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ABSTRACT

One class of interactive, computer-based learning environments (microworlds) for the exploration of school mathematics (and science) entails the incorporation of appropriate concepts within the engaging context of self-directed discovery learning. The objective of this research was to investigate and describe in detail the constructive learning process of children who interacted with a computer microworld that incorporated the central objectives and relations of transformational geometry. The qualitative model in this research focused on discerning changes in students' content knowledge, changes in their strategies for using the microworld environment, and changes in their goal behavior. Included in the report are the theoretical framework, methods of data collection, preliminary results, and examples of microworld activities. Even though the small sample size of this case study (n=12) prevented tests for statistical significance, specific results included an increase of 9 percent on the most difficult transformation task (identifying rotations) to an increase of 25 percent on the easiest task (executing reflections). Further, the results yield support for previous conclusions that students require meaningful problem-solving activities to facilitate the construction of new knowledge. (11 references) (JJK)

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The Design and Analysis of a Mathematical Microworld

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Objectives

In recent years, a growing number of researchers and educators have engaged in the construction of interactive, computer-based learning environments for exploring mathematics and science. One emerging class of such environments consists of those which embody mathematical or scientific concepts in a context which is engaging to the learner, and which allows for a certain degree of self-directed exploration or discovery of the implicit ideas and processes. These computer programs have been variously termed "microworlds", "simulation environments" or "functional learning environments", and a new body of data and theory is accumulating on how such environments can support children's learning. (e.g., diSessa, 1982; Dugdale, 1981; Newman, 1985; Thompson, 1987; White, 1981). The objective of the research reported here was to investigate, describe, and attempt to account in detail for the learning of children who interacted with a computer microworld which embodied the central objects and relations of transformation geometry. An additional focus of the research was on the principles involved in the design of such environments, and on the particular ways in which microworlds are used by children in learning about a new domain.

Aims of the Research

The aims of the research project were to:

(a) Design, test and refine a computer microworld for transformation geometry, working initially with a large group of children (a total of 65 students);

(b) Use the microworld with a small number of students for in-depth data collection;

(c) Build a qualitative model or "learning paths chart" (diSessa, 1982) in order to characterize the nature of the children's learning in the domain, and to describe how the microworld was used by the students in constructing their understandings.

The qualitative model focused on discerning changes in students' content knowledge in the domain, in their strategies for using the microworld, and in their goals. This report will focus primarily on changes in content knowledge.

Theoretical Framework

The work described here is based within the framework of constructivism; that is, upon the presupposition that learners bring to the learning situation a range of existing beliefs, ideas, and knowledge, both implicit and explicit, which are used in the process of building "new" knowledge. This knowledge is constructed within meaningful, problem-solving contexts. As Vergnaud states (1982, p. 31), "knowledge emerges from problems to be solved and situations to be mastered." Constructivism is concerned with the learner as an active participant in bringing meaning to his or her experiences, rather than as a passive recipient of information. diSessa and others (diSessa, 1982; Kliman, 1987; Lawler, 1985) focus in fine detail on the prior constructions and sequences of partial understanding which are involved in children's learning in a new domain. One aim of the research reported here was to carry out a detailed "genetic analysis" of children's learning in transformation geometry, as a particular case study of learning viewed from a constructivist perspective. This work also takes place within the context of research into the principles involved in the design of technological artifacts for learning (diSessa, 1985; diSessa & Abelson, 1986; Newman, 1985; White, 1981). The microworld described here was intended both to be a research tool for investigating how children learn about transformations, and also as an example of the principled design and analysis of a computer environment for learning in mathematics.

