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ABSTRACT

This monograph summarizes selected major activities, trends, issues, and recommendations related to curriculum, instructional materials and instruction in K-12 mathematics education that have been documented in the literature. The technique used for selecting trends, issues, and recommendations was to identify relevant literature that had published during recent years and selected documents referenced in these sources; determine the agreement or disagreement regarding trends, issues, and recommendations; select those that appeared most frequently and/or those that were indicated as possibly most influential; and select examples of curricula, programs, materials and instruction to illustrate trends, issues, and recommendations cited. Sections include: (1) "What Are the Conditions Creating a Demand for Change?"; (2) "What Is the Status of Mathematics Education in Elementary and Secondary Schools?"; (3) "Curricular Frameworks: Goals, Content, and Experience for Precollege Mathematics Education"; (4) "Research Related to Learning, Curriculum, Instructional Materials, and Instruction"; (5) "Development and Implementation of Curricula and Instructional Materials for Precollege Mathematics"; and (6) "Summary and Recommendations for the Reform of K-12 Mathematics Curriculum, Instructional Materials, and Instruction." A 166-item bibliography is provided. (KR)

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TRENDS AND ISSUES IN MATHEMATICS EDUCATION: CURRICULUM AND INSTRUCTION

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Preface

This monograph summarizes selected major activities, trends, issues and recommendations related to curriculum, instructional materials and instruction related to K-12 mathematics education that have been documented in the literature during the past few years.

The technique used for selecting trends, issues, and recommendations was to: (1) identify relevant literature that had been published during recent years and selected documents referenced in these sources; (2) determine the agreement or disagreement regarding trends, issues, and recommendations; (3) select those that appeared most frequently and/or those which were indicated as possibly most influential; and (4) select examples of curricula, programs, materials and instruction, to illustrate trends, issues, and recommendations cited.

A selected bibliography used in preparing the publication is included at the end of the monograph.

Table of Contents

Preface i

I. What Are the Conditions Creating a Demand for Change? . . 1

Conditions Creating a Demand for Change 1

Trends and Issues 4

II. What Is the Status of Mathematics Education in Elementary and Secondary Schools? 7

Achievement 7

National Elementary and Secondary Mathematics Enrollments 8

Opportunity to Learn 13

Comparisons of Time for Mathematics in K-12 Education in the U.S. and other Countries 14

Inclusion of Topics in U.S. Textbooks and Instructional Emphasis 14

Correlations of Mathematics Achievement from National and International Studies 15

Trends and Issues 16

III. Curricular Frameworks: Goals, Content, and Experiences for Precollege Mathematics Education 19

NCTM Curriculum and Evaluation Standards 20

Reshaping School Mathematics 29

State Frameworks and Curriculum Guides 30

Curriculum Development Project Frameworks 30

Goal and Content Statements of Associations and Commissions 31

Trends and Issues 32

IV. Research Related to Learning, Curriculum, Instructional Materials, and Instruction 35

Research on Learning 35

Research Related to Curriculum 38

Instructional Materials and Delivery of Instruction . . 40

Instruction and Learning Climate 42

Variables Related to Increased Achievement 43

Assessment and Evaluation 43

Teacher Characteristics, Behaviors and Preparation . . 44

School Practices 44

Community/Home Variables 44

Trends and Issues 45

V. Development and Implementation of Curricula and Instructional Materials for Precollege Mathematics . . 47

Revising and Strengthening Mathematics Curriculum . . . 47

Modifying Instruction 50

Revising High School Graduation and College Entrance Requirements 50

Devising Programs to Recruit and to Hold Minority and Female Students 51

Expanding the Curriculum and Extracurricular Programs to Include Contests and Competitions 51

Developing Special Programs for Mathematics Outside of School Hours 51

Developing Special Schools that Include an Emphasis on Mathematics 52

Accountability and Evaluation 52

Staff Development 53

Resources for Supporting Development, Dissemination and Implementation 53

Trends and Issues 54

VI. Summary and Recommendations for the Reform of K-12 Mathematics Curriculum, Instructional Materials, and Instruction 57

Conditions Creating a Demand for Change 57

Status of Mathematics Curriculum, Instructional Materials and Instruction in Elementary and Secondary Schools 58

Curricular Frameworks: Goals, Content, and Experiences for Precollege Mathematics Education 59

Research Related to Learning, Curriculum, Instructional Materials, Instruction, Evaluation, and School/Community Variables 60

Current Activities to Create Desired Changes in Curriculum, Instructional Materials, and Instruction. . 61

VII. References 64

I. WHAT ARE THE CONDITIONS CREATING A DEMAND FOR CHANGE?

The recent mathematics education literature stresses the need for change in the content of mathematics (the curriculum) and the way mathematics is taught (pedagogy).

Conditions creating a demand for change and some of the changes desired have been documented in reports, including The Agenda for Action (NCTM, 1980), A Nation at Risk (National Commission on Excellence in Education, 1983), Educating Americans' for the 21st Century (National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1983), Academic Preparation for College (College Board, 1983), The Underachieving Curriculum (McKnight, et. al., 1987), The Mathematical Sciences Curriculum K-12: What is Fundamental and What is Not (Commission on Precollege Education in Mathematics, Science and Technology, 1982), New Goals for Mathematical Sciences Education (Conference Board of the Mathematical Sciences, 1984), School Mathematics: Options for the 1990's (Romberg, 1984), Everybody Counts (Mathematical Sciences Education Board, 1989) and the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), and Reshaping School Mathematics: A Philosophy and Framework for Curriculum. (National Research Council, 1990). Other publications documenting the need for change are America's Next Crisis (Aerospace Education Foundation, 1989), One-Third of a Nation (American Council of Education, 1988), America in Transition: The International Frontier (National Governors Association, Task Force on Research and Technology, 1989), Investing in People (U.S. Commission on Workforce Quality and Labor Market Efficiency, 1989) Workforce 2000: Work and Workers for the Twenty-First Century (Johnston, William B. and Packer, Arnold E. (Eds.), 1987), and Educating Scientists and Engineers, Grade School to Grad School (U.S. Congress, Office of Technology Assessment, 1988).

Conditions Creating a Demand for Change

Conditions creating a demand for changes in precollege mathematics include seven main areas. These are: (1) changes in the world society, (2) changes in international competitiveness, (3) changes in the role of technology, (4) changes in the need for mathematics, (5) changes in mathematics and how it is used, (6) research on curriculum, learning and instruction, and (7) a discrepancy between changes desired and current school programs and student achievement (the current status of precollege mathematics.)

Changes in the World Society and the United States

Several writers, including Naisbitt (1982), Toffler (1985), and others have indicated that the United States and the developed world are shifting from an industrial to an information society. The new society uses information for much of the capital and raw material, and communication as a new means of production. Change is being accelerated by developments in both communication and computer technology. The older industrial economy is changing and new information-based economies are being developed.

These changes have created the need for individuals with the ability to continue to learn, to adapt to changing conditions, to produce new knowledge, and to acquire knowledge and skills needed in the current societal transition. These changes require increased, as well as different, knowledge and skills related to mathematics.

Demographics are also changing in the U.S. An increasing percentage of our work force will come from minorities and white females, rather than white males. Appropriate mathematics education needs to be provided for all students if the U.S. is going to have an adequate work force for the 1990's and beyond.

Changes in International Business, Marketing and Competitiveness

During the past 20 years there has been a significant change in economic competition among the nations of the world. Many developed and developing countries are becoming more productive and are creating marketing programs that are global in scope. These countries are also developing educational programs to produce better educated work forces and citizens.

Data indicate that U.S. students are not achieving in mathematics as well as students from many countries with whom the U.S. competes now or with whom it will compete for world markets (McKnight, et. al., 1987). U.S. business and industry leaders indicate that at the current time many of the workers they employ are not educated with sufficient mathematics to function effectively; as a result, business and industry spend billions of dollars to educate workers to a knowledge and skill level they need. In addition, more people are needed in the mathematics pipeline to provide a sufficient number of high-quality people at advanced degree levels for higher education, business and industry, and government.

There is a clear need to provide all students with the mathematical knowledge and skills they will need in the new global environment. Needs to prepare students to a higher level of achievement and to maintain more students in the mathematics pipeline also exist.

Changes in the Role of Technology and Use of Technology in Schools and in Society

The development of new technology during the past 15 years has changed how mathematics is used, what mathematics is important, and how mathematics is pursued. Major changes should be made in the way mathematics is taught from grades K-12 by using various types of calculators, including graphic calculators, and computers. Major changes should also be made in the curriculum, both in terms of the content taught and how the content is presented.

It has been estimated that nearly 30 percent of classroom time could be reclaimed by appropriate use of technology. Technology should also be used to individualize instruction to help all students learn more effectively and efficiently, understand concepts better, learn to solve problems more effectively, and prepare them for the work and non-work environments in which they will be using technology.

Changes in the Need for Mathematics Knowledge and Skills for Everyday Living and for Jobs

Major changes have taken place, and will continue to occur, in terms of knowledge and skills required for the work force. Analytical skills are increasingly needed for many jobs. Knowledge of computer-related concepts and processes are frequently required, and an awareness of these concepts and processes is helpful in many more positions.

A higher level of mathematical knowledge and skills is now needed for everyday living and for effective citizenship in our society. The ability to analyze and interpret information in the mass media and in a variety of databases and the ability to make business and financial decisions require more and different mathematics than are currently taught and better skills than are frequently learned.

Changes in Mathematics and How It is Used

During the past 20 years, there have been many changes in mathematics. Some of these changes have been brought about by the development of computers and the resulting need for different mathematics as well as changes in the way mathematics has been pursued. Other changes have been created by the development and emphasis of different mathematics for research, business, and industry. Changes in mathematics have been documented in reports by several groups including the NCTM and the Mathematical Sciences Education Board.

Research on Curriculum, Learning, Instructional Materials and Instruction

Knowledge of how students learn and how the curriculum, instructional materials, and instruction can help improve learning continues to increase. Fundamental ideas regarding how students construct their own knowledge, the role and sequencing of materials, the effectiveness of some instructional procedures and the use of technology require changes in mathematics curriculum and instruction. Relatively few schools are using materials and providing instruction in ways that are consistent with research on effective and efficient learning.

A Discrepancy Between Changes Desired and Current School Programs and Student Achievement

Data presented in Section 2 summarize some of the information on student achievement and school practices in the U.S.A. Evidence is clear that many students are not achieving either traditional goals or newer goals. Evidence is also clear that many school programs are not emphasizing or attaining many of the important traditional goals or newer goals.

Trends and Issues

These changing conditions have created a need to examine past and current goals, curricula, and programs for precollege mathematics to determine changes that are desired and possible. Educational research has been developing a knowledge base for mathematics education that provides a basis for the improvement of curricula, instructional materials, and instruction; the changing conditions and new goals also have created the need for additional research to help guide future efforts.

Trends

1. There is general consensus that changing conditions create a need for substantial modification of precollege mathematics education.

2. There is growing consensus on the conditions creating a need for change.

Issues

1. What mathematics education is needed for all students at the K-12 level?

2. What mathematics education should be provided for special groups at the K-12 level?

3. What changes in current precollege mathematics education are needed to respond to these conditions?

4. Is there a need for mechanisms and processes to determine how successful educational interventions are in providing solutions?

