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ABSTRACT

The way in which the partial credit model is being used in one large education system to analyze performance on standard assessment tasks undertaken by all students in their final year of high school is described. The assessment tasks include investigative projects, laboratory tasks, worked problems, drawings and studio work, essays, and paper-and-pencil tests. The initial assessments teachers make using shared assessment criteria and ten performance levels for each task are verified by procedures that involve meetings of teachers, the sampling and checking of student work from each school, and visits to schools to discuss grade allocation where necessary. The partial credit model will be used to assist in the definition and description of grade levels for each task and to monitor the ongoing application of assessment criteria. The use of the model to develop and refine assessment criteria and grade descriptions for a mathematics problem-solving and modeling task during a 2-week period is described. This new system of assessment will be introduced in the state of Victoria (Australia) in 1991-92 and will apply to over 40,000 students each year. Five tables and three figures are included. The Student Booklet of the Victorian Certificate of Education 1990 Pilot Program, Mathematics, is provided. (Author/SLD)

Using the Partial Credit Model to Analyze Performance Assessments

Geofferey N. Masters
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Abstract

This paper describes how the partial credit model (PCM) is being used in one large education system to analyze performances on standard assessment tasks undertaken by all students in their final year of high school. These tasks include investigative projects, laboratory tasks, worked problems, drawings and studio work, essays and paper-and-pencil tests. For many tasks, classroom teachers make initial assessments of students' performances using shared assessment criteria and ten performance levels (E to A+) for each task. Teachers' initial assessments are 'verified' by procedures that involve meetings of teachers, the sampling and checking of student work from each school, and visits to schools to discuss grade allocations where necessary. The partial credit model is being used to assist in the definition and description of grade levels for each task and to monitor the ongoing application of assessment criteria. This paper describes the use of the model to develop and refine assessment criteria and grade descriptions for a mathematics problem-solving/modelling task undertaken by students over a two-week period.

Introduction

Many large-scale assessment programs are taking steps to broaden the range of assessment methods they use in an attempt to better reflect curriculum priorities and emphases. These initiatives are based on a recognition that existing assessments--usually involving only paper-and-pencil tests--cover a limited part of school learning and so can distort curriculum priorities. In a search for alternative or complementary methods of assessment, education systems in a number of countries are experimenting with assessments of students' abilities to manipulate materials, construct models, play instruments, develop artworks, and solve practical problems.

Because performance assessments of this kind usually cannot be conducted under traditional test conditions and may require individual observation, possibly over a period of time, some assessment programs are giving classroom teachers a greater role in this process. In these programs, assessments of students' practical performances are based not so much on right/wrong answers to test questions as on professional judgements of the qualities of students' performances on one or more defined assessment tasks.

Paper presented at the annual meeting of the American Educational Research Association, Chicago, April 3-7, 1991.

If performance assessments are to replace or complement paper and pencil tests in large-scale assessment programs, then they must provide credible, reliable measures of student achievement in those areas of learning for which they are developed. Methods of analysis of multiple-choice tests will be inappropriate for these new forms of assessment. Instead, there will be a need for methods of analyzing and validating graded judgements of students' performances.

This paper describes how an item response model for graded judgements is being used for the analysis of performance assessments in one large education system. In this system, all students in their final year of high school undertake a set of standard assessment tasks in each of the subjects they study. Classroom teachers monitor the completion of tasks and provide an initial grade for a student's performance on a scale of E to A+ taking into account a list of provided criteria for the award of grades on each task. This paper shows how the partial credit model was used in the development of a mathematics assessment task to study teachers' use of provided criteria, to define grade levels, and to develop guidelines for grade allocation.

The 'Common' Assessment Tasks of the VCE

In 1991-92, a new system of student assessment will be introduced for students in their final two years of high school in the state of Victoria, Australia. Students satisfactorily completing high school (some 40,000 each year) will be awarded the Victorian Certificate of Education (VCE). Under the VCE, students in the last two years of high school will typically complete 24 half-year 'units' which must include English (4 units); Australian Studies (2); Arts/Humanities (2), Mathematics/Science/Technology (4).

Students undertaking VCE studies designed for the final year of high school will undertake a series of four (but sometimes three) 'common' assessment tasks (CATs) in each of their school subjects. Common assessment tasks take a variety of forms, from completing projects, to developing folios of artwork or writing, to more traditional paper and pencil tests. Each CAT has been designed by the Victorian Curriculum and Assessment Board (VCAB) and is intended to provide comparable student assessments across the state.

Students' performances on many common assessment tasks are given an initial grade by classroom teachers. To assist teachers in this process and to establish a degree of comparability of teachers' assessments, VCAB provides a set of *criteria* to be used in the assessment of each task. In a meeting at the beginning of the school year, subject-area teachers from nearby schools meet to review these criteria and to practise grading samples of student work representative of the ten grade levels E, E+, D, D+, C, C+, B, B+, A, and A+ on each task.

Teachers are also given *grade descriptions* of typical performances associated with grades E/E+, D/D+, C/C+, B/B+ and A/A+.

Later in the year, when teachers' initial grades are submitted to VCAB, work from each school is sampled (2 per grade level) to verify that teachers are using criteria and grade descriptions consistently across schools. Where inconsistencies are noted, schools can be asked to submit further graded work on that CAT and are able to request assistance with any work they find difficult to grade. Over a period of several months, visits to schools are then made to assist teachers in this process. At a final meeting towards the end of the school year, all student work on a task is brought to a local verification meeting of teachers where further sampling and checking of grades is carried out prior to the submision of final grades to VCAB for inclusion on students' certificates.

