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AUTHOR	Mullis, Ina V. S.; Dossey, John A.; Owen, Eugene H.; Phillips, Gary W.	
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ABSTRACT

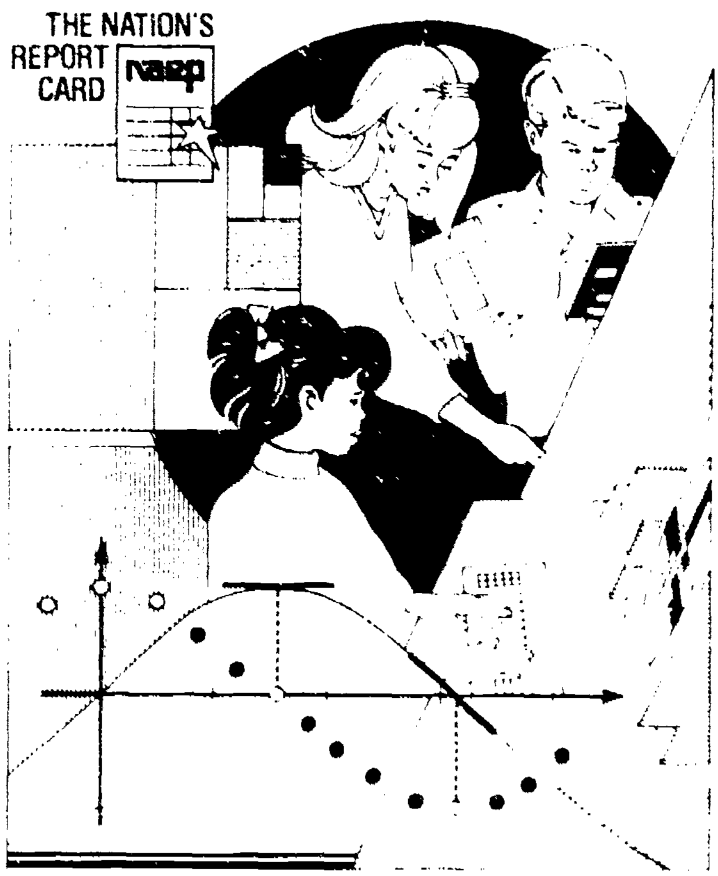
The National Assessment of Educational Progress (NAEP), also known as "The Nation's Report Card," is a congressionally mandated survey of educational achievement of American students in a variety of curriculum areas and of changes in that achievement across time. Part 1 of this report consists of the results of NAEP's 1990 national mathematics assessment of nationally representative samples of students in grades 4, 8, and 12. Part 2 consists of the results of NAEP's 1990 "Trial State Assessment Program," a voluntary eighth-grade mathematics assessment administered to representative samples of public school students at grade 8 in 40 participating states and jurisdictions. This is the first time NAEP has provided results by state. The Executive Summary (this volume) is organized as follows: (1) overall mathematics performance in the United States; (2) performance in the mathematics content areas for the nation; (3) mathematics performance by demographic subgroups; (4) home support for school: (5) mathematics performance at grade 8 in the states (based on the State Trial Assessments); (6) curricular emphases in content areas at grade 4 and 8 in the nationa and at grade 8 in the states; (7) curricular emphasis in skill areas at grades 4 and 8 in the nation and at grade 8 in the states; (8) student course taking at grade 12 in the nation; (9) summary of mathematics curriculum and course taking across the grades; (10) mathematics instruction at grades 4 and 8 in the nation and at grade 8 in the states (e.g., ability grouping, instructional materials, calculators and computers, student and teacher attitudes). (For a summary of results, see the abstract for the entire report.) (JJK/WTB)

NATIONAL CENTER FOR EDUCATION STATISTICS

The STATE of Mathematics Achievement

Executive Summary

NAEP's 1990 Assessment of the Nation and the Trial Assessment of the States



Prepared by Educational Testing Service under Contract with the National Center for Education Statistics
Office of Educational Research and Improvement • U.S. Department of Education

What is The Nation's Report Card?

THE NATION'S REPORT CARD, the National Assessment of Educational Progress (NAEP), is the only nationally representative and continuing assessment of what America's students know and can do in various subject areas. Since 1969, assessments have been conducted periodically in reading, mathematics, science, writing, history/geography, and other fields. By making objective information on student performance available to policymakers at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement is collected under this program. NAEP guarantees the privacy of individual students and their families.

NAEP is a congressionally mandated project of the National Center for Education Statistics, the U.S. Department of Education. The Commissioner of Education Statistics is responsible, by law, for carrying out the NAEP project through competitive awards to qualified organizations. NAEP reports directly to the Commissioner, who is also responsible for providing continuing reviews, including validation studies and solicitation of public comment, on NAEP's conduct and usefulness.

In 1988, Congress created the National Assessment Governing Board (NAGB) to formulate policy guidelines for NAEP. The board is responsible for selecting the subject areas to be assessed, which may include adding to those specified by Congress; identifying appropriate achievement goals for each age and grade; developing assessment objectives; developing test specifications; designing the assessment methodology; developing guidelines and standards for data analysis and for reporting and disseminating results; developing standards and procedures for interstate, regional, and national comparisons; improving the form and use of the National Assessment; and ensuring that all items selected for use in the National Assessment are free from racial, cultural, gender, or regional bias.

The National Assessment Governing Board

NATIONAL CENTER FOR EDUCATION STATISTICS

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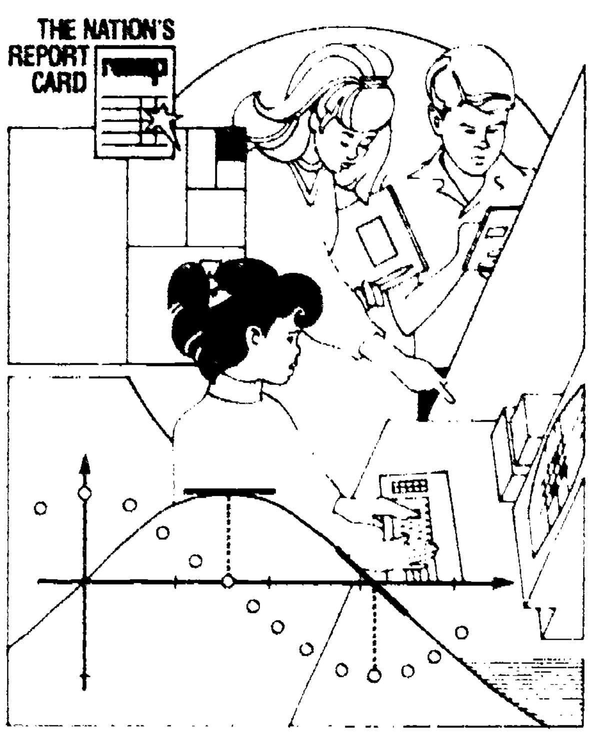

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The States participating in the NAEP Trial State Assessment of Mathematics are listed alphabetically in both the composite report of results for the Nation and all State participants, and in the Executive Summary. Copies of individual State reports are available directly from the participating States (for ordering information, please contact the assessment division of your State Department of Education). For ordering information on the composite report or for single copies of the Executive Summary while supplies last, write:

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Executive Summary

The State of Mathematics Achievement
NAEP's 1990 Assessment of the Nation and the Trial Assessment of the States

INTRODUCTION

The mathematical skills of our nation's children are generally insufficient to cope with either on-the-job demands for problem solving or college expectations for mathematical literacy.[1] Because of the emergence of the importance of mathematics to so many areas of education, citizenship, and careers, business and industry spend billions in training, colleges and universities devote large amounts of resources to remediation, and still the United States is having difficulty maintaining its competitive edge in the global marketplace.[2]

Not only are students generally ill equipped to cope confidently with the mathematical demands of today's society, such as the graphs that permeate the media and the regulations and procedures that underlie credit cards, discounts, taxation, insurance, and benefit plans, further, relatively small numbers of students persevere in the study of higher mathematics. Approximately half the students leave the mathematics pipeline each year.[3] For example, of the nearly 10 million secondary school students who study mathematics each year, fewer than 800 eventually receive doctorates in the mathematical sciences, and this number has been declining since the 1970s.

A number of publications addressing this national problem have been issued, including the landmark effort of the mathematics teachers to set

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[1] *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*, Lynn Steen, editor (Washington, DC: National Research Council, National Academy Press, 1989).

[2] *The Business Roundtable Participation Guide: A Primer for Business on Education* (New York, NY: National Alliance of Business, 1990).

[3] *Moving Beyond Myths: Revitalizing Undergraduate Mathematics* (Washington, DC: National Research Council, National Academy Press, 1991).

standards for the mathematics curriculum and for teaching mathematics.[4] These efforts and others recommend ways of teaching and learning that rely on the application of mathematics to relevant everyday problems and situations, that foster students' thinking skills, and that push them to use their minds to solve problems in unfamiliar and new settings and discover alternative solutions. These initiatives also describe the benefits provided by calculators and computers to relieve the tedium of hand calculations, to provide a basis for more complex problem-solving situations, and to engage students in mathematics learning.

Finally, the large gaps in achievement and interest in mathematics between Asian/Pacific Islander and White students and their Black and Hispanic counterparts, and to some extent between male and female students, have been widely documented.[5] There has also been considerable research showing that the differences in mathematics achievement by minority and female students may be linked to differences in motivation.[6] Teachers' and parents' expectations, school and home climate, and content and delivery of instruction may tend to seriously impede the number of minorities and females who pursue mathematics studies with sufficient interest, motivation, and preparation. Moreover, parents may often accept and even expect that their children will perform poorly in mathematics, because the parents "could never do math either."

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[4]*Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1989).

Professional Standards for Teaching Mathematics (Reston, VA: National Council of Teachers of Mathematics, 1991).

[5]*Everybody Counts: A Report to the Nation on the Future of Mathematics Education*, Lynn Steen, editor (Washington, DC: National Research Council, National Academy Press, 1989).

[6]Floretta Dukes McKenzie, "Education Strategies for the '90s" in *The State of Black America* (New York, NY: The National Urban League, Inc., 1991).

Elizabeth Fennema, "Justice, Equity, and Mathematics Education" in *Mathematics and Gender*, Elizabeth Fennema and Gilah C. Leder, editors (New York, NY: Teacher College Press, 1990).

THE NATIONAL EDUCATION GOALS

In 1990, the President and the governors adopted six ambitious education goals to be met by the year 2000. Two explicitly mention mathematics education:

- American students will leave grades four, eight, and twelve having demonstrated competency in challenging subject matter including English, mathematics, science, history, and geography; and every school in America will ensure that all students learn to use their minds well, so they may be prepared for responsible citizenship, further learning, and productive employment in our modern economy.

- U.S. students will be first in the world in science and mathematics achievement.

The remaining four goals address improving children's readiness for school learning, increasing the high-school graduation rate, adult literacy, and freeing the schools from drugs and violence.

THE 1990 NAEP MATHEMATICS ASSESSMENT

For more than 20 years, the National Assessment of Educational Progress (NAEP) has been monitoring the educational achievement of American students and changes in that achievement across time. However, as part of the 1990 mathematics assessment of fourth, eighth, and twelfth graders, a new dimension was added to NAEP whereby states (including the District of Columbia) and territories could, on a voluntary basis, participate in the mathematics assessment of eighth graders. The assessment was designed to provide state-level data comparable to results for the nation and other participating states and territories. The Trial State Assessment Program provides information about mathematics achievement as well as programs and practices in mathematics instruction.

This summary describes the results of NAEP's assessment of fourth, eighth, and twelfth graders nationwide, as well as for the 40 participants in the 1990 Trial State Assessment Program in eighth-grade mathematics.[7]

The Trial State Assessment participants include:

Alabama	Iowa	Ohio
Arizona	Kentucky	Oklahoma
Arkansas	Louisiana	Oregon
California	Maryland	Pennsylvania
Colorado	Michigan	Rhode Island
Connecticut	Minnesota	Texas
Delaware	Montana	Virginia
District of Columbia	Nebraska	West Virginia
Florida	New Hampshire	Wisconsin
Georgia	New Jersey	Wyoming
Hawaii	New Mexico	
Idaho	New York	
Illinois	North Carolina	Guam
Indiana	North Dakota	Virgin Islands

The mathematics achievement results for the nation and the participating states and territories are supported by extensive contextual information collected from the students, their teachers, and the administrators in their schools. Together, these data provide the richest source of information ever assembled about mathematics education in our country.

Developing and implementing the 1990 Trial State Assessment Program was a considerable undertaking involving participation and teamwork from the federal government, the states, the schools, the students, mathematics educators, and measurement and assessment experts. Every effort was made to ensure the

[7]For detailed results and an explanation of procedures see the full report, Ina V.S. Mullis, John A. Dossey, Eugene H. Owen, and Gary W. Phillips, *The State of Mathematics Achievement: NAEP's 1990 Assessment of the Nation and the Trial Assessment of the States* (Washington, DC: National Center for Education Statistics, 1991).

reliability and credibility of the results, including a congressionally mandated independent evaluation.[8]

The assessment was based on a framework and questions developed through a process of successive reviews by mathematics educators, measurement specialists, state representatives, and other interested parties. The materials were given to representative samples of students across the country-- including 26,000 students in 1,300 private and public schools nationally and, in addition, to approximately 2,500 students in about 100 public schools in each of the 40 participating states and territories.

The NAEP data are designed to provide a detailed portrait that can be used in examining where the nation is in relation to its overarching goals for mathematics education and how far mathematics educators have moved toward meeting their standards. The results can also be used by each state to determine in a general sense what its students know and can do in mathematics and how this compares to the nation and other states. The data also permit an analysis of the distribution of achievement, resources, and practices among demographic subgroups in the nation and the states. This information can be used to monitor students' progress in achieving what has been recommended for reform in school mathematics, to explore issues of equity in opportunity to learn mathematics, and to examine both school and home contexts for educational support.

