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AUTHOR Engelbrecht, Nancy; And Others
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ABSTRACT

This module is the fifth in a series of 12 learning modules designed to teach occupational mathematics. Blocks of informative material and rules are followed by examples and practice problems. The solutions to the practice problems are found at the end of the module. Specific topics covered include calculator multiplication of decimals, multiplication of measurements, significant digits, and accuracy of a measurement. (YLB)

Project Director
Ron Vordorstrasse

Project Secretary
Jan Wisialowski

Technical Consultant
Ray Plankinton

Technical Writers
Nancy Engelbrecht
Lynne Graf
Ann Hunter
Stacey Oakes

Module 5--Calculator Multiplication of Decimals and Applications

Multiplication is a third of the basic operations. It is the best operation to use in situations which require repeated addition of a constant value.

Suppose that 8 sheets of steel, each 2.5 mm thick, are in a storage bin. How thick a stack would they form if placed on top each other? The thickness of the stack can be found by adding 2.5 + 2.5 + ... etc, until eight 2.5's had been used. This addition provides a sum equal to the multiplication of 8 by 2.5 mm:

$$2.5 \text{ mm} + 2.5 \text{ mm} + \ldots + 2.5 \text{ mm} = 8(2.5 \text{ mm}) = 20 \text{ mm}$$

Situations which can be thought of as containing rows and columns may also be calculated using multiplication. A plate which has 3 rows of holes, 5 holes per row, will contain $5 + 5 + 5 = 3(5) = 15$ holes.

There are several commonly used ways to write multiplication. When two letters, like m and n, are used to represent the numbers to be multiplied in a formula, the multiplication of m by n is usually written as mn. With number values like m=3.25 and n=16.7, the various ways that multiplication can be written include the preferred forms:

$$mn = m(n) = 3.25(16.7)$$
$$mn = (m)(n) = (3.25)(16.7)$$

Forms which may be confusing in some situations are:

$$mn = m \times n = 3.25 \times 16.7$$
$$mn = m \cdot n = 3.25 \cdot 16.7$$

The examples and illustrations in these Lessons will write the multiplication of two or more numbers using the parentheses form.

The numbers being multiplied are called FACTORS and the answer to the multiplication problem is called the PRODUCT. The x key on a calculator is used to perform multiplication.

EXAMPLE 1: Use a calculator to compute (3.25)(16.7).

Solution:

Directions		Key Strokes	Display
Enter 3.25		$\boxed{3}\,\boxed{.}\,\boxed{2}\,\boxed{5}$	3.25
Multiply	$\boxed{x}$		3.25
Enter 16.7		$\boxed{1}\,\boxed{6}\,\boxed{.}\,\boxed{7}$	16.7
End problem	$\boxed{=}$		54.275
The product of (3.25)(16.7) is 54.275			

EXAMPLE 2: Calculate (14.07)(9.11)

Solution:

Direction		Key Strokes	Display
Enter 14.07		$\boxed{1}\,\boxed{4}\,\boxed{.}\,\boxed{0}\,\boxed{7}$	14.07
Multiply	$\boxed{x}$		14.07
Enter 9.11		$\boxed{9}\,\boxed{.}\,\boxed{1}\,\boxed{1}$	9.11
End problem	$\boxed{=}$		128.1777
The product of (14.07)(9.11)=128.1777			

The multiplication of two or more numbers, when some of them are measurements, has a very important effect on the unit of measure. An application of multiplication with which most people are familiar is the calculation of area. Observe what happens to the unit of measure when one computes the area of a rectangle that is 3 inches long and 2 inches wide. The area of a rectangle is found by area = (length)(width). Using the above dimensions, area = (3 in.)(2 in.). The number value from (3)(2) is 6 and the unit of measure for area is found by (inch x inch) = square inch. Square inch is also written as in^2. The next example contains several illustrations about the unit of measure which results from multiplication of measurements. Study them closely.

EXAMPLE 3: Compute the following products involving measurements.

