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ABSTRACT

Mathematical word problems are notoriously difficult to solve for students at all grade levels. Performance deficiencies have been attributed to students lacking abstract logico-mathematical knowledge or to insufficiently developed language comprehension skills. At least four general sources of difficulty can be distinguished in mathematical word problems: (1) the verbal formulation of the problem text; (2) the structure of the underlying episodic problem situation; (3) the conceptual logico-mathematical or arithmetical knowledge about set relations; and (4) the arithmetic problem solving skills that are required to perform counting operations or to resolve equations. This paper describes a process model Situation-Problem-Solver (SPS) computer program that constructs a problem representation based on strategies that take into account the specific situational structure as well as its wording. Discussed in this study is the role of two linguistic and of one situational factors on comprehension difficulty of simple mathematical word problems. The linguistic form factors that were manipulated concerned the narrative focus of the problem episode and the problem question. The results of this experiment indicate the script factor to be a dominant source of comprehension difficulty. (KR)

2)

Understanding Word Arithmetic Problems.

Linguistic and Situational Factors.

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Textual and situational factors in solving mathematical word problems. [1] [2]

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Mathematical word problems are notoriously difficult to solve for students at all grade levels. What makes word problems more difficult than comparable problems presented in a numeric format? The basic assumption that guided our theorizing in the past few years is that explanations of solving mathematical word or situation problems and its development roughly fall in two categories (Cummins-Dellarosa, Kintsch, Reusser & Weimer 1988): Either performance deficiencies are attributed to lacking abstract logico-mathematical knowledge, or to insufficiently developed language comprehension skills. At least four general sources of difficulty can be distinguished in mathematical word problems: (a) the verbal formulation of the problem text, (b) the structure of the underlying episodic problem situation, (c) the conceptual logico-mathematical or arithmetical knowledge about set relations, and (d) the arithmetic problem solving skills that are required to perform counting operations or to resolve equations.

Conceptual versus linguistic factors in word problems

Recent research on simple word problems including several computer simulation models (Riley, Greeno & Heller 1983; Briars & Larkin 1984; Kintsch & Greeno 1985), has focused mainly on the conceptual structure, i.e. on the abstract logico-mathematical knowledge underlying the solving of word problems (Cummins, Kintsch, Reusser & Weimer 1988). Correspondingly, many empirical studies have revealed, for different age levels, the importance contribution of the abstract conceptual problem type (change, combine, compare), together with the location of the unknown quantity, to

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2 This work is supported from the Swiss National Science foundation (Grant No 1.448.0.86). - I am very grateful to Marianne Reusser-Weyeneth and to Ruedi Stüssi for their substantial assistance in conducting the experiment reported in this study, and to Fritz Staub for discussions and for comments on this paper. Finally, I am grateful to Pietro Balinari for his assistance in performing the rank analysis of variance on the IBM main frame computer of the University of Bern.

problem difficulty and to the nature of children's errors (For an overview see Carpenter & Moser 1982, 1984; Riley 1979; Riley & Greeno 1988; Reusser 1989b).

So far, less emphasis has been put, however, on other, non-mathematical task characteristics as, e.g., on lingusitic form factors (language knowledge) and on episodic or situational factors (world knowledge) which constitute the presentational structure (Clark 1983), or the overall verbal setting of mathematical word problems.

The following set of problem statements which mirror the same change problem type (change-4) with a given start set and a given result set and an unknown tranfer-out set, only represents a small subset of almost numerous possible verbal formulations of the same episodic problem situation (some of the linguistic form variations are put in parenthesis; every problem can be stated with or without a problem question).

* (First) Alice had 9 cookies. (Then) She (Alice) gave some cookies to Peter. Now she (Alice) has 4 cookies. How many cookies did she give to Peter?

* Alice had 9 cookies. She gave some away. Now she has 4. How many cookies did Alice give away?

* Alice had 9 cookies. Now she has only 4.

* Alice got 9 cookies from Bruce. She gave some of them to Peter. Now Alice still has 4 cookies left. How many cookies did Peter get (from Alice)?

* Recently Bruce gave 9 cookies to Alice. Shortly after that, Alice had (only) four cookies left. She gave some cookies to Peter. How many fewer cookies does Alice have now than she got from Bruce (than she had first)?

* Recently Bruce gave 9 cookies to Alice. Shortly after that, Alice had (only) four cookies left. She gave some cookies to Bernadette. How many more cookies did Alice have first than she has now?

* Peter got some cookies from Alice. Now Alice has (only) four cookies left. First Bruce gave 9 cookies to Alice. How many cookies did Alice give to Peter?

* Alice has a sweet tooth. From the 9 cookies she got from Bruce, she has only 4 left.

* Alice has 4 cookies. (Before) She had 9. How many cookies did she give to her friend Peter?

* Peter likes cookies a lot. Sometimes he gets cookies from Alice. Today Alice got 9 cookies from Bruce. Then she met Peter. Now she has (only) 4 (left).

* After giving some cookies to Peter, Alice counts four cookies in her bag. Bruce gave her 9 cookies this morning. If we know that Alice did not eat a single cookie yet, how many cookies did Peter get?

The problem formulations differ in many subtle ways, mainly in the explicitness of the mathematical problem structure stated, in the presence and quality of the problem question, in quality and degree of elaboration, coherence, completness, of the situation description, in the kinds of presuppositions that are made by the verbal setting, and in the sequential order of mention of the situation elements.

There is evidence that task variables beside the conceptual structure can have a significant influence on problem difficulty. In a study from Hudson (1983) dramatical changes in performance were found if the identical problem situation "There are five birds and three worms" was presented to children of different age levels with either of two questions:

(a) How many more birds are there than worms?
(b) How many birds won't get a worm?

Correct performance with question (a) ranged from 17% to 64%, with question (b) from 83% to 100%. While with the unfamiliar question (a) the correct parsing of the complex linguistic form "have-more-than" is required, the more familiar question (b) allows to simulate a real world one-to-one correspondence of hungry birds that eat the worms, or want to eat them, respectively. This important observation suggests that word arithmetic problem solving is not just a matter of the right arithmetic strategies being cued by the right linguistic expressions, but that an understanding of the situation as a whole can constrain and facilitate the problem solving (Reusser 1985).