5. Are there other current major conditions that should influence precollege mathematics programs that should be considered?

6. Are there emerging conditions that should be considered?

7. These conditions are not all unique to mathematics education. How should the mathematics educational community and others organize to determine what should be done in a systematic way to address these conditions?

8. What should be the roles of (1) federal, state, and local governments and (2) the private sector in guiding and developing changes in precollege mathematics programs?

II. WHAT IS THE STATUS OF MATHEMATICS EDUCATION IN ELEMENTARY AND SECONDARY SCHOOLS?

Analyses of student mathematics achievement in U.S. schools indicate that American students are not learning several concepts and skills as well as desired. Analyses also indicate that U.S. students are not achieving as well on many important concepts and skills as students in several other industrialized countries.

Additional data indicate that the mathematics curriculum, instructional materials, and instruction tend to introduce less new material early and be more repetitive than the curricula, instructional materials, and instruction in several other countries. Data also indicate that some of the concepts and skills desired do not receive sufficient emphasis in U.S. curricula, instructional materials, and instruction and that the time U.S. students are involved in mathematics instruction is less than the time students in several other industrialized countries are involved in instruction.

Recent data indicate that most U.S. schools follow traditional instructional patterns and make relatively little regular use of technology such as calculators and computers; very few schools have curricula especially designed to capitalize on the useful features of new technology throughout their programs.

Achievement

SAT Tests

From 1961 to 1980 there was a consistent decline in scores on the SAT mathematics tests. From 1980 to 1989 some of the loss has been recovered as scores have increased (National Report on College Bound Seniors, 1987).

National Assessment of Educational Progress (NAEP)

National Assessment of Educational Progress (National Assessment of Educational Progress, 1988) has surveyed mathematics achievement in 1973, 1978, 1982, 1986, and 1988. Math test scores for age 9 showed small, but significant improvement from 1973 to 1988. Proficiency scores ranged from about 219-225. Proficiency scores for age 13 have also shown a general upward trend with scores in the 260's to just over 270. Proficiency scores for age 17 showed a slight decline from 1973 to 1982 with increases in 1986 and 1988; proficiency scores for age 17 students have ranged between about 299 and 305.

In general, gains have been made on items reflecting skills (routine manipulation), knowledge (primarily memory) and skill in the use of a calculator. Downward trends have generally been on items measuring higher-order thinking skills (problem solving and reasoning), applications, and understanding.

The achievement gap between advantaged and disadvantaged groups has narrowed, with strong gains by Blacks (Westat, Inc., 1988). Achievement scores of Hispanic students have not shown the same gains; this is possibly due to an increase in the number of students for whom the English language has not been the first language.

Achievement by males was slightly higher than that for females in 1986. Nine-year-old boys increased their performance in 1986 to equal that of girls. Thirteen-year-old boys' performance now surpasses scores of girls. Boys' scores, though significantly higher than those of 17-year-old girls, are still below their 1973 scores.

International Assessments

U.S. students have not done well on mathematics tests when compared with other countries studied (McKnight, 1987).

In the Second International Mathematics Study one group of students (12- or 13-year-old students) was assessed on the topics of arithmetic, algebra, geometry, statistics, and measurement. Japan obtained the highest achievement scores on all five topics. U.S. students were (1) slightly above the international average on computational arithmetic (computation), (2) well below the average in non-computational arithmetic (problem solving, etc.), (3) about average in algebra, and (4) in the bottom 25 percent of all countries in geometry.

A second group of students (end of secondary school experience) was assessed on number systems, sets and relations, algebra, geometry, elementary functions and calculus, and probability and statistics. Hong Kong received the highest scores on all topics while Japan was a close second. Achievement of the U.S. calculus classes was at or near the average achievement for advanced secondary school mathematics students. Achievement of the U.S. precalculus students was substantially below the international average.

Other reports have indicated similar patterns. In general they show U.S. students' achievement to be lagging behind students from many industrialized nations. The reasons for this condition are examined in this section and sections that follow.

National Elementary and Secondary Mathematics Enrollments

What mathematics have American students been experiencing? Reports from several studies provide some general information.

Table 2.1 compares the percentage of high school graduates who took selected mathematics courses in 1982 and 1987 (Westat, Inc., 1988). Differences between 1982 and 1987 enrollments were all significant at the .05 level except for calculus. These data indicate the percentage of students taking mathematics courses has been influenced by increased course requirements imposed by school districts, states, and colleges and universities. Enrollments in higher level courses (with the exception of calculus) have shown the greatest percent of increase. The data indicate, however, that only a small percentage of high school graduates is taking advanced math courses beyond Algebra II.

Table 2.1

Percentage of High School Graduates Who Took Selected Mathematics Courses, 1982 and 1987

Courses Taken	1982	1987	Percentage Point Change from 1982-1987
Algebra I	65.1	77.2	+12.1
Geometry	45.7	61.0	+15.3
Algebra II	35.1	46.1	+11.0
Trigonometry	12.0	20.4	+8.4
Pre-Calculus	5.8	12.4	+6.6
Calculus	4.7	6.1	+1.4

Source: Westat, Inc., 1988

Preliminary data from schools for 1987-88 and 1988-89 indicate enrollments have continued to increase (Council of Chief State School Officers, November, 1989). These continued increases appear to be due to new and continuing local, state, and college and university requirements.

Table 2.2 compares the percentage of high school graduates who took selected mathematics courses in 1982 and 1987 (Westat, Inc., 1988).

Table 2.2

Percentage of High School Graduates Who Took Selected Mathematics Courses, by Sex, 1982 and 1987

Courses	Male	Female
1982 Graduates		
Algebra I	73.3	66.8
Geometry	45.0	46.4
Algebra II	35.3	34.9
Trigonometry	12.9	11.2
Pre-calculus	6.0	5.5
Calculus	5.3	4.2
1987 Graduates		
Algebra I	75.8	78.5
Geometry	61.1	60.8
Algebra II	44.1	47.9
Trigonometry	21.9	19.0
Pre-calculus	13.5	11.3
Calculus	7.6	4.7

Source: Westat, Inc., 1988

Male enrollment increases between 1982 and 1987 were significant at the .05 level for all courses. Female enrollment increases between 1982 and 1987 were significant at the .05 level for all courses except calculus. Course enrollments for males and females did not differ greatly except for calculus.

Table 2.3 compares the percentage of high school graduates who took selected mathematics courses in 1982 and 1987 by race and/or ethnic background (Westat, Inc., 1988). Enrollment increases were significant at the .05 level for all courses except calculus for Whites, Blacks, and Hispanics. Enrollment increases were significant for Asian students for all courses, except Algebra I. Black and Hispanic enrollments in courses above algebra remain much lower than enrollments of Whites and Asians. Asian student enrollments are substantially higher than those of other groups analyzed.

Table 2.3

Percentage of High school Graduates Who Took Selected Mathematics Courses, by Race/Ethnicity, 1982 and 1987 *

	White	Black	Hispanic	Asian
<u>1982 Graduates</u>				
Algebra I	68.1	57.5	55.1	66.2
Geometry	51.2	28.5	25.8	64.3
Algebra II	38.7	24.2	20.8	56.4
Trigonometry	13.6	6.0	6.4	28.2
Pre-calculus	6.7	2.2	3.0	13.7
Calculus	5.5	1.4	1.8	13.2
<u>1987 Graduates</u>				
Algebra I	78.2	70.7	76.6	66.2
Geometry	64.2	43.6	44.3	82.3
Algebra II	51.4	32.3	33.2	68.3
Trigonometry	21.7	12.3	11.5	47.0
Pre-calculus	13.0	5.0	8.0	41.3
Calculus	5.9	2.4	4.1	33.0

Source: Westat, Inc., 1988

* While the percentages of Asian students enrolling in mathematics courses is high, the actual <u>number</u> of students enrolling in mathematics courses is low because Asians comprise a small percentage of the total school enrollment.

Table 2. compares the percentages of high school graduates by academic track who took selected mathematics courses in 1982 and 1987 (Westat, Inc., 1988) Enrollments of students in the Academic Track increased significantly between 1982 and 1987 at the .05 level for all courses except calculus. Enrollments for students in the other tracks increased significantly between 1982 and 1987 for Algebra I at the .05 level.

Table 2.4

Percentage of High School Graduates Who Took Selected Mathematics Courses, by Academic Track, 1982 and 1987

	Track		
Mathematics Courses	Academic	Vocational	Other
1982 Graduates			
Algebra I	76.7	55.4	55.2
Geometry	73.6	17.1	26.3
Algebra II	58.3	11.5	18.9
Trigonometry	24.5	1.1	3.4
Pre-calculus	11.8	0.5	1.5
Calculus	10.6	0.2	0.5
1987 Graduates			
Algebra I	81.3	67.6	68.7
Geometry	80.2	20.4	29.7
Algebra II	63.6	11.9	16.0
Trigonometry	31.5	1.8	3.2
Pre-calculus	19.0	0.4	1.7
Calculus	10.0	0.0	0.2

Source: Westat, Inc., 1988

Table 2.5 provides data on the number of credits earned by high school graduates in 1982 and 1987 (Westat, Inc., 1988). The data indicate that the average student earned about one more semester credit in 1987 than in 1982.

Table 2.5

Average Number of Credits Earned by High School Graduates in Various Subject Fields, 1982 and 1987

Subject Field	1982 Graduates	1987 Graduates	Change 1982-1987
English	3.8	4.05	+0.25
History	1.68	1.91	+0.23
Social Studies	1.42	1.44	+0.02
Mathematics	2.54	2.98	+0.45
Computer Science	0.11	0.42	+0.31
Science	2.19	2.63	+0.44
Foreign Language	1.05	1.47	+0.44
Non-Occup. Voc. Ed.	1.84	1.66	-0.19
Occup. Voc. Ed.	2.14	2.09	-0.05
Arts	1.39	1.41	+0.02
Physical Education	1.93	2.00	+0.07

Source: Westat, Inc., 1988

Opportunity to Learn

Several studies have indicated that studying specific content, prior knowledge (having studied content), and time for learning the subject relate to achievement (Dossey, el at, 1988; Oakes, 1987; McKnight, 1987).

National Data on Time Devoted to Mathematics in U.S. Schools

There has been a significant increase in mathematics requirements in the last eight years. Approximately 30 states currently have requirements related to the amount of time required for elementary school mathematics. From 1980 to 1984, 36 states increased graduation requirements; from 1984 to 1987 an additional seven states increased graduation requirements in mathematics. State standards listed recommendations of 45 to 60 minutes per day for grades K-3 and from 50 to 60 minutes for grades 4-6. The average reported time has been about 38 minutes per day in grades K-3 and about 49 minutes per day in grades 4-6. (Weiss, 1987)

Recommendations for grades 7-9 was from 50 to 60 minutes per day. Average reported time has been about 50 minutes per day.

In 1988, it was estimated that, on the average, schools required slightly in excess of two years of mathematics for high school graduation. This was up from 1.6 years in 1982 and 1.9 years in 1985. (Science and Engineering Indicators, 1989).

An average of 31 percent of the high schools were estimated to be offering calculus in 1986. It was most commonly offered in large schools (57 per cent), suburban schools (54 percent), medium-sized schools (48 percent) and urban schools (39 percent). It was least likely to be offered in rural (18 percent) and small-sized schools (22 percent). Small rural schools were least likely to offer calculus, but a substantial number of urban schools also did not offer calculus (Science and Engineering Indicators, 1989). The percent of students who graduated having enrolled in various courses was presented earlier in this section.

