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ABSTRACT

The purpose of this study was to determine whether students from different academic settings differ in their usage of metacognitive strategies in mathematical problem solving, and if they do, what the differences are. A questionnaire was administered to students at various levels from academic tracks and academic streams requiring them to self-report on: (1) their metacognitive beliefs; (2) their usage of metacognitive strategies in mental tasks involving memory, problem solving, and comprehension; and (3) their attitudes towards the learning of various academic subjects. Twenty items from the questionnaire were categorized according to orientation, organization, execution, verification, and beliefs. Within each category, the frequency of usage of these metacognitive strategies as reported by students were averaged, analyzed, and interpreted. An introduction, objectives, method, results, discussion, and research implications are included. The means of items by stream and level are appended. (KR)

STUDENTS' METACOGNITION IN MATHEMATICAL PROBLEM SOLVING

Paper presented at the Australian Association for Research in Education Annual Conference 28 Nov - 2nd Dec 1989

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ABSTRACT

This study aims to determine whether students from different academic settings differ in their usage of metacognitive strategies in mathematical problem solving, and if they do what the differences are.

Metacognition is considered by most educationists as an element necessary for many cognitive tasks. For example, in problem solving, it has been said that possessing knowledge alone is insufficient and problem solvers need to exhibit higher level cognitive skills like "self-regulation skills" (also known as metacognitive strategies) for successful problem solving (Gagne, 1985; Gagne & Glaser, 1987). Metacognitive strategies or "executive skills" as referred by Sternberg (1983) are generalized skills required for planning, monitoring, controlling, selecting and evaluating intellectual activities.

Over the past two years, a study on students' metacognitive strategies has been carried out with over a thousand secondary and pre-university students from 12 schools. A questionnaire adapted from Biggs (1987) was administered to students at various levels (Grade 8, Grade 10, Grade 11), from academic tracks (General, Science, Arts) and academic streams (Special, Express, and Normal) requiring them to self-report on their metacognitive beliefs; their usage of metacognitive strategies in mental tasks involving memory, problem solving and comprehension; and their attitudes towards the learning of various academic subjects.

20 items from the questionnaire were categorized following the framework proposed by Garofalo and Lester (1985). Within each stage, the frequency of usage of these metacognitive strategies as reported by the students were averaged, analysed and interpreted.

Some of the findings that emerged were:

(a) Normal stream students exhibited a lower usage of metacognitive strategies as compared to those in the Express and Special streams.

(b) Metacognitive strategies used by Normal stream students tended to be of the "surface" type.

(c) There is no significant difference in the frequency of usage of metacognitive strategies between students from different academic tracks.

(d) Students from different levels (Secondary 2, Secondary 4, and Pre-University) exhibited similar frequency of usage of metacognitive strategies in problem solving.

The implications of these findings on future research and development projects as well as the teaching of metacognitive strategies will be discussed in the paper.

STUDENTS' METACOGNITION IN MATHEMATICAL PROBLEM SOLVING

Problem solving is a complex task involving many types of knowledge and skills. Skills in planning, monitoring and revising strategies are as important as having a large domain of knowledge. It is undeniable that problem solving requires specialized knowledge such as linguistic, factual , schematic, strategic and procedural knowledge (Mayer, 1987). A number of researchers have also included metacognitive knowledge as another important factor that differentiates between the good and the average problem solver (Gagne, 1985; Briars & Larkin, 1984, Lester 1982). Past studies on metacognition have concentrated on tasks involving reading and memory work and little work is done with metacognition in problem solving. In mathematics, there is much interest to make students aware of metacognition and to develop their metacognitive skills. Lately researchers have begun to look at metacognitive skills in problem solving and have started to develop theoretical frameworks (Garofalo & Lester, 1985; Lester, 1985, Schoenfeld, 1985).

What is metacognition?

Metacognition is generally considered as "knowing about knowing" or what Schmitt and Newby (1986) refer to as "a body of knowledge that reflects knowledge itself". In other words, metacognition involves knowing the cognitive processes associated with an instructional task, and being able to use and monitor appropriate cognitive processes during the task. Although metacognition has been loosely defined, most psychologists (e.g. Brown, 1978; Flavell, 1976) consider metacognition to consist of two separate but related aspects (a) **knowledge about cognition** and (b) **regulation of cognition.**

Knowledge about cognition implies that a person is knowledgeable about variables that will affect one's instructional performance in a learning situation or during an instructional process. Drawing from their research on metamemory, Flavell (Flavell, 1987; Flavell & Wellman, 1977) suggested three variables that could influence a person's performance. They are **person** variables, **task** variables and **strategy** variables. Extending from metamemory research, Lester (Garofalo and Lester, 1985; Lester, 1985) felt that these variables will also influence one's performance in problem solving.

Knowledge about person variables involves knowing one's own cognitive resources, strengths, weaknesses and cognitive abilities and aptitudes. Garofalo and Lester (1985) proposed that in mathematics, this knowledge should include one's beliefs of own mathematical ability, affective characteristics such as motivation and anxiety, and the relationship between mathematics achievement and achievements in other subject areas. In mathematical problem solving, awareness of personal variables that could affect one's performance could play a major role in the success of the problem solving task. Identifying one's weaknesses or strengths in certain topics of mathematics, realizing that one is careless in computation and tends to make computational mistakes, and recognizing that one is weak in processing spatial and visual information, are some examples of this aspect of knowledge.

Knowledge about task variables implies that the individual knows the type of cognitive demands required in an instructional situation. Students' awareness of the different effects of semantic and syntactic structures (e.g., vocabulary, extraneous information and order of events) on the difficulty of word problems is an example of task knowledge. There is empirical evidence which shows that students are aware of the type of cognitive demands in solving word problems. Garofalo & Lester(1985) cited findings from their research which showed young children believed that (a) the type of numerical information in a word problem is an indicator of the difficulty of the word problem, (b) word problems are harder to solve than computation problems, (c) word problems can be solved by directly applying one or more arithmetic operations and (d) the correct operations depend on identifying the right "key words".

When an individual knows what strategies to use during an instructional situation to obtain the best results, then he is said to possess knowledge about strategy variables. Slife, Weiss, and Bell (1985) found that students who were mathematically weak but with IQ similar to regular students, had less knowledge about their problem-solving skills. Peterson and associates (Peterson, 1988; Peterson & Swing, 1982; Peterson, Swing, Stark, & Waas, 1984) investigated the effects of various metacognitive processes students used during a normal classroom mathematics instruction. In their studies, they videotaped the lessons and employed a stimulated recall procedure to probe students' cognitive processes when they were engaged during teacher instruction and also during seatwork. Among the many findings, they found that students were knowledgeable about cognition as shown by the various strategies used, such as, applying information at specific level, reworking problems, rereading the text found in problems, relating new information to prior knowledge, trying to understand the lesson or trying to solve a problem by using a specific operation.

Regulation of cognition

The regulation aspect of metacognition involves the type of decision behaviors exhibited in order to plan, monitor and evaluate one's action. Sternberg (1983) relates these types of behaviors as executive skills and proposes certain training strategies for the development of these executive skills. Although these skills are trainable, it is also believed that the regulation process is controlled by one's cognitive knowledge (Kluwe, 1987). In mathematics problem solving, for example, a student who believes that he/she tends to make computation mistakes and thus slows down and proceeds cautiously during the computation part of problem solving and rechecking the answers, is said to be exhibiting this executive skill.