Results pointing into a similar direction were found in a study by De Corte, Verschaffel, and De Win (1985). The authors manipulated the degree of explicitness in which the semantic relations between the quantities in a problem were stated. They found that rewording problems in such a way as to more explicitly or unambiguously stating set relations facilitated correct solution of young elementary school children.

These results, among other effects of linguistic and situational factors (Reusser & Staub 1989; c.f. also the studies from Hayes & Simon 1977; Nesher & Katriel 1977) on word problem difficulty, suggest that one should become more aware of *wording* or *framing* a particular logico-mathematical structure.

The neglect of linguistic and situational factors in mathematical word problems is both true for a large body of current theorizing and the majority of existing empirical work. Numerous experimental studies have been conducted using the same linguistically and semantically impoverished set of problems (which contain little verbal information beyond what is directly relevant to the arithmetic) and existing psychological process models are - if not lacking any assumptions at all - apparently weak in their text and situation comprehension apparatus. Either, as in the model of Riley et al., language comprehension is completely bypassed, or, as in the model of Briars & Larkin, text processing assumptions are specifically tailered to the employed problem set.

Only the Kintsch & Greeno model (1985) which is based on the strategic theory of discourse comprehension developed by Van Dijk & Kintsch (1983), includes a worked out text comprehension component. Following this approach and complementary to a purely logico-mathematical explana-tion of problem solving difficulties of young children, Cummins-Dellarosa, Kintsch, Reusser & Weimer (1988) have put forward (and partly tested) the hypothesis that much of the difficulty children experience with certain word problems can be attributed to a difficulty in comprehending complex or ambiguous language. Moreover, in her implementation of the Kintsch & Greeno model, Dellarosa (1986; see also Kintsch 1987), was able to simulate errors which were due to linguistic (lexical) misunderstandings, as e.g. to the faulty parsing of arithmetic key words like "some", "altogether", or "have-more-than".

Although the Kintsch & Greeno model and its implementation by Dellarosa (1986) is well-suited to explore certain types of lingistically based errors, there are also limitations of this approach, mainly related to the weak and lacking language- and knowledge-based reasoning capabilities of the model. Basically, Kintsch & Greeno follow a one-step approach of mathematization by jumping (via some arithmetic strategies) directly from the propositional text base to a mathematical problem model. As in the other simulation models mentioned above, no attempt is made in this model to understand the situation described by a word problem. This direct translation from text propositions to equations (see also Bobrow 1964), where a word problem is only understood in terms of its arithmetic structure on the basis of a few key words (see Nesher & Teubal 1975), may be a good idea for experts in the problem domain, as e.g. for older children who know very well the problem types with their associated mathematical reasoning strategies. It is probably not a good idea for the modelling of the behaviour of very young children, who neither possess spezialized arithmetic strategies nor powerful verbal and world-knowledge based reasoning strategies yet, to skip the representation of the non-arithmetic situation.

This leads to our own theorizing about the role that *linguistically induced understanding of the situation* (states, events, actions) plays in solving word problems. (What is the nature a of a "model" that is constructed from a word problem and that underlies solution?) What are the determinants of its construction? In sum: What are the features of a model that constructs a problem representation based on strategies that take into account the specific situational structure as well as its wording.

Guided by the epistemological intuition that early mathematical learning is embedded in everyday acting and reasoning (Piaget 1950; Aebli 1980), and that mathematical operations are the developmental derivatives of concrete, sensorimotor acting and manipulating, a process model was developed in which the construction of an *episodic, i.e. non-mathematical situation model* is[3] the core level of representation as a mediating level of understanding, situated between the propositional text base and the arithmetical problem model.

From text to situation to equation: a process model

In our process model SPS (Situation-Problem-Solver) of understanding and solving word arithemtic problems (Reusser 1989a,b) we have combined elements of the model of children's arithmetic problem solving by Kintsch & Greeno (1985) with ideas about the strategic nature of mathematization as a process of elaborative and goal-directed text comprehension and situation comprehension and about the teaching of that process. For a large class of word-problems this means to transform (translate) a natural language text with its characteristic fuzziness, into the canonical representation of a formal equation. The theoretical issues that motivates our research concerns the nature of the "model" that is constructed from a word problem

SPS takes as input elementary addition and subtraction word problems, "understands" and solves them by means of various types of lexical, syntactic, semantic and pragmatic strategies, and finally, gives back a numerical answer embedded in an answer sentence that takes account of the problem situation. SPS is using several levels of comprehension, i.e. consists of a progressively deepening of the understanding of a problem text. This is reflected in the model by the construction of a multilayered problem representation achieved by a hierarchical structure of macrostrategies (Figure 1). A first macrostrategy guides the construction of a propositional structure of case frames (text base sensu Kintsch 1974) by using a set of lexically based propositional

meaning postulates. A second macrostrategy - refering to the essential part of *situation comprehension* in SPS - creates a mental model of the action or situation structure as a whole (the episodic problem model). After the situation model has been built, a third

[3] From an educational or instructional point of view: it *should* be.