The components -- social, economic, instructional, and political -- that contribute to effective mathematics learning are massive in number. Yet information related to many of these factors has been collected, and the results provide extensive material for analysis by all concerned with improving mathematics education in our nation. The NAEP data do not suggest a "quick fix" for improving mathematics education; in fact, the assessment was not designed to determine causal relationships. The results do show, however, quite clearly and in some detail that mathematics education in our nation and in our states is far from the vision described in the recommendations for reform of what mathematics education could be. The findings further underscore the large differences in achievement and instructional contexts among some segments of our population, particularly Black and Hispanic students and

[8]Panel on the Evaluation of the NAEP Trial State Assessment Project, *April 1991 Interim Report on the Evaluation of the 1990 Trial State Assessment* (Palo Alto, CA: National Academy of Education, 1991).

students attending schools in our disadvantaged urban areas, as compared to Asian/Pacific Islander and White students and those attending schools in advantaged urban communities.

OVERALL MATHEMATICS PERFORMANCE IN THE NATION

TABLE 1 presents the average mathematics proficiency for nationally representative samples of fourth, eighth, and twelfth graders, as well as the percentages of students performing at or above four anchor levels on the 0 to 500 NAEP mathematics proficiency scale.

TABLE 1 Overall Mathematics Proficiency

		Grade 4	Grade 8	Grade 12
Average Proficiency		216 (0.7)	265 (1.0)	295 (1.1)
Level	**Description**	**Percentage of Students at or Above**		
200	**Simple Additive Reasoning and Problem Solving with Whole Numbers**	72 (1.1)	98 (0.4)	100 (0.0)
250	**Simple Multiplicative Reasoning and Two-Step Problem Solving**	11 (0.6)	67 (1.1)	91 (0.6)
300	**Reasoning and Problem Solving Involving Fractions, Decimals, Percents, Elementary Geometry, and Simple Algebra**	0 (0.0)	14 (1.1)	46 (1.4)
350	**Reasoning and Problem Solving Involving Geometry, Algebra, and Beginning Statistics and Probability**	0 (0.0)	0 (0.1)	5 (0.6)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the estimate for the whole population is within plus or minus two standard errors of the value for the sample. When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. Although no fourth-grade students achieved at or above Level 300, a few eighth graders (0.3 percent) did perform at or above Level 350. However, percentages less than 0.5 percent are rounded to 0 percent.

To summarize the levels on the NAEP scale, a panel of 19 distinguished mathematics educators analyzed the assessment questions to provide the anchor descriptions. Based on their collective experience with mathematics curriculum and classrooms, the panel further characterized Level 200 as material typically covered by the third grade, Level 250 as material generally covered by the fifth grade, Level 300 material as content introduced by the seventh grade, and Level 350 as content generally covered in high-school mathematics courses in preparation for the study of advanced mathematics.

Fourth Grade. Approximately 72 percent of the fourth graders demonstrated the ability to consistently solve simple addition and subtraction problems with whole numbers--material typically covered by the third grade. However, 11 percent demonstrated a grasp of multiplication and two-step problems--material often included in the fifth grade. No fourth graders attained Level 300 on the NAEP scale, which would have indicated a consistent grasp of fractions, decimals, percents, and simple algebra. This finding is understandable, considering the composition of the current curriculum in this country.

Eighth Grade. Virtually all the eighth graders (98 percent) demonstrated a grasp of the third-grade material typified by Level 200--adding and subtracting with whole numbers. Two-thirds showed that their mathematics understanding included consistent success with multiplication and division of whole numbers, or problems involving more than one step (typically fifth-grade content). Only 14 percent consistently demonstrated successful performance with problems involving fractions, decimals, percents, and simple algebra--topics generally introduced by the seventh grade. No eighth graders showed the breadth of understanding necessary to begin the study of relatively advanced mathematics (Level 350).

Twelfth Grade. All the high-school seniors demonstrated success with the third-grade material. However, 91 percent showed mastery of the fifth-grade content, indicating that not all students are graduating from high school with a grasp of how to apply the four basic arithmetic operations to solve simple problems with whole numbers. Fewer than half the high-school seniors (46 percent) demonstrated a consistent grasp of decimals, percents, fractions, and simple algebra, and only 5 percent showed an understanding of geometry and algebra that suggested preparedness for the study of relatively advanced mathematics.

These figures show that many students appear to be graduating from high school with little of the mathematics understanding required by the fastest growing occupations or for college work.[9] Approximately half the twelfth graders graduating from today's schools appear to have an understanding of mathematics that does not extend much beyond simple problem solving with whole numbers.

PERFORMANCE IN THE MATHEMATICS CONTENT AREAS FOR THE NATION

The national assessment was designed to measure mathematics proficiency in six content areas, including numbers and operations; estimation; measurement; geometry; data analysis, statistics, and probability; and algebra and functions.[10] FIGURE 1 shows that twelfth graders had approximately the same average proficiency in each of these areas, but there were some differences at grades 4 and 8.

At grade 4, students' performance was relatively lower in numbers and operations and estimation and relatively higher in measurement. At grade 8, average proficiency was slightly higher in numbers and operations and estimation than in the other content areas. These findings fit with the current school mathematics curriculum, which emphasizes arithmetic knowledge in the earlier years of schooling. In each content area, twelfth graders performed more similarly to eighth graders than eighth graders did to fourth graders, suggesting that as presently configured, the mathematics curriculum facilitates more learning in the lower grades.

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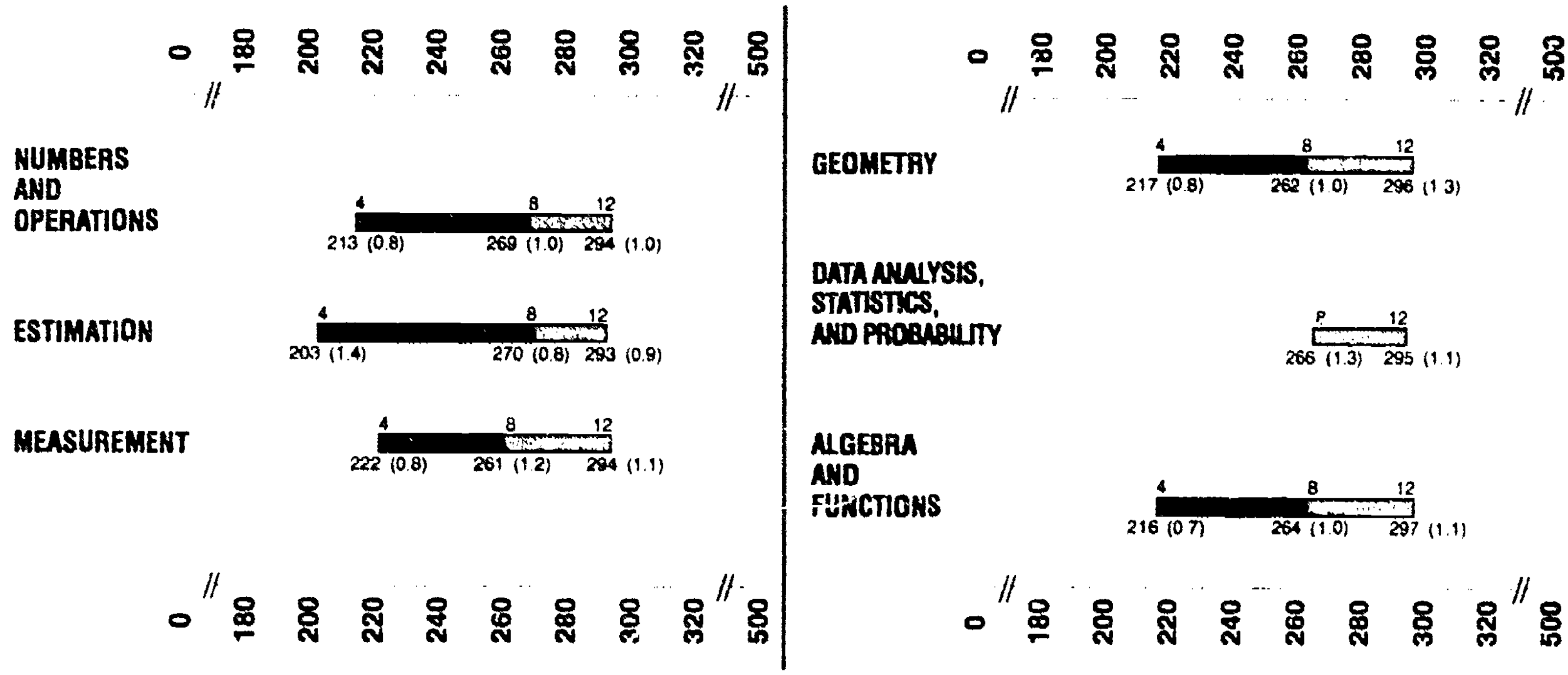
[9] *Workforce 2000: Work and Workers for the 21st Century* (Indianapolis, IN: Hudson Institute, 1987).

Moving Beyond Myths: Revitalizing Undergraduate Mathematics (Washington, DC: National Research Council, 1991).

[10] In creating the overall scale, the content scales were weighted as specified in *Mathematics Objectives, 1990 Assessment*. These were numbers and operations--45 percent at grade 4, 30 percent at grade 8, and 25 percent at grade 12; measurement--20 percent at grade 4 and 15 percent at grades 8 and 12; geometry--15 percent at grade 4 and 20 percent at grades 8 and 12; data analysis, statistics, and probability--10 percent at grade 4 and 15 percent at grades 8 and 12; and algebra and functions--10 percent at grade 4, 20 percent at grade 8, and 25 percent at grade 12. In addition, the estimation scale was constructed based on materials in a special paced-audiotape study administered only to national samples at all three grades.

FIGURE 1

Average Proficiency in Matnematics Content Areas
at Grades 4, 8, and 12



The standard errors of the estimated proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest. the value for the whole population is within plus or minus two standard errors ot the estimate for the sample.

Although questions requiring students to construct their responses were included for all content areas covered by the assessment, a special study of some open-ended questions was conducted for national samples of students. Among these items, one which was given at all three grades follows. It demonstrates the difficulty students had in applying basic mathematics skills. Thirty-seven percent of the fourth graders, 66 percent of the eighth graders, and 77 percent of the high-school seniors accurately determined the cost of the meal from the menu.

LUNCH MENU

Soups—Made by Our Chef Daily

Onion Soup .... .80
Soup of the day .... .70

Grilled Sandwiches

Beefburgers, cooked to order; 2.15
1/4 lb of the finest beef available, seasoned
to perfection, and served on a lightly buttered bun
Beefburger with Fries 2.70
Grilled Cheese 1.50
Grilled Ham and Cheese 2.50

Cold Sandwiches

Sliced Turkey 2.30
Turkey Salad 1.75
Chicken Salad 1.75
Tuna Fish Salad 1.90

Beverages

Tea .... .65
Cola .... .60
Milk .... .50

Desserts

Ice Cream (vanilla, chocolate, strawberry) 1.10
Pie (checkerboard) 1.75

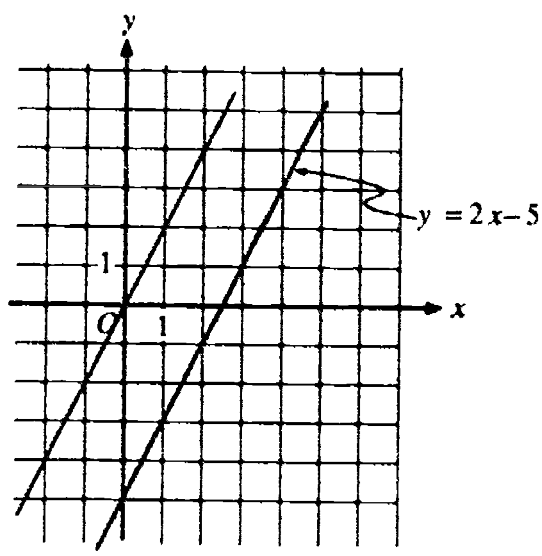
According to the menu above, what is the cost of the following order?

Soup of the day.....................
Beefburger with Fries................
Cola.............................

Total: $4.00

15

When the mathematics became at all complicated, performance fell off dramatically, even for twelfth graders. For example, high-school seniors had considerable difficulty with the following set of questions.



a. On the axes above, <u>draw</u> a line parallel to $y = 2x - 5$ that goes through the origin O.

b. On the line below, <u>write</u> an equation of the new line.

Equation: y = 2X

Only 32 percent of the high-school seniors drew the new parallel line on the graph, when a correct response essentially required the ability to find the origin 0 on the graph, the ability to find the existing line on the graph, and an understanding of the term "parallel." Sixteen percent of the twelfth graders answered both parts of this question correctly. Three peicent of the students provided the equation of the new line but did not draw it correctly.

MATHEMATICS PERFORMANCE BY DEMOGRAPHIC SUBGROUPS

Much of the concern about low mathematics achievement is centered on the particularly low achievement of Black and Hispanic students, and many recommendations for reform address this situation. The NAEP results by demographic subgroup enumerated below substantiate the concern that the goal of mathematics for all students is not being met, particularly for Black and Hispanic students and for students attending schools in our inner cities.

- In general, Asian/Pacific Islander and White students demonstrated the highest average mathematics achievement overall and in each of the separate mathematics content areas. Between the two groups, Asian/Pacific Islander students tended to outperform White students. Hispanic and Black students showed much lower average proficiency overall and in the content areas, but Hispanic students tended to perform better than the Black students did. The achievement gaps between Asian/Pacific Islander and Black students were large. For example, 70 percent of the Asian/Pacific Islander twelfth graders demonstrated a grasp of fractions, decimals, percents, and simple algebra (Level 300), compared to only 16 percent of the Black twelfth graders.

- At grades 4 and 8, there seemed to be few gender differences, except males had higher average proficiency in measurement and estimation. However, at grade 12, males showed an advantage in every content area except algebra and functions. The gender differences in overall performance were most noticeable at the higher anchor levels on the scale.

