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ABSTRACT

This report examines the contribution of secondary vocational education to learning outcomes. It is targeted to research scholars with professional interests related to vocational education, effective schooling, learning, and methodology. The first of six chapters is an "Introduction" (Lawrence Hotchkiss, Jeanne Desy). "The Effects of Vocational Education: Theory" (Desy) presents a comprehensive review of theoretical viewpoints that describe what the role of vocational education in learning is expected to be. "The Effects of Vocational Education: Empirical Findings" (Desy) assesses empirical findings on the effects of vocational education. "Interpretation and Standardization of Interaction Effects" (Hotchkiss) summarizes the literature on interpretation and estimation of statistical interaction models. "Interaction in Dynamic Models" (Hotchkiss) analyzes the impacts of dynamic cycling of cause and effect on interaction specifications. "The Vocational Classroom and Student Learning Styles" (Hotchkiss, Desy) summarizes and integrates the earlier chapters. Student characteristics are considered in examining the question, Which students especially benefit from the vocational education experience? Ten pages of references are provided. (YLB)

ASSESSING THE CONTRIBUTION OF VOCATIONAL EDUCATION TO BASIC EDUCATION IN SECONDARY SCHOOLS

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FOREWORD

Most past research on the effectiveness of vocational education has evaluated wages, earnings, and the employment advantages of secondary vocational graduates. Although this work has produced valuable results, it has largely ignored the contribution of secondary vocational education to learning outcomes. This report, which is sponsored by the Office of Vocational and Adult Education, U.S. Department of Education, examines that contribution. The report is targeted to research scholars with professional interests related to vocational education, effective schooling, learning, and methodology. The theoretical base and methodology presented here for investigating learning outcomes also provide direction for guidance in secondary schools and for interpreting past and future research on the role of vocational education.

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EXECUTIVE SUMMARY

A wide array of claims have been made for vocational education, with economic benefits most often stressed. More recently, however, noneconomic and educational outcomes have been emphasized. Although critical theoretical and empirical issues regarding noneconomic outcomes of vocational education remain unresolved, substantially more work has been conducted on economic issues than on noneconomic issues. This document reports research designed to help diminish this imbalance. Expected noneconomic outcomes of vocational education are diffuse and difficult to verify, and no single research project could resolve all the outstanding unanswered questions. The work reported here has attempted to take strategic first steps.

First, theoretical expectations regarding the noneconomic outcomes of vocational education are codified and documented. It is found that vocational classrooms offer an environment that is substantially different from that of academic classrooms. Vocational classrooms engage students in "hands-on" applied learning that, it is argued in the literature, helps students to see the need for academic knowledge. However, some students benefit more from exposure to vocational education than others do.

Second, these basic ideas are formulated into a statistical interaction model. The model expresses the idea that the motivating power of vocational education complements academic courses for some students and thereby stimulates learning of academic material. Third, a review of empirical studies uncovers very little empirical investigation of the basic hypothesis. One of the critical reasons is that empirical assessment of interaction effects poses difficulties not encountered in traditional linear models. Fourth, methodological analysis of interaction effects are conducted. These analyses help to clarify meaningful interpretation of interaction effects and examine complexities that accompany study of interactions in explicit models of change over time. Literature on interpretation of interactions is summarized and extended by proposing devices that aid in the interpretations of interaction effects. Additionally, key difficulties in estimating interaction effects in dynamic models are identified and elementary solutions to those difficulties proposed. Much work on this issue remains to be done, however.

The chapters in this report are self-contained. Chapter 1 provides an introduction to the study. Chapter 2 presents various theoretical approaches to vocational education. It begins with a discussion of the outcomes historically claimed for vocational education, with emphasis on the noneconomic outcomes that have been more recently stressed. The model of learning proposed by Sorensen and Hallinan is presented next. This model suggests that how much students learn in the vocational education classroom will vary according to the individual's personal characteristics,

particularly ability and effort (a function of motivation). Examination of this interaction is essential to any understanding of the value added by vocational education. Value-added theory is discussed briefly, with reference to the mathematical issues involved in any model of change and to the significance of the value-added approach for vocational education. Finally, the vocational classroom is described, as it has been documented by Goodlad (1983) and Weber and associates (1988). The discussion shows that this educational opportunity emphasizes the technical practice of psychomotor skills, rather than verbal practice, and focuses on individual, competency-based learning. The vocational classroom is thus less competitive than the academic classroom and more structured and theoretically has special benefits for students who learn slowly, who need well-organized information, and who respond poorly to competitive reward structures.

Chapter 3 presents empirical findings on the effects of vocational education. Basic skills are treated separately from other noneconomic outcomes because of their special importance. Vocational students generally have below-average basic skills; there are not adequate entry data to determine the role secondary programs play in basic skills achievement. There is evidence that learning experientially results in more meaningful learning and retention (Ausubel 1980), increasing recall through both the generation affect and the variability effect (Ellis 1980). Scholarship on learning through application supports the contention that vocational education provides an important learning experience.

Chapter 3 also surveys current knowledge about other noneconomic effects, finding that little research has been performed and few effects are well documented. An overview of studies of economic effects reveals that definitive findings will depend on future studies of earnings profiles over time.

A further effect of vocational education is motivation. This curriculum is often claimed to provide motivation that may be instrumental in retention of dropout-prone students. This capacity to motivate does seem to exist, particularly in the intrinsic feedback of technical skills practice, which offers specific and proximal goals. The "relevance" students ascribe to vocational education (Goodlad 1983) is also motivating. Overall, there is little doubt that the vocational classroom offers unique opportunities to achieve and to build self-efficacy and general motivation for learning.

Chapter 4 summarizes the literature on interpretation and estimation of statistical interaction models. This chapter shows how to interpret the substantive importance of interaction by comparing the mean level of effects of x and y to the variation in those effects. Standardization formulae are proposed that, unlike the usual standardized regression coefficients, apply generally to nonadditive (or nonlinear) models and can be used to compare effects between independent variables with different measurement units.

Chapter 5 shows that even the simplest statistical interaction model (involving products) requires nonlinear estimation of parameters in a dynamic model. Various strategies for obtaining estimates are reviewed and practical methods of estimation are proposed.

Chapter 6 draws on the concept of learning styles to help integrate theory of vocational education into a broad intellectual context. It is difficult to choose a learning style model, as none are empirically based. Assessment instruments have been extensively criticized; of those in wide use, the MBTI comes well recommended. It may be best for vocational administrators and teachers to devise their own assessment instruments, combining these with interview and discussion with the student. A model is proposed as a preliminary attempt to integrate the main ideas of the earlier chapters. The model shows how to include basic elements of learning style and vocational theory in an expanded interaction specification.

CHAPTER 1

INTRODUCTION

by Lawrence Hotchkiss
and Jeanne Desy

The philosophy of vocational education rests on the idea that the vocational curriculum differs in significant positive ways from the academic and general curricula. Not only does it provide work-related skills and values, but, proponents claim, it motivates students to learn, particularly such types of students as those with a practical, concrete orientation. A concept such as calculation of an area that a student may find difficult or meaningless in a classroom may be of more obvious value in the actual real-world task of painting a room.

The argument is that, both directly and indirectly, vocational course work complements academic course work--each type of education enhances the value of the other. Assessment of this "complementarity hypothesis" poses a number of conceptual and methodological difficulties. This report presents substantive background to the problem and analyzes conceptual and methodological issues regarding the concept of statistical interaction that are closely connected to the substantive theory.

A primary motivation for embarking on such a study was that we do not currently have available principles that can be used to help youth with different interests, learning styles, and aptitudes choose curricula that best suit their unique needs. It is plausible, for example, that students with particular learning styles would improve their knowledge of grammar and writing more in business and commercial courses than they would in English courses that relied on sentence diagramming. However, we do not know enough about the way these individual differences may interact with the processes in the vocational classroom to systematically help youth make curriculum choices to their best advantage.

Two broad questions are implied by the above considerations. First, is there, in fact, a unique educational process and content characterizing vocational education that is particularly well suited to the learning styles, attitudes, and aptitudes of some students but not others? In other words, are our commonly held assumptions true? Second, if they are, then how can we know what mix of academic and vocational courses will most profit a particular student? Refining our ability to guide students into suitable curricula is particularly important in view of accusations that students may be "tracked" into vocational education with reference to extraneous background factors (Oakes 1986).

The specific research questions raised by these broad issues are far too extensive to be resolved in a single research report. The present report presents important steps toward systematizing the critical issues in ways that promote their analysis. First, this report presents a comprehensive review of theoretical viewpoints that describe what the role of vocational education in learning is expected to be. This review is intended to clarify the theory and ground it in a broad base of research literature. Second, this report assesses empirical findings on the effects of vocational education. Third, the report provides an exposition of interpretation of interaction models designed to express key aspects of vocational education theory. Fourth, the report evaluates methods for estimating dynamic interaction models. Since learning is dynamic and certainly must involve interactions between learner characteristics and mode of presenting material, this work is particularly critical to adequate empirical assessment of the role of vocational education in learning.

The chapters in this report represent separate areas of expertise and are written as stand-alone papers; their authorship is indicated. The substantive chapters examine the various claims made for vocational education and describe an interactive theory of learning as a basis for the methodological study. The significance of value-added theory for vocational education is considered. Classroom observation studies are used to define the distinctive educational process of the vocational education classroom as it differs from that of the academic classroom.

Chapter 4 presents a careful review and assessment of appropriate methods for specifying and interpreting interaction effects in structural models. Lucid interpretations of interaction effects have often been missing in past work, and this chapter attempts to clarify approaches for promoting clear interpretation and examination of interactions. Chapter 5 analyses the impacts of dynamic cycling of cause and effect on interaction specifications. The analysis shows that interaction specification in a dynamic structural model introduces complexities that do not occur for dynamic linear models. Practical methods for handling these complexities are proposed. The final chapter summarizes and integrates the earlier chapters. Student characteristics are considered in examining the question, What students especially benefit from the vocational education experience?

CHAPTER 2

THE EFFECTS OF VOCATIONAL EDUCATION: THEORY

by Jeanne Desy

Claims for Vocational Education

Swanson (1982) remarks that it is surprising "that the search for a definition of vocational education is still underway, and that there is such diversity of perceptions about what it is or should be" (p. 36). McKinney and Fornash (1984), in their discussion of the outcomes of vocational education, comment similarly that "historically, there has been little agreement among vocational educators as to what outcomes should be expected. Vocational education has never had a single spokesperson or a prevalent view regarding outcomes" (p. 57). They add that vocational education operates in a complex, changing context and must attempt to meet a variety of expectations, with which Swanson agrees, asserting that "vocational education is not a system" (p. 40) but a collection of varying enterprises.

Nevertheless, much informal theorizing regarding the role and function of vocational education has taken place, and vocational programs continue to operate on a theory, if imperfectly articulated, of what vocational education can and should do. Grubb (1979) maintains that through cycles of criticism and change certain claims have always been made on behalf of vocational education, and that these form its core. The problems it promises to solve, he says, are economic, educational, and social. Grubb emphasizes social and economic outcomes in his discussion, outcomes that have historically been emphasized. For example, the 1914 report of the Commission on National Aid to Vocational Education, on which the Smith-Hughes Act of 1917 was based, described vocational education as "a wise business investment for this Nation" (Lazerson and Grubb 1974, p. 117) that would solve numerous social and economic problems by creating a supply of trained workers. In the same era, Charles Prosser talked of "fitting the great mass of our people for useful employment" (Lazerson and Grubb 1974, p. 135).

This emphasis on labor market outcomes continues today (see Miller and Budke, n.d.; American Vocational Association 1981). Oakes (1986) classifies claims for vocational education as noneducational and educational and says that the latter set of claims "receives much less consideration in either practice or research" (p. 34). Evans (1984) says that virtually all research and evaluation has centered on individual, labor market-related outcomes.

In 1984, Pratzner and Russell published the results of a small Delphi survey of national opinion leaders that addressed the appropriate role of vocational education. The consensus was that

> secondary-level vocational education is a multi-purpose program intended to provide technical skills, and at the same time, foster good work attitudes, facilitate the transfer of skills needed in all jobs, motivate disenchanted learners, enhance basic skills attainment, serve as an exploration into the careers arena, and compensate for discrimination in society against special needs populations. (p. 28)

This recent statement indicates the wide variety of needs vocational education is expected to fulfill. Similarly, the AVA's 1981 <u>A National Agenda for Vocational Education</u> discusses an impressive array of "appropriate national concerns" addressed by vocational education. These include, but are not limited to, economic revitalization, energy conservation, and maintenance of defense capability. <u>A National Agenda</u>, however, also mentions human development benefits specific to the vocational classroom--a reality-based standard of performance, pride in work, purpose, and development of a range of talents and interests.

Farley (1979) developed 252 questions representing hypothetical outcomes of vocational education mentioned in the literature. She divides these outcomes in terms of their effects on the individual, society, and special needs populations, and then into economic and noneconomic effects. The 23 noneconomic outcomes hypothesized for secondary students include only 3 related directly to learning:

- Skills of inquiry, that is, systematic information-seeking
- Scores on verbal and achievement tests (the phrasing suggests these may be lower than the scores of peers in other curricula)
- The ability to communicate in a variety of modes

From Farley's list, Darcy (1980) selected 15 outcomes of vocational education judged to be important, appropriate, amenable to empirical study, and useful for policy. Only one of these is related directly to educational outcomes, improving basic skills.

McKinney and Fornash (1983) asked experts in 6 fields to write papers discussing 18 outcomes of vocational education selected from Farley's list by project staff and 9 other vocational educators for their historic, individual, social, and economic importance. The only learning-related outcomes on the list were motivation for educational and occupational achievement and increased awareness of need for basic academic skills.

Campbell and Panzano (1985) divide educational outcomes into individual and societal outcomes. (Interestingly, this exploration of quality in vocational education lists more of the former than the latter, a distinctly modern emphasis.) The list of competency development (as opposed to personal) outcomes includes several work-related skills as well as basic skills, information skills, and problem-solving skills.

In a discussion of the noneconomic benefits of vocational education, Silberman (1980) sets up an "intrinsic perspective" that sees vocational education as a means to education in the broad sense of full human development. This paper makes explicit a number of claims for the process of vocational education that have been the subject of little rigorous investigation, but are often assumed to be true. Vocational education, Silberman says, offers "activities that are intrinsically meaningful and absorbing. . .[and emphasize] the playful aspects of work" (p. 43). He considers vocational education's opportunities for human development and personal satisfaction to be significant, and applauds the quality of the learning environment; the tangible product and the pleasure of the process offer intrinsic motivation. Vocational education, Silberman claims, enables personal growth through a sense of competence, opportunities for creative expression, discipline and the integrity of good work, teamwork, and altruism.

The Unfinished Agenda (National Commission on Secondary Vocational Education 1984) similarly assumes that the vocational classroom meets different student learning needs than the academic classroom. This document refers to "vocational education's potential to respond to diverse learning styles." The discussion assumes that the vocational classroom process includes--

- immediate feedback;
- applied learning;
- cooperative, small-group activities;
- individualized learning;
- mastery learning;
- activities the student considers relevant;
- real-world experiences (especially through cooperative education);
- tactile and kinesthetic learning;

- opportunities to be creative; and
- productive tasks.

In these activities, the report claims, lie the real strength of vocational education--"its ability to motivate students" (p. 5).

Figure 1 lists some of the educational claims made for vocational education. These have been grouped in categories suggested by Gagne (1977). This grouping highlights the prevalence of claims made that certain desirable attitudes are more likely to be learned in the vocational than in the academic classroom. These attitudes, and other capabilities learned, are said to be fostered by the educational opportunities unique to the vocational classroom. As a glance at this lists suggests, these opportunities derive from technical skills practice, the distinguishing feature of the vocational classroom.