This process will replace the current system of written examinations in each subject. This year, common assessment tasks will be taken by all students studying mathematics, English and Australian studies. In 1992, common assessment tasks will be introduced for the other 41 high school subjects.

Mathematics CAT 2: "Challenging Problem"

Students taking final-year mathematics studies will complete four common assessment tasks:

CAT 1. Investigative Project
CAT 2. Challenging Problem
CAT 3. Facts and Skills Task
CAT 4. Analysis Task

Two of these CATs will be completed under test conditions: the Facts and Skills Task (multiple-choice test) and the Analysis Task (worked problems). The other two tasks will be completed over a period of several weeks and will be the subject of the grading and verification procedures described above. Students will choose from one or more 'blocks' of mathematics (*Space and Number, Change and Approximation,* or *Reasoning and Data*) and undertake the four CATs appropriate to those blocks.

The focus of this paper is on the use of mathematics CAT 2 (Challenging Problem). This task is intended to assess students' abilities to undertsand mathematical problems, to use a number of strategies in solving problems and/or constructing models, and to interpret results obtained during the problem-solving process. In particular, CAT 2 is designed to provide students with an opportunity to:

Read and understand a problem;
Formulate and interpret a problem mathematically;

Use an appropriate problem-solving and/or modelling strategy;
Try simple cases;
Find patterns;
Formulate hypotheses;
Simplify complex situations;
Define important variables;
Find proofs or explanations;
Interpret solutions.

(Victorian Curriculum and Assessment Board, 1990)

The 'Challenging Problem' task requires each student to undertake a problem-solving/modelling activity and to prepare a report on that activity. Students undertake a task selected in consultation with their teachers from a list of four problems set by VCAB for each mathematics block. The four problems set by VCAB for the *Reasoning and Data* block in 1990 are shown in the Appendix.

Students attempt problems over a 2-week period. It is intended that they should spend from 6 to 8 hours on the problem they choose, about 4 to 6 of which should be during class time. Students may collaborate on the initial stages of their work, but each student must carry out the analysis, interpretation and writing up of the work individually. Students are encouraged to include in their reports a statement of the problem, the conclusions reached, an account of the process by which the conclusions were reached, an evaluation of the solution obtained, suggestions for directions in which further work could proceed, an indication of how computers or calculators were used (if appropriate), and appendices (e.g., raw data, computer printouts).

Teachers are required to monitor the development of each student's work, sighting plans and drafts and recording this process. The work is assessed only if the teacher can attest that, to the best of his or her knowledge, the submitted work is the student's own.

Teachers make an initial assessment of each student's work, rating it on a number of provided criteria. Teachers judge performances as *NOT SHOWN*, *LOW*, *MEDIUM*, or *HIGH*. They then give the work an overall grade on a scale of E to A+. Students who do not satisfy criteria for the achievement of an E receive a result of Ungraded (UG).

Preliminary Criteria and Grade Descriptions (1988)

Work on the development of the Chalienging Problem task began in 1988 when a set of problems was developed and administered to a group of several hundred students as part of a preliminary trial. In 1989, further trial testing took place in nine high schools. In 1990, several thousand students were assessed by these new assessment procedures as part of a pilot study. In 1991, for the first time, all students taking mathematics as part of the Victorian Certificate of

Education will attempt one of the Challenging Problems made available by VCAB for each of *Space and Number*, *Change and Approximation*, and *Reasoning and Data*.

This paper outlines some of the preliminary thinking and analyses carried out in 1988 to address the development of assessment criteria and grade descriptions for the Challenging Problem task. It also shows the revised criteria and grade descriptions to be used in the full-scale implementation of this assessment system in 1991.

VCAB staff working on the devlopment of the Challenging Problem task first identified a set of illustrative examples of the kinds of problems to be presented to students (see the examples in the Appendix) and then developed a preliminary set of criteria that teachers might use in assessing students' attempts at such problems. The first pass at a set of criteria is shown in Table 1. Each of these criteria is intended to describe an aspect of a student's attempt at a problem that teachers might observe and rate using the four categories described above.

Initially, 26 criteria were developed and grouped under three headings: Presentation of Report, Mathematical Content, Process of Problem Solving. In the initial trials of the Challenging Problem in 1988, teachers used these 26 criteria to judge students' work.

As part of the feedback on how these 26 criteria were used by teachers, and to provide input into the next phase of task development (the definition and description of grades on this task) teachers' ratings were analysed using the Rasch partial credit model (Masters, 1980, 1982). The results of the analysis are shown in Table 2. The 'thresholds' shown here are estimates of positions on an assumed underlying mathematics problem-solving continuum. Students with ability estimates above -4.84, for example, are estimated to have a greater than .5 probability of achieving a rating of LOW or better on criterion 1; students with estimates above -1.22 are estimated to have a greater than .5 probability of achieving a rating of MEDIUM or better; and students with estimates greater than 2.33 are estimated to have a greater than .5 probability of achieving a rating of HIGH on criterion 1.

