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ABSTRACT
        Although mathematics can be described as a formalized
language, the signs and symbols of mathematical representation are
subject to individual interpre.ation, which, if inaccurate, can lead
to inhibitions about and interferences with the mathematical learning
process. The purpose of this study was to explore communication in an
intermediate-level college discrete mathematics classroom and to
elucidate possible differences in interpretation that could detract
from the attainment of an accurate shared meaning, as well as the
effects of those differences upon achievement levels. Transcripts of
multiple interviews with 12 purposively chosen student volunteers
were analyzed for discrepant, imitative, or interpretive analyses of
the course material, and comparisons were made between students' test
results and the various types of students' explanations. When
students' test scores and final interview ratings were examined, no
significant patterns were noted beyond the fact that high achieving
students differed little from the other students in terms of
frequency of interpretative answers to interview questions. (JJK)

Semiotics in the

College Mathematics Classroom

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Abstract

This paper focused on students' understandings of lectures
in an intermediate college mathematics class, Finite Mathematics.
Segments of the videotaped class focusing on symbolic notation in
the lecture were replayed to the instructor and to students
individually. They were each interviewed and asked their
explanation of the material being presented. Transcripts of the
explanation interviews were analyzed for discrepant, imitative,
or interpretive analyses of the material that was presented.
Finally, test results over the material were compared with types
of explanation.

Semiotics in the

College Mathematics Classroom

At 10:07, three minutes before the scheduled start of
class, Laura, the instructor, arrives. She smiles at
the scattering of students and talks a little about the
weather and her walk across campus. A few students
approach her to ask about the recent test, a class date
they will miss... At 10:10 the class begins; about
fifty of the original 90 students who were enrolled ten
weeks ago are present today for the introduction of new
material on Markov chains. At 10:34, Lorrie, one of
two African American students in the study, arrives.
Paola, a Hispanic student who is also in the study, is
among those who miss today's class.

Recent reports on the status of education in the United

States have called attention to curriculum at the elementary,

secondary, and the postsecondary levels. In particular, of

sixteen major reports published since 1983, nearly all focused on

the importance of mathematics and mathematics achievement to the

country's future (Shane, 1987). Many students who come to

college have insufficient mathematics preparation at the

secondary level. Thus most colleges offer courses to prepare

students for college level work.

The success rate of students in these classes is typically

50% or less. A disproportionately high percentage of women and

minorities fail such courses (Hackett, 1985; Melange, 1987).

Unfortunately, these mathematics courses serve as educational

and vocational "gate keepers." A student who is unsuccessful at

mastering mathematics skills looses the opportunity to enroll in

a broad range of college courses. Thus their choice of career is

limited.

While many expressed concern about difficulties college students have with mathematics, there is little research on the mathematics experience at the postsecondary level. Although there has been a very specific focus on students who have what may be termed "math anxiety" (Stodolsky, 1985; Williams, 1988), little research has been conducted on learning of mathematics in the college classroom. Additionally, most studies of the college classroom tend to focus on quantitative relationships between evaluations of the classroom environment and students' grades, ignoring finer nuances of transmission of meaning and actual learning. Mentowski and Chickering (1987) have called for a higher education research agenda that places greater emphasis on learning in the college classroom.

Communication in American society is dominated by mathematical and verbal symbols. However, the signs and symbols that are used to communicate about mathematics are subject to individual interpretation (Cunningham, in press; Eco, 1976). Indeed, mathematics may be described as a formalized language (Eco, 1976). It is possible that misinterpretation of signs (written, verbal, and nonverbal) in a mathematics classroom may lead to mistaken notions regarding the nature of mathematics. Such inaccuracies about the nature of a domain of knowledge can inhibit or interfere with learning (such as incorrect knowledge about the nature of mathematics) (Alexander & Judy, 1988).

Symbols commonly employed by instructors to communicate in

mathematics classes may be misunderstood by students in those
classes. Such differences in linguistic interpretations have
been shown to cause problems for language minority students in
mathematics classes (Cocking & Mestre, 1988). It is possible
that such differences can cause problems for other students as
well.

The purpose of this study was to explore communication in an
intermediate level college mathematics classroom. The following
questions guided the study. What are the signs that are used to
communicate in the classroom? How does the instructor of the
class interpret those signs? How do students in the class
interpret those signs? Do differences in interpretation of signs
in a mathematics classroom vary by success levels of students?
Which students are more likely to share the instructor's
interpretation of those signs?

