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AUTHOR	Engelbrecht, Nancy; And Others
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ABSTRACT

This module is the 11th in a series of 12 learning modules designed to teach occupational mathematics. Blocks of informative material and rules are followed by examples and practice problems. The solutions to the practice problems are found at the end of the module. Specific topics covered include multiplication, powers, calculator use, and roots. (YLB)

Project Director

Ron Vorderstrasse

Project Secretary

Jan Wisialowski

Technical Consultant

Ray Plankinton

Technical Writers

Nancy Engelbrecht

Ann Hunter

Stacey Oakes

Lynne Graf

MODULE 11--POWERS AND ROOTS

The four basic operations used during number computation are add, subtract, multiply and divide. The numbers used in a multiplication problem are called FACTORS. Multiplication calculations in which one particular number (factor) is to be used repeatedly are fairly common. The use of a particular factor several times in the same multiplication occurs often enough that there is a short-hand notation to communicate that situation.

The multiplication (8)(8) is written as 8^2.
The multiplication (4)(4)(4) is written as 4^3.
The multiplication 3^5 means (3)(3)(3)(3)(3).
The multiplication 1.72^3 means (1.72)(1.72)(1.72).

The number which counts how many times a particular factor occurs in a multiplication is called an EXPONENT. The repeating factor is called the BASE. The name POWER is used to refer to the combination of base with exponent and also to the final number result.

EXAMPLE 1: Write the following multiplications in exponent notation and use ordinary multiplication to compute the product. Identify the numbers which are the base, exponent and power.

(a) (5)(5)(5)

(b) (11)(11)

(c) (2.1)(2.1)(2.1)(2.1)

Solutions:

(a) $(5)(5)(5) = 5^3$
$(5)(5)(5) = 125$
The base is 5.
The exponent is 3.
The power is 125 or 5^3.

(b) $(11)(11) = 11^2$
$(11)(11) = 121$
The base is 11.
The exponent is 2.
The power is 121 or 11^2.

(c) $(2.1)(2.1)(2.1)(2.1) = 2.1^4$
$(2.1)(2.1)(2.1)(2.1) = 19.4481$
The base is 2.1.
The exponent is 4.
The power is 19.4481 or 2.1^4

A general symbolic way of writing an exponent notation number is y^x. This particular letter combination for powers will be used in Module 11 because the same notation is found on one of the function keys of most calculators.

Special word names are used to read calculations which have exponents of 2 and 3. The power 11^2 is read as "eleven squared" and 8^2 is read as "eight squared". When reading a number with an exponent of 2, the word name of the base number is followed by the word SQUARED. When the exponent is 3, the base is said to be CUBED. Read 5^3 as "five cubed" and 1.72^3 is "one and seventy-two hundredths cubed". When any number other than 2 or 3 is the exponent in y^x, then it is read as the "base to the xth power". Read 2.1^4 as "two and one tenth to the fourth power" and read 3^5 as "three to the fifth power".

EXAMPLE 2: Write word names of the powers:

(a) 15^3
(b) 7.01^2
(c) 0.2^9

Solutions:

(a) 15^3 is fifteen cubed.

(b) 7.01^2 is seven and one hundredth squared

(c) 0.2^9 is two tenths to the ninth power.

PRACTICE PROBLEMS:

1. Given (9)(9)(9),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

2. Given (6)(6)(6)(6),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

3. Given (0.7)(0.7)(0.7),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

4. Given (12)(12),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

5. Given (1)(1)(1)(1)(1)(1)(1),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

6. Given (3.9)(3.9),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

7. Given (11.1)(11.1)(11.1),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

8. Given (24)(24),
   (a) calculate the ordinary multiplication.
   (b) write the multiplication in exponent form.
   (c) identify the base number.
   (d) identify the exponent.
   (e) write the word name of the exponent form.

The calculator which was recommended to be used with these Modules contains a special function key which will perform all powers. This is the y^x key. To use the y^x key to compute a power, begin by entering the base value called y. Next press the y^x key and then enter the value of the exponent x. The = key will conclude the input and the calculated value of the power will be displayed. For a few special exponents, there are alternatives to using the y^x which may be faster or more efficient. The key x^2 is the fastest way to compute the square of a number. The key stroke sequence to square a number using the y^x key and the x^2 key on your calculator is shown in Examples 3 and 4.

EXAMPLE 3: Compute 13.9^2 by using the y^x key and the x^2 key. Verify the result by using ordinary multiplication.