FIGURE 1

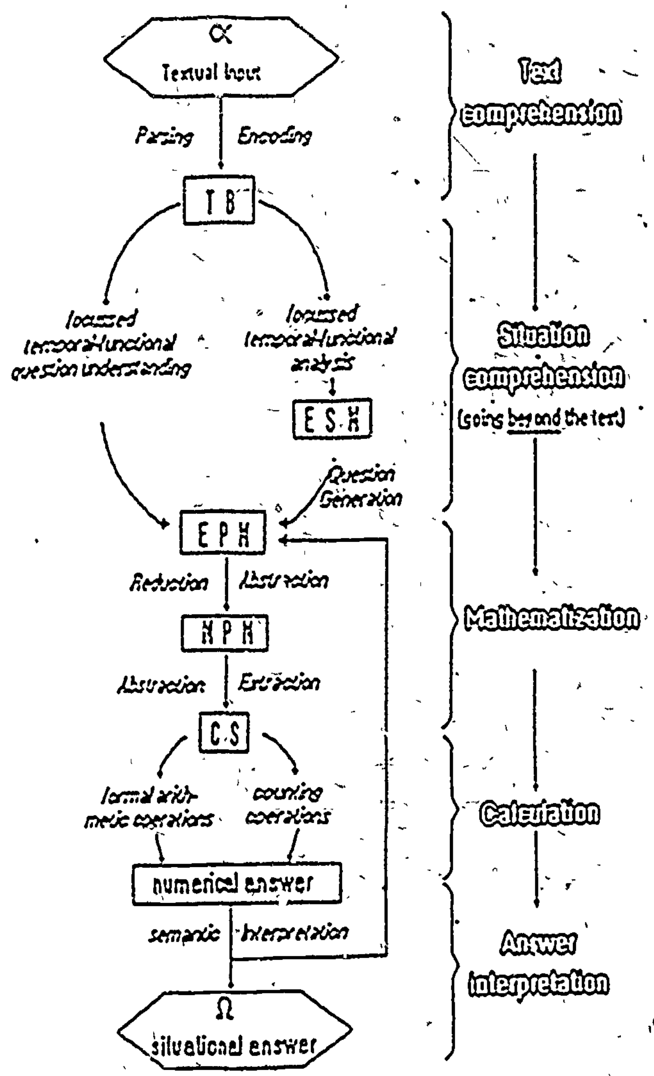

Figure 1 : Architecture and Macrostrategies of SPS

TB	:	Text Base
ESM	:	Episodic Situation Model
EPM	:	Episodic Problem Model
MPM	:	Mathematical Problem Model
CS	:	Connective Structure or Solution Equation

macrostrategy, which is mathematically oriented, reduces the episodic problem representation to its abstract mathematical gist (called the mathematical problem model) by mapping the semantic relations between the states and actions in the situation model onto an abstract structure of set relations.[4] After the arithmetic problem model is abstracted from the situation, a forth macrostrategy extracts the numeric structure or equation and solves it by means of various substrategies. Finally, a fifth macrostrategy generates an answer sentence which refers back to the question or the gap in the situation model and gives meaning to the numeric answer.

SPS is implemented as a production system in Interlisp-D.[5] It currently solves an open class of about sixty word problems, most of them belonging to the overall type of *change* problems.[6] SPS tolerates a great deal of variability in problem wording, and is capable of doing a lot of world knowledge inferencing, mainly associated with the molecular analysis of time and function in the action episodes. Because of its molecular design, SPS can simulate for every word problem several (up to twelve!) comprehension paths i.e. sequences of language and world knowledge based (micro-) inferential steps.

Empirical work

Word problems provide a valid discourse entity and an interesting case in text comprehension, because the goal of comprehension is clearly defined. Moreover there exist objective measures as to whether or not that goal was achieved.

However, it should be noted, that empirical testing of a cognitive simulation model is not a trivial task (see Ohlsson 1988). There is no clear-cut methodology so far as how to test complex cognitive simulation models.

The basic psychological and instructional hypotheses associated with the idea of an intervening episodic situation model in SPS states that, for most problem solvers, linguistically cued situational understanding is not (and in an instructional context: should not be) a superfluous, but a necessary and helpful achievement (see e.g.

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4 With Aebli (1980) we see operations as *abstract actions*: Concrete actions as , e.g., to give, get, or lose, bear an abstract relational ideas which can be expressed by the abstract meaning of mathematical operations as, e.g., adding or subtracting

5 The computer program runs on every XEROX AI workstation.

6 In change problems, an initial state undergoes a transfer-in or transfer-out of items, and the cardinality of either the initial set, the transfer set oder the resulting set must be computed.

Hudson's data). The theoretical and empirical issue is that the logico-mathematical structure of a word problem is merely one constraining factor of getting at the right arithmetic strategies. Other important, and probably underestimated factors which constrain and therby facilitate problem understanding and solving, are the situation as a more or less familiar and vivid episodic event-structure and its wording.

In a series of empirical studies[7] we have manipulated (beside variations of the problem type) various factors of the textual and presentational structure of word problems as e.g. the order of mention of the elements in a problem statement, the degree of textual elaboration and connectivity, the presence or absence of a problem question, and the narrative focus of the story protagonist. Results, as available so far, give support to the hypothesis that more elaborated or more explicit problem texts (with respect to set relations and to mathematical actions) facilitate problem solving - as we think, by leading to the construction of more appropriate mental situation models. In the following sections, I report one of our experiments.

Experiment: Narrative focus and familiarity of script in understanding and solving word problems.

In the experiment to be described here, we were looking at the role of two linguistic, and of one situational factor on comprehension difficulty of simple mathematical word problems. The linguistic form factors that were manipulated concerned the *narrative focus* of the problem episode and the problem question. That is, we were looking at actor-perspective, from which a story problem was narrated. While these factors can be manipulated without changing the problem situation, a script factor that varied, the relationship of the co-actors in the story (familiar versus unfamiliar relation of coactors), changed the situational structure of the story.

Because it is our main interest to study text and situation comprehension difficulty rather than arithmetic skills and in order to minimize the undesired consequences of kinds of a speed-accuracy tradeoff, which we certainly had to expect if working with reaction times of unexperienced very young children, only adults (university students) participated in the experiment.[8]

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7 English versions of the papers are in preparation (Reusser & Staub; Reusser 1989c; see also Cummins et al. 1988).

8 The expectation was that adults would produce ceiling effects regarding frequency of correct solutions, but a reasonable variance regarding solution time.

We predicted that solution performance would vary with both factors. First, problems with the narrative focus consistent with the focus of the main actor or protagonist of the story should be easier than problems with inconsistent (i.e. changing) narrative focus. Second, problems with coactors interrelated in familiar ways (as, e.g., father-/mother-son-relationship) should be easier than problems with completely unrelated coactors. The first prediction can be derived from our explicit simulation model (and various comprehension errors therefore can be simulated by SPS). The second prediction, while being consistent with the theoretical assumptions of SPS and only matches the more general spirit of situation comprehension of the model.

Method

Apparatus and materials. The 32 story problems from Table 1 served as stimulus material. Basically, all problems consist of variations of four instances of *Change* problems (Change 3 to Change 6; see Riley et al. 1983), in which a starting set undergoes a transfer-in or transfer-out of items, and the cardinality of the start set or initial state, or of the transfer set must be computed given information about two of the three relevant sets. In order to make the individual problems more distinct from each other, the problem types (as defined by the direction of transfer and the location of the unknown quantitaty) are systematically confounded with the order of mention or textual order of the situational elements (initial state or action, transfer action, resulting state). All change 3 problems are associated with the textual order 2-3-1 [9], all change 4 problems with the order of mention 3-2-1, change 5 problems with 2-3-1, and change 6 problems with 3-1-2.