Comparisons of Time for Mathematics in K-12 Education in the U.S. and Other Countries

Mathematics requirements in other countries tend to be higher, especially for grades 4-12. The amount of mathematics taken by most students in several major industrialized countries is nearly double U.S. elementary requirements and substantially exceeds the time of mathematics courses taken by U.S. students in grades 9-12 (Travers, 1986; McKnight, 1987; Science and Engineering Indicators, 1987; and Science and Engineering Indicators, 1989).

Inclusion of Topics in U.S. Textbooks and Instructional Emphasis

Data from research studies on materials and instruction (Science and Engineering Indicators, 1987; Science and Engineering Indicators, 1989; Travers, 1986, McKnight, 1987; Weiss, 1987; and Dossey, et. al. 1988) indicate that elementary mathematics textbooks in the United States usually introduce less new content than textbooks in other countries and involve more repetition and delay of topics than textbooks used in other countries. Textbooks are also not organized and constructed to take advantage of systematic use of calculators and computers. Because instruction in most U.S. schools tends to be based on textbooks, reports of instructional emphasis generally reflect the content provided in the textbooks.

At the secondary school level, textbooks have not kept pace with new topics; many do not have sufficient numbers and variety of applications, do not integrate mathematics concepts, and are not written and organized to make effective use of technology.

Because most instruction is based on the textbook, reports of instructional emphasis at the secondary level also tend to reflect the textbooks used.

Correlates of Mathematics Achievement from National and International Studies

Analyses of assessment data, both U.S. and international, provide a number of variables that correlate with mathematics achievement. The following variables have been identified:

1. Students who have taken more courses in mathematics generally have scored higher on general achievement tests in mathematics than those who have taken fewer.

2. Opportunity-to-learn (amount of time the student's teachers have emphasized or taught the content) has correlated positively with increased achievement in mathematics. International studies strongly support this correlate: highest ranking countries on opportunity-to-learn generally had the highest ranking scores.

3. Amount of time allocated for study has tended to correlate with achievement in mathematics.

4. Recency of study (use of information) has correlated with increased mathematics achievement. This variable relates also to opportunity-to-learn. The amount of recent study also has related positively to achievement.

5. Depth of coverage relates positively to mathematics achievement on content covered.

6. Countries that have a more rapid pace of instruction, especially in lower grades, have tended to have higher achievement. Samples of students from Japan and China show achievement levels exceeding U.S. students in the early elementary grades.

7. Students whose parents had higher levels of education generally had higher levels of achievement.

8. Students whose parents encouraged them to take mathematics courses tended to have higher achievement.

Trends and Issues

Data reported and analyzed in several studies provide an indication of the status of mathematics K-12 programs, opportunity to learn, enrollments, and achievement in U.S. schools. Studies also provide comparisons of U.S. data with programs, achievement, and instructional emphasis in other countries.

Trends

1. Achievement tests indicate U.S. students are not learning many desired concepts and skills.

2. Trends in achievement scores for U.S. students indicate scores have been within a close range of scores for each grade level from 1978-1988. There have been no substantial increases or decreases.

3. Achievement scores for Blacks have increased but a substantial gap still exists between their scores and those of Caucasians.

4. Achievement scores of Hispanics have not shown the same gains as those of Blacks.

5. The percentage of students enrolled in secondary school mathematics courses has increased during the past eight years. The percent of males increased significantly for all courses and the percent of females for all courses except calculus.

6. International assessments indicate U.S. students are not achieving in precollege mathematics as well as students in several other industrialized countries.

7. Analyses of achievement data from the U.S. and other countries identify several correlates related to higher achievement scores and suggest some possible modifications for U.S. programs that might help improve achievement scores.

Issues

1. Do the assessment tests cited in this section represent the important mathematics concepts and skills? If not, how can national assessment instruments be developed to represent desired learning?

2. If the assessment instruments are valid, what changes in U.S. mathematics programs will provide better achievement for all students?

3. How can the schools, the home, and the community work together to encourage more minority students to continue their study of mathematics?

III. CURRICULAR FRAMEWORKS: GOALS, CONTENT, AND EXPERIENCES FOR PRECOLLEGE MATHEMATICS EDUCATION

Section II identified trends and issues related to current courses in mathematics and achievement in mathematics. Among the issues raised were if the goals of the curriculum were appropriate and whether the curriculum was providing and emphasizing appropriate content and skills for today and for the future.

Several reports published since 1980 identify desired goals for precollege mathematical education and mathematical knowledge and skills recommended. Most of these recommendations are based on the changing conditions identified in Section I. Among these reports have been <u>An Agenda for Action</u> (NCTM, 1980), <u>The Mathematical Sciences Curriculum K-12: What is Still Fundamental and What Is Not</u> (Conference Board on the Mathematical Sciences, 1982), <u>Academic Preparation for College</u> (College Entrance Examination Board, 1982), <u>Educating Americans for the 21st Century</u> (National Science Board, 1983), <u>New Goals for Mathematical Sciences Education</u> (Conference Board of the Mathematical Sciences, 1984), <u>School Mathematics: Options for the 1990's</u> (Romberg, 1984), <u>Curriculum and Evaluation Standards for School Mathematics</u> (NCTM, 1989), <u>Everybody Counts: A Report to the Nation on the Future of Mathematics Education</u> (National Research Council, 1989), and <u>Reshaping School Mathematics: A Philosophy and Framework for Curriculum</u> (National Research Council, 1990).

The mathematics education community, with strong leadership from NCTM and the Mathematical Sciences Education Board and with substantial support from the federal and state governments and private foundations, has developed curricular frameworks for K-12 mathematics education that suggest desired goals, content, instruction, and evaluation for mathematics education programs.

States have been active in developing their own guides and frameworks and also adapting guides and frameworks to the NCTM recommended <u>Standards</u>.

Curriculum development projects have also developed frameworks, several of which are based on the NCTM <u>Standards</u>. Some curriculum development projects have focused on K-12 programs, though most are designed for fewer grades, usually elementary or middle schools.

While some of these development projects are working on plans for implementing reform ideas, others are not. Some projects are producing instructional materials, evaluation instruments, and recommendations for instruction, while others are not.

NCTM Curriculum and Evaluation Standards

The Curriculum and Evaluation Standards were the result of an elaborate process of deliberation and consensus building. The Standards were an extension of several previous reports. (See the list of earlier mathematics education publications at the beginning of this section.) Although there is continuing debate on general goals and content of precollege mathematics, the general themes of the Standards have been widely supported and are being used as the basis for reform of mathematics curriculum and instruction by many states including California, New York, Oregon, Texas, and Wisconsin.

The Standards specify five general goals designed to help students gain mathematical power and become mathematically literate. Mathematical power denotes an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems. This notion is based on recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context. In addition, for each individual, mathematical power involves the development of personal self-confidence (NCTM, 1989). Mathematically literate denotes an individual's ability to explore, to conjecture, and to reason logically, as well as to use a variety of mathematical methods to effectively solve problems. By becoming literate, mathematical power should develop (NCTM, 1989).

The five goals listed in the Standards (NCTM, 1989) are:

1. Learning to value mathematics. Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society, and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities.

2. Becoming confident in one's own ability. As a result of studying mathematics, students need to view themselves as capable of using their growing mathematical power to make sense of new problem situations in the world around them. To some extent, everybody is a mathematician and does mathematics consciously. To buy at the market, to measure a strip of wallpaper or to decorate a ceramic pot with a regular pattern is doing mathematics. School mathematics must endow all students with a realization that doing mathematics is a common human activity. Having numerous and varied experiences allows students to trust their own mathematical thinking.

3. Becoming a mathematical problem solver. Development of each student's ability to solve problems is essential if he or she is to be a productive citizen. We strongly endorse the first recommendation of An Agenda for Action (National Council of Teachers of Mathematics, 1980, p. 2): "Problem solving must be the focus of school mathematics." To develop such abilities, students need to work on problems that may take hours, days, and even weeks to solve. Although some may be relatively simple exercises to be accomplished independently, others should involve small groups or an entire class working cooperatively. Some problems also should be open-ended with no right answer, or even need to be formulated.

4. Learning to communicate mathematically. Development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking.

5. Learning to reason mathematically. Making conjectures, gathering evidence, and building an argument to support such notions are fundamental to doing mathematics. In fact, demonstration of good reasoning should be rewarded even more than students' ability to find correct answers.

The Standards offer a framework for curriculum development--a logical network of relationships among identified topics of study. Although they specify the key elements of a high-quality school mathematics program, they neither list topics for particular grades nor show a "scope and sequence" chart. Instead, the 40 curriculum standards discuss the content for three grade-level groups: K-4, 5-8, and 9-12. The 14 evaluation standards provide strategies to assess the curriculum, instruction, and program (Suydam, 1990).

The first three curriculum standards for each grade level and three of the evaluation standards deal with problem solving, communication, and reasoning. A fourth curriculum standard, thematic connections is predicated on the belief that mathematics must be approached as a unified whole. Consequently, curricula should deliberately include instructional activities to reveal the connections among ideas and procedures in mathematics and applications in other subject-matter areas (Suydam, 1990).

For each grade-level group, nine or ten content standards supplement the first four curriculum standards. Concepts and processes recommended vary by level. Mathematical outcomes for students, the focus of the standard, discussion of what the standard means, and examples of how the content can be taught are provided.

Implementation of the Standards requires several modifications in most current programs.

Content recommended differs substantially from that included in many textbooks and mathematics programs. A listing of content and experiences to receive increased attention and decreased attention by grade clusters is listed below (NCTM, 1989).