- Consistently, those students attending schools in advantaged urban communities had the highest average proficiency and those in disadvantaged urban schools the lowest average proficiency. Those students in extreme rural schools or schools in other community types performed somewhere in between the two urban groups.

- Students in the Southeast had the lowest average achievement overall and in each of the content areas. At grade 12, for example, 16 to 23 percent fewer students in the Southeast attained Level 300 than did students in the other regions of the country.

- At grade 12, students in academic school programs and with plans to attend a four-year college after high school had substantially higher average mathematics achievement than students in general or vocational/technical prog ms or those planning to enter the work force upon high-school graduation. Yet the average proficiency of students in academic programs was barely above Level 300 (material typically introduced by the seventh grade).

As shown in TABLE 2, when average proficiency by school was calculated, 10 percent of the high school seniors in the top one-third of the schools demonstrated breadth of mathematical understanding (Level 350). Even in the higher-performing schools, relatively few twelfth graders appear to be prepared for the study of relatively advanced mathematics.

TABLE 2 Average Proficiency and Percentage of Students at or Above Four Anchor Levels on the NAEP Mathematics Scale for the Top One-Third of the Schools and the Bottom One-Third of the Schools

	Percent of Students	Average Proficiency	Percentage of Students at or Above			
			Level 200	Level 250	Level 300	Level 350
Grade 4						
Top One-Third Schools	34 (2.6)	232 (0.8)	90 (1.1)	22 (1.4)	0 (0.0)	0 (0.0)
Bottom One-Third Schools	29 (2.4)	198 (1.2)	46 (2.0)	3 (0.6)	0 (0.0)	0 (0.0)
Grade 8						
Top One-Third Schools	29 (3.8)	284 (1.3)	100 (0.2)	88 (1.3)	29 (2.3)	1 (0.3)
Bottom One-Third Schools	33 (2.8)	246 (1.3)	94 (1.3)	44 (1.8)	4 (0.5)	0 (0.0)
Grade 12						
Top One-Third Schools	35 (3.9)	312 (1.0)	100 (0.0)	97 (0.6)	66 (1.5)	10 (1.1)
Bottom One-Third Schools	25 (2.9)	273 (1.1)	100 (0.2)	77 (1.7)	18 (1.2)	1 (0.3)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable.

In the lower-performing schools, fewer than half the fourth graders demonstrated a systematic grasp of addition with whole numbers (Level 200), and less than half of the eighth grader showed consistent success in two-step problem solving with whole numbers (Level 250). Only 18 percent of the graduating seniors demonstrated understanding of fractions, decimals, percents, and simple algebra (Level 300). Two-thirds of the Black students and nearly half the Hispanic students at all three grades attended lower-performing schools, as did about half to two-thirds of the students attending schools in disadvantaged urban communities.

HOME SUPPORT FOR SCHOOL

Parents are children's first teachers and should remain instrumental to their children's educational success.[11] Whether their children are in public or private schools, parents can support learning in many ways, including monitoring homework, turning off the television in favor of reading or other literacy-related activities, and making sure that students are attending school. The NAEP data, however, suggest that sizable proportions of students are in home situations that are less than ideal for fostering school learning.

- Students in homes with resource materials such as newspapers, magazines, and books had higher average mathematics proficiency, as did students who read more pages each day for school and homework. Those students with access to fewer resource materials and who did less daily reading for school had lower average proficiency.

- Similarly, students who did homework on a daily basis tended to have higher proficiency than those who did not do homework, particularly at grades 8 and 12.

- The impact of parents' level of education was once again reinforced by NAEP mathematics results. Students with well-educated parents had significantly higher achievement than did students with less well-educated parents.

- Fourth and eighth graders attending Catholic schools and other private schools had higher proficiency than did students attending public schools, but at grade 12, the difference was greatly reduced.

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[11]Carnegie Council on Adolescent Development, *Turning Points: Preparing American Youth for the 21st Century* (New York, NY: Carnegie Corporation of New York, 1989).

James P. Comer, "Home, School, and Academic Learning" in *Access to Knowledge: An Agenda For Our Nation's Schools*, John T. Goodlad and Pamela Keating, editors (New York, NY: College Entrance Examination Board, 1990).

The Harvard Education Letter, "Parents and Schools" (Cambridge, MA: Harvard University Press, November/December 1988).

- Students with both parents in the home had higher mathematics achievement, but only about three-fourths of the students at all three grades reported having both parents in the home.

- Eighth and twelfth graders who attended school regularly also performed better on the mathematics assessment. Yet 22 percent of the eighth graders and one-third of the twelfth graders had missed three or more days of school in the month preceding the assessment.

- Finally, there was a negative association between mathematics proficiency and amount of television watched each day. At all three grades, students who reported watching six hours or more of television per day had substantially lower average mathematics proficiency than their classmates who watched less television. One-fourth of the students at grade 4 reported watching six or more hours of television each day.

MATHEMATICS PERFORMANCE AT GRADE 8 IN THE STATES

FIGURE 2 provides a method for making appropriate comparisons in average overall mathematics proficiency across the states (including the District Columbia) and territories participating in NAEP's 1990 Trial State Assessment Program. The states are listed by overall average mathematics proficiency. However, the information presented in FIGURE 2, which uses appropriate tests of statistical significance to determine when average proficiency between states differs, shows that it would be quite misleading to assign numerical rankings (1 to 40) to these results. As can be seen, the pattern for most states is one of having lower average proficiency than some states, the same average proficiency as some states, and higher average proficiency than some states. To find out how any one state performed in comparison to the other states, find the state's name in the left column in FIGURE 2 and then read across the figure.

Essentially, North Dakota, Montana, Iowa, Nebraska, Minnesota, and Wisconsin had similar overall average mathematics proficiency for public-school eighth graders, although Montana had higher average proficiency than did Minnesota and Wisconsin. Because the overall average proficiency from state to state tended to be very similar, and the degree of the measurement error was slightly different from state to state, this type of overlapping

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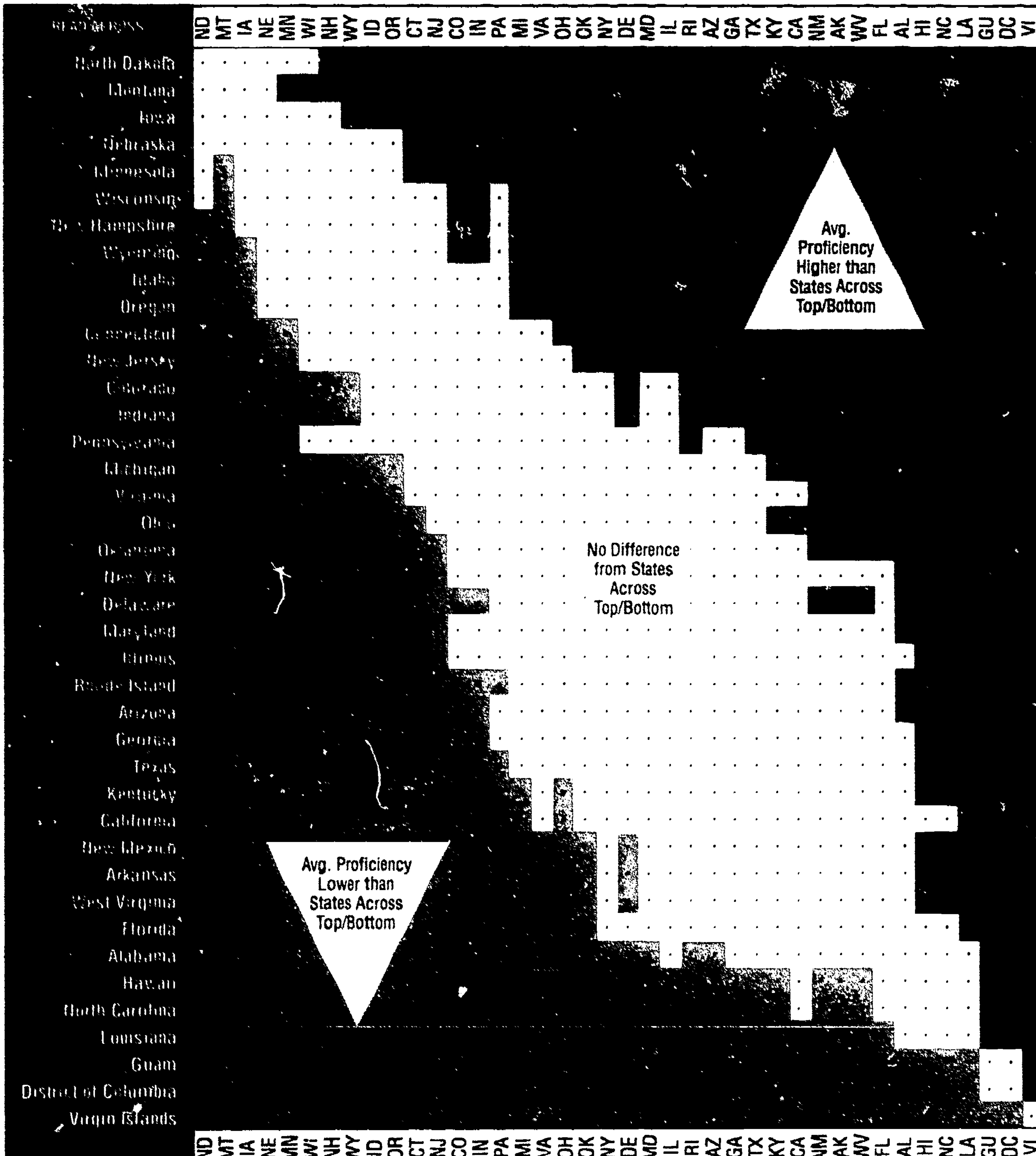
FIGURE 2

Comparisons of Overall Mathematics Proficiency
Based on Appropriate Tests of Statistical Significance



Note: Reading across, from left to right, this chart shows whether the average proficiency of each state or territory is lower than, the same as, or higher than that of other participants.

*Significance determined by an application of the Bonferroni procedure based on 780 comparisons by comparing the difference between the two means with four times the square root of the sum of the squared standard errors.

For any given state:

- Overall average proficiency statistically significantly higher than comparison state.
- No statistically significant difference from comparison state.
- Overall average proficiency statistically significantly lower than comparison state.

prevailed across the assessment results. For example, performance in New Hampshire, Wyoming, Idaho, and Oregon, did not differ from that in Nebraska, Minnesota, and Wisconsin.

However, there was considerable difference between overall average mathematics proficiency in the higher-performing states and overall average mathematics proficiency in the lower-performing states. An examination of contextual background data from the NAEP assessment and other sources suggests that the higher-performing states tended to have had fewer students in large-city schools, fewer students in free-lunch programs, smaller percentages of Black and Hispanic students, smaller percentages of students watching six hours or more of television each day, and larger percentages of students with both parents in the home. Higher-performing states also tended to be less densely populated in general. The lower-performing states tended to be in the Southeast. The District of Columbia and the two participating territories (Guam and the Virgin Islands) were also among the lower-performing participants. The Virgin Islands participated in the 1990 Trial State Assessment Program despite losing five weeks of school prior to the mathematics assessment as a result of Hurricane Hugo.

TABLE 3, which presents state-level results in alphabetical order, provides the overall average proficiency for each state and territory and the percentage of students performing at or above each anchor level on the NAEP scale. TABLE 3 also provides national and regional results for a subset of the grade 8 national data that provides a better basis for making state-to-nation comparisons.[12] Thus, these national and regional results differ from those presented previously. When considering results for the nation and its regions, it is best to use the data already presented. When comparing state results to the nation or a region, it is best to use the accompanying results in the tables.

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[12]Whereas the results for eighth graders presented for the 1990 national assessment are based on the full NAEP samples, including eighth graders in both public and private schools who were assessed during January to mid-May, those used for national comparisons in the Trial State Assessment Program involve only eighth graders attending public schools who were assessed during a shorter January to mid-March time period (also a nationally representative sample). The 1990 Trial State Assessment Program was conducted during the month of February, and only public school students were assessed.