The two fit statistics INFIT-T and OUTFIT-T on the right of Table 2 provide an indication of the fit of each criterion to the model (see Wright and Masters, 1982). When data fit the PCM, these statistics have an expected value near zero and standard deviation near one. There is a tendency for the statistics in Table 2 to take negative values, possibly indicating a lack of independence among these 26 criteria. This may be a special problem among Process criteria such as 'working systematically', 'keeping a record of work done', 'using appropriate strategies', and 'using appropriate techniques'. The criteria with the largest negative statistics are the *Overall* criteria 5 and 9. These criteria are not designed to be independent of criteria 1, 2, 3, 4, 6, 7 and 8, but to be summaries of students' performances on these criteria. The relatively poor fit of criteria 5 and 9 raises a question about the role and value of summary criteria

TABLE 1
PRELIMINARY CRITERIA FOR MATHEMATICS CAT 2
(CHALLENGING PROBLEM)

Presentation of Report

1. Clear statement of problem.
2. Clear statement of conclusions.
3. Organisation of material.
4. Representation of mathematical data (graphs, tables, etc.)
5. OVERALL assessment of presentation.

Mathematical Content

6. Level of mathematics used.
7. End point reached in the problem.
8. Effective use of mathematical language, symbols, conventions.
9. OVERALL assessment of mathematical content.

The Process of Problem Solving

10. Comprehension, ability to translate to special cases.
11. Quality of questions posed.
12. Extensions explored or suggested.
13. Trying simple cases.
14. Devising or using a model or diagram.
15. Working sytematically.
16. Keeping a record of work done.
17. Using appropriate problem-solving strategies (e.g., trial & error, working backwards).
18. Using appropriate mathematical techniques.
19. Using appropriate tools (e.g., computers and calculators).
20. Carrying out calculations.
21. Finding patterns.
22. Formulating hypotheses / making conjectures.
23. Looking for counterexamples / testing hypotheses.
24. Clarity of explanation.
25. Quality of explanation (e.g., mathematical proof vs. testing many cases).
26. OVERALL assessment of problem solving or modelling process.

TABLE 2
ANALYSIS OF PRELIMINARY CRITERIA
MATHEMATICS CAT 2
(CHALLENGING PROBLEM)

CRITERION	THRESHOLDS 1	2	3	INFIT T	OUTFIT T
1	-4.84	-1.22	2.33	1.7	2.2
2	-4.49	-0.61	2.08	1.8	1.3
3	-4.53	-0.47	2.94	2.0	1.4
4	-2.97	0.40	3.44	-0.5	-0.4
5	-4.34	0.06	3.01	-2.6	-2.1
6	-3.59	0.49	5.17	1.1	1.0
7	-2.97	1.03	5.34	1.5	1.7
8	-2.19	0.67	4.68	-1.0	-1.0
9	-2.89	0.94	4.37	-2.5	-2.0
10	-3.05	-1.06	3.56	0.0	-0.1
11	-1.21	-0.42	1.33	-0.2	0.6
12	-2.05	-0.31	1.78	-0.5	-0.3
13	-5.16	-1.49	2.51	-2.4	-1.8
14	-3.24	-1.07	1.31	-0.4	-0.2
15	-2.01	-1.03	1.90	-2.5	-1.2
16	-4.69	-0.90	2.12	-1.2	-1.0
17	-3.13	0.26	3.38	-1.6	-1.2
18	-3.20	0.82	2.04	-1.4	-0.8
19	-3.13	-0.77	2.68	0.3	0.4
20	-3.20	-0.26	4.11	-0.4	-0.4
21	3.40	-0.57	3.89	-0.7	-0.7
22	-4.02	2.58	3.27	0.0	0.2
23	-2.58	3.17	3.86	-0.9	-0 8
24	-3.20	1.28	4.01	0.7	0.3
25	-2.23	1.64	3.47	1.1	0.1
26	-2.97	-0.51	3.96	1.4	0.6
Mean			0.00	-0.3	-0.2
SD			0.79	1.4	1.1

in a list of this kind, and the generally negative fit statistics for the Process criteria raise a question about the value of having so many closely-related criteria.

Other feedback was obtained from teachers on the relevance and ease of using individual criteria. Over the developmental phase of this work (1988-90), feedback from teachers and feedback of a statistical kind were used to substantially revise and reduce the number of criteria for the Challenging Problem task.

The results of calibrating the 26 criteria (Table 2) were used to construct the map in Figure 1. This map plots the *LOW*, *MEDIUM* and *HIGH* thresholds from Table 2 for each criterion. An inspection of the map in Figure 1 shows that students generally achieved higher ratings on 'Presentation of Report' criteria (1 to 5) than on 'Mathematical Content' criteria (6 to 9). Students also found it easier to achieve *MEDIUM* or *HIGH* ratings on general problem-solving processes such as working systematically, keeping records, and using appropriate strategies, techniques and tools (criteria 13 to 19) than on more specific tasks such as carrying out calculations, finding patterns and counterexamples, and formulating hypotheses (criteria 20 to 23).

In addition to providing insight into the ways in which teachers have used these 26 criteria and the associated rating points, Figure 1 provides suggestions on how grade levels might be developed and defined. This is illustrated in Table 3 where three positions on the underlying continuum (-4 logits, 0 logits, +4 logits) have been considered. Students located in the vicintiy of -4.0 on this continuum could probably be expected to receive an overall grade of E for this task; students in the vicinity of 0.0 may receive a C or C+; students around +4.0 are likely to receive a grade of A or perhaps A+.

