DOCUMENT RESUME

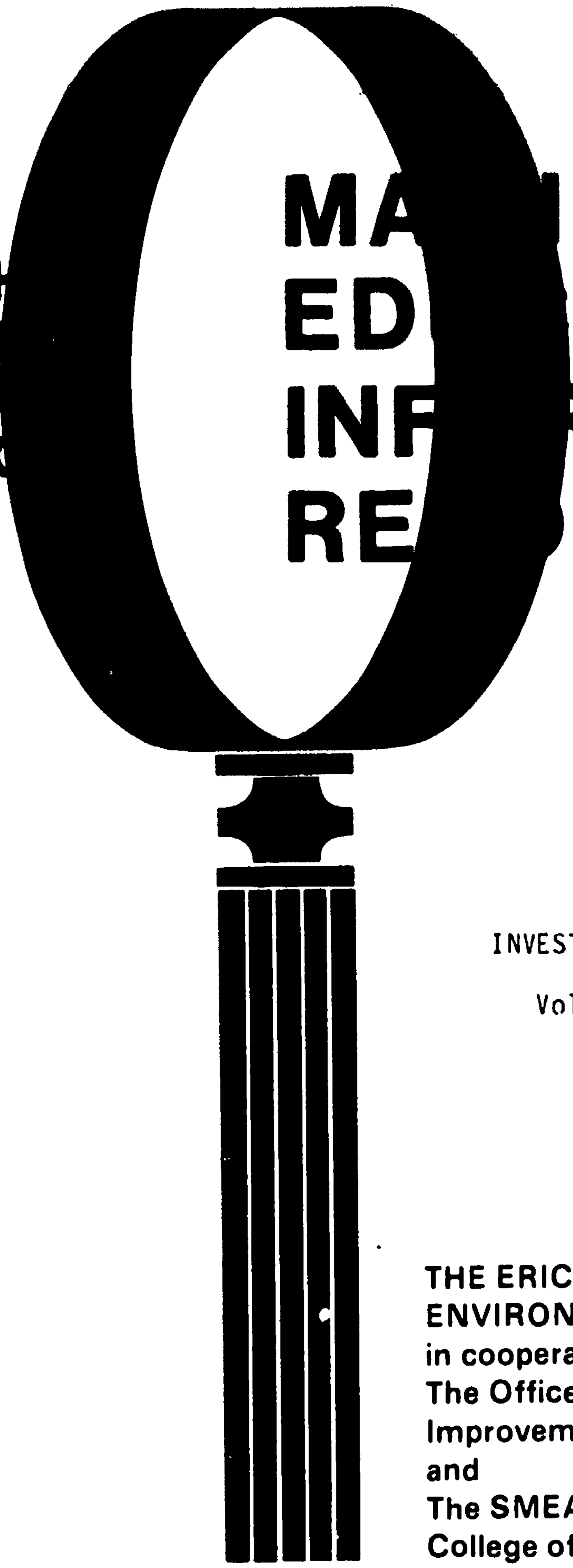
ED 331 693 SE 050 575

AUTHOR Suydam, Marilyn N., Ed.; Kasten, Margaret L., Ed.
TITLE Investigations in Mathematics Education. Volume 21, Number 3.
INSTITUTION Ohio State Univ., Columbus, Ohio. Information Reference Center for Science, Mathematics, and Environmental Education.
PUB DATE 88
NOTE 63p.; For previous edition see ED 302 415.
AVAILABLE FROM SMEAC Information Reference Center, 1200 Chambers Road, Room 310, Columbus, OH 43212 (U.S. subscription $8.00, $2.75 single copy).
PUB TYPE Collected Works - Serials (022) -- Reports - Research/Technical (143)
JOURNAL CIT Investigations in Mathematics Education; v21 n3 Sum 1988

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Arithmetic; Assignments; Cognitive Ability; Computation; Educational Research; *Elementary School Mathematics; Elementary Secondary Education; Mathematical Logic; *Mathematics Education; *Problem Solving; Ratios (Mathematics); Remedial Mathematics; *Secondary School Mathematics; Teaching Methods; Word Problems (Mathematics)
IDENTIFIERS Mathematics Education Research; Sequences (Mathematics)

ABSTRACT

This publication is a compilation of abstracts and critical comments for 12 published investigations in mathematics education. Information for each study includes: purpose; rationale; research design and procedures; findings; and interpretations. For each study abstractor's comments also provide a brief critique. Topics include: planning problem solving instruction; word problem difficulty; cognitive levels; the development of function concepts; proportion problems; counting skills; problem assignment; sequencing; the effect of teacher errors; and logic errors. Also provided are lists of mathematics education research studies indexed by "Current Index to Journals in Education" and "Resources in Education" for January through March, 1988. (CW)



MATHEMATICS EDUCATION INFORMATION REPORT

INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 21, Number 3 - Summer 1988

THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
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The Office of Educational Research and Improvement, U.S. Department of Education
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College of Education, The Ohio State University

INVESTIGATIONS IN MATHEMATICS EDUCATION

Published quarterly by

SMEAC Information Reference Center
The Ohio State University
1200 Chambers Road, Room 310
Columbus, Ohio 43212

Telephone: (614) 292-6717

With the cooperation of the ERIC Clearinghouse for Science, Mathematics and Environmental Education

Volume 21, Number 3 - Summer 1988

Subscription Price: $8.00 per year. Single Copy Price: $2.75
$9.00 for Canadian mailings and $11.00 for foreign mailings.

INVESTIGATIONS IN MATHEMATICS EDUCATION

Summer 1988

Burns, Robert B. and Lash, Andria A. NINE SEVENTH-GRADE TEACHERS' KNOWLEDGE AND PLANNING OF PROBLEM-SOLVING INSTRUCTION. Elementary School Journal 88: 369-386; March 1982.

Abstract and comments prepared for I.M.E. by BOYD HOLTAN, West Virginia University.

1. Purpose

The purpose of the study was to examine how teachers' conceptions about teaching mathematics influence the manner in which they plan instruction in mathematical problem solving.

2. Rationale

Although teacher planning has long been identified as a major contributor to effective teaching, only recently have investigators begun to systematically examine teacher planning. One of the factors is teacher knowledge. Two types of knowledge were considered in this study: pedagogical knowledge, or knowledge about how to organize and manage instruction, and pedagogical content knowledge, or knowledge about how to teach a specific subject matter. The first deals with ways to structure and conduct lessons, while the latter deals with ways to teach specific topics within a subject. How does this knowledge affect the planning of teacher instruction?

3. Research Design and Procedures

The investigators interviewed nine seventh-grade teachers to obtain their views on teaching mathematics, problem solving, and typical classroom operation. A three-hour workshop was held with the teachers in which they were given information about problem solving and a large number of arithmetical problems. Some non-routine problems were included. The teachers were given stipends and a month to plan a six-lesson unit on problem solving. Four problem-solving skills were to be taught in the unit. They were: (1) identify information necessary to solve a problem that is not in the problem statement, (2) separate relevant and irrelevant information in the problem statement, (3) identify the intermediate step in a multiple-step problem, and (4) represent information in a problem statement in a table or diagram. After planning the lessons, the teachers were interviewed a second time about how and what they had planned for their problem-solving unit. The teachers were observed during each of their instructional units, but the data are reported elsewhere (Burns & Lash, 1986).

4. Findings

The first interview suggested that teachers in the study had a consistent pedagogical knowledge about how to teach mathematics, but had a limited pedagogical content knowledge about how to teach problem solving. Showing students how to do problems and allowing them to practice similar ones was the commonly accepted teaching technique. The teachers indicated that they would focus more on how to do mathematics than on understanding mathematics. When asked to define problem solving, the responses were quite varied, ranging from textbook-type definitions to equating problem solving with simple word problems.