TABLE 3 | **Overall Average Mathematics Proficiency and Anchor Level Results**

GRADE 8 PUBLIC SCHOOLS	Average Proficiency	Percentage of Students at or Above Four Anchor Levels on the NAEP Mathematics Scale			
		Level 200	Level 250	Level 300	Level 350
NATION	261 (1.4)	97 (0.7)	64 (1.6)	12 (1.2)	0 (0.2)
Northeast	269 (3.4)	99 (0.6)	72 (4.8)	16 (2.7)	0 (0.5)
Southeast	253 (2.7)	94 (2.2)	52 (3.2)	8 (1.8)	0 (0.0)
Central	265 (2.6)	98 (0.9)	70 (3.2)	12 (2.5)	0 (0.2)
West	261 (2.6)	97 (1.0)	63 (2.8)	12 (2.4)	0 (0.4)
STATES					
Alabama	**252 (1.2)**	**96 (0.7)**	**52 (1.7)**	**7 (0.7)**	**0 (0.1)**
Arizona	**259 (1.2)**	**98 (0.3)**	**61 (1.9)**	**10 (1.0)**	**0 (0.1)**
Arkansas	**256 (0.9)**	**97 (0.5)**	**57 (1.6)**	**7 (0.7)**	**0 (0.0)**
California	**256 (1.3)**	**95 (0.9)**	**56 (1.6)**	**11 (1.0)**	**0 (0.1)**
Colorado	267 (1.0)	99 (0.3)	72 (1.5)	14 (0.9)	0 (0.0)
Connecticut	270 (1.1)	98 (0.4)	72 (1.4)	19 (1.0)	0 (0.1)
Delaware	261 (0.7)	97 (0.5)	60 (1.2)	13 (0.9)	0 (0.2)
District of Columbia	231 (0.7)	86 (0.8)	23 (1.0)	2 (0.5)	0 (0.1)
Florida	255 (1.2)	96 (0.7)	54 (1.7)	10 (1.0)	0 (0.0)
Georgia	**258 (1.3)**	**96 (0.5)**	**59 (1.6)**	**12 (1.1)**	**0 (0.1)**
Hawaii	**251 (0.6)**	**93 (0.6)**	**49 (1.0)**	**10 (0.6)**	**0 (0.2)**
Idaho	**272 (0.7)**	**100 (0.2)**	**79 (1.0)**	**15 (0.9)**	**0 (0.1)**
Illinois	**260 (1.7)**	**96 (0.8)**	**64 (2.1)**	**12 (1.1)**	**0 (0.1)**
Indiana	**267 (1.1)**	**99 (0.4)**	**71 (1.5)**	**14 (1.2)**	**0 (0.1)**
Iowa	278 (1.0)	100 (0.1)	84 (1.3)	21 (1.4)	0 (0.2)
Kentucky	256 (1.1)	98 (0.5)	57 (1.7)	8 (0.8)	0 (0.0)
Louisiana	246 (1.2)	94 (0.8)	43 (1.8)	4 (0.6)	0 (0.0)
Maryland	260 (1.4)	96 (0.5)	61 (1.8)	14 (1.2)	0 (0.1)
Michigan	264 (1.1)	98 (0.3)	67 (1.5)	13 (1.0)	0 (0.1)
Minnesota	**276 (0.9)**	**99 (0.3)**	**82 (1.0)**	**20 (1.1)**	**0 (0.1)**
Montana	**280 (0.8)**	**100 (0.1)**	**88 (0.9)**	**23 (1.4)**	**0 (0.1)**
Nebraska	**276 (0.9)**	**99 (0.3)**	**81 (1.2)**	**21 (1.2)**	**0 (0.2)**
New Hampshire	**273 (0.8)**	**100 (0.2)**	**79 (1.2)**	**17 (1.1)**	**0 (0.2)**
New Jersey	**269 (1.0)**	**99 (0.4)**	**72 (1.5)**	**19 (1.3)**	**0 (0.2)**
New Mexico	256 (0.8)	98 (0.5)	56 (1.3)	8 (0.8)	0 (0.0)
New York	261 (1.3)	96 (0.6)	62 (1.9)	13 (1.0)	0 (0.1)
North Carolina	250 (1.0)	94 (0.6)	49 (1.4)	7 (0.7)	0 (0.0)
North Dakota	281 (1.2)	100 (0.2)	88 (1.4)	24 (1.7)	0 (0.4)
Ohio	264 (1.0)	98 (0.3)	67 (1.3)	12 (0.9)	0 (0.0)
Oklahoma	**263 (1.2)**	**99 (0.4)**	**67 (1.7)**	**10 (1.0)**	**0 (0.0)**
Oregon	**271 (1.0)**	**99 (0.2)**	**76 (1.4)**	**18 (1.0)**	**0 (0.1)**
Pennsylvania	**266 (1.6)**	**98 (0.4)**	**69 (2.1)**	**15 (1.2)**	**0 (0.1)**
Rhode Island	**260 (0.5)**	**96 (0.5)**	**61 (0.8)**	**12 (0.8)**	**0 (0.1)**
Texas	**258 (1.3)**	**97 (0.8)**	**56 (1.8)**	**10 (0.9)**	**0 (0.1)**
Virginia	264 (1.5)	98 (0.4)	64 (1.6)	15 (1.6)	1 (0.4)
West Virginia	256 (0.9)	98 (0.4)	56 (1.4)	7 (0.8)	0 (0.0)
Wisconsin	274 (1.3)	99 (0.3)	80 (1.4)	20 (1.4)	0 (0.2)
Wyoming	272 (0.6)	100 (0.1)	80 (1.0)	15 (0.7)	0 (0.1)
TERRITORIES					
Guam	**231 (0.6)**	**81 (1.0)**	**28 (0.8)**	**3 (0.4)**	**0 (0.1)**
Virgin Islands	**218 (0.5)**	**78 (1.5)**	**11 (0.8)**	**0 (0.2)**	**0 (0.0)**

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages less than 0.5 percent were rounded to 0 percent.

More than 90 percent of the students across the states and sometimes all (or nearly all) reached Level 200, except in the District of Columbia and the two territories. Thus, most eighth graders attending public schools in the participating states demonstrated a grasp of additive reasoning with whole numbers typical of materials generally covered by the third grade. Conversely, very few eighth-grade students attending public schools, if any, reached Level 350 across all the states and territories participating in the Trial State Assessment Program.

The large variability in performance within each state or territory and the differences in achievement across participating entities are illuminated by the differing percentages of students who performed at or above Levels 250 and 300. For example, in the District of Columbia, Guam, and the Virgin Islands, the percentages of public-school eighth graders attaining Level 250 or above ranged from 11 to 28 percent and for the participating states, the percentage of students attaining Level 250 ranged from 43 percent in Louisiana to 88 percent in North Dakota and Montana. Thus, while most of the students in some states demonstrated a grasp of mathematics that included multiplicative reasoning and two-step problem-solving with whole numbers, in other states and territories, far fewer eighth graders had reached this level of understanding. Similarly, the percentages of eighth graders attending public schools performing at or above Level 300 ranged from 0 to 24 percent, showing that in some states and territories, very few eighth graders demonstrated a grasp of decimals, fractions, percents, and simple algebra. However, in North Dakota (24 percent) and Montana (23 percent) almost one-fourth of the eighth graders demonstrated this understanding

TABLE 4 summarizes the average proficiency in each of the five mathematics content areas for each of the states. North Dakota, Iowa, and Montana were the higher performing states in numbers and operations, although Nebraska, Minnesota, and Wisconsin did not have lower average proficiency than Iowa or Montana. North Dakota, Montana, Iowa, Nebraska, Wisconsin, New Hampshire, and Minnesota all had similar average proficiency in measurement, although Montana had higher average proficiency than did Minnesota. Also, a number of other states had average proficiency that did not differ from average proficiency in Iowa, Nebraska, Wisconsin, New Hampshire, and Minnesota. Although this type of overlapping prevailed across the average proficiency results for the content areas, Montana, North Dakota, and Iowa had higher average proficiency in geometry than many other participating states and territories. North Dakota, Montana, Iowa, and Minnesota generally had higher average proficiency in data analysis, statistics, and probability. In algebra and functions, Montana, North Dakota, Iowa, Minnesota, and Nebraska were the higher-performing states.

TABLE 4 | **Average Proficiency in Mathematics Content Areas**

GRADE 8 PUBLIC SCHOOLS	Numbers and Operations	Measurement	Geometry	Data Analysis, Statistics, and Probability	Algebra and Functions
NATION	266 (1.4)	258 (1.7)	259 (1.4)	262 (1.8)	260 (1.3)
Northeast	271 (3.1)	266 (4.7)	268 (3.6)	273 (3.6)	267 (3.4)
Southeast	259 (2.9)	246 (3.8)	249 (2.6)	250 (3.3)	254 (2.7)
Central	270 (2.7)	263 (3.4)	262 (3.1)	265 (3.2)	263 (2.1)
West	264 (2.6)	258 (3.0)	260 (2.6)	262 (3.6)	259 (2.4)
STATES					
Alabama	259 (1.2)	247 (1.4)	248 (1.2)	251 (1.6)	251 (1.4)
Arizona	264 (1.2)	257 (1.4)	258 (1.1)	258 (1.4)	258 (1.3)
Arkansas	262 (0.8)	253 (1.2)	253 (1.0)	254 (1.2)	253 (1.1)
California	259 (1.2)	252 (1.5)	255 (1.3)	254 (1.7)	256 (1.3)
Colorado	269 (1.0)	265 (1.3)	266 (1.1)	269 (1.1)	268 (1.1)
Connecticut	273 (1.0)	269 (1.5)	268 (1.1)	272 (1.4)	268 (1.2)
Delaware	265 (0.8)	258 (1.0)	256 (0.7)	261 (1.0)	260 (1.0)
District of Columbia	238 (0.8)	221 (1.0)	229 (0.9)	222 (1.1)	235 (1.1)
Florida	260 (1.2)	251 (1.4)	251 (1.3)	255 (1.5)	255 (1.3)
Georgia	263 (1.2)	252 (1.5)	256 (1.3)	260 (1.5)	257 (1.5)
Hawaii	258 (0.9)	249 (0.8)	252 (0.7)	242 (1.0)	249 (0.8)
Idaho	274 (0.8)	270 (1.0)	269 (0.8)	274 (0.9)	269 (0.9)
Illinois	265 (1.7)	258 (2.0)	256 (1.7)	262 (2.0)	260 (1.7)
Indiana	271 (1.2)	263 (1.3)	264 (1.1)	269 (1.4)	265 (1.2)
Iowa	283 (1.0)	277 (1.5)	275 (1.3)	281 (1.2)	274 (1.1)
Kentucky	261 (1.2)	253 (1.5)	253 (1.2)	257 (1.3)	256 (1.1)
Louisiana	253 (1.1)	241 (1.5)	242 (1.3)	243 (1.6)	245 (1.3)
Maryland	264 (1.4)	256 (1.7)	256 (1.4)	260 (1.5)	263 (1.6)
Michigan	268 (1.2)	260 (1.3)	262 (1.0)	264 (1.4)	264 (1.2)
Minnesota	279 (1.0)	272 (1.1)	273 (1.1)	279 (0.9)	274 (0.9)
Montana	282 (1.0)	279 (1.4)	280 (0.8)	282 (0.8)	278 (0.9)
Nebraska	279 (1.0)	274 (1.4)	273 (1.1)	279 (1.0)	273 (1.0)
New Hampshire	275 (1.0)	272 (1.3)	272 (1.0)	276 (0.9)	271 (1.0)
New Jersey	274 (1.1)	267 (1.4)	266 (1.1)	270 (1.3)	268 (1.1)
New Mexico	258 (0.8)	253 (0.8)	257 (0.9)	253 (1.1)	256 (1.0)
New York	263 (1.3)	255 (1.6)	259 (1.4)	263 (1.7)	260 (1.2)
North Carolina	255 (1.0)	241 (1.1)	249 (1.0)	247 (1.3)	251 (1.0)
North Dakota	286 (1.1)	280 (1.9)	278 (1.3)	286 (1.5)	275 (1.1)
Ohio	268 (1.0)	259 (1.2)	260 (1.1)	266 (1.2)	262 (1.0)
Oklahoma	268 (1.2)	258 (1.5)	256 (1.4)	264 (1.8)	262 (1.2)
Oregon	273 (1.0)	269 (1.3)	270 (0.9)	274 (1.3)	270 (1.1)
Pennsylvania	270 (1.5)	265 (2.0)	263 (1.7)	268 (1.9)	265 (1.6)
Rhode Island	264 (0.6)	256 (0.5)	258 (0.6)	258 (0.6)	261 (0.6)
Texas	262 (1.2)	253 (1.4)	258 (1.4)	256 (1.7)	256 (1.5)
Virginia	268 (1.4)	259 (1.8)	261 (1.5)	264 (1.8)	265 (1.6)
West Virginia	260 (0.9)	252 (1.3)	254 (0.9)	256 (1.2)	254 (1.0)
Wisconsin	278 (1.2)	273 (1.7)	272 (1.3)	277 (1.4)	271 (1.3)
Wyoming	275 (0.7)	270 (0.9)	270 (0.6)	274 (0.7)	270 (0.7)
TERRITORIES					
Guam	239 (0.7)	227 (0.9)	236 (0.8)	213 (0.8)	230 (0.7)
Virgin Islands	227 (0.6)	214 (1.3)	222 (0.8)	196 (1.2)	218 (0.8)

The standard errors of the estimated proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample.

In general, for both overall mathematics proficiency and for average proficiency in the content areas, the performance by demographic subgroups within each state reflected the achievement gaps described for the nation. However, there was tremendous variation from state to state in composition of the population of public school eighth graders by racial/ethnic subgroup, type of community, level of parents' education, the amount of reading resource materials in the homes, absenteeism, and even television-viewing habits.

CURRICULAR EMPHASES IN CONTENT AREAS AT GRADES 4 AND 8 IN THE NATION

To collect information about students' curriculum in the content areas covered by the 1990 NAEP assessment at grades 4 and 8, students' teachers were asked to estimate the degree of instructional emphasis they placed on each of the various content areas for which mathematics educators recommend a broad and balanced approach.[13] These results are summarized in TABLE 5. In addition, eighth and twelfth graders were asked about their course taking in mathematics.

[13] *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1989).

TABLE 5 Summary of Teachers' Reports on the Instructional Emphasis Placed on Each of Five Content Areas

	Heavy Emphasis		Moderate Emphasis		Little or No Emphasis	
Numbers and Operations						
Grade 4	42 (2.1)	213 (1.4)	33 (2.2)	216 (1.3)	25 (2.1)	213 (1.7)
Grade 8	51 (2.7)	264 (1.3)	35 (2.4)	270 (1.8)	14 (1.3)	292 (2.7)
Measurement						
Grade 4	18 (2.0)	221 (2.3)	67 (2.3)	244 (1.0)	15 (2.0)	222 (2.4)
Grade 8	17 (1.7)	264 (1.3)	50 (2.6)	260 (1.6)	33 (2.5)	274 (2.9)
Geometry						
Grade 4	8 (1.1)	214 (3.3)	53 (2.6)	219 (1.1)	39 (2.6)	218 (1.5)
Grade 8	27 (2.2)	263 (2.0)	49 (2.5)	262 (1.6)	23 (2.4)	265 (3.6)
Data Analysis, Statistics, and Probability						
Grade 4*	11 (1.3)	--	29 (2.5)	--	59 (2.4)	--
Grade 8	14 (1.7)	271 (3.0)	30 (2.2)	269 (3.0)	56 (2.8)	266 (1.8)
Algebra and Functions						
Grade 4*	1 (0.4)	212 (6.2)	15 (1.7)	215 (2.0)	84 (1.7)	218 (0.8)
Grade 8	50 (2.1)	278 (1.7)	33 (2.1)	256 (2.1)	17 (1.8)	246 (2.5)

*At grade 4, for data analysis, statistics, and probability and for algebra and functions, the question was phrased to cover only introductory concepts. (--) Because of too few questions in that area, no proficiency scale was developed at grade 4 for data analysis, statistics, and probability. The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. Population percentages may not total 100 percent due to rounding

According to their teachers, the greatest percentage of fourth graders (42 percent) were receiving heavy instructional emphasis in numbers and operations. Less than one-fifth were receiving heavy emphasis in any other mathematics content area. However, two-thirds were receiving moderate instructional emphasis in the area of measurement and the majority were receiving moderate emphasis in geometry.