This self-regulation process is important for successful problem solving. Schoenfeld(1985) after analysis of college students' protocol of their problem solving processes, concluded that at the self-control level, the lack of monitoring and assessing the situation could lead to failure in problem solving. Despite its importance, this cognitive procedure is not clearly demonstrated by young children and college students. Garofalo & Lester (1985) in their research found that young children did not routinely analyze information provided in the problem and did not monitor progress or validate the results. Similarly, Stifle et al. (1985) found that young students who were not mathematically inclined did not monitor their progress during problem solving. College students too, were not very efficient in regulating their problem solving behaviors. Schoenfeld (1985) found that the overall quality of college students' monitoring, assessing and executive decision-making in problem solving was relatively poor.

Metacognition in problem solving

Using Polya's (1957) heuristic problem-solving model as a foundation, Lester and associates (Lester,1985; Garofalo & Lester, 1985) proposed a cognitive-metacognitive framework for performance in various mathematical tasks. The framework consists of four cognitive components of orientation, organization, execution and verification. The four components correspond to Polya's four phases of problem solving of understanding, planning, carrying out the plan, and looking back. However, Lester differentiates his framework from Polya's as he believes that his "model purports to describe the categories of the cognitive component in terms of points during problem solving where metacognitive actions might occur" (Lester, 1985; p. 62). The four components can be briefly described as follows:

Orientation : At this stage, students need to assess and understand the problem. The skills exercised at this stage would be those of comprehension; analysis of information; assessment of familiarity of problem and task difficulty and the formation of internal representation.

Organization: This involves identifying goals then planning for the whole task and sub-tasks in order to achieve the goals and sub-goals.

Execution: The monitoring of behaviors exhibited in the execution of the plans falls into this category. It includes monitoring computation actions, maintaining progress towards the goal and assessing trade-off decisions between factors influencing the success of the problem-solving process.

Verification: This stage involves the monitoring and evaluating of the three components of orientation, organization and the execution of the whole problem-solving process.

Each component is controlled by metacognitive decisions made by the individual and the type of decisions will depend on his/her own knowledge of metacognition. Thus an individual's metacognition knowledge of person variables, task variables and strategy variables will influence the individual's action in the four components. For example, in the cognitive component of orientation, an individual may want to rephrase the text in order to help him/her understand the problem situation better or if the individual believes that he is better at processing visual information, he/she may reorganize and represent the text information visually. Thus, an individual with better metacognitive knowledge can use his/her executive decisions for better planning, execution, and monitoring of the problem solving process and, hopefully, achieve a higher success in solving problems. The depth of one's metacognitive knowledge can influence the type of strategies one uses for monitoring and regulating cognition during problem solving. For example, in the orientation component, an individual may use different types of strategies: "surface" strategy such as re-reading the problem, or "deep" strategy such as recalling old materials to link new materials found in the problem or "achieving" strategy such as analyzing and representing problem information in another format. Biggs (1987) defined surface strategies as superficial strategies and are reproduced through rote learning, deep strategies as meaningful ones that require the involvement of previous relevant knowledge while achieving strategies are those which involve organizing one's time and working space, as exhibited by a model student.

The amount of metacognitive knowledge, the frequency of use of metacognition and effectiveness of executive monitoring depend on a number of factors. Currently, research indicates strongly that the level and the execution of executive control process depends upon the type of task and the level of expertise of the individual (Lawson, 1984). Lawson after reviewing the literature concluded that the relationship between age and level of executive functioning is a complex one. He believes that executive functioning is task specific; it is related to the amount of "expertise" and is not directly proportional to age. Drawing from literature, he cited studies that showed young children who were experts at a particular task exhibiting better executive functioning than adult novices. Young children also exhibited some forms of cognitive monitoring. For example, in reading, children as young as fourth graders were already able to perform executive activities when reading text. Even poor readers were able to monitor their comprehension process although the processes were not very effective. However, when it comes to problem solving, young children may not be aware of metacognitive activities as older children (Kluwe, 1987). The proficiency of self-regulation process is not dependent on age per se but dependent on the experience and the knowledge base of the individual. Lawson, very appropriately sums up that "The adult novice will have the benefit of greater experience in executive processing and greater experience as a problem solver, both of which may compensate to some extent for the poverty of his or her knowledge base on a particular task " (p. 97).

While there are extensive studies on metacognition carried out with experts and novice, with academically-disabled students (Slife, Weiss & Bell, 1985) and with young children (e.g., Cross & Paris, 1988), there are insufficient studies carried out with youths from different academic backgrounds. This is important as knowledge gained in this area could provide teachers with some guidelines on what to teach to students with different academic backgrounds. However, knowledge in this area is lacking and there are a number of unanswered questions on the effects of academic settings on students' metacognition. For example, do students from different grade levels exhibit different amounts of cognitive knowledge? Do students from lower grade levels exhibit less frequent use of metacognitive skills such as monitoring, planning and verifying their answers when solving mathematics problems? What type of strategies do different grade-level students employ? Are the strategies surface type, deep, or achieving ones? Do students from different streams and different academic tracks exhibit different frequency of usage of metacognitive processes?

Objectives of this study

This study intends to investigate the metacognitive processes used by secondary school students in mathematics. Specifically, it seeks to answer the following questions:

1. How frequently do students employ metacognitive strategies during mathematics problem solving?

2. Do students from different academic settings (academic stream, academic tracks and grade levels) differ in their usage of metacognitive strategies?
3. Do students from different academic settings use different types of strategies (surface, deep or achieving strategies) ?

Academic setting of Singapore secondary schools

The secondary schools system in Singapore is organized along different grade-levels, academic tracks and streams. Streaming based on abilities is practiced in Singapore. For example, students can enter secondary schools only after they have passed their Grade 6 standardized examination. Based on their result students are then streamed to different curricula to cater for different paces of learning and aptitudes. There are three streams and they are:

(i) the Special Assisted Program (SAP), a four-year curriculum for Grade levels 7 to 10. Students in SAP program have to offer two languages (English Language and Chinese Language) at the first-language level. Only the top 10% of the Grade 6 student population is given the option to join the SAP program.

(ii) an Express Stream, which also offers a four-year curriculum for Grade levels 7 to 10 but in the Express stream, students will offer English as the first language level and another language at the second-language level. Both the SAP program students and the Express stream students will sit for the same standardized examination at the end of the fourth year.

(iii) a Normal stream, a 5-year curriculum for Grade levels 7 to 11 where students will offer two languages, English at the first language and another language at the second-language level. At Grade 10, students will sit for an examination specially for students from this stream and if they pass well, they will then be allowed to proceed to Grade 11 and sit for the same examination as the SAP program and the Express stream students at the end of the 5th year.

Within the SAP and Express streams, students can pursue different academic tracks, namely, a Science track, an Arts track and a General track. Students from different tracks follow different curriculum : the curriculum in the science track emphasizes the physical and biological sciences, the arts curriculum emphasizes social studies and English literature while the General curriculum is a broad curriculum which includes one science subject and some arts subjects. Mathematics is compulsory for all students at all levels and in all tracks.

After the completion of their secondary school education, students with good results can then opt to continue their education at the pre-university level (Grades 12-13). The pre-university curriculum is a matriculation course for admission to the local universities.