All problems used in the present study consisted of two actions (an action generating the initial state or the start set, and a transfer-in/out action) and the resulting state. All problems therefore contained three coactors, one of them serving as the *protagonist*, or the *main character* of the story. The protagonist of a story is the mathematically relevant actor, who participates in *every* partial action or state of the story as the person who gets, receives, or simply posesses a certain amount of items (while the two subordinate coactors only participate in one partial action of the story). For example, in the problem

> Today Silvia got seven marbles from Urs. Yesterday Hans gave some marbles to Silvia at the playground. Now Silvia has fifteen marbles. How many marbles did Silvia get at the playground?

9 I.e. the transfer action is mentioned first in the problem text, followed by the initial state and the resulting state.

Table 1: Set of 32 problems used in the experiment (cont.)

Unrelated coactors

CH3 Today Ruedi got some marbles from Hannah at the playground. Now Ruedi has eleven marbles. Yesterday Ruedi got eight marbles from Daniel. (E:NF-P)

Today Hannah gave some marbles to Ruedi at the playground. Now Ruedi has eleven marbles. Yesterday Daniel gave eight marbles to Ruedi. (E:NF-CA)

* How many marbles did Ruedi get at the playground? (Q:NF-P)
* How many marbles did Hannah give away at the playground? (Q:NF-CA)

CH4 Now Sonja has nine marbles. Today Sonja gave some marbles to Marianne at the playground. Yesterday Sonja got twelve marbles from Nathalie. (E:NF-P)

Now Sonja has nine marbles. Today Marianne got some marbles from Sonja at the playground. Yesterday Nathalie gave twelve marbles to Sonja. (E:NF-CA)

* How many marbles did Sonja give away at the playground? (Q:NF-P)
* How many marbles did Marianne get at the playground? (Q:NF-CA)

CH5 Today Silvia got seven marbles from Urs. Yesterday Silvia got some marbles from Hans at the playground. Now Silvia has fifteen marbles. (E:NF-P)

Today Urs gave seven marbles to Silvia. Yesterday Hans gave some marbles to Silvia at the playground. Now Silvia has fifteen marbles. (E:NF-QA)

* How many marbles did Silvia get at the playground? (Q:NF-P)
* How many marbles did Hans give away at the playground? (Q:NF-CA)

CH6 Now Anette has nine marbles. Yesterday Anette got some marbles from Werner at the playground. Today Anette gave three marbles to Reto. (E:NF-P)

Now Anette has nine marbles. Yesterday Werner gave some marbles to Anette at the playground. Today Reto got three marbles from Anette. (E:NF-CA)

* How many marbles did Anette get from Werner at the playground? (Q:NF-P)
* How many marbles did Werner give to Anette at the playground? (Q:NF-CA)

Related coactors

CH3 Today Ruedi got some marbles from the gransmother at the playground. Now Ruedi has eleven marbles. Yesterday Ruedi got eight marbles from his grandfather. (E:NF-P)

Today the grandmother gave some marbles to Ruedi at the playground. Now Ruedi has eleven marbles. Yesterday the grandfather gave eight marbles to Ruedi. (E:NF-CA)

* How many marbles did Ruedi get at the playground? (Q:NF-P)
* How many marbles did the grandmother give away at the playground? (Q:NF-CA)

CH4 Now Sonja has nine marbles. Today Sonja gave some marbles to her best friend at the playground. Yesterday Sonja got twelve marbles from her mother. (E:NF-P)

Now Sonja has nine marbles. Today her best friend got some marbles from Sonja at the playground. Yesterday the mother gave twelve marbles to Sonja. (E:NF-CA)

* How many marbles did Sonja give away at the playground? (Q:NF-P)
* How many marbles did the friend get an the playground? (Q:NF-CA)

CH5 Today Silvia got seven marbles from the teacher. Yesterday Silvia got some marbles from his godfather at the playground. Now Silvia has fifteen marbles. (E:NF-P)

Today the teacher gave seven marbles to Silvia. Yesterday the godfather gave some marbles to Silvia at the playground. Now Silvia has fifteen marbles. (E:NF-QA)

* How many marbles did Silvia get at the playground? (Q:NF-P)
* How many marbles did the godfather give away at the playground? (Q:NF-CA)

CH6 Now Anette has nine marbles. Yesterday Anette got some marbles from the babysitter at the playground. Today Anette have three marbles to her sister. (E:NF-P)

Now Anette has nine marbles. Yesterday the babysitter gave some marbles to Anette at the playground. Today her sister got three marbles from Anette. (E:NF-CA)

* How many marbles did Anette get from the babysitter at the playground? (Q:NF-P)
* How many marbles did the babysitter give to Anette at the playground? (Q-NF-CA)

Table 1: Set of 32 problems used in the experiment

The 32 problems can be obtained from the table by combining - for each problem type - four conditions of narrative focus (two focus conditions of story episode by two focus conditions of question) with two conditions of coactor-relationship or script (Related versus unrelated coactors).
E: Episode; Q: Question; NF-P: Protagonis-Related Narrative Focus; NF-CA: Coactor-Related Narrative Focus; E:NF-P: Protagonist-Related Episode; E:NF-CA: Coactor-Related Episode; Q:NF-P: Protagonist-Related Question; Q:N F-CA: Coactor-Related-Question

Silvia serves as the protagonist, because she is the actor who gives rise to a coherent mathematical action. While she occurs in every sentence of the verbal statement (either in topic or non-topic position), the coactors Urs and Hans are mentioned in the problem only once. Their contribution to the episode is essential (giver, receiver of items), but subordinated to the action episode as a whole.