Eighty-four percent of the fourth-grade students were receiving little or no instructional emphasis in introductory concepts pertinent to algebra and functions. The average proficiency of the fourth graders did not tend to differ by the instructional emphases provided in the content areas, except in measurement, where those students whose teachers provided a moderate degree of emphasis had higher proficiency than either the students whose teachers provided heavy emphasis or little or no emphasis.

At grade 8, the teachers reported a less balanced approach in their instructional emphases across the mathematics curriculum. Half the students were receiving heavy emphasis in numbers and operations and half were receiving heavy emphasis in algebra. About one-fourth were receiving heavy instructional emphasis in geometry and relatively few were receiving heavy instructional emphasis in the other two content areas.

This pattern reflects tracking eighth graders into at least three different courses--eighth-grade mathematics, pre-algebra, and algebra. The percentages of students in these courses and their proficiency are presented in TABLE 6.

TABLE 6 Average Proficiency in Algebra and Functions by Algebra Course Taking: Grade 8

	Eighth-Grade Mathematics		Pre-Algebra		Algebra	
	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Nation	58 (1.5)	255 (1.0)	22 (1.3)	274 (1.5)	16 (1.0)	297 (2.2)
White	55 (1.8)	261 (1.2)	23 (1.7)	279 (1.3)	18 (1.3)	301 (2.2)
Black	68 (2.8)	235 (1.8)	21 (2.1)	255 (3.2)	7 (1.3)	265 (6.1)
Hispanic	69 (2.3)	243 (1.7)	17 (1.8)	263 (2.3)	10 (1.3)	274 (4.6)
Asian/Pacific Islander	35 (6.8)	263 (4.6)	24 (5.3)	281 (7.8)	38 (5.3)	310 (4.9)
Advantaged Urban !	48 (5.2)	271 (4.6)	25 (4.4)	287 (2.4)	26 (2.5)	306 (7.0)
Disadvantaged Urban	64 (3.5)	243 (2.6)	17 (2.4)	260 (5.7)	15 (2.0)	285 (3.7)
Extreme Rural	62 (7.6)	253 (3.1)	21 (4.2)	270 (4.7)	13 (4.9)	291 (3.4)
Other	59 (1.7)	254 (1.4)	23 (1.6)	274 (2.4)	15 (1.1)	297 (2.4)
Public Schools	60 (1.6)	253 (1.1)	21 (1.4)	274 (1.4)	15 (1.0)	298 (2.4)
Private Schools	47 (4.2)	270 (2.0)	27 (2.8)	277 (2.5)	23 (2.8)	294 (3.8)
Male	59 (1.5)	255 (1.1)	21 (1.3)	276 (1.6)	16 (1.1)	300 (2.4)
Female	58 (1.8)	254 (1.2)	23 (1.5)	273 (1.6)	16 (1.2)	294 (2.4)
Northeast	59 (3.7)	259 (2.1)	18 (2.7)	279 (3.2)	18 (2.1)	299 (4.0)
Southeast	57 (3.3)	244 (2.5)	29 (3.8)	271 (2.3)	12 (1.9)	294 (3.5)
Central	59 (2.5)	260 (1.7)	22 (2.1)	276 (1.7)	15 (1.8)	296 (3.6)
West	58 (2.6)	255 (2.3)	19 (1.9)	273 (2.2)	19 (2.3)	299 (4.4)
Did Not Finish High School	74 (2.7)	241 (1.7)	18 (2.5)	267 (3.8)	3 (0.7)	269 (8.3)
Graduated High School	66 (2.2)	250 (1.3)	21 (2.2)	267 (2.0)	9 (1.0)	281 (3.2)
Some Education After High School	58 (2.0)	261 (1.2)	24 (1.8)	277 (1.6)	15 (1.3)	298 (2.0)
Graduated College	48 (1.9)	263 (1.5)	24 (1.6)	280 (1.5)	25 (1.5)	303 (2.4)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. Percentages may not total 100 percent because a few students reported taking other mathematics courses. !Interpret with caution--the nature of the sample does not allow accurate determination of the variability of the results for this population subgroup.

More than half of the students reported taking eighth-grade mathematics, 22 percent pre-algebra, and 16 percent algebra. However, 38 percent of the Asian/Pacific Islander eighth graders and approximately one-fourth of the students attending schools in advantaged urban areas, those in private schools, and those with at least one parent who had graduated from college were taking algebra. In contrast, two-thirds of the Black eighth graders were in eighth-grade mathematics classes, as were two-thirds of the Hispanic students. This was also true for students whose parents had at most a high-school education and for those eighth graders attending schools in disadvantaged urban areas.

Because students tend to be assigned to eighth-grade mathematics courses based on their previous achievement, it follows that the mathematics proficiency of students in pre-algebra and algebra courses was higher than that of students in eighth-grade mathematics classes. It may also help explain why, as was shown in TABLE 5, the students receiving heavy emphasis in numbers and operations tended to have lower average proficiency than those receiving less emphasis. The students with higher proficiency tend to have been assigned to pre-algebra and algebra courses and were receiving heavy instructional emphasis in algebra.

CURRICULAR EMPHASES IN CONTENT AREAS AT GRADE 8 IN THE STATES

TABLE 7 presents the course-taking results for eighth graders attending public schools across the states. In each participating state, higher average mathematics proficiency was associated with each successively higher level mathematics course. However, this relationship did not necessarily hold across states. For example, the District of Columbia had the largest percentage of public-school eighth graders taking algebra (32 percent), but its average mathematics proficiency was among the lowest of the participating states and territories. North Dakota, on the other hand, had one of the lowest percentages of eighth graders taking algebra (8 percent), but its overall average proficiency was one of the highest.

Teachers' reports about the percentages of students receiving heavy instructional emphasis across the states in the five mathematics content areas are presented in TABLE 8. The instructional emphasis reported by teachers in the states mirrored the course-taking results, with students receiving the most instructional emphasis in either numbers and operations or algebra and functions. In 15 states, the majority of the eighth graders attending public schools were receiving heavy emphasis in numbers and operations. Only in Colorado, Wisconsin, New Hampshire, Minnesota, Maryland, and Oregon were less than 40 percent of the students receiving heavy emphasis in numbers and operations.

With the exception of Georgia and the Virgin Islands, less than one-third of the eighth graders in any participating state were receiving heavy instructional emphasis in the area of measurement. Compared to the emphasis placed on numbers and operations, the eighth graders in public schools across the states also were receiving much less emphasis in geometry, although this tended to differ considerably from state to state. For example, in New York, 40 percent of the eighth graders attending public schools were receiving heavy emphasis in geometry, compared to 20 percent or fewer of the students in 22 of the other participating states. Few states were giving much emphasis to eighth-graders' learning in data analysis statistics, and probability. In 17 states, 10 percent or fewer of the students were given heavy instructional emphasis in this area.

In 16 states, the majority of the eighth graders attending public schools were receiving heavy emphasis in algebra and functions. In all the participating states except Hawaii, at least one-third of the students received heavy instructional emphasis in algebra and functions. However, across the states, considerable percentages of eighth graders, from 8 to 36 percent, were receiving little or no emphasis in algebra and functions.

TABLE 7 | **Students' Reports on Their Current Mathematics Course**

GRADE 8 PUBLIC SCHOOLS	Eighth-Grade Mathematics		Pre-Algebra		Algebra	
	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
NATION	62 (2.1)	251 (1.4)	19 (1.9)	272 (2.4)	15 (1.2)	296 (2.4)
Northeast	63 (5.8)	259 (2.9)	16 (3.9)	278 (6.7)!	18 (3.3)	297 (3.6)
Southeast	64 (3.7)	241 (3.4)	23 (4.4)	269 (4.6)	11 (2.2)	296 (4.8)!
Central	58 (4.8)	255 (3.1)	22 (4.3)	276 (3.1)!	15 (2.8)	289 (5.4)
West	63 (2.7)	252 (2.4)	15 (2.7)	266 (3.6)	17 (1.8)	299 (4.5)
STATES						
Alabama	66 (2.5)	243 (1.6)	20 (1.9)	266 (2.1)	11 (1.2)	287 (3.0)
Arizona	48 (1.5)	246 (1.3)	29 (1.5)	266 (1.6)	18 (1.3)	289 (2.4)
Arkansas	72 (2.2)	248 (1.0)	16 (1.9)	270 (2.3)	10 (1.1)	289 (2.4)
California	59 (1.9)	242 (1.1)	21 (1.4)	272 (2.2)	16 (1.0)	293 (2.0)
Colorado	46 (2.5)	255 (1.4)	32 (2.1)	270 (1.2)	18 (1.1)	295 (2.0)
Connecticut	50 (1.9)	251 (1.3)	30 (1.8)	280 (1.0)	17 (1.0)	308 (1.1)
Delaware	48 (1.2)	243 (0.7)	25 (1.2)	264 (1.3)	24 (0.9)	295 (1.7)
District of Columbia	57 (1.0)	217 (0.6)	10 (0.6)	241 (1.7)	32 (0.9)	253 (1.4)
Florida	63 (1.6)	242 (1.4)	19 (1.2)	271 (1.8)	14 (1.0)	298 (1.8)
Georgia	57 (2.5)	244 (1.1)	28 (1.9)	271 (1.8)	14 (1.3)	300 (2.4)
Hawaii	61 (1.0)	237 (0.6)	24 (0.9)	273 (1.3)	10 (0.6)	298 (2.2)
Idaho	47 (1.1)	264 (0.7)	32 (1.2)	271 (1.1)	18 (1.1)	301 (1.2)
Illinois	63 (2.4)	251 (1.7)	18 (2.0)	268 (3.7)	18 (1.3)	290 (2.6)
Indiana	68 (2.1)	256 (1.1)	16 (1.6)	262 (2.2)	13 (1.1)	306 (2.4)
Iowa	69 (2.8)	272 (1.1)	19 (2.7)	287 (2.1)	10 (1.0)	311 (2.4)
Kentucky	67 (2.2)	247 (1.1)	18 (1.7)	270 (1.9)	12 (1.2)	289 (2.2)
Louisiana	53 (2.9)	238 (1.5)	34 (2.6)	251 (2.3)	12 (1.1)	265 (4.2)
Maryland	38 (2.0)	237 (1.4)	32 (1.4)	261 (1.6)	27 (1.5)	291 (1.7)
Michigan	59 (2.8)	253 (1.4)	24 (2.1)	272 (1.5)	14 (1.4)	300 (2.1)
Minnesota	54 (3.0)	266 (1.3)	25 (2.4)	281 (1.1)	17 (1.4)	303 (1.6)
Montana	59 (2.4)	271 (1.0)	26 (1.9)	281 (1.1)	12 (1.5)	299 (3.8)
Nebraska	66 (2.5)	271 (1.2)	20 (2.1)	277 (1.4)	11 (1.0)	307 (2.0)
New Hampshire	55 (1.3)	262 (1.0)	28 (1.0)	280 (1.0)	14 (0.9)	308 (1.6)
New Jersey	55 (2.2)	255 (1.2)	24 (2.1)	275 (2.0)	18 (1.1)	306 (1.4)
New Mexico	62 (1.2)	247 (0.7)	23 (1.1)	265 (1.5)	11 (0.6)	288 (1.9)
New York	73 (1.8)	252 (1.4)	8 (1.2)	273 (2.7)	13 (1.1)	291 (2.7)
North Carolina	58 (1.8)	234 (1.1)	22 (1.4)	262 (1.4)	17 (1.3)	290 (1.3)
North Dakota	73 (2.0)	277 (1.4)	17 (1.9)	289 (2.4)	8 (1.0)	307 (4.4)
Ohio	63 (2.2)	254 (1.2)	20 (2.0)	270 (1.9)	16 (1.1)	300 (1.5)
Oklahoma	53 (2.7)	254 (1.5)	30 (2.7)	267 (1.8)	13 (1.1)	290 (2.8)
Oregon	43 (1.5)	254 (1.2)	30 (1.2)	278 (1.4)	20 (1.1)	305 (1.5)
Pennsylvania	49 (2.5)	246 (1.6)	24 (2.2)	275 (1.5)	25 (1.6)	298 (1.4)
Rhode Island	52 (1.1)	243 (0.7)	29 (0.8)	272 (0.9)	18 (0.8)	298 (1.7)
Texas	72 (2.0)	249 (1.4)	14 (1.5)	274 (2.8)	12 (1.0)	296 (1.6)
Virginia	46 (2.0)	244 (1.5)	35 (1.8)	271 (1.5)	16 (1.0)	305 (2.4)
West Virginia	63 (2.0)	244 (1.2)	19 (1.8)	267 (1.3)	16 (1.2)	291 (1.8)
Wisconsin	68 (2.5)	266 (1.4)	17 (1.8)	284 (2.3)	13 (1.3)	307 (1.9)
Wyoming	48 (1.0)	266 (0.9)	31 (0.9)	270 (1.1)	16 (0.8)	303 (1.2)
TERRITORIES						
Guam	77 (1.0)	225 (0.8)	12 (0.7)	255 (2.1)	7 (0.6)	260 (4.1)
Virgin Islands	88 (0.7)	216 (0.6)	3 (0.5)	*** (***)	8 (0.6)	240 (4.3)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. The percentages may not add to 100 percent because a small number of students reported taking other mathematics courses. ***Sample size insufficient to permit reliable estimate. There were fewer than 62 students. ! Interpret with caution - the nature of the sample does not allow accurate determination of the variability of this estimated statistic.