(a) (4 inch)(7 inch)

(b) (6 mm)(3 mm)

(c) (4)(5 inch)(2 inch)

(d) (3 mm)(7 mm)(4 mm)

Solution:

(a) (4 inch)(7 inch) = (4)(7)(inch x inch)
= 28 square inches
= 28 in^2

(b) (6 mm)(3 mm) = (6)(3)(mm x mm)
= 18 square mm
= 18 mm^2

(c) (4)(5 inch)(2 inch) = (4)(5)(2)(inch x inch)
= 40 square inches
= 40 in^2

(Note that the 4 was not a measurement.)

(d) (3 mm)(7 mm)(4 mm) = (3)(7)(4)(mm x mm x mm)
= 84 cubic mm
= 84 mm^3

(Note that when three mm measurements are multiplied, the result has the unit cubic mm or mm^3.)

EXAMPLE 4: Work is computed by the multiplication of the weight moved times the distance it is moved. How much work is performed when 7 lb (pounds) is moved 4 feet?

Solution:

Work = (4 feet)(7 lb)
= (4)(7)(ft x lb)
= 28 ft-lb

EXAMPLE 5: The circumference c of a circle of diameter d is computed using the formula:
circumference = π times the diameter = πd.
Compute the circumference of a circle of diameter 12.3 mm.

Solution: circumference = $\pi(d)$
The key stroke to display π on a calculator varies by brand. The key strokes for π shown below works on many calculators.

Direction		Key Strokes	Display
Enter π		2nd EXP	3.141592654
Multiply	x		3.141592654
Enter 12.3		1 2 . 3	12.3
End problem	=		38.64158964

The diameter has mm unit of measure and π does not have a unit of measure. The circumference will have the same unit of measure as the diameter: $c \approx 38.6$ mm

A person who works with measurements needs to train themselves to pay special attention to the unit of measure resulting from a calculation. The ending unit of measure is perhaps even more important than the number value. What would you do if your boss asks you to "make me one which measures 9?" You should respond something like, "9 what?" Is that mm or in. of length or area using mm^2 or in^2 or sho.. size or what?

PRACTICE PROBLEMS:

Calculate the following multiplications.

1. (23.01)(0.843)
2. (654)(2.45)
3. (0.0063)(39.2)
4. (29.1)(8.307)(100)
5. (8320)(1.35)(6.6)
6. (π)(7.36)(0.025)

Multiply the given measurements. Be sure to include the correct unit of measure with your answer.

7. (16 in.)(3 in.)
8. (4 cm)(15 cm)
9. (7)(4 mm)
10. (20)(3 in.)
11. (2 mm)(4 mm)(6 mm)
12. (3)(2 in.)(3 in.)
13. (4 cm)(2 cm)(8 cm)
14. (5 in.)(5 in.)(4 in.)
15. (8 ft)(4 lb)
16. (7)(2)(4 mm)

EXAMPLE 6: Use a calculator to compute the area of a rectangle with length 47.95 in. and width 3.017 in.

Solution:

Directions		Key Strokes	Display
Enter 47.95		$\boxed{4}\ \boxed{7}\ \boxed{.}\ \boxed{9}\ \boxed{5}$	47.95
Multiply	$\boxed{x}$		47.95
Enter 3.017		$\boxed{3}\ \boxed{.}\ \boxed{0}\ \boxed{1}\ \boxed{7}$	3.017
End problem	$\boxed{=}$		144.66515
The product of (47.95 in)(3.017 in) is 144.66515in^2			

The answer 144.66515 in^2 contains 8 digits. Can the original 4 digit length measurements produce an area result which contains 8 accurate digits? The American Society for Testing and Materials says NO! The accuracy resulting from calculations must reflect the accuracy of the instruments a technician uses in making measurements. The accuracy should not improve because of the multiplication operation. There will be more information about Example 6 after learning how to determine accuracy.

Much of the data used by the technician is the result of a measurement. Since measurements are never exact, technicians must work with approximate measurements. The ACCURACY of a measurement is found by counting the numbers of reliable digits contained in the measurement. These reliable digits are called SIGNIFICANT DIGITS.