Method

Subjects

Over 2500 students from nine secondary and four pre-university junior colleges participated in the research on learning and teaching strategies. Using the stratified sampling method, the subjects were selected. Within each category of schools and pre-university colleges, the schools were randomly selected and within each school, the classes of students from each stream, level and academic track, were randomly chosen. Whole classes were used in the survey and in each class, one third of the students was randomly assigned to answer the Language form questionnaire on learning strategies, another third answered the Science and Mathematics form, and the rest answered the Social Studies form.

Seven hundred and seven students answered the Science and Mathematics form. Out of this, 37 sets of data were incomplete thus leaving a sample size of 670. The 670 students came from

(a) three streams, namely, Special Assistance Programme (SAP), Normal Stream (a 5-year secondary school education) and Express stream (a 4-year secondary school education);

(b) three grade levels (Grade 8, Grade 10, and Grade 12);

(c) three academic tracks (Arts, Science and General).

The distribution of students for each stream, level and academic tracks is given in Appendices A, B and C.

Instrument

The instrument used is the Study Skill Questionnaire (Chang, 1988; 1989). There were three forms, each pertaining to the study of specific subject areas, namely, Language, Science and Mathematics, and Social Studies. Within each form, there were three sections in the questionnaire with the first two sections being common to all the three forms. The first two sections contained items on learning strategies, attitude towards learning and their motives for learning and they were drawn from the Learning Process Questionnaire (Biggs, 1987). The third section contained items that were specific to the content area. For example, in the Science and Mathematics form, students were asked about the frequency of usage of metacognitive strategies in solving mathematical and science problems while in the Language form students were asked about their metacognitive strategies in reading comprehension and in listening.

This study reports only on the students' returns in the Science and Mathematics form and on the section asking students about their metacognitive strategies in problem solving. There were 20 items related to strategies used in mathematical problem solving and for the purpose of this study, the items were classified into five sections. The first four sections followed the cognitive-metacognitive framework suggested by Garofalo and Lester (1985) with four items in each component. The fifth section of items measured students' beliefs in strategies which would help them in problem solving.

(i) Orientation component. The items here concentrated on the process of reading and understanding of the problem (e.g. I analyze and try to understand the information given and draw inferences).

(ii) Organization component. The items in this section concentrated on the approach and the planning for execution of procedures (e.g. I turn an argument over in my mind a number of times before accepting it).

(iii) Execution component. The items tried to determine how students execute the plan during problem solving (e.g. I find that drawing diagrams helps me to solve problems).

(iv) Verification component. The items were directed at finding out the frequency various strategies were used to check answers and procedures (e.g. I check over my test to avoid making mistakes).

(v) Beliefs - The items determined the beliefs students have concerning mathematics problem solving (e.g. I believe there is only one best way in solving a problem).

The questionnaire had been pilot tested, validated and used in a number of research studies (Chang, 1988; 1989).

Procedure

The questionnaire required students to rate each item on a 5-point Likert scale, with a score of 5 indicating a frequently-used metacognitive strategy while a score of 1 indicating a rarely-used or never-used strategy. The questionnaire was administered to the whole class by the class teacher. Most students were able to finish answering the questionnaire within a one-period lesson. The class teacher explained some phrasing of items to students who could not understand the item.

Results

There are five sets of subscores with a maximum of 20 points per set. There is a score for each of the four problem solving components (orientation, organization, execution and verification) and one score for students' problem-solving beliefs. Analysis of variance (ANOVA) with significant level of 0.05 was carried out using the mean scores as the dependent variable. Three separate analyses were conducted with different independent variables, namely, stream; level; and academic track. The results of each analysis are described below.

Stream. The three streams of Express, Normal and Special are applicable to secondary schools only. Data from Grade 12 students were not included in the analyses.

The means of all the problem phases were found to be statistically different. In all the four phases, Normal stream students scored lower than students from the Express and the SAP stream indicating that Normal stream students had reported less frequent use of metacognitive strategies than students from SAP and Express Stream students (Figure 1). A follow-up test using Duncan's test showed that the means of SAP students and Express students were not significantly different.

The score in the verification component was higher compared to the three other phases. The means for the three phases of orientation, organization and execution were around 12.5 while the means for the verification component were around 15.5.

Based on classification by Bigg (1987), each item in the questionnaire was classified as either surface, deep or achieving strategy. Appendix A shows the classification of individual items together with the means of each item for the three streams of students. On the analysis of individual item, it was found that Normal stream students used surface strategies more often than deep or achieving strategies. For example, they reported that they used surface strategies like " I need to attend to the instructions carefully in order to get the required results" (mean = 3.57) more frequently than to deep strategies like "I analyze and try to understand the information and draw inferences" (mean = 3.03).

Level. The means of each component are shown in Figure 2. Statistically, there was no difference in the frequency of usage of metacognitive strategies between students from different levels, viz. Grade 8, Grade 10 and Grade 12. Again, the means for the verification component (averaging 16.0) were higher than the means of the other phases (averaging 12.5).

