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## ABSTRACT

Structural equation modeling has become increasingly popular as a technique for the analysis of non-experimental data in the social sciences. This increase can be traced in part to the development of computer programs that facilitate structural equation modeling. The Linear Structural Relations (LISREL) program is the most widely used tool for implementing structural equation models. LISREL is applicable across a wide range of study, but is most useful in analyzing social science data. Some characteristics of the LISREL program are examined. The general LISREL model is explained along with methods for establishing the identification and goodness-of-fit of the overall model and its individual parameters. It is concluded that the most important aspect of LISREL methodology is its flexibility; almost any type of problem involving regression or factor analytic procedures can be computed using LISREL. LISREL models should be theory driven, but should not be used to explore the grosser aspects of a theory. LISREL may be seen as a revolutionary synthesis of classical measurement theory/methodology with computer technology. Although LISREL methodology requires a good deal of technical understanding, it currently is in a form that is accessible to a large number of users. (SLD)

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Using LISREL: Some Notes on Technique and Interpretation

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### Abstract

Structural equation modeling has become increasingly popular as a technique for the analysis of non-experimental data in the social sciences. This increase can be traced in part to the development of computer programs which facilitate structural equation modeling. The LISREL (Linear Structural Relations) program is the most widely used tool for implementing structural equation models. LISREL is applicable across a wide range of study, but is most useful in analyzing social science data. This paper examines some of the characteristics of the LISREL program. The general LISREL model is explained along with methods for establishing the identification and goodness-of-fit of the overall model and its individual parameters.

### Some Notes on the Use of LISREL

The linear structural relations (LISREL) computer program is a powerful tool for the analysis data according to specific, theory-driven models which can be represented by a system of structural equations. LISREL allows for the analysis of many different standard and non-standard models including factor analysis, regression analysis, recursive and non-recursive path analysis, and many others. The program also allows for the comparison of these models across sample populations (Joreskog & Sorbom, 1985). LISREL, therefore, can be of great use to researchers who need a way in which to test theoretical relationships among variables in a concise manner. LISREL is particularly useful in the Social Sciences and has been used in studies ranging from psychopharmacology, child development, gerontology, sociology, health behavior, and psychology (Agren et al., 1986; Anderson, 1987; Bentler, 1987; Crano, 1987; Liang et al., 1987; Rothman, 1983; Volkan, 1987). The purpose of this paper is to provide a guide to the use of and interpretation of LISREL methodology.

The LISREL VI program compares the fit of a model to the data by the use of maximum likelihood analysis of structural equations with latent variables. This method allows a simultaneous comparison of the fit of both the structural and the measurement components of the model. This method of analysis uses only the factorally validated portion of the observed variables to estimate the structural components of a model and is therefore more precise than standard regression procedures. This method also allows for comparisons between a number of models which all may appear theoretically valid (Bentler, 1980). Therefore, even though a general model under analysis is assumed to be confirmatory, the

configuration of components within the general framework may be tested against each other in an exploratory fashion. Lomax (1982) stated that ". . . the major goal of LISREL-type structural equation modeling is confirmatory, in the sense of substantiating some theory, and exploratory, in the sense of making finer theoretical distinctions than were initially hypothesized" (p. 4). This use of the LISREL program corresponds to Joreskog's view that LISREL is both exploratory and confirmatory (Joreskog, 1978).

The LISREL program also allows for the specification of error variance and the correlation of error variance. The addition of correlated measurement errors in a model can sometimes be meaningfully interpreted and can often allow the researcher to achieve a better fit of the model to the observed data. This procedure of getting a better fit of the model through the use of error variance corresponds to the derivative analysis suggested by Lomax (1982). The derivative analysis can be done during the preliminary testing of the measurement components of the model. The derivative analysis allows for measurement errors to be correlated as long as the correlated error terms make theoretical sense and their addition gives a significant increase in the Chi square difference between models.

The LISREL program follows conventional path analysis terminology. Variables within squares represent measured or observed variables and the circled variables represent the latent variables of the model. The latent X variables are notated as KSI and the latent Y variable is notated as ETA. The arrows drawn from both the KSI and ETA variables to the observed X and Y variables indicate that it is the factor loadings which are used to estimate the correlation matrix among the observed variables. Each of these arrows in the model represents the factor

loading of an observed variable and the factor from which the variable is drawn and is notated as LAMBDA ( $LAMDA^X$  for X variables and  $LAMBDA^Y$  for Y variables). One of the LAMBDA coefficients is usually set to 1.00 for each latent variable in the model. Therefore, in a two factor model, there are two LAMBDA coefficients set to 1.00. These coefficients are set to 1.00 following Joreskog's (1978), and Joreskog and Sorbom's suggestion (1985) that the largest hypothesized factor loadings be used to set the metric of the factor loadings.