All the digits of a measurement are considered significant except for zeros used solely to place the decimal point. In a measurement recorded as 0.0820 in., the 2 leading (left-hand) zeros are not significant. Their only purpose is to show where the decimal point is in comparison to the other digits.

RULE 1: Zeros leading on the left of a decimal measurement are not significant.

When the measurement of 49.6 inches is rounded to the nearest inch, we write 49.6 in. ≈ 50 inches. Since rounding was to units place, the ending zero of the 50 in. is a reliable (significant) digit. But, when the rounding of 49.6 in. is written as 50 in., it appears that the rounding was to tens place or nearest 10 inch. There is a special way to tag the ending zeros of whole numbers when they are significant. This is done by placing a bar above the zero. The rounding of 49.6 inches to the nearest inch is written as 50 inches.

> **RULE 2:** Zeros ending (on the right) a whole number which are not tagged with a bar are not significant.

> **RULE 3:** All nonzero digits are significant and all zeros not excluded by Rule 1 or Rule 2 are significant.

The ACCURACY of a measurement is found by counting the number of reliable digits, called SIGNIFICANT DIGITS, which the measurement contains.

EXAMPLE 7: A list of original measurements are shown in the left-hand column. The second column has the significant digits boxed. The third column gives the accuracy of the measurement. See column four for explanations.

Original Measurement	Signif. Digits	Accuracy Count	Explanation
406.2 in.	$\boxed{406.2}$	4	The zero is not a leading or ending zero. Rule 3.
31.00 mm	$\boxed{31.00}$	4	This is not a whole number. Not Rule 2.
0.0260 in.	$0.0\boxed{260}$	3	Rule 1
$86{,}0\bar{0}0$ rev	$\boxed{8600}\,00$	4	Whole number with one untagged ending zero. Thus Rule 2.
0.0006 in	$0.000\boxed{6}$	1	Rule 1.
40600 mm^2	$\boxed{406}\,00$	3	Rule 2.
$61\bar{0}$ in^2	$\boxed{610}$	3	Rule 3.

PRACTICE PROBLEMS: Give the ACCURACY of each of the measurements.

17. 115 V

18. 6972 m

19. $44\bar{0}0$ ft

20. 0.0040 g

21. 8030 mm
22. 173.4 mi
23. 47,000 lb
24. 320,070 ft
25. 61$\bar{0}$ L
26. 1,0$\bar{0}$0 hr
27. 100.020 in.
28. 0.001005 m
29. 30.0 min
30. 0.02040 m

Calculations during which measurements are to be multiplied or divided require that the accuracy of the various measurements be compared. Specifically, it is important to identify the accuracy of the least accurate measurement.

The LEAST ACCURATE MEASUREMENT of a set of measurements is the one with the least number of significant digits.

EXAMPLE 8: Given the measurements, find the number of significant digits for each and the least accurate measurement.

Measurement	Number of Significant Digits	
13.00 mm	4	
0.006 mm	1	Least accurate
0.140 mm	3	
3400 mm	2	

EXAMPLE 9: Given the measurements, find the number of significant digits for each and the least accurate measurement.

Measurement	Number of Significant Digits	
0.737 mm	3	
0.94 mm	2	Least accurate
16.01 mm	4	
140 mm	2	Least accurate

PRACTICE PROBLEMS: In each set of measurements, find the measurement (or measurements in case of tie) which has the least accuracy.

31. 15.5 in., 0.053 in., 0.04 in.

32. 635 ft, 40$\bar{0}$ ft, 240 ft, 560$\bar{0}$ ft

33. 15.27 mm, 631.3 mm, 20.0 mm, 37.7 mm

34. 14.7 in., 0.017 in., 9.0 in., 0.810 in.

35. 4.9 kg, $67\bar{0}$ kg, 0.043 kg, 9.17 kg

The American Society for Testing and Material's has adopted the following rule to follow when measurements are used in multiplication and division.

RULE OF ACCURACY FOR MULTIPLICATION OF MEASUREMENTS

To multiply measurements
1. First, multiply all the measurements as given.
2. Round the final product to the same number of significant digits as the measurement which has the least number of significant digits.

