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ABSTRACT

Numerous designs using analysis of variance (ANOVA) to test ordinal hypotheses were assessed using a Monte Carlo simulation. Each statistic was computed on each of over 10,000 random samples drawn from a variety of population conditions. The number of groups, population variance, and patterns of population means were varied. In the non-null patterns, the power of the alpha-corrected tests was extremely low. Extremely low power was also evident for both the uncorrected and the no significance test for ordering technique. The linear trend test exhibited the highest power in almost every fully monotonic circumstance. The tau and monotonic trend tests also had high power in most fully monotonic population means instances, but they varied as to which was more powerful. Of the tests for disconfirmations of monotonicity, the curvilinearity test for detecting inversions or ties was too often significant in monotonic but non-linear cases. Combined tests were preferred when they had high acceptance values (power) for the fully monotonic cases, but low acceptance values when disconfirmations were present. Ultimately, the choice of technique comes down to the balance between power when the ordinal hypothesis is correct versus spurious acceptance of the ordinal hypothesis when it is slightly in error. Sweep tests, particularly the linear trend test, have the needed power, but lack the ability to alone detect disconfirmations of monotonicity. The tests of disconfirmation that were evaluated showed far too many false positives to warrant wide acceptance. Six data tables are included, and the Statistical Analysis System program used to run the Monte Carlo study and the program used to calculate taus are outlined. (TJH)

ORDINAL HYPOTHESIS IN ANOVA DESIGNS: A MONTE CARLO STUDY

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A familiar occurrence in experimental designs using multiple ($k > 2$) groups is that the investigator has an ordinal hypothesis, that is that *a priori* considerations lead the investigator to make predictions about the ordering of the groups on the dependent variable. Thus, the investigator is able to predict which group's population mean should be lowest, which next lowest, etc. The predictions are typically strictly ordinal, with no implications that the group differences are equally spaced. The investigator wishes a statistical test to detect whether or not the ordinal hypothesis is accurate (significant) and/or whether any significant deviations from the predicted ordering are in evidence.

Ordinal hypotheses may arise both in one-way and in factorial ANOVA designs. In factorials, several possibilities exist for the nature of the ordinal hypothesis. One is that the ordinal predictions are made for the main effect means. A second possibility is that the ordinal hypothesis is specific to within a certain row or column (i.e., a simple main effect means). Typically, when this happens, different orders are predicted within different rows or columns (if not, the main effect ordinal hypothesis is identical to the simple main effect ordinal hypothesis). The last possibility is that the ordinal hypothesis orders cells relative to those in different rows or columns, as well as those within its row or column. In this case, the design is one in which *manipulations* are factorially arranged, but predictions are not. For analysis purposes, it is best to "un-factorialize" the cells, and treat the data as if it arose from a one-way design, with k = number of cells. Such a case may arise, of course, even when all factors have two levels, since the number of cells is greater than two.

The investigator's ordinal hypotheses may not be able to *completely order* the groups' population means. For example, one may predict that groups B and C are both higher than A, but not be able to predict the order of B relative to C. Note that, typically, the investigator is not actually predicting that the B and C have equal population means, but rather is simply *not making a prediction* as to their ordering, preferring to predict only that each is higher than A. In the case of *incomplete ordering*, if the groups fall into only two "clusters", there would be some consensus among data analysts that the solution is to use planned contrasts.

It is useful to distinguish whether the *a priori* considerations which lead to the ordinal hypothesis are *separable* or *coherent*. In the case of separable hypotheses, different hypotheses or theories may lead to different subsets of the predicted orderings. Thus one "theory" may be the basis for the prediction that group A has a lower mean than group B, while a *different* theory may have generated the prediction that group B is less than group C, and so forth. Alternatively, there may be *one coherent* theory that generates *all* the orderings. The separable case is straitforward: A separate planned contrast should be used to test each pair of means, since each contrast has implications for a different theory. The coherent theory test is more complicated: the entire ordering should be tested with one statistical test, since the theory is confirmed or disconfirmed in its entirety. We devote the rest of this paper to this more complicated coherent theory case.

The standard ANOVA textbooks written by and for psychologists (i.e., Hays, 1973; Kirk, 1968; Keppel, 1982; Myers, 1979; Winer, 1971) do not even mention that such a problem exists, let alone discuss possible solutions. The published statistical literature on this problem is also sparse, with the exception of the work by Bartholomew (1961, 1959a,b). However, since their proposals have seen few applications in psychological research (perhaps because their approach requires new tables not widely available, and because their technique is not incorporated into any statistical software) we will not further consider their proposals.

Survey of Journals

In order to ascertain how common the ordinal hypothesis problem is, and to assess how researchers approach [it in the absence of guidance from statisticians], we undertook a journal survey. All 1987 issues of two journals which tend to publish the results of psychological experiments, Journal of Experimental Social Psychology, and Memory and Cognition, were examined[1]. The results presented in Table 1, showed the problem is a very common one, with no standard solution approach.

Of the 73 articles we evaluated, the authors predicted an ordinal pattern of three or more population means in 42 cases (57.5%). In just over half of these cases, the the groups were completely ordered--with no "ties." Most of these involved main-effect trends in factorial designs. As shown, the incompletely ordered ordinal hypothesis [such as when two or more groups are predicted as medium] was also very common, but we do not consider it further here since the case is more complicated.

Insert Table 1 About Here

Our survey also revealed that, in the absence of definitive advice from statistical authorities, investigators have used a variety of techniques which, on the surface, appear reasonable. Table 2 discloses the choices researchers have used[2].

Insert Table 2 About Here

Of 22 articles in which the authors had hypotheses which completely ordered the groups, almost one-third of the time (n=7), the authors conducted only an omnibus significance test with no follow-ups[3]. When follow-ups were done, researchers tended to use alpha-corrected post-hoc procedures (n=8). In 6 cases, the authors conducted t-tests between adjoining groups to test their predictions, and in only 1 case did the author specifically test for the hypothesized trend. Thus, "decisions" about trend hypotheses are often made on the basis of statistics that ignore the overall "pattern" of the group means.

Disconfirmations of Monotonicity

An important distinction involves whether the investigator wishes to detect any disconfirmations of monotonicity. The investigator may wish to detect only whether the overall "sweep" of the population means agrees with the predicted ordering. This involves knowing whether there is simply a positive rank-order correlation between the predicted ordering and the actual population means. Another possibility is that the investigator wants to detect any instance of disconfirmation of monotonicity, i.e., wants to know if the rank order correlation between the predicted ordering and the actual population means is anything less than +1.0. This alternative seems more likely, since the coherent hypothesis being evaluated completely orders the population means; thus, the investigator will want to detect any incorrectly predicted ordering of the population means, since even one tends to disconfirm the hypothesis. Two types of disconfirmations are possible: Ties occur when two population means predicted to differ are actually equal, while inversions occur when a pair of population means predicted to be in one order are actually in the opposite order. There may be one instance of population mean ties or more than one. Inversions, too, might have one or more instances and/or several different degrees. For example, with $k = 3$, population means of 0, .3, 2 would represent no disconfirmations of monotonicity; 0, 0, 2 would represent ties; 0, 2, 1.8 would represent an inversion of mild degree; 0, 2, 1 would be an inversion of moderate degree; 0, 2, .2 would be an inversion of strong degree; and 1, 2, 0 would represent two inversions.

Evaluating Ordinal Hypotheses

The inference problem in evaluating ordinal hypotheses is the usual one that the hypothesis is about population means but only sample means are obtainable. The sample means may frequently not be in the same order as the population means. Statistical techniques to test an ordinal hypothesis must be evaluated against the usual criterion of achieving a high probability of making correct decisions regarding the hypothesis. Thus, when the population means actually are in the hypothesized order, statistical operations on the sample means that have a high probability of accepting the hypothesis (i.e., high power) should be preferred. On the other hand, when the hypothesized order is not accurate, we prefer techniques with a low probability of accepting the hypothesis. Unlike the simple inference case, this situation obtains not only when the null is true, but also when any disconfirmations of monotonicity (i.e., any ties and inversions) exist for some or all of the groups.