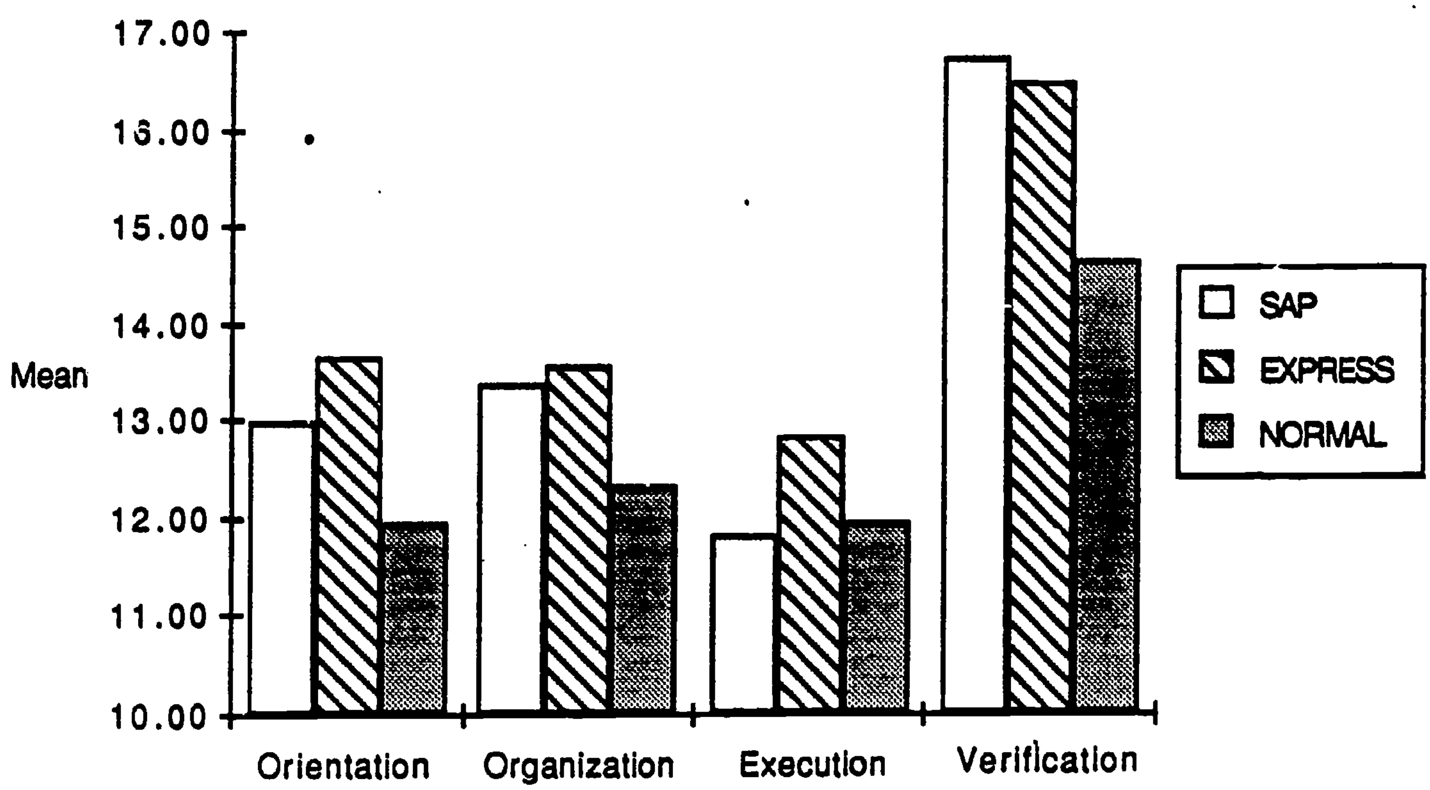


Figure 1: Means of each component by Stream (n=550)

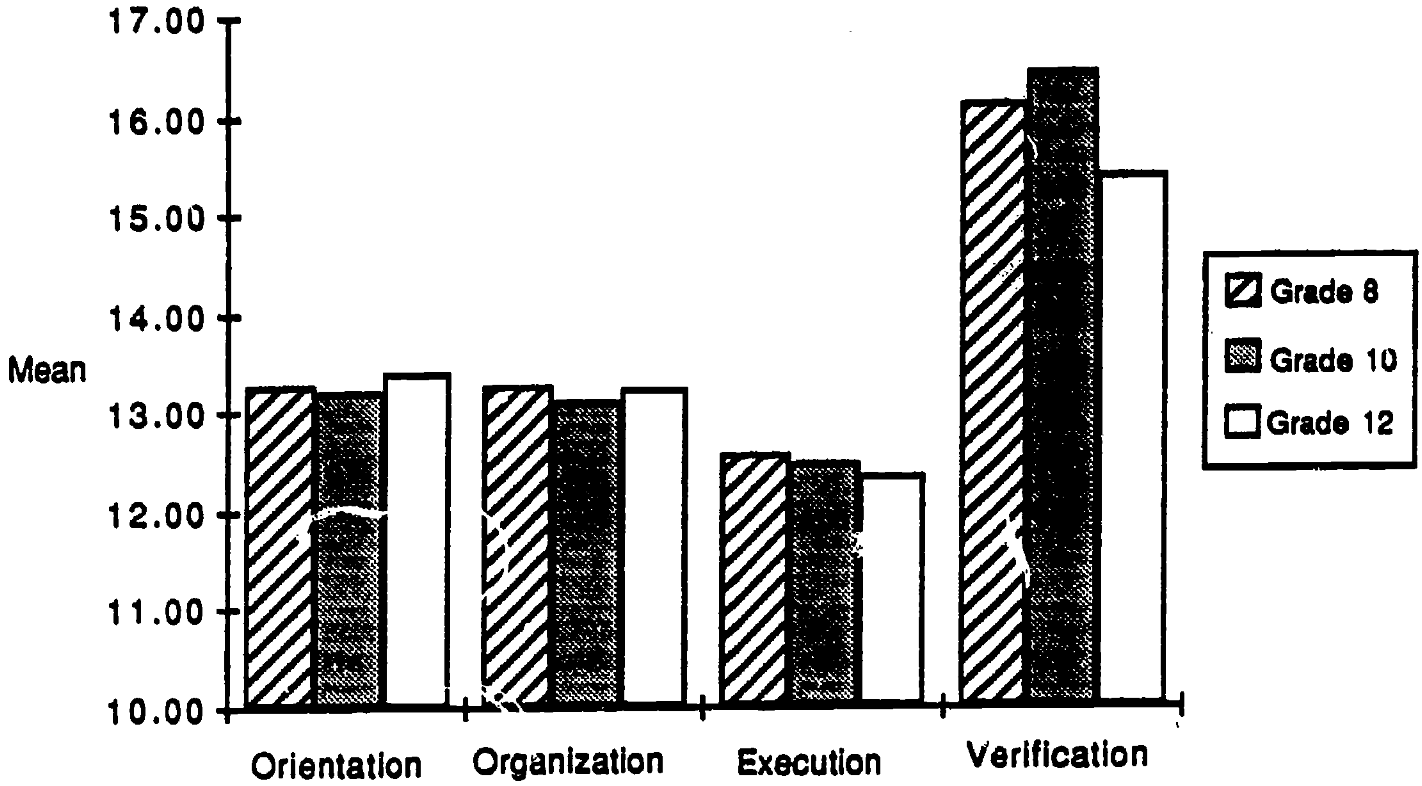


Figure 2: Means of each component by level (n= 670)