The second interview indicated that concerns about how to teach specific problem-solving content were not a major part of the teachers' planning. Their concerns focused on the collection of materials and the organization of worksheets. Planning for a problem-solving unit was little different from what the teachers normally did for other content topics. The teachers did not take into account the unique structure of the content as it related to the planning for the lessons.

5. Interpretations

The teachers appeared to be quite consistent in using the same "show and practice" instructional sequence no matter what the content to be taught. The investigators called this "default" planning as the teachers fell back on certain teaching techniques even though they were not always the best techniques for the specific subject matter being presented. They also suggested that alternate instructional techniques may require more planning time than many teachers feel they have available.

Abstractor's Comments

Although the number of teachers in the study was small, the study is certainly thought-provoking for mathematics educators who are interested in improved planning and instruction. The study indicates that teacher planning is generally not unique, no matter what the content is to be taught. This may be due to lack of planning time and less than optimal planning habits. Although there were only nine teachers interviewed, they were consistent in their responses, except when they defined problem solving. The power of habits and attitudes of the teachers was very strong as the teachers planned the mathematics instruction in problem solving. The study points to the difficulties in helping teachers improve instruction, since not only pedagogical knowledge must be understood, but also its relationship to the content that is to be taught.

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Shulman, L. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15: 4-14.

Caldwell, Janet H. and Goldin, Gerald A. VARIABLES AFFECTING WORD PROBLEM DIFFICULTY IN SECONDARY SCHOOL MATHEMATICS. Journal for Research in Mathematics Education 18: 187-196; May 1987.

Abstract and comments prepared for I.M.E. by CHARLES E. LAMB, The University of Texas at Austin.

1. Purpose

The purpose of the study was to compare the relative difficulties for junior and senior high school students of four types of verbal problems. The four types of problems were abstract factual (AF), abstract hypothetical (AH), concrete factual (CF), and concrete hypothetical (CH).

2. Rationale

Problems were designed using the following definitions: an abstract problem deals only with abstract or symbolic objects; a concrete problem uses a real situation regarding real objects; a factual problem describes a situation; and a hypothetical problem describes a situation as well as a possible change that does not actually occur in the problem. Three models were described and used to predict relative difficulty of problem types. The developmental theory would suggest CF problems as the easiest, CH and AF of intermediate difficulty, and AH problems as the most difficult. This situation should be present at the elementary school level but differences should decrease as students pass into high school. The direct translation model predicts AF problems easiest, AH and CF problems to be of intermediate difficulty, and CH problems to be the most difficult. Finally, the construction of a nonverbval internal representation model includes CF, CH, and then AF and AH problems in order of increasing difficulty.

A previous study by Caldwell and Goldin (1979) indicated general support for the developmental model at the elementary level. The present study was intended to extend these findings to the secondary level.

3. Research Design and Procedures

Instrumentation

Problem sets were constructed so as to accentuate the variables under consideration. The sets used were published previously (Goldin and Caldwell, 1984). Computational trends were used to determine if students could perform calculations similar to those required in the verbal problems on the tests.

Sample

"The sample selected was representative of course enrollments in general mathematics, college preparatory mathematics, and advanced mathematics" (p. 190).

Procedure

Students took the tests over two consecutive days. Testing order was random and data were only used when students had tested on both days. The design was a 3 X 2 X 2 X 2 X 2 X 2 multifactor analysis of variance with repeated measures on the two experimental factors. The factors were grade level, sex, test order (Part I or II), computational performance, abstract or concrete, factual or hypothetical.

Five analyses were performed:

1) junior high school;

2) senior high school;

3) problem sets common to elementary and junior high;

4) problem sets common to junior high and secondary; and

5) problem sets common to all three.

4. Findings

Results included analysis of data for 813 junior high students and 274 senior high students. Concrete problems were significantly less difficult than abstract problems at both levels. The differences decreased with increasing grade level. Factual problems were significantly less difficult than hypothetical problems. There were significant interactions between the two variables.

5. Interpretation

The authors report that the study best fits the predictions of the internal model as well as giving some support for the developmental level.

Major conclusions were:

a. Abstract problems are in principle accessible to young children;

b. "Teachers should not conclude on developmental grounds that word problems that are abstract or hypothetical are necessarily beyond their students' reach" (p. 195).

c. "It appears that the greater difficulty of hypothetical problems occurs almost exclusively in concrete contexts" (p. 195).

Abstractor's Comments

a. Word problems are a very important issue in mathematics education.

b. The concrete-abstract and factual-hypothetical dimensions are useful ideas when considering world problems.

c. The study was very ambitious. Maybe it tried to do too much. The analyses and all the tables were hard to follow and monitor.

d. The discussion seems to lack pertinent information at several points. The reader is referred several times to previous studies and publications. More information would have been helpful.

e. The study is based on a dissertation from 1977. Isn't ten years a little too long before journal publication?

f. The writing style is difficult to follow. Ideas seem to jump around a lot.

g. The discussion section is easier to follow and makes some important statements.

h. More explanation of suggestions for further research would have been in order.

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Caldwell, J. H. and Goldin, G. A. (1979). Variables affecting world problem difficulty in elementary school mathematics. Journal for Research in Mathematics Education, 10: 323-336.

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Cobb, Paul. CONCRETE CAN BE ABSTRACT: A CASE STUDY. Educational Studies in Mathematics 17: 37-48; February 1986.

Abstract and comments prepared for I.M.E. by F. ALEXANDER NORMAN, University of North Carolina at Charlotte.

1. Purpose

The purpose of this report is to illustrate, by way of a case study, the author's contention that some students who have developed relatively advanced conceptualizations of number and number operations might nonetheless express these concepts through relatively primitive problem-solving behaviors. Furthermore, the author conjectures that students fitting this characterization will, given appropriate instruction, develop rapidly in constructing problem-solving strategies more in consonance with their conceptualizations.

2. Rationale

The underlying philosophical view taken in this study is fundamentally constructivist and the analysis reflects a perspective based in the theory of children's concepts of counting espoused by Steffe et al. (1983).

3. Research and Design Procedures

A 20-minute clinical interview with one student and a 45-minute teaching session with her a week later comprised the investigation. At the time of the study, the subject, Melissa, had neared the latter part of her first-grade year. Both the interview and the teaching session were videotaped to allow for extensive reflective analysis. As with most studies of this genre, the interview consisted of asking the student to solve a variety of counting-based tasks (finding sums, missing addends, or differences), and analyzing the student's problem-solving activity. The purpose of the analysis was to "infer meanings she [Melissa] established in arithmetical situations (p. 38)." A typical task involved objects hidden beneath two cloth screens. Melissa was told that there were thirteen objects hidden, nine of which were under one of the screens. How many then were under the second?

4. Findings

The interview revealed that her solutions to tasks were based on finger representations and that she was able to separate the initial finger pattern (for 9) from the items she subsequently counted. It was determined that there were also finger patterns that she could not establish (e.g., those beyond 15).

Within the analytical framework of the study, Melissa's responses were viewed as intentional acts expressive of an underlying conceptual structure. She was seen as having attained the operative stage in Steffe et al.'s model of counting types. Thus, despite employing the relatively primitive solution strategies involving finger patterns (as opposed to counting-on or counting backwards), the author speculated that observations of her behavior suggested she had already constructed the requisite conceptual operations for counting-on or counting-down-from and so would need only to "express number in terms of alternative sensory-motor material, actual or represented counting activity, in order to produce these more sophisticated solution methods (p. 40)." Consequently, given that the appropriate conceptual structures were in place, appropriate guidance should result in Melissa's acquisition of these counting strategies.

