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ABSTRACT

This module is the seventh in a series of 12 learning modules designed to teach occupational mathematics. Blocks of informative material and rules are followed by examples and practice problems. The solutions to the practice problems are found at the end of the module. Specific topics covered include fractions, fraction to decimal conversion, reducing and raising fractions, measurement, mixed numbers, and improper fractions. (YLB)

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Module 7 Fractions and Conversion To Decimal

A fraction is a number of the form $\frac{m}{n}$ used to indicate that some whole thing has been divided into n equal parts. The m of the fraction shows how many of the equal parts are being considered. The fraction $\frac{7}{32}$ suggests that 7 of 32 equal parts are under consideration. A fraction like $\frac{12}{16}$ is a single number which is written using two whole numbers and a separation line. The separation line is called the fraction bar. This fraction bar serves as a grouping symbol to separate the two whole numbers. The whole number below the fraction bar is called the DENOMINATOR of the fraction. The denominator tells how many parts a whole thing has been divided into. The whole number above the fraction bar is called the NUMERATOR of the fraction. The numerator reports how many parts were counted for consideration.

	m	numerator
Fraction	———	fraction bar
	n	denominator

PROPER fractions have numerators which are less than their denominator.

Two fractions which have the same value are called equal fractions. Equal fractions often look different because of unequal denominators. A quick method of testing two fractions to determine if they are equal is to use cross-multiplication.

To test $\frac{3}{4}$ and $\frac{12}{16}$ for equal value, consider the cross-multiplication $\frac{3}{4} \times \frac{12}{16}$. Because the values (3)(16) = 48 and (4)(12) = 48 are equal, the fractions are equal: $\frac{3}{4} = \frac{12}{16}$.

The values of two fractions can be compared using the following rule.

Rule for Comparing Two Fractions

When comparing the two fractions $\frac{a}{b}$ and $\frac{c}{d}$, compute the cross-multiplications

$\frac{a}{b} \times \frac{c}{d}$

1) If ad = bc, then they are equal fractions. $\frac{a}{b} = \frac{c}{d}$

2) If ad is less than bc, then the fractions are not equal

Follow the use of numerator a to determine how to compare the values of the fractions. The numerator a was used in the cross-product ad, resulting in the smaller of the two cross-products. The numerator a is then in the smaller fraction. Therefore, $\frac{a}{b}$ is less than $\frac{c}{d}$ and this is written as $\frac{a}{b} < \frac{c}{d}$.

3) If ad is greater than bc, then the fractions are not equal.
Here the numerator a is part of the greater cross- product ad and a will be the numerator of the larger fraction. When $\frac{a}{b}$ is greater than $\frac{c}{d}$, the comparison is written $\frac{a}{b} < \frac{c}{d}$.

EXAMPLE 1: Compare the values of the following pairs of fractions. Describe their relative size using one of the symbols =, < , or > .

a) $\frac{1}{4}$, $\frac{8}{32}$

b) $\frac{7}{16}$, $\frac{23}{64}$

c) $\frac{3}{8}$, $\frac{13}{32}$

d) $\frac{41}{64}$, $\frac{5}{8}$

Solution:

(a) The cross-products are (1)(32) and (4)(8).
(1)(32) = 32 and (4)(8) = 32.
Since the cross-products are equal, the fractions are equal.

$$\frac{1}{4} = \frac{8}{32}$$

Solution:

(b) The cross-products are (7)(64) and (16)(23).
(7)(64) = 448 and (16)(23) = 368.
The cross-products are not equal.
The numerator 7 is used in the larger cross-product (7)(64)=448 and is the numerator of the largest fraction.

$$\frac{7}{16} > \frac{23}{64}$$

Solution:

(c) The cross-products are (3)(32) and (8)(13).
(3)(32) = 96 and (8)(13) = 104
The cross-products are not equal.
The numerator 3, used in 3(32) = 96 produced the smaller cross-product and is from the smallest fraction.

$$\frac{3}{8} < \frac{13}{32}$$

Solution:

(d) The cross-products are (41)(8) and (64)(5).
(41)(8) = 328 and (64)(5) = 320
The fractions are not equal.
The 41 numerator is part of the larger cross-product and is the numerator of the larger fraction.

$$\frac{41}{64} > \frac{5}{8}$$

PRACTICE PROBLEMS: Compare the values of the following pairs of fractions. Describe their relative size using one of the symbols =, <, or >.