In addition to the techniques attempted by researchers, a number of additional techniques to test completely ordered coherent ordinal hypotheses appear reasonable. The options fall into four logical groupings. First is the category of no significance tests for order. Here, (A) the investigator performs only the omnibus test in one-way ANOVA, and then if significant, simply observes whether the sample means order as predicted. While seeing frequent use in the journals we surveyed, this technique should have limited appeal, since there is no significance test for the ordinal hypothesis.

Second, is the category of the <u>uncorrected alpha multiple</u> significance tests. While a number of variants of this approach are possible, we consider only three: (B) All pairwise comparisons, where the investigator tests the significance of all means expected to differ with (uncorrected alpha) t-tests or contrasts. In addition, a set of mutually orthogonal planned contrasts could be used. One such set which addresses the ordinal hypothesis is (C) the "right-hand difference contrasts" (with four groups: -3, 1, 1, 1; 0, -2, 1, 1; 0, 0, -1, 1); another is (D) the "left-hand difference contrasts" (-1, -1, -1, 3; -1, -1, 2, 0; -1, 1, 0, 0). In using any of these three techniques, all contrasts in the set are tested -- and <u>all</u> need to be significant to support the hypothesis -- at the uncorrected alpha values.

The third category is the alpha corrected tests, corrected for alpha inflation because of the multiple tests performed. Again, a wide variety of such techniques are available. However, extensive research (Carmer & Swanson, 1973; Cohen, 1962, 1973, 1977; Petrinovich & Hardyck, 1969; Ryan, 1959) have been conducted on the properties of the various alternatives. Accordingly, we drop from consideration tests, such as the

Student-Neuman-Keuls and the Duncan test, of questionable validity or inadequate performance. Only three are considered further: (E) Fisher's protected LSD, in which uncorrected alpha paired comparisons are conducted only if the omnibus test is significant. This test has relatively high power but inadequate Type I error protection against partially true nulls. (F) Dunn's (or Bonferroni's) procedure. In this test, one corrects alpha for the number of tests being performed. Since only means predicted to be adjacent by the ordinal hypothesis need to be compared, the correction should be equal to the number of groups less one. Using this value should make the procedure more powerful. (G) Tukey's HSD procedure. Monte Carlo investigations frequently favor this multiple comparison technique (Carmer & Swanson, 1973; Elinot & Gabriel, 1975; Petrinovich & Hardyck, 1969). Again for the tests in this category, <u>all</u> the tests need to be significant to accept the hypothesis.

The final category are the procedures which specifically examine the group-to-group function. Ferguson (1976), the only text we could find that proposed a technique to test ordinal hypotheses, recommends a (H) Kendall's tau with ties, with the X variable for each score being the predicted rank order of the group. Similarly, Edgington (1969) recommends a Spearman's rho, which will have the same significance value since it has an identical sampling distribution (Siegel, 1956). It should be noted that these tests can only evaluate overall sweep; they cannot detect disconfirmations of monotonicity (ties or inversions). (I) The linear trend test can also assess sweep by testing whether the best-fitting straight line through the population means has zero slope.

A new test to evaluate sweep, a test we term the monotonic trend test (J), is also in the last category. This test examines the summed, signed group to group sample mean increase then divides by the appropriate standard error. Specifically, if the groups are arranged so that the group predicted to have the lowest population mean is designated group 1, with sample mean $\overline{X}_1$, the group predicted to have the second lowest population mean is designated group 2, with sample mean $\overline{X}_2$, etc., then the extent of the monotonic trend is gauged by the summed, signed group-to-group increase:

$$\sum_{i=1}^{i=k-1} (\overline{X}_{i+1}-\overline{X}_i),$$

Since the last term in each element of the expansion is also the first element of the next element of the expansion, but of opposite signs, all middle terms drop out, leaving:

$$\overline{X}_k-\overline{X}_1.$$

Since the monotonic trend turns out to be nothing more than the difference between two (of k) sample means, its standard error is well known. Dividing this monotonic trend index by this standard error yields the t-test for the monottonic trend:

$$t_{mono} = \frac{\bar{X}_k - \bar{X}_1}{\sqrt{MS_E \cdot \sum_i \left(\frac{1}{N_i}\right)}},$$

with degrees of freedom equal to:

$$df = \sum_i N_i - k.$$

To test disconfirmations of monotonicity, the linear trend test may be used in combination with (K) the test for curvilinearity. This numerator SS for this F-test is $SS_{between}$ - SS_{linear}, while the numerator _df_ = k - 2. Its denominator is MS_E. A reasonable approach might be to only deem the hypothesis fully supported when the linear trend test is significant and the curvilinear test is not. Since support for the hypothesis requires _non_-significance, this test might reasonably use standard (K1) _or_ high alpha (K2). Unfortunately, the curvilinearity test is also sensitive to non-linear monotonicities, such as concave functions, which do not disconfirm an ordinal hypothesis.

Another possible test for disconfirmations are paired comparison (t-) tests when _any inversions_ of _sample_ means occur, to test whether the inversion is significant. We term such tests _reversal_ tests. These might use standard (L1) or high alpha (L2). Either the high or standard alpha test might also use Bonferroni corrected alpha (L3-L4).

A reasonable approach might be to only deem the ordinal hypothesis fully supported when both a sweep test is significant and any disconfirmation tests are not. Thus combined tests might be conducted which involve linear trend tests (I) in combination with both curvilinearity tests (of either standard alpha, K1) or high alpha (K2) and with reversal tests (L1-L4). This yields combinations M1-M6, respectively. Alternatively, the monotonic trend test (J) might be used in combination with the reversal test (L1-L4), yielding tests N1-N4, respectively.

Method

To test approaches (A) through (N4), a Monte Carlo simulation was conducted, with each statistic (A-N4) computed on each of 10,000 random samples drawn from a variety of population conditions. Three population conditions were varied: (1) the number of groups, k: 3, 5 or 9. These values represent a wide range of those typically involved in ordinal hypotheses. (2) We also varied the population variance, σ^2. We chose three values, high, medium and low. The values were chosen in such a way that, in combination with constant sample size and range of population means, they led to low (near .3), medium (near .7) and high (near .95), respectively, power for the omnibus F-test. Low power of .3 for an omnibus test may be typical for many studies (Cohen, 1999; 1999) in the social sciences. Higher values of power, such as .7, are recommended; .95 represents a desirable extreme. Different values for σ^2-power were chosen to examine how power differentially impacted the different tests. (3) A large variety of patterns of population means. Included were (a) null hypothesis true; (b) completely monotonic increasing (with a variety of shapes, such as concave, ogive, linear, approximately linear, etc.); and disconfirmations of both varieties: (c) ties; and (d) inversions. The ties and inversions we used had a variety of degrees and instances (see page 6). Altogether 38 different population mean conditions were created, each with three levels of σ^2-power. These population mean conditions, together with our mnemonic name for the pattern, are presented in Table 3.

Insert Table 3 About Here

For each combination of population mean pattern, number of groups, and σ^2-power, the proportion of times the hypothesis was accepted over the 10,000 samples by each of the statistical tests A-N4 was calculated by computer. We wrote computer programs to do the sampling and compute the significance of the various tests. The programs were written in SAS language, and are included in the Appendix.