In the week prior to the teaching session, Melissa had begun to develop solution methods which incorporated counting strategies rather than finger patterns. For example, when solving the missing addend task 14+_=18, she did not construct a finger pattern, but rather represented the missing addend as a sequence of counting acts. She counted from fourteen, "1, 2, 3, 4" and stopped, apparently recognizing (as a rhythmic or temporal pattern) that this sequence brought her to eighteen. However, during the teaching session there arose cases (e.g., 29+_=35, to which she responded 5) in which Melissa was unable to recognize the correct temporal pattern. The investigator then led Melissa to discover that she could keep track of her counting by using her fingers. After this she had no problem solving missing addend tasks in this way. Initially, in solving difference tasks, such as 17-5=_, Melissa still relied on finger patterns. Further intervention led Melissa to construct a counting backwards strategy for solving these problems.

Analysis of the teaching session suggested that Melissa had already constructed a concept of number as an abstract arithmetical object and that her advance towards new problem-solving strateges was not predicated on the development of new conceptual structures, but rather consisted of finding different ways to express those which she had already built up. These advances were the result of Melissa's encountering an anomalous situation. That is,

> ...her advances were triggered by perceived limitations in her current methods, not by the demonstration of alternative methods. And she became aware of and was able to overcome these limitations in an instructional context because she had constructed the prerequisite conceptual operations.

This view of anomaly-generated development is tied to the more general argument of Kuhn (1970) regarding the construction of new paradigms in the evolution of scientific thought. Additionally, some of Melissa's solutions also exemplified the process of problem elaboration (Silver, 1983), in which one reflects on the occurance of anticipated difficulties or unexpected situations.

5. Interpretations

Why, if Melissa had previously constructed sophisticated conceptual structures, did she express these in relatively primitive ways? Most likely, there were two ways in which Melissa's classroom experiences engendered her perceptually-based methods. First, Melissa was accustomed to having manipulatives available to aid her in solving number tasks. Second, Melissa's teacher focused on correct answers rather than the problem solving process, thus lending support to the continuance of Melissa's less sophisticated methods and providing no incentive to construct others.

The researcher's analysis provides an alternative interpretation of Melissa's problem-solving behavior and "suggests the value of going beyond the surface level...[by] attempting to infer both children's purposes and intentions and their anticipations (p.45)" regarding the outcomes of their activity. Using this analysis one can identify those whose methods accurately reflect their level of conceptual development and those who express relatively complex conceptualizations in primitive ways. This latter group can, through appropriately orchestrated learning experiences, be encouraged to construct problem-solving methods which are reflective of their conceptual structures. Furthermore, it is reasonable to interpret children's problem-solving behaviors within the context of the "paradigm" under which they operate - for children doing mathematics this paradigm is represented by their general beliefs about mathematics.

Abstractor's Comments

- Research in early number harbors two quite different approaches to the characterization of children's conceptual development of number. On the one hand, advances in development are identified with the emergence of novel or different problem-solving behaviors. On the other hand, characterizations of the level of conceptual development take into account not only the overt problem-solving strategies of the student being interviewed, but also contextual factors and the investigators' own divinations of the meaning which the student establishes in the particular arithmetic situation being analyzed. Each approach has its advantages and disadvantages, and both reflect a particular mythology of the mind. The Steffes and Cobbs of this world fall into the latter category. There is considerable debate regarding the constructivist philosophy which undergirds this research as well as potential problems with the validity and generalizability of inferences drawn from investigations involving so few subjects. Furthermore, when the inferences drawn are so very much dependent on the researcher's particular conceptualizations of another's conceptual framework...well, things become a bit less tidy.

However, this said, if one once accepts the thesis that the researcher can identify conceptual structures through inferences of intentions, anticipations, and purposes (Steffe et al., 1983), then Cobb presents quite a convincing and reasonable analysis in this case study. I must confess, that the insight and clarity of his arguments dispelled many of the reservations that cropped up periodically. Most importantly, the findings here suggest that evaluating one's conceptual development solely on the basis of overt problem-solving behaviors might not be a tenable position.

The implications for instruction, however, are not so clear. Recall that Melissa's reliance on primitive finger pattern solution strategies may have been due to the availability of number cubes for solving problems for which her methods alone would have been insufficient. Cobb's contention is that by not being forced to express her conceptualizations in other ways, Melissa exhibited behavior that did not adequately reflect the sophistication of those concepts. Cobb's only comment here is the obvious, "...[do not] banish manipulatives from the classroom...[yet there are] unfortunate consequences that can arise when a child is allowed to rely unduly on manipulatives (p. 45)." Cobb does suggest that discovery methods and Socratic methods of instruction are appropriate for students like Melissa who have already developed the prerequisite concepts and might rapidly acquire new solution methods. It is unlikely though that there are many teachers with the expertise to identify the Melissas in their classrooms, much less to have the time to work them through an appropriate teaching session.

While the paper was excellently written and cogently analyzed, it includes two points, minor though they be, that are bothersome enough not to be dismissed off-hand. First, consider the following assertion, "The time it took her to solve this task (50 seconds) indicates that she did not merely remember the method the investigator had attempted to teach her. Instead, she constructed it for herself...(p. 43)." While this assertion may not be unreasonable, the presumption of a temporal relationship involving remembrance and the construction of solution methods is unsupported. Would the same inference have been drawn if her response had required only 30 seconds? or 40? Are there no other possible explanations for the time it took her to respond?

Second, despite Feyerabend's (1970) contention that Kuhn's thesis is applicable to learning in general, Cobb may be stretching that thesis a bit in trying to link it to the evolution of one child's solution methods. If I may be allowed to stretch an analogy a bit myself, this strikes me as akin to the old and discredited (but no less popular) theories of biological recapitulation (Gould, 1977). Is it really necessary, or merely vogue, to appeal to such a grandiose theory when simple assimilation/accommodation will suffice?

Implicitly Cobb's paper is a perfect expression of the work of Steffe, his students (among whom Cobb is perhaps the most prominent), and those who have been converted to his particular brand of exploring the conceptual development of early number. Reading this report one can not help but have a better understanding of their approach and motivations. Earlier I used the phrase "mythology of the mind." In part, this may reflect perceived ambiguities and the uncertainties inherent in trying to model that which can not be modelled - but one can say that, like the mythology of atomic structure postulated by Bohr, this particular mytholgy seems to be addressing and answering important questions.

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Steffe, L. P., von Glasersfeld, E., Richards, J., and Cobb, P. (1983). Children's counting types: Philosophy, theory, and applications. New York: Praeger.

Davidson, Philip M. EARLY FUNCTION CONCEPTS: THEIR DEVELOPMENT AND RELATION TO CERTAIN MATHEMATICAL AND LOGICAL ABILITIES. Child Development 58: 1542-1555; December 1987.

Abstract and comments prepared for I.M.E. by KAREN SCHULTZ and HWE-YOUNG LEE, Georgia State University.

1. Purpose and Rationale

The focus of this article is the development of function concepts and their relation to mathematical and logical abilities generally acquired during 5 to 7 years of age. The primary purpose of the study was to explore the origin of operations. It was expected that performance on operation tasks are related to children's function abilities. A second purpose was to explore the relations among separate function notions. If functions lay the groundwork for operations, it was thought important to determine whether function abilities reflect a generalized concept, or if they occur within relatively discrete domains. The possible connection between functions and numerical concepts is of particular interest, because even elementary constructions of quantities appear to depend on function properties.

The first hypothesis of this study was that competence on function tasks would be related to competence in numerical reasoning. This hypothesis is based on previous observations that the one-to-one correspondence used to establish cardinal equivalence of two sets entails the idea of single-value mapping. Counting to establish cardinal or ordinal value involves one-to-one mappings between count words and objects counted, and, the counting activity itself expresses a particular function which yields its unique successor. Moreover, the property of composibility suggests a possible basis for binary numerical operations. A second hypothesis was that operations on classes and relations are constituted from children's developing function notions. By this proposal, performance on function tasks should be related to the inclusion, vicariance, and seriation abilities tested. Function tasks were designed to assess cognition of the general form of function properties, without involving explicit numerical, classification, or seriation concepts.