Table 3 shows typical patterns of ratings observed at these three positions on the problem-solving continuum. A student estimated to be at -4.0 on this continuum is likely to achieve only a small number of *LOW* ratings, most probably on criteria such as 1, 2, 3 and 5. A student estimated to be near 0.0 on the continuum can probably be expected to achieve a relatively even mix of *LOW* and *MEDIUM* ratings, probably with the higher ratings on criteria relating to presentation of report and general problem-solving processes (rather than mathematical content and specific problem-solving behaviors). Finally, at +4.0 logits, a student is likely to achieve almost entirely *HIGH* ratings, with a small number of *MEDIUM* ratings, probably on mathematical content criteria.

In deciding how E to A+ grades are to be defined, consideration must be given to the proportions of students likely to achieve the resulting grades. Under the VCE, grades will not be defined normatively in the sense that fixed proportions of students will receive each grade. Rather, it is the intention that grades will be defined to provide a reasonable distribution over

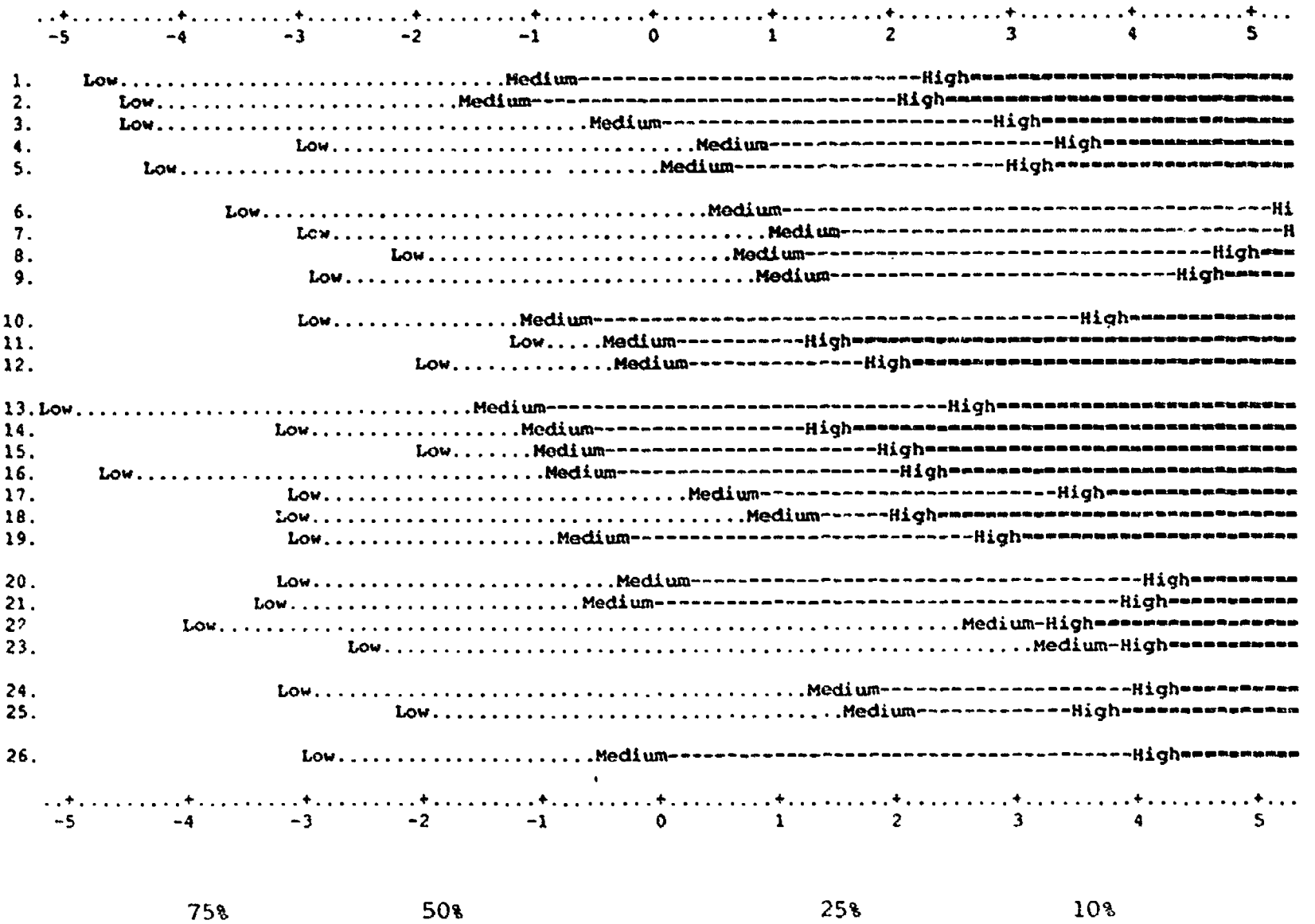

Figure 1. Map of Criteria for Mathematics CAT 2 (Challenging Problem)

TABLE 3
SOME TYPICAL RATING PATTERNS

	-4.0				0.0				4.0			
Criterion	NS	L	M	H	NS	L	M	H	NS	L	M	H
1		L					M					H
2		L					M					H
3		L					M					H
4						L						H
5		L				L						H
6						L					M	
7						L					M	
8						L					M	
9						L					M	
10							M					H
11							M					H
12							M					H
13		L					M					H
14							M					H
15							M					H
16		L					M					H
17						L						H
18						L						H
19							M					H
20							M				M	
21							M					H
22		L				L						H
23						L						H
24						L						H
25						L						H
26							M					H

the available grade levels. At the bottom of Figure 1, cumulative percentages of students achieving at various levels on the problem-solving continuum are displayed. These percentages provide a guide to how grades might be defined. They also indicate that in 1988, some 25 percent of students were unable to achieve a rating of *LOW* on more than a handful of these 26 criteria. This may be explained in part by the existence of one uncooperative school in the 1988 sample, but it also suggests there may be a need for more lenient criteria.