For all the statistical tests with the exception of tau (I), individual observations were not needed for the computations, only sample means. The sample means were randomly sampled from the sampling distribution, which, assuming normal population, was normal, with an expected value equal to μ_i and a standard error of σ. In such a case, we used $df_E = \infty$. The program is in Appendix A. For the tau program, Appendix B, individual observations were needed. We used an N of 10 per group, and sampled them from a normal distribution with variance equal 10 $\sigma^2 \cdot N$. The single df tests (contrasts, paired comparisons, Fisher's LSD, tau and the linear, and monotonic trend test) used one-tailed criteria. The remaining tests (omnibus, curvilinearity, Dunn's and Tukey's HSD) used two-tailed criteria. Standard alpha was .05; high alpha was .25.

Results & Conclusions

Tables 4-6 display the results of tests A-G, with low power, medium power, and high power, respectively. For the null hypothesis true cases, the omnibus is accepted near .05 of the time. With k = 3, the probability of the sample means ordering 1, 2, 3, was near .1666, as it should be (1/3!). The righthand and lefthand tests (C and D) for k = 3 had probability near .0025, as they should ($.05^2$).

Insert Tables 4-6 About Here

In the non-null patterns, the power of the alpha-corrected tests (E-G) was extremely low, generally 0.00. Only with high power and k =3, did power grow above .10, and then only for two patterns (linear and concave). Notice that in these instances, by comparison, the omnibus F-test had power near .95. Extremely low power was also evident for the uncorrected (B-D) and even, generally, the no significance test for ordering technique (A). The low power was caused by the fact that the frequency of the sample means even ordering as did the populations -- let alone all differences being significant by <u>any</u> criterion -- was very low, the moreso the greater the number of groups and/or the population variance. Only in the high power case with k = 3 did the no significance test for ordering technique (A) have power exceeding .42.

Insert Tables 7-9 About Here

Accordingly, only the tests in Category 4 (H-J) had appreciable power, as displayed in Tables 7-9, typically well above the power for the omnibus F-test. Of these, the linear trend test (I) had the highest power in almost every fully monotonic circumstance. The tau (H) and monotonic trend tests (J) also had high power in most fully monotonic population means instances, but varied between themselves as to which was more powerful.

Of the tests for disconfirmations of monotonicity, the curvilinearity test (K) for detecting inversions or ties was, as expected, too often significant in monotonic but non-linear cases. For example, with high power, even the standard alpha test had several instances of being significant more than 40% of the time, though the population means were fully monotonic. Three of the four reversal tests (L1, L3 and L4) did not have this problem.

Insert Tables 10-12 About Here

Tables 10-12 present the combined test results. Combined tests are preferred which have high acceptance values (power) for the fully monotonic cases, but low acceptance values when disconfirmations are present. All test except M4 and N2 tended to have the first quality. Although all the tests had desirable low values with moderate or strong degrees of inversions, all were considerably higher than desirable for ties and for mild cases of inversions. No clear basis for choice for general use among the combined approaches is apparent from the Tables.

Ultimately, the choice of technique comes down to the balance between power when the ordinal hypothesis is correct vs spurious acceptance of the ordinal hypothesis when it is slightly in error. Clearly all techniques in the first three categories are far too underpowered to recommend. The sweep tests, especially the linear trend test, have the needed power, but, by themselves, lack the ability to detect disconfirmations of monotonicity. The tests of disconfirmation we evaluated showed far too many false acceptances with ties or when the inversions were mild to find wide acceptance. Thus, more research is needed to develop more sensitive tests of disconfirmation of montonicity to be used in combination with a sweep test.

Notes

1. One issue of Memory & Cognition was unavailable for coding.

2. No distinction between ordinal predictions derived from coherent or separable theories was made in our journal survey. While we have argued that this is an important distinction for the choice of analytic techniques, it proved to be too difficult for a reader not versed in the substantive area of each research article to determine.

3. This seems to suggest that researchers are not even doing statistics, but merely "looking" for the predicted pattern in the data. This isn't the case. In at least 3 of these 7 articles, the omnibus test was non-significant, and the researcher did not do any follow-up tests for the trend. Thus, the trer hypothesis was rejected in these cases.

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Table 1. Articles with Ordinal Hypotheses (k > 2), N=73.

Design[1]	Complete Ordering	Incomplete Ordering
One-Way ANOVA	7 (17)[2]	2 (2)
Factorial Main Effects	10 (17)	1 (1)
Simple Main Effects	4 (4)	1 (1)
Cell by Cell	1 (3)	16 (29)
No Order Hypothesis	12 (32)	
Non-ANOVA Designs	20 (61)	

Notes.

1. Design refers to the experimental organization of the cells which were hypothesized to be ordered.

2. The numbers in parentheses refer to the number of experiments with ordinal hypotheses that fell into each design category.

Table 2. Approaches Used for Testing Ordinal Hypotheses.

Technique Used	Articles	Experiments
Omnibus Test Only	7	11
Omnibus & Post-hocs	8	16
Uncorrected T-tests	6	13
F-Linear Trend	1	1

Table 3. Population Conditions Varied for Monte Carlo Study.

Groups	σ^2s	Population Conditions - Group Means								
3	.67, .26, .13	Null True	1	1	1					
		Monotonic Linear	0	1	2					
		Mono Non-Linear	0	.3	2					
		Disconfirmation	0	0	2					
		(mild)	0	2	1.7					
		(moderate)	0	2	1					
		(strong)	0	2	.3					
5	2.65, 1.02, .53	Null True	2	2	2	2	2				
		Montonic Linear	0	1	2	3	4				
		Mono Non-Linear	0	.1	.2	.5	.4				
		(concave)	0	.2	.4	1	4				
			0	.5	1	2	4				
		(ogive)	0	.5	2	3.5	4				
			0	.1	.2	3.8	4				
		(irregular)	0	.2	3	3.4	4				
		Disconfirmation	0	2	2	3	4				
		(weak)	0	0	3	3	4				
		(mild)	0	2	1	3	4				
		(moderate)	2	0	1	3	4				
			0	1	4	3	2				
		(strong)	2	0	3	4	1				
			0	3	4	2	1				
9	10.87, 4.63, 2.60	Null True	4	4	4	4	4	4	4	4	4
		Montonic Linear	0	1	2	3	4	5	6	7	8
		Mono Non-Linear	0	.2	.4	.7	1	1.4	2	3	8
		(concave)	0	.2	.4	.7	1	1.4	2	3	8
			0	.4	.8	1.4	2	2.8	4	6	8
		(ogive)	0	.4	.8	2	4	6	7.2	7.6	8
			0	.2	.4	.7	1	2.8	7.2	7.6	8
		(irregular)	0	.8	1.4	1.6	3	4.4	5.6	6.2	8
		(weak)	0	1	2	2	4	5	6	7	8
			0	2	2	2	4	6	6	8	8
		(mild)	0	1	2	3	5	4	6	7	8
			0	2	1	3	5	8	6	7	4
		(moderate)	0	1	2	3	8	7	6	5	4
			2	3	5	4	0	1	6	7	8
		(strong)	8	0	1	2	3	4	5	6	7
			6	2	5	7	1	0	3	4	8
			6	2	5	7	1	0	3	4	8