2. Research Design and Procedures

Seventy-two children from 5-0 to 7-10 (M = 6-3) were tested individually in a laboratory setting. An equal number of boys and girls participated, with both sexes represented evenly across the age range. A majority of subjects were of middle-class backgrounds. Three different sets of materials were used in testing each of eight categories of function, number, and logic concepts. Order of task administration was counterbalanced across age and sex of subject. Standard methods of presentation were used for each task, followed by open-ended questioning that encouraged children to justify or elaborate on their responses. In all tasks, children were given the opportunity to repeat their efforts until the interviewer judged that an accurate indication of ability had been obtained. Testing sessions were recorded on videotape. The average time required for testing was 3 1/2 hours per child, divided into two or three shorter sessions during the course of a week.

a. Function Tasks

Three categories of tasks were used: functions defined as exchanges of properties; functions defined as rotations of regular polygons (displacements); and morphisms, or functions that preserve an equation or relation. In exchange functions, three tasks were introduced as games. In each problem, the child's objective was to compose functions in a sequence so as to obtain a result that was not possible in a single exchange. In displacement functions, congruence motions of regular polygons constituted functions defined on the set of vertices. These tasks tested children's ability to reason about the effects of such mappings, and their compositions. Morphisms were divided into element mapping and structure mapping. It is of interest to investigate whether children's function notions include intuitions about morphism properties because this is an important class of functions from the formal point of view.

b. Numerical Tasks

Two aspects of numerical knowledge were investigated: the construction of invariant quantities and the construction of a system with binary operations. These were assessed by number conservation problems and arithmetic problems, respectively. Numerical conservation included three problems. In each, two qualitative transformations were performed on the materials and two conservation judgments requested. Arithmetic operations were embodied in three sets of manipulative objects in a context for presenting addition, subtraction, multiplication, and division problems.

c. Logic Tasks

Class notions were assessed by means of inclusion and vicariance problems and order notions by means of seriation problems. Children's performances were scored directly from videotape using a coding scheme developed during pilot testing.

3. Findings

Orderly developmental trends were found in function task performance, with younger children manifesting limited success through trial-and-error strategies and older children achieving substantial success with anticipatory strategies. Moreover, certain function abilities were associated with the numerical domain, whereas others were associated with the logical domain. These findings were consistent with the developmental model of Piaget and associates, according to which cognition of functions lays the groundwork for reversible operations, but also suggest that this development occurs through parallel processes within separate conceptual domains.

Abstractors' Comments

This study makes an excellent contribution to our understandings of early function concepts. It is a masterpiece of detail and breadth in all aspects of the rationale, design, and interpretation of findings. The reader will need ample time, though, to absorb the detail presented. Particular attention to the task descriptions is a must for those interested in studying early function concepts.

Though the tasks appeared appropriate, the reader does question why the particular tasks were chosen. And if more inferences are given about why the investigator divided this experiment into these three tasks categories, the reader might be better able to interpret and accept the results of the data. Moreover, if the results of the experiment were more simply expressed, it might be better understood by the reader with less background knowledge.

Finally, because all subjects are so young (5 to 7 years of age), the question of allowing adequate time per test session is ' .ised. It appears that the younger the subject, the greater would be the effect of time on relations between variables.

Fisher, Linda C. STRATEGIES USED BY SECONDARY SCHOOL MATHEMATICS TEACHERS TO SOLVE PROPORTION PROBLEMS. Journal for Research in Mathematics Education 19: 157-168; March 1988.

Abstract and comments prepared for I.M.E. by NADINE BEZUK, San Diego State University.

1. Purpose

This study examined the strategies used by secondary school mathematics teachers to solve and to teach proportion problems. Three research questions were investigated:

a. Can teachers solve selected proportion problems?
b. What strategies do teachers use to solve proportion problems?
c. What strategies do teachers say they would use in teaching proportion problems?

2. Rationale

Few textbooks clearly integrate proportion problems and solution strategies for solving these problems. Many textbooks only teach the cross multiplication algorithm (If a/b = c/x, solve by equating the cross products and solving for x); few textbooks relate direct and inverse variation to the cross-multiplication strategy. But students effectively use a wide variety of strategies, many of which are less formal and often more intuitive. Some researchers have recommended the inclusion of student-generated strategies in instruction in order to build conceptual understanding. This inclusion, however, demands that teachers be able to use these strategies and to recognize that these strategies are consistent with proportional reasoning.

3. Research Design and Procedures

a. Subjects: The subjects were 20 secondary (grades 6-12) mathematics teachers who teach in ten secondary schools in a medium-sized city in northern Florida. Sixteen of the teachers were secondary school mathematics teachers; four of the middle school teachers had added middle or junior high school mathematics certification to their elementary credential. Twelve of the teachers were middle school mathematics teachers (4 males and 8 females), and eight of the teachers were high school teachers (2 males and 6 females). This distribution of teachers by sex and middle or high school level was representative of these groups throughout the school system as a whole.

b. Instrument: The investigator chose four basic problems, two of which were direct proportions (height and percent problems) and two of which were inverse relationships (work and wheel problems). Two problems of each basic type were selected, yielding a total of eight problems.

c. Procedure: Each teacher was interviewed individually. Interviews lasted approximately 1 hour and were tape-recorded. The teachers were asked to solve four problems "in the way that seemed most natural to them" (Task 1). They were then asked to explain how they would teach the height and wheel items (Task 2). The teachers were then shown a sample problem (involving saving money) with three different correct solutions, and were asked to solve the second set of four problems by using proportion strategies (Task 3).

d. Strategy classification scheme: The investigator classified the strategies used into nine categories: no answer, intuitive, additive, proportion attempt, incorrect other, proportion formula, proportional reasoning, algebra, and correct other. Three raters independently classified each subject's solutions to each problem. Initial agreement of these classifications was 70%; after group discussion, all three raters agreed on 90% of the responses. Classifications which were still disputed were examined by the author and either categorized if 3-of-4 agreement existed (6% of the responses), or classified as "other" if no agreement existed (4% of the responses).

4. Findings

a. Can teachers solve selected proportion problems? The teachers' performance on the direct proportion items was perfect, except for computation errors. The inverse proportion items were more difficult, especially the work problems, which less than 50% of the teachers answered correctly. The most common error was that the teachers tended to assume the relationship was direct and then did not realize that their answer was unreasonable. The teachers were slightly more successful on the wheel problems (12 and 14 teachers out of 20 answered each problem correctly on Tasks 1 and 3, respectively), but they considered this problem type the most difficult.

b. What strategies do teachers use to solve proportion problems? The teachers tended to choose the proportion formula to solve the height problem, an algebraic strategy for the percent and wheel problems, and an incorrect proportion strategy for the work problem. No teachers used the additive strategy, which was expected. When the teachers were specifically asked to use proportion strategies (Task 3), many of the teachers solved the problem in more than one way, preferring to find the answer and then try an alternate solution that used a proportion. The percent of responses classified as proportional increased from 51% on Task 1 to 71% on Task 3, when they were specifically asked to use a proportion strategy. It was interesting to note, however, that three of the teachers were frustrated by trying to use the proportion strategy on specific problems, and one teacher reported disliking to use proportion.

c. What strategies do teachers say they would use in teaching proportion problems? Eighty percent of the teachers said they would use the same strategy in teaching proportion problems as they use to solve the problems. Only three teachers reported that they would use less formal strategies when teaching proportion problems. It was interesting to note that several teachers expressed concern about whether students really understand what they are doing when they solve proportion problems, yet only three teachers reported that they would use less formal strategies when teaching proportion problems.