Figure 1

For the most part, the claims just discussed are not substantiated in the works cited, but presented as theory. A later section of this review considers the evidence for these claims.

Theory of Learning

For the most part, learning results from the interaction of the individual with the environment (Gagne 1977; Bower and Hilgard 1981). The nature of this process and the conditions that best foster learning are explained by numerous models (such as reinforcement, insight, and information processing) with which educators are generally familiar. For the purpose of this study, one model is especially interesting--that described by Sorensen and Hallinan (1986). Their model, in its simplest form, is as follows:

$$dy(t) = s\ dv(t)$$

where

$dy(t)$ = increase in student achievement in a small period of time (dt)

s = a measure of relevant student characteristics

$dv(t)$ = new material taught in dt.

By definition, s determines how much is learned of what is taught.

Capabilities Produced	
Intellectual skills	Darcy 1980, McKinney and Fornash 1983, Pratzner and Russell 1984, Campbell and Pazano 1985
Verbalizable information	Campbell and Pazano 1985
Cognitive strategies	Campbell and Pazano 1985
Motor skills	*Unfinished Agenda*
Attitudes	
Teamwork, cooperativeness	Silberman 1980 *Unfinished Agenda*
General motivation to learn	Pratzner and Russell 1984 McKinney and Fornash 1983
Sense of personal competence	Silberman 1980
Discipline	Silberman 1980
Educational Opportunities Provided	
Individualized learning	*Unfinished Agenda*
Mastery learning	*Unfinished Agenda*
Tactile and kinesthetic learning	*Unfinished Agenda*
Real-world performance	AVA 1981, *Unfinished Agenda*
Productive activities	*Unfinished Agenda*
Intrinsically meaningful (relevant) activities	Silberman 1980 *Unfinished Agenda*
Creative expression	Silberman 1980
Immediate feedback	*Unfinished Agenda*

Figure 1. Educational claims made for vocational education (excluding work-related outcomes).

> Variation in academic achievement among students will . . . depend on variation in the ability and effort of those students, and on variation in the amount taught that represents opportunities for learning. (p. 524)

Sorensen and Hallinan's model deals with a specified period of time (such as a class period) during which a learning opportunity is available. That opportunity varies, of course, between schools, curricula, teachers, and even instructional groups within a classroom.

The Sorensen-Hallinan model is connected to theory of the benefits of vocational education at a strategic point. As noted above, a primary claim of vocational theory is that the "hands-on" approach in vocational classrooms motivates students to learn. Motivated students should exert more effort and be more attentive to subject content. Thus, the value of s should be raised for particular students in vocational classrooms who are receptive to the "hands-on" approach. The Sorensen-Hallinan model is important in the present report for a second reason; it expresses an explicit dynamic model of learning that provides a good foundation for more elaborate theorizing.

Research on aptitude-treatment interaction has encompassed a wide variety of instructional treatments. For instance, learning opportunities may be inductive (discovery) or didactic, and numerous studies have examined the interaction of these treatments with ability as measured by IQ. The educational opportunity is also characterized by classroom dynamics, which can be seen as democratic or dominative in varying degrees. Teacher characteristics, another significant element of the learning opportunity, include divergent (versus conventional) thinking and other elements of cognitive style, such as conceptual level and field-independence.

To each learning opportunity, the individual student brings a certain ability and effort. The variable s is described as standing for "all characteristics of students and their situation relevant for how much they learn of what is taught" (p. 523). Obviously, students bring to the learning situation numerous relevant personal characteristics, including physical limitations, prior knowledge, cognitive skills, and a number of capabilities and modes of processing often discussed under the rubric of learning style.

Identifying, formulating, and measuring even the most important of these characteristics for use in a model of this sort is a tremendous task. Cronbach and Snow (1977) comment that "arguments about the complexity of the ability domain never end" p. 152). The literature on personality variables that affect academic achievement is equally voluminous. Much of it, according to

Cronbach and Snow, has focused on anxiety, "a syndrome of self-deprecation, expecting to fail" (p. 393), which has a significant effect on the student's reaction to demanding tasks. Among the other variables that appear to affect a student's response to an educational opportunity are sociability, adjustment/neuroticism, preference for certain instructional methods, and compatibility with instructor's basic beliefs. Demographic variables--age, gender, socioeconomic status--are also important, but, Cronbach and Snow suggest, they are most useful when translated into findings about the individual's psychological qualities.

The effort a student brings to the learning situation is an expression of motivation. Some of the student's motivation will be supplied by interaction with the learning opportunity; as Ausubel (1968) says, motivation is inherent in the learning situation that produces educational achievement. (Other aspects of motivation are briefly discussed in chapter 3.)

The foregoing describes some elements of two extremely complex variables--educational opportunity and student characteristics. A major task in working with interaction formulae is the selection of measures to represent these variables. Fundamental considerations in this choice will be discussed in the remainder of this chapter.

The Value Added by Vocational Education

During the past decade a new tactic has been proposed and explicated for evaluating ongoing educational practices--the "value-added" approach (Bryk, Strenio, and Weisberg 1980; Osegwih 1985, 1986; Evans and Hunter 1979). Ewell (1987) reports that assessment is increasingly emphasized and notes that the Fund for the Improvement of Postsecondary Education recently funded 11 projects on this subject. The concept of "value-added" assessment, Ewell adds, is supported by some 65 percent of college administrators.

The value-added terminology carries with it intriguing connotations, but the associated methodology is somewhat narrowly defined. As explicated by Bryk and associates (1980), the concept relies on a simple model of growth in the absence of educational intervention. Value added is defined as "the degree to which those exposed to the educational 'treatment' grow more rapidly than they otherwise would have." This comparison is substituted for comparison to a control group. Individual growth in the absence of the treatment is estimated by extrapolating the trend line derived from the growth model.

Adding to the general difficulty with extrapolating a trend line, the basic model proposed by Bryk and colleagues is based on a linear trend line. Although a straight line may fit an observed curve over a restricted range quite well, it usually diverges rapidly from the curve when both lines are extended beyond the time points used to fit the data. Bryk and associates recognize these points and offer suggestions for resolving them, but their papers are best viewed as sources of ideas rather than as providing a rigorous framework for assessing value added by education.

Mathematical models of change based on differential equations offer a much more general framework for assessing value added than the models proposed in the value-added literature. The linear growth model proposed by Bryk can be expressed as a special, particularly simple differential equation. Moreover, more useful expressions of change are well understood in the mathematics literature.

Some form of value-added assessment is no doubt useful to vocational education. As has been noted elsewhere in this chapter, meaningful assessment of vocational students' achievements is impossible without pretest data. Evans and Hunter (1979) have cautioned that

> the precise measurement of value added by education . . . needs to be tackled by the educational economist, the educational psychologist, the educational philosopher, and the educational sociologist. All four groups need to be involved, because we should be concerned not only with the value added in terms of dollars, but also in terms of personal worth and worth to society. (p. 68)

Osigweh (1985) similarly refers to the assessment of students' "mental as well as personal development" (p. 131).

Although the issues of value-added assessment are complex, the concept has immediate utility for educators. It encourages teachers to think not in terms of a predetermined objective for a student, but in terms of adding to the value of that individual student's education. Such a reorientation would focus more attention on the claims made for vocational education. It would also enable assessment of individual outcomes without one-on-one comparison to outcomes for students in other curricula.

The Vocational Classroom

The classroom was examined in A Study of Schooling, on which numerous articles have been based and which is described in Goodlad's A Place Called School (Goodlad 1983). Goodlad and his associates collected contextual data on over 1,000 elementary and secondary classrooms during the 1976-1978 school years. The overall data for secondary schools (which does include a proportional number of vocational classrooms) draws a familiar portrait of the high school classroom. Nearly 70 percent of total class time involves talk, and "teachers 'out-talk' students by a ratio of nearly three to one" (Sirotnik 1983). Most students are involved in passive learning, usually as a total class. Fewer than 5 percent work individually, fewer than 10 percent in small groups.

In a striking passage, Goodlad describes the general passivity of the classroom.

> Three categories of student activity marked by passivity--written work, listening, and preparing for assignments--dominate. . . . The chances are better than 50-50 that if you were to walk into any of the classrooms in our sample, you would see one of these three activities under way. . . . We saw a contrastingly low incidence of activities invoking active modes of learning. Except in the arts and vocational education, students were not very often called upon to build, draw, perform, role play, or make things. (Goodlad 1983, p. 105)

Table 1 uses the data summarized in Sirotnik's (1981) technical report to compare academic and vocational classrooms. The column headed "academic" includes English, math, social studies, science, and foreign language classrooms. The second and third columns contain the means for English and math classrooms.

Table 1

Two major differences between academic and vocational classrooms are immediately noticeable. First, the vocational classroom is less auditory; there is substantially less learning through listening to words, represented in the second category. In fact, the vocational classroom is less verbal altogether, giving less time given to discussion, reading, and even test-taking than does the academic classroom. Second, in the vocational classroom considerably more time is devoted to practice of psychomotor skills. These findings support one of the key claims of vocational educators cited earlier in this paper--that vocational classrooms offer a distinctive style and context. This emphasis on practice probably is the single most important feature of the vocational classroom, with implications of motivation and with special appeal of students with particular learning styles.

In math courses, a higher than average amount of time is given to lecture, and 27.2 percent of class time is spent on written assignments. English courses are also highly verbal; 6.1

TABLE 1
SECONDARY VOCATIONAL AND ACADEMIC CLASSROOM ACTIVITIES

Activity	Academic	English	Math	Vocation
Setup, cleanup, instructions	12.3	13.8	11.5	12.9
Explain, lecture, read aloud	30.6	27.2	35.3	19.0
Demonstration	1.4	0.7	3.0	2.1
Discussion	6.6	9.7	2.2	3.5
Simulation/role playing	.1	0.3	0.0	0.0
Reading	2.4	6.1	0.4	.8
Work on written assignments	17.8	18.4	27.2	17.7
Practice, performance psychomotor	4.1	1.8	3.3	29.6
Practice, performance verbal	6.3	1.3	0.8	2.2
Taking test or quiz	8.0	9.1	8.5	3.9
Audio visual	2.7	4.1	0.0	1.9
Off task	7.3	7.2	7.6	5.8
Discipline	.1	0.0	0.0	0.2

SOURCE: Adapted from Sirotnik (1981).
NOTE: Columns may not total 100, due to rounding.

percent of the time is spent on reading and 9.7 percent on discussion. As a later section will review, these modes of learning are especially difficult for some students who may have difficulty acquiring basic skills unless they can learn them another way.

In summarizing the data for all classrooms, Sirotnik (1983) says,

> the modal classroom patterns consist of (1) the teacher explaining or lecturing to the total class or to a single student, asking direct, factual questions, or monitoring students; and (2) the students ostensibly listening to the teacher or responding to teacher-initiated interaction. (p. 21)

The data show that the vocational classroom offers an alternative to this pattern.

Technical Skills Practice

Practice is the "applied learning" of which The Unfinished Agenda speaks. It is also the distinguishing feature of the vocational classroom, lending it numerous characteristics of importance to learning, and affecting instructional style. Sirotnik's data (see table 1) show that psychomotor practice occupies only 4.1 percent of the time in academic classrooms and 29.6 percent of the time in vocational classrooms. Only about 2.6 percent of basic skills (English and math) instructional time involves psychomotor practice.

Table 2 displays these findings in terms of percentage of engaged learning time--that time not spent in setup and cleanup, discipline, and miscellaneous off-task activities. Viewed this way, the difference is even more striking: academic classes overall spent 5.1 percent of their engaged learning time on psychomotor practice, compared to 36.5 percent for vocational classes, and spend more time on verbal practice.

TABLE 2

SECONDARY VOCATIONAL AND ACADEMIC CLASSES
PERCENTAGE OF ENGAGED LEARNING TIME
DEVOTED TO PRACTICE OR PERFORMANCE

	Practice Performance-Verbal	Practice Performance-Psychomotor
Academic	7.9	5.1
Vocational	2.8	36.5

SOURCE: Adapted from Sirotnik 1981.

Weber et al. (1988) recently completed a 2-year study of the processes that take place in secondary vocational classrooms. Data were collected in 120 schools and approximately 900 classrooms through questionnaires, interviews, and direct observation. This study showed an even higher relative proportion of vocational class time spent in the practice of technical skills--57 percent, with 29 percent spent on theory-related situations.

How much does time spent on practice vary from one vocational service area to another? Vocational classrooms were observed by Halasz, Behm, and Fisch (1984). Table 3 presents their findings in regard to technical skills practice. In all three areas examined, about half of all learning time was devoted to practice.

TABLE 3

SECONDARY VOCATIONAL SERVICE AREAS
PERCENTAGE OF TIME ON TASK
SPENT ON THEORY AND PRACTICE

Service Area	Theory	Practice
Agriculture (2 classes	30.66	55.49
Business and Office (3 classes)	23.99	51.59
Trade and Industrial (4 classes)	33.08	50.65
average for all	29.24	52.58

SOURCE: Adapted from Halasz, Behm, and Fisch (1984).

Classroom organization is also important to learning. For the great majority of all classroom time--75 percent--secondary students work as a total class. Students overall work individually less than 5 percent of the time and in small groups less than 10 percent of the time (Sirotnik 1983).

The vocational classroom shows a markedly different pattern. The secondary vocational students observed by Halasz, Behm and Fisch (1984) spent 80 percent of their time in class working individually or in small groups. Weber et al. (1988) found that vocational students worked in small groups 24 percent of the time, compared to 11 percent for nonvocational students. Further, vocational students worked on activities that provide a high degree of autonomy 56 percent of class time; nonvocational students did so only 11 percent of the time. Grouping has direct implications for instructional method. Because vocational students spend so much time working individually, one-on-one instruction is far more common than lecture. Halasz, Behm, and Fisch (1984) found that secondary vocational teachers spent about 23 percent of their time providing one-on-one instruction and less than 3 percent lecturing.

Competency-Based Vocational Education

Between 60 and 80 percent of all secondary vocational classrooms use competency-based vocational education--a highly systematic approach to mastery learning (Weber et al. 1988). Mastery learning assumes that most students can learn

> if instruction is approached sensitively and systematically, if students are helped when and where they have learning difficulties, if they are given sufficient time to achieve mastery, and if there is some clear criterion of what constitutes mastery. (Bloom 1976, p. 4)

CBVE accommodates individual differences in learning style far more readily than does a group instructional format. In particular, as Knaak (1983) notes, CBVE allows students to learn at their own rates. Some students need much more time than others to learn, a difference observed by Bloom (1976), who found that some individuals took five times as long as others to master a task. In fact, adequate <u>time</u> to learn may be more important than intelligence as measured by standard tests (Shuell 1986). Time to access information may also be a significant factor in achievement. Hunt, Frost and Lunneborg (1973) tested the amount of time subjects required to access overlearned information; postsecondary students who scored in the lower quartile on a standard verbal intelligence test took about 40 percent longer to do so than students in the upper quartile. The vocational classroom that uses CBVE offers students the opportunity to learn and to access and apply what they know at their own rates.

Flexible rate of learning is not the only important feature of CBVE. Competency-based instruction provides a logical, systematic structure for learning that makes learning easier for students with certain cognitive characteristics. Snow (1977) found the performance of low-ability students greatly improved in a relatively structured learning environment. A leading learning style theorist, Gregorc (1982), describes the learning style that emphasizes sequential, as opposed to random, ordering. A preference for sequence, he says,

> disposes your mind to grasp and organize information in a linear, step-by-step, methodical, predetermined order. Information is assembled by gathering and linking elements of data . . . in a chain-like fashion. (p. 5)

CBVE is characterized by the use of fixed performance criteria, and an independent reward structure is built into any such teaching system (Slavin 1977). The student succeeds not through others' success (the cooperative structure) and not through besting others (the competitive structure) but through meeting defined criteria.