Solution:

Directions	Key Strokes	Display
Enter base 13.9	[1] [3] [.] [9]	13.9
Power key	[y^x]	13.9
Enter exponent 2	[2]	2.
End calculation	[=]	193.21
Enter 13.9	[1] [3] [.] [9]	13.9
Square	[x^2]	193.21
Verify		
Enter 13.9	[1] [3] [.] [9]	13.9
Multiply	[x]	13.9
Enter 13.9	[1] [3] [.] [9]	13.9
End problem	[=]	193.21

EXAMPLE 4: Compute 2045^2 using the key y^x and verify the result by using the x^2 key.

Solution:

Directions	Key Strokes	Display
Enter base 2045	[2] [0] [4] [5]	2045.
Power key	[y^x]	2045.
Enter exponent	[2]	2.
End problem	[=]	4182025.
Verify		
Enter 2045	[2] [0] [4] [5]	2045.
Square	[x^2]	4182025.

The key y^x is used to compute any power, x, of the number y. Some special exponents, like the square, can be computed with fewer key strokes than is required with the y^x key. In later lessons it will become important that you have learned to perform calculator computations in the most efficient way.

EXAMPLE 5: Calculate 40.17^2 in the most efficient way and verify the result.

Solution: The x^2 key is the most efficient.

Directions	Key Strokes	Display
Enter 40.17	[4] [0] [.] [1] [7]	40.17
Square	[x^2]	1613.6289
Verify		
Enter base 40.17	[4] [0] [.] [1] [7]	40.17
Power	[y^x]	40.17
Enter exponent 2	[2]	2.
End problem	[=]	1613.6289

EXAMPLE 6: Calculate 1.96^5 using the y^x key.

Solution: You can not use x^2 key since the exponent is not 2.

Directions	Key Strokes	Display
Enter base 1.96	[1] [.] [9] [6]	1.96
Power	[y^x]	1.96
Enter exponent 5	[5]	5.
End problem	[=]	28.9254655

PRACTICE PROBLEMS: Use your calculator to compute the following powers:

9.	42.9^2	10.	0.105^2	11.	2.57^4
12.	9.03^3	13.	603.2^2	14.	7.1^4
15.	12^5	16.	1^{34}	17.	1.075^{12}
18.	5.35^2	19.	0.812^6	20.	3.14^7

All previous examples and problems had whole number exponents of 2 or larger size. Powers which have the exponents of 0 and 1 can also be computed on your calculator using the key y^x.

EXAMPLE 7: Use key y^x to compute these first powers:

(a) 670^1

(b) 4.92^1

(c) 0.038^1

Solution: The calculator key stroke sequence and results are:

(a) 670 y^x 1 = 670.

(b) 4.92 y^x 1 = 4.92

(c) 0.038 y^x 1 = 0.038

The results of this example correctly illustrate that the first power of a given number is the same as that given number. In general symbols, $y^1=y$. The exponent of 1 is rarely written on a given number since the exponent of 1 does not result in changing the given number into a different number. A factor in a multiplication which is to be used one time in the multiplication doesn't need an exponent of one in order for the person doing the calculation to understand that there is one of that factor to be used.

EXAMPLE 8: Use key y^x to compute these zero powers:

(a) 43^0

(b) 9.8^0

(c) 0.082^0

Solution: The calculator key sequence and results are:

(a) 43 y^x 0 = 1.

(b) 9.8 y^x 0 = 1.

(c) 0.082 y^x 0 = 1.

Yes, the zero power of a number is 1.

Exponents can also be negatives of whole numbers. The numbers which are negatives of whole numbers are those written like -1, -2, -3, -4,···. Exponents which are negative whole numbers do not tell you how many times the base is a factor in a multiplication. A person cannot count with negative whole numbers.

Powers which have -1 (negative one) as the exponent are equal to the inverse of the base number. That would mean that:

$5^{-1} = \frac{1}{5} = 1 \div 5$, $18^{-1} = \frac{1}{18} = 1 \div 18$, and in general, $y^{-1} = \frac{1}{y} = 1 \div y$.

An exponent of -1 inverts the base number.

The calculation of y^{-1} on a calculator can be achieved in three distinct ways. Powers can always be computed using the y^x key. In using key y^x where the exponent is negative, be sure the sign of the exponent is entered AFTER the number digits of the exponent have been input. The second and most efficient method is to use key 1/x. Using $y^{-1} = \frac{1}{y}$ to mean $1 \div y$ is the third method.

Example 9 will demonstrate each of the three methods. Example 10 will use only the 1/x key.

