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ABSTRACT

This module is the 12th in a series of 12 learning modules designed to teach occupational mathematics. Blocks of informative material and rules are followed by examples and practice problems. The solutions to the practice problems are found at the end of the module. Specific topics covered include order of operations, grouping, and use of the calculator. (YLB)

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MODULE 12 ORDER OF OPERATIONS

Calculation problems which contain several operations that are to be performed must be written and worked in a specific order. To better understand why there must be an agreement on the order of performing a series of operations, consider the problem of calculating

$$72+3(5)-6^2$$

This problem contains four operations: add, multiply, subtract and the square. A few of the possible orders of calculation include the cases:

Case 1: left to right order

$72+3(5)-6^2$	$= 75(5)-6^2$	addition
	$= 375-6^2$	multiplication
	$= 369^2$	subtract
	$= 136,161$	square

Case 2: multiply and then use a left to right order

$72+3(5)-6^2$	$= 72+15-6^2$	multiply
	$= 87-6^2$	add
	$= 81^2$	subtract
	$= 6561$	square

Case 3: square and multiply followed by add and subtract

$72+3(5)-6^2$	$= 72+3(5)-36$	square
	$= 72+15-36$	multiply
	$= 87-36$	add
	$= 51$	subtract

For this problem of four operations, there are several other possible operation orders and perhaps as many as a dozen different results. Noone, who had learned much in the previous eleven lessons, would suggest that all of these results are correct. Actually, a calculation problem can not be allowed to have more than one possible correct value. Mathematics is supposed to be like a laboratory in which you construct, on paper, the outcome of your design. For one set of input values, there is only one correct output result. Everyone who correctly works a particular problem will get the same single correct result. It is important that the person who writes problems and the people who calculate problems have an agreement (rule) about how a series of operations is to be ordered. The complete rule for the order of operations is as follows:

RULE: ORDER OF OPERATIONS

1. Perform the operations enc.osed by the innermost grouping symbols according to the order outlined in steps 2 through 5.

2. Perform all the powers and roots.

3. Perform the first multiply OR divide as you read from left to right. Continue with multiply or divide until all are performed.

4. Perform the first add OR subtract as you read from left to right. Continue with add or subtract until all are performed.

5. Return to step 1 if the next level of grouping symbols enclose operations. Return to step 2 if no grouping symbols enclose an operation.

As you are no doubt aware from earlier discussions about grouping symbols, one common type of grouping symbol is the parentheses: (4+17). Parentheses are also used to surround a number to be used with the multiplication operation as in (4)(17). The parentheses in (4)(17) are not being used for the purpose of a grouping symbol. No operation is "inside" the first pair nor the second pair of parentheses. When an operation symbol is inside the left and right of a pair of parentheses, (4+17), then the use of parentheses is that of a grouping symbol. Parentheses and other grouping symbols will be discussed later in greater detail.

The first problems with which we will apply the order of operations rule will not contain any grouping symbols. But there will be parentheses in some problems. When there is no operation enclosed by a grouping symbol, then the order rule begins with Step 2 and ends with Step 4.

When computing a sequence of operations without the aid of a calculator, perform only one operation at each pass through the problem. Rewrite the complete problem during each operation. Write a detailed step by step problem solution. Do not look for short-cuts.

EXAMPLE 1: Compute $84-(12)(4)+15$

Solution:

Step 1 of the order rule does not apply.
Parentheses are not being used as grouping symbols.
Step 2 of the order rule does not apply.
The calculation does not contain power or root.
Step 3 calls for multiplication or division to be performed.

$$84-(12)(4)+15 = 84-48+15$$

Step 4 calls for addition or subtraction to be performed.

$$\begin{aligned} 84-48+15 &= 36+15 \\ &= 51 \end{aligned}$$

EXAMPLE 2: Compute $102\div6-(3)(4)+8$

Solution:

Step 1 of the order rule does not apply.
Parentheses are not being used as grouping symbols.
Step 2 of the order rule does not apply.
The calculation does not contain power or root.
Step 3: Perform multiplication or division

$$\begin{aligned} 102\div6-(3)(4)+8 &= 17-(3)(4)+8 \\ &= 17-12+8 \end{aligned}$$

Step 4: Perform add or subtract

$$\begin{aligned} 17-12+8 &= 5+8 \\ &= 13 \end{aligned}$$

EXAMPLE 3: Compute the correct value of the problem from page 1 used to illustrate the need for an order rule:

$$72+3(5)-6^2$$

Solution:

Step 1 of the order rule does not apply.
Parentheses are not being used as grouping symbols.
Step 2: Perform powers and roots

$$72+3(5)-6^2 = 72+3(5)-36$$

Step 3: Perform multiplication or division

$$72+3(5)-36 = 72+15-36$$

Step 4: Perform addition or subtraction

$$\begin{aligned} 72+15-36 &= 87-36 \\ &= 51 \end{aligned}$$

EXAMPLE 4: Compute $7+3(4)^3\div12$

Solution:

Step 1 of the order rule does not apply.
Parentheses are not being used as grouping symbols.

Step 2: Perform all the powers and roots

$7+3(4)^3\div12 = 7+3(64)\div12$

Step 3: Perform multiply or divide

$7+3(64)\div12 = 7+192\div12$

$= 7+16$

Step 4: Perform add or subtract

$= 23$

EXAMPLE 5: Compute $30\div62-\sqrt{25}+3(6)$

Solution:

Step 1 of the order rule does not apply.
Parentheses are not being used as grouping symbols.

Step 2: Perform all the powers and roots

$30+62-\sqrt{25}+3(6) = 30+36-\sqrt{25}+3(6)$

$= 30+36-5+3(6)$

Step 3: Perform multiplication or division

$30+36-5+3(6) = 30+36-5+18$

Step 4: Perform add or subtract

$30+36-5+18 = 66-5+18$

$= 61+18$

$= 79$

PRACTICE PROBLEMS: Use the order of operations rule to calculate the following:

1. $12-(4)(2)-1$
2. $4+(5)(3)-2$
3. $28\div4+(9)(3)$
4. $4(2)(6)-12\div4$
5. $6(8)\div2(\ )\div12(6)$
6. $8-3(7)\div21-7$
7. $3^3-2(3^2)-3+7(2^3)$
8. $5+3(7^2)-6-2\sqrt{121}$
9. $2\sqrt{36}+3(5)-4^2\div2$
10. $15+6^2\div3-3\sqrt{25}$

Calculators are designed to automatically follow the Order of Operations Rule. This means that as the sequence of number, operation, number, operation, number and so on of a calculation are entered, the calculator will perform the operations of highest priority first and delay performing operations of lower priority.

Some problems will need to be rewritten to match the key strokes which enter the numbers and operations into a

calculator. Replace with the multiplication symbol x those parentheses used only to surround a number being used in multiplication.

EXAMPLE 6: Use a calculator to compute the value 38+9(12)-21÷3

Solution:

Write problem with x for multiplication instead of the parentheses.

38+9(12)-21÷3 = 38+9x12-21÷3

Directions	Key Strokes	Display
Enter 38	3 8	38.
Add	+	38.
Enter 9	9	9.
Multiply	x	9.
Enter 12	1 2	12.
Subtract	-	146.
Enter 21	2 1	21.
Divide	÷	21.
Enter 3	3	3.
End Problem	=	139.

The calculator delayed the addition because the next operation was a multiplication and multiplication has higher priority than addition. As soon as the subtract symbol is entered, the calculator completes the previous multiplication and addition (146) because subtract is a low priority operation. The subtraction is delayed when division is entered because divide is of higher priority than subtract.