Figure 1 illustrates the central themes of the research. A mathematical microworld is created which embodies the central objects and relations of some subdomain of mathematics in a form accessible to new learners. A characteristic feature of microworlds is that they link multiple representations of the objects and relations, often using a symbolic system (for example, a set of simple Logo commands) linked to a graphical representation (for example, a visual display of the effects of the transformations). The learner interacts with the microworld, by playing games or solving problems, and in the process constructs an initial, working model of the mathematical domain in question. The learner will base his or her actions on this initial model, and will at times be surprised to find that his/her expectations about how an operation works are not borne out when the operation is used in the microworld. The microworld thus provides feedback that the learner must interpret, and can use to revise his/her conceptual model of the domain. It is important to note that this feedback comes about as a natural consequence of using the microworld, and is not preprogrammed, as in traditional computer-assisted instruction tutorials. The feedback from the microworld often requires the learner to reconcile or re-link an understanding of the symbolic with the visual representation (e.g., linking an equation with its graph, as in Green Globs, or a transformation with its visual display, in the current work). The learner again acts in the microworld, now using his/her revised conceptual model, and this cycle continues until an understanding of the domain which is adequate for carrying out the tasks embodied in the microworld is constructed. Ideally, the learner's conceptual model will be closer to the idealized or expert's model of the domain following experience with the microworld. If it is not, then the microworld and its associated activities must be reevaluated and redesigned.