EXAMPLE 9: Calculate 14^{-1}.

Solution:

Directions	Key Strokes	Display
Enter base 14	[1] [4]	14.
Power	[y^x]	14.
Enter positive 1	[1]	1.
Change sign	[+/-]	- 1.
End problem	[=]	0.071428571
Enter base 14	[1] [4]	14.
Invert	[1/x]	0.071428571
Enter 1	[1]	1.
Divide	[÷]	1.
Enter base 14	[1] [4]	14.
End problem	[=]	0.071428571

EXAMPLE 10: Compute 0.347^{-1} by the most efficient calculator method.

Solution: The most efficient calculation of 0.347^{-1} uses key 1/x.

Directions	Key Strokes	Display
Enter base 0.347	[.] [3] [4] [7]	0.347
Invert	[1/x]	2.88184438

PRACTICE PROBLEMS: Calculate these special powers:

21. 16^0	22. 13.7^1	23. 9.06^1
24. 42.3^0	25. 6.5^0	26. 0.03^1
27. 4^{-1}	28. 0.8^{-1}	29. 26.3^{-1}
30. 1.8^{-1}	31. 0.045^{-1}	32. 6.08^{-1}

The exponent in y^x can be any type of number; positive, negative, whole number, decimal or fraction. However, the value of y is generally considered to be only positive. When the y of y^x is negative, some decimal values of exponent x will cause the calculator to display an error message E. Fractions for either the base value or exponent value should be changed into decimals before computing the power.

EXAMPLE 11: Compute $9.15^{2.16}$ to four significant digits.

Solution:

Directions	Key Strokes	Display
Enter base 9.15	9 . 1 5	9.15
Power	y^x	9.15
Enter exponent 2.16	2 . 1 6	2.16
End problem	=	119.3079994
Four significant digit answer		119.3

EXAMPLE 12: Compute $0.86^{-3.42}$ to four significant digits.

Solution:

Directions	Key Strokes	Display
Enter base 0.86	. 8 6	0.86
Power	y^x	0.86
Enter positive 3.42	3 . 4 2	3.42
Change sign	+/-	- 3.42
End problem	=	1.675001872
Four significant digit answer		1.675

EXAMPLE 13: Compute $(1\frac{5}{16})^{\frac{3}{4}}$ to four significant digits.

Solution: The fractions must be changed into decimals. Compute the exponent decimal first and store its value into the calculator memory. The decimal for the base is computed second so that it is on the screen display when the calculation is ready for the power key. If the calculations are done in the other order, then one of the decimal values will need to be written on paper and re-entered.

Directions	Key Strokes	Display
Change 3/4 into decimal		
Enter numerator 3	[3]	3.
Divide	[÷]	3.
Enter denominator	[4]	4.
End	[=]	0.75
Store exponent x	[Sto]	0.75
Change 1 5/16 into decimal		
Enter numerator 5	[5]	5.
Divide	[÷]	5.
Enter denominator 16	[1] [6]	16.
Add	[+]	0.3125
Enter whole number 1	[1]	1.
End	[=]	1.3125
Power	[y^x]	1.3125
Recall exponent	[Rcl]	0.75
End	[=]	1.226237192
Four significant digit answer		1.226

PRACTICE PROBLEMS: Compute the following powers. Round your answers to four significant digits.

33. $6.23^{3.15}$	34. 9.2^{-4}	35. $0.024^{-3.5}$
36. $0.95^{-0.95}$	37. $23.1^{1.5}$	38. $15.3^{0.6}$
39. $1.52^{-6.9}$	40. $(\frac{3}{40})^{-2}$	41. $(3.6)^{2\frac{1}{2}}$
42. $(3\frac{5}{8})^{2.5}$	43. $(0.27)^{-1\frac{1}{3}}$	44. $(4\frac{1}{3})^{3\frac{1}{2}}$

Whole number exponents count how many times, x, that a factor, y, is to be used in a multiplication y^x. The calculation process and the number result is referred to as being a POWER. There is a calculation which is the exact opposite called ROOT. The operation of squaring (second power) and the operation called square root are exact opposites. The square of 16 results in a number that has 16 and 16 as its two identical factors: (16)(16)=256. The square root of 16 is one of two identical factors of 16. The square root of 16 is 4 since (4)(4)=16.

The symbols which ask for the square root of 16 are $\sqrt{16}$. The square root of the number y is one of two identical factors of y.