1. $\frac{3}{8}$, $\frac{7}{16}$
2. $\frac{7}{16}$, $\frac{27}{64}$
3. $\frac{7}{16}$, $\frac{1}{2}$
4. $\frac{3}{4}$, $\frac{48}{64}$
5. $\frac{3}{16}$, $\frac{6}{32}$
6. $\frac{1}{4}$, $\frac{11}{32}$
7. $\frac{9}{32}$, $\frac{1}{4}$
8. $\frac{13}{64}$, $\frac{7}{32}$
9. $\frac{15}{16}$, $\frac{60}{64}$
10. $\frac{27}{32}$, $\frac{3}{4}$

Testing the fractions $\frac{6}{8}$, $\frac{12}{16}$, $\frac{48}{64}$, $\frac{3}{4}$, and $\frac{24}{32}$ will show that they are all of the same value. That is, $\frac{6}{8} = \frac{12}{16} = \frac{48}{64} = \frac{3}{4} = \frac{24}{32}$. The best fraction to use from this list will be a different selection for different situations. One of these fractions is considered to be the simplest fraction of all the fractions which have this value. A fraction is said to be in LOWEST TERMS when the numerator and denominator cannot both be divided exactly by a whole number greater than 1. The simplest fraction is $\frac{3}{4}$ for the above set of equal fractions. Each of the other fractions can be reduced to $\frac{3}{4}$.

Reducing a fraction is the process of dividing both the numerator and denominator of the fraction by the same whole number (greater than 1). The quotients obtained must be whole numbers so they can be used as the numerator and denominator of the new fraction. The goal of reducing a fraction is to reach lowest terms.

EXAMPLE 2: Reduce the following fractions to lowest terms.

a) Simplify $\frac{12}{16}$

b) Reduce $\frac{30}{48}$

c) Simplify $\frac{90}{150}$

Solution:

a) 4 will divide into both 12 and 16.

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4}$$

$$= \frac{3}{4}$$

The only whole number which divides into both 3 and 4 is 1 (one), so $\frac{3}{4}$ is the lowest terms.

Solution:

b) 2 will divide into both 30 and 48

$$\frac{30}{48} = \frac{30 \div 2}{48 \div 2}$$

$$= \frac{15}{24}$$

Both 15 and 24 can be divided by 3

$$\frac{15}{24} = \frac{15 \div 3}{24 \div 3}$$

$$= \frac{5}{8}$$

The only whole number which divides both 5 and 8 is 1, so $\frac{5}{8}$ is in lowest terms. The reduced form solution can also be achieved in one step if a person notices that 6 divides into both 30 and 48.

$$\frac{30}{48} = \frac{30 \div 6}{48 \div 6}$$

$$= \frac{5}{8}$$

Solution:

c) Find some whole number, preferably the largest possible, which divides both 90 and 150.

$$\frac{90}{150} = \frac{90 \div 10}{150 \div 10}$$

$$= \frac{9}{15}$$

Look for some whole number which divides 9 and 15.

$$\frac{9}{15} = \frac{9 \div 3}{15 \div 3}$$

$$= \frac{3}{5}$$

The only whole number which divides both 3 and 5 is 1, so $\frac{3}{5}$ is the simplest form.

The process of raising fractions to higher terms is just as important as that of reducing fractions. Raising a fraction to higher terms results from designing an equal fraction that has larger whole numbers in the numerator and denominator than were in the original fraction. A fraction is changed into an equal fraction with higher terms by multiplying both the numerator and denominator by the same whole number greater than 1.