5. Interpretations

Less formal strategies were rarely used by these teachers to solve proportion problems. Even though they were shown two fairly informal proportion strategies, few teachers used them, instead switching in most cases to a proportion formula. If less formal strategies must be taught to students, then teachers must be made aware of the need to teach alternate strategies and must accept the alternate strategies themselves. Teachers' reliance on the proportion formula seemed to limit their consideration of alternate strategies which might be more accessible. Teachers need to be aware of research results regarding students' strategy choices and errors. Teachers need to broaden their recognition of proportional situations. And teachers need more or better instruction on proportion and direct and inverse variation.

Abstractor's Comments

This research addressed an important component of instruction on proportional reasoning: teachers' use of various strategies. The author's opinion (and that of other researchers) that students' existing strategies should be capitalized on in instruction is consistent with that of Ginsburg (1977). This inclusion, however, demands that teachers be able to use these strategies with understanding and connect these strategies with more formal strategies of direct and inverse variation as well as with the cross-multiplication algorithm. This research did not examine teachers' abilities to do this.

This research tells us much about the strategies teachers use to solve proportion problems, but the quality of teachers' understanding of these strategies is still to be determined. Subjects' use of strategies immediately after they were shown them does not indicate their understanding of them (nor does their telling an interviewer that they "felt comfortable using proportion"). Teachers were not asked to explain how or why their chosen strategies worked. These data are a necessary component in an investigation of teachers' proficiency in solving and teaching proportion problems.

It was reasonable to include middle school teachers in the sample, since instruction on proportions is usually introduced in middle school. It would have been interesting, however, to have examined the responses of the two groups of teachers separately. This opportunity was not capitalized upon in the analysis, or at least was not reported.

There was one limitation in the design of the testing instrument: the type of numeric ratio was not held constant throughout all problems. The percent problems used integral ratios within rate pairs (see Karplus, 1983), while the three other problem types used non-integral ratios. These two types of numeric ratios have been shown (Rupley, 1981) to be of unequal difficulty (integers within is easier) and also have been shown to affect students' use of solution strategies (Bezuk, 1987). This oversight muddies the interpretation of the data.

The three different correct solutions that were shown to the teachers involve three proportional reasoning strategies: factor of change, unit rate, and the cross-multiplication algorithm. It is not clear exactly how these strategies were presented to the teachers. The author does not state if they were explained verbally to the teachers at all, and if so, if the explanations were procedural or conceptual. It is not clear if the teachers understood these strategies or just rotely applied them.

In Task 3, teachers were presented with three different proportion strategies and were asked to solve problems by using one of these strategies. It would have been interesting if the author had reported the frequency of teachers' use of each of these three strategies, rather than just reporting the most frequently used strategy.

Teachers' frequent use of proportion formulas (using variables) and the algebraic strategy raises concern regarding their abilities to teach these strategies to children, especially given the tendency for teachers to teach the same strategies that they use. Is the algebraic strategy accessible to most students in the grades taught by these teachers? This question was not addressed.

Why did the author only ask teachers to explain how they'd teach two of the problem types? An answer regarding their teaching of all four problem types would have been more interesting and helpful.

The inverse proportion items were more difficult, especially the work problem, in which the teachers tended to assume the relationship was a direct proportion and then did not realize that their answers were unreasonable. These difficulties may indicate weakness in teachers' ability to consider the reasonableness of results in teaching and solving proportions.

The author has conducted a thoughtful study examining one aspect of secondary teachers' use of solution strategies in solving proportion problems. Any suggestions from the author regarding directions for preservice and inservice teacher education to prevent or correct the deficiencies reported in this study would have been welcome. Teacher educators must now address this question.

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Newman, Richard S.; Friedman, Carol A.; and Gockley, David R. CHILDREN'S USE OF MULTIPLE-COUNTING SKILLS: ADAPTATION TO TASK FACTORS. Journal of Experimental Child Psychology 44: 268-282; October 1987.

Abstract and comments prepared for I.M.E. by CAROL A. THORNTON and MICHAEL K. POTTS, Illinois State University, Normal.

1. Purpose

The purpose of the study was to determine conditions under which young children enumerate by counting in multiples.

2. Rationale

Previous research has shown three distinct processes involved for enumeration: subitizing, counting, and estimating. Subitizing is a relatively automatic appraisal of small numerosities, usually up to 4 or 5. In more difficult problem situations where there are constraints of both time and demands of accuracy, advanced procedures such as repeated addition and counting in multiples may be utilized. While many studies have looked into children's use of the three basic skills of enumeration, few have examined the more advanced procedures (i.e.. counting in multiples). This study concerns itself with the various task conditions which influence children's spontaneous use of such advanced, yet still developing enumeration skills.

3. Research Design and Procedures

Subjects consisted of 38 children, half in kindergarten and half in first grade, consisting of 20 boys and 18 girls with a mean age of 6 years 2 months. The children all attended a middle-class elementary school in Suffolk County, New York. A further prerequisite was that the children be able to count accurately by ones to at least 36.

Each child was seen individually in sessions of 20-30 minutes. In the first two sessions children were asked to judge, as accurately and as quickly as possible, how many objects appeared on a computer screen. The "objects" were an array of dots varying in overall numerosity (12, 24, and 36), subgroupings numerosity (2, 3, or 4), and type of arrangement (random, clustered, and rectangular). Upon the verbal response of the child, the examiner hit a space bar on the computer which both erased the current array as well as recorded the time of response. Over the first two sessions children were allowed practice trials, were presented with two different arrays of random order, and were exposed to foils (which prevented recognition of distinct levels of the task factors).

In the third session children were exposed to several arrays and again asked to judge, as accurately and quickly as possible (though with no timer operating), how many dots were in each display. While the display remained on the screen, children were then asked (1) how they determined their answer and (2) to report any other ways they could have enumerated the display. Finally, at the end of the third session, children were shown additional displays having clusters of size 2, 3, or 4 and numerosity of 39 or 40. The children were instructed explicitly to count aloud by twos, threes, or fours. Here accuracy, not speed, was emphasized. This last procedure provided an independent assessment of children's multiple-counting skill which could not bias their earlier, spontaneous performance.

4. Findings

In conducting the statistical analyses, children were broken into skillful and nonskillful multiple-counting groups. Children in the nonskillful group were those incapable of counting aloud in one or more of the multiple-counting tasks (when asked to count up by twos, threes, or fours). Overall, the vast majority (97%) of the nonskillful children's categorizable strategies are counting by ones, regardless of numerosity, type of presentation, or size of subgroupings. A majority (71%) of the skilled children's categorizable strategies was also counting by ones, also regardless of numerosity, type, or size. Of the small number of cases where multiple counting was reported in use by the skilled children, it was apparent that strategies were chosen in a systematic way (i.e., dependent on the numerosity, type, and size of the display). There was a tendency for small numbers of objects, as opposed to large numbers of objects, to elicit the multiple-counting skills among those .uren proficient in multiple-counting (the skilled group). The s.me was found to be true for objects arranged in clusters, as opposed to a random fashion. When the overall number of objects to be counted was large or when the size of the subgroupings was small, it was found that a clustered arrangement was more facilitati of multiple counting than even a rectangular arrangement. With a large overall number of objects which was either random or rectangular, however, the tendency was to count by ones for both the skilled and unskilled groups. It was suggested that children counted by ones in this circumstance in an effort to avoid anticipated errors such as losing on. s place, skipping objects, or counting objects more than once.

It was further found that regardless of the child's skill level in multiple counting, there was greater accuracy with small, versus large, numerosities and with systematic, versus random displays. While the skilled children preferred multiple counting for clustered rather than rectangular arrangements under conditions mentioned above, there was no advantage in using the skill for either accuracy or response time. It was hypothesized that the developing nature of the multiple-counting skill resulted in no clear advantage at this stage. Nonskillful children displayed a tendency to take longer to enumerate rectangles than clusters, though no difference in accuracy was found. It was suggested that, while not specifically measured, the nonskillful children allocated more time to what they perceived as a more difficult task.