Method

The primary data collection technique employed was similar
to Guba and Lincoln's hermeneutic dialectic (Guba, 1988) and
Glazer and Strauss' (1967) constant comparative method. However,
the purpose of this study was not to negotiate consensus or
shared meaning; rather, the purpose was to elucidate differences
in interpretation (dicensus) that could inhibit the achievement
of shared meaning.

The Setting

The setting was a college level mathematics class entitled

"Finite Mathematics" at a midwestern research university. The

class included business as well as social science majors, many of

whom had already successfully completed one semester of calculus.

It fulfilled general university distribution requirements for

mathematics. Additionally, it was an alternative choice to

second semester calculus for many of the business majors in the

class.

One of four major class segments, covering material on

Markov Chains, was chosen for this study. Based on the

instructor's knowledge, it was decided that this was material

with which students would have had the least previous exposure.

Other segments of the class included topics on probability,

statistics, systems of equations, matrix algebra, and linear

programming.

Markov chains are stochastic processes (one definition of

stochastic process is "An experiment which consists of a

sequence of subexperiments" (Maki & Thompson, 1989:99). A

stochastic process is called a Markov chain if 1) it includes a

fixed number of states, and at each stage the process is in one

of those states, and 2) the conditional probability of a

transition from any given state to any other state depends only

on the two states and not on the preceding transitions (Maki &

Thompson, 1989).

As an example of a Markov chain, consider a basketball

player shooting free throws. If she makes a free throw, the

probability that she makes the next one is .7, the probability

that she misses is .3.   If she misses a free throw, the

probability that she makes the next one is .6 and that she misses

the next one also, .4.   This Markov chain has possible two

states, make (1) or miss (2).   Information about this Markov

chain may be represented symbolically in one of three ways:

   a)   By listing the probabilities:

      P_{12} =   .3   (probability of going from make to miss)

      P_{11} =   .7   (probability of going from make to make)

      P_{21} =   .6   (probability of going from miss to make)

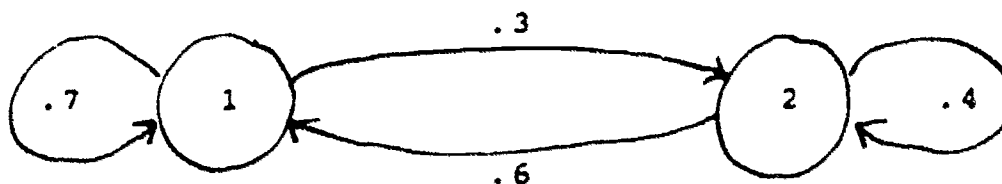 P_{22} =   .4   (probability of going from miss to miss)

   b)   By creating a transition matrix:

$$P \quad = \quad \begin{bmatrix} .7 & .3 \\ .6 & .4 \end{bmatrix}$$

      (The first element in the second row indicates

      the probability (.6) of going from miss to make.)

   c)   By drawing a transition diagram:

      This seemingly heavy use of notation is not unusual in a

mathematics classroom.   Thus, mathematics affords a unique

opportunity to study students' interpretations of signs.

The Instructor

      The instructor, Laura, holds two master's degrees, one in

mathematics, and one in mathematics education. She has fifteen year's experience teaching college level mathematics. She has authored several mathematics textbooks and teacher's guides. Additionally, she has won several awards for outstanding teaching in the mathematics classroom. For the past eight years she has been the coordinator of the mathematics learning skills program as well as a lecturer in mathematics at the university under study.

Researcher as Instrument

The researcher holds a master's degree in mathematics and a Ph.D in education and has seven years' experience teaching college level courses. The author has many publications focusing on a variety of college student outcomes and research methods. Recent publications focus on success in low level college mathematics classrooms.

Sample

The researcher visited a class session and explained the purpose and procedures for the research project. Students who volunteered were given a form to fill out. The instructor and the researcher used purposive sampling to select 12 students from the volunteers to participate in the project. Respondents were chosen to include male and female as well as culturally diverse students at various levels of achievement within the class. The sample included 7 women and 5 men. Two were African American and one was Hispanic American. Students were selected from the top, middle, and low ranges of achievement in the class.

At the end of the term, final grades earned by the students included, one A, one A-, two B+s, one B, one B-, two Cs, two C-s, one D+, and one D-.