[In the 1990 pilot study, the percentages of students achieving the available grades (aggregated across CATs within each subject) were:

	NA*	UG	E/E+	D/D+	C/C+	B/B+	A/A+
Aust Stds	9	3	15	23	25	16	9
English	6	1	9	23	32	21	8
Math	7	6	15	25	23	15	9

*NA means work was not presented for grading.]

Revised Criteria and Grade Descriptions (1991)

On the basis of these analyses and subsequent analyses of revised criteria in 1989 and 1990, a final set of criteria for mathematics CAT 2 were developed for use with all students from 1991. The final set of criteria are shown in Table 4. As can be seen from this table, the number of criteria for the Challenging Problem task has been reduced from 26 to 15; the *Overall* criteria developed in 1988 (but displaying misfit in Table 1) have been removed; and the specific process criteria (carrying out calculations, finding patterns, formulating hypotheses, looking for counterexamples) have been removed from the final list.

Using feedback on how these revised criteria functioned in 1989-90, a map has been constructed linking *LOW-MEDIUM-HIGH* ratings of the 15 criteria to define E to A+ grade levels. This map is shown in Figure 2. It shows, for example, that to achieve a grade of E, a student will typically be required to achieve a *LOW* rating on about half the criteria (most probably criteria 1, 2, 4, 5, 7, 12 and 13). To achieve a grade of E+, a student will require several additional *LOW* ratings (probably on criteria 3, 8 and 14).

To assist teachers in making judgements about appropriate grades, the typical patterns of ratings associated with the ten grade levels have been displayed as in Figure 3. The picture in Figure 3 is a guide to the allocation of grades C and C+. Boxes marked with an X indicate the kind of rating pattern associated with a grade of C. The shaded boxes indicate the additional expectations for a grade of C+.

TABLE .
CRITERIA FOR MATHEMATICS CAT 2
(CHALLENGING PROBLEM)

Defining the Problem

1. Clear definition of what is required.
2. Definition of important variables, assumptions and constraints.
3. Identification of nature of solution sought.

Solution and Justification

4. Production of a solution which addresses the problem.
5. Degree of mathematical formulation of problem.
6. Appropriate use of mathematical language, symbols, and conventions.
7. Accuracy of mathematics.
8. Interpretation of mathematical results.
9. Depth of analysis of problem.
10. Quality of justification of solution.

The Solution Process

11. Usefulness of questions asked.
12. Relevance of mathematics used.
13. Generation and analysis of appropriate information.
14. Recognition of the relevance of findings.
15. Refinement of definition of problem.

Criterion	Grades									
	E	E+	D	D+	C	C+	B	B+	A	A+
1	Low				Medium			High		
2	Low					Medium				High
3		Low				Medium			High	
4	Low					Medium				High
5	Low					Medium			High	
6			Low		Medium			High		
7	Low			Medium				High		
8		Low					Medium			High
9			Low			Medium			High	
10				Low				Medium		High
11			Low		Medium				High	
12	Low					Medium			High	
13	Low				Medium				High	
14		Low					Medium			High
15				Low				Medium		High
	E	E+	D	D+	C	C+	B	B+	A	A+
Score	7	10	13	17	21	27	29	36	40	45

Figure 2. Criteria-Grade Map, Mathematics CAT 2 (Challenging Problem)

GRADE: C ☒ C+ ■

Defining the problem

	High	Med	Low	Not shown	
clear definition of what is required	☐	☒	☐	☐	1
definition of important variables, assumptions and constraints	☐	■	☒	☐	2
identification of nature of solution sought	☐	■	☒	☐	3

Solution and justification

	High	Med	Low	Not shown	
production of a solution which addresses the problem	☐	■	☒	☐	4
degree of mathematical formulation of problem	☐	■	☒	☐	5
appropriate use of mathematical language, symbols, and conventions	☐	☒	☐	☐	6
accuracy of mathematics	☐	☒	☐	☐	7
interpretation of mathematical results	☐	☐	☒	☐	8
depth of analysis of problem	☐	■	☒	☐	9
quality of justification of solution	☐	☐	☒	☐	10

The solution process

	High	Med	Low	Not shown	
usefulness of questions asked	☐	☒	☐	☐	11
relevance of mathematics used	☐	☒	☐	☐	12
generation and analysis of appropriate information	☐	☒	☐	☐	13
recognition of the relevance of findings	☐	■	☒	☐	14
refinement of definition of problem	☐	☐	☒	☐	15

Figure 3. Teachers' Guide to Grades of C / C+

Teachers using these guides will rarely encounter students with exactly the rating patterns displayed in the teacher guides. In general, teachers will be required to make judgements in which they compare students' ratings on criteria with the provided templates to arrive at an appropriate grade. As outlined above, teachers' interpretations and use of criteria and the grades they assign will be verified through meetings of teachers, sampling, and visits to schools.