1. Changes in content experiences and emphasis in K-4 mathematics:

   a. Increased attention:

      1. Number
         - Number sense
         - Place-value concepts
         - Meaning of fractions and decimals
         - Estimation of quantities

      2. Operations and Computation
         - Meaning of operations
         - Operation sense
         - Mental computation
         - Estimation and the reasonableness of answers
         - Selection of an appropriate computational method
         - Use of calculators for complex computation
         - Thinking strategies for basic facts

      3. Geometry and Measurement
         - Properties of geometric figures
         - Geometric relationships
         - Spatial sense
         - Process of measuring
         - Concepts related to units of measurement
         - Actual measuring
         - Estimation of measurements
         - Use of measurement and geometry ideas throughout the curriculum

      4. Probability and Statistics
         - Collection and organization of data
         - Exploration of chance

      5. Patterns and Relationships
         - Pattern recognition and description
         - Use of variables to express relationships

6. Problem Solving
   - Word problems with a variety of structures
   - Use of everyday problems
   - Applications
   - Study of patterns and relationships
   - Problem-solving strategies

7. Instructional Practices
   - Use of manipulative materials
   - Cooperative work
   - Discussion of mathematics
   - Questioning
   - Justification of thinking
   - Writing about mathematics
   - Problem solving approach to instruction
   - Content integration
   - Use of calculators and computers

b. Decreased Attention

1. Number
   - Early attention to reading, writing, and ordering numbers symbolically

2. Operations and Computation
   - Complex paper-and-pencil computations
   - Isolated treatment of paper-and-pencil computations
   - Addition and subtraction without renaming
   - Isolated treatment of division facts
   - Long division
   - Long division without remainders
   - Paper-and-pencil fraction computation
   - Use of rounding to estimate

3. Geometry and Measurement
   - Primary focus on naming geometric figures
   - Memorization of equivalencies between units of measurement

4. Problem Solving
   - Use of cue words to determine which operation to use

5. Instructional Practice
   - Rote practice
   - Rote memorization of rules
   - One answer and one method
   - Use of worksheets
   - Written practice
   - Teaching by telling

2. Changes in Content, Experiences and Emphasis in 5-8 Mathematics

a. Increased Attention

1. Problem Solving
   Pursuing open-ended problems and extended problem-solving projects
   Representing situations verbally, numerically, graphically, geometrically, or symbolically

2. Communication
   Discussing, writing, reading, and listening to mathematical ideas

3. Reasoning
   Reasoning in spatial contexts
   Reasoning with proportions
   Reasoning from graphs
   Reasoning inductively and deductively

4. Connections
   Connecting mathematics to other subjects and to the world outside the classroom
   Connecting topics within mathematics
   Applying mathematics

5. Number/Operations/Computation
   Developing number sense
   Developing operation sense
   Creating algorithms and procedures
   Using estimation both in solving problems and in checking the reasonableness of results
   Exploring relationships among representations of and operations of whole numbers, fractions, decimals, integers, and rational numbers
   Developing an understanding of ratio, proportion, and percent

6. Patterns and Functions
   Identifying and using functional relationships
   Developing and using tables, graphs, and rules to describe situations
   Interpreting among different mathematical representations

7. Algebra
   - Developing an understanding of variables, expressions, and equations
   - Using a variety of methods to solve linear equations and informally investigate inequalities and nonlinear equations

8. Statistics
   - Using statistical methods to describe, analyze, evaluate, and make decisions

9. Probability
   - Creating experimental and theoretical models of situations involving probabilities

10. Geometry
    - Developing an understanding of geometric objects and relationships
    - Using geometry in solving problems

11. Measurement
    - Estimating and using measurement to solve problems

12. Instructional Practices
    - Actively involving students individually and in groups in exploring, conjecturing, analyzing, and applying mathematics in both a mathematical and a real-world context
    - Using appropriate technology for computation and exploration
    - Using concrete materials
    - Being a facilitator of learning
    - Assessing learning as an integral part of instruction

b. Decreased Attention

1. Problem Solving
   - Practicing routine, one-step problems
   - Practicing problems categorized by types (e.g. coin problems, age problems)

2. Communication
   - Doing fill-in-the-blank worksheets
   - Answering questions that require only yes, no, or a number as responses

3. Reasoning
   Relying on outside authority (teacher or an answer key)

4. Connections
   Learning isolated topics
   Developing skills out of context

5. Number/Operations/Computation
   Memorizing rules and algorithms
   Practicing tedious paper-and-pencil computations
   Finding exact forms of answers
   Memorizing procedures, such as cross-multiplication, without understanding
   Practicing rounding numbers out of context

6. Patterns and Functions
   Topics seldom in the current curriculum

7. Algebra
   Manipulating symbols
   Memorizing procedures and drilling on equation solving

8. Statistics
   Memorizing formulas

9. Probability
   Memorizing formulas

10. Geometry
   Memorizing geometric vocabulary
   Memorizing facts and relationships

11. Measurement
   Memorizing and manipulating formulas

12. Instructional Practices
   Teaching computations out of context
   Drilling on paper-and-pencil algorithms
   Teaching topics in isolation
   Stressing memorization
   Being the dispenser of knowledge
   Testing for the sole purpose of assigning grades

3. Changes in Content, Experiences, and Emphasis in 9-12 Mathematics

a. Topics to Receive Increased Attention

1. Algebra
   - The use of real-world problems to motivate and apply theory
   - The use of computer utilities to develop conceptual understanding
   - Computer-based methods such as successive approximations and graphing utilities for solving equations and inequalities
   - The structure of number systems
   - Matrices and their applications

2. Geometry
   - Integration across topics at all grade levels
   - Coordinate and transformation approaches
   - The development of short sequences of theorems
   - Deductive arguments expressed orally and in sentence or paragraph form
   - Computer-based explorations of 2-D and 3-D figures
   - Three-dimensional geometry
   - Real-world applications and modeling

3. Trigonometry
   - The use of appropriate scientific calculators
   - Realistic applications and modeling
   - Connections among the right triangle ratios, trigonometric functions, and circular functions
   - The use of graphing utilities for solving equations and inequalities

4. Functions
   - Integration across topics at all grade levels
   - The connections among a problem situation, its model as a function in symbolic form, and the graph of that function

5. Statistics

6. Probability

7. Discrete Mathematics

8. Instructional Practices
   - The active involvement of students in constructing and applying mathematical ideas
   - Problem solving as a means as well as a goal of instruction
   - Effective questioning techniques that promote student interaction
   - The use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole-class discussions, project work)
   - The use of calculators and computers as tools for learning and doing mathematics
   - Student communication of mathematical ideas orally and in writing
   - The establishment and application of the interrelatedness of mathematical topics
   - The systematic maintenance of student learning and embedding review in the context of new topics and problem situations
   - The assessment of learning as an integral part of instruction

b. Topics to Receive Decreased Attention

1. Algebra
   - Word problems by type, such as coin, digit, and work
   - The simplification of radical expressions
   - The use of factoring to solve equations and to simplify rational expressions
   - Operations with rational expressions
   - Paper-and-pencil graphing of equations by point plotting
   - Logarithm calculations using tables and interpolation
   - The solution of systems of equations using determinants
   - Conic sections

2. Geometry
   - Euclidean geometry as a complete axiomatic system
   - Proofs of incidence and betweenness theorems
   - Geometry from a synthetic viewpoint
   - Two-column proofs
   - Inscribed and circumscribed polygons
   - Theorems for circles involving segment ratios
   - Analytic geometry as a separate course

3. Trigonometry
   - The verification of complex identities
   - Numerical applications of sum, difference, double-angle, and half-angle identities
   - Calculations using tables and interpolation
   - Paper-and-pencil solutions of trigonometric equations

4. Functions
   - Paper-and-pencil evaluation
   - The graphing of functions by hand using tables of values
   - Formulas given as models of real-world problems
   - The expression of function equations in standardized form in order to graph them
   - Treatment as a separate course

5. Instructional Practices
   - Teacher and text as exclusive sources of knowledge
   - Rote memorization of facts and procedures
   - Extended periods of individual seatwork practicing routine tasks
   - Instruction by teacher exposition
   - Paper-and-pencil manipulative skill work
   - The relegation of testing to an adjunct role with the sole purpose of assigning grades

Reshaping School Mathematics

Reshaping School Mathematics (Mathematical Sciences Education Board, 1990) provides a philosophy for teaching mathematics and a framework for the curriculum. The framework is based on a need for changes in mathematics education due to changes in society. Transitions needed include: (1) greater breadth of mathematical sciences; (2) more students who take more mathematics; (3) increased use of technology; (4) more active learning; (5) enhanced professionalism for teachers; (6) increased need for higher-order thinking skills; and (7) more sophisticated means of assessment (Mathematical Sciences Education Board, 1990).

Reshaping School Mathematics provides a framework for elementary school mathematics, middle school mathematics, and secondary school mathematics.

Emphasis for elementary school mathematics includes (1) developing number sense (representation, operations and interpretation); (2) providing an effective foundation for all aspects of mathematics including arithmetic, geometry, measurement, data analysis, probability, and discrete mathematics; (3) use of calculators; and (4) use of real objects and real data (Mathematical Sciences Education Board, 1990).

Middle school mathematics should build on the elementary foundation and expand the students' experiences. Emphasis should include (1) the practical power of mathematics and (2) reinforcement of other school subjects and vice versa (Mathematical Sciences Education Board, 1990).

The transition from elementary to secondary school mathematics should be characterized by a shift from concrete objects to abstract symbols. Developing fluency with symbols and other abstract entities--which can be geometric, algebraic, or algorithmic--must be a central aim of secondary school mathematics. Emphasis of the secondary mathematics curriculum should include (1) developing number sense (representation, operations, and interpretation), (2) introducing the entire spectrum of mathematical sciences (algebra, geometry, data analysis, discrete mathematics, and optimization), (3) stressing that reasoning is the standard of truth, and (4) mathematics every year in the curriculum (Mathematical Sciences Education Board, 1990).

State Frameworks and Curriculum Guides

More than 40 states have developed curriculum guides and/or state frameworks to influence the local school curricula and instruction. States with detailed frameworks include California, New York, Wisconsin, and Michigan. As this publication goes to press, there is activity in over 20 states to modify current curriculum guides or frameworks to analyze their curriculum guides and frameworks based on the NCTM Standards and other reports to determine how their current materials related to these reports and what they should do based on these analyses.

States that relate statewide assessment to their frameworks and curriculum guides have been effective in increasing the use of frameworks by local school districts.

Curriculum Development Project Frameworks

Some curriculum projects have developed frameworks for their programs.

The University of Chicago School Mathematics Project (UCSMP) is developing a K-12 program emphasizing a number of features including: (1) facilitating the teaching of geometry and algebra; (2) using calculators throughout the curriculum; (3)

teaching "reward" areas of mathematics (statistics, probability, discrete mathematics); (4) teaching mathematics with an application orientation; (5) utilizing microcomputers as an integrative tool to expand mathematics experiences beyond textbook presentations; and (6) using a cumulative review strategy to reinforce learning (UCSMP, 1990).

Several curriculum development projects being supported by NSF have also developed guidelines or frameworks for their work. Most of the projects are focused on the elementary school and middle school grades. Their frameworks are in general agreement with the NCTM Standards.

Goal and Content Statements of Associations and Commissions

Mathematics education and mathematics organizations and commissions have been leaders in producing statements to guide mathematics education curriculum development and instruction. Statements of the National Council of Teachers of Mathematics (NCTM) and the Mathematical Sciences Education Board (MSEB) have been presented. Closely allied with NCTM and the MSEB have been other mathematics education associations (such as The National Council of Supervisors of Mathematics), mathematics associations (such as the Mathematical Association of America and The American Mathematical Society), and other education associations.

Another statement that has been used as a guide for designing mathematics programs has been Essential Mathematics for the Twenty-First Century: The Position of the National Council of Supervisors of Mathematics (National Council of Supervisors of Mathematics, 1989). This statement identified what the organization felt students needed in mathematics for responsible adulthood.

Twelve interrelated components of essential mathematics were identified and defined. Included were: (1) problem solving; (2) communicating mathematical ideas; (3) mathematical reasoning; (4) applying mathematics to everyday situations; (5) alertness to the reasonableness of results; (6) estimation; (7) algebraic thinking; (8) measurement; (9) geometry; (10) statistics; (11) appropriate computation skills; and (12) probability.

Suggestions are also given for extending the essentials for students with interests in fields of mathematics, science and engineering. Among the content areas stressed are discrete mathematics and calculus.