Three experimental factors were implemented in the present study, two of them rather linguistic, one rather situational in nature. While the first and the second factors are associated with the narrative perspective or point of view from which either the story episode in a problem is presented, or the final question is posed, the third factor is associated with the familiarity of the employed story scripts. The following factors have been systematically varied: - *Factor I*: We call the *narrative focus (NF) of a story episode* (E) *protagonist-related* (P) or *consistent*, if the protagonist in every partial action or state is in topic position, or serves as the grammatical subject of the respectives sentences in the verbal problem statement (E:NF-P). The narrative focus of a story episode is called *coactor-related (CA) or inconsistent*, if the coactor in sentence topic position is not identical with the protagonist, or is changing within the same story episode (E:NF-CA). -*Factor II*: The *narrative focus of the final question statement* (Q) of a story problem is said to be protagonist-related (Q:NF-P) if in the question statement of the problem the main story character is in topic position. Complementary, the narrative focus of the final question is said to be *coactor-related* (Q:NF-CA), if a subordinate coactor of the story is in topic sentence position. *Factor III* (script factor): The relations between the coactors in a problem are said to be *unfamiliar* if the coactors are represented by three *unrelated proper names or individuals*, one of which serving as the main character of the story. In contrast, the relations between the coactors are called *familiar* if only the protagonist of the story is introduced by his/her proper name, and the two coactors are introduced by some obvious *relationship* in which they stay to the protagonist (being a relative, a friend, a teacher, or a functionally defined partner in an interaction, as e.g., a salesperson in a store). While, in the first case, the three coactors are almost completely unrelated in the text (beside the fact that they transfer items between each other), in the second case, the text invites the reader to form a familiar script of personal relations between the protagonist and his/her coactors of the story episode.

In Table 1, the 32 problems are constructed by combining - for each of the four problem types - four conditions of point of view (two levels of episode focus by two levels of question focus) with two conditions of coactor-relationship. In order to make the individual problems somewhat more distinguishable from each other, some further textual variability was introduced across single problems by varying the names of individuals, objects, locations, and of temporal specifications.

Subjects. Seventy-six adult students from different departments (law, medicine, sciences, humanities) of the University of Bern voluntarely served as participants in the study.

Procedure. The experimental factors were crossed to form a 2 x 2 x 2 x 4 design, the two focus factors and the problem type being within-subject factors, the script factor (related versus unrelated coactors) being a between-subject factor. Each subject received half of the 32 problems, i.e. 16 problems, one problem from each problem type (change 3-6) under four of the experimental conditions.

Subjects were tested individually in a quiet room in front of a computer screen.[10] The sessions approximately lasted twenty to thirty minutes, and began with verbal instructions concerning the general apparatus and the problem solving tasks. All detailed instructions as well as the problems including four practice problems were presented to subjects on the screen of an AT personal computer. During the instructional phase, subjects were tought to push the ENTER key as soon as they had solved a problem, then to type in the numerical solution in an blank screen-field, and (if ready to go on) to push the ENTER key again in order to get the next task on the screen. Problem presentation was randomized for each subject. Numerical solutions as well as solution times (the main dependent variable in this study) were recorded by the computer.

Results

Solution accuracy. As expected from our subjects, proportion of correct answers were generally high (change 3: 97%; change 4: 95%; change 5: 96%). The highest percentage of incorrect solutions occurred with change 6 problems (78%). Five subjects with more than three wrong numerical solutions were abandoned from further analysis of solution times.

Solution time. Solution times produced by subjects as a function of narrative focus of both episode and question and of coactor-relationship (script) were computed and are presented in Table 2. Following Conover & Iman (1981), a rank transformation procedure (replacing original observations with their respective ranks) was applied in order to reduce the influence of extreme values. An analysis of variance was then conducted on these ranks using as between-subject factor coactor-relationship (Unrelated or Related), and as within-subject factors narrative focus of both story

10 Ruedi Stüssi wrote the Pascal programm by which the experiment was run on the computer.

Table 2
Mean solution times (sec)

	Q:NF-P Protagonist-Related Question	Q:NF-CA Coactor-Related Question	Mean
UCA: Script with Unrelated Coactors			
E:NF-P: Protagonist-Related Episode	26,02	32,78	29,40
E:NF-CA: Coactor-Related Episode	31,39	30,34	30,86
Mean	28,70	31,56	30,13
RCA: Script with Related Coactors			
E:NF-P: Protagonist-Related Episode	22,06	23,01	22,54
E:NF-CA: Coactor-Related Episode	23,04	24,33	23,68
Mean	22,55	23,67	23,11
Mean	25,76	27,78	26,77 (Sd: 17,8)

Note: Cell means are based on an average of 142 observations (4 problems from each of 37 (UCA-condition) or 34 (RCA-condition) subjects, respectively, under each cell condition). NF: Narrative focus; E: Episode; Q: Question; P: Protagonist; CA: Coactor.

episode and question (Consistent or Inconsistent), and problem type (Change 3 to Change 6), with repeated measures on the latter three variables.

The analysis returned three significant main effects and four interactions. The first main effect was of script or coactor-relationship, F(1,69)=10.41, p<.002, indicating that solving problems with script-unrelated coactors took considerably longer than problems with script-related coactors (mean difference: 7 seconds). The second was the main effect of narrative focus of the question, F(1,69)=4.73, p<.04, indicating that understanding and solving problems took longer when the quantitative question was formulated from the unusual point of view of a coactor. The third was a main effect of problem type, F(3,207)=28.29, p<.000, indicating Change 6 as being the most difficult, and Change 3 as the easiest of the employed problems.[11]

While no significant main effect of narrative focus of the story episode was observed, F(1,69)=2.78, p<.10, there was a significant interaction (Figure 2) between episode focus and question focus, F(1,69)=6.51, p<.02. All problems with either one or both episode and question focus inconsistent or coactor-related (coactors in sentence topic position) were hard to understand, whereas the problems consistently narrated from the point of view of the protagonist took considerably less time to get at the solution. Narrative focus of the story episode also interacted with problem type, F(3,207)=4.76, p<.005 (Figure 3). This was the only interaction that was observed between problem type and experimental variables.

A significant interaction was also returned from the analysis between script versions (related versus unrelated coactors) and narrative focus of question, F(1,69)=5.24, p<.03 (Figure 4), but not between script and narrative focus of story episode, where the analysis of variance yielded an F close to zero for the interaction. These results indicated a triple interaction between script, story episode and question focus. Although this interaction was not statistically significant, F(1,69)=3.05, p<.08, there both was a statistical tendency and a clear pattern of results observable as it is shown in Figure 5-1 and 5-2. Under the condition of related coactors, comprehension difficulty (which was generally rather low) was only marginally modified by the point of view of narration (Fig. 5-2). However, under the condition of unrelated coactors (Fig. 5-1), not only problem difficulty was generally much higher, but there was also an interaction between focus of story episode and focus of question. According to this interaction, problems with both protagonist-related episode and protagonist-related question were the easiest, but were still more difficult than all cases in the condition of script-related

11 Because problem type and text order (order of mention) were confounded in this experiment, it is hard to speculate about the reasons of exactly this profile of problem difficulties.