The KSI variables can either be defined as independent or dependent. Dependent KSI variables are notated with a correlation arrow between these variables. The GAMMA coefficients relate the KSI variable(s) to the ETA variable and represent the structural component of the model. The ETA variable is the latent Y variable and the arrows from it to the observed Y variables represent the LAMBDA factor loadings, which are interpreted like the factor loadings for the X variables. The DELTA coefficients represent the error variances and covariances for the X variables, and the correlations among the DELTA terms are represented by the THETA DELTA matrix. Likewise, the EPSILON coefficients represent the error variances and covariances of the Y variables, and the correlations among them are represented by the THETA EPSILON matrix. The ZETA coefficient represents the error in the structural equation and is sometimes called the disturbance term.

#### Identification of the Model

In LISREL it is necessary to establish the identification of the models according to methods suggested by Joreskog and Sorbom (1985), Joreskog (1978) and Lomax (1982). Without identification of the model it is not possible to tell whether or not the model is indeterminate.

An indeterminant model is one in which any number of alternative estimates are acceptable as parameter values (Long, 1983). There are three conditions which are used to establish identification of a model. The first two conditions are relatively easy to ascertain and involve the constraint of model parameters. The third condition is the most difficult to establish. This method for checking identification using another more complicated method also suggested by Joreskog and Sorbom (1985).

The first condition for identification is called the order condition. This condition is a necessary, but not a sufficient condition for identification of a model. In other words, even if the order condition is fulfilled, the model still may not meet sufficient conditions for identification. A model is said to fulfill the order condition if the number of parameters estimated in the model is less or equal to the number of equations in the model. This relationship can be expressed in the algebraic form:

$$t \leq q(q + 1)/2$$

where  $t$  is the number of parameters to be estimated and  $q$  is the number of equations in the model (Long, 1983, p. 42; Joreskog & Sorbom, 1985, p. 1.22). The example model considered in this paper fulfilled the necessary condition of identification.

The sufficient conditions for identification are difficult to establish and must be mathematically resolved from the variance/covariance equations in the model. Joreskog recognized that this type of calculation would be beyond the ability of most users of LISREL and he built a check of the sufficient conditions for identification into the LISREL program. This method works by checking the positive definiteness of the information matrix. If the information

matrix is positive definite, then the model is most likely identified. Joreskog claims that although this method is not completely reliable, it usually works in practice. Most users of LISREL assume their models are identified if the program does not give them a non-positive definite warning (Joreskog & Sorbom, 1985, p. 1.23). All LISREL models should fulfill this condition of identification.

The last method of establishing identification is somewhat more involved and is often used only in the final, stages of model development. This method for checking identification requires running the model with all parameters set to reasonable values. The program then gives the estimated SIGMA variance/covariance matrix as output. The SIGMA matrix is then used as input data for the program and the previously set parameters are set free. The program is then run and if the model is identified, the free parameters will correspond closely to the original reasonable estimates used to produce the SIGMA matrix. The final, best-fitting model can be checked for identification in this way.

#### Overall Assessment of Fit

A number of methods of establishing the overall assessment of fit of a model can be used with the LISREL program. The foremost of these is the Chi square/df ratio. Many writers (Hoelter, 1983; Long, 1983) including Joreskog and Sorbom (1985) have determined that the Chi square parameter is insufficient as a statistical test of a model. Instead, it is to be understood to be an assessment of fit. In this context, the use of the Chi square is only of use if the overall fit of one model is compared with that of another, nested model (Bentler, 1980). In addition, the Chi square (and the maximum likelihood [ML] estimates) should be considered valid only if the structural equation component of

the model can be shown to meet the assumptions of multivariate normality. If the multivariate distribution is unknown, then the Chi square must be used with caution as an indicator of goodness of fit. The way to approach this problem is to generate estimates for the model using the unweighted least squares (UL) procedure (which does not assume multivariate normality) before obtaining the maximum likelihood solution. The UL parameter estimates are then compared with the ML estimates. If both estimates give similar solutions, then it is likely that the ML estimation procedure is unaffected by the distribution of the data. Under these conditions the Chi square can be safely used as a test of the relative goodness of fit among the models tested. It should be noted, however, that the UL estimation procedure is scale dependent. This necessitates the use of the correlation rather than the covariance matrix for analysis in order to assess the variables on a standard scale. Since the UL estimates are compared with the ML estimates for consistency, the ML estimates must also be derived from the correlation matrix. Joreskog and Sorbom (1985) also suggest that the correlation matrix be analyzed when the units of measurement in the analysis are arbitrary. The use of the correlation matrix in the analysis does not pose a problem as long as models are not compared across groups. In addition, the Chi square should always be examined in light of the other goodness of fit indices when multivariate normality is in question (Fornell, 1983; Joreskog & Sorbom, 1985, pp. I.38-I.40; Long, 1983, pp. 47-48). The fit of a model should not necessarily be interpreted as "good" just because the Chi square/df ratio is very small. It is possible that the fit may be trivial due to small values among the observed relationships or high colinearity among the latent variables. A low Chi square/df ratio and thence a high probability are not as