TABLE 8 | **Teachers' Reports on Placing Heavy Instructional Emphasis on Specific Content Areas**

GRADE 8 PUBLIC SCHOOLS	Numbers and Operations		Measurement		Geometry		Data Analysis, Statistics, and Probability		Algebra and Functions	
	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
NATION	49 (3.8)	260 (1.8)	17 (3.0)	250 (5.6)	28 (3.8)	260 (3.2)	14 (2.2)	269 (4.3)	46 (3.6)	275 (2.5)
Northeast	41 (6.9)	268 (2.9)	32(11.5)	257(11.7)!	46(11.9)	264 (6.1)!	12 (6.1)	*** (***)	52(11.5)	273 (6.6)!
Southeast	59 (7.3)	256 (3.1)	13 (6.8)	242 (7.6)!	22 (7.0)	253 (7.5)!	19 (5.9)	274 (5.8)!	42 (6.0)	277 (5.6)
Central	54 (7.2)	264 (4.3)	17 (5.7)	247(12.5)!	26 (7.0)	261 (7.9)!	12 (2.5)	262 (7.5)	50 (7.6)	273 (3.6)
West	42 (7.4)	257 (3.6)	11 (2.8)	251 (7.7)!	24 (6.3)	260 (2.8)!	14 (3.7)	264(10.6)!	43 (5.6)	277 (5.2)
STATES										
Alabama	58 (3.0)	254 (1.8)	24 (3.3)	244 (3.7)	26 (3.0)	251 (2.4)	11 (1.8)	242 (5.6)	41 (3.0)	266 (1.8)
Arizona	52 (3.3)	259 (1.9)	10 (1.6)	250 (4.5)	14 (1.8)	260 (3.7)	7 (1.3)	252 (3.9)	51 (2.6)	271 (2.0)
Arkansas	60 (3.3)	259 (1.2)	17 (2.7)	246 (3.4)	16 (2.5)	254 (2.7)	9 (2.3)	259 (5.1)!	33 (2.8)	273 (2.1)
California	40 (3.1)	251 (1.7)	21 (2.5)	246 (2.7)	25 (3.1)	259 (2.7)	17 (2.7)	263 (5.0)	46 (2.4)	273 (2.4)
Colorado	37 (3.0)	262 (1.7)	7 (1.2)	259 (4.5)	20 (3.1)	269 (2.4)	14 (2.0)	271 (2.8)	51 (3.5)	276 (1.7)
Connecticut	41 (3.4)	266 (1.9)	28 (3.3)	263 (3.8)	27 (2.9)	268 (2.5)	16 (3.2)	279 (3.3)	48 (2.6)	287 (1.6)
Delaware	43 (1.5)	255 (1.3)	20 (1.1)	251 (2.2)	17 (0.9)	258 (1.9)	17 (0.7)	274 (2.0)	39 (1.1)	265 (1.5)
District of Columbia	47 (0.9)	231 (1.4)	25 (0.8)	217 (1.8)	25 (0.9)	229 (1.9)	31 (0.8)	220 (1.7)	46 (1.0)	251 (1.4)
Florida	56 (2.4)	253 (1.6)	19 (2.3)	240 (2.9)	18 (2.4)	255 (2.7)	16 (2.0)	256 (3.1)	42 (2.2)	279 (2.0)
Georgia	57 (2.7)	255 (1.6)	33 (2.6)	242 (2.2)	30 (2.6)	255 (2.5)	24 (2.6)	258 (3.0)	47 (2.2)	272 (2.0)
Hawaii	46 (1.0)	246 (1.2)	15 (0.6)	239 (2.5)	17 (0.7)	264 (1.7)	9 (0.6)	250 (3.2)	29 (0.8)	263 (1.4)
Idaho	48 (1.6)	271 (1.1)	10 (1.1)	266 (2.5)	14 (0.7)	269 (2.2)	9 (0.8)	273 (3.3)	56 (1.5)	281 (0.9)
Illinois	41 (4.3)	257 (2.7)	17 (3.4)	235 (8.0)!	29 (4.0)	258 (3.6)	14 (3.0)	253 (8.3)!	55 (3.5)	272 (2.2)
Indiana	55 (2.6)	266 (1.9)	9 (1.9)	255 (4.2)!	15 (2.4)	263 (2.8)	4 (1.3)	262 (5.0)!	45 (2.9)	284 (1.9)
Iowa	48 (4.1)	278 (1.7)	14 (2.8)	272 (4.7)	25 (3.5)	282 (2.8)	4 (1.7)	293 (8.8)!	49 (4.4)	284 (2.1)
Kentucky	58 (3.8)	255 (1.5)	19 (3.0)	257 (3.4)	25 (3.4)	256 (2.5)	15 (2.7)	262 (2.9)	46 (2.9)	272 (1.8)
Louisiana	57 (4.4)	248 (1.5)	13 (2.3)	232 (5.2)	14 (2.4)	238 (4.1)	11 (2.2)	243 (7.4)	59 (2.7)	252 (1.6)
Maryland	35 (2.6)	249 (1.9)	21 (2.6)	237 (3.9)	22 (2.5)	254 (3.1)	14 (2.0)	257 (4.5)	51 (2.4)	283 (2.3)
Michigan	44 (3.7)	259 (2.3)	12 (2.2)	247 (4.6)	20 (2.9)	261 (3.0)	10 (2.1)	259 (7.4)!	47 (3.0)	277 (2.2)
Minnesota	36 (3.3)	275 (1.6)	12 (2.2)	266 (4.1)	19 (3.0)	270 (2.5)	8 (1.6)	287 (3.3)!	50 (3.2)	285 (1.5)
Montana	40 (2.8)	280 (2.0)	9 (1.0)	277 (5.7)	31 (2.5)	266 (1.5)	13 (2.3)	267 (3.0)	56 (3.0)	281 (1.5)
Nebraska	41 (3.0)	277 (1.4)	12 (2.3)	276 (3.2)	19 (2.8)	279 (1.8)	8 (1.5)	287 (3.3)	51 (3.5)	282 (1.9)
New Hampshire	36 (1.6)	269 (1.9)	15 (0.9)	261 (2.0)	27 (1.4)	272 (2.2)	16 (0.8)	269 (3.6)	47 (1.9)	284 (1.8)
New Jersey	50 (3.4)	263 (1.5)	24 (3.1)	255 (3.2)	37 (3.2)	264 (1.7)	14 (1.8)	263 (3.7)	55 (2.6)	280 (2.0)
New Mexico	54 (1.2)	254 (1.0)	16 (1.1)	245 (3.1)	25 (1.1)	256 (2.0)	14 (0.9)	255 (3.3)	53 (1.2)	267 (1.4)
New York	44 (3.7)	255 (2.2)	13 (2.3)	258 (4.9)	40 (3.0)	265 (2.7)	24 (2.8)	272 (3.9)	49 (3.0)	274 (2.0)
North Carolina	49 (2.7)	246 (1.4)	17 (2.3)	228 (3.2)	17 (2.4)	254 (2.5)	13 (2.2)	251 (4.0)	44 (2.6)	273 (1.8)
North Dakota	49 (3.2)	283 (1.9)	13 (2.6)	277 (5.0)!	23 (3.0)	280 (1.8)	9 (2.6)	286 (3.7)!	56 (3.4)	281 (1.2)
Ohio	48 (3.7)	261 (1.8)	17 (2.8)	243 (4.2)	23 (3.1)	264 (2.7)	13 (2.3)	270 (4.4)	50 (3.0)	277 (1.8)
Oklahoma	58 (3.6)	263 (1.4)	11 (2.5)	258 (3.5)!	17 (2.8)	262 (2.4)	5 (1.6)	264 (6.7)!	56 (3.4)	270 (1.6)
Oregon	34 (3.0)	267 (2.3)	13 (2.2)	265 (4.7)	19 (2.1)	271 (2.9)	17 (1.8)	287 (3.7)	43 (2.7)	289 (1.5)
Pennsylvania	47 (3.0)	260 (1.7)	15 (2.2)	252 (3.7)	17 (2.7)	259 (2.6)	6 (1.1)	266 (3.5)	48 (2.8)	283 (1.9)
Rhode Island	52 (1.0)	252 (0.7)	13 (0.5)	250 (2.6)	17 (0.7)	261 (2.1)	10 (0.5)	274 (2.6)	43 (1.0)	266 (1.1)
Texas	61 (3.5)	257 (1.7)	29 (3.7)	246 (3.0)	37 (3.0)	257 (2.4)	20 (2.5)	259 (4.4)	52 (2.8)	284 (1.9)
Virginia	46 (2.4)	256 (1.8)	12 (2.0)	245 (3.9)	18 (2.1)	266 (3.5)	10 (1.8)	270 (5.0)	52 (2.3)	282 (2.3)
West Virginia	48 (3.7)	255 (1.6)	13 (2.4)	241 (3.6)	14 (2.6)	252 (2.5)	8 (2.0)	259 (3.7)!	41 (2.6)	275 (1.7)
Wisconsin	37 (3.4)	272 (1.9)	11 (2.5)	264 (4.3)!	17 (2.7)	278 (2.9)	6 (1.8)	284 (3.7)!	48 (3.7)	284 (2.2)
Wyoming	42 (1.2)	274 (0.9)	7 (0.4)	268 (3.7)	15 (0.9)	274 (1.5)	6 (0.7)	276 (2.6)	48 (1.3)	282 (1.3)
TERRITORIES										
Guam	56 (0.8)	231 (1.0)	24 (0.7)	233 (2.0)	22 (0.9)	253 (1.6)	12 (0.8)	248 (3.4)	37 (0.8)	255 (1.1)
Virgin Islands	53 (1.1)	227 (1.1)	35 (0.7)	216 (1.6)	11 (0.2)	219 (1.6)	11 (0.4)	197 (2.8)	47 (0.8)	227 (1.0)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. ***Sample size insufficient to permit reliable estimate. There were fewer than 62 students. ! Interpret with caution - the nature of the sample does not allow accurate determination of the variability of this estimated statistic.

CURRICULAR EMPHASIS IN SKILL AREAS AT GRADES 4 AND 8 IN THE NATION AND AT GRADE 8 IN THE STATES

Because the recommendations for mathematics education reform stress altering curricular and instructional emphases to help students learn to reason, to think productively, and to communicate in mathematical situations, NAEP asked teachers of fourth and eighth graders participating in the assessment to indicate the degree of emphasis they placed on four skill areas -- learning mathematics facts and concepts, learning procedures needed to solve problems, developing reasoning ability to solve problems in unique or unfamiliar situations, and learning how to communicate ideas in mathematics effectively. The results for grades 4 and 8 for the national samples of students in public and private schools are presented in TABLE 9.

TABLE 9 Teachers' Reports of Mathematics Skills Emphasized

	Heavy Emphasis		Moderate Emphasis		Little or No Emphasis	
	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Learning Facts and Concepts						
Grade 4	91 (1.4)	217 (0.7)	8 (1.3)	218 (2.9)	0 (0.2)	--
Grade 8	57 (3.0)	266 (1.8)	6 (4.3)	265 (1.6)	7 (1.1)	274 (5.3)
Learning Skills and Procedures						
Grade 4	85 (1.4)	218 (0.8)	15 (1.4)	215 (1.9)	0 (0.1)	--
Grade 8	68 (2.8)	265 (1.6)	29 (2.7)	266 (1.7)	3 (0.8)	270 (5.3)
Developing Reasoning and Analytic Ability						
Grade 4	41 (2.3)	217 (1.2)	49 (2.3)	217 (1.1)	11 (1.6)	218 (1.9)
Grade 8	28 (4.0)	274 (2.1)	42 (2.7)	263 (1.4)	14 (1.4)	253 (2.5)
Learning How to Communicate Ideas Effectively						
Grade 4	36 (3.0)	216 (1.3)	45 (2.7)	218 (1.3)	19 (2.3)	219 (1.5)
Grade 8	37 (2.9)	269 (2.5)	45 (2.9)	266 (1.5)	17 (1.7)	261 (1.7)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. When the proportion of students is 0 percent, the standard error is inestimable. However, in the table, percentages less than 0.5 percent were rounded to 0 percent.

At both grades 4 and 8, students across the nation were receiving much more emphasis on learning facts and concepts and learning procedures than they were on learning to reason or to communicate in mathematics. Considerable percentages of fourth and eighth graders were receiving heavy emphasis on both learning facts and concepts and learning procedures. These emphases appeared to exist across classrooms, regardless of students' ability levels.

In contrast, teachers at both grades reported providing fewer than half their students with heavy instructional emphasis on developing reasoning and analytic ability. Further, they reported that more students in high-ability classes received emphasis in this area than did those in other classes. For example, 69 percent of the eighth graders in high-ability classes as compared to 28 percent in low-ability classes, received heavy emphasis on reasoning and analytic ability. According to teachers' own reports, 30 percent of the eighth graders in low-ability classes were receiving little or no emphasis in reasoning strategies.

At both grades, teachers reported that approximately one-third of their students were receiving heavy emphasis in mathematics communication, and that about one-fifth were receiving little or no emphasis. Although there seemed to be little difference in the degree of this emphasis according to class ability level at grade 4, more eighth graders in high-ability classes than in low-ability classes received emphasis in how to communicate mathematically.

The results for the eighth-grade students attending public schools across the states are shown in TABLE 10 for teachers' reporting a high degree of emphasis in each skill area. In all the participating states except the District of Columbia, teachers of eighth graders attending public schools reported that more students were receiving emphasis in facts and procedures skills than in reasoning and communication, although the pattern was less clear-cut in California, Colorado, Maryland, and Oregon. In only California, Colorado, the District of Columbia, Georgia, and Maryland were even half the eighth graders receiving heavy instructional emphasis in how to apply their reasoning to solve new problems. Across the states for public-school students at grade 8, even less emphasis tended to be placed on the ability to communicate mathematics ideas effectively than on developing the reasoning ability necessary to apply mathematics to unfamiliar and unique situations.