EXAMPLE 3: Change the terms of each fraction so that its new denominator is 64.

a) $\frac{7}{16}$

b) $\frac{7}{8}$

c) $\frac{1}{4}$

Solution:

a) The 16 denominator must be multiplied by 4 to make a new denominator of 64.

$$\frac{7}{16} = \frac{(7)(4)}{(16)(4)}$$

$$= \frac{28}{64}$$

Solution:

b) The 8 denominator needs to be multiplied by 8 to get 64 in the denominator.

$$\frac{7}{8} = \frac{(7)(8)}{(8)(8)}$$

$$= \frac{56}{64}$$

Solution:

c) The 4 denominator divides into 64 exactly 16 times. The 4 must be multiplied by 16 to get 64.

$$\frac{1}{4} = \frac{(1)(16)}{(4)(16)}$$

$$= \frac{16}{64}$$

PRACTICE PROBLEMS: Reduce each fraction to its lowest terms.

11. $\frac{15}{20}$ 12. $\frac{30}{48}$ 13. $\frac{12}{128}$

14. $\frac{6}{24}$ 15. $\frac{60}{320}$ 16. $\frac{38}{64}$

17. $\frac{15}{40}$ 18. $\frac{24}{96}$ 19. $\frac{270}{480}$

20. $\frac{36}{72}$

Change each of the given fraction into sixteenths (denominators of 16).

21. $\frac{3}{4}$ 22. $\frac{5}{8}$ 23. $\frac{1}{2}$

24. $\frac{7}{8}$ 25. $\frac{1}{4}$

Change each fraction into a fraction with denominator of 32.

26. $\frac{3}{8}$ 27. $\frac{3}{16}$ 28. $\frac{3}{4}$

Fractions in a measurement are frequently encountered when the English measurement system is being used. When the distance or length dimensions for a machine part are given using the English inch measurement unit, these measurements can be given either in fraction form, like $\frac{3}{8}$ inch, or in decimal form, 0.375 inch. Most simple measurement instruments, which record length in the fractional inch scale, will allow size to be determined to the nearest 64th (sixty-fourth) inch. Table 1, at the end of Module 7, contains the decimal equivalent for every proper fraction of denominator 64. For each number of 64ths, Table 1 has its decimal equivalent and all the possible reductions of sixth-fourths into a fraction of smaller denominator. Table 1 is used to read fraction and decimal equivalents.

EXAMPLE 4: Use Table 1 to give the reduced fraction and decimal equivalent of the given fractions.

a) $\frac{18}{64}$

b) $\frac{56}{64}$

c) $\frac{24}{32}$

Solution:

a) $\frac{9}{32}$ is the reduced fraction. 0.28125 is the equivalent decimal.

Solution:

b) $\frac{7}{8}$ is the reduced fraction. 0.875 is the equivalent decimal.

Solution:

c) $\frac{3}{4}$ is the reduced fraction. 0.75 is the equivalent decimal.

A MIXED NUMBER is the combination of a whole number and a proper fraction. Examples of mixed numbers include $2\frac{1}{2}$, $3\frac{5}{8}$, and $5\frac{7}{32}$.

A mixed number which contains a fraction in 64ths, or a reduction of 64ths, can be changed into a decimal using Table 1. The whole number part of the mixed number is repeated in the whole number part of the decimal. The fraction of the mixed number is converted to a decimal by Table 1 and attached to the whole number.

EXAMPLE 5: Use Table 1 to give the decimal equivalent of the mixed numbers.

a) $4\frac{3}{16}$

b) $7\frac{23}{32}$

c) $6\frac{3}{8}$

Solution.

a) Working with the fraction and then whole number:

$$\frac{3}{16} = 0.1875$$

Attaching whole number 4 to decimal 0.1875 gives $4\frac{3}{16} = 4.1875$

Solution:

b) The fraction of the mixed number is $\frac{23}{32} = 0.71875$

With whole number 7, $7\frac{23}{32} = 7.71875$

Solution:

c) Since $\frac{3}{8} = 0.375$, then $6\frac{3}{8} = 6.375$

PRACTICE PROBLEMS: Use Table 1 to determine the decimal value of these proper fractions and mixed numbers.