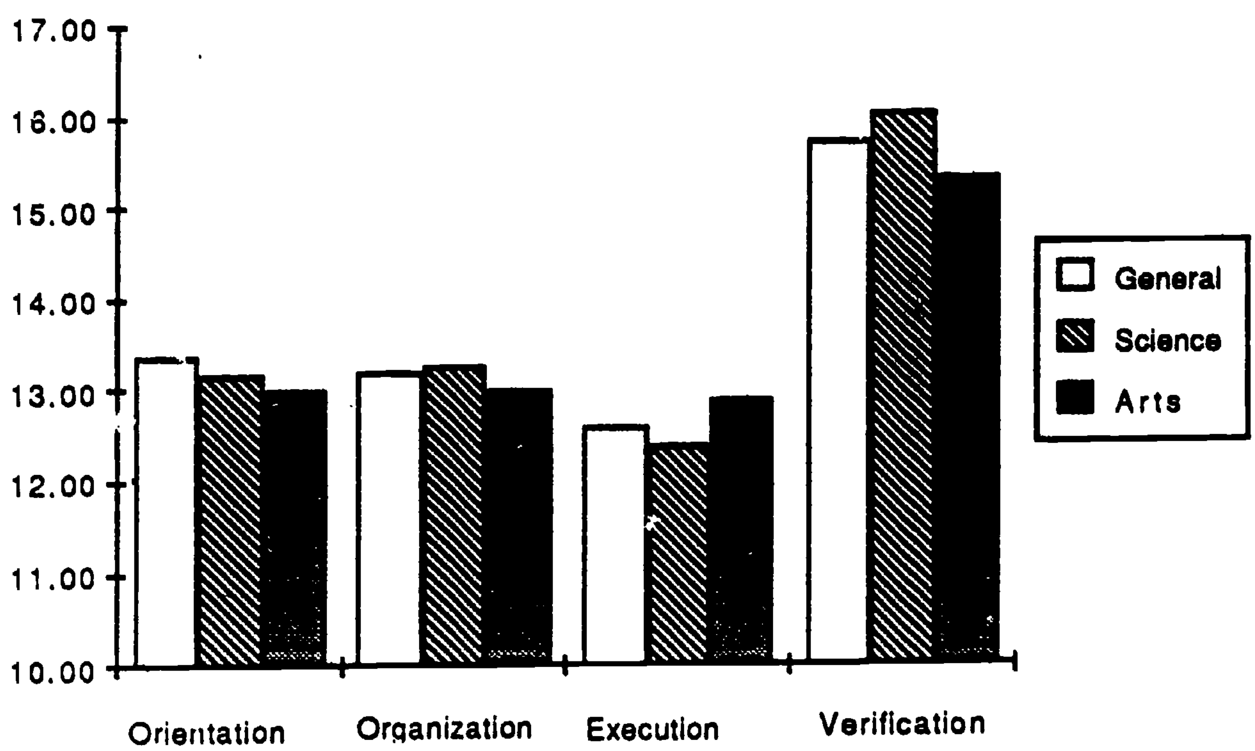


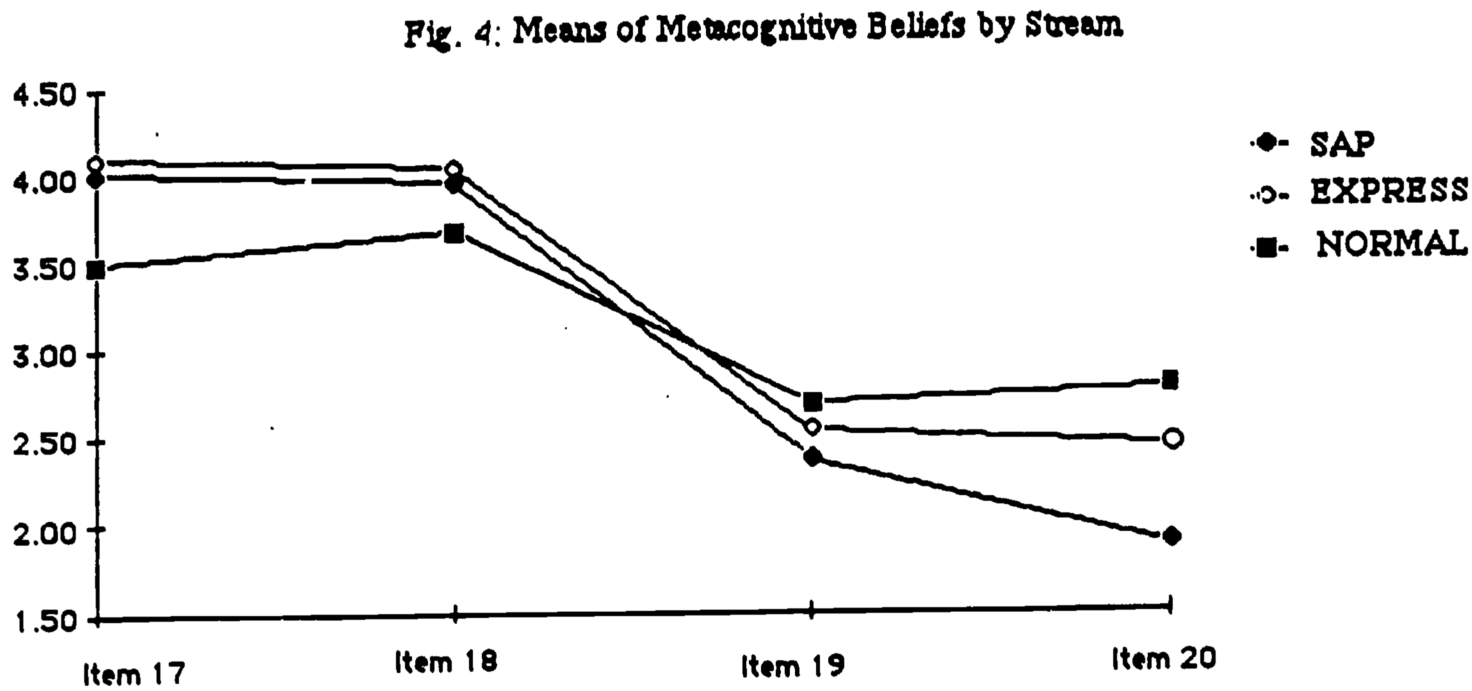
Figure 3: Means of each component by Academic Track (n= 640)

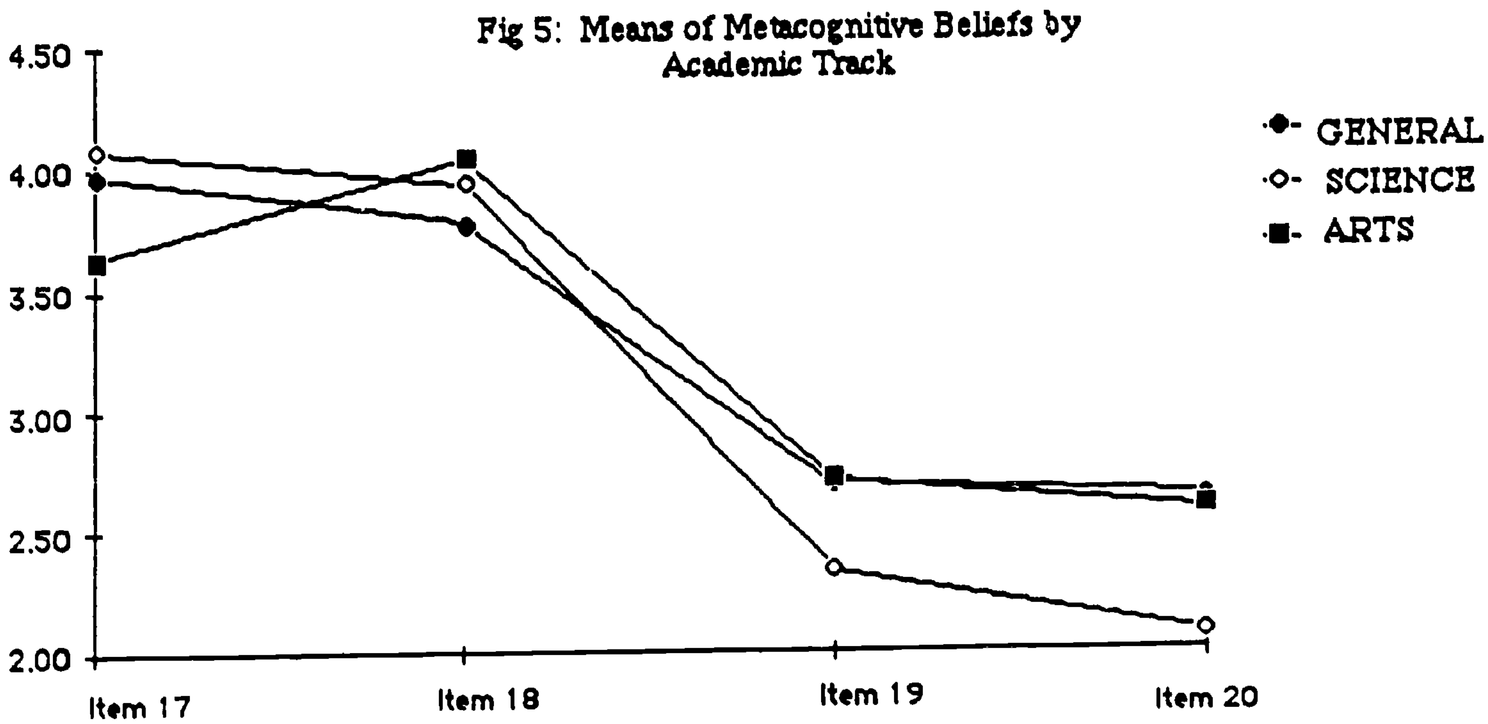
Academic Track. The means of the four phases are shown in Figure 3 and the means were found to be statistically not different for all the three tracks. The means for the verification component were higher than the means for the other three components (averaging 13.0).