Table 4. Results of Tests under Low-Power Population Conditions

					UNCORRECTED			*POST-HOCS*		
POP	K	OMNI	ORDR	O&O[1]	PAIRS	RIGHT[2]	LEFT	LSD	BONF	TUKEY
Null	3	0520	1667	0083	0000	0021	0024	0000	0000	0000
	5	0507	0075	0002	0000	0000	0000	0000	0000	0000
	9	0506	0000	0000	0000	0000	0000	0000	0000	0000
Linear	3	3096	6155	2339	0123	0981	0915	0123	0026	0000
	5	3004	1057	0478	0000	0024	0028	0000	0000	0000
	9	3314	0002	0000	0000	0000	0000	0000	0000	0000
Concave	3	3622	5425	2210	0112	1320	0523	0112	0025	0000
	5	3483	0457	0184	0000	0051	0002	0000	0000	0000
	5	3195	0568	0210	0000	0030	0004	0000	0000	0000
	5	2981	0868	0379	0000	0041	0006	0000	0000	0000
Ogive	5	3651	1077	0508	0000	0023	0024	0000	0000	0000
	5	5057	0816	0482	0000	0047	0009	0000	0000	0000
Irreg	5	4293	0974	0529	0000	0023	0040	0000	0000	0000
Concave	9	2703	0000	0000	0000	0000	0000	0000	0000	0000
	9	3255	0004	0002	0000	0000	0000	0000	0000	0000
Ogive	9	4704	0004	0001	0000	0000	0000	0000	0000	0000
	9	5246	0002	0000	0000	0000	0000	0000	0000	0000
Irreg	9	3408	0000	0000	0000	0000	0000	0000	0000	0000
Weak	3	4102	4548	1979	0076	1360	0308	0076	0017	0002
	5	2735	0841	0299	0000	0015	0013	0000	0000	0000
	5	4153	0892	0444	0000	0024	0010	0000	0000	0000
	9	3425	0002	0001	0000	0000	0000	0000	0000	0000
	9	3740	0004	0001	0000	0000	0000	0000	0000	0000
Mild	3	3719	3607	1354	0028	0166	0887	0028	0008	0000
	5	2992	0674	0252	0000	0014	0013	0000	0000	0000
	9	3218	0001	0001	0000	0000	0000	0000	0000	0000
	9	3292	0002	0001	0000	0000	0000	0000	0000	0000
Medium	3	3195	1729	0399	0003	0024	0276	0003	0000	0000
	5	3117	0240	0074	0000	0008	0000	0000	0000	0000
	5	2963	0175	0038	0000	0000	0009	0000	0000	0000
	9	3359	0000	0000	0000	0000	0000	0000	0000	0000
	9	3242	0001	0000	0000	0000	0000	0000	0000	0000
	9	3287	0001	0001	0000	0000	0000	0000	0000	0000
Strong	3	3583	0594	0082	0000	0001	0060	0000	0000	0000
	5	2973	0036	0003	0000	0000	0000	0000	0000	0000
	5	3093	0021	0001	0000	0000	0001	0000	0000	0000
	9	3366	0000	0000	0000	0000	0000	0000	0000	0000
	9	3307	0000	0000	0000	0000	0000	0000	0000	0000

Notes 1. O & O represents testing the ordinal hypothesis with an omnibus test only, and if significant, merely observing whether the group means fall into the predicted order.

2. Right & Left refer to uncorrected tests of the right- & left-handed difference contrasts (explained in text).

Table 4 (continued).

		TREND TESTS			DISCONFIRMATION TESTS					
POP	K	TAU	F-LIN	MONO	F-NON		REVERSAL		BONF-REV[3]	
Null	3	0423	0503	0503	0503	2501	1033	4863	0505	2701
	5	0489	0487	0470	0495	2481	1939	7662	0543	2648
	9	0499	0466	0470	0503	2536	3541	9551	0519	2636
Linear	3	4466	5279	5279	0487	2474	0119	1236	0049	0475
	5	5713	6164	5368	0506	2409	0740	4685	0158	1076
	9	7303	7612	5249	0470	2518	2341	8717	0260	1619
Concave	3	4446	5329	5329	1037	3493	0272	1834	0133	0879
	5	4289	4912	5380	1701	4694	1317	6122	0319	1794
	5	4623	5235	5343	1263	4108	1100	5727	0274	1568
	5	5293	5778	5359	0713	2941	0840	5020	0188	1223
Ogive	5	6439	6870	5344	0562	2619	0809	4798	0181	1195
	5	6846	7344	5348	1417	4245	1310	5803	0336	1782
Irreg	5	6539	7016	5357	0952	3378	1079	5369	0228	1515
Concave	9	5033	5276	5265	1150	3918	2715	9060	0348	1927
	9	6863	7193	5355	0668	3044	2355	8803	0310	1634
Ogive	9	8486	8684	5292	0635	2811	2373	8713	0284	1670
	9	8353	8608	5223	1114	3812	2613	8908	0359	1916
Irreg	9	7226	7577	5386	0556	2687	2382	8762	0300	1690
Weak	3	4383	5309	5309	1710	4561	0490	2649	0243	1350
	5	4923	5394	5366	0645	2918	0930	5193	0193	1342
	5	6393	6891	5371	0908	3465	1129	5536	0282	1632
	9	7416	7812	5365	0553	2565	2299	8729	0280	1625
	9	7529	7792	5301	0622	2832	2545	8785	0301	1804
Mild	3	3293	4317	4317	2124	5139	0823	3496	0454	2025
	5	4849	5382	5355	0973	3437	1413	5950	0429	1905
	9	7213	7413	5319	0562	2596	2336	8764	0295	1673
	9	5393	5624	2159	1522	4564	3905	9477	0682	2991
Medium	3	1683	2212	2212	3224	6362	2176	5803	1366	4099
	5	3413	3858	2220	1880	5034	2555	7475	0918	3232
	5	2656	3100	2074	2256	5516	2456	7762	0757	3185
	9	4416	4689	2216	2062	5168	3548	9500	0583	2608
	9	3626	3859	3569	2332	5605	3966	9569	0792	3031
	9	2219	2430	0307	2917	6303	6295	9843	2352	5472
Strong	3	0539	0844	0844	4484	7585	4261	7921	3104	6408
	5	0919	0976	0188	3239	6533	5190	9291	2498	6050
	5	0579	0743	1148	3422	6758	3449	8907	1250	4372
	9	0539	0634	1124	3544	6900	6016	9872	1810	5021
	9	0539	0648	1174	3496	6809	6027	9879	1848	4991

Note 3. The first comlumn of each of the disconfirmation headings refers to the performance of the test with normal alpha ($\alpha = .05$) while the second column represents the performance of the test using a purposefully inflated alpha ($\alpha = .25$).

Table 4 (continued).

COMBINED TESTS (TREND TESTS & DISCONFIRMATION TESTS)

POP	K	LINEAR and NONLINEAR		REVERSAL		BONF-REV		MONOTONIC and REVERSAL		BONF-REV[4]	
Null	3	0476	0383	0503	0475	0503	0494	0503	0475	0503	0494
	5	0458	0359	0456	0276	0480	0439	0424	0234	0456	0407
	9	0444	0338	0343	0055	0448	0376	0328	0040	0451	0360
Linear	3	5016	4010	5269	5100	5275	5233	5269	5100	5275	5233
	5	5851	4696	5874	3912	6110	5721	5140	3542	5320	5015
	9	7242	5696	5975	1116	7427	6485	4252	0904	5151	4574
Concave	3	4779	3492	5297	4931	5315	5188	5297	4931	5315	5188
	5	4104	2632	4512	2505	4844	4339	4858	2549	5271	4646
	5	4566	3084	4864	2876	5145	4689	4904	2811	5243	4705
	5	5350	4111	5480	3498	5718	5316	5066	3255	5291	4921
Ogive	5	6482	5053	6462	4111	6791	6262	5168	3583	5315	5056
	5	6316	4257	6621	3567	7188	6331	4975	3085	5266	4817
Irreg	5	6329	4631	6434	3707	6902	6180	5077	3234	5310	4931
Concave	9	4663	3164	4046	0639	5122	4428	4014	0622	5104	4395
	9	6714	4992	5650	1019	6993	6131	4272	0842	5219	4606
Ogive	9	8119	6260	6737	1193	8447	7317	4319	0924	5174	4636
	9	7636	5324	6459	1036	8314	7054	4130	0818	5073	4453
Irreg	9	7165	5543	5918	1070	7388	6417	4291	0882	5246	4611
Weak	3	4384	2914	5243	4606	5279	5016	5243	4606	5279	5016
	5	5046	3799	5065	3194	5329	4909	4968	3033	5279	4781
	5	6259	4499	6317	3563	6753	6047	5018	3145	5287	4863
	9	7392	5831	6147	1114	7619	6654	4290	0922	5238	4603
	9	7314	5573	5962	1067	7580	6504	4140	0865	5178	4488
Mild	3	3408	2084	4237	3588	4286	4009	4237	3588	4286	4009
	5	4865	3539	4845	2735	5242	4643	4677	2583	5141	4461
	9	6998	5521	5862	1073	7227	6315	4268	0896	5178	4564
	9	4762	3053	3612	0395	5323	4105	1556	0235	2092	1718
Medium	3	1513	0802	2091	1572	2152	1877	2091	1572	2152	1877
	5	3160	1921	3185	1394	3657	2992	2031	1095	2176	1958
	5	2384	1386	2652	1083	2974	2470	1805	0833	1985	1695
	9	3718	2213	3257	0330	4471	3652	1597	0223	2120	1770
	9	2932	1718	2524	0243	3600	2856	2279	0228	3300	2594
	9	1731	0885	1058	0075	1965	1278	0236	0036	0300	0258
Strong	3	0483	0231	0758	0498	0800	0634	0758	0498	0800	0634
	5	0657	0346	0678	0193	0853	0596	0182	0099	0186	0176
	5	0508	0247	0631	0242	0717	0589	0859	0273	1046	0797
	9	0403	0191	0332	0028	0549	0375	0559	0035	0980	0659
	9	0415	0199	0352	0017	0557	0404	0585	0028	1003	0705