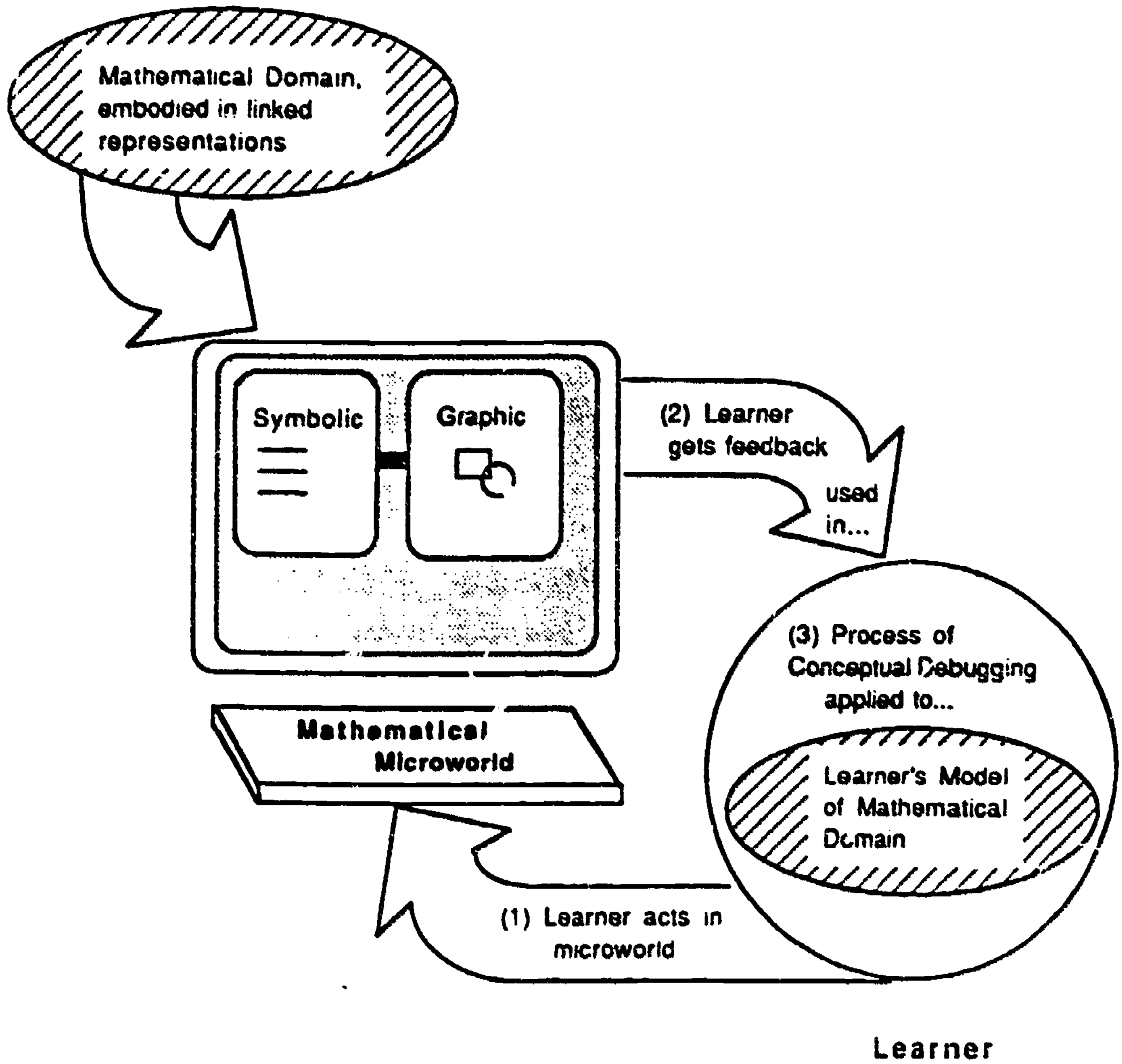


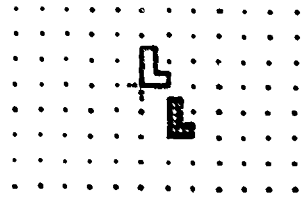
Figure 1: Central Themes of the Research

Methods and Data Sources

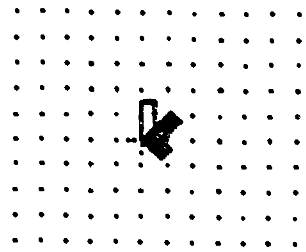
As described by Confrey (1990, p. 13), "Methodologically, constructivist research demands extensive interactions with students over months and years and the detailed analysis of videotapes." The study reported took place over a period of a month and a half, and involved 12 middle-school students, ages 11 to 14, from a small private elementary/middle school in California. The students, who had had no previous instruction in the topic, were introduced to transformation geometry in two initial classroom sessions of 40 minutes each. They then met with the investigator once a week, in pairs, to work with the computer microworld to carry out a sequence of problem-solving activities for five more weeks, making a total of approximately seven hours of work with the transformations. Pencil-and-paper measures, intended to capture the students' capabilities to work with transformations apart from the computer context, were administered during the initial and final after-school sessions. One such measure, the written final exam, included 3 items out of 24 which were completely new applications of the transformations. The bulk of the data, however, was not written, but consisted of videotaped and transcribed protocols of the students' interactions with each other, with the investigator, and with the microworld.

The microworld, called TGEO, was programmed in Logo and run on a Macintosh computer. It instantiated three euclidean transformations, translation, reflection and rotation, as well as change of scale (see Figures 2 and 3). The students were able to carry out any transformation by typing its name and input parameters in a Logo-style command, and the resulting mapping of the plane was shown graphically in an adjacent window. Thus, the microworld consisted of a dynamically-linked pair of representations for the transformations, one representation being a set of symbolic commands and the other a visual depiction of the motion or change.

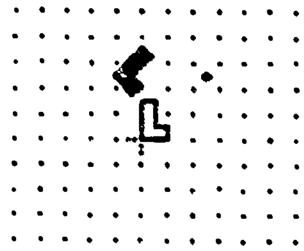
SLIDE 10 -20
slides the shape 10 *across* and 20 *down*



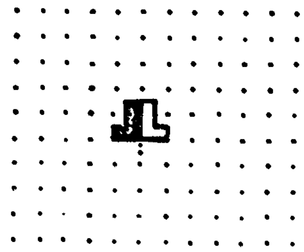
PIVOT 45
pivots the shape 45 degrees *clockwise* around the bottom corner.
PIVOT -45 would turn *counterclockwise.*



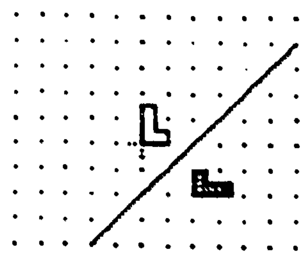
ROTATE 25 25 45
places a center point at 25 across, 25 up and then rotates the plane 45 degrees clockwise around that point



FLIP
flips the shape over its long side

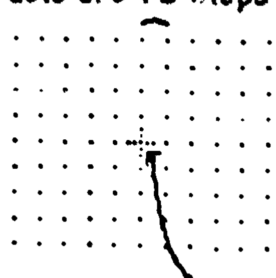


REFLECT 20 0 45
places a mirror line which goes through the point 20 across, 0 up with the heading 45, then flips the plane over that line.