The vocational classroom that uses CBVE is not necessarily free of competitiveness. Competition is encouraged in any classroom in which teachers grade on the curve, which means that one student's success is linked to another student's failure (Michaels 1977). One outcome of competitiveness is that students may not only withhold help from their peers but may even hinder one another (Slavin 1980). It is understandable, then, that peer sanctions oppose high performance in competitive reward structures. This peer opposition to success is very important in schools (Coleman 1961). Its special importance in delinquent groups (Graubard 1969) may mean that students with delinquent behavior stand to benefit particularly from competency-based vocational education in a noncompetitive classroom.

Competitiveness increases individual performance under some circumstances, but when competitors are poorly matched there is very little motivation to achieve (Slavin 1980). It is not difficult to imagine this dynamic in the traditional academic classroom. Suppose students in a high school civics class are discussing a current event. Low-verbal students who need time to access what they know and time to reflect may perceive themselves at such a disadvantage that they make no effort to join in. Other personal characteristics, such as low self-esteem, may exaggerate a student's inability to perform in competitive situations of this kind. CBVE, however, allows the student to perform without this competitive disadvantage.

Competency-based learning, then, has features of special importance for students who learn slowly, those who learn best when information is strictly organized, and those whose learning is negatively influenced by competitive reward structures.

CHAPTER 3

THE EFFECTS OF VOCATIONAL EDUCATION: EMPIRICAL FINDINGS

by Jeanne Desy

Basic Skills in the Vocational Curriculum

Too little information is available on the basic skills achievements of vocational students. Corman found in her 1980 review of research that there were no data comparing vocational students' basic skills at entry and completion, and that almost nothing was known about the differential competencies of students in different programs.

Overall, the basic skills levels of vocational students are "generally below the average of the entire student population" (Greenan 1983). These students score about one standard deviation below academic students (ibid) and one-half standard deviation below all secondary students (Weber and Silvani-Lacey 1983) on basic skills measures.

Can basic skills be taught effectively in vocational programs? Weber and Silvani-Lacey examined empirical data on 11 programs for potential and actual dropouts. They found that when these students participated in vocationally oriented programs with explicit basic skill components, their basic skill levels increased an average of half a standard deviation. They stressed that these successful programs used "an integrated vocational-academic approach to curricular content and structure" (p. 11).

In recent years, the need to integrate basics into the vocational curriculum has been much discussed. Classroom observation has revealed a dismaying lack of emphasis on basic skills. Halasz, Behm and Fisch (1984) found that the secondary vocational students they observed spent only .7 percent of their classroom time on basic skills.

There are practical and immediate reasons for emphasizing basic skills in the vocational classroom. Bishop (1986) points out that basic skills improve productivity in many jobs, including jobs for which high school vocational education prepares students. Greenan (1983) focused on the student's need for core generalizable skills required simply to succeed in the secondary vocational program.

Greenan, among others, has urged that basic skills "be related directly to vocational education programs and services and the expected outcomes" (p. 47). Pratzner (1984) also argues for broadly applicable skills but believes "the focus should be in strengthening vocational education's contributions to general education" (p. 5).

This last comment probably reflects the prevailing attitude toward basic skills in the vocational classroom. In discussing future trends, Lewis (1986) notes that pressure from the education reform movement has been pushing vocational education "away from specific occupational training and toward general educational goals" (p. 2). Feldman (1986) also sees the basic skills issue in a broad context:

> As education has become more universal, as we have sought to make education more accessible to more people, the need to integrate vocational and liberal education has become more urgent. (p. 53-54)

Learning through Application

In her extensive review of research, Corman (1980) found that "evidence to support or refute the claim that basic skill instruction in a vocational context improves basic skills of vocational students is extremely limited" (p. 22). Yet, there is a widespread belief that, as Bishop (1986) states,

> vocational courses sometimes contribute more to the development of basic skills than watered-down courses in academic subjects. (p. 99)

Ludeman (1976) assessed the mathematics performance of 16,000 students in Minnesota and concluded that vocational education students perform somewhat below average in working with more advanced and theoretical concepts and better than average in practical applications of math. In the absence of data on entry-level skills, this is only an indication that the actual use of basic skills may facilitate certain kinds of learning, but that concept is in line with common sense thinking about learning through doing.

Experiential learning has also provided some indication of the value of learning through application. In a passage worth quoting, Hedin (1983) claims that learning through doing has serious educational value.

> Schools, especially secondary schools, give priority to teaching and learning through symbolic, abstract, assimilative systems. How successfully they do so is open to question. But even if some success is granted, the schools clearly have fallen short in helping students develop their active experiential modes of learning. They may have learned to read a book and to comprehend a

> lecture and may have accumulated a storehouse of disparate facts, but how practiced are they in applying this knowledge, how capable of reading and comprehending their own experiences--the skills and processes they will be called on to use most often in their adult lives? Thus, to urge more active participation is not just to make a plea for more satisfying educational experiences, or even more broadly educated adults. It is to take a stand on the fundamental skills which students must have for thinking about and dealing with their world. (p. 12)

Hedin reviews a number of studies of the impact of experience on academic learning, dealing with programs that are not in the vocational education curriculum, but that offer learning-through-work experiences outside of the classroom. Such programs usually give students the opportunity to apply what they know, and in this important respect are similar to vocational education. Hedin concludes that experiential learning can--

- increase motivation,
- involve students in gathering original data and in reality testing,
- give students experience in transferring knowledge to new situations,
- engage students in critical thinking and problem solving, and
- provide useful feedback.

She points out that "the most well documented and strongest academic benefits of experiential learning come from a form of it that is most 'school like' in orientation" (p. 20), a comment that has important implications for vocational education.

Research on learning-in-work also suggests that application is a meaningful element in certain kinds of learning. Crowe and Harvey (1980) studied the retention of mathematical concepts in a traditional school program and a learning-in-work program. They found that although basic skills could be learned equally well in either environment, the pattern of learning-forgetting differed, in particular for math concepts. In discussing these results, Miguel and Crowe (1983) suggest that the two environments have a complementary effect and learners may benefit from the alternation of these opportunities.

Ausubel (1980) states that "learning-in-work environments are more likely to result in meaningful learning and retention" (p.

E-20). Noting that application is a necessary part of vocational training, he points out that it must be accompanied by expository teaching methods and carried out with genuine understanding. Still, he says, "knowledge acquired in 'real-life' learning environments has greater proximate, immediate, perceptible, and personalized relevance" (p. E-21).

Ellis (1980) examines the ways learning through experience influences retention. Citing research in this area, he claims that recall is substantially enhanced if we generate an answer (the generation effect). Task variety also increases performance and recall (the variability effect).

The teaching-learning process is clearly different in the vocational classroom, involving much more skills practice and application of knowledge than the academic classroom. It seems reasonable to generalize that this difference can and does assist the educational process in ways that have not always been systematically studied but are, nevertheless, real.

Other Noneconomic Effects

To date there has been little empirical research with the primary objective of assessing noneconomic outcomes of vocational education. The idea that the benefits of schooling of any kind--academic or vocational--must be assessed against economic criteria is deeply embedded in American culture (see Spring 1976). Yet, confining attention to direct economic benefits measured by wage and hours differentials is unduly restrictive. Haveman and Wolfe (1984), for example, catalogue 21 potential noneconomic benefits of schooling. These include quality of child care, marital choice and stability, crime reduction, social cohesion, charitable giving, and capacity to learn.

Grasso and Shea (1979a) focus on economic outcomes of vocational education, but they also examine outcomes such as post-high school training and education and various psychological attitudes, including belief in adequacy of schooling, perceived economic well being, and perceived chance of reaching occupational goals. They found that a vocational program of study in high school depressed the amount of schooling completed after leaving high school, even after controlling for educational aspirations expressed while in high school. This finding held for the total sample of males and females when the contrast was to college preparatory students. Also, males in vocational programs reported less college than those in the general program, although females did not.

Using self-reported curriculum, Grasso and Shea (1979a) also found that vocational students are less likely to be dissatisfied with their level of educational achievement than general students (except black males). Male vocational students expressed greater satisfaction with the progress of their careers than did general students. Differences between vocational and general students in educational expectations measured after high school, satisfaction with their high school education measured after leaving high school, and perceived likelihood of realizing occupational goals were negligible.

Two studies from the National Center for Research in Vocational Education examine the effects of vocational curriculum on selected noneconomic outcomes. Campbell and Basinger (1985) report small and generally insignificant regression coefficients associated with vocational concentrator, limited concentrator, and concentrator-explorer where attendance at any postsecondary institution was the dependent variable. Their estimates rely on NLS Youth data and Probit analysis. Academic curriculum, they found, positively affects postsecondary attendance for all but minority males.

Hotchkiss and Dorsten (1986) reports small negative effects of vocational education on test scores in the HS&B sample. He also reports negative effects on level of educational and occupational expectation, and on college attendance. The negative effects on college attendance and years of schooling completed are replicated by Hotchkiss in the NLS sample.

One of the strongest claims for vocational education has been that it helps to prevent school dropout (Mertens, Seitz, and Cox 1982)--a pivotal hypothesis. A massive literature has arisen in connection with the dropout problem; yet in spite of the pervasive view among vocational educators that vocational education helps to prevent dropping out, few studies not specifically designed to assess vocational education outcomes include any mention of vocational education. For example, of nine papers in the spring 1986 Teacher's College Record, a special issue devoted to dropout, only one pays any attention to vocational education (Hamilton 1986). Hamilton's paper is interesting in view of the paucity of research attention outside of vocational education paid to the influence of vocational studies on dropping out. Hamilton reviews a number of programs designed to prevent dropping out. In spite of the inconclusive nature of research on the topic, Hamilton concludes that vocational education is an important component of most of the successful dropout prevention programs.

Economic Effects

Early research on the economic benefits of vocational education concluded that the benefits were generally small (Grasso and Shea 1979; Woods and Haney 1981; Meyer and Wise 1982; Meyer 1982.) Most of the early research was based on student self-report of vocational track and omitted any consideration of whether the job a youth obtained following high school was training related. More recent studies have used a conceptually sophisticated schema based on high school transcripts to define vocational study in high school. Campbell and coauthors (1985a; 1986) find strong effects of vocational training on labor market outcomes after leaving high school, provided that training related employment is secured.

There are serious limitations to past work. First, none of the studies assess economic benefits for more than a few years following high school; empirical studies demonstrate the importance of observing earnings profiles over time (Meyer 1981; Haller and Spenner 1977). The conclusion to be garnered from both theory and observation is that adequate assessment of the impacts of education and training on economic outcomes requires lengthy time series on the economic outcomes.

Furthermore, high school education is a complex system. To date no study has adequately integrated tests of various aspects of vocational education. The Campbell studies (Campbell et al. 1985a; 1986), the most adequate evaluation of labor market outcomes of vocational education, do not investigate effects of vocational specialty.

Adequate assessment of the impact of vocational education requires not only detailed data, but also careful formulation of concepts and theory. An important aspect of continuing research on the economic impact of vocational education, therefore, must be careful attention to the theoretical basis of the empirical research. In spite of the strong a priori reasons for the expectation that different types of individuals benefit to varying degrees from vocational education, little empirical investigation of this issue has been carried out. Such research will require large samples as well as precise measurements that are not available even in existing national surveys such as The High School and Beyond (HS&B) and the Class of 1972.

The Motivating Power of Vocational Education

As noted earlier, one important claim for vocational education is that it motivates students in ways the other curricula cannot. An examination of the literature on motivation suggests that this is probably true. Vocational education offers both incentives and tasks that have the potential to motivate students who are unmotivated to learn in the traditional classroom environment.

Motivation is not usually seen as an isolated feeling or fixed personality trait, but as a behavioral pattern subject to change. The motivational cycle can be represented in a simplified form in which personal investment leads to performance, and the reinforcing payoff leads to further investment.[1]

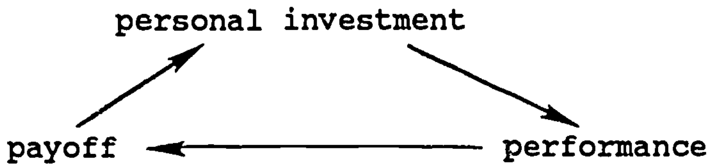

Figure 2. The motivational cycle

Maehr and Archer (1987) present a more complex model that includes intervening factors that influence performance. Personal investment, according to Maehr and Archer, is evidenced by choice (decision to do the task), persistence, and activity level. This motivation not only leads to performance, but will be influenced in the future by the perceived outcome (the payoff; the performance appraisal). The performance itself may also be intrinsically satisfying and thus increase motivation for future action.

When educators discuss motivation, it is with a concern to harness and enhance existing motivation. This can be done through supplying (extrinsic) incentives and building (intrinsic) task and achievement motivation.

Many of the claims for vocational education's motivational power refer to the intrinsic and extrinsic rewards of technical skills practice. Mastery learning, especially competency-based vocational education (CBVE), uses goals that are specific and proximal (quickly achieved). Much research has established that

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[1] It is interesting to note that a system such as shown in figure 2 will, if represented by a system of three linear differential equations with constant coefficients, achieve a stable equilibrium provided that the product of the feedback loops of each variable on itself is negative and larger in absolute value than the product of effects of the variables on one another (as depicted by the arrows). The larger the cycle of reinforcing effects shown in figure 2 relative to the negative feedbacks of each variable on its own change over time, the higher the equilibrium level of all three variables. Viewed in this way, the goal of education is to structure learning so as to achieve a maximum equilibrium level. In concrete terms, this means that effective education (1) enhances the connection between personal investment of students and their performance, and (2) ensures a close connection between performance and payoff. The connection between payoff and personal investment probably is not subject to outside influences.

the properties of specificity and proximity are motivating. To begin with, people are more likely to accept goals when they believe they can attain them (Mento, Cartledge, and Locke 1980). Perhaps for this reason, specific performance standards lead to higher performance than do general goals (Locke et al. 1981); the same is true of proximal goals (Bandura 1977). Goals with these characteristics, because they are achievable, also promote self-efficacy (Bandura and Schunk 1981), and self-efficacy affects subsequent achievement (Schunk 1984a). Well-designed mastery learning makes use of these motivating elements.

Feedback is usually integral to technical skills practice; results are often apparent as the student works, even without the teacher's appraisal. Ausubel (1980) notes that feedback not only supplies cognitive reinforcement, but enhances existing motivation by increasing attention, self-confidence, and perseverance. The motivational cycle in technical skills practice may be very short; the student sees results (is able, in word processing, to move a block of text to another place in the file), feels more competent (immediate payoff), and is moved to increase personal investment in further performance (learning to move text to a new file). The same cycle operates in an academic classroom, but there it is possible for a student to work without feedback for weeks on end.

This lack of feedback was one of the chief criticisms of the academic classroom made by Goodlad and others who worked on A Study of Schools. Sirotnik (1983), with reference to data from that study, says,

> corrective feedback is rarely observed, particularly at the secondary level. Providing corrective feedback in combination with additional information designed to help students understand and correct their mistakes is almost nonexistent. (p. 20)

In technical skills practice, the perceived outcome is close to or coincident with the performance of the task.