Performance between groups showed that the skilled group was faster. They were also more accurate than the nonskillful group, though only on displays with subgroups of twos. While the nonskillful group overwhelmingly favored counting by ones, they did occasionally experiment with multiple counting by twos on tasks with display subgroups of two. However, they utilized what was perhaps an unpracticed skill at the cost of accuracy, where their performance was significantly less accurate than the skilled group. On other tasks (where the nonskillful groups counted by ones), accuracy for the groups did not differ.

5. Interpretations

The authors make several summarizing statements relative to the results of their study. First, they state that the findings suggest that small numerosity and clustered displays facilitate the use of multiple counting in first and second graders. The findings are also said to illustrate a certain flexibility in early strategy use. That is, children systematically applied the skill of multiple counting under certain task conditions while depending on counting by ones under other specific conditions.

A larger issue which the authors bring up for further study pertains to uncovering the relationship between subitizing and the development of number concept. A subject for further study is the process of quantification that allows a small number of objects to be perceived as a discrete unit of quantity. This may have implications for both the development of multiple counting as well as the conceptual understanding that accompanies the skill.

Abstractors' Comments

The study was a descriptive one, and provided insights into children's informal counting strategies. The authors clearly found that the kindergarten and first-grade children of their study did not spontaneously use multiple counting skills a majority of the time, regardless of skilled versus nonskilled group placement. They were successful in their effort of determining several task conditions when multiple counting was utilized at this early stage. One implication of the study, not stated by the authors, relates to the emergence of multiple counting patterns noted in the study. The fact that multiple counting was typically not used by kindergarten and first-grade students may serve to support the formal introduction of multiplication beyond Grade 1.

The authors give the impression that the conditions surrounding the spontaneous expression of multiple counting at the early ages are relevant to some later development of number concept or other skills. It would have been helpful had they outlined more completely their perceptions of how the development of multiple counting skills influences later mathematical development. It also would have been useful had they specified more carefully possible directions of future research in this area.

Overall the study appeared to have been well thought out by the researchers. Adequate control was built into the study to prevent confounding influences. The brief description of the subject pool, however, does not necessarily convince the readers of the generalizability of the study. Sufficient information (i.e., SES, racial representativeness) was not provided to determine whether students were "typical." The presence or absence of prior instruction background related to the counting tasks was also not given. This may have influenced "spontaneous" expression of multiple counting. Many students in the <u>Mathematics Their Way</u> curriculum, for example, skip count by 2s at an early age. It is difficult to conjecture whether different results might have occurred had multiple counting been emphasized as a necessary skill or whether the children were simply not ready at the grade levels described?

In sum, the authors have done an excellent job of describing the various conditions under which spontaneous multiple counting occurs. The interactions of display conditions and their influences on multiple counting are fully described by the authors, while only outlined here. The future implications based upon results obtained here should provide a more complete picture of the development of enumeration by multiple counting.

Reference

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Perry, Michelle. PROBLEM ASSIGNMENT AND LEARNING OUTCOMES IN NINE FOURTH-GRADE MATHEMATICS CLASSES. Elementary School Journal 88: 413-426; March 1988.

Abstract and comments prepared for I.M.E. by CHRISTINE BROWNING and RUTH ANN MEYER, Western Michigan University.

1. Purpose

The stated purpose of this study was to investigate the problems that students were assigned from their mathematics texts and the influence of problem assignment on learning outcomes.

2. Rationale

Since students spend over half the time of their mathematics classes in solving problems assigned by their teachers, the author argues that one should examine these problems when focusing on how children learn mathematics. Problem "dimensions" such as complexity, frequency of presentation, content, etc., may contribute to mathematical learning. This study examined whether a relationship existed between the problems that students were assigned and student learning.

3. Research Design and Procedures

The following three questions were investigated:

a. What problems do nine fourth-grade teachers assign?

b. How does children's initial knowledge affect what problems teachers assign?

c. What are the relative effects of students' initial knowledge and of problems assigned during the fourth grade on students' learning?

Nine fourth-grade teachers and their classes participated in the study. The sample of 204 students was diverse in both economic and cultural backgrounds.

After reviewing the fourth-grade mathematics textbooks used by the teachers of the sample, the author developed a coding scheme based upon the mathematics topics usually taught in fourth-grade classrooms. She then constructed a 50-item entry-level test and 50-item year-end test based on these topics. The coding scheme was used to code the problems that participating teachers assigned to their students throughout the year of investigation.

Each mathematics classroom was observed eight times throughout the year. The observers wrote running narrations of instructional activities which were later coded as explanations, recitations and questions-and-answers, work check, supervised seatwork, unsupervised seatwork, and noninstructional interaction. Regression analysis was performed to determine how entry-level scores and problem coverage were related to learning outcomes. To take into account the possibility of interaction between students' entry levels and problem coverage, a third variable, entry by coverage, was added to the analysis. For further investigation of the interrelationships among children's entry-level scores, problem coverage and outcomes, two classrooms with similar entry characteristics but different patterns of problem assignments were examined more thoroughly.

4. Findings

When the 50 test problems were classified as review or new problems the following results were obtained:

a. For the entry-level test
   1) An average of 79% of the 16 review problems were solved correctly.
   2) An average of 15% (5.15) new problems were solved correctly.

b. For the year-end test

1) An average of 94% of the 16 review problems were solved correctly.

2) An average of 45% of the new problems were solved correctly.

The teachers gave students an average of 1,682 review and 2,945 new problems to solve during the year. Students spent about 44% of their time on review problems. Across classrooms there was a significant positive relationship between students' initial knowledge of review topics and teachers' coverage of review topics.

New topics comprised 56% of the fourth-grade curriculum across the nine classrooms. The correlation between students' initial knowledge of new topics and teachers' coverage of that material was not significant.

In the regression analysis, the three predictor variables, entry-level scores, problem coverage, and the interaction between these two variables accounted for 41% of the variance in year-end new scores and 35% of the variance in year-end review scores. Entry-level scores of the new problems and problem coverage were significant positive contributors to the variance in year-end new scores, whereas the interaction between these two variables was not significant. All three predictor variables were significant contributors to the variance in year-end scores of the review problems. The relation to year-end review scores was positive for entry-level scores and coverage and negative for the interaction between entry-level scores and coverage.

Since classrooms 3 and 6 had similar entry characteristics, came from the same district, and used the same textbook, they were selected for further analysis. Year-end results for review problems were similar for both classes, yet dramatic differences occurred in the year-end results for new problems. Year-end means for the two classes were 10.71 and 17.8 (SDs. of 4.1 and 7.9, respectively). These two classes also differed in the number and types of problems assigned, with the class with the higher year-end mean having solved more new and diverse problems than the other class.

Tabulations of the observers' data showed that the children of this study spent over two-thirds of an average class period solving assigned problems; the remainder of the class time was split between explanation and directions (about 8%) and noninstructional activities (about 18%).

5. Interpretations

For the teachers of this sample it appeared that performance on the entry-level test had little influence on what students were actually taught in their fourth-grade mathematics classrooms. Although students performed well on the entry-level review problems, they were assigned numerous review exercises throughout the year.

The comparison between the two classes suggests that the type of problems assigned may contribute to student learning. If students are only required to solve simple routine type problems, success on new, perhaps more complicated problems, will be minimal. It is noted by the author that solution of simpler problems may be accomplished within the solution of more difficult ones and that the assignment of difficult problems is partially in the hands of the teacher. The comparisons demonstrated that solving more new problems than review problems allows students to increase their understanding of both new and review topics.

The author concluded with the suggestion that teachers should make use of more new and complex problems within the curriculum which would accomplish both tasks of reviewing old and learning new material.

Abstractors' Comments

The results of this study support our experiences that if students are to increase their mathematics learning they must solve many good problems and it is the responsibility of the teacher to assign good problems. Furthermore, more elementary teachers should shift the focus away from assigning numerous review problems toward assigning problems that cover new content.