Lengths and Angles (Headings)

The dots are 10 steps apart



center at (0,0)

Headings (0 is North)

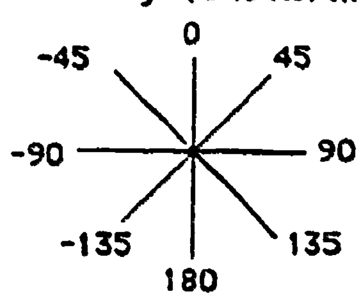
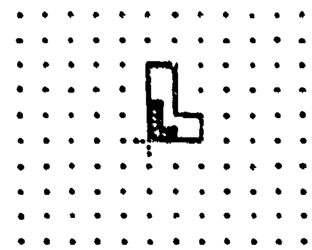
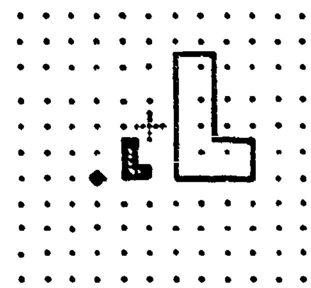


Figure 2 : Help Sheet for Euclidean Transformations

SIZE 2
enlarges shape by a factor of 2,
starting from the bottom corner



SCALE -20(-20)3
enlarges shape by a factor of 3,
from a fixed point at (-20, -20)



Size and Scale

Figure 3: Change of Scale Transformations

The microworld itself was supplemented by activities and problems, both computer-based and worksheet-based. The sequence of activities carried out by the students, with minimal intervention by the investigator, included:

(1) Free exploration of the commands for the euclidean transformations (approximately 30 minutes);

(2) The Match game, in which the transformations were used to achieve the goal of superimposing two congruent shapes on the screen (1 1/2 hours; see Figure 4);

(3) Worksheets asking the students to use the microworld to find and write down inverses and combinations for each type of transformation (1 hour, see Figure 5);

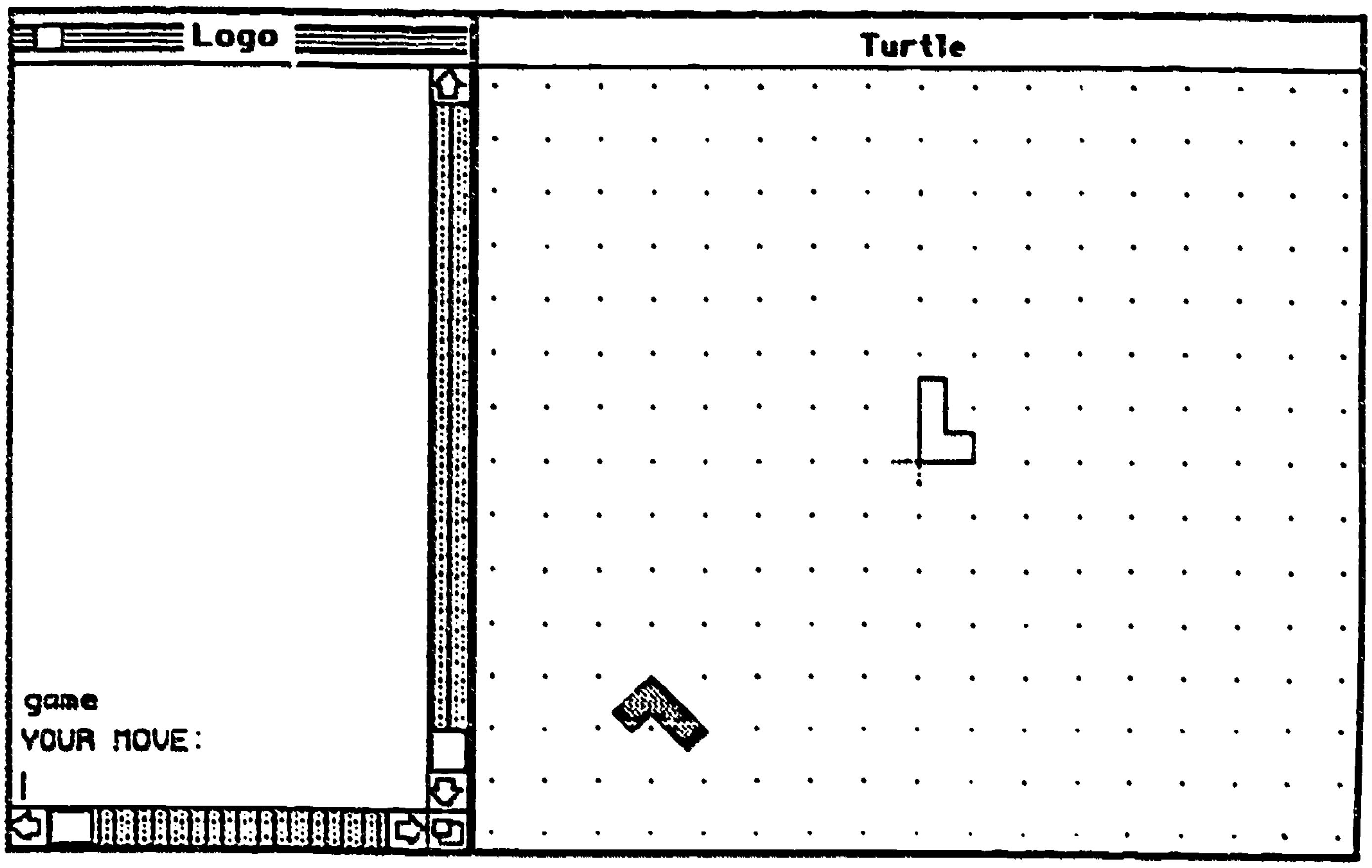
(4) Worksheets asking the students to use the transformations to describe the symmetries of a set of 17 figures (1 hour);