TABLE 10 | Teachers' Reports on Placing Heavy Instructional Emphasis on Specific Mathematics Skills and Abilities

	Learning Mathematics Facts and Concepts		Learning Skills and Procedures Needed to Solve Problems		Developing Reasoning Ability to Solve Unique Problems		Learning How to Communicate Ideas in Mathematics Effectively	
GRADE 8 PUBLIC SCHOOLS	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
NATION	55 (4.2)	264 (2.1)	67 (3.9)	264 (1.9)	45 (3.4)	269 (2.7)	37 (3.6)	264 (3.1)
Northeast	57(12.8)	268 (6.5)!	69(13.7)	269 (4.8)!	47(10.7)	272 (8.0)!	25 (8.7)	259(15.2)!
Southeast	69 (9.1)	260 (2.8)	74 (8.2)	258 (2.9)	49 (6.8)	265 (5.1)	47 (8.0)	256 (4.3)!
Central	41 (7.7)	263 (5.1)	55 (6.2)	264 (3.9)	29 (4.7)	273 (5.6)	23 (6.1)	264 (7.8)!
West	53 (5.7)	265 (4.3)	69 (5.4)	265 (3.7)	54 (6.0)	270 (4.4)	46 (5.8)	271 (5.0)
STATES								
Alabama	60 (4.2)	253 (1.6)	70 (3.2)	254 (1.5)	48 (3.7)	258 (1.8)	43 (3.8)	257 (2.0)
Arizona	58 (3.0)	258 (1.8)	60 (2.7)	259 (1.7)	43 (2.7)	267 (2.4)	38 (2.9)	263 (2.6)
Arkansas	61 (3.5)	256 (1.2)	65 (3.7)	256 (1.3)	36 (3.1)	266 (1.7)	31 (3.4)	263 (2.4)
California	64 (2.9)	257 (1.8)	61 (3.4)	256 (1.8)	50 (2.9)	268 (2.2)	41 (3.4)	264 (2.5)
Colorado	52 (3.2)	265 (1.8)	64 (3.1)	267 (1.6)	50 (3.1)	273 (1.5)	45 (2.9)	272 (1.5)
Connecticut	53 (3.3)	270 (1.8)	61 (3.2)	272 (1.5)	47 (3.1)	282 (1.6)	41 (3.1)	279 (1.7)
Delaware	60 (1.8)	263 (1.1)	61 (1.7)	265 (1.1)	47 (1.5)	273 (1.3)	37 (1.4)	275 (1.7)
District of Columbia	62 (1.1)	232 (0.9)	68 (0.9)	233 (1.0)	65 (1.1)	236 (1.0)	63 (1.0)	236 (1.0)
Florida	62 (3.1)	257 (1.6)	68 (2.8)	258 (1.8)	46 (2.6)	267 (2.0)	43 (3.1)	262 (2.0)
Georgia	65 (2.8)	255 (1.5)	72 (2.5)	258 (1.6)	50 (2.4)	265 (1.9)	52 (3.2)	261 (1.8)
Hawaii	60 (0.9)	252 (0.9)	66 (1.0)	253 (0.9)	42 (0.8)	264 (1.2)	34 (1.0)	264 (1.3)
Idaho	59 (1.4)	271 (0.9)	65 (2.3)	273 (1.0)	39 (1.5)	280 (1.1)	41 (2.1)	277 (1.2)
Illinois	61 (4.0)	261 (2.5)	69 (3.8)	261 (2.3)	48 (3.9)	271 (1.9)	35 (3.8)	265 (3.3)
Indiana	65 (3.6)	265 (1.7)	69 (3.3)	268 (1.8)	36 (3.4)	281 (2.7)	35 (3.7)	276 (2.8)
Iowa	54 (4.2)	278 (1.6)	64 (4.0)	279 (1.5)	38 (4.2)	284 (2.0)	28 (3.6)	285 (2.5)
Kentucky	72 (3.5)	256 (1.5)	69 (3.3)	256 (1.5)	44 (3.4)	265 (1.8)	44 (3.7)	262 (2.4)
Louisiana	64 (3.7)	244 (1.6)	68 (3.8)	245 (1.6)	38 (3.9)	251 (1.9)	40 (4.3)	248 (2.4)
Maryland	55 (2.8)	260 (1.9)	64 (2.7)	261 (1.8)	53 (2.8)	271 (2.3)	48 (3.2)	268 (2.2)
Michigan	56 (3.4)	265 (1.8)	64 (3.4)	266 (1.8)	43 (3.6)	271 (2.4)	35 (3.2)	270 (2.8)
Minnesota	47 (3.3)	278 (1.6)	62 (3.7)	277 (1.3)	36 (3.3)	283 (1.9)	29 (3.4)	282 (2.3)
Montana	52 (2.9)	280 (1.2)	58 (2.5)	280 (1.2)	45 (2.8)	284 (1.2)	33 (2.0)	286 (1.6)
Nebraska	57 (2.3)	276 (1.4)	62 (3.2)	279 (1.2)	39 (3.1)	283 (1.4)	31 (2.9)	282 (1.3)
New Hampshire	53 (1.9)	274 (1.1)	62 (1.8)	273 (1.3)	45 (1.6)	282 (1.3)	37 (1.7)	281 (1.4)
New Jersey	70 (3.0)	270 (1.8)	72 (2.9)	269 (1.5)	49 (3.5)	278 (2.2)	49 (3.6)	276 (2.2)
New Mexico	61 (1.1)	256 (1.1)	70 (1.0)	256 (0.9)	48 (1.5)	262 (1.2)	40 (1.4)	263 (1.4)
New York	56 (3.6)	261 (2.1)	63 (3.6)	260 (2.1)	41 (3.0)	271 (2.4)	37 (3.5)	264 (2.6)
North Carolina	59 (3.4)	250 (1.5)	65 (3.4)	251 (1.3)	46 (3.2)	262 (1.7)	44 (3.1)	258 (1.7)
North Dakota	49 (3.7)	283 (2.1)	64 (2.6)	284 (1.5)	33 (2.9)	288 (1.6)	25 (2.9)	286 (1.9)
Ohio	59 (3.4)	265 (1.9)	67 (3.5)	266 (1.4)	42 (3.6)	273 (2.3)	36 (4.0)	271 (2.3)
Oklahoma	64 (3.7)	263 (1.3)	68 (3.5)	265 (1.3)	41 (3.4)	270 (1.8)	40 (4.0)	269 (1.7)
Oregon	62 (3.2)	272 (1.7)	58 (3.3)	272 (1.6)	49 (3.2)	281 (1.7)	36 (2.6)	279 (2.2)
Pennsylvania	65 (3.4)	270 (1.9)	75 (2.2)	267 (1.8)	48 (3.8)	275 (2.5)	43 (3.5)	275 (2.8)
Rhode Island	59 (1.2)	260 (0.8)	65 (1.0)	260 (0.8)	43 (1.3)	274 (1.1)	37 (1.3)	266 (1.2)
Texas	61 (3.1)	258 (1.8)	68 (3.4)	258 (1.7)	45 (3.1)	261 (2.2)	42 (3.1)	257 (2.6)
Virginia	64 (2.9)	265 (1.7)	75 (2.6)	263 (1.5)	46 (2.5)	275 (2.4)	46 (2.9)	271 (2.4)
West Virginia	62 (3.9)	256 (1.2)	69 (3.3)	257 (1.2)	44 (3.5)	265 (1.9)	38 (3.1)	263 (2.2)
Wisconsin	53 (4.1)	273 (1.8)	63 (3.6)	274 (1.9)	38 (2.9)	283 (1.7)	24 (3.0)	282 (2.9)
Wyoming	48 (1.9)	273 (0.9)	61 (1.8)	272 (0.9)	37 (1.2)	280 (1.1)	37 (1.1)	279 (1.0)
TERRITORIES								
Guam	42 (0.8)	230 (1.0)	37 (0.8)	241 (1.2)	15 (0.7)	258 (2.6)	19 (0.4)	243 (1.8)
Virgin Islands	43 (0.9)	214 (0.0)	55 (0.9)	217 (0.7)	30 (0.7)	221 (0.8)	38 (0.6)	221 (1.0)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. ! Interpret with caution - the nature of the sample does not allow accurate determination of the variability of this estimated statistic.

STUDENT COURSE TAKING AT GRADE 12 IN THE NATION

The twelfth graders were asked about their course taking in algebra, calculus, geometry, and statistics/probability in grades 9 through 12. The results showed a substantial increase in average proficiency with each course taken in the sequence from no algebra, pre-algebra, Algebra I, Algebra II, Algebra III/pre-calculus, and calculus. However, as shown in TABLE 11, which provides the percentages of students taking these courses by demographic subgroup, students in subgroups with lower average mathematics proficiency showed considerable attrition in the course-taking pipeline.

In general, few high-school seniors had taken Algebra III/pre-calculus, and even fewer reported going on to calculus. However, nearly one-fourth of the Asian/Pacific Islander twelfth graders had taken Algebra III/pre-calculus or both Algebra III/pre-calculus and calculus, as well as 18 percent of the students attending schools in advantaged urban communities, 18 percent of the students in the Northeast, 19 percent of those having at least one parent who had graduated from college, and 19 percent of those in academic high-school programs. In contrast, 6 percent of the Black twelfth graders and 8 percent of the Hispanic twelfth graders had taken Algebra III/pre-calculus courses. Similarly, very few students whose parents were less well-educated had taken Algebra III/pre-calculus, and virtually none of the students in general or vocational/technical high school programs reported taking these more advanced courses. On the other hand, for Algebra III/pre-calculus and calculus, there were no gender differences in either course taking or average proficiency.

Regarding other high-school mathematics courses, there was also a strong relationship between students' geometry proficiency and whether they had studied geometry and trigonometry. However, 28 percent of the high-school seniors had not studied a year of geometry, 55 percent had studied geometry but not trigonometry, and only 17 percent had additional course work in trigonometry. For geometry, course-taking patterns by subgroup tended to parallel those for algebra, except fewer females were likely to go on to trigonometry. Only about 12 percent of the high-school students reported even a semester of course work in statistics and probability. In statistics and probability, there were few differences in course-taking patterns by subgroup, except students in the Northeast were more likely to have taken a semester of course work than students in the other three regions of the country.

TABLE 11 Algebra and Calculus Course Taking: Grade 12

	Have Not Studied Algebra	Only Taken Pre-Algebra	Only Taken Algebra I	Taken Algebra II but not beyond	Taken Algebra III or Pre-Calculus but not Calculus	Taken Calculus
	Percent of Students	Percent of Students	Percent of Students	Percent of Students	Percent of Students	Percent of Students
Nation	9 (0.7)	8 (0.5)	27 (1.0)	43 (1.3)	9 (0.7)	4 (0.4)
White	8 (0.8)	8 (0.6)	26 (1.2)	45 (2.2)	10 (0.8)	4 (0.4)
Black	10 (1.3)	9 (1.2)	34 (2.0)	41 (2.2)	5 (1.0)	1 (0.4)
Hispanic	14 (1.9)	12 (1.5)	30 (2.1)	36 (2.5)	5 (1.0)	3 (0.8)
Asian/Pacific Islander	5 (1.3)	8 (3.8)	23 (3.8)	41 (4.7)	17 (3.6)	7 (3.2)
Advantaged Urban !	4 (1.2)	8 (1.6)	22 (2.4)	48 (2.2)	12 (1.9)	6 (1.6)
Disadvantaged Urban	9 (1.6)	10 (1.8)	34 (3.4)	39 (2.5)	5 (1.0)	3 (0.9)
Extreme Rural !	13 (2.4)	8 (2.6)	25 (2.5)	45 (3.4)	7 (2.3)	2 (0.8)
Other	8 (0.7)	8 (0.6)	27 (1.2)	43 (1.7)	10 (0.9)	4 (0.4)
Public Schools	9 (0.8)	9 (0.6)	28 (1.1)	42 (1.4)	8 (0.8)	3 (0.4)
Private Schools	2 (0.7)	4 (1.1)	23 (2.1)	50 (2.9)	16 (1.9)	5 (1.1)
Male	10 (0.9)	8 (0.6)	27 (1.1)	41 (1.4)	9 (0.8)	4 (0.5)
Female	7 (0.7)	9 (0.7)	28 (1.4)	45 (1.4)	9 (0.7)	3 (0.4)
Northeast	8 (1.1)	6 (0.7)	27 (2.3)	41 (3.4)	12 (1.5)	6 (1.0)
Southeast	10 (1.6)	6 (0.7)	26 (2.2)	49 (2.8)	7 (1.2)	2 (0.3)
Central	9 (1.8)	11 (1.0)	29 (1.9)	39 (1.9)	9 (1.4)	3 (0.5)
West	7 (0.9)	10 (1.3)	27 (1.7)	44 (2.0)	8 (1.3)	3 (0.7)
Did Not Finish H.S.	20 (2.1)	14 (1.9)	34 (2.3)	28 (3.1)	3 (1.0)	1 (0.2)
Graduated H.S.	13 (1.5)	11 (1.2)	32 (1.7)	37 (2.1)	6 (0.7)	1 (0.3)
Some Ed. After H.S.	6 (0.8)	9 (1.1)	27 (1.7)	46 (2.0)	9 (1.0)	3 (0.5)
Graduated College	5 (0.7)	5 (0.5)	24 (1.3)	48 (1.6)	13 (1.0)	6 (0.7)
Academic	2 (0.3)	3 (0.4)	22 (1.4)	54 (1.7)	14 (0.9)	5 (0.6)
General	16 (1.5)	15 (1.3)	35 (1.6)	30 (1.3)	3 (0.6)	1 (0.3)
Vocational/Technical	25 (2.1)	16 (2.1)	39 (3.2)	19 (2.3)	1 (0.4)	0 (0.3)

Courses taken were defined as those subjects studied for at least one year. The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. Population percentages may not total 100 percent due to rounding. !Interpret with caution--the nature of the sample does not allow accurate determination of the variability of the results for these population subgroups.

