathematical skills and knowledge" (Yager & Penick, 1987:13).

On the other hand, our own research has shown that students get involved in and research a wide range of topics that interleaves the life with the physical sciences. We have to keep in mind, too, that it is impossible for teachers to teach and students to learn everything of importance. It has been suggested to center the curriculum around essential questions and to give students and teacher "the intellectual freedom to go where [these] essential questions lead, within bounds set by the general questions, themes, and concepts of the syllabus" (Wiggins, 1989, p. 47). This point has been supported by intensive anthropological work on arithmetic practice in everyday contexts (Lave, 1988). Lave's research supports the claim that knowledge takes on a process character rather than being primarily a factual commodity or a compendium of facts. She suggested that problem solving should be conceptualised as situated activity where means and ends are inseparably fused; she also underscored the importance of learning to set goals and to structure problems; and she contrasted it to traditional view of "problem-solving" that emphasized the search for algorithms to goals set by problem givers.

Collaboration

At present it is not clear how the joint work in teams increases the abilities of the individuals and it may be difficult to determine who does what during the activities. Past research (Newman, Griffin, & Cole, 1984 ; Forman & Cazden, 1985) and the observations during the present study have shown that through the negotiation of the goals, sharing in the procedures, and checking activities the student groups arrived at research products that none of them could have achieved on their own. Researchers operating from a Vygotskian perspective reason that the increase in social resources first give rise to a social accomplishment (inter-personal) which may then aid in the construction of intra-personal knowledge and skills (Forman & Cazden, 1985; Wertsch & Stone, 1985).

What emerges from our observations is that the activity structures described seem to provide ideal resources for sharing in discourse among students (novices in the field) or between the teacher and the students. The objects and events during the experiments, the collaborative writing activities on lab reports, and the negotiation during whole class discussions provide for contexts for the social construction of meaning. In addition, the concrete objects students manipulate become mediators for the negotiation of meaning during conversations and become tools for reasoning about the phenomena observed (Pea, Sipusic, & Allen, 1990). Although a full analysis of the proposed activity structures is still pending, the present analysis suggests that concept mapping and concrete modeling are effective tools for the construction of knowledge in collaborative situations. While doing these activities, the students engaged in conversations about the meaning of terms and models and they use the resources provided to arbitrate meaning (Pea, Sipusic, & Allen, 1990; Roschelle, 1990). Robust understanding is likely to occur when students are required to explain, elaborate, or defend their position to others. In trying to explain, students may feel the necessity to evaluate, integrate, and elaborate on knowledge in new ways (Brown, 1988). In addition, the interaction with peers and teachers forces students to reflect upon their own understanding. Because of this, teachers and students cycle through processes of situated experience and unsituated abstraction which is a necessary prerequisite for the construction of robust knowledge (Collins, 1990). Social interactive methods such as the ones described, during which students engage in discourse, set the stage for the construction of understanding acceptable by a scientific community.

Teacher as Researcher

The observations in our own classrooms, the audio and video taping of our lessons, the interviews and consultations forced us to reflect on our practices in the classroom. In turn, these reflections affected our teaching and curriculum planning activities. Of particular impact to our work appeared to be our collaborative efforts which allowed us to construct meaning together in the various stages of teaching: from the of planning larger units and daily activities via the implementation of these activities in the classroom to the reflection upon both planning and implementation. Although there are considerable time constraints if such work is done without an outside observer such as a university faculty, it is our feeling that such work is essential to change in the classroom. Of particular value in our own situation seemed to be the fact that the trained researcher was part of the school, knew the school environment with its specific physical and social situation.

Our situation can be understood from the methodological perspectives of the teacher-as-researcher (Stenhouse, 1975) or the reflective practitioner (Schön, 1983; 1987). From both perspectives, research on school learning should be based in the classroom. Schön argued that the problems in complex fields such as education can only be understood through the work of the acting practitioner. Thus, as practitioners encounter problems, they frame their situation by constructing generative metaphors from their past experience and knowledge, their repertoire of exemplars, systematic knowledge, and patterns of knowing-in-action to be applied to the novel problem. This framing leads to action research with outcomes upon which the practitioner reflects. After analyzing and reflecting, practitioners will loop back for further inquiry. This process of inquiry can be enhanced using various techniques that provide external and fixed representation of the setting in which the practitioner acts.