(5) Exploration of new transformations, namely, change of scale (1 hour).

In addition to the videotapes collected during the study, the sequences of commands input by the students when playing the Match game were captured on the computer and analyzed for changes in strategies. The written worksheets and the final exam were also analyzed for error patterns. These data were used to describe changes in performance on transformational tasks over the course of the study, and to construct a qualitative learning paths chart for the domain.

Results

The results of the study suggest that the microworld and the associated activities were very effective in assisting the students to construct a working knowledge of the transformations. The average performance of the students on the written final examination was 70% correct . Furthermore, by the time of the final session, all of the students were able to carry out any of the transformation on the computer without error or hesitation. In terms of specific transformations, the change from initial to final session in the percentage of correct responses on the worksheets ranged from an increase of 9 percent on the most difficult task (identifying rotations) to an increase of 25.3 percent on the easiest task (executing reflections). (Tests of statistical significance were not performed on this data because of the small sample size and because the primary objective was to build a detailed qualitative model, referred to as a "learning paths chart").



Object of Game: To superimpose shapes by applying a sequence of transformations

Figure 4: Sample Match Game

Name(s): Gavin
Shannon

Date: 5/24/88

Worksheet 2: COMBINING TRANSFORMATIONS

For each pair of transformations below, find a single move which gives the same result. Write down your answers.

To find the answers, try typing both transformations on one line, hit enter, and observe the result. Then type RESET to bring the shape back where it started, leaving a shadow behind. Try out moves till you find a single one with the same result.

a. Slide 20 30 Slide 50 10 — Single Move: slide 70 40

b. Slide 10 50 Slide -40 10 — Single Move: Slide -30, 60

c. Slide A B Slide X Y — Single Move: Slide (A+X), (B+Y)

d. Pivot 45 Pivot 45 — Single Move: pivot 90

e. Pivot 270 Pivot -90 — Single Move: Pivot 180

f. Pivot A Pivot B — Single Move: Pivot (A+B)

g. Rotate 20 0 90 Rotate 20 0 45 — Single Move: Rotate 20 0 135

h. Rotate 0 0 120 Rotate 0 0 -60 — Single Move: Rotate 0 0 60

i. Rotate X Y A Rotate X Y B — Single Move: Rotate X Y (A+B)

j. Flip Flip — Single Move: slide 0, 0 or [illegible]

k. Reflect 0 0 0 Reflect 20 0 0 — Single Move: Slide 40, 0

l. Reflect 0 10 90 Reflect 0 30 90 — Single Move: slide 0, 40

m. Reflect 10 20 90 Reflect 10 20 0 — Single Move: Rotate 10, 20, 180

On the back, write down some pairs of your own and their single moves. Make sketches.