Procedure

During the first week of classes of the spring semester demographic data and information regarding students' high school exposure to mathematics were collected by the instructor. Additionally, scores on the Mathematics Skills Assessment Test and an arithmetic pretest are coded on students' records. (This information is collected in a standardized way for all low and intermediate level mathematics classes).

During the semester, four class lectures within the "Markov chain" segment of the class were videotaped. Simultaneously, the researcher viewed the class noting the instructor's as well as students' uses of signs in the classroom (writing on the board, verbal explanation, verbal cues, nonverbal cues).

Immediately following the class, the instructor and the researcher viewed portions of the videotape. The researcher asked the instructor questions about the signs that were employed in the just-completed class session. The questions were simplistic and were intended to be of a type that students in the class might also be able to answer. The same day, half of the students were interviewed using the same questions. During the interview students were shown the same segments of the tape and asked their interpretation of the signs being employed by the instructor. All interviews were recorded.

Following the next class session, the remaining half of the sample was interviewed following the procedures described above. Each student participated in interviews for two class sessions (one introductory material and a second higher level material) for a total of 24 student interviews. All interviews were transcribed and are currently being analyzed. The analysis will be compared with students' achievement in the class.

Data Analysis

Data analysis began with the first interview. This is consistent with the constant comparative method described by Glaser and Strauss (1967). As the interviews were conducted, hypotheses were formed that were then researched in later interviews. For example, it became apparent during the first round of interviews that a hypothesis set forth initially (high achieving students' explanations of signs would most closely resemble the instructor's explanations) seemed to be too simplistic. During the second round of interviews, an alternative hypothesis, that high achieving students put their own interpretations on explanations of mathematic phenomena, was explored.

Following the interviews, the audiotapes were transcribed to hard copy. In an initial reading of the transcripts, codes were made in the margins to identify types of symbols used in the class and to categorize student responses. The explanations were initially coded as "wrong", "as an 'I don't know,'" as "imitations" of the instructor's explanations, or as

"interpretation" that went beyond the instructor's own

explanations.   During a second round of coding, impressions were

refined and examples were identified for types of symbols that

were being used in the classroom.

Establishing Trustworthiness

Lincoln and Guba's (1985) criteria for establishing

trustworthiness in naturalistic inquiry guided this study.

Credibility was established through the use of prolonged and

persistent engagement, triangulation, peer debriefing, and

member checks.   One instructor and twelve students were engaged

in 26 interviews to elicit their personal explanations of the

material being presented in class.   Triangulation was employed

through observations of four classes (two on introductory

material, two advanced material), the instructor's and students'

personal explanations of the class, and the use of multiple

respondents.

The student sample as well as the instructor conducted

"member checks."   The instructor and the interviewed students

were sent copies of the transcripts of their own interviews as

well as copies of the preliminary results.   They were asked to

contact the researcher with any questions, comments, or

challenges to the material as presented.

The instructor also served as peer debriefer for the study.

She continually served as a person to bounce ideas off during the

data collection.   She was asked to read and respond to the final

report.

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For transferability, attempts were made to be as accurate as possible about the content of the course, the classroom setting, and the skill level of the students involved. Thus the reader should be able to judge the applicability of the findings to their own settings of interest.

For dependability and confirmability, an external audit was carried out by an external auditor, a doctoral student who was familiar with the methods of naturalistic inquiry and who had full access to the videotapes, audiotapes, transcriptions of interviews and the coding of information.

Results

The questions that were used to guide the study initially form the bases for the analysis of data and discussion of the results.

What are the signs or symbols that are used to communicate in the classroom? How does the instructor of the class interpret those signs? How do students in the class interpret those signs?

Within this Finite Mathematics classroom, signs or symbols were observed that could be categorized in three ways: mathematical vocabulary, disciplinary assumptions, and axioms or theorems. Examples of the symbols and signs used in each of these categorizations as well as the instructor's (Laura) and students interpretations of the symbols used follows.

Mathematical Vocabulary. Within the class mathematical meanings attributed to words formed a vocabulary that students

had to comprehend in order to understand the material. Several

of these were evident in the videotaped segments. In any

discussion of Markov chains Laura frequently used the term

"state."

Me: In this part of the tape you are talking
           about states. What do you mean by states?
           What is that?