A distinction is commonly made between the incentive motivation (or reinforcement) supplied by external performance standards and the more internalized task and achievement motivation. Task motivation theory emphasizes cognitive motives and feeling states such as curiosity, exploration, and mastery (Gagne 1977). Katz and Kahn (1978) describe this motivation:

> People do derive important satisfaction in the expression of their skills, in interesting and challenging work, and in the sense of accomplishment from successful performance of such tasks. (p. 366)

Task mastery is ego-enhancing (Ausubel 1968); the perception of personal competence directly affects self-esteem. This means that "unmotivated" students (students who are not motivated initially to invest in learning) can develop motivation through performance. The positive feelings experienced with achievement (the payoff) can lead to increased interest in performance and become "a most dependable source of continuing motivation" (Gagne, 1977, p. 291). This effect depends on attribution; achievement motivation is highest in people who attribute their success to something they have done (Weiner 1984). The concrete tasks of technical skills practice are related to the student's effort in a very direct, obvious fashion. It is easy to see who controls the outcome.

One of the most nebulous and persistent claims made for vocational education is "relevance." Perceived relevance is a matter of the value attached to the task, a value that has reference to personal needs and goals. The student decides to invest energy in a task because a cognitive appraisal suggests that a desirable payoff will result, desirable with reference to personally held (not socially mandated) goals.

When "relevance" is cited, students may be referring to emotional satisfactions inherent in the task ("I like to work with wood") or to outcome expectations ("This will get me a job with a builder"). In any case, the student is expressing the belief that this course or curriculum offers something of real personal value--not someone else's outcomes, but his or her own internalized goals. Obviously, schools must demand many tasks of students in which students have no such interest; educators, representing society, design curricula with reference to distant outcomes young people are not equipped to evaluate. But the student's perception of relevance, when it exists, should be seen as a present motivation that can be a powerful force for learning.

There is evidence that students in general do see vocational training as relevant to their needs. A Study of Schooling asked 15,000 students what function seemed to be most emphasized at their schools--intellectual development, social responsibility, personal development, or vocational development--and what should be emphasized. The students said that the intellectual is most emphasized, and that the vocational should be given equal or greater weight (Goodlad 1983).

The motivating power of vocational coursework can also be inferred from other data in Goodlad's study. The students surveyed perceived themselves as having the greatest decision-making role in three subjects--the arts, physical education, and vocational education. Perceived locus of control is one of the most important factors in achievement behavior, according to Maehr and Archer (1987).

These subjects were also seen as "more satisfying" than academic subjects (Goodlad, p. 115). Students liked vocational and career education more than any subject except the arts _and_ rated it third in importance, after math and English, among all subjects. In terms of interest, vocational and career education was again in the top three, after the arts and physical education. Goodlad comments that a similarity between these three subjects and foreign language study (also well-liked, "is that they provide models against which to check one's own performance rather quickly" (p. 119). In other words, these subjects provide the frequent reinforcement and feeling of self-efficacy that builds achievement motivation.

A study by Campbell et al. (1987) discusses "the conventional wisdom that tangible tasks in which accomplishment can readily be perceived are sources of greater satisfaction for most people than abstract and uncertain accomplishments." This study briefly examined student attitudes toward learning, using information obtained from debriefing interviews with observers in Weber's Classroom Dynamics Study of nearly 700 vocational classrooms (1988).

The pattern Campbell et al. found confirms Goodlad's data: in both area vocational schools and comprehensive high schools, students in vocational classrooms seemed to have a more positive attitude toward learning than their academic counterparts. Furthermore, overall school climate (learning environment) was rated significantly higher for area vocational schools than for comprehensive high schools.

Maehr and Archer (1987) state that "at the heart of personal investment theory is the contention that students invest themselves in learning tasks as they see meaning in those tasks" (1987, p. 30). They also point out that, while motivation is not the only variable in classroom achievement, it is extremely important, accounting for between 11 and 20 percent of the variance in achievement. Moreover, motivation is a variable over which educators have some influence (in contrast to such factors as SES or ethnicity). The motivating aspects of the vocational classroom experience offer students a valuable opportunity and incentive to achieve and to build self-efficacy and general motivation for learning.

CHAPTER 4

INTERPRETATION AND STANDARDIZATION OF INTERACTION EFFECTS

by Lawrence Hotchkiss

A variety of substantive contexts in sociology and other social science disciplines leads one to expect nonadditive and/or nonlinear specifications in a structural model. For example, Sorensen and Hallinan (1978, 1986) propose a formal model postulating that "ability and effort" interact with "opportunities for learning" to generate learning outcomes. The relationship between age and earnings is routinely specified as parabolic (Mincer 1974). Race and gender interactions in stratification processes have been the subject of much study (Treiman and Terrell 1975; Portes and Wilson 1976; Parcel and Mueller 1973. Educational theory and research frequently have postulated aptitude-by-treatment interactions in determining learning outcomes (Cronbach and Snow 1977). The often cited Cobb-Douglas production function, though linear in the logarithms of all variables, is both nonadditive and nonlinear with respect to each independent variable in its antilog representation. Bohrnstedt and Marwell (1977) give several additional examples of theory that predicts nonadditive relationships.

There is little doubt that social science theorizing often suggests nonadditive and/or nonlinear relationships, yet few such relationships are well established in the literature. It appears likely that one important reason is that methodological and conceptual difficulties are exacerbated with each departure from the standard linear specification.

In recent years, important steps have been taken to help improve interpretation and testing of nonlinear and nonadditive models. In perhaps the most important paper on the topic, Stolzenberg (1979) shows appropriate methods for interpreting and testing both nonlinear and nonadditive regression models. He defines effect as a partial derivative and suggests useful standardization procedures. Marsden (1981) clarifies procedures for testing for interaction effects and gives a cogent exposition of appropriate interpretation of product terms in regression equations. Allison (1979) corrects past misperceptions regarding statistical tests for interaction effects and argues persuasively that all "main effect" terms generally must be retained in a regression model containing one or more product terms--to adjust for the arbitrary metrics of the independent variables.[1]

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[1] Allison argues that main effect terms may be omitted only in special cases where all independent variables are ratio scales (origin naturally defined) and theory indicates that the main effects should be omitted.

Tate (1984) demonstrates the advantages and limitations of subtracting the mean from each independent variable before forming a product term and shows the correspondences between parameters estimated using an arbitrary metric for the right-side variables and parameters estimated with all right-side variables centered to zero mean. Stimson, Carmines, and Zeller (1981) also review appropriate test procedures for use with polynomial regression and demonstrate how interpretation of the results may be simplified by translation of scale on the X-axis.

Bohrnstedt and Marwell (1977) develop key algebraic relationships involving the reliability of a product term. Heise (1986) shows how to correct interaction models for measurement error, allowing for correlated errors, and presents simulation results of the performance of his formulae.

In summary, three important achievements have resulted to date from work on nonlinear specifications. First, appropriate procedures for testing nonlinear/nonadditive models have been clarified. Second, appropriate methods for interpreting effects from nonlinear/nonadditive models have been elucidated. This achievement is particularly important in view of the confusion that formerly reigned in the usage of terms like "main effect" and "interaction effect." Finally, important initial steps have been taken toward assessing the impact of measurement error on estimation of nonlinear/nonadditive models.

In spite of encouraging progress, there remain important issues that deserve attention. First, methods for assessing the magnitude and variation of effects over a sample or population have not been fully expounded. Second, the impact of dynamic cycling of "cause and effect" on estimation strategies does not appear to be widely recognized. Linear specification of a dynamic model remains linear even after any number of cause-and-effect cycles, but nonlinear/nonadditive specifications do not retain their original functional form (Tuma and Hannan 1984). Third, much remains to be investigated regarding the impact of measurement error on nonlinear/nonadditive estimation.

This paper addresses the first issue, primarily because it is, by far, the most tractable of the three and because clear interpretation of nonlinear/nonadditive effects provides an important basis for addressing the other two issues. When the linear model is not in place, effects are no longer constant over a population. Analysts, therefore, must decide how to summarize them. This paper shows how to calculate useful summaries of the magnitude and variation of effects and suggests standardization procedures for assisting with the interpretation of results. Revised formulas for calculating standardized coefficients in nonlinear and nonadditive equations are shown to be at once invariant under linear change of scale and easily interpretable.

As might be anticipated, the summary measures often depend in a straightforward way on coefficients associated with various terms in an equation. Thus, these coefficients are given clear meaning. For example, the revised standardized coefficients are consistent with Stolzenberg's definition of standardized effects. The standardized coefficient associated with a linear term in a simple product model is shown to be the average effect in standardized metric (standard deviation units), and the absolute value of the standardized coefficient associated with the product term is shown to be the standard deviation of the effects over the sample (or population).

Continuous Variables and Interaction

Consider first the simple product model to represent the interaction of the effects of two independent variables (x and z) on an outcome (y).

$$y = a + bx + cz + dxz + u \qquad [1]$$

where a, b, c, d are constant parameters and u is a disturbance <u>independent</u> of x and z. All variables are assumed to be continuous variables. Frequently, a third independent variable is defined by the product of x and z--$v = xz$, and [1] is rewritten

$$y = a + bx + cz + dv + u. \qquad [1a]$$

This form of the model can be deceiving because it generates the temptation to interpret the multiplier coefficients (b, c, d) as independent indexes of "main effects" (b and c) and "interaction effects" (d).

The confusion stemming from this type of interpretation is so common that the main point of clarification given by Stolzenberg (1979), Allison (1977), Marsden (1981), Tate (1984), and others bears repeating. The key point is this: the variable v cannot vary if both x and z are constant because $v = xz$. Therefore, it is incorrect to interpret d in the usual sense in <u>linear</u> regression, as the "effect" of u with x and v constant. Likewise, for the same reason, the coefficient b cannot be viewed as the "main effect" of x with <u>both</u> z and v constant, nor can c be interpreted as the effect of z with x and u constant. An important derivative point is that a simple translation of scale (adding/subtracting a constant from all values) changes the coefficients associated with the linear terms in the equation (b and c).

By the partial derivative rule (Stolzenberg 1979), the effect of x on y in equation (1) is--

$$q_{yx} = b + dz \qquad [2a]$$

and the effect of z on y is

$$q_{yz} = c + dx \qquad [2b]$$

where q_{yx} is used to denote the effect of x on y and q_{yz} denotes the effect of z on y. Neither one of these effects is constant across individuals, unlike effects in a linear model. It therefore seems useful to summarize their central tendency and variation. The mean and standard deviation are the two most commonly used measures of central tendency and variation. Their values for q_{yx} and q_{yz} are

$$Eq_{yx} = b + dEZ \qquad [3a]$$

$$Eq_{yz} = c + dEx \qquad [3b]$$

$$\sigma(q_{yx}) = |d|\sigma(z) \qquad [4a]$$

$$\sigma(q_{yz}) = |d|\sigma x). \qquad [4b]$$

where E is the expected value operator, and $\sigma(.)$ stands for standard deviation. It is useful to note that the average effects are precisely the effects evaluated at the mean of the independent variables. Evaluating effects at the means is a common practice among economists when interpreting probits, logits, log transformed equations, and many other nonlinear functional forms.

If x and z are standardized to zero mean and unit variance, equations (1) and (2) still hold, but (3) and (4) simplify considerably. To avoid confusion, let p_{yx}, p_{yz} indicate effects when all variables are standardized to zero mean and unit variance, and let the corresponding coefficients be indicated by an asterisk. The mean and standard deviation of effects can now be written as follows:

$$Ep_{yx} = b^* \qquad [5a]$$

$$Ep_{yz} = c^* \qquad [5b]$$

$$\sigma(p_{yx}) = \sigma(p_{yz}) = |d^*| \qquad [6]$$

Now the terms commonly called "main effects" (b^* and c^*) and "interaction effects" do have a clear interpretation. When all variables are standardized, the so-called main effects are average effects, and the interaction effect is the standard deviation of those effects.

Marsden (1981) argues that "it is probably best to abandon efforts to compare the relative importance of main effects and interaction terms" (p. 110). His argument is certainly correct in the sense that neither main-effect nor interaction-effect terms index effects as defined by the partial derivative rule. Moreover, the relative magnitude of the two types of effects can be altered by a simple translation of scale. On the other hand, equations (5) and (6) do provide a viable basis for making comparisons between the main effects and interaction effects. When variables are in standardized metric, the comparison gives the degree of variation in effects relative to the mean effect. The coefficient of variation ($\sigma(p_{yx})/Ep_{yx} = |d^*|/b^*$), for example, provides a convenient numeric summary. As will be seen momentarily, however, the simple expression for the standard deviation of effects given by equation (6) becomes substantially more complex when more independent variables are added to the model.

It is critical to distinguish between the coefficients in a nonlinear model where all variables are standardized (such as equation [1] subject to-- $\mu_y = \mu_x = 0$, and $\sigma_y = \sigma_x = \sigma\varsigma = 1$) and the standardized coefficients that result from applying the formula for a linear model to coefficients in a nonlinear model like (1). It is the latter type of coefficient that is routinely produced by regression programs. These coefficients are virtually uninterpretable for a nonlinear model. They are calculated on the assumption that x, z, and v (e.g., [1a]) each has a standard deviation of unity. However, since v = xz its variance is as follows:

$$\sigma_v^2 = Ex^2z^2 - c_{xz}^2 \quad \Bigg| \quad \begin{array}{l} \mu_x = \mu_z = 0 \\ \sigma_x = \sigma_z = 1.0 \end{array}$$

where c_{xz} stands for covariance of x and z. If x and z are independent and $\sigma_x = \sigma_z = 1$, then Ev^2 will equal one; otherwise, it generally will not. Hence, in general, assigning a variance of unity to v is inconsistent with setting the variance of x and z to unity. Consequently, standardized coefficients based on the assumption of unit variance for v have no clear interpretations.

Simple formulas can be derived to calculate a*, b*, c*, and d* from a, b, c, and d. These are--

$$a^* = \frac{1}{\sigma_y}[a - (\mu_y - b\mu_x - c\mu_z - d\mu_x\mu_z)] \qquad [7a]$$

$$b^* = \frac{\sigma_x}{\sigma_y}(b \quad d\mu_z) \qquad [7b]$$

$$c^* = \frac{\sigma_z}{\sigma_y}(c + d\mu_x) \qquad [7c]$$

$$d^* = \frac{\sigma_x\sigma_z}{\sigma_y}d \qquad [7d]$$

Note that a^* generally will not be zero. Only if x and z are uncorrelated, so that $Exz = \mu_x\mu_z$, will $a^* = 0$.

The effects of the model in standard form can readily be seen by writing (1) again for the transformed variables and calculating the partial derivatives

$$y^* = a^* + b^*x^* + c^*z^* + d^*x^*z^* \qquad [8]$$

$$p_{yx} = b^* + d^*z^* \qquad [9]$$

where y^*, x^*, and z^* have all been standardized to zero mean and unit variance, and p_{yx} denotes the standardized effect of x on y.[2] An entirely parallel result holds for the effect of z^* on y^*. By substituting (7b) and (7d) into (9), one finds that

$$p_{yx} = (\sigma_x/\sigma_y)\ q_{yx}\text{, and} \qquad [10a]$$

$$p_{yz} = (\sigma_z/\sigma_y)\ q_{yz} \qquad [10b]$$

which is precisely the standardization proposed by Stolzenberg (1979). Stolzenberg applies the rules of differential calculus to arrive at the results in [10]; relying on rules of differentiation is more elegant and more general than the derivation shown here, but it is less transparent.

The generality of Stolzenberg's result is worth emphasizing, and its basis bears review and amplification. Consider arbitrary linear transformations on x and y--

$$x_t = g_o + g_1x$$

$$y_t = h_o + h_1y$$

and let y be a partial function of x--

$$y = f(x,z)$$

so that the effect of x on y is

$$q_{yx} = \frac{\partial f}{\partial x}$$

Applying standard rules of differentiation one finds that the effects in the new metrics (y_t, x_t) are

$$q_{y_tx_t} = \frac{h_1}{g_1}\frac{\partial f}{\partial x}. \qquad [11]$$

---

[2] The notation $q_{y^*x^*}$ would be more consistent with the operations leading from (1) to (8), but p_{yx} is more convenient.