Finally, students' grades on the Challenging Problem task will be accompanied by a set of *Grade Descriptions* to provide users of the Certificate with an indication of students' levels of performance on each task. The *Grade Descriptions* for the Challenging Problem task are shown in Table 5. These descriptions have been developed from Figure 2 and describe typical performances (i.e., typical ratings on these criteria) for students achieving each grade.

References

Caulley, D., Orton, J, & Clayton, L. (1988). *A report on an evaluation of the 1988 VCAB trials of certain common assessment tasks (CATs)*. Bundoora: School of Education, La Trobe University.

Cropley, M. & Reynolds, C. (1989). *Report on the trials of common assessment tasks (CATs), 1988*. Melbourne: Victorian Curriculum and Assessment Board.

Masters, G.N. (1980). A Rasch model for rating scales. Doctoral dissertation, University of Chicago.

Masters, G.N. (1982). A Rasch model for partial credit scoring. *Psychometrika*,

Masters, G.N. & Hill, P.W. (1988). Reforming the assessment of student achievement in the senior secondary school. *Australian Journal of Education*, 32, 274-86.

McGaw, B., Eyers, V., Montgomery, J., Nicholls, B. & Poole, M. (1990). *Assessment in the Victorian Certificate of Education:* Report of a review commissioned by the Victorian Minister for Education and the Victorian Curriculum and Assessment Board, Melbourne.

Symons, M.R. (1990). *An evaluation of the common assessment tasks (CATs) trials for mathematics*. Bundoora: School of Education, La Trobe University.

Victorian Curriculum and Assessment Board. (1988). *Assessment for the Victorian Certificate of Education*, Melbourne.

TABLE 5
GRADE DESCRIPTIONS
MATHEMATICS CAT 2
(CHALLENGING PROBLEM)

GRADE	GRADE DESCRIPTION
A/A+	Elaborated the problem effectively. (M/H 2,15; H 1,3) Demonstrated understanding of the complexity of the problem. (H 9) Formulated the problem mathematically at each stage of the elaboration. (H 5) Applied appropriate mathematical knowledge and problem-solving techniques to find solutions. (M/H 4; H 6,7,11,12,13) Justified solutions obtained and explained their relevance to the problem. (M/H 8,10; H 14)
B/B+	Elaborated the problem effectively. (M 2,15; M/H 1,3) Demonstrated some understandings of the complexities of the problem. (M 9) Formulated some aspects of the problem mathematically. (M 5) Applied appropriate mathematical knowledge and problem-solving techniques to find solutions. (M 4,11; M/H 6,7,12,13) Provided some justification of solutions obtained and explained their relevance to the problem. (M 8,10,14)
C/C+	Defined the problem, demonstrating some understanding of its complexity. (L/M 2,3,9; L 15; M 1) Expressed elements of the problem mathematically. (L/M 5) Applied appropriate mathematical knowledge and problem-solving techniques to find partial solutions. (L/M 4; M 6,7,11,12,13) Explained the relevance of the results obtained to the problem defined. (L 8,10; L/M 14)
D/D+	Explored initial stages of the problem. (L 1,2,3) Applied some relevant mathematical knowledge and problem-solving techniques to find partial solutions. (L 4,5,6,9,11,12; L/M 7,13) Partially explained the relevance of the results obtained to the problem. (UG/L 10,15; L 8,14)
E/E+	Explored initial stages of the problem. (L 1,2; UG/L 3) Applied some relevant mathematical knowledge and problem-solving techniques to work towards solutions. (L 4,5,7,12,13; UG/L 8,14)

Victorian Curriculum and Assessment Board

Victorian Certificate of Education
1990 Pilot Program
VCE Mathematics

Common Assessment Task 2 – Challenging Problem

REASONING AND DATA

DESIGNATED PERIOD
Starting date: Wednesday 22 August 1990
Completion date: Wednesday 5 September 1990

STUDENT BOOKLET

CONTENTS

Prescribed conditions: instructions to students

The following conditions will apply to the 1990 Challenging Problem common assessment task.

- You must attempt to solve one of the four challenging problems set and must complete a written report of your solution according to the format provided in the document titled 'Format for written reports' (page 7).
- The report format specifies a number of sections which are designed to ensure your report is as thorough as possible and to assist the assessment of your report. Note particularly the two sections 'Solution and justification' and 'Solution process'. The first of these should contain your solution to the problem, and your justification of the solution. This is where your mathematical conclusions are presented. The second should be a record of your progress towards your solution. It should be developed from the rough notes which you make and keep while working on the problem. The report should normally be no more than 1000 words, or up to six pages in length, plus any necessary appendices.
- All work must be completed in the two weeks from Wednesday 22 August to Wednesday 5 September 1990.
- You are expected to spend a total time of between six and eight hours on the task. You should be given the opportunity to work on the task during four to six hours of class time.
- This assessment task must be done individually. In some cases you may cooperate with other students in the initial information and data gathering stages of the problem. Your report must be based on your own analysis and interpretation of the chosen problem and you must be able to show that you understand everything in your report. You must sign the declaration of authenticity (page11) indicating that all unacknowledged work is your own. Your teacher can give you general advice on problem-solving strategies but not specific help to find a solution to your chosen problem.
- It is your responsibility, through discussion and through work done in class time, to demonstrate to the teacher that your work on the problem and your report is in fact your own work. You should have at least one consultation with your teacher during the period allowed for the task in order to discuss your progress with the problem. Keeping all of your rough notes, even those which do not seem to lead anywhere, may be helpful in providing your teacher with information about your progress. Your teacher will observe your work and will discuss it with you to ensure that the work is your own and will then sign a statement on the sheet provided inside the back cover of this booklet as to the authenticity of your work. If your teacher is not satisfied that the work is your own and is therefore unable to sign the declaration of authenticity, your report will not be assessed and you will be awarded NA (not assessed).
- Once the declaration of authenticity has been signed by the teacher, you may complete the details on the cover sheet which you must attach to your completed report. (The school will retain the authentication statement.)
- Your written report must be submitted to the teacher by Wednesday 5 September 1990.