Laura: Well, when you are talking about Markov
           chains, you are always doing them within a
           system. You always have something that is
           happening and that system is either in one
           outcome or another outcome and in this case
           we only had two outcomes... Each of those
           outcomes becomes a state.

     One student stated that she didn't know what a state was.

The other eleven students who were interviewed about this segment

of the videotape indicated an understanding of the term   Bill

was an A- student and Jason, an A student.

Bill: ...I think she was talking about just the
           fact that the field (goal) kicker either made
           it, in the state of making it, or in the
           state of not making it. If that is what she
           means, I wouldn't use that term, but I think
           that is what she is talking about. I've kind
           of let that slide by on the board, I was
           just trying to see what she was doing on the
           board.

Jason: They're the different conditions that .
           apply... there are different conditions in
           the process. Different times, time stages.

While Bill's answer differed from the instructor Laura's in that

it was specific to the problem that was presented, Jason's answer

was less exact. He seemed to be grasping at random from phrases

he heard in class.

     Other, lower achieving students in the class were clearer

than Jason in their explanations.

Pati: I think a state is the position that... the
          condition of the chain before going on to
          another position or condition... or the n xt
          link in the series from one kick to the next
          kick.

Paola: I think she meant, like kind of a place, ...
          not even a place, the action in the present,
          the condition.

While these students' explanations were not as elaborate as

the instructor's, not as tied to the actuality of the specific

problem as Bill's, they demonstrated an understanding of the

term.

Disciplinary Assumptions. An example of a symbol that

incorporated a disciplinary assumption was evident when Laura,

the instructor, was asked, "what does that transition diagram

mean?" Part of her answer, "It is just another way of

summarizing the information. ..." provided an example of the

kinds of assumptions that those who use mathematics might make

about the signs with which they communicate. The transition

diagram is sometimes a complex combination of circles, arrows,

and numbers, that provide a description of a Markov chain -- a

very efficient way of presenting a wealth of information.

Six students were asked what was meant by the transition

matrix on the blackboard in the videotaped segment of class. One

student (a minority student who came in 24 minutes late to class)

said she didn't know. Another student gave a simplistic

explanation that was correct but provided no details. Three

students gave very literal interpretations of the actual diagram

at which we were looking. One student, with the lowest grade in

the class, provided a more elaborate, abstract explanation that

indicated a good understanding of what the diagram was meant to

represent:

Mitchell: It's a graph of the relationship between two
           events and the potential linkages between
           them... the linkages are indicated by the
           direction arrows and next to each of those is
           the probability...of going from state one to
           state two, state one to state one (etc.)...

        A second example of the use of symbols in the classroom that

incorporated disciplinary assumptions occurred when Laura was

solving a general expression (deriving a formula) for a

probability matrix.

Me: Why are there letters all over the board?
           Before there were numbers and decimals that
           you were adding up and here it just seems
           messier.

Laura: ...if we did a specific case, all we would be
           able to do would be to come up with a
           specific two step matrix. And what we want
           is a general expression, kind of like a
           formula for finding it and so we have used
           general expressions for the probabilities.

        Six students were also interviewed about the videotaped

segment on solving for a general formula. The best response was

given by the Hispanic student who earned a C- for the course:

Paola: She is trying to prove something by using
           letters that you could substitute numbers for
           in any case.

Answers from other students ranged from good "I think she is

trying to indicate what the numbers represent" to inadequate.

Nanette: ...she's just using two different states,
            cause she is just showing three people in
            going from the first state to the same
            state...

Jason: She is using variables to make it simpler, or
            to use it as an example, to have it, to show
            us another way of doing it.

Interestingly, answers given by Nanette, a B+ student, and Jason,

an A student, were not nearly as clear as Paola's.

    Axioms and Theorems. Some of the explanation that the

instructor used relied on axioms or theorems studied earlier in

the semester. Probability, a topic that had been covered in an

earlier segment of the class, was an important tool in the study

of Markov chains. At one point in the interview Laura was asked

why she totalled a series of probabilities.

Laura: Because the probability of a set of disjoint
            outcomes is the sum of each probability.

Some students' answers in this section, while they were not able

to quote the axiom, reflected an understanding of the basic

concepts.

Terry: I guess to get the total probability of an
            event happening. And when you have several
            different combinations you just add the
            probability of each individual happening.

Mark: We were looking for the probability that he
            makes the last and those were four situations
            in which he made the last one, so in order to
            get the whole situation she needs to add up
            all the probabilities.