If the constant $h_1 = 1/\sigma_y$ and the constant $g_1 = 1/\sigma_x$, as is the case when all variables are standardized to unit variance, then the standardized effect is

$$p_{yx} = \frac{\sigma_x}{\sigma_y}\frac{\partial f}{\partial x} = \frac{\sigma_x}{\sigma_y} q_{yx}.$$

Thus, the results in (10) are completely general. They apply irrespective of the functional form of the structure. They also apply more generally to linear transformations that do not follow the zero mean-unit variance convention.

In fact, these results apply even more generally to any one-to-one transformations on x and y. Let

$$x_t = g(x)$$

$$y_t = h(y)$$

$$y = f(x,z)$$

$$y_t = h\{f[g^{-1}(x_t),z]\} = f_t(x_t,z).$$

Then the effect in the new metric is a generalization of (11), as follows:

$$q_{y_t x_t} = \frac{h'(y)}{g'(x)}\frac{\partial f}{\partial x} = \frac{h'(y)}{g'(x)} q_{yx} \qquad [12]$$

where the primes indicate the derivative of the function.

Two cautionary notes are in order here. First, the transformation functions g(x) and h(y) must be one-to-one over their domains in a given application. Thus, $h(y) = y^2$ is not permitted because its inverse, which has two values for each y, is not a function. However, if y is nonnegative in an application (e.g., wage), then y^2 is permitted. Second, the functional form of the structural equation in the transformed scales may be quite different than in the original metrics. For example, let

$$x_t = g(x) = \sqrt{x} \qquad 0 \leq x$$

$$y_t = \ln y \qquad 0 < x$$

$$y = f(x) = x^2.$$

In this case, the function connecting y_t to x_t is quite different from the original function:

$$y_t = f_t(x_t) = 4 \ln x_t$$

Even when the transformation functions are linear, the form of the function connecting y_t to x_t may not appear like the function connecting y to x. For example, a simple exponential function in the original metric appears substantially more complex in the transformed metric:

$$y = ae^{bx}$$

$$x_t = g_o + g_1x$$

$$y_t = h_o + h_1y$$

$$y_t = \frac{-h_o + ae^{b(x_t-g_o)/g_1}}{h_1}$$

$$= \alpha + \beta e^{\gamma x_t}, \text{ with } \alpha = \frac{-h_o}{h_1}$$

$$\beta = \frac{a}{h_1} e^{-bg_0/g_1}$$

$$\gamma = b/g_1$$

Consider again the matter of summarizing the magnitude and variation of effects in a nonlinear, nonadditive model. A simple generalization of the two-variable model in (1) includes all first-order interactions for several independent variables--

$$y = a + \sum_j b_jx_j + \sum_{j\leq k}\sum c_{jk}x_jx_k + u \qquad [13]$$

where j, k index the independent variables; j, k = 1, 2, . . . , J. Note that (13) permits squared terms as well as products. The average effect and standard deviation of those effects can be calculated as follows:

$$Eq_{yj} = b_j + \sum_k c_{jk}\mu_k + 2c_{jj}\mu_j \qquad [14a]$$

$$\sigma(q_{yj}) = \sqrt{\sum_k c^2_{jk}\sigma^2_k + 4c^2_{jj}\sigma^2_j + 2\sum_{km}\sum c_{jk}c_{jm}\sigma_{km} + 4c_{jj}\sum_k c_{jk}\sigma_{jk}} \qquad [14b]$$

where the sums over k and m exclude values where k = j and m = j, respectively. Again, these formulas simplify if all variables are standardized to zero mean and unit variance:

$$Ep_{yj} = b^*_j \qquad [15a]$$

$$\sigma(p_{yj}) = \sqrt{\sum_k c^{*2}_{jk} + 4c^{*2}_{jj} + 2\sum_{km}\sum c^*_{jk}c^*_{jm}\rho_{km} + 4c^*_{jj}\sum_k c^*_{jk}\rho_{jk}} \qquad [15b]$$

where the double summation runs over values of k < m, ρ stands for population correlation, the coefficients marked by an asterisk represent values estimated from data for which all variables are in standardized form, and the sums over k and m exclude cases for which k = j and m = j, respectively.

As shown in (14b) and (15b), the simple relationship between the product coefficients and the standard deviation of effects found in equations (4) and equations (6) becomes substantially more complex when more than two independent variables and their products and/or squares are included in a model. It is clear, though, that the coefficients of the product terms do provide a rough index of the contribution of one variable to variation in effects of other variables. For example, given that $c_{11} = \rho_{12} = 0$, then the contribution of x_2^* to variation over the population in effects of x_1^*, on y is given unambiguously by c_{12}^*.[3]

When ρ_{12} is not zero, however, x_2^* does not make a separable contribution to the variance of effects of x_1^*, on y^*. This is precisely the same ambiguity that arises in more general cases of variance partitioning. It therefore seems preferable to avoid the variance partition approach. Instead, it is sensible to rely on a partial derivative definition. For example, the contribution of x_2 to the effect of x_1 on y may be defined as the second derivative of (13), first with respect to x_1, (giving the effect of x_1 on y), then with respect to x_2 (showing the impact of x_2 on the effect of x_1 on y). For equation [13], one has --

$$\frac{\partial^2 y}{\partial x_j \partial x_k} = c_{jk}.$$

These "effects on effects" cannot be readily compared unless all variables appear in a standard metric. Again, the coefficients calculated when all variables are standardized offer a potentially useful aid to interpretation. It is worth reiterating that these coefficients are not the same standardized coefficients as those produced by most regression software.

The simplicity of (15a) as compared to (14a) is one of two reasons for being interested in the standardized coefficients. The second reason is that effect magnitudes are roughly comparable among independent variables when all variables are converted to the standardized metric. The standardized coefficients for model (13) can be calculated as follows:

$$a^* = [a - (\mu_y - \sum_j b_j\mu_j - \sum_{j<k}\sum c_{jk}\mu_j\mu_k)]/\sigma_y \qquad [16a]$$

$$b_j^* = \frac{\sigma_j}{\sigma_y}[b_j + \sum_k c_{jk}\mu_k] \qquad [16b]$$

---

[3] The coefficient c_{12}^* also indexes the contribution of x_1^* to variation in the effects of x_2^* on y^*.

$$c^*_{jk} = \frac{\sigma_j \sigma_k}{\sigma_y} c_{jk} \qquad [16c]$$

The formulas for the standard deviation of effects given by (14b) would be tedious to use for models with many variables and many nonlinear, nonadditive terms. Equation (15b) is somewhat simpler than (14b), but it also could be tedious to use. A relatively simple alternative to finding the algebraic expression for the mean and standard deviation of effects and using that expression to calculate numerical values of Eq and (q) is as follows: Use a computer to (1) calculate numeric values of derivatives of *y* with respect to each x, and then (2) use standard computer software to summarize those effects. Means and standard deviations, as well as summary statistics such as median, minimum, and maximum, could be produced routinely.

Noncontinuous Variables and Interactions

If any variable in a structural relation is not continuous, then the partial-derivative definition of an effect does not apply to that variable. If a variable is not continuous, then it may be categorical (nominal scale), ordinal, or interval defined by a count (e.g., frequency of drug use). The noncontinuous variable(s) may be either independent or dependent. Generally, when observed dependent variables are not continuous, a latent probability model can be devised for which the theoretical specification applies to a latent probability of being in each category. In this case, any interpretation that applies to continuous dependent variables can be applied to the latent probabilities. The discussion in this paper therefore is focused on interpretations of effects in cases where independent variables are not continuous.

Consider first the case of one dummy coded dichotomous independent variable (x) and one continuous independent variable (z). The effect of x now is defined as the difference between the value of y with x=1 and the value of y with x=0, given that both z and u are constant. The effect of x is $q_{yx} = b + dz$, which is the same expression as for the case of continuous x (see eq. [2a]). Its interpretation, however, is different from that of continuous x. With dichotomous x, the effect is the difference between those in the category coded 1.0 and those in the category coded 0. This difference is a continuous function of z. For continuous x, in contrast, the effect is a partial derivative.

The effect of z can still be defined as a partial derivative, and its expression is precisely the same as for the case when x is continuous; however, the effect of z takes on only two values:

$$\begin{aligned} q_{yz} &= c + dx \\ &= c + d \qquad \mid x = 1 \\ &= c \qquad\qquad \mid x = 0 \end{aligned}$$

The equations for average effects and the standard deviation of effects over the population given by equations (3) and (4) still apply. Likewise, the formulas for calculating standardized coefficients (eqs. [7]) also apply. However, effects of standardized dummy variables probably are not as easy to interpret as effects in the zero-one metric. The reason is that the zero-one coding preserves easy comparisons among categories of a nominal scale. There is no reason why the categorical variables cannot be left in the zero-one metric and the other variables standardized. It is only necessary to take care when interpreting the results to note what is being compared to what.

When one uses more than two dummy variables to represent a categorical variable with three or more categories, then some generalization of the interpretations for a dichotomy are needed. Suppose that each of the first J-1 categories of a nominal scale variable having J categories are represented by a dummy variable denoted x_j. If comparisons between any one of the first J-1 categories and the Jth category are desired, precisely the same principles apply as for a dichotomous nominal scale. For any comparison between two of the first J-1 categories, differences between coefficients are required. Consider the following structural equation used to represent effects of a single three-category nominal scale:

$$y = a + b_1x_1 + b_2x_2 + cz + d_1x_1z + d_2x_2z + u \qquad [17]$$

The effect of being in category 1 instead of category 3 (reference category) is

$$q_{y,x_1-x_3} = b_1 + d_1z \qquad [18]$$

where the notation q_{y,x_1-x_3} indicates the effect of being in category 1 instead of category 3. This result is analogous to the case of a "natural" dichotomy. However, the effect of being in group 1 instead of group 2 is not precisely analogous:

$$q_{y,x_1-x_2} = b_1 - b_2 + (d_1-d_2)z \qquad [19]$$

Likewise, the effect of z now is a little more complex than before--

$$q_{yz} = c + d_1x_1 + d_2x_2. \qquad [20]$$

This effect can take on three values, one for each category of the nominal-scale variable:

$$q_{yz} = c \qquad | \; x_1 = x_2 = 0 \qquad [21a]$$

$$q_{yz} = c + d_1 \qquad | \; x_1 = 1 \qquad [21b]$$

$$q_{yz} = c + d_2 \qquad | \; x_2 = 1 \qquad [21c]$$

In the case of an interval but noncontinuous independent variable (x) the effect is defined as the difference between y at one designated value of x and the value of y at some other designated value of x. For an equation such as (1), this becomes

$$y_1 - y_0 = b(x_1 - x_0) + dz(x_1 - x_0)$$

$$\Delta y = (b+dz)\Delta x; \; \Delta y = y_1 - y_0$$

$$\Delta x = x_1 - x_0.$$

If $x = 1$, then this result reduces to the same expression as (2a). In the completely general case with $y = f(x,z)$, the effect of having value x_1 on x instead of value x_0 is:

$$q_{y,x_1 - x_0} = f(x_1,z) - f(x_0,z)$$

An Example

A long-standing claim among vocational educators is that vocational education enhances the value of academic education by helping to motivate students. The basic idea is that the "hands-on" experience of vocational courses reveals the concrete application of the abstractions of academic study, thereby stimulating students' interest in academic topics. An important corollary to this claim is that most or all students could benefit from taking both academic and vocational courses (Silberman 1980, 1982; National Commission on Secondary Vocational Education 1984).

A typical specification of the effects of academic and vocational courses on, say, verbal test score might be linear--

$$y = a + b_1A + b_2V + c'x + u \qquad [22]$$

where

y = verbal test score

A = number of academic courses (Carnegie units)

V = number of vocational courses (Carnegie units)

x = a column vector of "controls"

u = random disturbance

a, b_1, b_2 = scalar constants

c = a column vector of coefficients conformable with x (prime indicates transpose)

The maximum number of courses is constrained to some number T. Hence, $A + V \leq T$. For expository reasons, set $A + V = T$.[4] With this constraint, then the optimum combination (to maximize y) of academic and vocational courses is to either (1) take all academic courses if $b_1 > b_2$, or (2) take all vocational courses if $b_2 > b_1$. This conclusion violates common sense and disregards the claim of vocational educators that most students benefit (in the sense of eq. [22], learn more) if they take some academic and some vocational courses.

A simple alternative to the linear specification in (22) is to posit an interaction between academic and vocational courses of the following form:

$$y = a + b_1A + b_2V + b_{12}A \cdot V + c'x + u \qquad [23]$$

where b_{12} is a constant with $b_{12} > 0$, and $b_{12} > |b_2 - b_1|/T$ and the other terms were defined in conjunction with (22). Equation (23), given that b_{12} is positive as hypothesized, expresses the claim of vocational educators that vocational education complements or enhances the value of academic education--that is, the effect of academic courses is, in fact, higher to the extent that one takes vocational courses--

$$q_{yA} = b_1 + b_{12}A \qquad (b_{12} > 0).$$

With equation (23) we also find that the optimum combination of academic and vocational courses falls somewhere between the extremes, given the constraint $A + V = T$. The optimum proportion of vocational courses is found to be

$$\text{optimum } (V/T) = [1 + (b_2 - b_1)/Tb_{12}]/2.$$

Although the idea that different types of courses may complement each other [in the sense of (23)] is prominent in vocational education, the basic hypothesis obviously has a much broader potential application. For example, it seems likely that physics and math courses would be complementary. That is, taking physics courses should enhance what is learned from math courses, and vice versa.

To illustrate the interpretations of interaction specifications developed in this paper, two equations of the form of (23)

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[4] This constraint presumes that all courses are classified as either academic or vocational, and all students take the maximum number of courses (T). Generalization of this simplifying assumption is straightforward.

were estimated from the High School and Beyond (HS&B) sample. In the first equation, a composite verbal test score is the dependent variable. English credits earned during the last 2 years of high school are taken to represent A in equation (23), and credits in clerical business and office courses taken during the last 2 years of high school are used to represent V in equation (23). These specific academic and vocational courses are used rather than all academic and vocational courses to avoid excessive diffusion of whatever interaction effect may occur. The second equation estimates a math x physics interaction. Here A is replaced by math credits in the last 2 years of high school, V is replaced by physics credits in the same time period, and the dependent variable is the HS&B math test score. In both cases, the complementarity hypothesis implies that the coefficient on the product term (b_{12}) is positive.

The dependent variables in both equations were measured in the first follow-up of the HS&B survey, when most members of the sophomore cohort would have been high school seniors. The verbal test score consists of the combined reading, vocabulary, and writing HS&B tests. The math test score consists of the combined HS&B math tests (computational, theoretical). Base-year measures of the dependent variable (when respondents were high school sophomores) were entered as one right-side variable in each equation. In addition, 24 control variables, all measured at base year, were included in each equation.[5]

Since the HS&B survey now is familiar to most social scientists, only a brief description is provided in this paper. In 1980, samples of the sophomore and senior classes of 1,015 U.S. high schools were surveyed. Calculations reported here are based on data from the sophomore cohort. There were 29,737 members of the sample in 1980. Almost all of these individuals were surveyed again in 1982, when most of them were seniors in high school. Transcript data were targeted to be collected from 18,427 of the original sample. Of the 18,427 transcripts, 15,941 are usable. After eliminating from the sample respondents who did not participate in the first follow-up, and cases with missing data on the dependent variables, the number remaining for analysi is 11,903.