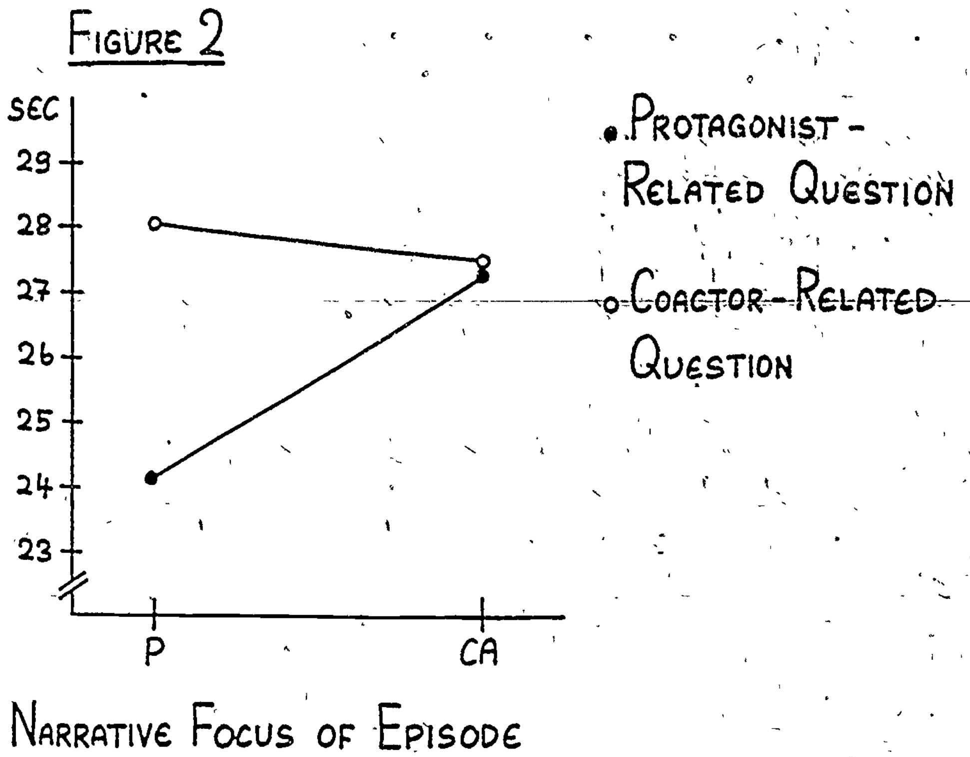


Fig. 2
Solution time as a function of narrative focus of question and episode.

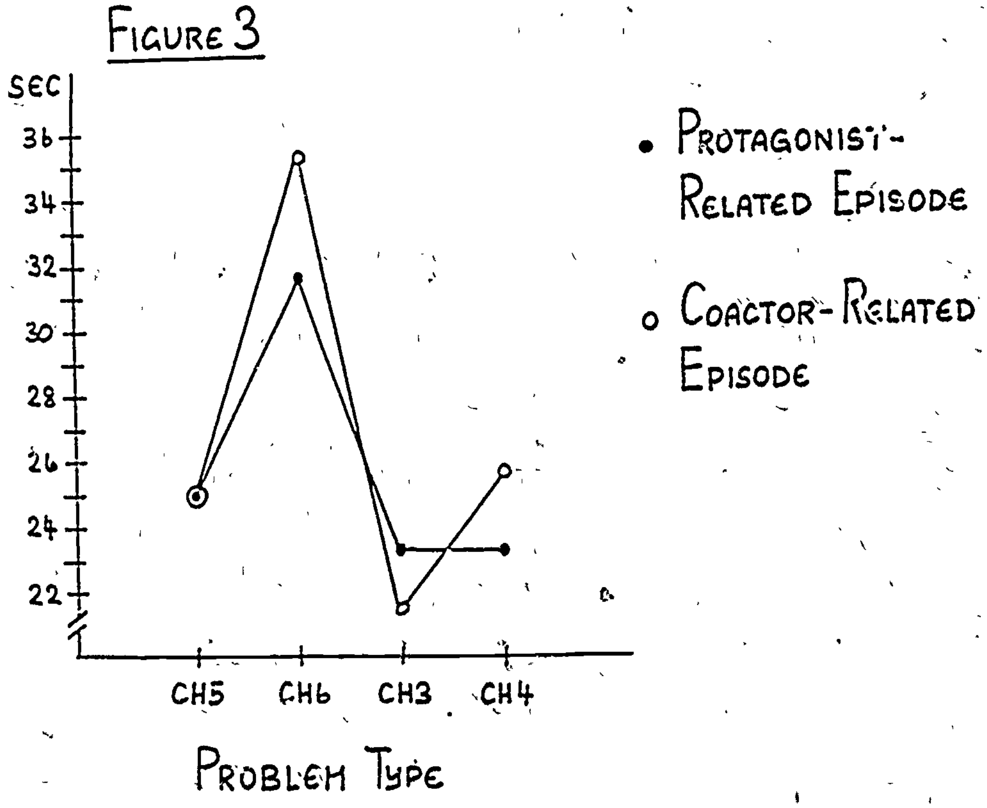


Fig. 3
Narrative focus of episode and problem type (Change 4 - 6)