EXAMPLE 7: Use a Calculator to compute 18.23-(34.5)(0.25)+8.75

Solution:

Rewrite 18.23-(34.5)(0.25)+8.75 as 18.23-34.5x0.25+8.75

Directions	Key Strokes	Display
Enter 18.23	1 8 . 2 3	18.23
Subtract	-	18.23
Enter 34.5	3 4 . 5	34.5
Multiply	x	34.5
Enter 0.25	0 . 2 5	0.25
Add	+	9.605
Enter 8.75	8 . 7 5	8.75
End	=	18.355

EXAMPLE 8: Compute $\sqrt{6.76}+0.9(4.3^3)\div 6.08$ correct to three significant digits

Solution: The square root key operates after the number is entered. Notice the key sequence in the solution below. The parentheses are not entered. They are being used to separate the factors of a multiplication. The problem is rewritten as

$$\sqrt{6.76}+0.9\times 4.3^3\div 6.08$$

Directions	Key Strokes	Display
Enter 6.76	6 . 7 6	6.76
Square root	$\sqrt{x}$ or 2nd x^2	2.6
Add	+	2.6
Enter 0.9	0 . 9	0.9
Multiply	x	0.9
Enter 4.3	4 . 3	4.3
Power	Y^x	4.3
Exponent 3	3	3.
Divide	÷	71.5563
Enter 6.08	6 . 0 8	6.08
End	=	14.36912829
Rounded to three significant digits		14.4

PRACTICE PROBLEMS: Use a calculator to compute the value

11. $12-(4)(2)-1$ 12. $4+(5)(3)-2$

13. $28\div4+(9)(3)$ 14. $4(2)(6)-12\div4$

15. $6(8)\div2(9)\div12(6)$ 16. $8-3(7)\div21-7$

17. $3^3-2(3^2)-3+7(2^3)$ 18. $5+3(7^2)-6-2\sqrt{121}$

19. $2\sqrt{36}+3(5)-4^2\div2$ 20. $15+6^2\div3-3\sqrt{25}$

21. $22.8+4.1(7.6)-8\div3\ 2$ 22. $29.75-4\sqrt{28.09}\ -3.75$

Round the final result to three significant digits

23. $3.62+1.7(2.15^2)-6.9\div9$ 24. $5.8-0.4\sqrt{1.6}\ +3.2^4$

The parentheses in $(24+18)\div7$ surround the operation + and are being used as grouping symbols. The purpose of the grouping symbol is a message that the operation inside the grouping symbol has the highest priority and is to be performed before any operations outside the grouping symbol. In calculating $(24+18)\div7$ the addition is performed before the division. Without the parentheses (), division is performed before addition.

Most scientific calculators have a left parenthesis key, (, and a right parenthesis key,). These keys are used to enter the parentheses grouping symbol and any other grouping symbols.

EXAMPLE 9: Compute $8-7(9-6\div2)\div21+5$

Solution:

Step 1: Perform the operations subtract and divide which are inside the parentheses using the Rule of Order: That is, divide before subtract

$8-7(9-6\div2)\div21+5 = 8-7(9-3)\div21+5$

$= 8-7(6)\div21+5$

Step 2 does not apply

Step 3: Perform multiplication or division

$8-7(6)\div21+5 = 8-42\div21+5$

$= 8-2+5$

Step 4: Perform add or subtract

$8-2+5 = 6+5$

$= 11$

The steps of a calculator solution of 8-7(9-6÷2)÷21+5 are

Directions	Key Strokes		Display
Enter 8	8		8.
Subtract	-		8.
Enter 7	7		7.
Multiply	x	NOTICE OPERATION	7.
Left paren	(		0.
Enter 9	9		9.
Subtract	-		9.
Enter 6	6		6.
Divide	÷		6.
Enter 2	2		2.
Right paren	)		6.
Divide	÷		42.
Enter 21	2 1		21.
Add	+		6.
Enter 5	5		5.
End Problem	=		11.