Challenging problem

1. Tattslotto

In a game similar to Tattslotto, three balls are drawn out of a barrel containing twenty balls. The balls are numbered from 1 to 20. An observer has been keeping careful records of the game over a long period of time. During this time the observer has found that adding together the highest and lowest numbers drawn gave 21 more often than any other number. It was also found that in approximately 50 per cent of the draws the sum of the highest and lowest numbers was between 17 and 24 inclusive. Can you explain these observations?

In Tattslotto itself, six numbers, excluding the supplementaries, are drawn from the numbers one to forty-five inclusive. What is the most probable value of the sum of the highest and lowest numbers and what is the probability of this value?

2. Bad queues

A number of people line up in a queue to buy movie tickets which cost $10.50 each. Eight people are left in the queue when the cashier runs out of change. Of these eight people, four have exactly $10.50 (a ten dollar note and a fifty cent coin) while the remaining four have $11.00 (a ten dollar note and a one dollar coin). A combination of the people in the queue for which the cashier would not be able to give the correct change to a person with $11.00 is termed a 'bad queue'.

- You are required to devise an efficient and systematic way of calculating the number of bad queues, and thus to find the probability that the arrangement of the people forms a bad queue.
- Suppose instead that twelve people remain in the queue, six of whom have $10.50 and six $11.00, in the same notes and coins as before, when the cashier runs out of change. Apply your method to calculate the probability of a bad queue forming in this case.

Can you express your answer for the number of bad queues in both cases as one simple formula?

Challenging Problem - continued

3. Number sentence

In this sentence there are 2 zeros, 3 ones, 0 twos,
0 threes, 2 fours, 0 fives, 1 six, and 0 sevens.

The above sentence is not true. In fact it has 4 zeros, 1 one, 2 twos, 1 three, 0 fours, 0 fives, 0 sixes, and 0 sevens.

In this sentence there are ... zeros, ... ones, ... twos,
... threes, ... fours, ... fives, ... sixes, and ... sevens.

This is the same sentence, but with the numbers replaced by gaps. There are eight gaps which are to be filled with numbers which make the sentence true.

Here is a similar kind of sentence. This one has four gaps.

In this sentence there are ... zeros, ... ones, ... twos,
and ... threes.

The problem is to investigate sentences like these to find out if it is possible to fill the gaps with numbers which make the sentences true.

You should explore sentences of different length (that is, with different numbers of gaps and where the first gap is for the number of zeros and subsequent gaps are labelled consecutively).

For some sentences there may be more than one solution. For each sentence you explore you should establish how many different ways there are to fill the gaps to create a true sentence. You should give an explanation of your findings and justify any conclusions you reach.

4. Estimating population size

One method of estimating the total number of a breed or type of animal living in any one place involves capturing a random sample of the animals, tagging them, releasing them, then later capturing another random sample and counting how many of these animals have tags and how many do not. Useful data can be gained by repeating this procedure a number of times.

The data provided below refers to two different animal populations. In this problem you are asked to find methods to use the information provided to estimate the population sizes, to put upper and lower bounds on your estimates, to identify the simplifying assumptions that have been made, and, if possible to test their accuracy.

Case 1

Sixty adult frogs are released into an isolated and previously frog-free dam. It is believed that, provided the frogs survive the first couple of days, then they should survive for at least the next two months.

Two weeks after the frogs are released, an attempt is made to find out how many frogs are still alive. Twenty adult frogs are captured, tagged and released again. The next day, 30 adult frogs are captured, of which 15 already have tags.

Case 2

The brown *Antechinus* (pronounced 'An-te-ki-nus') is a mouse-sized marsupial found in most forest areas in south-eastern Australia. Although very common, it is rarely seen because of its size and because it tends to be active only at night.

The brown *Antechinus*' short mating season is in late winter after which every male dies, leaving only pregnant females in the population. This simple population structure makes the *Antechinus* an important animal for scientific study.

The following data were collected in May before the breeding season, when the young of both sexes born in the previous breeding season, and older, second-year females, were present. The data were collected by trapping for five days on a five-hectare grid at Sherbrooke Forest near Melbourne.

	Day 1	Day 2	Day 3	Day 4	Day 5
New capture	22	11	16	5	6
First recapture	-	7	10	11	5
Second and subsequent recapture	-	-	3	5	8

(Data provided by Zoology Department, Monash University)

END CHALLENGING PROBLEM

Format for written reports

The report should be written at the conclusion of your work on the chosen problem. It should be as concise as possible, and should be no more than 1000 words, or up to six pages in length, plus any necessary appendices. The cover sheet provided inside the back cover of this booklet must be completed and attached to the front of the report. The report should consist of the following.