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[5] These controls are 8 region dummies; rural residence dummy; urban residence dummy; missing data dummy for residence; race dummy (1=black); ethnic dummy (1=Hispanic); an SES scale including parental occupation, education, number of siblings, housing characteristics, and possessions; log of family income; missing-data dummy for income, lagged math <u>and</u> verbal test scores; dropout dummy; an index of significant-other variables for education; educational expectation; occupational expectation; perceived college ability; grade point average; and hours of homework per week. All of these variables were measured from the base year student questionnaire, except the two test scores. The tests are specially designed tests for the HS&B. Identical versions were administered at base-year and first follow-up.

Missing data were handled by deleting cases with missing dependent variables and substituting the mean for missing values of the independent variables.

Table 4 goes here

Table 4 displays mean and standard deviations of the primary variables in the analyses, and table 5 presents effect estimates. Estimates were calculated separately for females and males because of the high likelihood that the processes under study here differ according to gender. Columns entitled "standardized coefficient" contain entries that were calculated according to equations (7), and the columns labeled "linear leg. standardized coefficient" were taken directly from the computer output. They were calculated with the usual formula for the standardized regression coefficient in a <u>linear</u> specification.

Table 5 goes here

The complementarity hypothesis does not fare very well in these data. In only one case does the coefficient associated with a product term achieve statistical significance and that coefficient is negative instead of positive as hypothesized. The significantly negative product coefficient occurs in the math equation for females. Using equation (3a), one finds that the average effect over the sample of females of an additional math credit on the math test score is

$$\begin{aligned}\bar{q}_{yx} &= b + d\bar{z} \\ &= 2.0838 - 0.8141(0.1028) \\ &= 2.0001\end{aligned}$$

and the sample standard deviation of those effects is [using (4a)]--

$$\begin{aligned}s(q_{yx}) &= |d|s(z) \\ &= 0.8141\ (0.3097) \\ &= 0.2521\end{aligned}$$

where the bar over q indicates sample mean, and s denotes sample standard deviation.

TABLE 4

MEANS AND STANDARD DEVIATIONS FOR PRIMARY VARIABLES

		Variables					
		English credits	Clerical credits	Math credits	Physics credits	Verbal test	Math test
Females	$\bar{X}$	1.5936	.7876	.7926	.1028	53.9520	51.1653
	SD	.8175	1.1302	.8015	.3097	9.0854	9.2671
Males	$\bar{X}$	1.5119	.1814	.9174	.1927	53.1871	52.9057
	SD	.8375	.3943	.8614	.4097	9.4921	10.2220

NOTE: The course credits are Carnegie units.

TABLE 5

TWO EMPIRICAL EXAMPLES OF INTERACTION

Effects of English and Clerical Credits on Verbal Test Score

Independent Variable	Females Unstandardized Coefficient	Females Standardized Coefficient	Females Linear Reg. Standardized Coefficient	Males Unstandardized Coefficient	Males Standardized Coefficient	Males Linear Reg. Standardized Coefficient
Eng. credits	0.4673***	0.0412	0.0419	0.6464****	0.0643	0.0570
Cler. credits	0.0736	0.0071	0.0092	-0.2569	0.0024	-0.0107
Eng. and cler.	-0.0105	-0.0011	-0.0024	0.2076	0.0072	0.0163

Effects of Math and Physic Credits on Math Test Score

Independent Variable	Females Unstandardized Coefficient	Females Standardized Coefficient	Females Linear Reg. Standardized Coefficient	Males Unstandardized Coefficient	Males Standardized Coefficient	Males Linear Reg. Standardized Coefficient
Math credits	2.0838****	0.1730	0.1802	2.1110****	0.1741	0.1779
Phys. credits	3.9650****	0.1109	0.1325	3.2217****	0.1206	0.1291
Math and Phys.	-0.8141*	-0.0218	-0.0511	-0.2332	-0.0081	-0.0181

NOTE: 1. See footnote 6 for a list of variables included in each equation.

* $p \leq .05$
** $p \leq .01$
*** $p \leq .001$
**** $p \leq .0001$

where y = math test, x = math credits, z = physics credits, and the circumflex indicates sample estimation. It is clear from these results that the magnitude of the interaction is not large, about one-eighth the size of the average effect. Precisely the same conclusion is reached by examining the "standardized coefficients," but the standardized coefficients facilitate the interpretation because they can be compared directly.[6]

In the data presented here the coefficients standardized by equations (7) generally are close in value to the linear regression standardized coefficients. In the case of the coefficients associated with the linear terms (b_1 and b_2), this similarity is due primarily to the relatively small magnitude of the interaction coefficients (b_{12}). In the case of the interaction coefficients, the discrepancies between the two types of standardizations are altogether due to the degree to which standard deviations of the independent variables differ from unity (see eq. [7d]). With the particular data in the example, the b_{12} coefficients standardized by formula (7d) are about half the size of those standardized by the usual algorithm. This magnitude of discrepancy would likely have important bearing on substantive interpretations in many settings.

There are a number of possible reasons why the complementarity hypothesis is not supported in these data. One of the most likely is that the hypothesis is false, at least with respect to the gross level of aggregation of educational experience represented by credits in high school courses. It is likely that much more explicit connections between subjects taught in different classes than are typically made by teachers would be necessary for the hypothesis to be true.

With respect to math and physics, the negative coefficients (significant only for females) on the product term suggest that "information overload" may be operating. That is, after a certain peak in the amount of exposure to technical material, students' capacity to absorb the material may decline.

There also are several possible methodological reasons why the complementarity hypothesis is not supported in the data. Probably the most important is specification error due to inadequate modeling of a learning process that describes how learning occurs over short (relative to time between measurements) periods of time (see Sorensen and Hallinan 1978). Additionally, effects of measurement error and missing data on estimates of interaction models are not thoroughly understood. It seems likely that their

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[6] For example, their ratio ($b^*/|d^*|$) gives the ratio of the mean effect to the standard deviation of those effects; this ratio has the same value as the ratio of unstandardized mean effects to the standard deviation of the unstandardized effects.

impact is more severe and unpredictable in nonlinear models than in the usual linear specification.

Discussion

This paper is intended to review and clarify existing expositions of the interpretation of interaction specifications in structural models. The paper extends current work in several respects. The most notable are (1) explication of the relationships between coefficients in a product model and the mean and variation of effects over a population (or sample) and (2) development of standardization formulas that facilitate interpretations of interaction coefficients. It is worth reiterating here that the usual "standardized betas" produced by most computer software are difficult or impossible to interpret when the structural model contains a product term.

The exposition in this paper has focused on an interaction specification as expressed by a product term in a structural model. Although this relatively narrow focus makes the exposition clear and easy to follow, it tends to mask the generality of most of the discussion. Specific formulae to calculate average effects, standard deviations of effects (and/or second partial derivatives), and standardized coefficients developed for the product models do not, of course, apply to any other type of model. However, the main principles discussed in this paper apply generally. It is often possible to derive simple calculating formulae for the average and standard deviation of an effect. For example, given a second degree polynomial ($y = a + bx + cx^2$), the effect is $q_{yx} = b + 2cx$, the average effect is $b + 2c\mu_x$, and the standard deviation of the effects is $2|c|\sigma_x$. Standardized coefficients for this simple model also are readily derived--$b^* = \sigma_x b/\sigma_y$; $c^* = \sigma_x^2 c/\sigma_y$ (where the asterisks indicate standardized coefficients). In many cases, however, it probably would be easier to calculate averages and standard deviations of effects numerically with a computer. Standardized coefficients also can be calculated readily by standardizing all variables prior to estimation, but in doing so it essential to modify the functional form to offset the change of scale.[7]

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[7] For example, nonpositive arguments are inadmissable for the log function.

CHAPTER 5

INTERACTION IN DYNAMIC MODELS

By Lawrence Hotchkiss

In sociology, as in other social sciences, there is ample reason to be interested in relationships between variables in cases where the relationship is conditional--that is, the relationship changes from one condition to another. Such relationships often are described as <u>interaction</u> in the statistical literature.

Interaction has been studied in a variety of contexts (Bohrnstedt and Marwell 1977). In sociology, mobility (attainment) models frequently are depicted as contingent on personal characteristics such as race and gender (e.g., McClendon 1976; Hout and Morgan 1975; Featherman and Hauser 1976). Stolzenberg (1974) argues that education and labor market experience interact in their influence on earnings. In economics both production functions and utility functions generally are sensible only if they contain some interaction component. The Cobb-Douglas production function is the most elementary example of this type of interaction.

Educational researchers have also hypothesized interactive relationships. Formal theory of learning (e.g., Sorensen and Hallinan 1978, 1986) hypothesizes that "opportunities for learning" interact with "ability and effort" to produce the rate of learning. Educational theory and research often postulate (but seldom find) "aptitude-by-treatment" interaction (Cronbach and Snow 1977). Vocational educators frequently argue that the applied aspects of vocational training help to motivate students and thereby enhance what students learn from academic courses (National Commission on Secondary Vocational Education 1984).

In the past several years, important steps have been taken to elucidate the proper meaning and interpretation of interaction effects (Stolzenberg 1979; Marsden 1981; Tate 1984; Allison 1977). The most important accomplishment has been to clarify the interpretation of structural relations that incorporate interaction. Stolzenberg's paper (1979) is particularly cogent in this respect, because it provides a thorough development of the idea that an effect in the presence of interaction with continuous functions is clearly defined by a partial derivative. Interpreting effects as partial derivatives avoids the confusion that often accompanies terms like "main effect" and "interaction effect."

It is useful to summarize and formalize the ideas regarding interaction that appear in the literature just cited. Let y be a continuous variable, and let x be a second variable whose effect on y we wish to assess. Define z to be a vector of other variables that influence y. Suppose that the dependence of y on x and z is summarized in the following functional relation:

$$y = f(x,z) \qquad [1]$$

where f is a function. If x is continuous, then the effect of x on y (q_{yx}) is a partial derivative. If f is not continuous (e.g., x is a nominal scale), then the effect of x on y must be defined as a difference, since the partial derivative no longer exists. These ideas may be summarized as follows:

$$q_{yx} = \partial f(x,z)/\partial x \quad | \text{ f continuous} \qquad [2a]$$

$$q_{yx} = \Delta y = y_1 - y_0 \quad | \text{ f not continuous} \qquad [2b]$$

$$= f(x_1,z) - f(x_0,z)$$

where y_1 and y_0 are values of y at x_1 and x_0, respectively. A convenient way to define interaction is to stipulate that the effect of x on y is a function of (one or more elements of) z--

$$\text{x interacts with z} \longleftrightarrow q_{yx} = g(x,z) \qquad [3]$$

where g is some function. The critical aspect of the formalization is that q_{yx} depends on z. The fact that it <u>may</u> also depend on x is incidental here.

The recent literature on interaction effects also has helped to clarify how to estimate and test the significance of coefficients associated with product terms in a structural relationship (e.g., Allison 1977; Tate 1984). Additionally, Bohrnstedt and Marwell (1978) and Heise (1986) have taken important first steps toward assessing the effects of unreliability on statistical estimates of parameters in equations that include product terms.

No paper has investigated the implications of dynamic structural models for estimation of interaction effects, however. This chapter provides an introduction to the complexities that arise when interaction is hypothesized in dynamic models. All social relationships are processes that unfold over continuous time, and we expect many interactions; hence, the topic of this chapter is particularly important in the quantitative study of social relationships.

The chapter begins with a discussion of discrete-time models as represented by difference equations and then presents analogous results for continuous-time models as represented by differential equations. There are two reasons to begin with difference equations. First, they are conceptually easier to grasp than differential equations; hence, they provide a good basis for introducing fundamental ideas. Second, starting with difference equations helps to show how use of differential equations may resolve a critical issue in longitudinal research with nonlinear functions--how should one proceed in the absence of knowledge of lag time between cause and effect? A major practical difficulty with difference equations is that one must know the length of the time lag between cause and effect in order to estimate their paramaters. On the other hand, the lag time is assumed to approach zero in differential equations; hence, it need not be specified by theory. An important result of this chapter is that differential equations may provide reasonable (though slightly biased) estimates of the parameters of a difference equation provided that there are several cause-and-effect cycles between measurement points. As few as five cycles are shown to yield reasonable results in Monte Carlo simulations of a fairly simple model. It is concluded, therefore, that lack of knowledge of precise lag time is not a critical barrier to estimating parameters of nonlinear dynamic models. All that is needed is the qualitative knowledge that lag time is short relative to the length of time between measurements.

Discrete Time--Difference Equations

The basic issue raised here can best be explicated by starting with a few simple relationships in which there is a finite lag time between "cause" and "effect." The (constant) lag time is set to unity, and the time points are represented as a sequence of integers--1, 2, . . . , n. First, as a basis for comparison, consider a linear difference equation in which change in y is depicted as a function of x and z and lagged y--

$$y_{j+1} = a + by_j + cx + dz \qquad [4]$$

where the subscripts represent time points, and, a, b, c, and d are constants. Note that x and z are constant over time but vary over individuals. Parents' education and occupational status at age 16 are typical examples of x and z; examples of y include test scores and level of occupational aspiration. The solution to (4) itself may be written as a linear function of lagged y, x, and z--

$$y_n = a^* + b^*y_1 + c^*x + d^*z \qquad [5]$$

where (given $b \neq 1$)--[8]

$$a^* = a(1-b^n)/(1-b)$$

$$b^* = b^n$$

$$c^* = c(1-b^n)/(1-b)$$

$$d^* = d(1-b^n)/(1-b).$$

The importance of this solution is as follows: No matter how many cause-and-effect cycles (n) elapse between measurements, the linear form of the fundamental model (4) is preserved. Thus, even after several cycles we can still use a linear regression (with appropriate assumptions about the disturbance) to study the impacts of x and z on y. In doing so, however, it is important to recognize that the regression coefficients do not directly estimate the fundamental parameters (a, b, c, d). Rather, they estimate accumulated effects over repeated cycles. It is necessary to know n before the fundamental parameters can be retrieved from the regression coefficients.

An interaction between x and z, say, of the following form--

$$y_{j+1} = a + by_j + cx + dz + gxz \qquad [6]$$

(g = constant coefficient) yields the following solution:

$$y_n = a^* + b^*y_1 + c^*x + d^*z + g^*xz \qquad [7]$$

The relations between the fundamental parameters (a, b, c, d) and the regression estimates (a^*, b^*, c^*, d^*) defined in conjunction with (5) hold here, and $g^* = g(1-b^n)/(1-b)$. Again, the functional form of the fundamental structure (6) is preserved in the estimating equation (7).

Now, suppose that x interacts with lagged y in the fundamental structure, as follows:

$$y_{j+1} = a + by_j + cx + dxy_j \qquad [8]$$

This model might be viewed as a representation of the idea that learning $[y_{j+1} - y_j = a + (b-1)y_j + cx + dxy_j]$ is a function involving the product of readiness and exposure. Here exposure is

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[8] These expressions follow from well-known results for linear first-order difference equations (see e.g., Yamane 1968).

the rate of presentation of new material x, assumed constant over time, and readiness is identified with current level of knowledge.[9]

The structure implied by (8) after just two cycles is

$$y_3 = a^* + b^*y_1 + c^*x + d^*xy_1 + g^*x^2 + h^*x^2y_1 \qquad [9]$$

where

$$a^* = a(1+b)$$

$$b^* = b^2$$

$$c^* = c(1+b) + ad$$

$$d^* = 2bd$$

$$g^* = cd$$

$$h^* = d^2.$$

In this case the functional form of the original structure (8) is altered. Two new terms appear in (9), one involving x^2 and one containing x^2y_1. Thus, using the same functional form in the estimating equation as one hypothesizes in theory would lead to a misspecified estimating equation.