29. $\frac{7}{16}$ 30. $\frac{15}{32}$ 31. $\frac{39}{64}$

32. $\frac{26}{32}$ 33. $1\frac{3}{4}$ 34. $2\frac{5}{8}$

35. $7\frac{11}{16}$ 36. $5\frac{31}{64}$ 37. $3\frac{29}{32}$

38. $4\frac{7}{32}$ 39. $6\frac{8}{16}$ 40. $8\frac{41}{64}$

EXAMPLE 6: Use Table 1 to find the reduced fraction or mixed number value for the decimals.

a) 0.65625

b) 0.328125

c) 4.03125

d) 9.375

Solution:

a) $0.65625 = \frac{21}{32}$

Solution:

b) $0.328125 = \frac{21}{64}$

Solution:

c) $0.03125 = \frac{1}{32}$, so

$4.03125 = 4\frac{1}{32}$

Solution:

d) $0.375 = \frac{3}{8}$, so

$9.375 = 9\frac{3}{8}$

Fractions in which the whole number in the numerator is greater than or equal to the whole number in the denominator are called IMPROPER FRACTIONS. Don't let the name of this kind of fraction influence your opinion about the usefulness of such fractions. If a family names their first son Pierre, that doesn't make him French. Pierre may choose to name his pet hamster Goliath. That won't make the hamster 7 foot tall and weigh-in at 300 pounds. Most names do not carry any special meaning about its owner. IMPROPER FRACTIONS are not IMPROPER. Improper fractions make good answers.

An improper fraction is often changed into a mixed number. To change an improper fraction into a mixed number using a calculator and Table 1:

1) Divide the numerator by the denominator. The whole number part of the decimal is the whole number part of the mixed number.

2) The decimal part is converted to a fraction using Table 1.

3) The whole number and the fraction are joined to form the mixed number.

EXAMPLE 7: Change the improper fraction $\frac{51}{8}$ into a mixed number.

Solution:

Directions	Key strokes	Display
Enter numerator	5 1	51.
Divide	÷	51
Enter denominator	8	8.
End divide	=	6.375

The whole number part of the mixed number is 6. Table 1 gives $0.375 = \frac{3}{8}$.

Therefore, $\frac{51}{8} = 6\frac{3}{8}$.

EXAMPLE 8: Change the improper fraction $\frac{307}{32}$ into a mixed number.

Solution:

Directions	Key strokes	Display
Enter numerator	3 0 7	307.
Divide	÷	307
Enter denominator	3 2	32.
End divide	=	9.59375

The whole number part of the mixed number is 9. Table 1 gives $0.59375 = \frac{19}{32}$.

Therefore, $\frac{307}{32} = 9\frac{19}{32}$.

PRACTICE PROBLEMS: Use Table 1 to find the reduced fraction or mixed number value for the decimals.

41. 0.8125 42. 3.5625 43. 4.125

44. 2.828125 45. 5.21875 46. 6.03125

47. 12.75 48. 7.875

Use a calculator and Table 1 to change each improper fraction into a mixed number.

49. $\frac{31}{8}$ 50. $\frac{47}{16}$ 51. $\frac{19}{4}$

52. $\frac{71}{16}$ 53. $\frac{105}{32}$ 54. $\frac{413}{32}$

55. $\frac{281}{64}$ 56. $\frac{745}{64}$

When a given decimal is not exactly oqual to any of the decimals provided in Table 1, the table can be used to determine an approximate fraction to the nearest 64th. The given decimal will fall between two consecutive table decimal entries. The entry just smaller will correspond to the next smaller fraction. The entry just larger will correspond to the next larger fraction. Subtraction of decimals will show which fraction is closest to the given decimal.