Trends and Issues

Trends

1. There is a growing consensus that guidelines and frameworks for K-12 mathematics education ought to have the following characteristics:

   a. be consistent with the nature of mathematics, knowledge, processes, organization and values;

   b. be consistent with the intellectual, social, emotional, and physical development of the learner;

   c. be consistent with research on learning, curriculum, and instruction;

   d. provide for the development of knowledge, skills, and attitudes for life-long learning;

   e. provide interdisciplinary experiences related to current and future life needs for solving personal and social problems;

   f. provide appropriate content, materials, and experiences for <u>all</u> students;

   g. provide an articulated and comprehensive K-12 program;

   h. provide experiences that stress the development of creative and critical thinking, problem solving, and decision-making skills;

   i. provide experiences that emphasize major integrating concepts and principles;

   j. provide experiences that stress the application of knowledge and skills to practical and theoretical problems.

   k. provide experiences that emphasize attitudes and values;

   l. provide experiences that emphasize communication;

   m. provide emphasis on content and activities consistent with the developmental levels of students;

n. provide emphasis on content and activities that consider a wide range of student abilities, interests, and goals to give all students opportunities to succeed with mathematics and to find applications for their learning;

o. provide emphasis on content and experiences with a high probability of being used outside of school;

p. provide instructional materials that are congruent with the goals and objectives of the curriculum;

q. stress evaluation that is congruent with goals, objectives, and instruction; and

r. provide staff development to assure effective implementation and improvement of the program or curriculum.

2. All guideline and framework teams agreed that there are problems with the content of the current mathematics curriculum and that it should be changed. Recommended changes in mathematics content tend to be similar for most teams.

3. All guideline and framework teams agreed that there should be changes in experiences provided for students in schools. Recommended changes tended to be similar.

4. All guideline and framework teams agreed that instructional materials and evaluation procedures consistent with their goals and objectives were needed.

5. All guideline and framework teams indicated there was a need to inform and influence school personnel regarding desired changes and what needed to be done to complement desired changes. The new frameworks represent a major change from those used in most schools.

Issues

1. Should there be a national curriculum?

2. Should states have different curricula?

3. Are these the frameworks that will be most useful for current and future mathematics education? What mathematics is important for all students? What other mathematics should be included?

4. Are the frameworks consistent with current research?

5. What content and what experiences should be emphasized at each grade level?

6. Why have frameworks from previous reform efforts had relatively little long-term impact on mathematics curriculum and instruction?

7. Can schools be expected to change their curricula to emphasize these frameworks if assessment instruments are not aligned with these frameworks?

8. What is the relationship of these frameworks for mathematics education to frameworks developed for other content areas such as social studies/social science, science and language arts?

9. What reforms are required to enable schools and teachers to provide a learning environment to accomplish the goals of the frameworks?

10. How can the frameworks be translated into school curricula, instructional materials, instructional practices, and evaluation procedures?

11. How will the various publics become aware of needed reform and participate in the reform activities?

IV. RESEARCH RELATED TO LEARNING, CURRICULUM, INSTRUCTIONAL MATERIALS, AND INSTRUCTION

Research on mathematics education continues to extend our knowledge of learning, the curriculum, instructional materials, and instruction. Some curriculum development efforts and course improvement projects are attempting to incorporate some of these findings into their work. In other cases, curriculum development and instructional improvement projects are proceeding with little evidence that they are incorporating recent research findings into their planning, products, and implementation.

There has been increasing awareness of the need to develop an understanding of the ecology of mathematics learning to help develop effective mathematics curricula, instructional materials, and instruction. Such an ecology of learning needs to establish understanding of the interactions of the student, teacher, curriculum, instructional materials, classroom including instruction, school, home, community, and higher education.

The literature indicates (1) a growing consensus about the nature of the learner; (2) a changing view of the curriculum; (3) a changing view of teaching; and (4) developing interest in the use of new technologies. Research on these themes is building a knowledge base for use by the field and is identifying additional research that needs to be conducted.

There are continuing concerns regarding: (1) the generalizability of research data; (2) the low impact of available research data on curriculum, instructional materials and instruction in the past; and (3) communicating new research data to interested people in a timely way and in forms so that it can be used.

Research on Learning

Research on learning has been increasing and provides suggestions for improving curriculum, instructional materials, and instruction. New goals for mathematics education, new technology, and research on various aspects of mathematics education and other areas of education have resulted in the identification of new agendas for research on learning related to mathematics education.

Several aspects of learning have been emphasized in recent research (Suydam, 1989; and Suydam and Crocker, 1990) and others have been identified as needed. These are briefly considered in this section.

Conceptual Development

Research data related to concept knowledge have been increasing. There is growing agreement (Reshaping School Mathematics, 1990) regarding the importance of considering how students construct knowledge for the design of curriculum materials and for the instructional process. Prior knowledge is extremely important in an individual's learning process. Most information is learned by connecting it with existing knowledge. Concepts are usually learned most effectively when they are taught in a variety of contexts and are used in a variety of ways.

Constructing new knowledge also requires reasoning skills to be able to process information being learned and to be able to use the information that has been learned.

Research on Constructing Concepts and Understanding

Students come to school with previously learned ideas and are continually exposed to ideas related to mathematics inside and outside the classroom. Research is being focused on learning how students construct new knowledge and the impact of prior knowledge and non-instructional knowledge on their learning. Implications for modifying curriculum, instructional materials, and instruction have been developed and are continuing to be explicated.

Mastery of Subject Matter

Understanding of mastery of mathematics subject matter is increasing. Research continues to support that the order of presentation (sequence), depth, and use of application have a substantial impact on both mastery at the time and learning and mastery of more advanced ideas at a later time.

Research on Reasoning and Problem Solving

Research on reasoning skills for both the processing of information for learning and for the use of information continues to be an area of substantial activity (Silver, in Cooney and Hirsch, 1990). The influence of reasoning skills on learning is becoming better described. The interactions of reasoning skills and types of knowledge being learned are also now better described. The role of reasoning skills in using knowledge is being explored, and research is providing useful information for the design of instructional materials and instruction.

The research that is being produced and synthesized indicates the importance of both content and specific reasoning skills. Students learn concepts more effectively in most contexts when they possess reasoning skills related to the knowledge being learned. They also can generally use reasoning skills more effectively when they have learned reasoning skills with appropriate context.

There is an increasing amount of research that indicates how reasoning and problem solving skills can be taught successfully (Schoenfeld, 1985; Charles and Silver, 1988, Noddings, 1988). Metacognition skills, knowing when and why procedures should be used, has been found to be important. These are skills seldom emphasized in mathematics instruction.

Spaced Learning Development and Reuse of Knowledge and Skills

Spaced learning and reuse of knowledge and skills (Dempster, 1988) have been found to be effective for helping to increase learning. This knowledge has implications for the design of curricula, instructional materials and instruction.

Fragmentation of Knowledge and Topics

Analyses of achievement test data (NAEP, 1988) comparisons of U.S. curricula with curricula in other countries and analyses of textbooks suggest that part of the explanation for achievement that is lower than desired for mathematics and the fact that scores are relatively stable is due to fragmentation of knowledge and topics in the elementary school curriculum, instructional materials, and instruction. A large number of concepts and topics are introduced, but not developed to the extent that students understand them well enough to use them or to retain them for further use. This requires a substantial amount of reteaching when the topic is approached later. Emphasizing few topics and covering them in more depth is being investigated.

Procedural Knowledge

Research evidence related to learning in mathematics is accumulating to indicate that a substantial amount of the pencil-and-paper activities related to mathematics instruction do not improve procedural knowledge for many students. Procedural knowledge, even when learned correctly, is not as useful in many situations as learning that stresses conceptual understanding (Romberg and Carpenter, 1986; Hiebert, 1986; Cooney, 1989).

Applications of Mathematics

Analyses of recent mathematics achievement indicate that many students do not do well on application problems (Dossey, et. al., 1988). National Assessment of Educational Progress (NAEP) data have indicated that this has been a continual problem. It is apparent from other research that students can be taught to see the use of mathematics and how to apply their knowledge and skills.

Students need to have experiences in using information to effectively retain and construct mental structures for use of information. They need to use and reuse information and skills frequently in a variety of situations to be able to retain important information and skills and to be able to use them in a variety of contexts.

Attitudes toward Mathematics

Research continues to identify the importance of students' attitudes toward mathematics and mathematics courses and their success in mathematics courses and enrollment in mathematics classes when enrollment is elective. Pivotal times appear to be: (1) early elementary grades, (2) late elementary school/middle school years, and (3) required and elective courses in the secondary schools.

Research Related to Curriculum

Recent research indicates that the mathematics education curriculum needs to be modified to help learners achieve desired results. Four aspects of the curriculum (emphasis, placement, treatment of topics, and integration) have been the subjects of a substantial amount of discussion and some research.

Emphasis

Analyses of goals, objectives, content, and experience for K-12 mathematics education (NCTM, 1989; Mathematical Sciences Education Board, 1989; McKnight, 1987; and Weiss, 1987) indicate substantial differences between recommendations for K-12 programs and recent and current curricula, instructional materials, and instruction. How mathematics should provide different emphases has been delineated in most detail recently by NCTM (1989) and the Mathematical Sciences Education Board (1989).

Placement of Content

Research on learning has been providing clues on order and placement of content in the curriculum. The use of calculators and computers also is influencing what content can be taught at different levels and alternative orders for instruction. Characteristics of concepts and skills, such as concrete-abstract, familiar-unfamiliar, reasoning skills involved, relationships to other concepts, and relevancy, affect the order in which content should be presented.

Research suggests that developing mastery of number sense and operations will occur if topics are taught in more depth and if computation is approached more slowly.

Treatment of Topics

Research has also been helping to indicate more effective ways of teaching topics to improve both learning and use of knowledge. Practices that have been found to make a difference include using calculators and computers, spaced learning, focusing on fewer topics with more depth (understanding, meaningful learning, integrating content, and using knowledge in a variety of contexts).

Time

Research has indicated that several time variables relate to increased learning and achievement. These include emphasis (time) devoted to learning the content, engaged time, recency of instruction, and courses completed. Research is exploring the impact of how time is used and its relationship to learning specific concepts and skills.

Background Knowledge and Skills

Increased emphasis should be given to mathematics, language arts and reading in the elementary school grades. The importance of establishing a good foundation during the early elementary school years has been consistently shown to be important for further learning (NAEP, 1988; Anderson, et. al., 1982; Dossey, et. al., 1988; Wittrock, et. al., 1986; Howe and Kasten, 1990). Students who fall more than one and a half years behind grade level during the elementary school years often are not able to maintain effective learning at higher grade levels (Wittrock, et. al., 1986; Howe and Kasten, 1990) in mathematics. Elementary school experiences are also important for establishing understanding of science concepts and developing needed skills for further learning.

Attitudes toward Mathematics

Increased emphasis should also be given to modifications of the curriculum that will help to improve student attitudes toward mathematics. Data (Anderson, et. al., 1988; Dossey, et. al., 1988; Howe and Kasten, 1990) have indicated students in upper elementary years and middle school years become less interested in mathematics as an area for future study and a possible career emphasis. This is particularly evident for Black and Hispanic students.

Tracking and Grouping of Students

Tracking has been found to have an effect on what students learn in mathematics, their interest in the future study of mathematics, their achievement in mathematics, and later enrollments in mathematics. Tracking often provides different expectations, different content, different materials, different instruction, and different opportunities than organizational arrangements emphasing courses for all students.

Instructional Materials and Delivery of Instruction

Research and development related to instructional materials and delivery of instruction have increased during the past four years. This work has not made an impact on a large number of classes because much of the work initiated recently is in a developmental state.