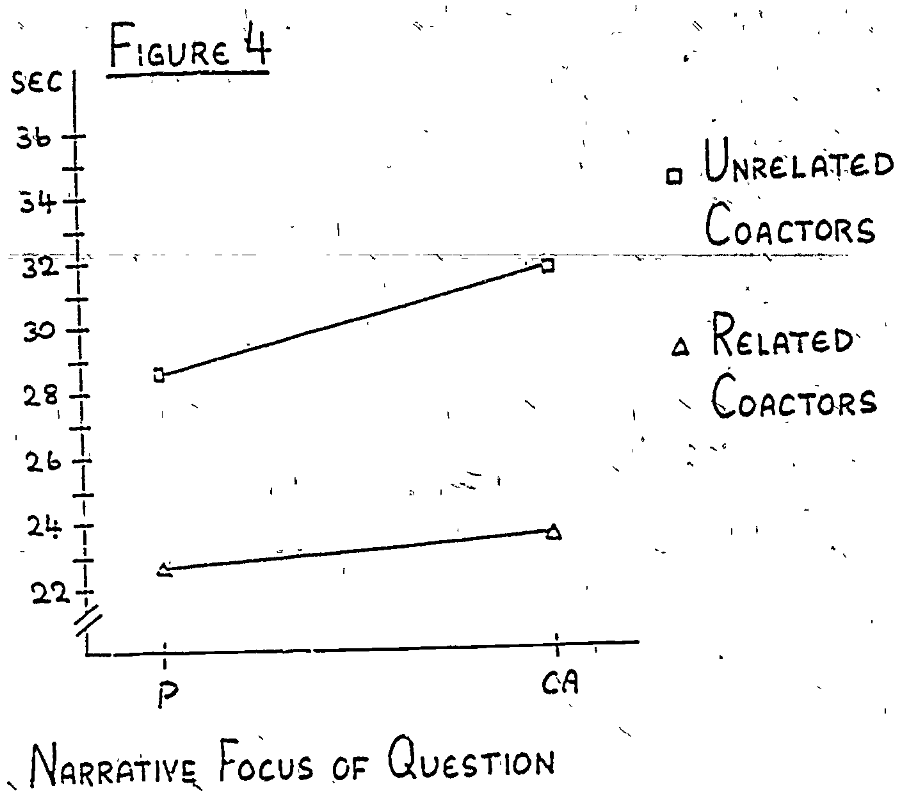
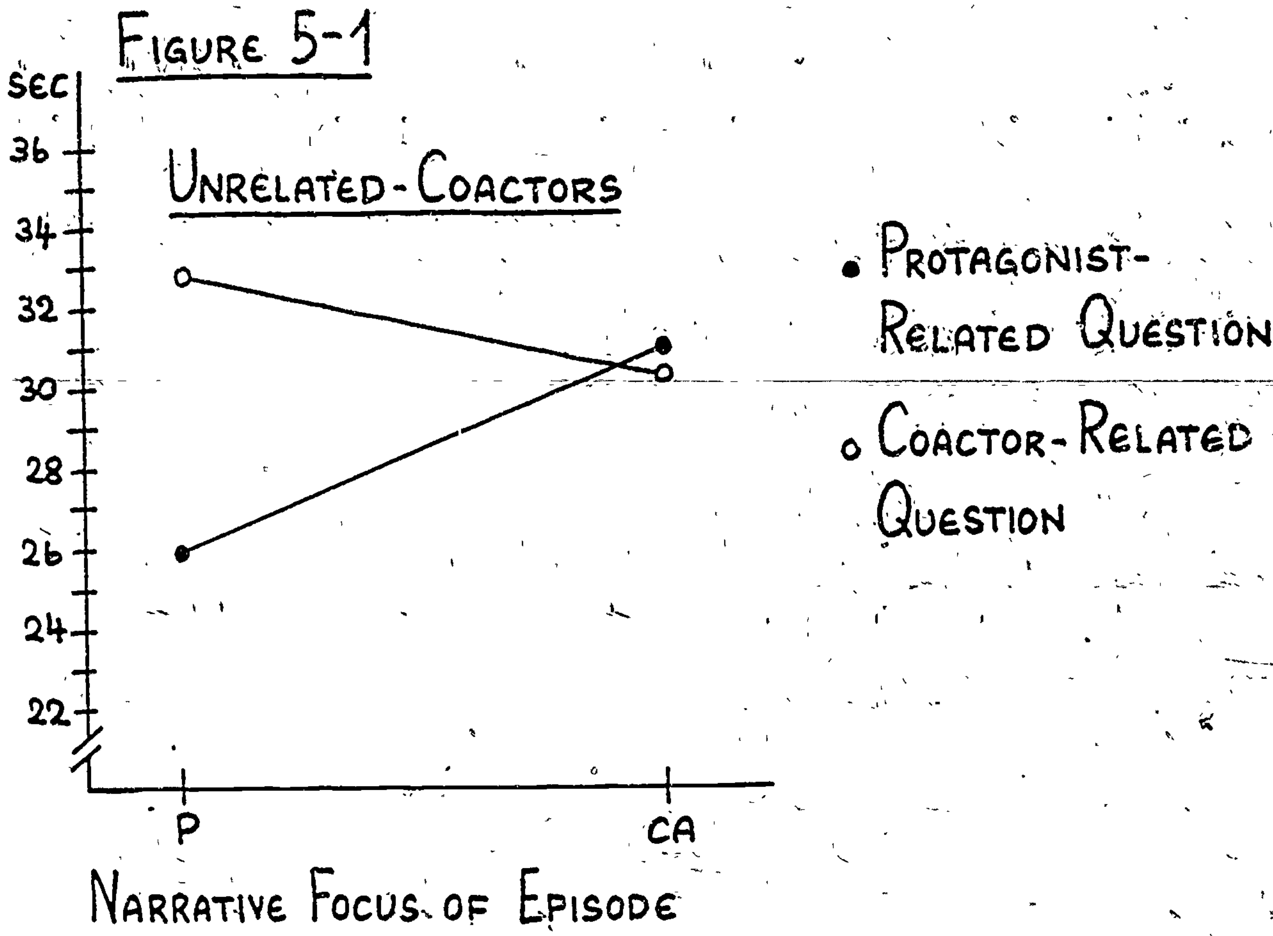


Fig. 4
Interaction between script (related vs unrelated coactors) and question focus.

coactors. Under all conditions of inconsistent focus, difficulty of comprehension, as measured by solution times, was considerably increased.

Discussion

The results of this experiment indicate the script factor (related versus unrelated coactors) to be a dominant source of comprehension difficulty. Problems with unrelated coactors were generally much harder to understand than problems with related coactors. Moreover, if the three coactors in the story script were related in familiar ways, narrative focus transformations hardly had an influence on problem difficulty. Not so in the case of completely unrelated coactors in the story scripts, where problem difficulty varied with the transformation of narrative focus.