Note 4. The first comlumn oi each of the headings refers to the performance of the test with normal alpha ($\alpha = .05$) while the second column represents the performance of the test using a purposefully inflated alpha ($\alpha = .25$) for the disconfirmation portion of the test.

Table 5. Results of Tests under Medium Power Conditions

POP	K	OMNI	ORDR	O&O	UNCORRECTED PAIRS	UNCORRECTED RIGHT	UNCORRECTED LEFT	POST-HOCS LSD	POST-HOCS BONF	POST-HOCS TUKEY
Null	3	0463	1669	0074	0001	0025	0020	0001	0001	0000
	5	0489	0066	0005	0000	0000	0000	0000	0000	0000
	9	0502	0000	0000	0000	0000	0000	0000	0000	0000
Linear	3	6984	8335	6113	0845	3097	3073	0845	0280	0046
	5	7161	2509	2012	0000	0216	0220	0000	0000	0000
	9	7244	0017	0015	0000	0001	0002	0000	0000	0000
Concave	3	7723	6498	5180	0466	4418	0961	0466	0148	0021
	5	7789	0632	0502	0000	0391	0000	0000	0000	0000
	5	7518	0988	0764	0000	0464	0011	0000	0000	0000
	5	7132	1874	1504	0000	0491	0043	0000	0000	0000
Ogive	5	8172	2418	2103	0000	0194	0210	0000	0000	0000
	5	9309	1018	0954	0000	0257	0018	0000	0000	0000
Irreg	5	8589	1738	1552	0000	0093	0242	0000	0000	0000
Concave	9	6213	0003	0002	0000	0003	0000	0000	0000	0000
	9	7109	0014	0013	0000	0000	0000	0000	0000	0000
Ogive	9	8863	0017	0017	0000	0000	0001	0000	0000	0000
	9	9291	0014	0013	0000	0004	0000	0000	0000	0000
Irreg	9	7283	0010	0009	0000	0000	0001	0000	0000	0000
Weak	3	8279	5062	4201	0263	4238	0482	0263	0075	0014
	5	6412	1717	1250	0000	0104	0212	0000	0000	0000
	5	8530	1461	1292	0000	0105	0121	0000	0000	0000
	9	7584	0026	0025	0000	0000	0000	0000	0000	0000
	9	7899	0019	0017	0000	0001	0000	0000	0000	0000
Mild	3	7738	3325	2484	0104	0193	2604	0104	0019	0004
	5	7017	1102	0831	0000	0108	0083	0000	0000	0000
	9	7236	0011	0011	0000	0000	0001	0000	0000	0000
	9	7217	0000	0000	0000	0000	0000	0000	0000	0000
Medium	3	6979	0804	0419	0003	0008	0441	0003	0000	0000
	5	7102	0193	0123	0000	0049	0000	0000	0000	0000
	5	7010	0103	0060	0000	000[illegible]	0027	0000	0000	0000
	9	7241	0000	0000	0000	0000	0000	0000	0000	0000
	9	7232	0000	0000	0000	0000	0000	0000	0000	0000
	9	7311	0000	0000	0000	0000	0000	0000	0000	0000
Strong	3	7682	0100	0021	0000	0000	0022	0000	0000	0000
	5	7016	0002	0001	0000	0000	0000	0000	0000	0000
	5	6996	0006	0002	0000	0000	0001	0000	0000	0000
	9	7275	0000	0000	0000	0000	0000	0000	0000	0000
	9	7183	0000	0000	0000	0000	0000	0000	0000	0000

Table 5 (continued).

POP	K	TREND TESTS TAU	F-LIN	MONO	DISCONFIRMATION TESTS F-NON		REVERSAL		BONF-REV	
Null	3	0380	0498	0498	0473	2384	0956	4776	0470	2546
	5	0449	0511	0517	0494	2544	1938	7763	0514	2684
	9	0499	0538	0521	0508	2551	3535	9569	0506	2606
Linear	3	8126	8692	8692	0495	2490	0026	0393	0005	0126
	5	8956	9356	8813	0493	2530	0386	3144	0066	0612
	9	9623	9752	8386	0518	2500	1791	8029	0186	1246
Concave	3	7979	8693	8693	1991	5047	0203	1409	0096	0656
	5	7493	8397	8741	4053	7334	1247	5882	0316	1710
	5	7959	8673	8791	2904	6194	0931	5203	0215	1310
	5	8646	9042	8723	1173	3867	0569	3860	0116	0842
Ogive	5	9389	9683	8828	0744	3041	0498	3489	0096	0748
	5	9529	9796	8740	3339	6528	1173	5578	0280	1624
Irreg	5	9436	9672	8737	1565	4533	0821	4488	0198	1181
Concave	9	7963	8432	8386	2265	5585	2417	8753	0281	1695
	9	9433	9598	8368	0997	3547	2053	8245	0208	1385
Ogive	9	9873	9947	8393	0826	3206	2003	8126	0218	1372
	9	9859	9945	8352	2127	5439	2449	8634	0311	1743
Irreg	9	9576	9761	8345	0675	2840	1971	8198	0222	1346
Weak	3	7879	8721	8721	3570	6764	0481	2424	0237	1316
	5	8346	8838	8810	0967	3416	0696	4156	0167	0984
	5	9369	9627	8695	1769	4831	1030	5004	0266	1446
	9	9669	9779	8389	0588	2722	1902	8190	0208	1341
	9	9723	9820	8423	0784	3157	2232	8350	0269	1604
Mild	3	6556	7645	7645	4582	7618	1064	4040	0585	2502
	5	8259	8808	8769	1835	4828	1815	5793	0669	2326
	9	9566	9695	8366	0646	2972	2219	8384	0289	1566
	9	8356	8658	3711	3314	6766	4767	9592	1134	3863
Medium	3	2689	3970	3970	6773	8982	4033	7599	2880	6108
	5	6346	7037	3983	4437	7571	4249	8398	2050	4963
	5	5079	5898	3931	5347	8247	3482	8574	1258	4334
	9	7399	7816	3738	4519	7722	3958	9575	0652	2970
	9	6346	6705	6253	5314	8267	5215	9763	1409	4282
	9	4036	4260	0223	6591	9007	8744	9979	5669	8258
Strong	3	0479	1045	1045	8413	9642	7663	9540	6600	8940
	5	1126	1560	0083	7288	9232	8106	9878	5614	8607
	5	0536	0921	1722	7437	9307	5528	9646	2570	6496
	9	0576	0767.	1663	7469	9376	8190	9980	3990	7494
	9	0576	0690	1629	7436	9301	8246	9973	3946	7522

Table 5 (continued).