EXAMPLE 9: For each given decimal, use Table 1 to find the fractions in 64ths which are the:

(1) next smaller fraction (mixed number)
(2) next larger fraction (mixed number)
(3) closest fraction (mixed number)

a) 0.6800
b) 3.1750
c) 5.6

Solution:

a) The decimal 0.6800 lies between 0.671875 and 0.6875

(1) The next smaller fraction is 0.671875 = $\frac{43}{64}$

(2) The next larger fraction is 0.6875 = $\frac{44}{64}$ = $\frac{11}{16}$

(3) The differences are
0.6800 - 0.671875 = 0.008125 and 0.6875 - 0.68 = 0.0075
The smaller difference is 0.0075

Closest fraction 0.6875 = $\frac{44}{64}$ = $\frac{11}{16}$

Solution:

b) The decimal in 3.1750 is 0.1750 which lies between 0.171875 and 0.1875

(1) The next smaller fraction is $0.171875 = \frac{11}{64}$.

The next smaller mixed number is $3\frac{11}{64}$

(2) The next larger fraction is $0.1875 = \frac{12}{64} = \frac{3}{16}$

The next larger mixed number is $3\frac{3}{16}$

(3) Computing differences:
0.1750 - 0.171875 = 0.003125 and
0.1875 - 0.1750 = 0.0125
The smaller difference is 0.003125.

The closest fraction to 0.1750 $= \frac{11}{64}$ The closest mixed number to 3.1750 is $3\frac{11}{64}$.

Solution:

c) The decimal part 0.6 lies between 0.59375 and 0.609375

(1) The next smaller fraction is $0.59375 = \frac{38}{64} = \frac{19}{32}$.

The next smaller mixed number is $5\frac{19}{32}$

(2) The next larger fraction is $0.609375 = \frac{39}{64}$

The next larger mixed number is $5\frac{39}{64}$

(3) Computing differences:
0.6 - 0.59375 = 0.00625 and 0.609375 - 0.6 = 0.009375
The smaller difference is 0.00625

The closest fraction is $0.59375 = \frac{19}{32}$ and the closest mixed number is $5\frac{19}{32}$.

PRACTICE PROBLEMS: For each decimal, use Table 1 to find the fractions in 64ths which are the:
(a) next smaller fraction,
(b) next larger fraction, and
(c) closest fraction.

57. 0.235 58. 0.872 59. 0.0825

60. 0.94375 61. 0.6050 62. 0.40055

For each decimal, use Table 1 to find the mixed numbers in 64ths which are the:
(a) next smaller mixed number,
(b) next larger mixed number, and
(c) closest mixed number.

63. 3.27875 64. 2.8945 65. 1.07625

66. 4.32750 67. 7.5675 68. 5.6025

Table 1 —FRACTIONS AND DECIMAL EQUIVALENTS

Fractions						Decimals
					1/64	0.015625
				1/32	2/64	0.03125
					3/64	0.046875
			1/16	2/32	4/64	0.062
					5/64	0.078125
				3/32	6/64	0.0937
					7/64	0.109375
		1/8	2/16	4/32	8/64	0.125
					9/64	0.140625
				5/32	10/64	0.15625
					11/64	0.171875
			3/16	6/32	12/64	0.1875
					13/64	0.203125
				7/32	14/64	0.21875
					15/64	0.234375
		1/4	2/8	4/16	8/32	16/64	0.25
					17/64	0.265625
				9/32	18/64	0.28125
					19/64	0.296875
			5/16	10/32	20/64	0.3125
					21/64	0.328125
				11/32	22/64	0.34375
					23/64	0.359375
			3/8	6/16	12/32	24/64	0.375
					25/64	0.390625
				13/32	26/64	0.40625
					27/64	0.421875
			7/16	14/32	28/64	0.4375
					29/64	0.453125
				15/32	30/64	0.46875
					31/64	0.484375
1/2		2/4	4/8	8/16	16/32	32/64	0.5
					33/64	0.515625
				17/32	34/64	0.53125
					35/64	0.546875
			9/16	18/32	36/64	0.5625
					37/64	0.578125
				19/32	38/64	0.59375
					39/64	0.609375
			5/8	10/16	20/32	40/64	0.625
					41/64	0.640625
				21/32	42/64	0.65625
					43/64	0.671875
			11/16	22/32	44/64	0.6875
					45/64	0.703125
				23/32	46/64	0.71875
					47/64	0.734375
		3/4	6/8	12/16	24/32	48/64	0.75
					49/64	0.765625
				25/32	50/64	0.78125
					51/64	0.796875
			13/16	26/32	52/64	0.8125
					53/64	0.828125
				27/32	54/64	0.84375
					55/64	0.859375
			7/8	14/16	28/32	56/64	0.875
					57/64	0.890625
				29/32	58/64	0.90625
					59/64	0.921875
			15/16	30/32	60/64	0.9375
					61/64	0.953125
				31/32	62/64	0.96875
					63/64	0.984375
2/2		4/4	8/8	16/16	32/32	64/64	1.000000