SUMMARY OF MATHEMATICS CURRICULUM AND COURSE TAKING ACROSS THE GRADES

In summary, the curriculum results--reported largely by teachers--depict a curriculum in which less than half the fourth graders were receiving heavy instructional emphasis in any of the five content areas. The most emphasis was in numbers and operations, followed by measurement, geometry, and data analysis (primarily reading graphs and tables). Few were introduced to the concepts underlying algebra. Almost all of the fourth graders were given heavy instructional emphasis in learning facts and concepts, and substantial proportions were given heavy emphasis in learning procedures. Much smaller percentages were given heavy instructional emphasis in reasoning and communicating in mathematics, although the emphasis in skill areas did not differ across the ability levels of students' classes.

At grade 8, students reported being in one of three courses, with more than half in eighth-grade mathematics and the remainder in pre-algebra or algebra courses. This data agreed with teachers' reports, which indicated that approximately half the students were receiving heavy instructional emphasis in numbers and operations and the other half were receiving heavy emphasis in algebra. Eighth graders were still being given much more instructional emphasis in facts and procedures than they were in mathematics reasoning and communication, and teachers reported differing amounts of emphases in these areas by the ability level of the class. Substantial percentages of eighth-graders in low-ability classes were being given no instructional emphasis in these areas highlighted in the research as necessary for effective mathematics teaching. Although there were variations, the curricular patterns shown for eighth graders across the nation attending public and private schools prevailed across the participating states for eighth-grade students attending public schools.

Nine percent of the high-school seniors reported never having taken algebra, and 43 percent reported persevering only through Algebra II before discontinuing their algebra coursework. Only 13 percent reported taking Algebra III/pre-calculus, and only 4 percent of these also taking calculus. The majority (55 percent) had taken geometry, but only 12 percent had studied even a semester of statistics.

Course-taking patterns, beginning with the differentiation of students in the eighth grade, were quite distinct by demographic subgroup, with proportionately more Asian/Pacific Islander students, those in advantaged urban schools, and those with more well-educated parents tending to continue in the mathematics pipeline. Black and Hispanic students, those attending schools in disadvantaged urban communities, and those with less well-educated parents displayed a disproportionately high rate of attrition. There appeared to be few gender differences in course-taking patterns.

MATHEMATICS INSTRUCTION AT GRADES 4 AND 8 IN THE NATION AND AT GRADE 8 IN THE STATES

Teachers reported that most students were working problems textbooks or worksheets on a daily basis and that more innovative activities were used less frequently. Small-group work and use of mathematics tools, such as geometric shapes and rulers were not widespread, nor was the use of calculators -- even by the eighth grade, where arithmetic should be well in hand.

Most students were never asked to write reports or do mathematics projects, and hardly any regularly used a computer in mathematics class.

These findings provide considerable contrast with the recommendations for revitalizing mathematics instruction described by educators and researchers.[14] According to their recommendations, effective mathematics instruction would include helping all students learn to think mathematically through group and individual projects that stress the application of mathematics, and incorporate the use of calculators and computers to engage students and facilitate their efforts with more complex problems and solutions.

ABILITY GROUPING FOR MATHEMATICS INSTRUCTION

- Although fourth graders were not typically grouped by ability as a matter of school policy, their teachers reported that more than half were in classes with students of similar ability, most probably as a result of external factors such as the socioeconomic standing of the community. At grade 8, more than two-thirds of the students were grouped as a matter of policy, since students were placed into differential mathematics curricula.

- In almost all participating states, with the exception of Montana, Nebraska, North Dakota, and the Virgin Islands, at least half of the eighth graders attending public schools were assigned to mathematics

[14]*Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1991).

Professional Standards for Teaching Mathematics (Reston, VA: National Council of Teachers of Mathematics, 1991).

Reshaping School Mathematics: A Philosophy and Framework for Curriculum (Washington, DC: Mathematical Sciences Education Board and National Research Council, National Academy Press, 1990).

Everybody Counts: A Report to the Nation on the Future of Mathematics Education, Lynn Steen, editor (Washington, DC: National Research Council, National Academy Press, 1989).

classes by some form of ability grouping. Connecticut, Hawaii, Maryland, Oregon, Rhode Island, and Guam grouped more than 85 percent of their eighth graders by ability.

- At grade 12, 58 percent of the students reported being enrolled in an academic high-school program, 34 percent in a general program, and 8 percent in a vocational/technical program. For those high-school seniors in an academic high-school program, less than three-fourths reported taking Algebra II.

INSTRUCTIONAL MATERIALS

- Teachers reported asking most students to work problems from textbooks on a daily basis (64 percent of the fourth graders and 71 percent of the eighth graders) and asking many students to work problems from worksheets at least several times a week (62 percent of the fourth graders and 38 percent of the eighth graders). The use of worksheets in eighth-grade classrooms was much more prevalent for low-ability than high-ability classes.

- Teachers reported that approximately half or more of the students worked in small groups at least once a week (63 percent of the fourth graders and 49 percent of the eighth graders), but students tended to report less small-group work -- 33 percent of the fourth graders and 28 percent of the eighth graders reported working in small groups on a weekly basis. Only 34 percent of the twelfth graders taking mathematics reported working in small groups at least once a week.

- Teachers reported using mathematics tools such as geometric shapes or rulers with approximately half the fourth graders and 28 percent of the eighth graders on a weekly basis. Here students were in better agreement with their teachers, with 43 percent of the fourth graders, 30 percent of the eighth graders, and 27 percent of the twelfth graders who were taking mathematics reporting that they used such tools on a weekly basis.

- In contrast to the heavy reliance on textbooks and worksheets, 70 percent of the eighth graders and twelfth graders who were taking mathematics reported that they never wrote reports or did mathematics projects. Teachers were less willing to report total absence of these activities but did report that 43 percent of the eighth graders were never asked to engage in these activities in mathematics class.

- The pattern of substantial reliance on textbooks and worksheets and limited use of some of the more innovative strategies recommended in the mathematics-reform literature also tended to prevail across the states participating in NAEP's 1990 Trial State Assessment.

TESTING IN MATHEMATICS CLASSES

- Teachers reported that 48 percent of the fourth graders and 60 percent of the eighth graders were given teacher-generated tests about once a week or even more frequently. They also reported occasional use of state- or district-mandated tests and administering other published tests.

- Fifty-two percent of the fourth graders, 71 percent of the eighth graders, and 70 percent of the twelfth graders taking mathematics reported being tested about once a week or even more frequently.

- Mathematics instruction in the participating states also appeared to be characterized by considerable testing. For example, in Alabama, Louisiana, New Hampshire, and Rhode Island, teachers reported administering teacher-generated tests to more than two-thirds of their eighth-grade students about once a week. In general, those students tested the most frequently tended to have lower average proficiency.

RESOURCES IN MATHEMATICS CLASSES

- According to their teachers, only 13 percent of the fourth-grade students and 19 percent of the eighth-grade students were in classrooms with all the necessary resources. Thirty-eight percent of the fourth graders and 28 percent of the eighth graders were in classrooms where teachers reported receiving only some or none of the necessary resources. For schools in disadvantaged urban communities, these figures were 48 percent and 40 percent, respectively, for fourth and eighth graders.

- In no single state participating in the 1990 Trial State Assessment Program, were more than one-third of the public-school eighth graders in classrooms where teachers reported receiving all the necessary resources. The students in the most poorly supplied classrooms tended to have lower average proficiency than their counterparts in classrooms where teachers reported receiving more resources.

CALCULATORS AND COMPUTERS IN MATHEMATICS CLASSROOMS

- According to their teachers, only 3 percent of the fourth graders and 19 percent of the eighth graders were permitted unrestricted use of calculators in mathematics class.

- For all but three state participants (the District of Columbia, Oregon, and Wyoming), fewer than one-third of the eighth graders attending public schools were permitted unrestricted use of calculators. In every participating Southeastern state, fewer than 15 percent of the students were permitted unrestricted use of calculators although states from other regions also reported similar policies. Within the states, the data consistently showed that eighth graders who were permitted use of calculators had higher average mathematics proficiency.

- Calculator use appears relatively infrequent in fourth-grade classrooms. According to their teachers, only 6 percent of the fourth graders were asked to use a calculator several times a week and 47 percent were never asked to do so. Sixty-two percent of the fourth graders reported never using a calculator for mathematics.

- At grade 8, teachers reported that 30 percent of the students used a calculator at least several times a week, while 22 percent never did. Thirty-nine percent of the eighth graders reported never using a calculator in mathematics class, as did 14 percent of the twelfth graders taking mathematics.

- Across the states, calculator use varied dramatically for eighth graders attending public schools. At the high end of the continuum were Iowa, Minnesota, Montana, Nebraska, North Dakota, Oregon, Wisconsin, and Wyoming, with the majority of both teachers and students reporting frequent calculator use. Students in these states also performed relatively well. The low-use states appeared to be concentrated in the Southeast, with another cluster in the Northeast.

- Although more than half of the fourth graders demonstrated some degree of success in using the four-function calculator provided for use with portions of the assessment, the eighth and twelfth graders had more difficulty with the scientific calculator that they were provided. Forty-four percent of the eighth graders and 30 percent of the twelfth graders demonstrated knowledge of both when and how to use a calculator.

- Except in the two territories, student success in calculator usage did not vary much across states--from 40 percent in the high-performance group in Hawaii to 56 percent in North Dakota.

- Computer use in mathematics classes was even more infrequent than calculator use. Most teachers reported that computers were difficult to access. However, in contrast to results for calculators, computers seemed to be used more frequently in the fourth grade than in the higher grades. Teachers reported that 49 percent of the fourth graders used a computer in mathematics class at least once a week, although they also reported that 26 percent never did. At grade 8, teachers reported that only 18 percent of the students used a computer at least once a week and that the majority (52 percent) never did.

- From students' perspective, 50 percent of the fourth graders, 69 percent of the eighth graders, and 66 percent of the twelfth graders taking mathematics reported never using a computer in mathematics class.

- For the states participating in the Trial State Assessment Program, teachers reported low availability of computers in public-school eighth-grade mathematics classrooms. The majority of the eighth graders never used a computer in mathematics class in many of the states.

STUDENTS' MATHEMATICS TEACHERS

- Students' mathematics teachers appeared to be experienced, reporting 14 years average teaching experience in mathematics at both grades 4 and 8.

- Four-fifths of the fourth graders were taught mathematics by White teachers and by female teachers. At grade 8, 91 percent were taught by White teachers, but only about half were taught by female teachers (58 percent).

- Sixty-four percent of the fourth graders were taught by teachers with the highest certification, although 83 percent were taught by teachers certified in education rather than mathematics. Sixty-five percent of the eighth graders were taught by teachers with the highest certification; 78 percent were taught by teachers certified in mathematics.

- Sixty-four percent of the fourth graders were taught mathematics by teachers with a bachelor's degree and 36 percent by teachers with a master's or specialist's degree. At grade 8, 55 percent of the students were taught by teachers with a bachelor's degree, 44 percent by teachers with a master's or specialist's degree, and 1 percent by teachers with a doctorate or professional degree.

- The vast majority of fourth graders (83 percent) were taught by teachers who had majored in education. In contrast, 39 percent of the eighth graders had teachers who were mathematics majors, 38 percent had teachers who were education majors, and 23 percent had mathematics teachers with some other major.

- Fourth graders had teachers who reported limited course work in mathematics. For example, 41 percent reported no course work in number systems and numeration, 62 percent no course work in geometry, 82 percent no course work in advanced algebra, and 53 percent no course work in probability and statistics.

- The teachers of eighth graders reported more course work, but still about one-fifth reported no course work in number systems and numeration or in geometry. Thirty percent reported no course work in advanced algebra, and 15 percent reported no course work in probability and statistics.

- Teachers of 31 percent of the fourth graders and 13 percent of the eighth graders reported no time spent on in-service education in mathematics or the teaching of mathematics during the last year before the assessment.

- Across the states participating in the Trial Assessment Program, there was tremendous variation in teachers' preparation and training, although the patterns tended to be similar to those for eighth graders nationally. Within states, there was a tendency for the better-performing students to have teachers with more depth and breadth of course work in mathematics, as well as more in-service education in mathematics.

STUDENTS' PERCEPTIONS OF MATHEMATICS

- In general, most students at all three grades had positive perceptions of mathematics, and positive perceptions were related to higher mathematics proficiency. Although two-thirds of the fourth graders agreed that they liked mathematics, only slightly more than half of the eighth or twelfth graders did.

- Sixty-two percent of the fourth graders, 63 percent of the eighth graders, and 57 percent of the twelfth graders either agreed or strongly agreed that they were good in mathematics. However, nearly one-fifth of the students at all three grades were either neutral or agreed that "Mathematics is more for boys than for girls."

- The vast majority at all three grades strongly agreed or agreed that mathematics was useful in solving everyday problems and that mathematics is used by almost all people in their jobs.

- In general, public-school eighth graders across the individual states followed the national pattern and reported positive perceptions of mathematics. Within each state, a higher degree of positive agreement was associated with higher proficiency, but this relationship did not hold across states, because more students in some of the lower-performing states and fewer students in some of the higher-performing states reported positive attitudes.

In summary, school mathematics across the nation at grades 4, 8, and 12, and in the public schools in the states at grade 8 appeared to be characterized by classrooms grouped by ability, where students were working on problems from textbooks and worksheets with considerable regularity. Many students also appeared to be tested on a weekly basis.

Teachers reported that resources were in short supply and that computers were difficult to access. Both teachers and students agreed that small-group work, use of mathematics teaching tools, and use of calculators were not widespread, and that students were rarely asked to write reports or do mathematics projects.

Considering the recommendations for reform suggesting pervasive changes in the delivery of mathematics instruction, this portrait suggests a challenge in moving mathematics instruction into alignment with current expectations, let alone expectations for the year 2000.

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