If one knew that the time elapsed between measurements were exactly twice the lag time in the basic structure, then an estimating equation like (9) could be used (along with appropriate constraints on the coefficients). All the fundamental parameters of (8) could then be calculated from the regression estimates of the coefficients in (9). However, it is important to realize that the functional form of the estimating equation changes for each cycling that occurs between measurement points. The general solution for (8) is as follows:

$$y_n = (a + cx)\frac{1-(b+dx)^n}{1-b-dx} + (b + dx)^n y_1 \qquad [10]$$

A nonlinear method of estimation obviously would be required here. Moreover, it would be necessary to know the value of n a priori.

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[9] This model contains the basic conceptual ideas of Sorensen and Hallinan (1978) but expresses them in a somewhat different functional form. Connecting equation (8) to a learning hypothesis is done purely for purposes of exposition and is not intended as a substantive alteration of the Sorensen-Hallinan paper.

If $-1 < b + dx < 1$, then as n goes to infinity equation (10) reduces to

$$y_n = (a+cx)/(1-b-dx) \quad | \; -1 < b + dx < 1. \qquad [10a]$$

This result can be viewed as a nonlinear reduced form. As with a linear reduced form it is not possible to identify all the structural parameters from (10a). It is, however, possible to identify relative values of the parameters. For example, one might estimate by nonlinear least squares a function of the following form: $y_n = (\alpha+\gamma x)/(1+\beta+\delta x)$. The ratio of these parameters to each other will be the same as the ratios of corresponding structural coefficients (e.g., $\beta/\delta = b/d$).[10]

One interesting aspect of equation (10) is that it does take on the same functional form as the basic structure (8), given that the exogenous variable x is a dummy variable. It is interesting to speculate that this observation may help to account for the fact that statistically significant interactions are more often reported for categorical variables such as race and gender than for approximate continuous variables such as education, occupational status, or test scores. The point here is that estimating equations with x a dummy variable assume the same functional form as the structural equation; hence, they are less likely to be misspecified than estimating equations with continuous x.

It is conceivable that an estimating equation with the same functional form as the basic structure (8) would provide an adequate approximation in many instances. Although such an estimating equation would be misspecified, given a large sample, it might be sufficient to detect the presence and direction of the interaction. Given the approximate nature of most theory predicting interaction, detecting its presence and sign may be an important achievement. The effectiveness of this strategy and others suggested here will be examined using Monte Carlo simulations in a later section of this paper.

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[10] Adding a "white noise" disturbance to equation (8) generates a version of (10) and of (10a) that includes a disturbance independent of x. Preliminary experiments using generated data and nonlinear least squares suggest that either (10) or (10a) might serve as a basis for estimating the parameters of the model. However, as noted in the text, not all the structural parameters are identified in (10a).

Continuous Time--Differential Equations

With the basic issue of this paper now in mind, it is useful to review the results for the case where "causal" lag time approaches (never quite reaches) zero. If one subtracts y_j from both sides of (4), the following result is obtained:

$$\Delta y = a + (b-1)y_j + cx + dz \quad [11]$$

(with $\Delta y = y_{j+1} - y_j$)). Since the time lag between time j and time j+1 is scaled to equal 1, the left side of (11) is a rate of change with respect to time. As the lag time approaches zero, (11) becomes a linear differential equation of the following general form:

$$dy/dt = p_o + p_1 x + p_2 z + qy \quad [12]$$

There are distinct conceptual and practical advantages of (12) as compared to (4) [or (11)]. First, time is conceptualized as continuous in (12) but is discrete in (4). Since time is, in fact, continuous, this is an important advantage of (12) over (4). Second, since (12) implies that the number of cycles between measurements approaches infinity, it is unnecessary to specify a priori the number of cycles. In practice, whenever the cycle time is short relative to the time elapsed between measurements, the solution to (12) provides a good basis for an estimation strategy.

The solution to (12) has been noted repeatedly in the sociology literature; it is

$$y(t) = p_o^* + p_1^* x + p_2^* z + q^* y(0) \quad [13]$$

where

$$p_o^* = \frac{p_o}{q}(e^{qt}-1)$$

$$p_1^* = \frac{p_1}{q}(e^{qt}-1)$$

$$p_2^* = \frac{p_2}{q}(e^{qt}-1)$$

$$q^* = e^{qt}$$

(see, e.g., Coleman 1968; Doreian and Hummon 1976; Tuma and Hannan 1984).[11] In a qualitative way, the same result found for a linear difference equation also holds for a linear differential equation--that is, the linear form hypothesized for the basic structure (12) also applies to the estimating equation.[12]

Again, as for the difference equation (8), interaction involving x and y--

$$dy/dt = p_o + p_1 x + q_o y + q_1 xy \qquad [14]$$

generates an estimating equation with a functional form quite different from the original structure (eq. [14]), one that is nonlinear in the parameters as well as in x and y--

$$y(t) = \frac{p_o + p_1 x}{q_o + q_1 x}\,[e^{(q_o+q_1 x)t} - 1] + e^{(q_o+q_1 x)t}\,y(0) \qquad [15]$$

where e is the base of the natural logarithm. Most of the conclusions regarding the solution to the difference equation (10) apply here. In principle, nonlinear estimation could be applied either directly to (15) or to a reduced form (letting $t \longrightarrow \infty$)--

$$y(\infty) = (p_o + p_1 x)/(q_o + q_1 x) \quad | \quad q_o + q_1 x < 0 \qquad [16]$$

It is easier to use (15) as a basis for estimating parameters than to use (10) (solution to difference equation), because it is not necessary to know the number of cycles to estimate (15). All one needs to know is that the recycle time is short relative to the length of the measurement interval (five cycles or more between measurements give a good approximation--see simulation results in the next section of this paper.)

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[11] It should be noted that setting t = 0 at the initial observation point is an arbitrary translation of the time scale. This scaling often is used in the literature for notational convenience, to avoid awkward expressions like $y(t+\Delta t) = f[y(t),x]$.

[12] See Tuma and Hannan (1984) for a review of statistical issues related to estimating (13). Basically, adding "white noise" to (12) transforms it into a stochastic differential equation. The solution is again linear and can be estimated consistently using OLS.

Monte Carlo Simulations

In order to convey some impressions of the behavior of alternative estimating strategies for testing interaction hypotheses in dynamic models, several simulations were carried out. Calculations are based on the difference equation (8) which expresses an interaction between lagged y and an exogenous variable x. Three estimating strategies are considered: (a) apply ordinary least squares regression to observed data using precisely the same functional form as in the structural equation (8); (b) use nonlinear least squares in conjunction with the general solution (10) to equation (8); and (3) use nonlinear least squares with the reduced-form (10a) solution to (8). In practice, the first option is by far the most likely to be applied. The adequacy of option (a) as an approximation depends on n, the number of cycles between measurement time points. Simulations were calculated for n = 1, n = 3, and n = 5.

The following algorithm was used to generate the observations:

$$y_1 = 3 + cx + u_o \qquad \sigma_{u_o} = \sqrt{5 - c^2}$$

$$y_{j+1} = a + by_j + cx + dxy_j + u_j \quad \mid j \geq 1$$

$$\sigma_{u_j} = 2.25$$

with x normed to zero mean and unit variance. The u_j also have zero mean. Both x and u are normally distributed. Samples of size N = 200 cases were generated with a time series of n = 100 for each case. Each estimating equation except the reduced form was based on two time points out of this series, thereby simulating the two-wave panel survey so commonly analyzed in sociological and educational research. The reduced form was estimated from data after 100 cycles (n = 100). Three different sets of values for the parameters c and d were used. Twenty-five samples were drawn for each set of simulations.[13]

The results, including the values of the parameters, are displayed in table 6. The lines labeled "mean" show the average value of the parameter estimate over the 25 samples. The lines

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[13] In some or possibly all of the cases considered here, it would be possible to find analytically the expected values and standard errors of the parameters. Even with the OLS lag-one estimates, however, the standard errors depend on the inverse of the X'X matrix. After several cycles, the entries of this matrix might not be easy to calculate analytically.

labeled "SD" show the standard deviations. The lines labeled "RMSE" give the root mean-square errors. Standard deviations summarize sampling stability of an estimate irrespective of bias. The RMSE provide a combined summary of sampling stability and bias. When there is no bias, RMSE = SD; otherwise, RMSE > SD. No statistics are summarized for the intercept (a) in the reduced-form estimates. Recall that the reduced-form is not identified--only the ratios of parameters are identified. For the simulations, a was set a priori to 1.0, hence, there is no sampling fluctuation, and the summary statistics are therefore not informative.

Table 6

Several interesting observations are contained in table 1. The most salient result is, as expected, that OLS yields highly biased estimates when the lag time does not equal 1. It is somewhat anomalous, however, that one is more likely to detect the presence of the interaction with OLS if the lag time is greater than 1 than if it equals 1. This anomaly is due to the fact that the bias on the OLS estimate of the interaction coefficient d is positive as lag time increases.

The finding that a badly misspecified OLS equation is still capable of detecting interaction is somewhat encouraging, since theory is seldom able to indicate the lag time between "cause" and "effect." In many instances, simply detecting the presence and direction of interaction may be an important accomplishment. However, it is easy to be overly sanguine in this respect. Specifications of the underlying structure other than the one used for the present simulation may not yield the same conclusion. Moreover, although qualitative conclusions regarding theory seldom require precise numerical estimates of structural coefficients, some reasonable approximation generally is necessary in order to assess the "substantive importance" of the interaction. It is difficult to use the OLS estimates in this way since their strengths depend on the unknown lag time and, unless lag = 1, they do not estimate any interpretable parameter.

In contrast to the OLS estimates, both of the nonlinear least squares (NLS) methods yield essentially unbiased and reasonably efficient parameter estimates. The empirical standard error (SD) estimates are of the same order of magnitude as are the OLS estimates for lag = 1.

It is interesting that the standard errors for the reduced-form NLS are uniformly smaller than other standard errors. Presumably this observation is due to the fact that only three parameters are estimated with the reduced form, since only three

TABLE 6

SIMULATION RESULTS BASED ON DISCRETE TIME LAGS

Estimation Method

Summary Statistics		OLS lag=1	OLS lag=3	OLS lag=5	NLS lag=5	NLS Reduced form (n=100)
a=1	Mean	0.926	2.151	2.774	1.048	--
	SD	0.181	0.291	0.381	0.263	--
	RMSE	0.196	1.193	1.824	0.267	--
b=.7	Mean	0.726	0.368	0.183	0.632	0.689
	SD	0.047	0.067	0.072	0.086	0.029
	RMSE	0.054	0.339	0.523	0.088	0.031
c=.15	Mean	0.133	0.364	0.556	0.100	0.133
	SD	0.252	0.381	0.359	0.209	0.163
	RMSE	0.253	0.437	0.543	0.215	0.164
d=.05	Mean	0.041	0.081	0.098	0.066	0.054
	SD	0.053	0.082	0.080	0.054	0.039
	RMSE	0.054	0.088	0.093	0.057	0.039
a=1	Mean	0.935	2.192	2.740	1.027	--
	SD	0.281	0.372	0.338	0.206	--
	RMSE	0.288	1.251	1.777	0.207	--
b=.7	Mean	0.711	0.354	0.214	0.699	0.695
	SD	0.063	0.091	0.071	0.058	0.020
	RMSE	0.063	0.359	0.492	0.058	0.020
c=.075	Mean	0.063	0.318	0.442	0.093	0.091
	SD	0.277	0.285	0.472	0.142	0.099
	RMSE	0.277	0.375	0.598	0.143	0.100
d=.10	Mean	0.099	0.182	0.223	0.097	0.100
	SD	0.057	0.075	0.072	0.029	0.008
	RMSE	0.058	0.111	0.143	0.029	0.008
a=1	Mean	0.928	2.192	2.777	1.041	--
	SD	0.181	0.288	0.376	0.263	--
	RMSE	0.195	1.189	1.821	0.266	--
b=.7	Mean	0.725	0.365	0.177	0.683	0.636
	SD	0.047	0.069	0.072	0.087	0.031
	RMSE	0.054	0.344	0.531	0.089	0.034
c=.3	Mean	0.272	0.631	0.891	0.261	0.268
	SD	0.247	0.383	0.343	0.187	0.142
	RMSE	0.249	0.508	0.687	0.191	0.146
d=.03	Mean	0.023	0.053	0.065	0.044	0.039
	SD	0.050	0.084	0.076	0.050	0.035
	RMSE	0.051	0.087	0.084	0.052	0.036

NOTES:

1. NLS stands for nonlinear least squares; OLS means ordinary least squares.

2. The sample size N = 200; the number of samples = 25.

3. SD = standard deviation, RMSE = root mean square error

are identified. To identify all the parameters, it is necessary to use the full solution to (8)--equation (10). However, this is not a practical strategy since one seldom knows the value of n a priori. It is therefore concluded that the reduced form equation (10a) provides a useful basis for exploratory analyses in which one is expecting interaction between an exogenous variable and a lagged dependent variable. This strategy certainly seems preferable to indiscriminate application of OLS to equation (8).

In the absence of theoretical specification of the lag time (which implies knowledge of n), a useful approximation might be to assume instantaneous effects (i.e., $\Delta t \longrightarrow 0$) and use the solution (15) to the differential equation (14). A simulation with lag = 5 and $a = 1$, $b = 0.7-1 = -0.3$, $c = 0.15$, and $d = 0.05$ was conducted. The results are displayed in table 2. The estimates of a and b appear to exhibit slight upward bias in absolute value; t-tests indicate statistical significance at $\alpha < 0.05$ in both cases. However, the magnitude of the bias is small, well within acceptable levels for most social science applications. Typical longitudinal social data are collected at widely spaced intervals, generally at least 1 year and often several years. In most situations, it therefore is likely that at least five cycles are completed between measurements. Consequently, a differential-equation model often may yield a good approximation even when a finite length of time between "cause" and "effect" is expected a priori.

Table 7

It is noteworthy that the standard errors on the interaction coefficient (d) generally are so large that the interaction coefficient would be unlikely to be detected in a sample of size 200. Assuming that this standard error declines approximately with the square root of N, then a sample size of 2,000 or more likely would be needed in order to produce reasonably stable estimates.

Summary and Discussion

This chapter investigates the impact of dynamic models on estimating interaction effects. Primary attention is devoted to interaction between one exogenous variable (constant over time but not over individuals) and one endogenous variable. A simple product model is used to represent the "causal" process that is hypothesized to occur over a period of time. It is observed that if longitudinal data are analyzed that have been collected at time points that are farther apart than the length of the "causal lag," then the functional form of the estimating equation is different

TABLE 7
SIMULATION RESULTS BASED ON CONTINUOUS-TIME APPROXIMATION

Statistic	Parameter a=1	b=0.7-1=-0.3	c=0.15	d=0.05
Mean	1.188	-0.354	0.122	0.066
Std. dev.	0.329	0.105	0.216	0.062
Root mean sq. error	0.378	0.118	0.218	0.064

than the functional form of the basic structure. This result stands in sharp contrast to the linear case, since a linear estimating equation is appropriate no matter how many "cause-and-effect" cycles occur between measurement points.

Simulation results for a difference-equation model reveal the following:

- Indiscriminate application of OLS using an estimating equation with the same functional form as the basic structure generates highly biased estimates of the structural coefficients. Moreover, the OLS coefficients do not estimate any easily interpretable quantity. However, the OLS may be sufficiently robust to detect the interaction--though other methods are preferable.

- Nonlinear least squares (NLS) estimates using the complete solution to the difference equation yield good estimates of the structure. A critical disadvantage of this method, however, is that it requires a priori knowledge of the length of the "causal lag."