COMBINED TESTS (TREND TESTS & DISCONFIRMATION TESTS)

				LINEAR and				MONOTONIC and			
POP	K	NONLINEAR		REVERSAL		BONF-REV		REVERSAL		BONF-REV	
Null	3	0470	0374	0496	0471	0497	0486	0496	0471	0497	0486
	5	0491	0374	0484	0259	0506	0466	0467	0225	0503	0435
	9	0517	0400	0400	0060	0521	0444	0365	0043	0508	0421
Linear	3	8260	6535	8684	8514	8691	8642	8684	8514	8691	8642
	5	8892	6990	9034	6601	9300	8849	8570	6418	8779	8419
	9	9248	7318	8035	1950	9576	8557	7050	1829	8252	7481
Concave	3	6977	4311	8603	7784	8655	8316	8603	7784	8655	8316
	5	4967	2208	7527	3816	8179	7162	7811	3869	8514	7431
	5	6181	3320	7995	4497	8531	7704	8107	4511	8642	7815
	5	7983	5548	8597	5798	8958	8382	8327	5697	8643	8134
Ogive	5	8966	6743	9232	6399	9596	9002	8544	6195	8780	8370
	5	6519	3400	8672	4388	9534	8243	7957	4262	8584	7625
Irreg	5	8160	5281	8917	5421	9494	8579	8212	5235	8626	7941
Concave	9	6522	3719	6491	1137	8211	7057	6489	1131	8169	7060
	9	8642	6193	7664	1715	9404	8294	6846	1634	8223	7352
Ogive	9	9124	6761	7964	1871	9732	8585	6968	1801	8242	7413
	9	7827	4532	7516	1363	9637	8219	6569	1327	8149	7099
Irreg	9	9107	6992	7868	1790	9549	8471	6874	1701	8183	7351
Weak	3	5630	2825	8465	7017	8613	7874	8465	7017	8613	7874
	5	8007	5841	8307	5431	8726	8082	8244	5342	8683	8012
	5	7924	4969	8691	4904	9387	8305	7969	4724	8518	7642
	9	9208	7119	7953	1801	9584	8494	6978	1702	8248	7409
	9	9055	6719	7646	1642	9560	8257	6749	1566	8234	7239
Mild	3	4184	1842	7214	5224	7432	6332	7214	5224	7432	6332
	5	7201	4583	7365	3951	8298	6939	7249	3866	8216	6820
	9	9066	6811	7574	1595	9422	8201	6679	1513	8135	7181
	9	5788	2795	4632	0387	7712	5409	2388	0283	3470	2697
Medium	3	1284	0399	3163	1613	3458	2346	3163	1613	3458	2346
	5	3920	1696	4379	1387	5840	3903	3069	1182	3681	2829
	5	2753	1031	4296	1145	5356	3843	3049	0950	3640	2781
	9	4282	1767	4894	0388	7346	5639	2534	0294	3540	2857
	9	3146	1157	3428	0203	5857	4024	3108	0185	5405	3676
	9	1465	0432	0635	0015	2001	0866	0122	0008	0197	0143
Strong	3	0160	0036	0565	0177	0715	0339	0565	0177	0715	0339
	5	0400	0105	0497	0051	0938	0385	0065	0021	0075	0058
	5	0213	0064.	0583	0123	0805	0502	0924	0151	1373	0787
	9	0206	0046	0219	0006	0536	0277	0454	0007	1108	0571
	9	0180	0066	0176	0007	0488	0232	0410	0011	1109	0540

Table 6. Results of Tests Under High Power Conditions

					UNCORRECTED			*POST-HOCS*		
POP	K	OMNI	ORDR	O&O	PAIRS	RIGHT	LEFT	LSD	BONF	TUKEY
Null	3	0472	1682	0080	0000	0019	0025	0000	0000	0000
	5	0517	0081	0004	0000	0000	0000	0000	0000	0000
	9	0529	0000	0000	0000	0000	0000	0000	0000	0000
Linear	3	9497	9495	9075	3122	5966	5962	3122	1667	0533
	5	9530	4298	4189	0000	0938	0918	0000	0000	0000
	9	9497	0064	0064	0000	0004	0003	0000	0000	0000
Concave	3	9757	7218	7066	1256	7903	1496	1256	0612	0187
	5	9796	0792	0781	0000	1638	0004	0000	0000	0000
	5	9684	1406	1368	0000	2167	0023	0000	0000	0000
	5	9507	3102	2998	0000	2332	0147	0000	0000	0000
Ogive	5	9855	3667	3631	0000	0668	0637	0000	0000	0000
	5	9984	1228	1226	0000	0511	0043	0000	0000	0000
Irreg	5	9930	2446	2429	0000	0274	0586	0000	0000	0000
Concave	9	9029	0006	0004	0000	0011	0000	0000	0000	0000
	9	9478	0037	0036	0000	0031	0000	0000	0000	0000
Ogive	9	9944	0066	0066	0000	0005	0007	0000	0000	0000
	9	9984	0022	0022	0000	0020	0000	0000	0000	0000
Irreg	9	9544	0054	0054	0000	0012	0002	0000	0000	0000
Weak	3	9881	4970	4912	0482	7201	0519	0482	0230	0064
	5	9262	2793	2642	0000	0416	1043	0000	0000	0000
	5	9938	1814	1806	0000	0340	0332	0000	0000	0000
	9	9636	0066	0064	0000	0006	0001	0000	0000	0000
	9	9773	0054	0054	0000	0003	0003	0000	0000	0000
Mild	3	9754	2880	2801	0121	0136	4823	0121	0050	0010
	5	9530	1098	1050	0000	0391	0209	0000	0000	0000
	9	9542	0048	0047	0000	0007	0006	0000	0000	0000
	9	9545	0001	0001	0000	0000	0000	0000	0000	0000
Medium	3	9507	0268	0220	0001	0002	0506	0001	0000	0000
	5	9535	0077	0071	0000	0095	0000	0000	0000	0000
	5	9562	0029	0024	0000	0000	0063	0000	0000	0000
	9	9531	0000	0000	0000	0000	0001	0000	0000	0000
	9	9529	0000	0000	0000	0000	0000	0000	0000	0000
	9	9537	0000	0000	0000	0000	0000	0000	0000	0000
Strong	3	9738	0000	0000	0000	0000	0006	0000	0000	0000
	5	9551	0000	0000	0000	0000	0000	0000	0000	0000
	5	9548	0001	0001	0000	0000	0000	0000	0000	0000
	9	9507	0000	0000	0000	0000	0000	0000	0000	0000
	9	9508	0000	0000	0000	0000	0000	0000	0000	0000

Table 6 (continued).