Although the article was interesting and has implications for classroom teachers, we do have the following questions and concerns:

a. What data did the author use for the coverage variable? Did she use the total number of assigned problems or the number of content topics or subtopics that teachers covered in their assignments? How did she distinguish topics and subtopics when counting her tallies for the assigned problems?

b. Figure 1 is confusing and adds little to understanding the data being reported. The author first created a bimodal situation with the new and review problems and then constructed Figure 1 to show that the distribution is indeed bimodal. The author's choice of scaling and insufficient labelling of axes makes the figure difficult to interpret.

c. A discussion of the interpretatio of the regression equation coefficients would have helped readers to appreciate why the author used regression analysis.

For a future study, further division of problem type might be considered. Review and some new problems within the fourth-grade curriculum could be viewed as "exercises"; whereas those problems that can be solved by using algorithms or processes not readily apparent could be viewed as "problem-solving situations." This further classification may provide more insight into the relationship between [illegible] learning.

Petty, Osmond S. and Jansson, Lars C. SEQUENCING EXAMPLES AND NONEXAMPLES TO FACILITATE CONCEPT ATTAINMENT. Journal for Research in Mathematics Education 18: 112-125; March 1987.

Abstract and comments prepared for I.M.E. by KAREN GEUTHER GRAHAM, University of New Hampshire, Durham.

1. Purpose

The purpose of this study was to investigate the effect of two instructional strategies, rational vs. randomly arranged sequences of examples and nonexamples, on sixth-grade students' acquisition of the concept parallelogram.

2. Rationale

Results from previous studies designed to study the use of nonexamples in the teaching of concepts are inconclusive. The authors state that only a few of these studies have involved the content area of mathematics and that most have involved methods of instruction which are atypical of most classroom environments.

This study worked from the premise that nonexamples are potentially useful in teaching mathematical concepts. Knowledge of various methods of presenting examples/nonexamples and how these methods interact with mathematical ability may be useful in designing mathematics curriculum and instruction.

3. Research Design and Procedures

The study involved 105 sixth-grade students from two school divisions in Canada; one situated in a metropolitan area and one in a rural area. The students were arranged in treatment groups using the technique of stratified random sampling. The strata were school (and class), sex, and mathematical ability.

The posttest-only control group design had three major phases: Day 1: preliminary instruction by the classroom teacher; Day 2: individual seatwork by students using self-instructional booklets designed to reflect the two instructional sequencings of nonexamples, rational vs. random; and Day 3: posttest. The preliminary instruction phase consisted of teacher-directed activities focusing on the parallelogram and its characteristics. The activities, conducted by the teacher with guidelines from the researchers, consisted of small-group exploration of similarities and differences between various parallelograms followed by large-group discussion of the results. The teacher used examples to clarify characteristics of the parallelogram and to draw attention to irrelevant attributes. Discussion of nonexamples during this phase was not allowed (p. 119). The class was 40 minutes long and students from both treatment groups received the same instruction.

During Day 2, students studied self-instructional booklets at their own pace during the 40-minute class period. T treatment booklets consisted of a definition of the concept, a list of defining attributes, four rational sequences of examples and nonexamples, and three interrogatory practice sequences. In a rational sequence an example is paired with a nonexample with similar irrelevant attributes. The pairs of instances within the sequence are then arranged in order from the least to the most difficult. Levels of difficulty were adapted from procedures given by Merrill and Tennyson (1977). Each of the three interrogatory sequences asked the student to decide whether or not a given figure was a parallelogram by responding to a list of questions involving the defining attributes.

To control for extraneous factors within the treatment materials, the control booklets contained the same definition, list of attributes, and instances as the experimental material. The difference between the booklets occurred in the sequencing of the examples and nonexamples. A table of random numbers was used to arrange the instances within each sequence of the control booklets.

The posttest, given on Day 3 of the study, was designed by the researchers to measure two levels of concept attainment, classificatory and formal. In the classificatory subtest, students were presented with 29 instances and asked to identify each as either an example or nonexample of a parallelogram. The formal subtest consisted of 24 multiple-choice questions designed to measure three objectives: the students' ability to discriminate and name attributes (12 questions), the students' ability to name irrelevant and relevant attributes of the concept (6 questions), and the students' ability to define the concept and evaluate instances (6 questions).

Results, at each level of concept attainment, were analyzed by a standard parametric analysis of variance with treatment and mathematics ability as the independent variables and score on the appropriate sub-posttest as the dependent variable.

4. Findings

The analysis of variance for the classificatory-level scores yielded a significant ($p < .01$) main effect for mathematical ability. Main effects for treatment and mathematical ability were found to be significant ($p < .01$) in the analysis involving the formal-level scores. A follow-up test using the partial omega squared statistic was conducted to estimate the proportion of total variance that is attributable to treatment effects. A "medium" effect of 6% was calculated for the formal-level scores.

5. Interpretations

Previous research has attested to the superiority of rational vs. random sequences of examples and nonexamples in enhancing concept attainment. The authors state that their results support conclusions from previous research at the formal-level but do not support previous conclusions at the classificatory-level.

Several implications for teaching geometric concepts to sixth-grade students and for designing instructional materials are presented. First, the authors suggest that prior to teaching a geometric concept, a careful analysis of the concept's attributes should be undertaken by the teacher. Secondly, rational sequences of examples and nonexamples can be used effectively in the teaching of geometry concepts. Finally, instructional materials that present a random sequencing of examples and nonexamples may be detrimental to the development of the formal level of concept attainment.

Abstractor's Comments

The authors acknowledged several limitations of their study which included the following: participating schools were not randomly selected, only one, fairly narrow, geometric concept was investigated, and mathematical ability was based on teachers' ratings of the students, not standardized test results.

There are additional questions and concerns raised by this study. First, the purpose and results of using a stratified random sampling technique are unclear. What was the purpose of using two schools? It also appears that an attempt was made to balance the strategy groups by sex. However, no mention is made of either sex differences or school differences in performance.

Second, the purpose of the preliminary instruction phase of the study is unclear. Why were the teachers not allowed to discuss nonexamples? Were the teachers observed during this phase to determine consistency of instruction?

Third, the following statement made in relation to the activities on Day 2 of the study is perplexing: "The students were allowed sufficient time (during a regular 40-minute class session) in which to study the material at their own pace" (p. 119). What does sufficient time mean? Wouldn't it vary from student to student? Does this statement mean that all students completed the booklet during the 40-minute period? If not, how was time required to complete the booklet related to mathematical ability? What happened with students who finished prior to the end of the 40-minute period? Did a student have the opportunity to go through the booklet more than once?

In addition, no pre-measure was given. Was it assumed that all sixth-graders have the same understanding of the concept parallelogram? Is this a valid assumption? Is the lack of a pre-measure related to the inclusion of a preliminary instruction phase? Could a pre-measure of mathematical ability, given by the researchers, have been used in place of teacher ratings?

Finally, a clearer description of how the relevant and irrelevant attributes were chosen would have been helpful in interpreting the results and designing future research and instruction. The authors state that "only the more confusing attributes that cause misconceptions could be dealt with in this study" (p. 123). What are these misconceptions? What supporting evidence was used?

Although the authors acknowledge limitations and caution against over-generalizing, their stated implications appear to go beyond the reported data. In particular, the statement that "too much dependence on such materials (where instances are arranged in some random order) may be detrimental to the development of concept attainment at the formal level" appears a bit strong, based on the available evidence. It would seem that before such statements can be made, replication of these results with larger samples at a variety of levels, a larger number of instances involving additional concepts, longer periods of time between instruction and testing, and clinical interviews which probe student understanding are needed.

The main value of this study lies in the materials developed for its instructional and testing components. They provide examples of the types of questions and activities that have the potential to expose alternative conceptions and assess higher levels of student understanding than many of the more traditional materials on the market today.