Instructional Material Research and Development

Research continues to be done on existing materials and developing new materials. Among the topics of research are stated and implied goals, alignment with curricula and tests, content organization, structure, readability, misconceptions in the materials, writing style, visual materials, activities included and packaging.

Efforts are being made to modify print materials based on research knowledge from both education and mathematics. The NSF is supporting a variety of curriculum development projects to address some of the problems identified in the research. The University of Chicago Mathematics Project is also considering research knowledge in their development of materials.

Use of Technology to Improve Learning

Research indicates that technology can provide ways of improving learning through creative modifications of curricula, instructional materials, and instruction. Some of the recent research findings and potentials for the use of technology are highlighted in this section.

Use of Calculators in Mathematics Instruction

Summaries of research on the use of calculators (Hembree and Dessart, 1986; Suydam, 1986) indicate that groups using calculators exceeded control groups (groups not using calculators) in almost all the reported studies. Effective use of calculators appears to develop achievement equally well or better than instruction without calculators, allow teaching of more content and problem solving, and provide additional time for modification of the current curriculum. In addition, use of calculators permits teaching some concepts and ideas earlier and in a different sequence.

The use of graphing calculators is being explored at the secondary school level. Results from early use are very promising.

Computers and Instruction

Computers have been found to have a positive effect on achievement and attitudes toward mathematics for some students. Research on computer use is growing.

Data indicate many students enjoy using computers. They enjoy being actively engaged; they can make mistakes without being embarrassed; they are in control with many programs; they are kept on task and motivated; and they often receive immediate feedback on what they have done.

Computers have been found to be successful for drill and practice to develop basic skills in mathematics.

Computer-assisted instruction (CAI) has been investigated as a way of improving instruction and learning of mathematics for many years. Although data are mixed on its use (Bangert-Downs, et.al., 1985), as more effective materials are developed and as CAI is applied to purposes for which it has been found to be effective, CAI has provided better achievement in a shorter period of time and/or developed better understanding.

Computers also have been used successfully for managing instruction (Computer-Managed Instruction), for simulations, to assist in solving problems, to develop models, and for obtaining information from databases. Computers have also been used successfully as a part of integrated learning systems.

Integrated learning systems have been developed, and several offer a variety of materials for mathematics education. Data indicate that materials have been effective in improving learning for several mathematics education objectives. Materials are available from companies including Computer Curriculum Corporation (CCC), Computer Networking Specialists, Inc., MECC, New Century Education Corp., Roach Org, Inc., Wasatch Education Systems, and WICAT Education.

Distance Education

Distance education is being used to provide a variety of resources for precollege education for several purposes. One of the major uses in the United States has been to assist rural schools by providing courses to augment the school curriculum. A second common use has been to provide enrichment experiences to a variety of schools for their more able learners.

Research indicates distance education has been effective for adult learners in a variety of settings (Moore, 1989). Data related to its use and effectiveness for K-12 education has been less conclusive, though reports indicate it offers many opportunities for schools and students that they can not obtain in traditional ways. A number of systems are available to schools, but most have limited materials and courses at this time.

Audiovisual Technology

Several technologies have been developed to the point that they are being used in classrooms for instruction, though not on a wide-scale basis. Videodiscs, interactive videodiscs, CD ROM, and interactive TV are among the technologies being used and which hold promise for modification of curriculum and instruction. There has been very little formal research on the use of these for mathematics instruction.

Instruction and Learning Climate

During the past ten years, there has been an increased amount of research related to classroom instruction and learning. In addition, many of these studies have been reviewed and synthesized to provide strategies for the application of research to practice. From this research, patterns of classroom activities and variables related to increased achievement have continued to be identified.

Knowledge regarding how students learn and construct their knowledge and how they use their knowledge indicates the importance to the learner of both subject matter and skills. New technology makes it possible to provide instructional experiences that could not be offered before. The importance of having teachers who understand the content and skills they are trying to teach has also been thrust into sharper focus; it is difficult to teach concepts and skills if you do not understand the concepts and skills.

Research on the climate for learning has also indicated practices that can improve learning and attitudes. High teacher expectations, clear goals and expectations, cooperative learning, feedback, experiences encouraging problem solving and higher-order thinking, use of manipulative materials, and a nonthreatening environment are some of the conditions that improve learning.

Analyses of current instruction (Weiss, 1987), however, indicate that most teachers use traditional instruction and are not making use of many instructional procedures that have been found to improve mathematics learning.

Classroom Instructional Activities Used

Research reports present a consistent picture of what mathematics instruction has been in many schools. Weiss (1987) and National Assessment of Educational Progress surveys (1988) indicate most instruction in mathematics classrooms includes the following: (1) using a mathematics textbook, (2) listening to a teacher provide explanations, (3) watching a teacher do problems at the board, and (4) working problems on a worksheet. More than 50 percent of the students reported never working in small groups. Most students reported never working on projects or doing laboratory activities.

Variables Related to Increased Achievement

Among the strategies and variables that have been related to increased achievement are: (a) homework assignments; (b) low absenteeism; (c) corrective measures for errors in learning; (d) high teacher expectations; (e) teachers' confidence that they can help students; (f) academic time; (g) engaged time; (h) classroom organization; (i) feedback on learning; (l) congruence of instructional materials, instruction, and evaluation; (m) cooperative learning techniques; (n) procedures to help students construct knowledge and to eliminate misconceptions; (o) direct instruction; (p) use of calculators; (q) preinstructional strategies (set-induction, focusing, advanced-organizers); (r) questioning strategies; (s) manipulative activities; (t) emphasizing reasoning skills and concepts; and (u) mastery learning approaches.

While most of the recent and current instructional improvement efforts have been at the elementary school level, secondary school students and programs have been included in recent research.

There is a developing consensus that recent research efforts provide knowledge about teaching and learning that can make a substantial impact on instruction. Some of the information is currently being applied; further work is needed to translate more of the information so that it can be used in practice and to determine effective combinations of variables to use.

Assessment and Evaluation

Data continue to indicate that many national and statewide evaluation programs and instruments do not measure the major current goals and objectives of mathematics education. They also differ markedly from proposed goals and objectives of newer frameworks.

Research data indicate that evaluation of programs and instruments needs to be congruent with the curriculum, instructional materials, and instruction in order that a curriculum program succeed. Teachers tend to emphasize what is being tested and students focus their time and attention on what is being tested.

Nearly all reports concerned with the topic of evaluation call for different evaluation instruments. The NCTM Standards clearly support this position with an entire section on evaluation.

Teacher Characteristics, Behaviors and Preparation

Trends and issues related to this topic are outside the scope of this publication. Research data clearly indicate, however, that effective teachers are a requirement for any curriculum to succeed.

Recent research information related to learning and instruction helps to identify teacher competencies needed for effective student learning. Preservice and inservice teacher education programs need to assess how current knowledge should influence programs to prepare effective teachers.

School Practices

Several school building practices are related to effective mathematics programs and higher student achievement. Among these variables are school leadership, articulation of instructional goals, time allocations for programs, class size, supervision practices, school and staff expectations, teacher stability, staff development activities, and resources (time, materials, personnel).

Learning is enhanced when the building as a unit is focused on providing a setting for maximizing learning.

Community/Home Variables

Research continues to show significant relationships between achievement/attitudes and community/home variables, particularly socioeconomic levels of the home and expectations of the home and community.

These data support the development of programs that involve the home and the community in school activities. The data also support the development of out-of-school programs for youth.

Trends and Issues

Trends

1. There is a growing body of literature and increasing consensus that research is changing how the learner is viewed and learning occurs.

2. There is a growing body of literature and increasing consensus that research on learning, and curriculum indicates the curriculum should be modified to aid and improve learning.

3. There is a growing body of literature and increasing consensus that research on learning, instructional materials, and instruction indicate that instructional materials and research on instruction should be modified to aid and improve learning.

4. There is a growing body of literature and increasing consensus that research on learning, curriculum, instructional materials, instruction, and evaluation indicates that evaluation instruments and procedures need to be modified to aid and improve learning, instruction, and programs.

5. Research continues to provide suggestions on ways to improve curriculum, instructional materials, and instruction.

6. Recent research has helped to identify areas needing more research and new agendas for research.

7. Research data indicate teacher knowledge and beliefs regarding mathematics curriculum, instruction, instructional materials, and evaluation influence how they teach.

8. Research data continue to indicate that early school learning and achievement have a strong relationship to later learning in mathematics.

9. Research data continue to indicate that school practices relate to mathematics learning and achievement. Specific school practices have been found to relate both.

10. Research data continue to indicate that community/home variables relate to mathematical achievement and learning. Specific variables have been found that relate to both.

11. Research continues to describe curricula, instructional materials, and instruction used in K-12 classrooms for mathematics education. These data indicate recent research has not made a major impact in many schools on any of the three.

12. The use of technology for mathematics instruction is slowly increasing in schools. Few schools have made major modifications in curriculum, instructional materials, and instruction based on use of technology.

Issues

1. Should the mathematics education community be encouraged to direct more of its research toward areas identified as new agendas and areas in need of research? If so, how?

2. Should procedures be established to support studies and replication of studies to permit greater generalization of data? If so, how?

3. What can be done to have research make a greater impact on curriculum instructional materials, and instruction?

4. What can be done to help synthesize completed research more effectively?

5. What can be done to identify the implications of synthesized research for curriculum, instructional materials, and instruction?

6. What can be done to communicate research results, synthesized research results, and implications of research to appropriate audiences in timely and effective formats?

7. Should regulations and incentives be used to encourage those involved in the design and production of curriculum and instructional materials to use research results in developing their products?

8. Should regulations and incentives be used to encourage those involved in selecting materials and providing instruction to use research results in their work?

V. DEVELOPMENT AND IMPLEMENTATION OF CURRICULA AND INSTRUCTIONAL MATERIALS FOR PRECOLLEGE MATHEMATICS

This section presents trends and issues related to recent and current activities to develop and implement curricula and instructional materials. Activities include (1) revising and strengthening mathematics curricula and developing instructional materials, (2) modifying instruction, (3) revising high school graduation and college entrance requirements, (4) devising programs to recruit and to hold minority students, (5) expanding the curriculum and extracurricular programs to include contests and competitions, (6) developing special programs for mathematics outside of school hours, (7) developing special schools that include an emphasis on mathematics, (8) accountability and evaluation and (9) staff development.

Revising and Strengthening Mathematics Curricula

There has been substantial activity during the past three years to address concerns related to precollege mathematics curricula and instructional materials.

State Curriculum Guides

There has been continued activity by states to develop or revise curriculum guides for mathematics. Approximately 30 states have recommended curriculum guides and over 20 have required guides; some have both. Only a few states do not have any form of guide for mathematics.

Guides vary in detail but are tending to include more recommendations on instructional objectives, instruction, and assessment based on research. There also is a current effort to review guides against the NCTM Standards and to modify some of the guides or to issue analyses of the guides and the Standards.

Materials to Support Comprehensive Curricular Frameworks

Several groups developing comprehensive curricular frameworks for mathematics for grades K-12, 1-6, and 7-12 were described in Section III.