We experienced in our research, as Wells (1990) suggested, that both written journals, as well as video and audio recordings can function as texts which can be read and reflected upon, and made as the basis for further classroom inquiry. Wells particularly high-lighted the use of written journals that may act as "cognitive amplifiers" through which the writer's understanding of the situation is significantly enhanced. In this process, the teacher as researcher and reflective practitioner will critically engage with various texts in ways that also serve as models for the engagement of the students in their classrooms. By reflecting on classroom practice collaboratively with colleagues, other educators, and with students, teachers become agents of change of in their own environment. A change which was brought about through their own action-research rather than being recommended and implemented by outside agents, whether from the same school system or some university.

CONCLUSION

A scientist modeling research skills, coaching students, and scaffolding their efforts to higher levels. In this process, the teacher functions as a resource and as a facilitator in the students' quest for new knowledge. As a consequence, students come to function more independently, chose their own paths of inquiry, determine their own research agendas, make their own discoveries. If we are to achieve the primary goal of all schooling, namely to prepare children for life, then we have to allow them to experience and develop the skills they need in the real world. Through constraining the tasks to some extend and through the teacher's presence as facilitator we can provide contexts that are constrained and with small enough complexity so that students can develop their skills with low risks of failure; we can provide contexts, in which students may experience the frustrations and the successes of those building new knowledge; and we can provide contexts in which our students think critically and generate new knowledge as a matter of course. In such an environment, teachers as researchers or reflective practitioners can model the very attitudes which are conducive to the development of skills and attitudes in their students. Collaborative talk about texts of various kinds in the context of meaningful joint activity will provide for effective learning environments of the nature presented throughout this paper.

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Figure 1

The system

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

can be solved in the following ways:

1. Solve line one for x and substitute into second line

$$x = \frac{e - by}{a}$$

$$f = c * \frac{e - by}{a} + dy$$

from which follows

$$y = \frac{af - ce}{ad - cb}$$

which can be solved for y. Substituting back into line one yields solution for x.

2. The solution can be written and solved in determinant form

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

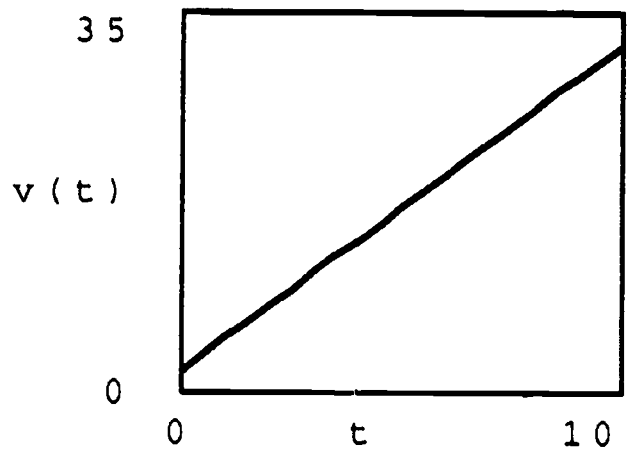
3. The solution can be written and solved in matrix form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} * \begin{pmatrix} e \\ f \end{pmatrix}$$

Figure 2

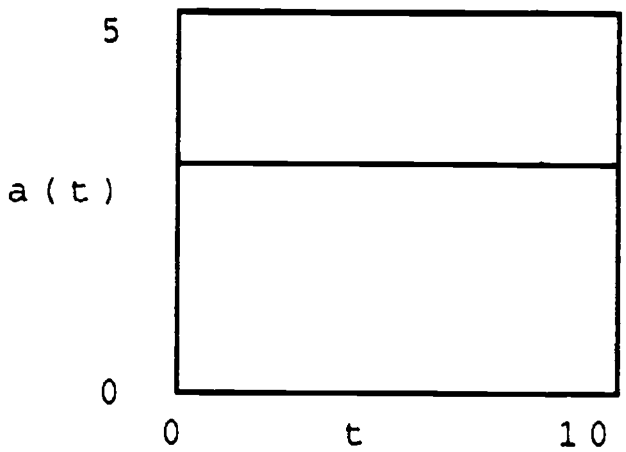
1. The acceleration is the rate of change of velocity.

2. Provided the following v(t) graph, the acceleration is given by its slope.



3. $$a(t) = \frac{v_2 - v_1}{t_2 - t_1}$$

4. Provided the graph under 2, the following represents acceleration.



5. $$a(t) = \frac{\Delta v}{\Delta t} \quad \text{or} \quad a(t) = \frac{dv}{dt}$$