Figure 5 : Combining Transformations Worksheet

The Learning Paths Chart for the Match Game

The learning paths chart for the Match game is based upon the collection of 43 games played by the students in Session 1 of the teaching experiment. In playing the game, the students could use any of the transformations they knew about, including slide and the simple, local transformations pivot and flip, as well as the more difficult rotate and reflect. The learning paths chart is organized in terms of the students' developing knowledge about each of the euclidean transformations, slide, rotate, and reflect.

The learning paths chart is presented in Figure 5. A key to reading the chart is given below:

General Organization:

• Within each strand, topics which appear near the top of the chart were observed earlier than those shown near the bottom.

Borders:

• Topics outlined with bold borders were observed in all of the students, and were judged to be robust elements of the students' knowledge in this context.

• Topics outlined with lighter borders were observed among some subset of the students, and indicate some of the variation in the approaches and knowledge of the different children.

• Topics outlined in gray borders indicate "bugs" in the children's understanding, that is, incomplete or unrefined models.

Connectors:

• Arrows between topics indicate a refinement or correction of a bug (note that most of the boxes do not have arrows; many items of knowledge seemed to develop in parallel).

• Black connecting lines indicate specific aspects involved in a complete understanding of the main topic (necessary knowledge).

• Gray connecting lines indicate related topics, where the related topics may not be necessary to understand or use the original topic (e.g., they may be particular versions of a strategy, additional theorems, or elaborations of an existing idea).