important as the change in the Chi square/df ratio from one model to another (Fornell, 1983; Joreskog, personal communication, 1986). The Chi square value is also sensitive to sample size as well as departures from normality and either of these constraints may raise the Chi square/df ratio. A number of authors have suggested that Chi square/df ratios ranging from 2 to 5 indicate a reasonable fit of the model to the data (Carmines & McIver, 1981; Wheaton et al., 1977). However, these criteria can sometimes be misleading (Hoelter, 1983). Thus it is possible that a model may be well fitted even though the Chi square/df ratio is large and significant.

In order to compensate for these problems in using the Chi square, a few indices have been developed which are not affected by either sample size or departures from normality. The first of these indices are the Goodness of Fit Index (GFI) and the Adjusted Goodness of Fit Index (AGFI). The AGFI is adjusted for the degrees of freedom in the model. These are measures of the variance and covariance accounted for by the model. The GFI should range between 0 and 1 and is of limited use in comparing models (Hoelter, 1983; Joreskog & Sorbom, 1985, p. I.40). The distribution of the GFI and the AGFI are unknown and therefore there is no known standard to which to compare them (Joreskog & Sorbom, 1985, p. I.41). Nevertheless, GFI coefficients of 0.9 and above are generally considered to represent an adequate fit of the model.

Another index of fit which takes into account the influence of sample size is the critical N (CN) suggested by Hoelter (1983). This index assesses the fit of a model relative to identical hypothetical models with differing sample sizes and is expressed as:

$$CN = \frac{(z_{crit} + \sqrt{2df - 1})^2 + G}{2X^2/(n-G)}$$

where  $z_{crit}$  is the critical value of the selected probability level and  $G$  is the number of groups analyzed simultaneously. Hoelter suggested that  $CN$  values exceeding  $200(G)$  indicate that a model adequately reproduces the covariance structure of the observed data and that the difference between  $SIGMA$  and the covariance structure of the observed data are trivial. Hoelter based this assumption on the fact that in a number of trials, using different models, he found that the average residual variation to be under 1% when  $CN$  values were equal to, or exceeded,  $200(G)$ . Although Hoelter stressed the tentative nature of this index, he believed it to be valuable when used in conjunction with other goodness of fit indices.

The last overall goodness of fit parameter to be discussed is the root mean square residual (RMSR), which is a measure of the average variance and covariance of the residuals. This measure of goodness of fit is of value in comparing different models which use the same data. The RMSR does not, however, give any information about the individual parameters which may contribute to the overall residual error.

#### Goodness of Fit of Individual Parameters

The overall goodness of fit parameters are important in comparing differences across models. Nevertheless, it is the assessment of fit of the individual parameters which is important in understanding the final model. These individual parameter goodness-of-fit indices provide detailed information about the fit of each parameter of the model. The individual goodness-of-fit indices also provide information about possible modifications of a model, although this information must be evaluated carefully (Joreskog, personal communication, 1986; Lomax,

1982). The indices of individual parameter fit given by the LISREL program are the standard error terms for each parameter, the normalized residuals (including the Q-plot), and the modification indices. The standard error terms provide a measure of the precision of each parameter (for ML only). These should be examined for any unusually large values which would be indicative of a problem in a parameter. The normalized residuals are perhaps the most important indices for assessing the fit of the model. A normalized residual value higher than 2 may indicate a specification error in the model, and the parameters of this error should be examined. The Q-plot gives a summary of the residual errors for all the parameters. The normalized residuals should fall along, or be slightly greater than, a  $45^\circ$  angle. If the normalized residuals are too steep, the model is over-fitted and there is a chance that more than one set of model parameters will account for the data. If the angle of the normalized residual plot is smaller than  $45^\circ$ , then the model is poorly fitted and there is probably some specification error. Periodicity in the Q-plot may be caused by specification error or non-normality of the data. Generally, a small amount of periodicity is noticeable in the Q-plot and is not a problem unless it is extreme. The modification indices are associated with the derivatives of the fitting function. The modification indices give an estimate of the amount of expected decrease in Chi square per 1 df if a parameter is set free in the model. In general, a modification index of 5 or more is indicative of a problem (Joreskog, personal communication, 1986). This index provides a means by which changes in the model may be judged; however, it should be used with care (Joreskog & Sorbom, 1985, p. 1.42).