Beliefs. Students' problem solving beliefs were investigated through four items (17, 18, 19, and 20). Figures 5, 6, and 7 show the means of the four items for stream, level and academic track. Students from the Normal stream, from Secondary Two and from General and Arts academic tracks believed that certain surface strategies were appropriate for developing problem solving skills. For example, in Item 19 they indicated that they memorized model answers more often than the other students. Similarly, in Item 20, more students from Grade 8, Normal stream, academic track of Arts and General, believed that there is only one best way to solve a problem. On the other hand, the Express and SAP students, students from the General and the Arts stream, and Grade 10 and Grade 12 students believed that certain deep strategies (e.g., they needed a lot of drill and practice; that it is important to be able to solve problems set in past-year examination) are important to their problem solving abilities. This is indicated by the higher ratings in Items 17 and 18.

Discussion

From the results, the mean scores for the four components are above the half-way mark of 10 indicating that students are conscious of metacognition and that they used strategies for monitoring and regulating the processes necessary for problem solving. Most students indicated that they practiced some of these metacognitive activities at least half the time when they are solving problems. Although the four components are equally important for problem solving, the





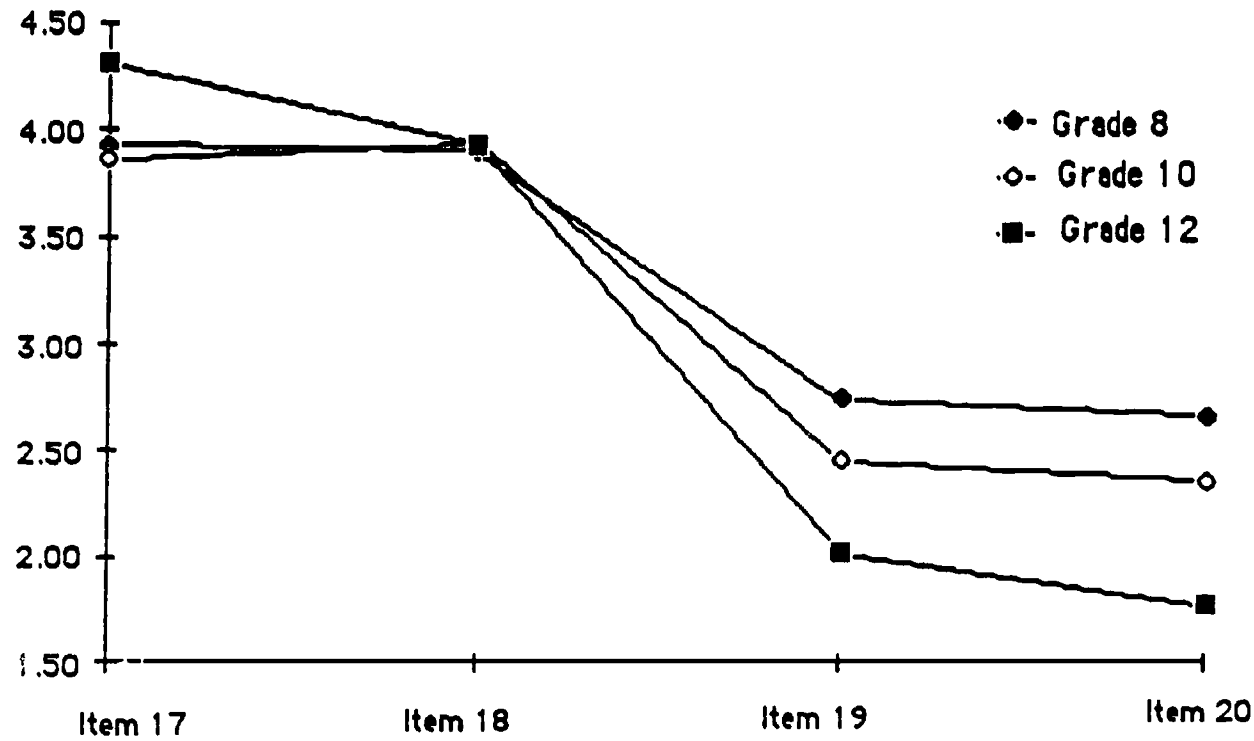
Item 17: In my revision, it is important to me to be able to solve problems set in past-year examinations.

Item 18: I need a lot of drill and practice in learning maths.

Item 19. There is only one best way in solving a problem.

Item 20: I memorize model answers.

Fig 6: Means of Metacogntivie Beliefs by Level



Item 17: In my revision, it is important to me to be able to solve problems set in past-year examinations.

Item 18: I need a lot of drill and practice in learning maths.

Item 19. There is only one best way in solving a problem.

Item 20: I memorize model answers.

students practice the verification process more frequently than the other components. This is indicated by the high scores for all the four items in this component. However, the four items of the questionnaire asked students on only one aspect of the verification component. The students were asked about the accuracy of their workings and the accuracy of their answers. This aspect of verification is emphasized during classroom instruction and hence, the high scores. Unfortunately, the questionnaire is unable to give us an insight about what really happened in their monitoring process during problem solving. A better method would be to use protocol analysis, interviews and observations as additional sources of data collection.

Although the results generally showed that students do practice metacognitive activities, certain groups of the student population were not very fluent in their usage. For example, the Normal stream students scored lower when compared to the SAP and Express stream students. This could be due to the selection process when students were streamed into SAP, Express or Normal. The Normal stream students follow a five-year secondary school education compared to the Express and the SAP stream students who follow a four-year secondary school education. The students entered the various streams based on the achievement scores in their primary school leaving examination. Thus academic ability could have an effect on the frequency of usage of metacognitive strategies and other researchers have reported similar findings (Chang,1989; Peterson, 1988; Slife et al., 1985). Also, the methods used in the teaching of students from different ability groups, the types of interactions between teachers and students, and the types of cognitive tasks could have caused the frequency of usage of metacognitive strategies. It has been observed that different teaching methods are used to teach students from low and high ability groups. Low academic ability students preferred high-structured instruction as compared to high academic ability students who preferred more flexible approaches to instruction

(Heinich, Molenda, & Russell, 1989). Maddus, Kellaghan and Schwab (1989) after reviewing literature on the effects of ability-grouping on achievement observed that:

> "Teachers in such (low-ability) classes tend to provide less appropriate instruction and resource materials; they tend to pace instruction too slowly; to ignore or underemphasize the substantive aspects of tasks, and to provide instructional materials that are less interesting and less challenging. ...Less task-related verbal interaction occurs between teachers and students in low-ability classes." (pp. 323)

The highly-structured instruction, the non-challenging tasks, and insufficient verbal interactions are some factors that do not encourage students to explore strategies or to discover other methods of seeking solutions to problems. This lack of environmental and instructional incentives could curb the development of metacognition in the low-ability students.