The discussion can now return to Example 6 and examine the calculator value of:

$$\begin{aligned} \text{area} &= (47.95 \text{ in.})(3.017 \text{ in.}) \\ &= (47.95)(3.017)\ (\text{in. x in.}) \\ &= 144.66515 \text{ in}^2 \end{aligned}$$

Both width and length measurements are of 4 digit accuracy. Therefore, the final product needs to be rounded off to 4 digit accuracy.

$$\begin{aligned} \text{area} &= 144.66515 \text{ in}^2 \\ &\approx 144.7 \text{ in}^2 \text{ at 4 digit accurate} \end{aligned}$$

EXAMPLE 10: Use the rule for measurements to complete the following calculations.

(a) (8.31 in.)(9.027 in.)

(b) (6)(4.17 mm)(91.62 mm)

(c) 2π(7.864 mm)

Solution:

(a) Final product will match the accuracy of the 8.31 in. (3 signf. digit)

$$\begin{aligned} (8.31 \text{ in.})(9.027 \text{ in.}) &= (8.31)(9.027)(\text{in. x in.}) \\ &= 75.01437 \text{ in}^2 \\ &\approx 75.0 \text{ in}^2 \text{ (3 digit accuracy)} \end{aligned}$$

(b) The 6 is not a measurement so accuracy does not depend upon it. The measurement of least accuracy is the 4.17 mm; 3 digit.

$$(6)(4.17\text{ mm})(91.62\text{ mm}) = (6)(4.17)(91.62)(\text{mm} \times \text{mm})$$
$$= 2292.3324\text{ mm}^2$$
$$\approx 2290\text{ mm}^2 \quad \text{(3 digit accuracy)}$$

(c) The only measurement being used is the 7.864 mm which has 4 digits. The key stroke to display π on a calculator varies by brand. The key strokes for π shown below work on many calculators.

$$2\pi(7.864\text{ mm}) = (2)(\pi)(7.864\text{ mm})$$

Directions		Key Strokes	Display
Enter 2		$\boxed{2}$	2.
Multiply	$\boxed{x}$		2.
Enter π		$\boxed{\text{2nd EXP}}$	3.141592654
Multiply	$\boxed{x}$		6.283185307
Enter 7.864		$\boxed{7}\ \boxed{.}\ \boxed{8}\ \boxed{6}\ \boxed{4}$	7.864
End Problem	$\boxed{=}$		49.41096926

Solution $2(\pi)(7.864\text{ mm}) \approx 49.41\text{ mm}$ (4 digit)

PRACTICE PROBLEMS: Calculate the following multiplications. Round all products to the appropriate accuracy. Give the correct unit of measure.

36. (126 m)(35 m)

37. (18.3 cm)(48.2 cm)

38. (4.7 mm)(82.15 mm)

39. (560 in.)(28.0 in.)

40. (460 in.)(235 in.)(361 in.)

41. (2460 mm)(960 mm)(1970 mm)

42. (0.045 in.)(0.0292 in.)(0.301 in.)

43. (12)(18.40 in.)(9.75 in.)

44. 2π(17.35 mm)

45. π(9.39 in.)(9.39 in.)

As you work through the next few examples and the practice problems which follow, remember what you are trying to learn. The purpose of these word problems is to become exposed to situations which are solved by multiplication. You are learning WHEN to multiply, not how. After completion of a problem, you should review the information which described WHEN a particular operation was the one to be used.

One clue which might be encountered that suggests multiplication is that the problem might also be solved by repeatedly adding a number to itself.

EXAMPLE 11: What would be a workers gross weekly pay if they earn \$6.13 per hour and work 42.25 hours during one week?

Solution: One could add \$6.13 + \$6.13 + \$6.13 +··· until the hourly wage had been used a number of times equal to the number of hours worked. Repeated addition is more quickly solved by multiplication.

$$(\$6.13)(42.25) = \$258.9925$$
$$\approx \$258.99$$

In the case of money, rounding the nearest cent is customary rather than the rule of accuracy.