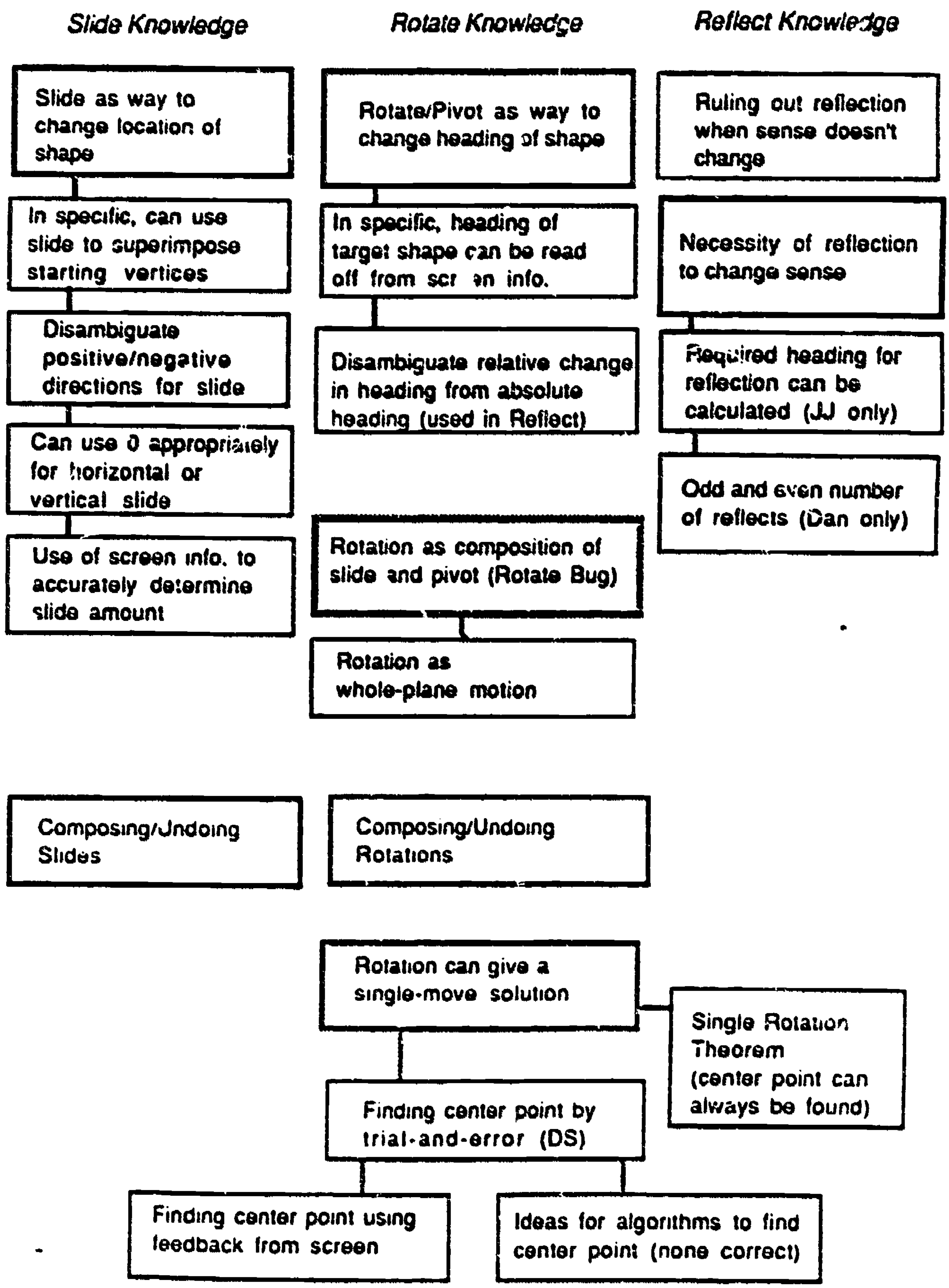


Figure 6 : Learning Paths Chart for Match Game

Changes in Content Knowledge

An important change in the students' thinking was a shift in thinking about each transformation as a procedure carried out on a discrete object, to thinking of the transformations more as objects in themselves. They also refined their initially vague models of each motion to include precisely the features which are necessary and sufficient to define it. This was most evident when the students investigated inverses and combinations of the transformations, as evidenced by comments like the following (the student is trying to find the combination of two reflections in intersecting axes, an original exploration carried out after completing the worksheet):

Jos: *"This is rotate. This calls for a rotate. The center point is still the same, you just have to find the amount of degree angle."*

Initially, students focused on features of the shape shown on the screen, such as its orientation or location; later the students discussed the transformations (the mapping or change) only, without regard to the features of the shape itself.

A similar progression showed up in a small number of students who initially misinterpreted rotate as a translation combined with a pivot, rather that a rotation of the entire plane around a fixed point. The students were able to use the visual feedback from the microworld to correct this misconception and to think of rotate in a more global sense. In general, the students used the microworld in a process of "conceptual debugging" in which their initial, partially-correct models of the transformations were reconciled by comparing them to the mathematically-correct models instantiated in the microworld.

Changes in Strategies

The strategies used over the course of the study progressed from an early but brief reliance on trial and error, to the use of visual feedback from the screen to improve the accuracy of inputs. The students soon realized that the transformations behaved in a logical and predictable way, and most spontaneously began to predict the outcome of the commands they entered while playing the game or completing the worksheets. Two pairs of students were able to carry out independent explorations of combinations of transformation after finishing the worksheets; these students showed the beginning of an inductive approach to discovering and describing patterns (or simple theorems) in transformation geometry.

Changes in Goals

The goals of the students had to be inferred from their comments and from any activities they carried out spontaneously. In the early activities, the goals appeared to consist of satisfying the investigator by getting the correct response, and also succeeding in playing the Match game with a good score. However, over time, many of the students appeared to be genuinely interested in understanding the transformations and in discovering patterns in the microworld. Some were interested in using the transformations to create visually-pleasing designs on the screen. Thus their goals eventually included some degree of self-directed activity, aside from the tasks assigned by the investigator.

Educational/Scientific Importance

The goal of this study was to add to our understanding of how children can construct an understanding of a new area of mathematics by interacting with a computer-based instantiation of the domain. The study traced the students' learning in transformation geometry, and also provided empirical data on how children made use of the computer environment to "debug" their understanding of the transformations. It supports previous work which suggests that students need to be engaged in meaningful problem-solving activities in order to construct new knowledge, and it constitutes a case study of designing such activities around an interactive computer microworld.

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