1. Statement of the problem
   - This should include a clear statement in your own words of what the problem is about, what assumptions you had to make and how you interpreted what you had to do.
   - It should also state what could constitute a solution to the problem.

2. Solution and justification
   - This section consists of a formal statement of your solution of the problem, together with explanations, proofs or other evidence to justify your solution.
   - An evaluation of the solution should also be given and, where appropriate, you should suggest how your results could be generalised and possible directions in which further work could proceed.

3. Mathematical tools used
   - This section should be a brief summary of the mathematical knowledge used in solving the problem.
   - If appropriate, an indication should also be given of how computers or calculators were used.

4. Solution process
   - This section should describe your attempts to solve the problem. It should show how you started off, how your understanding of the problem developed, what you tried at different stages and any important insights and breakthroughs which occurred as you sought a solution to the problem. It should highlight the problem-solving strategies and mathematical techniques you tried.
   - Any rough notes made in exploring the problem may be included as an appendix.

5. Acknowledgments
   - This section should be a brief acknowledgment of the ideas or other help provided by other students, specialist information sources, family, teachers and so on.

6. References (if used)
   - Only those references actually consulted and found useful should be included.

7. Appendices (if necessary)
   - Appendices may be included to help the reader to understand your report. They may be relevant to the 'Solution and justification' or 'Solution process' sections of the report. Include rough notes, any necessary lengthy calculations, computer programs, raw data or other essential resources.
   - Appendices should only be included if they have been specifically referred to in the text.
   - They should be numbered consecutively and each should bear a title.

END REPORT FORMAT
TURN OVER

Grade descriptors for CAT 2: Challenging Problem

A. Elaborated the problem effectively. Demonstrated understanding of the complexity of the problem. Formulated the problem mathematically at each stage of the elaboration. Applied appropriate mathematical knowledge and problem-solving techniques to find solutions. Justified solutions obtained and explained their relevance to the problem.

B. Elaborated the problem effectively. Demonstrated some understanding of the complexities of the problem. Formulated some aspects of the problem mathematically. Applied appropriate mathematical knowledge and problem-solving techniques to find solutions. Provided some justification of solutions obtained and explained their relevance to the problem.

C. Defined the problem, demonstrating some understanding of its complexity. Expressed elements of the problem mathematically. Applied appropriate mathematical knowledge and problem-solving techniques to find partial solutions. Explained the relevance of the results obtained to the problem defined.

D. Explored initial stages of the problem. Applied some relevant mathematical knowledge and problem-solving techniques to find partial solutions. Partially explained the relevance of the results obtained to the problem.

E. Explored initial stages of the problem. Applied some relevant mathematical knowledge and problem-solving techniques to work towards solutions.

END GRADE DESCRIPTORS

VCE Mathematics
1990 Pilot Program
CAT 2: Challenging problem

ASSESSMENT CHECKLIST

Block:	
Problem:	
Grade:	

VCAB School Number: ☐☐☐☐☐ Teacher attestation completed: Yes ☐ No ☐

Candidate Number: ☐☐☐☐☐☐☐ Teacher name:

Defining the problem

	High	Med	Low	Not shown	
clear definition of what is required	☐	☐	☐	☐	1
definition of important variables, assumptions and constraints	☐	☐	☐	☐	2
identification of nature of solution sought	☐	☐	☐	☐	3

Solution and justification

	High	Med	Low	Not shown	
production of a solution which addresses the problem	☐	☐	☐	☐	4
degree of mathematical formulation of problem	☐	☐	☐	☐	5
appropriate use of mathematical language, symbols, and conventions	☐	☐	☐	☐	6
accuracy of mathematics	☐	☐	☐	☐	7
interpretation of mathematical results	☐	☐	☐	☐	8
depth of analysis of problem	☐	☐	☐	☐	9
quality of justification of solution	☐	☐	☐	☐	10

The solution process

	High	Med	Low	Not shown	
usefulness of questions asked	☐	☐	☐	☐	11
relevance of mathematics used	☐	☐	☐	☐	12
generation and analysis of appropriate information	☐	☐	☐	☐	13
recognition of the relevance of findings	☐	☐	☐	☐	14
refinement of definition of problem	☐	☐	☐	☐	15

END ASSESSMENT CHECKLIST
TURN OVER

1990 Pilot Program
VCE Mathematics
Common Assessment Task 2 – Challenging Problem
REASONING AND DATA

Declaration of Authenticity

Student declaration

I declare that all unacknowledged work is my own.

Student's signature _____

Student's name (printed) _____

Date _____

Teacher declaration

I declare that to the best of my knowledge this work is the work of the student identified above.

Teacher's signature _____

Teacher's name (printed) _____

Date _____

If for any reason the teacher is unable to make such a declaration, he or she should attach a signed statement indicating why this is so.

NOTE : This declaration should be retained by the school.

(DETACH HERE)

1990 Pilot Program
VCE Mathematics
Common Assessment Task 2 – Challenging Problem
REASONING AND DATA

Cover Sheet for Report

Student number | | | | | | | |

VCAB school number (assessing school) | | | | | |

Title of problem: _____

Problem number: _____

Total number of pages

of report (including appendices): _____

NOTE: Attach this cover sheet to your report before handing it in to your teacher.