EXAMPLE 12: The weight W in pounds of a steel plate with length L = 7.31 in., width w = 6.05 in., thickness h = 0.25 in., and has density d = 0.2829 lb/in^3, is given by formula W = dLwh. Compute the weight of a steel plate with those dimensions.

Solution: When letters of a formula are written without separation by an operation, like in W=dLwh, then multiplication is used. The least accurate measurement is the 0.25 in. (2 signf. digits).

$$W = dLwh$$

$$= (0.2829)(7.31)(6.05)(.25) \text{ pound}$$

Directions		Key Strokes	Display
Enter .2829		$\boxed{.}\ \boxed{2}\ \boxed{8}\ \boxed{2}\ \boxed{9}$	0.2829
Multiply	$\boxed{x}$		0.2829
Enter 7.31		$\boxed{7}\ \boxed{.}\ \boxed{3}\ \boxed{1}$	7.31
Multiply	$\boxed{x}$		2.067999
Enter 6.05		$\boxed{6}\ \boxed{.}\ \boxed{0}\ \boxed{5}$	6.05
Multiply	$\boxed{x}$		12.51139395
Enter .25		$\boxed{.}\ \boxed{2}\ \boxed{5}$	0.25
End Problem	$\boxed{=}$		3.127848488
Solution: W ≈ 3.1 pounds			

PRACTICE PROBLEMS:

46. What would be a workers gross weekly pay if they earn $4.85 per hour and work 38.50 hours during one week?

47. What would be a workers gross monthly pay if they earn $5.35 per hour and work 157.25 hours during one month?

48. How high would a pile of 32 metal sheets be if each sheet is 0.045 in. thick?

49. Each cut on a lathe is 0.018 in. deep. By how much is the stock turned down after 14 cuts?

50. How much would 144 drill bits cost if they are price at $0.93 each?

51. The area of a rectangle is computed by length times width. Compute the area of a rectangle that is 14.75 inches long and 5.125 inches wide.

52. Compute the area of a rectangle that is 3200 mm long and 425 mm wide.

53. V=Lwh is the formula for the volume of a rectangular solid, where L is length, w is width, and h is height. Compute the volume of the rectangular solid with L=16.4 ft, w=8.6 ft, and h=6.4 ft.

54. Compute the volume of the rectangular solid with L=92.4 mm, w=61.25 mm, and h=12.50 mm.

55. Use the formula of Example 12 to compute the weight in pounds of a steel plate with length L=3.50 in., width w=2.2 in. and thickness h=0.40 in. that has density pf d = 0.2829 lb/in^3.

56. Use the formula of Example 12 to compute the weight in grams of a steel plate with length L=372.2 mm, width w=91.3 mm and thickness h=4.55 mm that has density of d = 38.9 g/mm^3.

SOLUTIONS TO PRACTICE PROBLEMS---Module 5

1.	19.39743	2.	1602.3	3.	0.24696
4.	24,173.37	5.	74,131.2	6.	0.578,053,048
7.	48 in^2	8.	60 cm^2	9.	28 mm
10.	60 in.	11.	48 mm^3	12.	18 in^2
13.	64 cm^3	14.	100 in^3	15.	32 ft-lb
16.	56 mm	17.	3	18.	4
19.	3	20.	2	21.	3
22.	4	23.	2	24.	5
25.	3	26.	2	27.	6
28.	4	29.	3	30.	4
31.	0.04 in.	32.	240 ft	33.	20.0 mm and 37.7 mm
34.	0.017 in. and 9.0 in.			35.	4.9 kg and 0.043 kg
36.	4400 m^2	37.	882 cm^2	38.	390 mm^2
39.	16,000 in^2	40.	$39,\bar{0}00,000$ in^3	41.	4,700,000,000 mm^3
42.	0.00040 in^3	43.	2150 in^2	44.	109.0 mm
45.	277 in^2	46.	$186.73	47.	$841.2[illegible]
48.	1.4 in.	49.	0.25 in.	50.	$133.92
51.	75.59 in^2	52.	1,360,000 mm^2	53.	$9\bar{0}0$ ft^3
54.	70,700 mm^3	55.	.87 pounds	56.	6,010,000 grams

END

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