The cube (third power) of 64 and the cube root of 64 ask for opposite operations. The cube of 64 is requested by 64^3 and equals 262,144. While the cube root of 64 is $\sqrt[3]{64}$ and equals 4 since (4)(4)(4) = 64. The cube root of the number y is one of three identical factors of y.

In general, symbols $\sqrt[x]{y}$ ask you to find one of x equal factors of y. The x is called the index of the root, $\sqrt{}$ is the radical sign and y is called the radicand.

The square root is the most common root level that occurs in calculations. Because square root is far more common than any other level, the index 2 of the radical is omitted. No other root level can have the index omitted. If a root has no index, then the root is understood to be a square root. The calculator key $\sqrt{x}$ computes only square roots of numbers. The calculator key $\sqrt[x]{y}$ computes square root, cube root and any other whole number root index. The index must be a positive whole number. The examples below will show how to invoke the root keys when root is the second operation by a calculator key.

EXAMPLE 14: Compute the value of $\sqrt{72.85}$ to four significant digits.

Solution:

Directions	Key Strokes	Display
Enter radicand 72.85	7 2 . 8 5	72.85
Call $\sqrt{x}$	2nd x^2	8.535221145
Four significant digit answer		8.535

EXAMPLE 15: Compute the value of $\sqrt[3]{6.983}$ to four significant digits.

Solution:

Directions	Key Strokes	Display
Enter radicand 6.983	6 . 9 8 3	6.983
Call $\sqrt[x]{y}$	2nd y^x	6.983
Enter index 3	3	3.
End	=	1.911381364
Four significant digit answer		1.911

EXAMPLE 16: Compute $\sqrt[5]{209.56}$ to three significant digits.

Solution:

Directions	Key Strokes	Display
Enter radicand 209.56	2 0 9 . 5 6	209.56
Enter $\sqrt[x]{y}$	2nd y^x	
Enter index 5	5	5.
End	=	2.912471458
Three significant digit answer		2.91

PRACTICE PROBLEMS: Compute the following roots correct to four significant digits.

45. $\sqrt{42.75}$

46. $\sqrt{0.0281}$

47. $\sqrt{963.7}$

48. $\sqrt[3]{51.44}$

49. $\sqrt[3]{7391.4}$

50. $\sqrt[3]{0.00918}$

51. $\sqrt[5]{3.918}$

52. $\sqrt[4]{1.948}$

53. $\sqrt[3]{378,000}$

54. $\sqrt{640,000}$

55. $\sqrt[3]{0.7392}$

56. $\sqrt{16.2893}$

57. $\sqrt[5]{4239.5}$

58. $\sqrt[3]{8373.2}$

59. $\sqrt[7]{0.00029}$

PRACTICE PROBLEM SOLUTIONS--MODULE 11

1. a) 729
   b) 9^3
   c) 9
   d) 3
   e) nine cubed

2. a) 1296
   b) 6^4
   c) 6
   d) 4
   e) six to the fourth power

3. a) 0.343
   b) 0.7^3
   c) 0.7
   d) 3
   e) seven tenths cubed

4. a) 144
   b) 12^2
   c) 12
   d) 2
   e) twelve squared

5. a) 1
   b) 1^7
   c) 1
   d) 7
   e) one to the seventh power

6. a) 15.21
   b) 3.9^2
   c) 3.9
   d) 2
   e) three and nine tenths squared

7. a) 1367.631
   b) 11.1^3
   c) 11.1
   d) 3
   e) eleven and one tenth cubed

8. a) 576
   b) 24^2
   c) 24
   d) 2
   e) twenty-four squared

9.	1840.41	10.	0.011025	11.	43.62470401
12.	736.314327	13.	363,850.24	14.	2541.1681
15.	248,832	16.	1	17.	2.381779599
18.	28.6225	19.	0.286639591	20.	3009.591395
21.	1	22.	13.7	23.	9.06
24.	1	25.	1	26.	0.03
27.	0.25	28.	1.25	29.	0.038022813
30.	0.555555555	31.	22.22222222	32.	0.164473684
33.	318.2	34.	0.0001396	35.	466,900
36.	1.050	37.	111.0	38.	5.138
39.	0.05563	40.	177.8	41.	24.[illegible]9
42.	25.02	43.	5.730	44.	169.4
45.	6.538	46.	0.1676	47.	31.04
48.	3.719	49.	19.48	50.	0.2094
51.	1.314	52.	1.181	53.	72.30
54.	800.0	55.	0.9042	56.	4.036
57.	5.315	58.	20.31	59.	0.3123