The National Council of Teachers of Mathematics has developed the Curriculum and Evaluation Standards and is working with states, local school districts, publishers, and curriculum development projects to modify curricula. Articles published in the NCTM journals the Arithmetic Teacher and the Mathematics Teacher have presented suggestions on implementing the standards in curriculum and instruction. NCTM Yearbooks have also included suggestions for implementing the Standards.

The Mathematical Sciences Education Board (MSEB) received a grant from the Carnegie Corporation to support work on strands of the curriculum. Strands to be included in the project include change, dimension, quantity, shape, and uncertainty. These strands could serve as examples of how totally new curricula could be developed.

The University of Chicago School Mathematics Project (UCSMP) is producing instructional materials for a K-12 program. The materials are designed to implement the framework described in Section III. The materials are different from those traditionally used in many classrooms: (1) they are designed to make effective use of the calculators; (2) applications receive a strong emphasis; (3) the computer is used throughout the program; (4) newer areas of mathematics are emphasized; (5) instructional procedures are modified to reinforce learning; and (6) several procedures for teaching arithmetic are modified. Special materials have also been developed for work with teachers.

Other Selected Curriculum Development Activities

Elementary/Middle School Materials

The National Science Foundation is providing support for the development of several elementary and middle school programs. All materials developed go through trials with pupils before they are released for widespread use by schools. Among the projects being supported are the following: (1) Development of a Logo-based Elementary School Geometry Curriculum, Kent State University, Kent, OH; (2) Used Numbers: Collecting and Analyzing Real Data, Technical Education Research Centers, Cambridge, MA; (3) Reckoning with Mathematics: Tools and Challenges for the Information Age, Educational Development Center, Newton, MA; (4) Calculators and Mathematics Project-- Los Angeles (CAMP-LA), California State University at Fullerton, Fullerton, CA; (5) K-6 Supplementary Mathematics Materials for a Technological Society, New York University, New York, NY; (6) A Revision of the Geometry and Measurement Strands, K-6, University of Georgia, Athens, GA; and (7) in addition, there are a variety of materials being developed by local schools, publishers, and producers of computer software and audiovisual materials.

A variety of materials is also being developed or modified by commercial publishers, producers of computer software, audiovisual producers, and integrated system companies. Local schools are producing or adapting materials to supplement current curricula. Eisenhower and Title II funds from the U.S. Department of Education have been used by several schools for this purpose.

There has been a definite increase in the development and modification of curriculum and instructional materials during the past several years for the elementary and middle schools. Most of the curricula and materials produced prior to 1988 do not reflect many of the recent recommendations from NCTM, MSEB, and research reports. Most of the materials produced recently have been for a few units on separate grade levels for one year and have not provided sufficient materials designed for an articulated program for several years; fitting pieces together in an effective and meaningful way becomes difficult for many schools.

Secondary School Materials

Several secondary school curriculum development and materials development projects have also been completed or are underway in addition to those cited earlier. The majority of the larger projects are by publishers, producers of computer software, and producers of audiovisual materials. There are also projects at (1) special schools for science and mathematics including those in North Carolina and Texas; (2) local schools; and (3) collaborative groups of schools. Funding for these has come from a variety of sources including publishers, government (federal and state), private foundations, business partnerships, and local schools. Most comments relative to elementary and middle schools also apply to secondary materials. Most of the materials produced prior to 1988 do not reflect many of the recent recommendations from NCTM, MSEB and research reports. Most of the materials also do not provide for an articulated program for several years.

It is difficult to implement recommendations such as those of NCTM's Standards or the MSE' without materials designed to facilitate learning acco.ding to the recommended programs. It is especially difficult to construct and to sustain learning if materials do not exist for multiple years.

Developing Materials and Programs that Use Electronic Media

There has been a substantial amount of development activity to produce software including supplemental software, software for microcomputers that include portions of a semester or more, software for microcomputers that include strands of material over several grade levels, and total courses for integrated learning systems. There have also been some excellent materials developed for television at both the elementary and secondary school levels.

There has also been development and experimentation with distance learning programs including districts in the states of Washington, Montana, Idaho, Nevada, Alaska, Illinois, North Carolina, Oklahoma, and others. Use has not been high, but advantages and disadvantages of distance learning are being learned, and use is increasing. Rural areas have generally been more involved in the use of distance education for mathematics education.

Television has received the most use, particularly in the elementary grades, and microcomputer use is steadily increasing. Problems frequently reported related to the use of technology include costs of equipment and materials (when available), teacher knowledge related to the technology and time to plan and use the technology, quality of materials, and "fit" between the school curriculum and the materials available.

Modifying Instruction

There has been substantial effort to assist teachers in learning about and using instructional materials and procedures that can be used to assist students in learning and becoming more interested in mathematics.

The Eisenhower Act of the U.S. Department of Education has supported inservice activities in every state. The National Science Foundation, private foundations, business and industry, and states have supported inservice programs; several collaborative efforts have been developed to link major urban school districts to share information. The Regional Educational Laboratories, supported by the U.S. Department of Education, have also been involved in assisting schools in modifying instruction.

Professional associations such as NCTM continue to focus on desirable instructional practices and recommended changes through meetings, conferences, publications, and activities with schools.

Reports suggest that changes are occurring where emphasis is given to instructional improvement and when resources and time are provided to make needed changes.

Revising High School Graduation and College Entrance Requirements

There has been a significant increase in mathematics requirements for high school graduation by state governments during the past several years. From 1980 to 1987, 46 states introduced or increased graduation requirements.

There also has been a trend for colleges and universities to increase the number of mathematics courses or years of mathematics required for admission. These requirements have caused many local school districts to raise the required number of mathematics courses for graduation in academic or college-bound programs.

Devising Programs to Recruit and to Hold Minority and Female Students

There has been increased effort and support to develop and maintain programs to interest minorities and females in mathematics, help them succeed in mathematics, and encourage them to continue in mathematics. Over 30 states have programs designed for these purposes.

Local schools, associations, colleges and universities, businesses, and foundations are also developing programs related to minority and female students. Intervention programs, if replicated with care and given stable funding, can make a difference. For example, the Southeastern Consortium for Minorities in Engineering (SECME), sponsored by universities and corporations, coordinates intervention programs across the southeast United States to reach over 200 schools, 27 universities, 45 corporations, and approximately 15,000 minority students a year.

Expanding the Curriculum and Extracurricular Programs to Include Contests and Competitions

Contests and competitions are receiving increased emphasis at the international, national, state, regional and local levels. The number of programs, the number of schools participating, and the number of students participating have generally been increasing. Recognition given to winners, especially those for international and national competition, has also been emphasized more in recent years.

As interest in these contests and competitions has increased, there has also been interest in school, community, and student variables related to schools that have been highly successful in these competitions.

Developing Special Programs for Mathematics Outside of School Hours

Special programs for students are being offered more frequently outside of school hours for able students, minorities, females, and students who need time to improve their knowledge and skills.

Summer programs are being offered by many school districts, colleges and universities, and states; at the current time more than 20 states offer summer programs. The formats for these programs vary from several days to as much as six weeks. Sites also vary; some are held at a local school, but many are offered at colleges and universities, camps, and research facilities. Funding for summer programs has also been increasing with federal, state foundation, and business support for many. The integration of science, computers, technology, and/or mathematics has been emphasized by many such programs.

After-school programs and Saturday programs are also being used to provide more time for mathematics and to provide more extensive experiences than the school can offer onsite. Many of these programs use local colleges, universities, and industries.

Special programs to help students who need more time to learn fundamental knowledge and skills have also been developed. While most of these programs focus on what the student needs to learn, some try to develop increased interest in mathematics by showing applications of mathematics and/or involving students in mathematics activities not usually encountered in the school.

Developing Special Schools that include an Emphasis on Mathematics

Special schools have been developed by several states and cities for mathematics. There are at least 12 states supporting or helping to support state schools that focus on mathematics. Several cities have magnet schools that focus on mathematics and that are supported in part by state funds but which are not state schools.

Although the number of state and locally supported schools for mathematics is increasing, the number is not increasing at a rapid rate.

Accountability and Evaluation

There is a strong consensus in the recent literature that changes are needed in testing and evaluation procedures to reflect desired goals. National Assessment of Educational Progress planners, IEA planners, NCTM Standards activities, and state guide development are examples of groups working on modifying evaluation instruments and procedures.

Changes being designed and implemented include emphases, types of items, procedures for collecting data, and use of data. There is an emphasis on newer content, using higher-order questions, using applications, and allowing students to use technology for questions, and designing some questions based on the use of technology.

Aligning the evaluation with the curriculum, instructional materials, and instruction is being emphasized, but data suggest this practice is not frequently followed. Use of assessment data to aid learning and the improvement of instruction, therefore, is frequently difficult and often suspect.

Staff Development

While staff development is not the focus of this publication, staff development has been identified as a major need for reform activities. It has received and is receiving strong attention and financial support. The amount of inservice education has increased dramatically with federal support from the Eisenhower Act and other programs from the U.S. Department of Education, National Science Foundation, U.S. Department of Energy, and NASA. In addition states, local schools, foundations and businesses are also providing support.

Identified teacher needs include those related to beliefs, methodology, and current knowledge of content, materials, and instruction.

Many reports indicate that previous reform efforts have failed to a large extent because teachers did not believe they needed to change instruction, were not aware of curricula materials or instructional procedures, or did not understand them, therefore they would not implement them properly and/or lacked sufficient knowledge and/or skill to instruct the class effectively.

Resources for Supporting Development, Dissemination and Implementation

Mathematics education has been receiving increased support for curriculum development, instructional material development, and implementation. Major increases have come from federal funds (NSF, U.S. Department of Education, and others), private foundations, and business and industry. Additional increases in support have been provided by some states.

The federal government has been supporting some dissemination activities through the U.S. Department of Education (Eisenhower Act, FIRST, NDN, ERIC), the National Science Foundation, and other agencies. States have also been providing resources for dissemination. In addition, new federal legislation is being considered to provide for additional dissemination of information regarding curriculum and instruction.

Professional associations have continued to focus on dissemination of information through conferences, meetings, and publications. Associations have also been involved in establishing networks, including electronic networks, to share information with potential users.

Data were not available to indicate whether local funds have been increased beyond the rate of inflation, though articles and reports continue to identify resources for the purchase of equipment, materials, and supplies as a problem; these items are usually obtained with local funds.

There has been a steadily increasing number of partnerships involving business, industry and schools. In many localities these arrangements have provided funding, materials, personnel, and other resources for assisting in the improvement of K-12 mathematics education.

Trends and Issues

Trends

1. Some curricula and instructional materials for K-12 mathematics are being developed or revised to reflect increased knowledge of how students learn mathematics, expanded uses of the mathematics sciences, more emphasis on higher-order learning skills, more emphasis on active learning, and more emphasis on newer topics.

2. Work is being done to improve assessment of learning and to develop indicators of effective programs at local, state, and national levels.

3. Relatively few mathematics curricula have articulated programs that include grades 1-12.

4. Use of technology for mathematics instruction is increasing, but slowly.

5. Funding support and opportunities for inservice education for teachers of mathematics have steadily increased during the past several years.

6. There has been continued development of programs for minorities and women to interest them in mathematics and to provide assistance.

7. The percentage of schools and agencies offering programs outside of school hours is increasing.

8. The percentage of schools participating in contests and competitions is increasing.

9. The number of special schools that emphasize mathematics has been increasing.

10. Support for curriculum development has been increasing for several years. The amount of support remains low for

the tasks identified as needed.