		TREND TESTS			DISCONFIRMATION TESTS					
POP	K	TAU	F-LIN	MONO	F-NON		REVERSAL		BONF-REV	
Null	3	0476	0494	0494	0487	2436	0957	4786	0498	2627
	5	0496	0465	0484	0533	2524	1981	7689	0502	2665
	9	0499	0513	0536	0533	2512	3579	9537	0529	2612
Linear	3	9659	9879	9879	0525	2513	0004	0080	0003	0022
	5	9963	9968	9881	0477	2480	0161	1875	0023	0268
	9	9969	9989	9701	0476	2469	1383	7338	0120	0942
Concave	3	9586	9885	9885	3503	6700	0136	1080	0065	0456
	5	9316	9740	9861	7008	9147	1073	5573	0271	1481
	5	9633	9845	9882	5181	8160	0801	4566	0169	1114
	5	9873	9935	9872	1768	4772	0338	2831	0059	0537
Ogive	5	9993	9989	9850	0988	3634	0340	2680	0076	0514
	5	9999	9999	9858	5940	8561	1103	5518	0272	1568
Irreg	5	9996	9993	9862	2792	6007	0647	3858	0138	0958
Concave	9	9443	9707	9719	4140	7405	2218	8561	0247	1563
	9	9923	9982	9687	1443	4446	1692	7720	0171	1164
Ogive	9	9996	10E3	9667	1116	3914	1758	7722	0189	1206
	9	9996	10E3	9642	3779	7150	2233	8402	0242	1567
Irreg	9	9966	9991	9680	0760	3065	1555	7491	0163	1052
Weak	3	9496	9882	9882	6249	8666	0547	2532	0279	1366
	5	9819	9891	9873	1430	4211	0581	3365	0153	0814
	5	9996	9992	9885	3273	6578	1020	4751	0256	1374
	9	9976	9997	9697	0659	2767	1518	7490	0151	1040
	9	9983	9995	9665	1075	3780	2127	8050	0293	1509
Mild	3	8726	9544	9544	7368	9265	1460	4648	0881	3005
	5	9783	9871	9882	3187	6555	2511	6406	1032	3071
	9	9963	9991	9666	0814	3250	2074	7823	0311	1484
	9	9656	9796	5414	5961	8632	6015	9769	1906	5135
Medium	3	3879	6347	6347	9214	9864	6219	8988	5069	7974
	5	8659	9167	6167	7399	9315	6258	9221	3924	6887
	5	7329	8398	6207	8434	9684	4894	9326	2036	5779
	9	9123	9377	5386	7465	9305	4452	9743	0826	3425
	9	8353	8747	8418	8264	9614	6600	9902	2532	5717
	9	5946	6036	0166	9186	9894	9801	9999	8533	9647
Strong	3	0449	1551	1551	9870	9996	9525	9967	9095	9860
	5	1349	2199	0040	9609	9951	9642	9993	8494	9777
	5	0426	1071	2487	9653	9968	7798	9932	4774	8461
	9	0486	0785	2185	9604	9955	9525	9997	6749	9215
	9	0486	0821	2177	9598	9949	9510	9998	6698	9174

Table 6 (continued).

COMBINED TESTS (TREND TESTS & DISCONFIRMATION TESTS)

				LINEAR and				MONOTONIC and			
POP	K	NONLINEAR		REVERSAL		BONF-REV		REVERSAL		BONF-REV	
Null	3	0472	0384	0493	0472	0494	0488	0493	0472	0494	0488
	5	0440	0360	0434	0251	0460	0420	0429	0229	0470	0408
	9	0487	0382	0390	0062	0493	0434	0391	0060	0514	0439
Linear	3	9357	7400	9875	9816	9876	9861	9875	9816	9876	9861
	5	9492	7496	9810	8112	9945	9704	9738	8087	9863	9635
	9	9513	7524	8607	2662	9869	9047	8424	2651	9592	8837
Concave	3	6430	3266	9770	8872	9831	9471	9770	8872	9831	9471
	5	2920	0836	8752	4405	9491	8371	8845	4412	9604	8448
	5	4741	1815	9072	5408	9684	8772	9112	5420	9722	8810
	5	8175	5193	9606	7146	9878	9410	9557	7129	9823	9364
Ogive	5	9001	6358	9649	7317	9913	9477	9556	7301	9791	9393
	5	4060	1439	8896	4482	9727	8431	8844	4480	9617	8393
Irreg	5	7202	3988	9347	6140	9856	9036	9269	6130	9741	8966
Concave	9	5694	2526	7590	1430	9469	8216	7605	1425	9488	8245
	9	8540	5540	8292	2280	9811	8819	8119	2270	9536	8615
Ogive	9	8884	6086	8242	2278	9811	8794	8071	2274	9505	8594
	9	6221	2850	7767	1598	9758	8433	7601	1595	9437	213
Irreg	9	9233	6930	8440	2509	9828	8942	8237	2493	9534	8702
Weak	3	3713	1319	9385	7445	9638	8593	9385	7445	9638	8593
	5	8476	5725	9329	6599	9746	9101	9309	6595	9723	9084
	5	6722	3420	8976	5247	9736	8622	8918	5241	9648	8578
	9	9338	7230	8480	2510	9846	8957	8290	2501	9557	8734
	9	8920	6216	7870	1950	9702	8488	7689	1939	9405	8269
Mild	3	2516	0705	8317	5301	8845	6870	8317	5301	8845	6870
	5	6728	3403	7427	3577	8868	6873	7422	3577	8867	6864
	9	9178	6743	7922	2177	9680	8510	7729	2165	9378	8283
	9	3951	1335	3922	0229	7935	4785	2673	0200	4729	3148
Medium	3	0488	0099	3105	0931	3912	1766	3105	0931	3912	1766
	5	2384	0627	3552	0763	5697	2965	3042	0727	4514	2602
	5	1324	0269	4623	0660	6894	3875	3767	0609	5302	3255
	9	2391	0667	5288	0256	8624	6242	3311	0213	5027	3812
	9	1509	0330	3094	0098	6602	3863	2936	0093	6295	3683
	9	0477	0059	0139	0001	0959	0247	0024	0001	0087	0033
Strong	3	0013	0000	0242	0025	0404	0086	0242	0025	0404	0086
	5	0085	0009	0150	0004	0535	0098	0018	0002	0030	0018
	5	0039	0001	0472	0044	0754	0379	0859	0053	1487	0657
	9	0032	0001	0064	0000	0308	0082	0168	0003	0896	0249
	9	0040	0006	0073	0001	0326	0117	0179	0002	0878	0283