Reference

Merrill, M. D., & Tennyson, R. D. (1977). Concept teaching: an instructional design guide. Englewood Cliffs, NJ: Educational Technology.

Williams, Paul D. EFFECTS OF INTENTIONAL TEACHER ERRORS ON ACHIEVEMENT OF COLLEGE STUDENTS IN REMEDIAL MATHEMATICS. Mathematics and Computer Education 21: 93-97; Spring 1987.

Abstract and comments prepared for I.M.E. by FRANKLIN DEMANA, The Ohio State University.

1. Purpose

The purpose of the investigation was to compare and contrast presentations that include intentional errors (Errant-Lecture Treatment) with regular presentations (Regular-Lecture Treatment) in college remedial mathematics classes. The researcher hypothesized that students receiving the Errant-Lecture treatment would score higher on course examinations than the students receiving the Regular-Lecture treatment.

2. Rationale

Evidence was presented from several previous research studies to demonstrate controversy about presentation techniques of non-examples in mathematics in order to justify his study.

3. Research Design and Procedures

The study involved two sections of a second remedial mathematics course for two-year technical majors at The Pennsylvania State University, Fayette Campus; each section met for 110 minutes twice a week for 10 weeks. Presentations to one group were intended to be error-free, and the students in the other group were told that the instructor would make at least five mistakes each class period. If students failed to spot an intentional error, the instructor would pause momentarily and, if necessary, present additional evidence (examples and non-examples) until the concept was clarified. Both sections were taught by the researcher using the same textbook.

The 35 students in the study were not selected at random, but the assignment of treatment to the two sections was random. A two-tailed t-test showed no significant differences between the two treatment groups according to the course averages in the previous mathematics course (also taught by the researcher). Both treatment groups took the same three- to four-item multiple-choice quizzes at the end of each section. The three lowest quiz scores of each student were discarded and a mean quiz score computed for each section. All students took the same comprehensive final examination and completed a course/instructor evaluation form. Partial credit was awarded on the final exam.

4. Findings

A two-tailed t-test was used to analyze the data. A significant difference between the two treatment groups was found on the course/instructor evaluation score but not on the mean section-quiz score or the final examination score.

5. Interpretations

In spite of the lack of higher scores on the achievement tests the researcher recommends the Errant-Lecture method because he found that a majority of these students actively participated in class, appeared to be thinking critically about mathematics, and had significantly higher mean attitude toward the course/instructor.

Abstractor's Comments

The Errant-Lecture instructional approach stimulated students to actively participate in class. However, the use of deliberate errors with remedial students may not be in their best interest. Such students often have major misunderstandings about mathematical concepts. Might it be that any method that produces active student participation will also produce better student attitude toward mathematics? Other methods - such as the use of calculators, computers, manipulatives, and so forth - should be used to stimulate student participation and contrasted with the Errant-Lecture method. If similar results are obtained, these other methods would be preferred by this evaluator.

The researcher's study does point out the need to make mathematics instruction more interesting for students. But what are the long range implications of an Errant-Lecture method? More research is needed before recommending an Errant-Lecture method of instruction.

Zepp, R.; Monin, J., and Lei, C. L. COMMON LOGICAL ERRORS IN ENGLISH AND CHINESE. Educational Studies in Mathematics 18: 1-17; February 1987.

Abstract and comments prepared for I.M.E. by JOHN A. DOSSEY, Illinois State University, Normal.

1. Purpose

The purpose of the study was to investigate whether differences in the learning of mathematics in a second language are due to differences in command of the second language or to inherent thinking and logical processes dependent on the student's mother language.

2. Rationale

The study grew out of concern over the mathematical education of students whose instruction takes place in a cultural and language setting different from that of their native language. Considerable research has been conducted relative to the general Whorf hypothesis (1956) that posits a person's thinking and logical decisions are guided by processes dominated by patterns based on the individual's native language. If such is the case, considerable adaptations in curriculum and instruction may be needed to accommodate the linguistic and cultural differences of mathematics teaching or learning situations in universities worldwide.

The specific research questions were:

a. Are difficulties with logical sentences language specific?

b. Do second-language learners experience different difficulties from first-language learners in interpreting logical sentences?

3. Research Design and Procedures

To test their hypotheses, the researchers chose to examine the following cases of possible logical misinterpretations:

a. Whether disjunction is treated as inclusive or exclusive.

b. If A v B is true and A is false, what can be concluded?

c. Questions involving misinterpretation of the implication:

   1) converse reasoning

   2) inverse reasoning

   3) failure to conclude from the contrapositive.

A test (Zepp, 1982, p. 209) covering these three logical interpretation areas was administered in English to 58 Chinese (Cantonese) speaking students, to 74 first-language English students in English, and to 82 first-language Chinese (Cantonese) students in English. The students were enrolled in quantitative business courses at the University of East Asia in Macau.

The test presented students with logical situations related to conditions placed on a set of cards having colors on both sides, given a statement expressing a relationship about the nature of the two colors found on the two sides of the cards.

Several possibilities for the results were envisioned:

a. Chinese and English response pattern differ

1) Bilinguals' response patterns like Chinese

2) Bilinguals' response patterns like English

3) Bilinguals' response patterns resemble neither

b. Chinese and English response patterns similar

1) Bilinguals' response patterns like native speakers'

2) Bilinguals' response patterns different from native speakers'.

Results 1c and 2b would be evidence that the differences, if any, are not Whorfian, but simply due to language deficiency. Result 2a would be evidence that there are no Whorfian differences. The remaining cases would support evidence of Whorfian differences.

An analysis of variance and correlation analyses were performed on the results of the tests measuring the items related to disjunctions and the items representing implications.

4. Findings

a. Disjunction

The overall scores on items dealing with the interpretation of the disjunction of the Chinese bilinguals who took the test in English were lower than the scores of the other two groups. However, the strategies chosen by the bilingual group were more similar to those o' the English-speaking group than they were to the Chinese-speaking group. Hence, no evidence of Whorfian differences was noted.

The second set of items dealing with disjunctive settings confronted student with the apparent contradiction resulting from being given the statement A v B is true, but A is false. The bilingual speakers tended to opt for indecision rather than an error pattern identifiable with either of the native-tongue groups. Such findings support the theory that the problems observed reside in "linguistic confidence" rather than in "linguistic competence." Such confidence may be a result of extensive language experience in the second language rather than deeper Whorfian differences.

b. Implication

On the overall test dealing with implications and their interpretations, no significant differences were noted in the first- and second-language speakers' performances. Analysis of the three subtest scores indicated that significant differences existed between the groups relative to items dealing with settings involving interpretation of the validity of the inverse of a valid implication. The bilingual students tended to exhibit a greater tendency to use the inverse reasoning as a valid form. However, their thought patterns were different from those of their Chinese counterparts. Thus, Whorfian differences were again ruled out as the "confidence issue" again was cited as the possible explanation.

5. Interpretations

The investigators concluded that deep differences explained by the Whorfian hypothesis were not observed in the data collected. Rather, they suggested that the learning/development of the logical [illegible] of sentences involving implication were similar in [illegible]

[illegible].

Abstractor's Comments

The study appears to have been carefully conceived and competently executed. The authors carefully considered alternative explanations and related their findings to them and to the previous work of others. However, the evidence they present must be interpreted with certain cautions:

a. only Chinese and English speaking individuals were involved and the bilingual pattern studied was Cantonese-speaking individuals learning in English

b. the participants performance was measured in a card-sorting setting, rather than one that is rich in context and cultural meaning

c. the participants were individuals who had already passed significant cultural and intellectual hurdles to be accepted into university programs of study.

While the findings and interpretations of the present study are compelling, caution must be exerted to keep from generalizing from the findings to settings far beyond those observed in the present, controlled, experimental setting.

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