It appears that academic ability influences metacognition in three ways: first, the lack of academic ability impedes students' knowledge of strategies; second, the lack of knowledge of metacognition leads to poor academic performance and third, the teaching styles used can discourage or encourage the development of metacognitive skills. Unfortunately, findings from this research cannot differentiate them.

Age and years of schooling are some other factors that could influence one's knowledge and application of metacognitive strategies. Awareness of metacognitive strategies starts at a very early age. Various studies on metamemory have shown that children as young as five years old are aware of strategies for recall (see Flavell & Wellman, 1977). It is also observed that older children are better at using various strategies for recall than young children (e.g., Brown, 1978). This is also true in reading comprehension. For example, Myers and Paris (1978) found out that 12 year-old students were more aware of the effects of various variables, such as their knowledge of content and their interests in the stories, on their comprehension than 8 year-old students. Biggs (1987) in his study, noticed that young students may have the awareness of the needs of monitoring and regulating their cognitive processes but may not have sufficient executive control over them. However, in this study there was no statistical difference in scores between students of different ages as students from Grade 8, Grade 10 and Grade 12 indicated similar frequency of usage in the four components. While the level of usage remains the same across students from different levels and academic tracks, the types of strategies used differed. The younger students, the Arts and General track students and Normal stream students tended to use more surface strategies. However, the use of surface strategy should decline as the level changed to the higher level. Similar observations were also noted by Biggs (1987). This study does support the fact that students become increasingly more aware of metacognition with increasing years of schooling.

Research Implications

Research on metacognition should be a multifaceted task using a variety of research methods, instruments and a sample with different academic backgrounds. This study is the initial phase of the research project on effectiveness of learning strategies and is based on students' self report in a questionnaire. However, the use of self-report questionnaire has its limitations. Ideally, this report should be supported from evidence from interviews and verbal protocols. These procedures would give us a better insight of the type of problem-solving and metacognitive activities used. Unfortunately, there are limitations to this procedure. This research method is time consuming and only a handful of students can be interviewed. In Singapore where students are not very vocal, gathering data using this method is problematic and students have found this method to be unnatural, mentally demanding and difficult as they find it difficult to verbally express themselves (Wong, 1990).

Teaching Implications

This study shows that students need guided instruction in the use of metacognitive strategies for problem solving. Besides, the emphasis on verification of solutions, the students reported less frequent use of monitoring strategies in planning, executing and orientation. As such teachers should consider incorporating strategies to help students develop metacognitive skills in these three areas. Generally, teachers do not introduce metacognition as a topic in a lesson but instead subsumed the concept of metacognition within the lesson content. Thus, to the students metacognition is taught unconsciously and the concept of metacognition could be lost amongst the more important subject matter. Instead, there should be conscious and direct effort by the teachers to introduce the concept of metacognition during the lesson. Students need to be informed of what metacognition is, how it works, when it works and be provided with some examples. At present, at the teacher training level, trainee teachers are exposed to a number of lectures on this aspect. They are encouraged to use various methods to achieve this. They could incorporate some of the teaching methods, activities, and approaches suggested by Callahan & Garofalo (1987), Long (1986) and Devine (1981).

Drawing from this study and other reports, lower ability students do not use strategies for metacognition as frequently as high ability students and this deficiency could lead to poor performance. They, therefore, need extra training in order to enable them to operate at the same level as the higher ability students. Various projects on teaching students thinking skills, reading skills and learning skills have been very successful. For example, Peterson and associates (cited in

Peterson, 1988) conducted an extensive project which helps fourth grade students to develop thinking skills in mathematics, and noted that low ability students benefitted more from the training. She said " the thinking skills training may have provided the low ability students with the thinking skills or cognitive strategies that they did not have, but that higher ability students did have already.... Acquisition of these strategies then permitted them to learn as effectively as the higher ability students within the class" (p. 10). Attempts have been made by schools and the Ministry of Education to introduce some of these projects to students. Some examples include: the publication of a handbook, *Learning Skills in Content Area*, for secondary school teachers to be used for conducting workshops on learning skills, the introduction of DeBono's CORT programme to 25 secondary schools, and training programmes to student-teachers on effective teaching and learning skills.

Conclusion

This study reports on the frequency of usage of metacognitive activities and the type of strategies used by students from different academic settings in mathematical problem solving. It is generally observed that students are aware of metacognition although students from the Normal stream seem to use strategies on metacognition less frequently. There is a declining use of surface strategies and increasing use of deep and achieving strategies as the level changed to the higher level. The results of this study warrant a need to introduce the teaching of metacognition to all students especially to low ability students.

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APPENDIX A : MEANS OF ITEMS BY STREAM (n = 550)

ITEM	ORIENTATION	TYPE		SAP n=110	EXPRESS n=279	NORMAL n=161
1	I spend time to recall key points	A	Mean	2.57	2.59	2.39
			S.D.	1.51	1.51	1.50
2	I read over my text a number of times to understand and identify the important points	S	Mean	3.39	3.42	3.15
			S.D.	1.46	1.38	1.42
3	I attend to the instructions carefully in order to get the required results.	S	Mean	3.62	4.02	3.27**
			S.D.	1.44	1.28	1.50
4	I anlayse and try to understand the information and draw inferences	A	Mean	3.39	3.64	3.13**
			S.D.	1.49	1.38	1.42
	ORGANIZATION					
5	I think of different ways to solve a problem	A	Mean	3.23	3.42	3.05*
			S.D.	1.50	1.40	1.42
6	I think through the problems before turning to others for help.	S	Mean	4.09	4.14	3.69**
			S.D.	1.07	1.10	1.27
7	I turn an argument over in my mind a number of times before accepting it.	A	Mean	3.07	3.29	2.81**
			S.D.	1.37	1.34	1.25
8	I recall the key points and write a brief outline of the problems.	D	Mean	3.00	2.73	2.79
			S.D.	1.49	1.42	1.30
	EXECUTION					
9	I understand better by comparing and contrasting 2 sets of data	D	Mean	2.67	2.99	2.85
			S.D.	1.53	1.39	1.40
10	When solving problems I tend to skip those that happen to be hard	D	Mean	3.23	2.97	3.15
			S.D.	1.45	1.50	1.33
11	I organize my work in neat steps to help me do better	S	Mean	3.56	4.04	3.44*
			S.D.	1.46	1.22	1.47
12	I draw diagrams to help me to solve problems.	D	Mean	3.22	3.27	2.99*
			S.D.	1.50	1.46	1.56

* $p < .05$
**$p < .01$
A=achieving, D= deep, S=surface

APPENDIX A : MEANS OF ITEMS BY STREAM (n = 550)

ITEM	VERIFICATION	TYPE		SAP	EXPRESS	NORMAL
13	I check my answers with the answers given in the book	S	Mean	4.46	4.26	3.58*
			S.D.	0.99	1.10	1.27
14	I check over my test to avoid making mistakes	D	Mean	4.20	4.07	3.78*
			S.D.	1.06	1.22	1.38
15	I make it a point to check my workings to a problem before handing in my paper.	A	Mean	4.06	4.26	3.74*
			S.D.	1.25	1.07	1.37
16	When a test is returned, I go over it careful correcting all errors and trying to understand why I made the original mistakes.	A	Mean	4.01	3.90	3.56*
			S.D.	1.10	1.18	1.36
	BELIEFS					
17	In my revision, it is important to me to be able to solve problems set in past-year examinations.	D	Mean	4.01	4.10	3.48**
			S.D.	1.28	1.14	1.32
18	I need a lot of drill and practice in learning maths	D	Mean	3.95	4.04	3.67**
			S.D.	1.40	1.21	1.36
19	There is only one best way in solving a problem	S	Mean	2.37	2.54	2.68
			S.D.	1.49	1.40	1.49
20	I memorize model answers	S	Mean	1.89	2.45	2.78*
			S.D.	1.41	1.45	1.43