- NLS applied to a reduced form yields good estimates of the ratios of the parameters of the structure, but the absolute values of the parameters are not identified. Two advantages of the reduced-form method are that a priori knowledge of the length of the causal lag is not necessary and sampling error is low.

- Estimates based on the solution to a differential equation provide reasonable approximations of the parameters of the difference equation when as few as five "cause-and-effect" cycles occur between measurements.

This last point is critically important. It implies that we do not necessarily need to know the length of the lag time between cause and effect in order to proceed. All that is necessary is to know that lag time is short relative to the time between measurements. Since lag time is almost never known in practice, it is concluded that differential equations provide a powerful tool for representing social processes.

The clearest implication is that interaction effects in a dynamic model introduce complexities that may not be anticipated by many practicing researchers In view of the fact that social processes do, in fact, operate over time, it seems likely that failure to detect and validate many hypothesized interactions is partly due to these complexities. It is important to keep in mind an accurate perspective on the results reported in this paper. First, the product model analyzed here is but a special case of nonlinear nonadditive dynamic models. Such models generally introduce enormous complexity into empirical analysis (see Tuma and Hannan 1984; Gandolfo 1981). Second, attention here is confined to a single equation with one endogenous variable and one exogenous variable. Estimating methods become substantially more complex when the exogenous variable(s) change over time and/or a nonlinear system of two or more endogenous variables is considered. Additionally, use of a product formulation to represent interactions is convenient and reflects common practice in the social sciences, but it is arbitrary since the functional form of the interaction is almost never derived from theory.

This chapter serves two important functions. First, it shows practical methods for estimating coefficients of a relatively simple dynamic interaction model. Second, it alerts researchers to important consequences of nonlinear specifications in dynamic models and thereby suggests the need for careful consideration of appropriate estimating strategies when such models are hypothesized.

CHAPTER 6

THE VOCATIONAL CLASSROOM AND STUDENT LEARNING STYLES

by Lawrence Hotchkiss and
Jeanne Desy

The earlier chapters of this report describe the characteristic vocational classroom and what it offers. In this classroom students learn through doing, often working alone and receiving one-on-one instruction. Mastery is emphasized, and structured competency-based learning may be used. Students read less, write less, and listen less than in the academic classroom and learn more often through tactile-kinesthetic modes. They work with a high degree of autonomy and make numerous decisions as they go. They report that this kind of learning is enjoyable and relevant. The accomplishment it affords them is ego enhancing, and the hands-on work provides incentive, task, and achievement motivation.

What type of student stands to benefit most from this educational opportunity? This question cannot be answered from the present base of theory and research. However, learning style theory appears to be particularly relevant to the question. In this final chapter, we review learning style theory and assess its usefulness as a theoretical basis for organizing theory of vocational education.

It is apparent that a student whose learning style is fairly well matched to the opportunity will learn more than a student with a poor match. The key issue is to achieve optimum matches. Some writers (see Kirby 1979) recommend challenging students--teaching through less-favored modes--to develop their flexibility in learning. Challenge matching for certain tasks can take place in the classroom, where the teacher can carefully regulate the teaching method with attention to individual needs. For purposes of placement, however, style matching is valuable, especially for students who need to experience success in learning. Lembke (1985) reviewed the extensive research on this topic and found that the evidence supported style matching: "when instruction was consonant with learning style, academic achievement was increased" (p. 62).

As the term suggests, learning style is a way of describing how an individual goes about learning. Keefe (1979) sees learning style as composed of "cognitive, affective, and physiological behaviors that serve as relatively stable indicators of how learners perceive, interact with, and respond to the learning environment" (p. 4).

As this definition suggests, "learning style" is a construct with a large and complex domain whose boundaries are not clearly defined. Simply establishing the content to be assessed is difficult, and the various models differ widely. Sewall (1986) says that most are based on the findings of cognitive style research, especially on Witkin's theory of field-dependence/-independence. McCarthy (1980) states that a number of models (including her own) are based on Jung's definitions of psychological types--thinkers, feelers, sensors, and intuitors. These categories are represented by McCarthy, Kolb, Gregorc, and others by the individual's relationship to two intersecting continua.

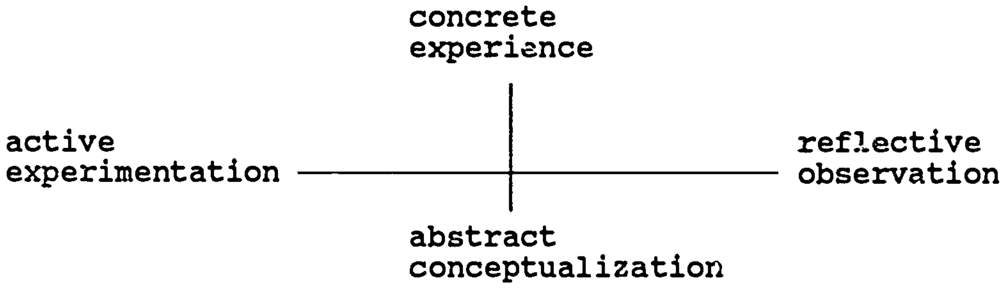

Figure 3. The Motivational Goals

Roughly, these axes represent perception (the vertical) and cognitive processing (the horizontal).

Models of learning style based on cognitive/processing preference describe four basic learning styles, located in the quadrants of the model. In Kolb's model, the style that best matches with the vocational classroom is that of the learner who perceives abstractly, that is, favors thinking over intuition, and processes actively. Kolb (1976) calls this learner the "Converger" and says "his greatest strength lies in the practical application of ideas" (p. 5). The Converger thinks deductively, does best when there is a single correct answer and would rather deal with things than people.

Gregorc (1982) defines processing somewhat differently, as linear or nonlinear ordering. His instrument would depict the ideal vocational student as Concrete Sequential. This student prefers to work tactilely and learn through hands-on experience. He or she likes to follow exact directions.

McCarthy (1980) describes a Common Sense Learner, who, like Kolb's Converger, is active-abstract. This student's strength is the "practical application of ideas" (p. 41), and he or she needs hands-on experiences. Unlike Gregorc, McCarthy describes this student as one who "resents being given answers." (p. 41)

This brief review of three similar models of learning styles demonstrates an important truth about this area: numerous conflicting models are in use, and many differ from one another more sharply than those mentioned above. For example, the Dunn's model incorporates a conglomeration of elements that are environmental, emotional, sociological, and physical. The student's preferences are surveyed on room temperature, structure, optimum time of day for learning, snacking while learning, and 19 other stimuli (Dunn 1984).

The Dunn's work is illustrative of a second difficulty with the literature on learning styles: the research base is often very thin or questionable. In a 1982 article titled "Teaching Students through Their Individual Learning Styles: A Research Report," Rita Dunn (National Association of Secondary School Principals 1982) presents the research behind the Dunn/Price Learnings Style Inventory (LSI). Of 14 studies cited, 8 are doctoral dissertations completed at St. John's University, where Rita Dunn teaches. Of the remaining five sources, three are dissertations, one deals with college students (the LSI is used for grades 3-12), and one is an article about high school students in which no population number is given. Lembke (1985) comments that "a self-fulfilling prophecy could be at work" (p. 65). Although this research base is questionable, it remains true that the Dunns have emphasized the empirical basis of their work more than any other learning style theorists.

Assessment of learning style is fraught with problems. Blakemore, McCray, and Coker (1984) noted that the 11 instruments they surveyed had several common problems. Most important--

- few have vocational relevance,
- little technical data supports their validity or reliability, and
- they "rely on the testee's ability to make accurate judgements about himself" (p. 28).

Davidman (1981) has pointed out that learning style inventories, such as the Dunn/Price LSI, can yield only "informed speculations," but are often taken to provide "precise, accurate conclusions about individual students' preferred learning styles" (p. 4).

Other critiques of the numerous assessment instruments available have also been harsh. For instance, in 1977 the American College Testing Program carried out a thorough study of cognitive style mapping based on Joseph Hill's reputable and well-developed model. Hill and others selected the ACT instruments that were refined and used in the study. The project found that those instruments lacked construct validity, although the selectors had confidence in their face and content validity. The writers note that

> in the research on educational cognitive style completed to date, little or no data have been presented to demonstrate the psychometric adequacy of the measures employed. In those instances where validity is

> addressed, it is nearly always content or face validity based on expert judgement. (American College Testing Program 1978, p. 53)

They recommend that the development of technically sound instruments be the priority for future research.

Nearly 10 years later, this situation seemed little changed. In a 1986 study, Sewall carefully evaluated four assessment tools. He found that

> evidence supporting the construct validity of the Myers-Briggs is minimal and practically nonexistent for the Kolb LSI, the Canfield LS and the Gregorc Style Delineator. (p. 59)

Sewall theorizes that the problem lies with both the models and the instruments. He further notes that these instruments may not be reliable and, like the ACT report, calls for further work on a valid measurement device.

The selection (or creation) of the most valid and reliable tool for measuring learning style will demand extensive research, given the intricate problems associated with learning style models and with instrument design. Of the most-discussed instruments the Myers-Briggs Type Indicator (MBTI) is most recommended. The model it is based on is a direct descendent of Jung's psychological type theory. The forced-choice, self-report inventory consists of four scales:

- Extraversion-Introversion
- Sensation-Intuition
- Thinking-Feeling
- Judgement-Perception

Sewall rates the MBTI highly for its ease of administration, scoring, and interpretation, and considers it "one of the better instruments currently available to assess learning style type" (p. 18).

The MBTI yields results that can indicate whether a student would fit the vocational classroom. The ideal student would favor "sensing" in perception, tending to "deal realistically, observantly and precisely with tasks. . .[and] with details and facts, preferring experience to theory" (Barger and Hoover 1984, p. 57). The student would favor thinking judgements over feeling judgements, tending to be logical and interested in causal relationships, and might be more introverted than extraverted, preferring to work alone.

Another approach to assessing the individual differences that might make a student more suitable for the vocational classroom has been suggested by the work of Anderson and Scott (1978). Their study of 100 secondary students in humanities and social science classes measured student involvement in learning--the percent of time the student was on task. Scholastic aptitude (APT) was measured by the verbal subtest of the Lorge-Thorndike Intelligence Test, and academic self-concept (ASC) was measured by the Scott Academic Self-Concept Scale. The analysis showed that low APT-ASC students were less responsive to lecture and audiovisual methods than other students. These students may do better in a classroom that offers increased opportunities for technical practice and one-on-one teaching.

Critics of learning style assessment agree that, as Sewall (1936) says, "a valid model and measurement device for learning style would be a powerful tool for the facilitation of learning" (p. 61). In its absence, several writers recommend a combination of formal and informal assessment. Davidman (1981) recommends that educators create their own "mini-questionnaires" (p. 4) and invest time in questioning individual students. Hunt (1981) suggests that teachers perform a reciprocal, transactional "reading" in the classroom.

Campbell, (1987) who has been extensively involved with assessment, suggests a similar process. Having observed that many students fill out self-report questionnaires quickly and without much thought, he suggests assessment for purposes of curriculum choice through an interview. The interviewer should develop and express genuine interest in the student as a person with a serious decision to make. The interview would begin with a statement of purpose, to help the student see the seriousness of the task. The guidance counselor or teacher would survey the student's academic history and ask open-ended questions about interests and activities. These and an informal atmosphere would be designed to put the student at ease and encourage free discussion. Next, the interviewer would verbally administer a simple, locally designed, forced-choice questionnaire. This should be designed to elicit learning preferences as well as general areas of interest. Davidman (1981) references a number of sources with helpful material on this subject.

As this discussion suggests, certain preferences flag a student as one who will profit from some degree of involvement in vocational education. (How concentrated that experience should be is a matter for the student and counselor to determine.) The candidate for vocational concentration as defined by Campbell, Orth, and Seitz (1981) would show the following characteristics:

- o Prefers to work with things rather than words
- o Needs autonomy, likes to work alone
- o Responds more to one-on-one instruction than whole class

- Learns best through hands-on (tactile-kinesthetic) modes

This student might demonstrate additional characteristics:

- Is not motivated to perform in the academic classroom
- Is less successful academically and needs mastery experiences
- Does not respond well to competition

The assessment process suggested here combines the student self-report characteristic of many learning style inventories (e.g., the Dunn-Price LSI) with teacher assessment. Davidman (1981) argues for a process of this kind, nested in "heightened individual dialogues" between students and teachers.

The assessment of individual differences, especially those imperfectly defined qualities that form learning style, is not a mature science. However, even the recognition that meaningful differences exist seems useful. Garger and Guild (1984) state that "an important goal is for teachers and students to accept and value the diversity of styles" (p. 12). Davidman and Chiarelott (1985) emphasize the importance of the concept of learning style for teachers when they anticipate the

> reconceptualization of the learning environment, and . . . the teacher's enhanced perception of the student as a learner who possesses educationally significant strengths, preferences, and interests. (p. 12)

In order to connect the material on theory of vocational education, learning styles, and the technical issues regarding interaction models it may be useful to formulate a model that expresses the key ideas as interactions. For illustrative purposes, assume that the outcome is a combination of basic and applied skills measured by a standardized test. For a particular student, we assume that optimum learning occurs with a particular mix of academic and vocational courses, as follows:

$$y = a + bA + cV + dAV + u \qquad (1)$$

where

y = learning outcome

A = academic courses/credits

V = vocational courses/credits

u = random disturbance

$a, b, c, d,$ = constant parameters

The complementarity between vocational and academic curriculum is expressed by the interaction term d. The value of d is expected to be positive. The larger d is, the more academic courses enhance students' ability to learn from vocational courses, <u>and</u> the more vocational courses improve student learning in academic courses. Presumably, the latter impact is due in part, to the motivation classrooms.

One important way in which learning styles enter this formulation is by influencing the degree to which hands-on and other approaches (e.g., CBVE) provide motivation. Thus, one may postulate that students who on the MBTI favor "sensing" and tend to "deal realistically, observantly and precisely with tasks" profit more from vocational education than other students. Denoting a variable indicating degree of sensing and realistic approaches to tasks by R, this theory can be expressed formally by setting both c and d in equation (1) to be positive functions of R. Linear formulation can be expressed as follows:

$$c = p + qR$$

$$d = g + hR$$

Inserting these expressions into (1) yields a relatively complex interaction model--

$$y = a + bA + pV + qRV + gAV + hARV + u. \qquad (2)$$

A more general model also would set b to a linear function of R, but one with a negative slope.

Although the model in (2) is substantially more complex than most formalized theory, it remains a substantial simplification of the informal theory. It does, however, serve two important functions for this report. First, it helps to integrate the chapters dealing with theory, substantive findings, and technical issues related to interactions. Second, it suggests important next steps in improving understanding the complex interplay between curriculum choices and individual characteristics.

Experience indicates that it would be advisable to add complex interactions to our assessment of vocational education in small steps. It seems advisable to focus first on equation (1). If a positive value for d cannot be established, there seems little point in trying to estimate the more complex model in (2). Preliminary evidence in this report indicates that establishing empirical support for the model in (2) may not be easy to achieve. If equation (1) is not supported by empirical evidence, then careful reexamination of the theory and educational practice would be the important next steps.

In the meantime, it is clear that the vocational curriculum differs significantly from the academic. In addition, we know enough about individual differences and learning to say with a fair degree of certainty that vocational education will benefit certain students far more than other curricula. The continued exploration of the interactions between vocational curriculum, academic curriculum, and student characteristics such as learning styles should produce two types of benefits. First, it will promote systematic inquiry into the relationships between curriculum choices and learning outcomes. Second, it will stimulate reflection about the primary considerations that must be accommodated in practical everyday curriculum choices.

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