Appendix A. SAS Program used to run Monte Carlo.

```
OPTIONS LINESIZE=79;
DATA A; INFILE CARDS MISSOVER;
ARRAY MN1 (J1) BAR1-BAR10;ARRAY MN2 (J2) BAR1-BAR10;
ARRAY MU (M) MU1-MU10;
*input this set of population means*;
INPUT K VAR POWN $ POPN $ FNCT05 FNCT25 FOCT TKCT MU1-MU10;
*initialize values*;
SIGLIN=0;SIGNON05=0;SIGMONO=0;ORDER=0;SIGRIGHT=0;SIGLEFT=0;
SIGTUK=0;SIGPAIRS=0;SIGNON25=0;SIGREV05=0;SIGREV25=0;SGREVC25=0;
SGREVC05=0;SIGNOTST=0;SIGBONF=0;SIGLSD=0;SIGOMNI=0;SGFLN25=0;
SGFLN05=0;SGFLR25=0;SGFLR05=0;SGFMR25=0;SGFMR05=0;SGFLRC25=0;
SGFLRC05=0;SGFMRC25=0;SGFMRC05=0;
*generate the next one of 10,000 samples*;
DO SAMPLE =1 TO 10000;
*get the sample means*;
DO J1=1 TO K; M=J1;MN1=SQRT(VAR)*RANNOR(0)+MU;END;
*test the omnibus null*;
SSB=(K-1)*VAR(OF BAR1-BAR10);
OMNI=(SSB/((K-1)*VAR) GE FOCT);SIGOMNI=SIGOMNI+OMNI;
*compute a prioris*;
ARRAY TC1 (I) T1C1-T1C9;
ARRAY TC2 (I) T2C1-T2C9; ARRAY RL (L) TC1-TC2; TN=1.645;
DO L=1 TO 2; DO I=1 TO K-1;RL=0;SWSQ=0; DO J1=1 TO K;WT=0;
IF L=1 THEN DO; IF J1=I THEN WT=-(K-I);IF J1 GT I THEN WT=1;END;
IF L=2 THEN DO; IF J1=(K-I+1) THEN WT=K-I;
IF J1 LT K-I+1 THEN WT=-1; END; RL=RL+WT*MN1;SWSQ=SWSQ+WT**2; END;
RL=(RL/(SQRT(VAR*SWSQ)) GE TN);END; END;
SIGRIGHT=SIGRIGHT+(SUM(OF T1C1-T1C9) = K-1);
SIGLEFT=SIGLEFT+(SUM(OF T2C1-T2C9)=K-1);
*COMPUTE LINEARS AND NONLINEARS; VLIN=0; DO J1=1 TO K;
VLIN=VLIN+(MN1*(J1-((K+1)/2))); END;
SSQW=K*(K-1)/12*(K+1);TLIN=VLIN/SQRT(SSQW*VAR);
SIGLIN=SIGLIN+(TLIN GE TN);
FNON=((SSB-((VLIN**2)/SSQW))/(K-2))/VAR;
SIGNON25=SIGNON25+(FNON GE FNCT25);
SIGNON05=SIGNON05+(FNON GE FNCT05);
*calculate the all pairwise tests*;
NPAIRS=0; NORDER=0; NTUK=0; DO J1=1 TO K-1; DO J2=J1+1 TO K;
NPAIRS=NPAIRS+((MN2-MN1) GE SQRT(VAR*2)*1.645);
NTUK=NTUK+((MN2-MN1) GE SQRT(VAR)*TKCT);
NORDER=NORDER+(MN2 GE MN1);
END; END; NP=K*(K-1)/2;
SIGPAIRS=SIGPAIRS+(NPAIRS=NP); SIGTUK=SIGTUK+(NTUK=NP);
ORDER=ORDER+(NORDER=NP); SIGLSD=SIGLSD+OMNI*(NPAIRS=NP);
SIGNOTST=SIGNOTST+OMNI*(NORDER=NP);
*calculate the adjacent groups tests including mono and reverse*;
BONF=0; J2=K; J1=1; TMONO=(MN2-MN1)/SQRT(VAR*2); NREV05=0; NREV25=0;
NREVC05=0; NREVC25=0; DO J1=1 TO K-1;J2=J1+1;
BONF=BONF+(PROBNORM((MN2-MN1)/SQRT(VAR*2)) GE (1-(.05/(K-1))));
NREV25=NREV25+(PROBNORM((MN1-MN2)/SQRT(VAR*2)) GE .75);
```

```
NREV05=NREV05+(PROBNORM((MN1-MN2)/SQRT(VAR*2)) GE .95);
NREVC25=NREVC25+(PROBNORM((MN1-MN2)/SQRT(VAR*2)) GE ((.75**(1/(K-1)))));
NREVC05=NREVC05+(PROBNORM((MN1-MN2)/SQRT(VAR*2)) GE ((.95**(1/(K-1)))));
END;
SIGBONF=SIGBONF+(BONF=K-1); SIGMONO=SIGMONO+(TMONO GE TN);
SIGREV25=SIGREV25+(NREV25 GE 1); SIGREV05=SIGREV05+(NREV05 GE 1);
SGFLN25=SGFLN25+(TLIN GE TN)*(FNON<FNCT25);
SGFLR25=SGFLR25+(TLIN GE TN)*(NREV25 =0);
SGFMR25=SGFMR25+(TMONO GE TN)*(NREV25 =0);
SGFLN05=SGFLN05+(TLIN GE TN)*(FNON<FNCT05);
SGFLR05=SGFLR05+(TLIN GE TN)*(NREV05 =0);
SGFMR05=SGFMR05+(TMONO GE TN)*(NREV05 =0);
SGREVC25=SGREVC25+(NREVC25 GE 1);
SGREVC05=SGREVC05+(NREVC05 GE 1);
SGFLRC25=SGFLRC25+(TLIN GE TN)*(NREVC25 =0);
SGFMRC25=SGFMRC25+(TMONO GE TN)*(NREVC25 =0);
SGFLRC05=SGFLRC05+(TLIN GE TN)*(NREVC05 =0);
SGFMRC05=SGFMRC05+(TMONO GE TN)*(NREVC05 =0);
END;
*print it out*;
FORMAT SIGOMNI ORDER SIGNOTST SIGPAIRS SIGRIGHT SIGLEFT SIGLSD SIGBONF
SIGTUK SIGLIN SIGNON05 SIGNON25 SIGMONO SIGREV05 SIGREV25 SGFLN05 SGFLN25
SGFLR05 SGFLR25 SGFMR05 SGFMR25 SGREVC05 SGREVC25 SGFLRC05 SGFLRC25
SGFMRC05 SGFMRC25 Z4.; OUTPUT;
CARDS; [see table 3 for data];
PROC PRINT NOOBS;
   VAR POPN K POWN SIGOMNI ORDER SIGNOTST SIGPAIRS SIGRIGHT
   SIGLEFT SIGLSD SIGBONF SIGTUK TAU SIGLIN SIGNON05 SIGNON25
   SIGMONO SIGREV05 SIGREV25 SGFLN05 SGFLN25 SGFLR05 SGFLR25
   SGFMR05 SGFMR25 SGREVC05 SGREVC25 SGFLRC05 SGFLRC25 SGFMRC05
   SGFMRC25 ;
```

Appendix B. Program Used to Calculate Taus.

```
OPTIONS LINESIZE=79;
DATA;INFILE CARDS MISSOVER;INPUT KK VAR POWN $ POPN $ M1-M9;
FILE PRINT;ARRAY TIES (T) T1-T100;
ARRAY GROUP (G) G1-G9;
ARRAY A (I1) A1-A180;
ARRAY B (I2) A1-A180;
ARRAY MU (K) M1-M9; NN=10;
AVEZ=0;SIGTAU=0;DO SAMPLE =1 TO 3000;
*get the random observations*;
DO I1=1 TO NN*KK; K=INT((I1-1)/NN)+1; A=RANNOR(10)*SQRT(VAR*NN)+MU; END;
*calculate s*;
S=0; DO I1=1 TO (KK-1)*NN; DO I2=INT((I1+NN)/NN)*NN+1 TO KK*NN;;
IF A < B THEN S=S+1; IF A > B THEN S=S-1; END; END;
*calculate number of sets of ties*;
T=0; DO I1=1 TO NN*KK-1;IF A =. THEN GO TO EL; T=T+1;TIES=1;
DO I2=I1+1 TO NN*KK; IF A = B THEN DO; TIES=TIES+1; B=.; END; END; IF
TIES=1 THEN T=T-1; EL:END;
*calculate standard error and z*;
N=KK*NN; DO G=1 TO KK; GROUP=NN; END; S1=0; S2=0; S3=0; S4=0; S5=0; S6=0;
NT=T; DO G=1 TO KK; S1=S1+GROUP*(GROUP-1)*(2*GROUP+5);
S3=S3+GROUP*(GROUP-1)*(GROUP-2);
S5=S5+GROUP*(GROUP-1); END; DO T=1 TO NT; S2=S2+TIES*(TIES-1)*(2*TIES+5);
S4=S4+TIES*(TIES-1)*(TIES-2);
S6=S6+TIES*(TIES-1); END;
VARIANC=1/18*(N*(N-1)*(2*N+5)-S1-S2)+(1/(9*N*(N-1)*(N-2))*S3*S4)
  +(1/(2*N*(N-1))*S5*S6);
Z=S/SQRT(VARIANC); AVEZ=AVEZ+(Z/3000);
SIGTAU=SIGTAU+(Z GE 1.645)*(10/3); END; SIGTAU=INT(SIGTAU);
OUTPUT; CARDS; [for data, see table 3];
PROC PRINT; VAR KK NN POWN POPN AVEZ SIGTAU;
```