* $p < .05$
** $p < .01$
A=achieving, D= deep, S=surface

APPENDIX B : MEANS OF ITEMS BY LEVEL (n = 670)

ITEM	ORIENTATION	TYPE		SEC. 2 (n=205)	SEC. 4 (n=345)	PRE -U (n=120)
1	I spend time to recall key points	A	Mean	2.71	2.69	2.70
			S.D.	1.53	1.50	1.60
2	I read over my text a number of times to understand and identify the important points	S	Mean	3.35	3.35	3.27
			S.D.	1.39	1.42	1.71
3	I attend to the instructions carefully in order to get the required results.	S	Mean	3.77	3.71	3.78
			S.D.	1.35	1.45	1.84
4	I anlayse and try to understand the information and draw inferences	A	Mean	3.44	3.46	3.67
			S.D.	1.40	1.42	1.50
	ORGANIZATION					
5	I think of different ways to solve a problem	A	Mean	3.35	3.25	3.35
			S.D.	1.42	1.45	1.50
6	I think through the problems before turning to others for help.	S	Mean	3.96	4.00	4.05
			S.D.	1.21	1.16	0.99
7	I turn an argument over in my mind a number of times before accepting it.	A	Mean	3.14	3.08	3.22
			S.D.	1.30	1.30	1.57
8	I recall the key points and write a brief outline of the problems.	D	Mean	2.82	2.79	2.64
			S.D.	1.38	1.41	1.43
	EXECUTION					
9	I understand better by comparing and contrasting 2 sets of data	D	Mean	2.92	2.91	2.84
			S.D.	1.41	1.40	1.59
10	When solving problems I tend to skip those that happen to be hard	D	Mean	3.29	2.93	2.09**
			S.D.	1.50	1.36	1.07
11	I organize my work in neat steps to help me do better	S	Mean	3.83	3.72	4.11
			S.D.	1.41	1.39	1.24
12	I draw diagrams to help me to solve problems.	D	Mean	3.08	3.40	3.40
			S.D.	1.53	1.50	1.40

* $p < .05$
**$p < .01$
A=achieving, D= deep, S=surface

APPENDIX B : MEANS OF ITEMS BY LEVEL (n = 670)

ITEM	VERIFICATION	TYPE		SEC. 2	SEC. 4	PRE-U
13	I check my answers with the answers given in the book	S	Mean	3.96	4.11	4.16
			S.D.	1.24	1.17	0.96
14	I check over my test to avoid making mistakes	D	Mean	4.09	3.96	3.68*
			S.D.	1.22	1.28	1.47
15	I make it a point to check my workings to a problem before handing in my paper.	A	Mean	4.05	4.06	4.03
			S.D.	1.28	1.21	0.96
16	When a test is returned, I go over it careful correcting all errors and trying to understand why I made the original mistakes.	A	Mean	4.03	3.72	3.54**
			S.D.	1.13	1.28	1.49
	BELIEFS					
17	In my revision, it is important to me to be able to solve problems set in past-year examinations.	D	Mean	3.93	3.86	4.31**
			S.D.	1.22	1.28	1.40
18	I need a lot of drill and practice in learning maths	D	Mean	3.89	3.94	3.92
			S.D.	1.41	1.20	1.50
19	There is only one best way in solving a problem	S	Mean	2.74	2.45	2.02
			S.D.	1.45	1.55	1.10
20	I memorize model answers	S	Mean	2.66	2.36	1.77*
			S.D.	1.53	1.40	1.01

* $p < .05$
**$p < .01$
A=achieving, D= deep, S=surface

APPENDIX B : MEANS OF ITEMS BY TRACK (n = 640)

ITEM	ORIENTATION	TYPE		GENERA (n=205)	SCIENCE (n=261)	ARTS (n=174)
1	I spend time to recall key points	A	Mean	2.71	2.61	2.89
			S.D.	1.51	1.51	1.52
2	I read over my text a number of times to understand and identify the important points	S	Mean	3.39	3.29	3.25
			S.D.	1.31	1.47	1.45
3	I attend to the instructions carefully in order to get the required results.	S	Mean	3.80	3.75	3.45
			S.D.	1.27	1.43	1.56
4	I anlayse and try to understand the information and draw inferences	A	Mean	3.46	3.51	3.42
			S.D.	1.34	1.44	1.53
	ORGANIZATION					
5	I think of different ways to solve a problem	A	Mean	3.31	3.30	3.31
			S.D.	1.41	1.47	1.37
6	I think through the problems before turning to others for help.	S	Mean	3.98	4.07	3.88
			S.D.	1.16	1.13	1.25
7	I turn an argument over in my mind a number of times before accepting it.	A	Mean	3.17	3.16	2.90
			S.D.	1.29	1.37	1.26
8	I recall the key points and write a brief outline of the problems.	D	Mean	2.73	2.75	2.89
			S.D.	1.29	1.47	1.38
	EXECUTION					
9	I understand better by comparing and contrasting 2 sets of data	D	Mean	2.89	2.86	3.05
			S.D.	1.33	1.47	1.45
10	When solving problems I tend to skip those that happen to be hard	D	Mean	3.27	2.97	2.92
			S.D.	1.41	1.52	1.27
11	I organize my work in neat steps to help me do better	S	Mean	3.94	3.85	3.63
			S.D.	1.28	1.41	1.39
12	I draw diagrams to help me to solve problems.	D	Mean	3.05	3.36	3.49*
			S.D.	1.51	1.51	1.49

* $p < .05$
**$p < .01$
A=achieving, D= deep, S=surface

APPENDIX B : MEANS OF ITEMS BY TRACK (n = 640)

ITEM	VERIFICATION	TYPE		GFNERA	SCIENCE	ARTS
13	I check my answers with the answers given in the book	S	Mean	3.90	4.17	3.97
			S.D.	1.24	1.14	1.23
14	I check over my test to avoid making mistakes	D	Mean	3.94	3.98	3.81
			S.D.	1.31	1.22	1.36
15	I make it a point to check my workings to a problem before handing in my paper.	A	Mean	4.04	4.07	3.96
			S.D.	1.24	1.21	1.32
16	When a test is returned, I go over it careful correcting all errors and trying to understand why I made the original mistakes.	A	Mean	3.84	3.82	3.60
			S.D.	1.21	1.20	1.42
	BELIEFS					
17	In my revision, it is important to me to be able to solve problems set in past-year examinations.	D	Mean	3.97	4.08	3.63*
			S.D.	1.15	1.28	3.57
18	I need a lot of drill and practice in learning maths	D	Mean	3.77	3.94	4.04*
			S.D.	1.40	1.28	1.18
19	There is only one best way in solving a problem	S	Mean	2.70	2.33	2.72*
			S.D.	1.39	1.44	1.55
20	I memorize model answers	S	Mean	2.64	2.07	2.59*
			S.D.	1.48	1.42	1.44

* $p < .05$
**$p < .01$
A=achieving, D= deep, S=surface