SOLUTIONS TO PRACTICE PROBLEMS—Module 7

1. $<$ 2. $>$ 3. $<$ 4. $=$ 5. $=$ 6. $<$

7. $>$ 8. $<$ 9. $=$ 10. $>$ 11. $\frac{3}{4}$ 12. $\frac{5}{8}$

13. $\frac{3}{32}$ 14. $\frac{1}{4}$ 15. $\frac{3}{16}$ 16. $\frac{19}{32}$

17. $\frac{3}{8}$ 18. $\frac{1}{4}$ 19. $\frac{9}{16}$ 20. $\frac{1}{2}$

21. $\frac{12}{16}$ 22. $\frac{10}{16}$ 23. $\frac{8}{16}$ 24. $\frac{14}{16}$

25. $\frac{4}{16}$ 26. $\frac{12}{32}$ 27. $\frac{6}{32}$ 28. $\frac{24}{32}$

29. 0.4375 30. 0.46875 31. 0.609375 32. 0.8125

33. 1.75 34. 2.625 35. 7.6875 36. 5.484375

37. 3.90625 38. 4.21875 39. 6.5 40. 8.640625

41. $\frac{13}{16}$ 42. $3\frac{9}{16}$ 43. $4\frac{1}{8}$ 44. $2\frac{53}{64}$ 45. $5\frac{7}{32}$

46. $6\frac{1}{32}$ 47. $12\frac{3}{4}$ 48. $7\frac{7}{8}$ 49. $3\frac{7}{8}$ 50. $2\frac{15}{16}$

51. $4\frac{3}{4}$ 52. $4\frac{7}{16}$ 53. $3\frac{9}{32}$ 54. $12\frac{29}{32}$ 55. $4\frac{25}{64}$

56. $11\frac{41}{64}$ 57. a) $\frac{15}{64}$ 58. a) $\frac{55}{64}$ 59. a) $\frac{5}{64}$

b) $\frac{16}{64}$ b) $\frac{56}{64}$ b) $\frac{6}{64}$

c) $\frac{15}{64}$ c) $\frac{56}{64}$ c) $\frac{5}{64}$

60. a) $\frac{60}{64}$ 61. a) $\frac{38}{64}$ 62. a) $\frac{25}{64}$ 63. a) $3\frac{17}{64}$

b) $\frac{61}{64}$ b) $\frac{39}{64}$ b) $\frac{26}{64}$ b) $3\frac{18}{64}$

c) $\frac{60}{64}$ c) $\frac{39}{64}$ c) $\frac{26}{64}$ c) $3\frac{18}{64}$

64. a) $2\frac{57}{64}$ b) $2\frac{58}{64}$ c) $2\frac{57}{64}$

65. a) $1\frac{4}{64}$ b) $1\frac{5}{64}$ c) $1\frac{5}{64}$

66. a) $4\frac{20}{64}$ b) $4\frac{21}{64}$ c) $4\frac{21}{64}$

67. a) $7\frac{36}{64}$ b) $7\frac{37}{64}$ c) $7\frac{36}{64}$

68. a) $5\frac{38}{64}$ b) $5\frac{39}{64}$ c) $5\frac{39}{64}$

END

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