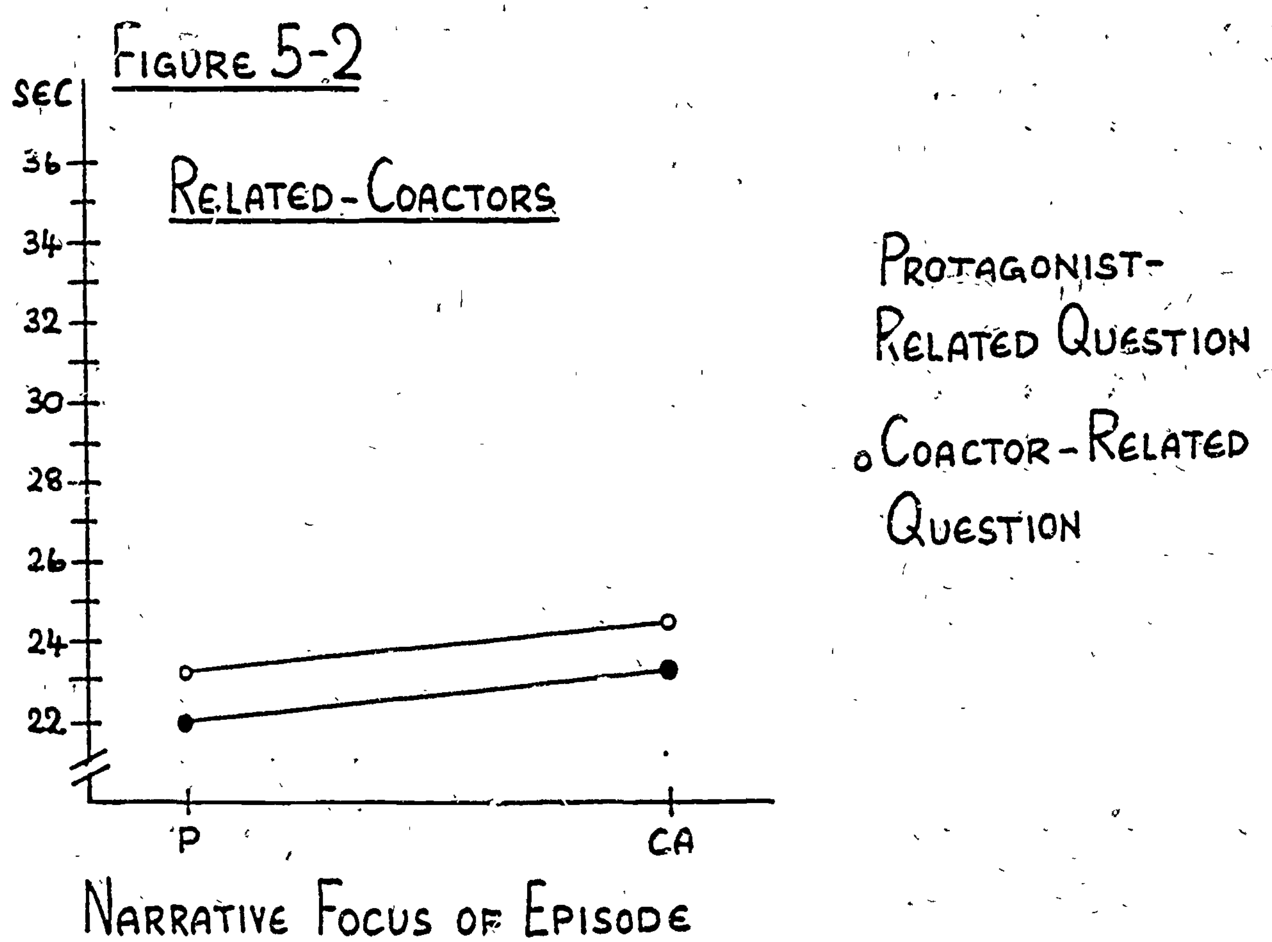


Fig. 5

Triple interaction between script, story focus and question focus.

A reasonable and theoretically consistent interpretation of this pattern of results (with respect to SPS) goes with the assumption that coactors relating to each other in familiar ways, are more easily contributing to the forming of a more familiar and more coherent story script than do coactors which are not related at all - and therefore are beneficial to the construction of a situation model. Subjects who are able to benefit from more coherent, more interrelated scripts, are able to construct a situation model with possibly less effort on the analysis of the (changing) narrative focus. The possible reason might be that in the story script related coactors, that are more distinct from each other than proper names, serve as powerful cues for the necessary topic identification strategies (or in terms of SPS: for protagonist or main character identification strategies) on the level of situation comprehension. In the condition of related actors, only the mathematical protagonist is introduced by its proper name, i.e., as a distinct, independent individuum. The two other coactors have no proper name in the problem statement. They are introduced by relational generic terms, as e.g. grandmother, father, godfather, teacher, etc. In the text world of the problem, they only exist in their relation to the (same) main character. That makes it easy to identify the character, who is introduced in the story episode by his proper name, as the mathematically relevant character, or protagonist, independent of his topic or non-topic position in a single sentence. To identify the protagonist in the stories with related coactors simply means to pick the participant who is introduced by his proper name as the protagonist, and to reconstruct the story from his perspective. This can not be done "one-line" (i.e. while while reading the problem text the first time) if three unrelated, completely distinct coactors are introduced. In this case, comprehension difficulty depends on the narrative focus in such a way that all problems versions containing a shift of narrative focus, were more difficult to comprehend in the experiment.

Why are word problems difficult? The present results support the usefulness of the theoretical perspective discussed in the introductory part of this paper. The results reported here suggest that, beside mathematical structure variables (as problem type or the location of the unknown quantity) and arithmetic skills, text comprehension or linguistically based factors and - mediated by these - situation comprehension factors figure heavily in word problem difficulty. In terms of our theoretical model SPS[12]: If narrative or discourse comprehension involves at least two related major components, first the translation of the surface form of a text into a text base, and second the construction of a mental model of the situation described by the text, then it is reasonable to assume that the more explicit the text base, that is cued by the explicit linguistic

12 And in accordance with the findings of the study of De Corte, Verschaffel & De Win (1985) on the influence of linguistic factors

features of the problem statement, is, the more easy the construction of the situation model will be.

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