Of the six students who answered this question, the one who

scored the highest grade, Bill (A-) and the student who scored

lowest, Mark (D-), were not as eloquent.

Bill: Because that is how you get the total

> prr ability, you have to add up each
> possibility that would satisfy the question
> you are answering, you have to add up all the
> probabilities.

Mark: To find the total of the thing she is
defining. Every chance that the individual
probability or different outcomes that affect
the condition of making the last kick... find
the probability of that one thing by adding
the final probability.

The ability to solve the problems did not seem to rely solely on

ability to recall axioms.

<u>Do differences in interpretation of signs in a mathematics</u>

<u>classroom vary by success levels of students? Which students</u>

<u>more likely to share the instructor's interpretation of those</u>

<u>signs</u>?

One assumption of the researcher was that relatively weak

students would exhibit gaps in the ways that they talked about

mathematics. The expectation was that the higher the achievement

level of the student, the more closely that student's

explanations and interpretations of the signs used in the class

would "match" the instructor's. In some instances, that was

true, but in general it was too simplistic a hypothesis.

According to initial analysis there were at least three

levels of interpretation emerging from responses in the

interview. At the lowest level, there were incomplete,

inaccurate, or no explanations about the signs used in response

to questions. However, at a second level, students gave

"reverbalizations" of what they had heard the instructor say in

class. Finally, a higher level of response included answers or

explanations of signs and symbols that involved interpretation
and moved beyond echoing the teacher's words. Students were able
to provide responses in their own terms.

When students' test scores and final scores were examined
there was almost no pattern of response in the above categories.
While the students who received the top grades in the class less
frequently provided wrong answers to questions, they did not
differ from the other students (middle and low grades) in terms
of frequency of interpretive answers.

For example, in the above section on "Disciplinary
Assumptions," a student who received a C- for the course, Paola,
provided the best explanation (and in her own words) for the
instructor's derivation of a general formula. The A student,
Jason, provided a weaker explanation. Yet, students who
understand the usefulness of deriving a general formula have
incorporated a disciplinary assumption into their own way of
thinking and learning.

Discussion

Tobias (1991) conducted a study that included highly
successful academics from the humanities taking calculus and
physics classes and highly successful mathematicians and
scientists taking humanities classes. She learned that each
group of scholars had difficulty with low level college material
across disciplinary groupings. Frequently, they reported having
trouble organizing information and sorting what was important

from what was unimportant.

One of the most striking findings from the study reported here was the difference in the ways that some high achieving and low achieving students in this mathematics class described the symbols employed in their classroom. Frequently, a low achieving student was able to describe accurately and eloquently the meanings of the symbols used in abstract terms. However, those same students', as indicated by test scores and final grades, were least successful in the classroom. By contrast, frequently the students who earned As in the class gave verbal explanations of symbols employed that were completely inaccurate.

It is possible that those who were high achievers verbally were not high achievers quantitatively and those at the highest level of the mathematics class did not have good verbal skills. This finding in itself was unsurprising - that the high verbal students may be low in quantitative skills. What was surprising was the frequency with which those students could better explain elements of their mathematics course better than high achievers in the same classroom.

Students' responses were categorized according to three subjectively developed categories: those that were wrong, responses that were reverbalizations of what the instructor had said, and interpretive - those that went beyond the instructor's explanation. Those categories seemed unrelated to students' success in the classroom as measured by scores on tests and final grade.

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This study raises two questions. First, if students grasp material conceptually and can explain the use of symbols in the abstract, what is it that prevents them from translating that grasp into ability as measured in the classroom? Second, is it important for those who can calculate the answers in a mathematics class to be able to explain the constructs about which they are calculating?

Students in this study had either successfully tested into this Finite Mathematics class or had succeeded in passing lower level prerequisite courses. The range of final grades for the students interviewed was A through D-. However, by the time this last segment of the mathematics class was reached, most of those who were unsuccessful had already withdrawn or failed out of the class. Perhaps those students would be unable to muster a verbal explanation of the symbols being used in class.

Further research might include students from an earlier point in the semester. This would enable the inclusion of a broader range of ability in the student sample. Another study might explore the degree to which students respond to questions in terms of merely describing symbols. That could be contrasted with explanations that include evidence of knowledge about what one does with those symbols.

The results of this research were the beginnings of an attempt to understand and overcome some of the problems with learning processes in college level mathematics classrooms.

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