### Summary

The most important aspect of LISREL methodology is its flexibility. Most any kind of problem involving regression or factor analytic procedures can be done using LISREL. Time series analysis and longitudinal type analyses can also be done. LISREL modeling parameters can be specified according to a given theory and will yield very precise results. LISREL, however, is not a panacea. LISREL models should be theory driven, and not used to explore the grosser aspects of a theory. Practitioners who employ LISREL in this fashion will quickly find themselves overwhelmed by the task of clearly interpreting their results.

LISREL methodology has spawned a generation of offshoots, each of which has tried to improve on the original (Bentler, 1985; Muthen, 1984). The LISREL program itself has undergone a number of revisions. In addition, newer versions of the LISREL program are available which do not require time consuming statistical programming. These programs, PRELIS and SIMPLIS, as well as LISREL, are available for IBM compatible computers from Scientific Software Incorporated, P.O. Box 536, Mooresville, IN 46158-0536.

LISREL can definitely be seen as a revolutionary synthesis of classical measurement theory and methodology, with computer technology. Although LISREL methodology requires a good deal of technical understanding, it has now arrived in a form that is accessible to a large number of users.

## References

- Agren, H., Mefford, I. N., Rudorfer, M. V., Linnoila, M., & Potter, W. Z. (1986). Interacting neurotransmitter systems: A non-experimental approach to the SHIAA-HVA correlations in human CSF. Journal of Psychiatric Research, 20(3), 175-193.
- Bentler, P. M. (1987). Drug use and personality in adolescence and young adulthood: Structural models with nonnormal variables. Child Development, 58, 65-79.
- Bentler, P. M. (1985). Theory and implementation of EQS: A structural equations program. Los Angeles: BMDP Statistical Software.
- Bentler, P. M. (1980). Multivariate analysis with latent variables: Causal modeling. Annual Review of Psychology, 31, 419-456.
- Carmines, E. G., & McIver, J. P. (1981). Analyzing models with unobserved variables: Analysis of covariance structures. In G. W. Bohrnstedt & E. F. Borgatta (Eds.), Social measurement: Current issues. Beverly Hills, CA: Sage Publications.
- Crano, W. D., & Mendoza, J. L. (1987). Maturational factors that influence children's positive behavior: Demonstration of a structural equation analysis of selected data from the Berkeley Growth Study. Child Development, 58, 38-48.
- Fornell, C. (1983). Issues in the application of covariance structure analysis: A comment. Journal of Consumer Research, 9, March, 443-447.
- Hoelter, J. W. (1983). Structural analysis of covariance and correlation matrices. Sociological Methods and Research, 11(3), 325-344.

- Joreskog, K. G. (1978). Structural analysis of covariance and correlation matrices. Psychometrika, 43(4), 443-477. Joreskog, K. G. (1986). Personal communication. LISREL training seminar, Washington, DC.
- Joreskog, K. G., & Sorbom, D. (1985). LISREL VI: Analysis of linear structural relationships by the method of maximum likelihood. Mooresville, IN: Scientific Software.
- Liang, J., Aseno, H., Bollen, K. A., Kapana, E. F., & Maeda, D. (1987). Cross cultural comparability of the Philadelphia Geriatric Center Morale Scale: An American Japanese comparison. Journal of Gerontology, 42(1), 37-43.
- Tomax, R. G. (1982). Methods and designs: A guide to LISREL-type structural equation modeling. Behavior Research Methods and Instrumentation, 14(1), 1-8.
- Long, J. S. (1983). Covariance structure models: An introduction to LISREL. Beverly Hills, CA: Sage Productions.
- Muthen, B. (1984). A general structural equation model with dichotomous, ordered categorical, and continuous latent variable indicators. Psychometrika, 49, 115-132.
- Rothman, M. L. (1983). Hierarchical comparisons of structural equation models: An application to models of health behavior (Doctoral dissertation, University of Washington). Ann Arbor, MI: University Microfilms International.
- Wheaton, B., Muthen, B., Alwin, D. F., & Summers, G. F. (1977). Assessing the reliability and stability in panel models. In D. R. Heise (Ed.), Sociological methodology. San Francisco, CA: Jossey-Bass.

Volkan, K. (1987). The relationship of health conception to health-promoting lifestyle among older adults (Doctoral dissertation, Northern Illinois University). Ann Arbor, MI: University Microfilms International.