11. Support for dissemination and implementation has increased in recent years, but the amount on a per school basis is very low.

<u>Issues</u>

1. Should a core curriculum be established for secondary school mathematics?

2. How can effective instructional materials be developed to implement curricular recommendations?

3. How can more effective instructional procedures be implemented in the schools?

4. How can more effective assessment procedures be developed?

5. How can more effective assessment procedures be implemented in the schools?

6. How can the use of technology be increased to improve the teaching and learning of mathematics?

7. How can needed improvements in mathematics education be financed?

8. Can significant and important changes be made in K-12 mathematics without substantial restructuring of schools?

9. How can federal, state, and local policies that <u>encourage</u> reform be enacted?

10. How can federal, state, and local policies that <u>sustain</u> learning improvement activities be educated?

11. How can reform activities in mathematics education be coordinated?

12. How can reform activities in mathematics education be coordinated with other school reform efforts?

VI. SUMMARY AND RECOMMENDATIONS FOR THE REFORM OF K-12 MATHEMATICS CURRICULUM, INSTRUCTIONAL MATERIALS, AND INSTRUCTION

The preceding sections presented information and trends related to: (1) conditions creating a demand for change; (2) the status of mathematics education in elementary and secondary schools; (3) curricular frameworks for precollege mathematics education; (4) research related to learning, curriculum, instructional materials and instruction; and (5) current activities to create desired changes in curriculum, instructional materials and instruction. The literature also identified recommendations for reform of mathematics education. A selection of recommendations suggested are identified in this section.

Conditions Creating a Demand for Change

Summary

Reports document at least seven conditions requiring a demand for major changes in mathematics education K-12. Included among those most frequently cited in the literature are: (1) changes in the world society and the United States; (2) changes in international business, marketing and competitiveness; (3) changes in the role of technology and the use of technology in schools and in society; (4), changes in the need for mathematics knowledge and skills for everyday living and for jobs; (5) changes in mathematics and how it is used; (6) research on curriculum, learning, instructional materials, and instruction; and (7) a discrepancy between changes desired and current school programs and student achievement.

Several of these conditions demand changes in other areas of the school program. While mathematics reform can be addressed specifically, it should also be considered as part of a total needed reform.

Recommendations for Reform

1. Changing conditions should each be analyzed to explicate what needs to be done in mathematics education to take advantage of new knowledge, provide needed content and experiences, and correct discrepancies between desired achievement levels and current achievement levels.

2. The information obtained from this analysis should be used to analyze the comprehensiveness of current frameworks for mathematics education for the development of new frameworks for mathematics education.

3. A mechanism should be established to determine progress related to meeting these needs and opportunities and

changes in conditions that present new needs and opportunities.

Status of Mathematics Curriculum, Instructional Materials and Instruction in Elementary and Secondary Schools

Summary

Analyses of student mathematics achievement in U.S. schools indicate that American students are not learning several concepts and skills as well as desired. Analyses also indicate that U.S. students are not achieving as well on many important concepts and skills as students in several other industrialized countries.

Additional data indicate that the mathematics curriculum, instructional materials, and instruction tend to introduce less new material early and be more repetitive than the curricula, instructional materials, and instruction in several other countries. Data also indicate that some of the concepts and skills desired do not receive sufficient emphasis in U.S. curricula, instructional materials, and instruction and that the time U.S. students are involved in mathematics instruction is less than the time students in several other industrialized countries are involved in instruction.

Recent data indicate that most U.S. schools follow traditional instructional patterns and make relatively little regular use of technology such as calculators and computers; very few schools have curricula especially designed to capitalize on the useful features of new technology throughout their programs.

Recommendations for Reform

1. Achievement data identified for four NAEP studies indicate very little change for all students at all grade levels tested. Major systemic reforms are needed in mathematics education to markedly improve learning and achievement.

2. Data indicate that early schooling in mathematics has a strong relationship to later achievement, particularly for low income and minority students. Any major reform needs to provide a special focus on the first three years of schooling to prepare all children adequately for continued learning. Mathematics, reading, and language skills are among the most essential learnings to be emphasized.

3. Assessment tests need to be developed that reflect current goals and objectives. Schools that emphasize and achieve these goals and objectives should be identified to aid continued improvement of all schools.

4. Current practices in most schools indicate that past reforms have not had a major impact on instruction and classroom practices. Barriers to change need to be addressed so that current reform efforts are more effective to producing changes in instruction and improvements in learning.

Curricular Frameworks: Goals, Content, and Experiences for Precollege Mathematics Education

Summary

The mathematics education community, with strong leadership from NCTM and the Mathematical Sciences Education Board and with substantial support from the federal and state governments and private foundations, has developed curricular frameworks for K-12 mathematics education that suggest desired goals, content, instruction, and evaluation for mathematics education programs.

States have been active in developing their own guides and frameworks and also adapting guides and frameworks to the NCTM recommended Standards.

Curriculum development projects have also developed frameworks, several of which are based on the NCTM Standards. Some curriculum development projects have focused on K-12 programs, though most are designed for fewer grades, usually elementary or middle schools.

While some of these development projects are working on plans for implementing reform ideas, others are not. Some projects are producing instructional materials, evaluation instruments, and recommendations for instruction, while others are not.

Recommendations for Reform

1. There is a need to involve all major stakeholders in reviewing the frameworks, establishing the need for the frameworks, identifying what the frameworks will accomplish, and identifying alternative ways they can be implemented.

2. There is a need to develop and to test materials, instructional procedures, and evaluation procedures in a variety of sites.

3. Effective prototype materials need to be shared widely with state and local school personnel so that they can be adopted and adapted.

4. Effective communication procedures need to be established for all personnel interested in continuing developments in mathematics education. The communication procedures should use both on-line and print techniques, be widely publicized, and permit multiple pathways for information exchange.

Research Related to Learning Curriculum, Instructional Materials, and Instruction, Evaluation, and School/Community Variables

Summary

Section four presents selected research on K-12 mathematics education. Research information is available to provide for significant improvement of cognitive and affective mathematics learning and achievement. Suggestions are available for modifying the curriculum, instructional materials, instruction, evaluation, and school and community activities to be more consistent with research on learning and achievement.

Some of the results of this research are being used by curriculum developers, developers of instructional materials, developers of evaluation instruments, and school personnel working to improve community, school and classroom activities. In general, however, relatively few instructional material developers are making substantial use of this knowledge and relatively few schools are making substantial use of available research information.

Recent research and new technologies have also established the need for new research agendas related to precollege mathematics education. As we learn more about the learner, curricula, instructional materials, instruction, and school/community variables that affect learning, there are needs and opportunities for research that can continue to help the education community to understand learning and to improve educational processes.

Recommendations for Reform

1. Support needs to be provided and mechanisms developed to make better use of available research knowledge for the improvement of K-12 curricula, instructional materials, instruction, and teacher education.

2. Support needs to be provided and mechanisms established to replicate previous studies that indicate promising practices for the improvement of mathematics education to determine the extent to which the funding can be generalized. These replications will probably achieve better results if conducted on an organized basis as opposed to an unorganized approach.

3. Support needs to be provided for research related to new goals and frameworks for mathematics education. Included are higher order learning, assessment, curriculum materials, role of oral and written communication in mathematics learning and use, effects of technology on mathematics instruction and learning, effect of a core curriculum for grades 9-12, policy related issues (outcomes/inputs; regulations, etc.), teacher knowledge, and prototype programs for accomplishing specific groups of students.

4. Support needs to be provided for research and development to develop new learning systems.

5. Expand and support ways of sharing information related to research on K-12 mathematics education. Current mechanisms do not reach enough people who should be informed and information frequently is not in the most useful form for specific groups of people (policy makers, curriculum developers, researchers, etc.). These activities if done right, require substantial staff, considerable amount of money, and probably can be most effectively developed and sustained with federal support.

Current Activities to Create Desired Changes in Curriculum, Instructional Materials, and Instruction

Summary

Efforts to develop new mathematics curricula and produce instructional materials have accelerated during the past several years. The activities of the NCTM, MSEB, UCSMP, and development efforts supported by NSF (particularly for elementary and middle schools) are developing procedures and materials to change curricula, instructional materials, and instruction.

In addition some local schools and collaborative groups of schools are working to change curricula and instructional materials.

Use of technology in instruction is increasing and materials being developed for mathematics instruction used in conjunction with technology is also increasing.

Efforts to change instructional procedures have increased during the past several years, largely due to an infusion of funds from the federal and state governments, foundations, and business and industry.

Programs for interesting and assisting minorities and females in mathematics have continued, as have programs sponsoring mathematics contests and competitions and activities outside school hours.

Special schools for mathematics continue to operate and a few new ones are being developed.

The need for changes in assessment and the uses of assessment has been recognized and several organizations, agencies, and groups are working to modify current practice.

Finally, support for development, dissemination, and implementation has been increasing, but the amount available per school is very small.

Recommendations for Reform

1. Several of the frameworks being developed for K-12 mathematics education lack details and ideas regarding implementation. Alternative articulate curricula need to be developed for clusters of grades (ideally K-12) to assist schools that want to implement the frameworks. Recent research on cognitive learning argues against fragmented, unrelated instruction; it also argues for strong programs in the early grades to aid concept development, developing and using reasoning skills, and developing positive attitudes.

2. Recent research indicates that tracking has negative as well as some positive impacts. The use of core curricula particularly in the secondary school needs to be tried in a variety of settings and evaluated.

3. Barriers to change identified in a variety of publications including those of the MSEB and NCTM need to be addressed and corrected. A substantial amount of knowledge has been developed on the change process during the past 30 years. This information should be considered in developing solutions to real and perceived barriers.

4. States should work with local school districts to help them align their goals, curricula, instructional materials, instruction, and evaluation/assessment. Work to align all aspects of the mathematics program can be a powerful force in reforming mathematics education.

5. States and local schools (especially large urban and county and parish districts) need to communicate what instructional materials they want to publish. Collaborative efforts between states and local schools should be established with publishers for the

production of materials. <u>Many states and metropolitan areas have more students than do the countries with whom the U.S. is compared in international studies</u>.

States and large school districts need to exert more leadership in assuring quality curricula, instructional materials, and instruction. Reports indicate school districts often will adopt frameworks, especially if useful materials are available to support frameworks.

6. The use of technology in instruction is increasing slowly. Major barriers include, lack of teacher knowledge related to effective use of technology such as computers and highly effective materials to use with the technology. Efforts should be provided to assist teachers and to provide more useful materials.

7. The impact of various special programs (contests, after hours programs, out-of-school programs, special schools programs for minorities, etc.) should be analyzed. Models that are effective for specific outcomes should be documented and information shared with schools. Models that are less effective for specific outcomes should also be identified and information should be shared with schools.

8. Support systems for schools interested in modifying their curricula and instruction need to be developed. Analyses of the new frameworks and many of the newer materials indicate that effective use in the schools will probably require some major modifications of classroom and school procedures.

9. Efforts to change assessment need to be accelerated. There is substantial evidence that tests are one of the many variables influencing curricula and instructional materials.

10. Staff development, both at the preservice and inservice levels, needs to focus on a vision of mathematics education and curricula, materials, instruction, and evaluation that will accomplish the desired goals. Teachers belief systems influence what they consider, what they use, and what they do.

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