EXAMPLE 10: Compute (19-8)(4÷3)÷5+12

Solution:

Step 1: Perform the operation which is inside each of the parentheses

(19-8)(4+3)÷5+12 = (11)(4+3)÷5+12
= (11)(7)÷5+12

Step 2 does not apply

Step 3: Perform multiplication or division
= 77÷5+12
= 15.4+12

Step 4: Perform add or subtract
= 27.4

Calculator solution of (19-8)(4+3)÷5+12

Directions	Key Strokes	Display
Left paren	(	0.
Enter 19	1 9	19.
Subtract	-	19.
Enter 8	8	8.
Right paren	)	11.
Multiply	x NOTICE OPERATION	11.
Left paren	(	0.
Enter 4	4	4.
Add	+	4.
Enter 3	3	3.
Right paren	)	7.
Divide	÷	77.
Enter 5	5	5.
Add	+	15.4
Enter 12	1 2	12.
End Problem	=	27.4

The most common grouping symbols are the parentheses which have been used in the previous two examples. Other grouping symbols include brackets [], braces {}, the bar in fractions and the root symbol.

EXAMPLE 11: Compute $\frac{36-6}{5+10} + 4\sqrt{16+9}$

Solution:

Step 1: The fraction bar is a symbol of grouping. Perform the operations which are above and below the fraction bar

$$\frac{36-6}{5+10} + 4\sqrt{16+9} = \frac{30}{15} + 4\sqrt{16+9}$$

The square root contains operation add and is therefore being used as a grouping symbol. Compute the operation inside the root sign before the root itself.

$$\frac{30}{15} + 4\sqrt{16+9} = \frac{30}{15} + 4\sqrt{25}$$

Step 2: Perform roots and powers

$$\frac{30}{15} + 4\sqrt{25} = \frac{30}{15} + 4(5)$$

Step 3: Perform multiplication or division

$$\frac{30}{15} + 4(5) = 2 + 4(5)$$
$$= 2 + 20$$

Step 4: Perform add or subtract

$$= 22$$

Calculator solution of $\frac{36-6}{5+10} + 4\sqrt{16+9}$

Since the calculator uses only parentheses for grouping symbols, the other types of grouping symbols will need to be replaced with parentheses. The fraction bar is a grouping symbol and also means division. Replace $\frac{36-6}{5+10}$ with $(36-6)\div(5+10)$.

The square root key of a calculator operates on the previous entry and not on the next entry. Because of this number before root operation, the sequence of keys is not in the same order as the problem is read. The key sequence for calculation of $\frac{36-6}{5+10} + 4\sqrt{16+9}$ is

$$\frac{36-6}{5+10} + 4\sqrt{16+9} = (36-6)\div(5+10)+4\times(16+9)\sqrt{}x$$

Directions	Key Strokes	Display
Enter (36-6)	(3 6 - 6)	30.
Divide	÷	30.
Enter (5+10)	(5 + 1 0)	15.
Add	+	2.
Enter 4	4	4.
Multiply	x	4.
Left paren	(	0.
Enter 16+9	1 6 + 9	9.
Right paren	)	25.
Square root	√ or 2nd x^2	5.
End Problem	=	22.

PRACTICE PROBLEMS: Use the order of operations rule to calculate the following: Round to three significant digits if necessary.

25. $38+9(8+4)-3(5-2)$

26. $7+6(3+2)-7-48\div(4+2)$

27. $3(6^2+13)+5-2(7-4)$

28. $1\div(1\div15+1\div30+1\div45)$

29. $\{8-3+2)[16-4(2)]\div2$

30. $\sqrt{49}-[65-3(21)]+8^2$

31. $\frac{24}{6-2}+6(2)-\frac{7+8}{3}$

32. $\frac{12.7}{1.5(72-54)}$

33. $\sqrt{24(24-15)(24-11)(24-22)}$

34. $\frac{2(156.8)}{(8.37)(7.19)}-2.81$

SOLUTIONS TO PRACTICE PROBLEMS--MODULE 12

1. 3	2. 17	3. 34
4. 45	5. 108	6. 0
7. 62	8. 124	9. 19
10. 12	11. 3	12. 17
13. 34	14. 45	15. 108
16. 0	17. 62	18. 124
19. 19	20. 12	21. 51.46
22. 4.8	23. 10.7	24. 110
25. 137	26. 22	27. 146
28. 8.18	29. 28	30. 60
31. 13	32. 0.470	33. 74.9
34. 2.40		

END

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